ASSESSMENT OF CASUALTIES FROM SMALL ARMS FIRE

U.S. ARMY STRATEGY & TACTICS ANALYSIS GROUP

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FOREWORD

It is generally conceded that small arms fire in actual combat situations is largely unaimed. Machine gun fire especially, is generally "sprayed" into a target area. The concept of a lethal area per round is therefore useful in assessing casualties caused by small arms. Utilizing formulas developed by Arthur D. Groves of Ballistic Research Laboratory, Aberdeen, Maryland, lethal areas per round have been computed for various tactical situations. The lethal areas and associated single-shot kill probabilities as functions of target size, troop posture and density, height of aiming point, dispersion of impact point, and range to target are tabulated.

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ASSESSMENT OF CASUALTIES FROM SMALL ARMS FIRE

1. General. Small arms fire may be either aimed or unaimed. One naturally prefers to think of rifle fire as aimed fire in combat, but it is generally conceded that machine gun and automatic rifle fire is more or less "sprayed" into an area in which a target is presumably located. Over a large number of combat situations, however, it is quite likely that even rifle fire is generally not precisely aimed. For this reason, the consensus of combat officers is that small arms fire is essentially unaimed fire over a specific small target area.

2. Gaming Assumptions. In assessing small arms fire in gaming operations, it is generally agreed that at some small unit of resolution, for example a platoon, the distribution of men may be considered uniform. Using this assumption and the one above, that small arms fire is unaimed over the platoon area, a mathematical model of assessment of small arms fire can be developed.

3. Generalized Assessment Model. One round of fire passing through a vertical target area, A_p, will have probability, P_{HK}, of causing a casualty. The concept of a horizontal lethal area, A_L,

will be employed, however, where A_L is a function of the following parameters:

θ = Angle of fall at mean range R

D = Depth of target in direction of fire

Y = Height of aiming point

σ = Standard deviation of Y

A_p = Vertical area of individual target

P_{HK} = Probability of casualty, given a hit thru A_p.

It will be assumed that all rounds land within or pass over the target area of width, W, and depth, D. Therefore, the probability that a single round causes a casualty is:

$$P_k = A_L / WD$$

To account for duplications of kill, i.e., overlapping of individual lethal arms, an exponential model is used to obtain the expected fraction kill in the target area:

$$F(K) = 1 - e^{-NP_k}$$

where

N = number of rounds fired into target area

Obviously, if NP_k is small, the first two terms in the series expansion of the exponential yields,

$$\text{Approx. } F(K) = NP_k$$

This can be very inaccurate, however, in cases of heavy fire.

4. Calculation of Lethal Area and Kill Probability. A mathematical model * has been developed for computing the lethal area, A_L, and the associated single shot kill probability, P_k, when the density of men per unit area, δ, is specified.

a. Parameters used in Computation

Let

$$a = \min\{H, D \tan \theta\}$$

$$b = \max\{H, D \tan \theta\}$$

$$\alpha = a/\sigma$$

$$\beta = b/\sigma$$

$$\gamma = Y/\sigma$$

$$M = \sigma A_p \cot \theta / H$$

For any X,

$$\Phi(X) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X} e^{-t^2/2} dt$$

$$\varphi(X) = \frac{1}{\sqrt{2\pi}} e^{-X^2/2}$$

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* U. S. Army, BRL Memo Rpt. No. 1452, "Effectiveness of Unaimed Small Arms Fire, " Arthur D. Groves, 3 January 1963.

b. <u>Algorithms for Computations</u>

(1) When the angle of fall is larger than 0.001, i.e., when $\tan\Theta \geq 0.001$ the lethal area and single shot kill probability are computed by:

$$A_L = P_{HK}M \Big[\varphi(-\gamma) - \varphi(\alpha-\gamma) - \varphi(\beta-\gamma) + \varphi(\alpha+\beta-\gamma)$$

$$-\gamma\,\Phi(-\gamma) - (\alpha-\gamma)\,\Phi(\alpha-\gamma) - (\beta-\gamma)\,\Phi(\beta-\gamma)$$

$$+(\alpha+\beta-\gamma)\,\Phi(\alpha+\beta-\gamma) \Big]$$

$$P_k = P_{HK} \Big[\Phi(\alpha+\beta-\gamma) - \Phi(-\gamma)$$

$$-\{\Phi(\alpha-\gamma+\delta M) - \Phi(-\gamma+\delta M)\} \exp\left\{\delta M \left(\frac{\delta M}{2} - \gamma\right)\right\}$$

$$-\{\Phi(\beta-\gamma) - \Phi(\alpha-\gamma)\} \exp\{-\delta M\alpha\}$$

$$-\{\Phi(\alpha+\beta-\gamma-\delta M) - \Phi(\beta-\gamma-\delta M)\} \exp\{-\delta M(\alpha+\beta-\gamma-\delta M/2)\} \Big]$$

