ROCK ISLAND ARSENAL
RESEARCH & ENGINEERING DIVISION
RESEARCH LABORATORIES

TECHNICAL NOTES

MISCELLANEOUS ARTILLERY PROBLEMS

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MISCELLANEOUS ARTILLERY PROBLEMS

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CONTENTS

$\alpha(t)$ = centroid of $B(t)$

$\beta(t)$ = centroid of $R(t)$

$R(t)$ = resisting force

Derivation of the Moment-Area Method for Application to the Design of Control Rods

Given the differential equation for recoil

$$M\ddot{x} = B(t) - R(t) \qquad (1)$$

Integrating

$$M\dot{x} = \int_a^t B(\tau)d\tau - \int_b^t R(\tau)d\tau \qquad (2)$$

where $B(t)$ is applied when $t = a$; and $R(t)$ is applied when $t = b$. Define

$$\int_a^t B(\tau)d\tau = G(t) \qquad \int_b^t R(\tau)d\tau = H(t) \qquad (3)$$

so

$$M\dot{x} = G(t) - H(t) \qquad (4)$$

Integrating

$$Mx = \int_a^t G(\tau)d\tau - \int_b^t H(\tau)d\tau \qquad (5)$$

Now, integrating by parts

$$Mx = \tau G(\tau)\Big|_a^t - \int_a^t \tau \frac{d}{d\tau} G(\tau)d\tau - \tau H(\tau)\Big|_b^t + \int_b^t \tau \frac{d}{d\tau} H(\tau)d\tau$$

or

$$Mx = t\,G(t) - \int_a^t \tau B(\tau)d\tau - t\,H(t) + \int_b^t \tau R(\tau)\,d\tau$$

Rewrite as

$$Mx = tG(t) - \frac{\int_a^t B(\tau)d\tau}{\int_a^t B(\tau)d\tau}\cdot\int_a^t \tau B(\tau)d\tau - tH(t) + \frac{\int_b^t R(\tau)d\tau}{\int_b^t R(\tau)d\tau}\cdot\int_b^t \tau R(\tau)d\tau \qquad (6)$$

Define

$$\alpha(t) = \frac{\int_a^t \tau B(\tau)d\tau}{\int_a^t B(\tau)d\tau} \qquad \beta(t) = \frac{\int_b^t \tau R(\tau)d\tau}{\int_b^t R(\tau)d\tau}$$

Note that $\alpha(t)$ and $\beta(t)$ are expressions for the centroids of the areas generated by the functions $B(t)$ and $R(t)$, at time t, measured from $t = 0$.

Now, write equation 6 as

$$Mx = tG(t) - \alpha(t)\cdot G(t) - tH(t) + \beta(t)\cdot H(t) \qquad (7)$$

$R(t)$ = resistance to recoil

t_R = recoil time

b = time when $R(t)$ is applied

α = centroid of $B(t)$ curve

β = centroid of $R(t)$ curve

or

$$M x = (t-\alpha)\int_a^t B(\tau)d\tau - (t-\beta)\int_b^t R(\tau)d\tau \tag{8}$$

At the end of recoil ($x = L$) and ($t = t_R$) the following relations hold

$$\int_a^{t_R} B(\tau)d\tau = \int_b^{t_R} R(\tau)d\tau = I \quad \text{(a constant)} \tag{9}$$

$$ML = [(t_R-\alpha) - (t_R-\beta)]\, I \tag{10}$$

or

$$\beta(t_R) = \frac{ML}{I} + \alpha(t_R) \tag{11}$$

Assume that the shape of the "total resistance to recoil" force, $R(t)$, is known. Then recalling

$$\beta(t) = \frac{\int_b^t \tau R(\tau)d\tau}{\int_b^t R(\tau)d\tau}$$

write

$$\beta(t_R)\cdot\int_b^{t_R} R(\tau)d\tau = ML + I\alpha = \int_b^{t_R} \tau R(\tau)d\tau$$

Now, from the two relations

$$I = \int_b^{t_R} R(\tau)d\tau \tag{12}$$

$$ML + I\alpha = \int_b^{t_R} \tau R(\tau)d\tau \tag{13}$$

the magnitude of $R(t)$ and t_R may be determined.

CASE 1: $a = b = 0$, $t_R > t_e$ where t_e is the time $B(t)$ acts. Assume the following data to be known.

$M = 3.8$ # sec²/in $I = 2009$ # sec $\alpha = 0.007$ sec $L = 48$ in.

