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DESIGN FOR MINIMUM WEIGHT

BY

D. C. DRUCKER AND R. T. SHIELD

U.S. Army Ordnance Corps
Office of Ordnance Research
Contract DA-19-020-ORD-3172
Project TB2-0001(1086)
Tech. Rept. No. 11 September, 1956

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DESIGN FOR MINIMUM WEIGHT*

by

D. C. Drucker** and R. T. Shield*** (Brown University)

__Summary.__ Design rather than analysis poses the real problem in machines and structures. The basis for an optimum design is, of course, a compromise between material and fabrication costs within some space limitation. It is always of interest, therefore, to know the minimum weight possible so that an appropriate design may be selected. A very simple formal approach to this problem is outlined for a perfectly plastic material with a completely general yield criterion and then specialized to sheets, membranes and plates. Some of the specific results have been established previously by more elaborate techniques, others appear to be new. Much more work must be done because in all cases the conditions found are sufficient rather than necessary and the question of existence of solutions arises. Also, in applications involving bending of homogeneous beams and plates the minimum is a relative one and not an absolute one as for space structures, sheets, and membranes.

*The results presented in this paper were obtained in the course of research sponsored by the Office of Ordnance Research.

**Chairman, Division of Engineering.

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<u>Introduction</u>. Many forward steps have been taken toward the highly desired goal of direct design of structures as contrasted with usual procedure of an informed guess followed by analysis. The complexity of elastic analysis is so great that it is not surprising to find design so strongly based on empiricism and arbitrary rule despite great effort to develop scientific procedures.[1][2]* The relative simplicity of limit analysis, on the other hand, offers hope that, based upon the same physical assumptions, true design techniques can be developed which will lead to economical as well as safe structures. Several authors have employed this approach to determine structures of minimum weight. Heyman[3], Foulkes[4], and Prager[5] have considered rigid frames with constant cross-section between joints. Hopkins and Prager[6][7] have investigated plates using an intuitive concept of the behavior of minimum weight structures, Freiberger and Tekinalp[8] obtained a direct answer with the calculus of variations.

The present paper is more in the spirit of Michell[9] in that a general result is found. It is subject to the same type of limitation as this early work. Although a sufficient condition for minimum weight is established, solutions satisfying this condition do not exist in general. Means of overcoming this difficulty will be considered in a later paper.

<u>Formal Approach</u>. The general problem considered here is to find the minimum weight or the minimum volume V_m of a structure of homogeneous material designed to carry a given distribution of loads. Although in some instances there may be no limitation on the extent of the structure, ordinarily it

*Numbers in brackets refer to the Bibliography at the end of the paper.

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must lie within some bounding volume V_T. The material will be idealized as perfectly plastic so that the results have the same physical validity as conventional limit or plastic analysis [10]. In fact, the main theorems of limit analysis [11] will be employed. As stated for a given structure they are:

1. If any distribution of stress σ_{ij}^{s} exists satisfying the equations of equilibrium and the boundary traction conditions and which is everywhere below yield $f(\sigma_{ij}^{s}) < k^2$, the structure is safe against collapse at the given loads.

2. Collapse will occur, if for any compatible distribution of strain rates ε_{ij}' and velocities u_i' considered as plastic only, the rate of work done by the external forces equals or exceeds the internal dissipation.

For a structure which is safe or just at the point of collapse Theorem 2 may be written in the standard notation with repeated latin subscripts denoting summation as

$$\int_A T_i u_i' dA \leq \int_V D(\varepsilon_{ij}')\, dV \tag{1}$$

where body forces have been neglected and continuous u_i' is assumed. In view of the normality condition [12] the dissipation function $D(\varepsilon_{ij}') = \sigma_{ij}\varepsilon_{ij}'$ is uniquely determined by the plastic strain rates ε_{ij}' even when the state of stress σ_{ij} may not be determined completely, Fig. 1. The tacit assumption is made for Theorem 1 and Inequality (1) that at least one safe structure exists and that the loading program is such that collapse has not occurred prior to reaching the existing loads T_i.