(2) When the fire is nearly horizontal, $\Theta \simeq 0$ and $\cot\Theta \to \infty$ therefore, the above forms cannot be used. In practice if $\Theta < 0.001$, special alogrithms are used:

$$A_L = (P_{HK}\,A_p\,D/H) \left[\Phi\left(\frac{H-Y}{\sigma}\right) - \Phi(-Y/\sigma) \right]$$

$$P_k = P_{HK}\left[1 - \exp(-\delta D A_p/H)\right] \left[\Phi\left(\frac{H-Y}{\sigma}\right) - \Phi(-Y/\sigma) \right]$$

(3) In the following computation, $\Phi(X)$ is approximated by the analytic function: $\Phi(X) = 1/(1-e^{-1.7X})$.

5. Sample Calculations. The STAG 7090 computer was used to calculate A_L and P_k for three tactical configurations, four personnel postures, and firing ranges up to 1500 meters. The angle of fall was taken from Firing Tables for 0.50 caliber, War Dept. FT 0.50AA-T-1, 1 Feb. 1946.

a. Tactical Configurations Assumed

Formation	W(m.)	D(m.)	δ(men/m^2.)
Attack	750	100	.0018
Defense	1500	100	.0009
Column	1200	20*	.0077

b. Postures Assumed. The individual soldier was assumed to present a vertical target two feet wide. The height of the individual target, H, is given in feet and meters and the assumed single shot conditional kill probability, P_{HK} is indicated.

Posture	H		Y(m)	A_p(m^2)	P_{HK}
	feet	m.			
Standing	5' 9"	1.75	1.0	1.07	.75
Crouching	3'	.915	.5	.56	.85
Prone	15"	.41	.5	.25	.80
Protected	4"	.1	.3	.06	.30

In all cases σ was assumed to be 0.5 m. The computed lethal areas and kill probabilities are shown in Tables 1-3.

* This assumes fire from the flank.

6. Approximations to Curves. The computed lethal areas plotted as curves of area versus mean range, R, are shown in Figures 1 and 2. In actual gaming practice linear approximations to these curves shown as dashed lines will be used in assessing the casualties from small arms. Analytic formulas for the various approximations are shown.

7. Casualty Assessment. For any given set of tactical mission, phase, and formation, a distribution of personnel among the various postures is assumed. The lethal area used against any particular target is then a weighted mean based on this distribution. For example, an infantry unit in the attack from line of departure to assault might have a composite posture such as:

45% standing, (i. e., walking)
40% crouching
5% prone
10% protected (i. e. in trucks)

Suppose the' the range = 750 meters. The weighted lethal area is computed as follows:

$$0.45 \times 37.89 = 17.05 \ m^2$$
$$0.40 \times 25.22 = 10.10 \ m^2$$
$$0.05 \times 13.60 = .28 \ m^2$$
$$0.10 \times 1.23 = .12 \ m^2$$
$$\text{Mean } A_L \quad 27.55 \ m^2$$

This value would be the value used in the casualty assessment formula of paragraph 3. to get the expected fraction of casualties:

$$F(K) = 1 - \exp(-NA_L/WD)$$

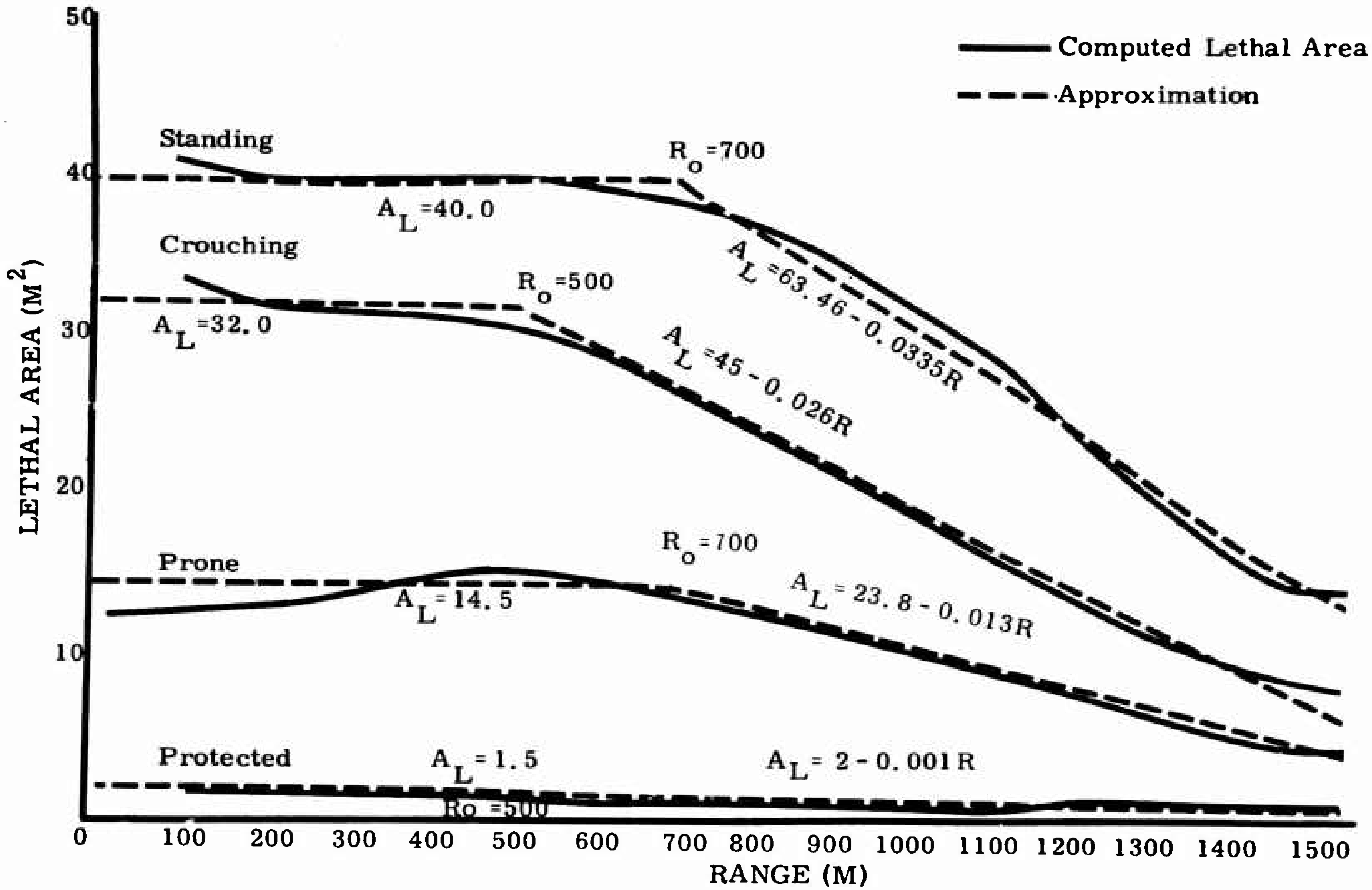

Figure 1. Lethal Area and Approximation - Attack or Defense Deployed Formation

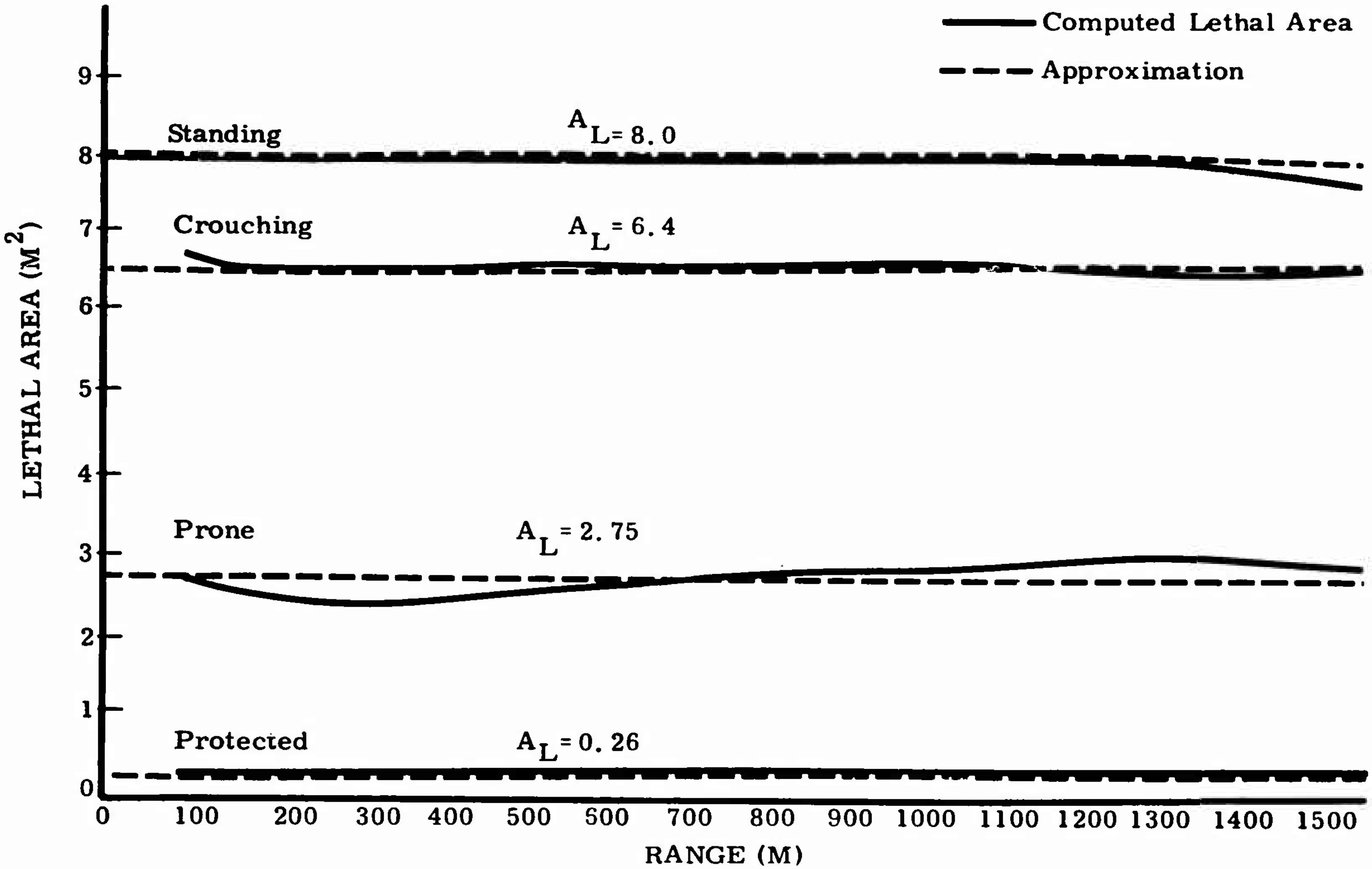

Figure 2. Lethal Area and Approximation - Column Formation, Flanking Fire

8. <u>Kill Assessment Using Lethal Area.</u> The formulas for A_L and P_k as computed from algorithms in paragraph 4 presume these values remain constant for the duration of engagement. Since the geometric extent of a tactical formation will remain fairly constant throughout the assessment period, the use of a lethal area is considered preferable to a constant kill probability which is based on a constant density. Obviously, if area remains constant, density decreases as casualties occur. In computing actual number of kills, therefore the fraction of kills is computed by formula in paragraph 7 above and multiplied by current strength to obtain expected number of casualties, K. $K = S \cdot F(K)$ where S is current strength in number of men.

In the tactical situation above assuming two platoons forward in a depth of 100 meters, current strength, 132 men. The casualties for 500 rounds of enemy small arms fire would be computed as follows:

W = 750 meters
D = 100 meters
S = 132 men
A_L = 27.55 square meters/round
N = 500 rounds

$$K = 132\,[1 - \exp(-500 \cdot 27.55/75{,}000)]$$
$$K = 132\,[0.168]$$
$$K = 22.2$$

The assessed casualties would be 22 men.