EXAMPLE 1:

Assume a continuous function for $R(t)$, namely

$$R(t) = R_0 \sin \frac{\pi}{t_R}t$$

then

$$I = R_0\int_0^{t_R} \sin\frac{\pi}{t_R}t\,dt = -R_0\frac{t_R}{\pi}\cos\frac{\pi}{t_R}t\Big|_0^{t_R} = \frac{2R_0 t_R}{\pi}$$

$$ML + I\alpha = R_0\int_0^{t_R} t\sin\frac{\pi}{t_R}t\,dt = R_0\Big[t\Big(-\frac{t_R}{\pi}\Big)\cos\frac{\pi}{t_R}t\Big]_0^{t_R} + R_0\frac{t_R}{\pi}\int_0^{t_R}\cos\frac{\pi}{t_R}t\,dt$$

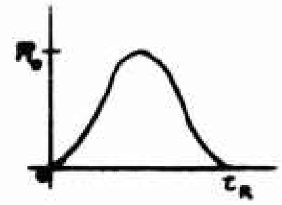
$$= R_0\frac{t_R^2}{\pi} + R_0\Big(\frac{t_R}{\pi}\Big)^2 \sin\frac{\pi}{t_R}t\Big|_0^{t_R}$$

$$= R_0\cdot\frac{t_R^2}{\pi}$$

Thus

$$\pi I = 2R_0 t_R \qquad \pi(ML + I\alpha) = R_0 t_R^2$$

or



$$R_0 = \frac{\pi I^2}{4(ML + I\alpha)} = 16,100$$

$$t_R = \frac{2}{I}(ML + I\alpha) = 0.196$$

By the Moment-Area Method (eq. 10)

$$ML = [(t_R - \alpha) - (t_R - \beta)] I$$

Obviously $\beta = \frac{1}{2} t_R$, therefore

$$ML = [(t_R - \alpha) - (t_R - t_R/2)] I$$

or

$$t_R = \frac{2}{I}(ML + I\alpha) = 0.196$$

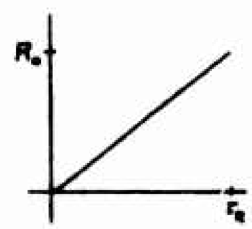
as before.

EXAMPLE 2:

Using the same data and assuming

$$R(t) = R_0 \frac{t}{t_R}$$

then



$$I = \frac{R_0}{t_R}\int_0^{t_R} t\,dt = \frac{1}{2} R_0 t_R$$

and

$$ML + I\alpha = \frac{R_0}{t_R}\int_0^{t_R} t^2\,dt = \frac{1}{3} R_0 t_R^2$$

thus

$$2I = R_0 t_R \qquad 3(ML + I\alpha) = R_0 t_R^2$$

or

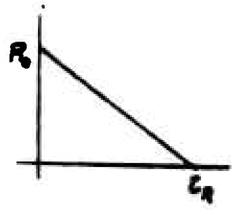
$$R_0 = \frac{4I^2}{3(ML + I\alpha)} = 27,400$$

$$t_R = \frac{3(ML + I\alpha)}{2I} = 0.147$$

EXAMPLE 3:

Using the same data and assuming

$$R(t) = R_0\left(1 - \frac{t}{t_R}\right)$$



now

$$I = \int_0^{t_R} R_0\left(1 - \frac{t}{t_R}\right)dt = R_0\left(t - \frac{t^2}{2t_R}\right)\Big|_0^{t_R} = \frac{1}{2}R_0 t_R$$

and

$$ML + I\alpha = \int_0^{t_R} t\cdot R_0\left(1 - \frac{t}{t_R}\right)dt = R_0\left(\frac{t^2}{2} - \frac{t^3}{3t_R}\right)\Big|_0^{t_R} = \frac{1}{6}R_0 t_R^2$$

thus

$$R_0 t_R = 2I$$

$$R_0 t_R^2 = 6(ML + I\alpha)$$

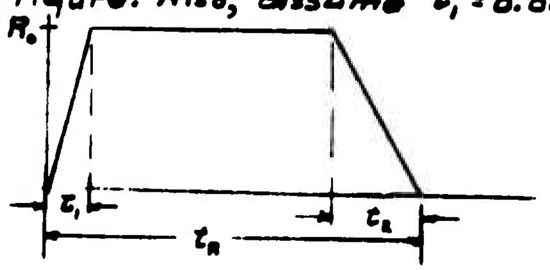
or

$$R_0 = \frac{2I^2}{3(ML + I\alpha)} = 13,680$$

$$t_R = \frac{3(ML + I\alpha)}{I} = 0.294$$

EXAMPLE 4

Given the same data with $R(t)$ defined by the accompanying figure. Also, assume $t_1 = 0.005$ and $t_2 = 0.010$.