Theorem 1 gives an upper bound on the minimum volume and Theorem 2 a lower bound. More detailed consequences of this approach in which body forces also are included will be treated in a subsequent paper. For the present consider a structure V_c in V_T which is just at the point of collapse under the given loads T_i. Labelling a collapse mode of the structure as $u_i^{\prime c}$, $\varepsilon_{ij}^{\prime c}$, Inequality (1) takes the special form

$$\int_A T_i u_i^{\prime c} dA = \int_{V_c} D(\varepsilon_{ij}^{\prime c})\, dV \qquad (2)$$

For any other structure V_s which is safe or just at the point of collapse

$$\int_A T_i u_i^{\prime c} dA \leq \int_{V_s} D(\varepsilon_{ij}^{\prime c})\, dV \qquad (3)$$

so that comparing (2) and (3)

$$\int_{V_c} D(\varepsilon_{ij}^{\prime c})\, dV \leq \int_{V_s} D(\varepsilon_{ij}^{\prime c})\, dV \qquad (4)$$

Clearly now if $D(\varepsilon_{ij}^{\prime c})$ = constant in V_c and no greater elsewhere in V_T then $V_c \leq V_s$ and $V_c = V_m$ the absolute minimum volume. Therefore a structure designed for such a continuous collapse mode will be of minimum weight. Here the term design implies that there is a state of stress corresponding to the collapse mode which will be in equilibrium and will satisfy the yield condition everywhere in V_c. Also, the boundary conditions on surface traction must be satisfied so that the existence of such a field is by no means assured and finding one will generally be very difficult.

The plane stress problem is not very different from the general three-dimensional one just discussed. Equilibrium and the conditions on boundary tractions must be satisfied by forces per unit length or stresses times sheet thickness, h, rather than by the stresses themselves. Strain rates in the plane $\varepsilon_{\alpha\beta}^{\prime}$ determine the rate of dissipation per unit volume

$D(\varepsilon'_{\alpha\beta})$ and the element of volume is replaced by h times the element of middle surface area dA

$$\int_L (T_\alpha h)u'^c_\alpha dL = \int_{A_c} D(\varepsilon'^c_{\alpha\beta})h_c dA \leq \int_{A_s} D(\varepsilon'^c_{\alpha\beta})h_s dA \tag{5}$$

Once again, a design for a continuous u'^c with D = constant in A_T, the maximum permissible extent of the middle surface area of the sheet, gives absolute minimum weight

$$\int_{A_c} h_c dA \leq \int_{A_s} h_s dA \tag{6}$$

where $A_c = A_m$ and A_s are contained in A_T.

The membrane problem gives an identical result to that for plane stress but in the derivation it is necessary to add the rate of work done by lateral pressure on the transverse displacement $\int_A pw' dA$.

Bending of Plates. A sandwich plate is, in a sense, a double membrane and the result of the previous section carries over directly. Design with constant rate of dissipation per unit volume leads to an absolute minimum volume of the cover plate material.

When solid or homogeneous beams or plates are considered it is obvious that the possibility of D = constant everywhere is ruled out. In bending, strain rates vary continuously from extension on one side of the plate through zero at the middle surface to compression on the opposite side. For thin plates, the variation is linear

$$\varepsilon'_{\alpha\beta} = \frac{2z}{h} \varepsilon'_{\alpha\beta})_o \tag{7}$$

where z is measured from the middle surface and $\varepsilon'_{\alpha\beta})_o$ is the maximum value of the strain rate.

The dissipation function will likewise vary linearly

$$D(\epsilon'_{\alpha\beta}) = \frac{2|z|}{h} D_o \qquad (8)$$

The procedure which established (4) and (5), leads with obvious modifications to

$$\int_A pw'^c dA = \int_V D(\epsilon'^c_{\alpha\beta})\, dV = \int_{A_c} \frac{D_o^c}{2} h_c\, dA \leq \int_{A_s} \frac{D_o^s}{2} h_s dA \qquad (9)$$

where w' is rate of deflection and p is the transverse load per unit area. Unfortunately, setting D_o^c constant does not give the desired result (6) because for a given deflection rate w' the maximum rate of dissipation per unit volume depends linearly upon the thickness of the plate. From (8)

$$D_o^s = D_o^c \frac{h_s}{h_c} \qquad (10)$$

However, taking D_o^c as constant is worth exploring as it does give some information about the volume needed

$$V_c = \int_{A_c} h_c dA \leq \int_{A_s} \frac{h_s^2}{h_c} dA \qquad (11)$$

The next step is to consider neighboring plates to V_c. Neighboring is defined as $h_s = h_c + \Delta h$, where Δh is small in the sense $\Delta h << h_c$. Therefore, the middle surface of all plates which may be considered must be the same; $A_c = A_s = A$. Ignoring the second order term, (11) becomes

$$0 \leq \int_A 2\Delta h\, dA \quad \text{or} \quad \int_A h_c\, dA \leq \int_A h_s\, dA \qquad (12)$$

for small changes in h. Constant D_o thus gives a relative minimum for plates of a given middle surface A.