TABLE 1. LETHAL AREAS AND KILL PROBABILITIES FOR ATTACK FORMATION

ATTACK FORMATION

DEPTH = 100.0000 DELTA = 0.0018 SIGMA = 0.5000

STANDING POSTURE

HEIGHT = 1.7500 Y = 1.0000 A_P = 1.0700 P_{HK} = 0.7500

R	TAN θ	A_L	P_K
100.0000	0.0006	41.0549	0.0700
200.0000	0.0013	40.0863	0.0712
300.0000	0.0021	40.1007	0.0719
400.0000	0.0033	39.9989	0.0718
500.0000	0.0040	39.8702	0.0714
750.0000	0.0080	37.8940	0.0651
1000.0000	0.0130	31.5807	0.0549
1250.0000	0.0212	21.4616	0.0376
1500.0000	0.0330	13.6901	0.0243

CROUCHING POSTURE

HEIGHT = 0.9150 Y = 0.5000 A_P = 0.5600 P_{HK} = 0.8500

R	TAN θ	A_L	P_K
100.0000	0.0006	33.7860	0.0576
200.0000	0.0013	32.1082	0.0488
300.0000	0.0021	31.8631	0.0487
400.0000	0.0033	31.1382	0.0484
500.0000	0.0040	30.5188	0.0480
750.0000	0.0080	25.2225	0.0438
1000.0000	0.0130	18.9398	0.0335
1250.0000	0.0212	11.9948	0.0210
1500.0000	0.0330	7.6294	0.0135

PRONE POSTURE

HEIGHT = 0.4100 Y = 0.5000 A_P = 0.2500 P_{HK} = 0.8000

R	TAN θ	A_L	P_K
100.0000	0.0006	13.1525	0.0224
200.0000	0.0013	13.5235	0.0237
300.0000	0.0021	14.3415	0.0223
400.0000	0.0033	15.1755	0.0210
500.0000	0.0040	15.3884	0.0206
750.0000	0.0080	13.5996	0.0208
1000.0000	0.0130	10.5385	0.0178
1250.0000	0.0212	6.8836	0.0113
1500.0000	0.0330	4.3725	0.0073

PROTECTED POSTURE

HEIGHT = 0.1000 Y = 0.3000 A_P = 0.0600 P_{HK} = 0.3000

R	TAN θ	A_L	P_K
100.0000	0.0006	1.2822	0.0022
200.0000	0.0013	1.3460	0.0029
300.0000	0.0021	1.4263	0.0023
400.0000	0.0033	1.4903	0.0019
500.0000	0.0040	1.4924	0.0018
750.0000	0.0080	1.2267	0.0019
1000.0000	0.0130	0.9379	0.0016
1250.0000	0.0212	0.6029	0.0010
1500.0000	0.0330	0.3832	0.0006

TABLE 2. LETHAL AREAS AND KILL PROBABILITIES FOR DEFENSE FORMATION

DEFENSE FORMATION

DEPTH = 100.0000 DELTA = 0.0009 SIGMA = 0.5000

STANDING POSTURE

HEIGHT = 1.7500 Y = 1.0000 A_P = 1.0700 P_{HK} = 0.7500

R	TAN θ	A_L	P_K
100.0000	0.0006	41.0549	0.0360
200.0000	0.0013	40.0863	0.0370
300.0000	0.0021	40.1007	0.0372
400.0000	0.0033	39.9989	0.0371
500.0000	0.0040	39.8702	0.0368
750.0000	0.0080	37.8940	0.0334
1000.0000	0.0130	31.5807	0.0281
1250.0000	0.0212	21.4616	0.0191
1500.0000	0.0330	13.6901	0.0123

CROUCHING POSTURE

HEIGHT = 0.9150 Y = 0.5000 A_P = 0.5600 P_{HK} = 0.8500

R	TAN θ	A_L	P_K
100.0000	0.0006	33.7860	0.0296
200.0000	0.0013	32.1082	0.0250
300.0000	0.0021	31.8631	0.0250
400.0000	0.0033	31.1382	0.0248
500.0000	0.0040	30.5188	0.0246
750.0000	0.0080	25.2225	0.0223
1000.0000	0.0130	18.9398	0.0170
1250.0000	0.0212	11.9948	0.0106
1500.0000	0.0330	7.6294	0.0068