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It is perhaps more customary to look at the plate problem in terms of moments $M_{\alpha\beta}$ and curvatures $k_{\alpha\beta}$ rather than stress and strain. The dissipation term $\int_V D(\epsilon'_{\alpha\beta})\,dV$ becomes $\int_A F(k'_{\alpha\beta})dA = \int_A \frac{F(k'_{\alpha\beta})}{h}\,h\,dA$ where the dissipation per unit area of middle surface $F(k'_{\alpha\beta}) = M_{\alpha\beta}k'_{\alpha\beta}$. The choice for minimum volume, $\int_A h\,dA$, would be

$$F(k'^{c}_{\alpha\beta})/h_c = \text{constant} \tag{13}$$

The two approaches are, of course, entirely equivalent as

$$2F(k'^{c}_{\alpha\beta})/h_c = D_o(\epsilon'^{c}_{\alpha\beta}) \tag{14}$$

A relative minimum weight design for plates is obtained, therefore, from a design based upon a deflection rate pattern w'^{c} which gives constant dissipation per unit volume at the surfaces of the plate or equivalently dissipation per unit area of middle surface a constant times the local thickness of the plate.

Although a relative minimum only is given by this procedure, the calculus of variations can do no better despite the far greater effort required. The basic result (13) was obtained for a smooth yield surface in Reference [8] using the calculus of variations. There was, however, no clear indication that an absolute minimum was not really found as well.

<u>Plane Stress Example - Circular Disc</u>. Fields of constant rate of dissipation do not generally exist and except in special instances cannot easily be found when they do. However, the circular annular disc of inner radius r_i and outer radius r_o loaded by uniform force per unit of length P on the inside and Q on the outside is particularly simple. A choice of the

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Tresca or maximum shearing stress criterion of yielding, Fig. 2, further reduces the difficulty.

Radial symmetry fixes σ_r and σ_θ as principal stresses and leaves but one equation of equilibrium

$$\frac{d(\sigma_r h)}{dr} + (\sigma_r - \sigma_\theta)\frac{h}{r} = 0 \quad \text{or} \quad \frac{d}{dr}(\sigma_r r h) - \sigma_\theta h = 0 \tag{15}$$

The constant rate of dissipation requirement is

$$D(\varepsilon'_{ij}) = \sigma_{ij}\varepsilon'_{ij} = \sigma_r\varepsilon'_r + \sigma_\theta\varepsilon'_\theta = \text{constant} \tag{16}$$

where in terms of the rate of radial displacement u'

$$\varepsilon'_r = du'/dr, \qquad \varepsilon'_\theta = u'/r \tag{17}$$

The stress point cannot lie on sides AF or CD, Fig. 2, because there the normality condition requires $\varepsilon'_r = 0$, and the constant dissipation rate requires $\varepsilon'_\theta = \text{constant} \neq 0$. These requirements are contradictory in view of (17). Sides BC and EF are prohibited because $\varepsilon'_\theta = 0$ forces $u' = 0$. The normality condition for sides AB and DE would require $\varepsilon'_r = -\varepsilon'_\theta$ and each must be constant if the dissipation rate is constant. This too is impossible in view of (17). The stress point, therefore, must be at a corner. Corners A and D are not permissible because with $\sigma_r = 0$, Equation (15) gives $h = 0$. At B or E, $\sigma_\theta = 0$ and $|\sigma_r| = \sigma_y$ so that from (15)

$$rh = \text{constant} \tag{18}$$

as shown in Fig. 3a. At C or F, $\sigma_r = \sigma_\theta = \pm\sigma_y$ so that from (15)

$$h = \text{constant} \tag{19}$$

as shown in Fig. 3b.

The velocity pattern is restricted by the normality condition and (16) and (17). At B and E, du'/dr must be constant, opposite in sign to u'/r and numerically as large or larger. Therefore,

$$u' = a(b - r), \qquad b/2 \leq r \leq b \tag{20}$$

where a and b are constants.

At C and F, $du'/dr + u'/r$ must be constant and both strain rates must be of the same sign.

$$u' = \frac{a}{r}(r^2 \pm c^2), \qquad r^2 \geq c^2 \tag{21}$$

The constant a in (20) and (21) is positive at B and F, negative at C and E.

The two basic solutions can be combined in only one way. The continuity of u' requires the constant thickness region to surround the variable thickness portion as shown in Fig. 4. All possible combinations of P and Q cannot be carried with the basic solutions of Fig. 3 alone. A flange at r_1 is often needed in addition, Fig. 4.

The minimum volume V_m required is listed in Fig. 4. Fig. 5 gives the ratio of the minimum volume to the volume of a disc of uniform thickness with the same strength. Although a rather unfair comparison it does emphasize the weight saving possibilities of careful design.

Conclusion. Plastic strain rate fields for which D, the rate of dissipation of energy per unit volume, is constant give minimum weight designs. For space structures, sheets, and membranes an absolute minimum is obtained with D a constant throughout the permissible volume. For beams and plates a relative minimum is found for D a constant at the surfaces of the plate.

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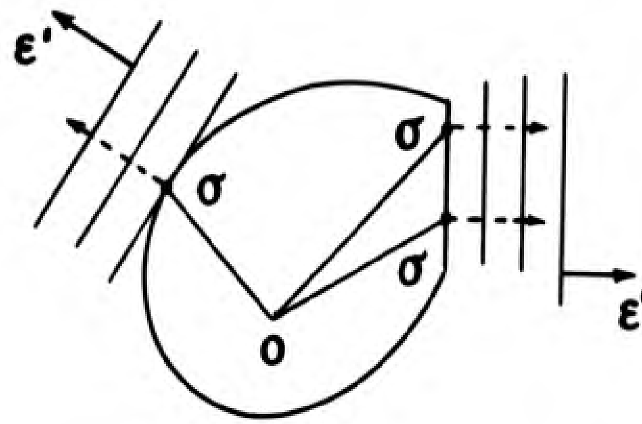


FIG. 1 DISSIPATION FUNCTION $D \equiv \sigma_{ij} \varepsilon'_{ij}$ IS DETERMINED UNIQUELY BY THE PLASTIC STRAIN RATE ε'_{ij}