PRONE POSTURE

HEIGHT = 0.4100 Y = 0.5000 A_P = 0.2500 P_{HK} = 0.8000

R	TAN θ	A_L	P_K
100.0000	0.0006	13.1525	0.0115
200.0000	0.0013	13.5235	0.0114
300.0000	0.0021	14.3415	0.0109
400.0000	0.0033	15.1755	0.0103
500.0000	0.0040	15.3884	0.0102
750.0000	0.0080	13.5996	0.0105
1000.0000	0.0130	10.5385	0.0090
1250.0000	0.0212	6.8836	0.0057
1500.0000	0.0330	4.3725	0.0036

PROTECTED POSTURE

HEIGHT = 0.1000 Y = 0.3000 A_P = 0.0600 P_{HK} = 0.3000

R	TAN θ	A_L	P_K
100.0000	0.0006	1.2822	0.0011
200.0000	0.0013	1.3460	0.0012
300.0000	0.0021	1.4263	0.0010
400.0000	0.0033	1.4903	0.0009
500.0000	0.0040	1.4924	0.0009
750.0000	0.0080	1.2267	0.0010
1000.0000	0.0130	0.9379	0.0008
1250.0000	0.0212	0.6029	0.0005
1500.0000	0.0330	0.3832	0.0003

TABLE 3. LETHAL AREAS AND KILL PROBABILITIES FOR COLUMN FORMATION (FLANKING FIRE)

COLUMN FORMATION

DEPTH = 20.0000 DELTA = 0.0077 SIGMA = 0.5000

STANDING POSTURE

HEIGHT = 1.7500 Y = 1.0000 A_P = 1.0700 P_{HK} = 0.7500

R	TAN θ	A_L	P_K
100.0000	0.0006	8.2110	0.0603
200.0000	0.0013	7.9939	0.0595
300.0000	0.0021	7.9989	0.0595
400.0000	0.0033	8.0055	0.0603
500.0000	0.0040	8.0088	0.0607
750.0000	0.0080	8.0199	0.0618
1000.0000	0.0130	8.0153	0.0621
1250.0000	0.0212	7.9628	0.0614
1500.0000	0.0330	7.7752	0.0585

CROUCHING POSTURE

HEIGHT = 0.9150 Y = 0.5000 A_P = 0.5600 P_{HK} = 0.8500

R	TAN θ	A_L	P_K
100.0000	0.0006	6.7572	0.0497
200.0000	0.0013	6.4301	0.0497
300.0000	0.0021	6.4328	0.0454
400.0000	0.0033	6.4342	0.0429
500.0000	0.0040	6.4334	0.0424
750.0000	0.0080	6.4076	0.0420
1000.0000	0.0130	6.3227	0.0419
1250.0000	0.0212	6.0550	0.0413
1500.0000	0.0330	5.4496	0.0394

PRONE POSTURE

HEIGHT = 0.4100 Y = 0.5000 A_P = 0.2500 P_{HK} = 0.8000

R	TAN θ	A_L	P_K
100.0000	0.0006	2.6305	0.0193
200.0000	0.0013	2.4592	0.0209
300.0000	0.0021	2.4980	0.0222
400.0000	0.0033	2.5559	0.0215
500.0000	0.0040	2.5893	0.0211
750.0000	0.0080	2.7698	0.0196
1000.0000	0.0130	2.9511	0.0184
1250.0000	0.0212	3.0826	0.0175
1500.0000	0.0330	2.9166	0.0176

PROTECTED POSTURE

HEIGHT = 0.1000 Y = 0.3000 A_P = 0.0600 P_{HK} = 0.3000

R	TAN θ	A_L	P_K
100.0000	0.0006	0.2564	0.0019
200.0000	0.0013	0.2429	0.0023
300.0000	0.0021	0.2471	0.0033
400.0000	0.0033	0.2534	0.0031
500.0000	0.0040	0.2570	0.0030
750.0000	0.0080	0.2758	0.0021
1000.0000	0.0130	0.2924	0.0017
1250.0000	0.0212	0.2975	0.0015
1500.0000	0.0330	0.2680	0.0016