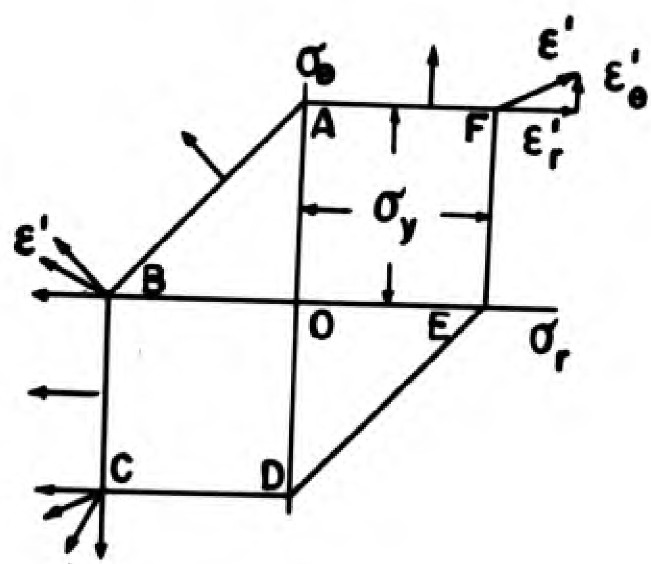


FIG. 2 TRESCA YIELD CRITERION— PLANE STRESS

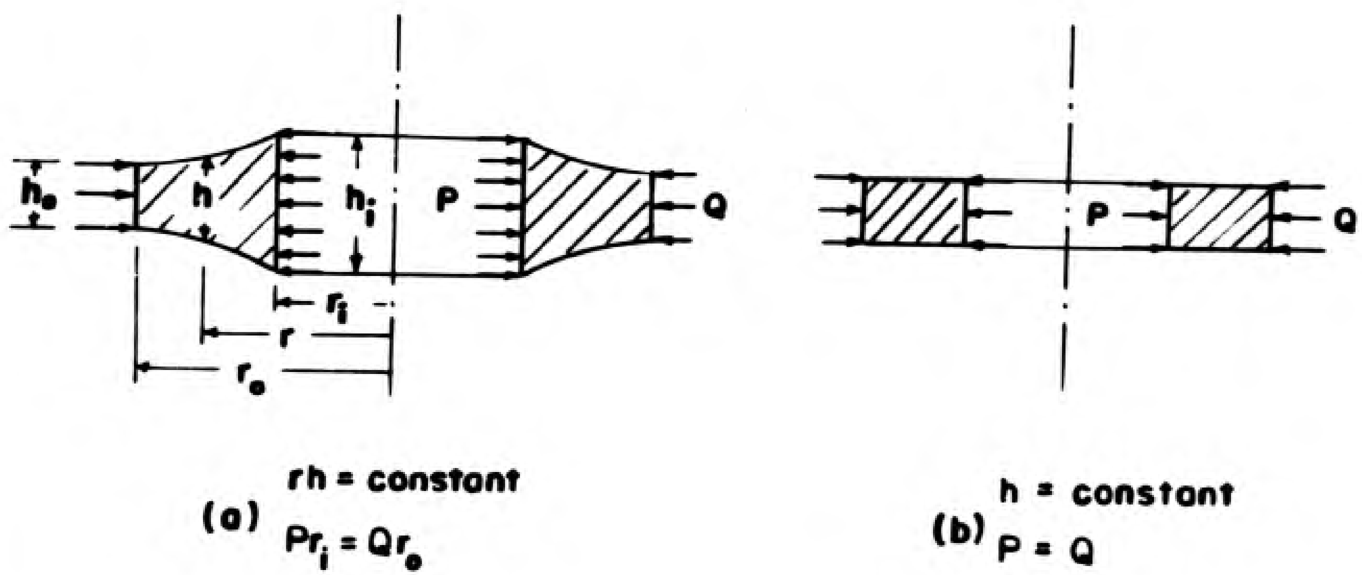
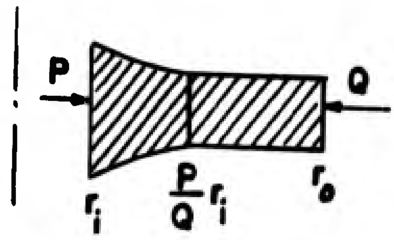


Fig. 3. Basic minimum weight solutions for circular disc under interior and exterior pressure $P = \sigma_y h_i$, $Q = \sigma_y h_o$.

I. P and Q of same sign $|P| > |Q|$

1. $P r_i < Q r_o$

   $P \leq 2Q$

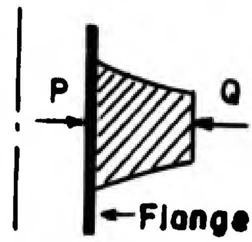


$$\frac{\sigma_y V_m}{P \pi r_i^2} = \frac{P}{Q} - 2 + \frac{Q}{P} \frac{r_o^2}{r_i^2}$$

$h = P/\sigma_y \quad h_o = Q/\sigma_y$

2. $P r_i > Q r_o$

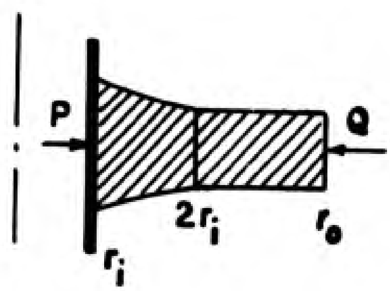
   $r_o \leq 2 r_i$



$$\frac{\sigma_y V_m}{P \pi r_i^2} = 2 - 2 \frac{Q}{P} \frac{r_o}{r_i} \left(2 - \frac{r_o}{r_i}\right)$$

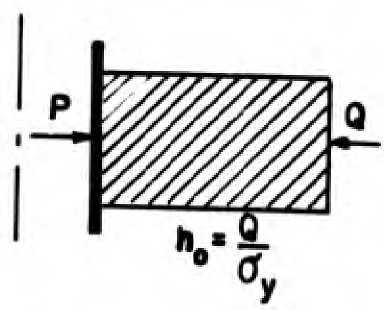
3. $P > 2Q$

   $r_o > 2 r_i$



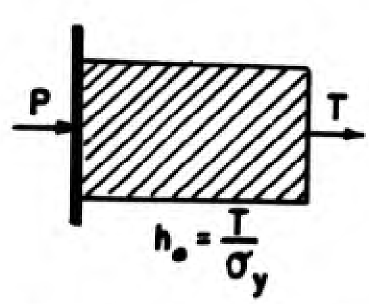
$$\frac{\sigma_y V_m}{P \pi r_i^2} = 2 + \frac{Q}{P} \left(\frac{r_o^2}{r_i^2} - 4\right)$$

II. P and Q of same sign $|Q| > |P|$



$$\frac{\sigma_y V_m}{Q \pi r_o^2} = 1 + \frac{r_i^2}{r_o^2} \left(1 - \frac{2P}{Q}\right)$$

III. P and Q of opposite sign, $Q = -T$



$$\frac{\sigma_y V_m}{T \pi r_o^2} = 1 + \frac{r_i^2}{r_o^2} \left(1 + \frac{2P}{T}\right)$$

FIG. 4 MINIMUM WEIGHT DESIGN FOR DISC

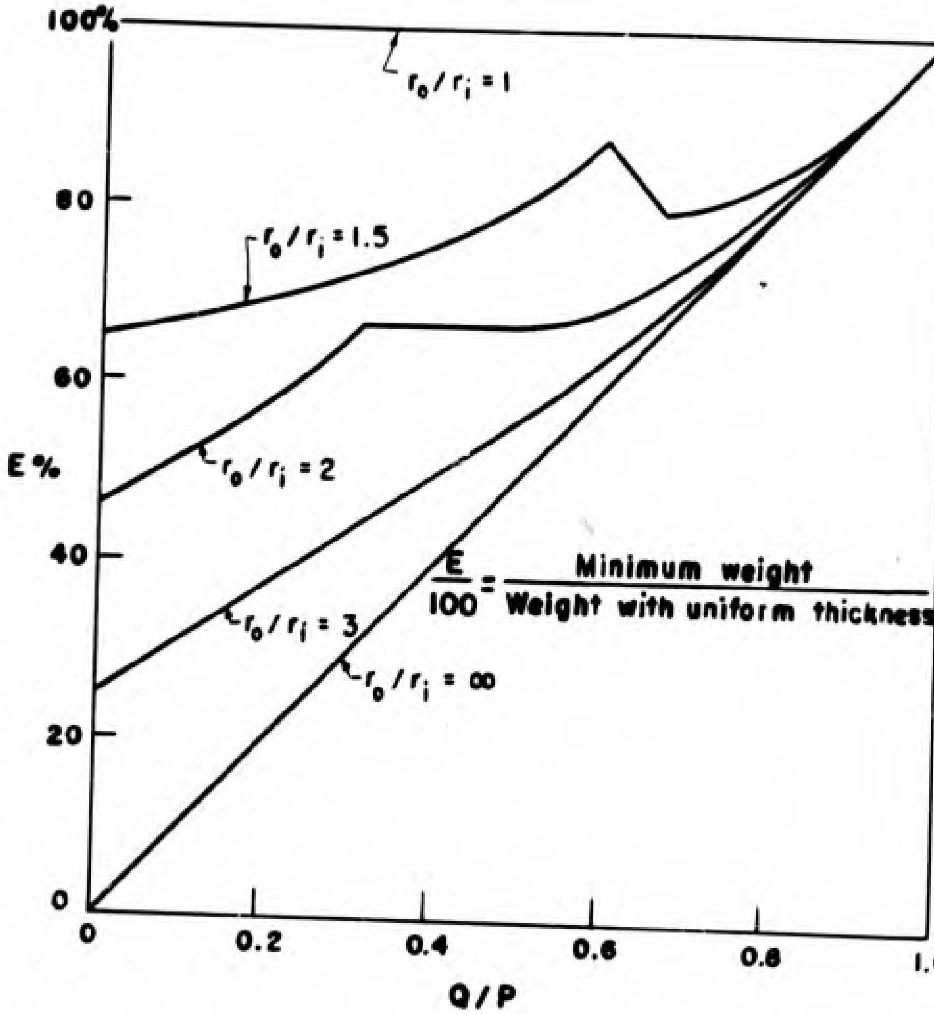


FIG. 5. COMPARISON OF MINIMUM WEIGHT AND WEIGHT OF DISC OF UNIFORM THICKNESS

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