Now

$$I = \int_0^{t_R} R(t)\,dt = \int_0^{t_1}\frac{R_0}{t_1}t\,dt + \int_{t_1}^{t_R - t_2} R_0\,dt + \int_{t_R - t_2}^{t_R} R_0\left(1 - \frac{t - t_R + t_2}{t_2}\right)dt$$
$$= \frac{1}{2}R_0 t_1 + R_0(t_R - t_2 - t_1) + \frac{1}{2}R_0 t_2$$

and

$$(ML + I\alpha) = \int_0^{t_R} t\,R(t)\,dt = \int_0^{t_1}\frac{R_0}{t_1}t^2\,dt + \int_{t_1}^{t_R - t_2} R_0\,t\,dt + \int_{t_R - t_2}^{t_R} R_0\cdot\frac{t_R - t}{t_2}\,t\,dt$$
$$= \frac{1}{3}R_0 t_1^2 + \frac{1}{2}R_0\left[(t_R - t_2)^2 - t_1^2\right] + R_0 t_2\left(\frac{1}{2}t_R - \frac{1}{3}t_2\right)$$

or

$$I = \frac{1}{2} R_o (2t_R - t_1 - t_2)$$

$$(ML + I\alpha) = \frac{R_o}{6}\left[3t_R(t_R - t_2) - t_1^2 + t_2^2\right]$$

Note: the same results would be obtained by the moment-area method of equation 8. That is:

$$ML = (t_R - \alpha)\int_0^{t_R} B(\tau)\,d\tau - (t_R - \beta)\int_0^{t_R} R(\tau)\,d\tau$$

or

$$ML = (t_R - \alpha) I - \Big\{\tfrac{1}{2} R_o t_1 \cdot (t_R - \tfrac{2}{3} t_1) + R_o (t_R - t_2 - t_1)\left[t_2 + \tfrac{1}{2}(t_R - t_1 - t_2)\right]$$
$$+ \tfrac{1}{2} R_o t_2 \cdot \tfrac{2}{3} t_2\Big\}$$

but

$$I = \frac{1}{2} R_o (2t_R - t_1 - t_2)$$

thus

$$ML + I\alpha = \tfrac{1}{2} R_o t_R (2t_R - t_1 - t_2) - \Big\{\tfrac{1}{2} R_o t_1 \cdot (t_R - \tfrac{2}{3} t_1) + R_o (t_R - t_2 - t_1)\left[t_2 + \tfrac{1}{2}(t_R - t_1 - t_2)\right]$$
$$+ \tfrac{1}{2} R_o t_2 \cdot \tfrac{2}{3} t_2\Big\}$$

Expanding and collecting terms

$$ML + I\alpha = \frac{R_o}{6}\left[3t_R(t_R - t_2) - t_1^2 + t_2^2\right]$$

as before.

Solving for t_R and R_o

$$t_R = \frac{6(ML + I\alpha) + 3It_2 + \sqrt{\left[6(ML + I\alpha) + 3It_2\right]^2 - 12I\left[I(t_2^2 - t_1^2) + 3(ML + I\alpha)(t_1 + t_2)\right]}}{6I}$$

So

$$t_R = 0.203 \qquad R_o = 10{,}280$$

Note the simplicity if $t_1 = t_2$. (Assume $t_1 = t_2 = 0.010$)

$$I = R_o (t_R - t_1)$$

$$(ML + I\alpha) = \tfrac{1}{2} R_o t_R (t_R - t_1)$$

So

$$t_R = \frac{2(ML + I\alpha)}{I} = 0.2016 \qquad R_o = \frac{I^2}{2(ML + I\alpha) - It_1} = 10{,}250$$

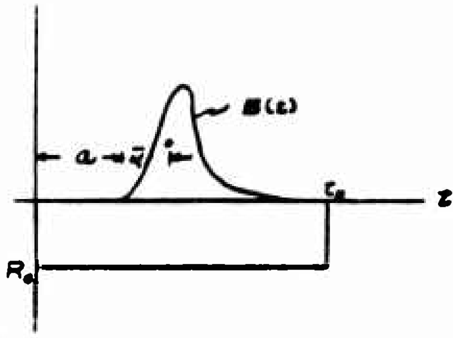
CASE 2: $a \neq 0,\ b = 0$ $\quad t_R - a > t_e$ Assume the following data to be known. (Firing-out-of-battery)

$$M = 3.8\ ^{\#sec^2}/_{in} \qquad I = 2009\ \#sec \qquad \bar{a} = 0.007\ sec \qquad L = 0$$

EXAMPLE 1

Define $x = -L'$ when $t = a$. Note: $\alpha = a + \bar{a}$

Now



$$M\ddot{x} = -R_o$$

$$M\dot{x} = -R_o t$$

$$Mx = -\tfrac{1}{2} R_o t^2$$

At the end of the stroke

$$ML + I\alpha = \tfrac{1}{2} R_o t_R^2 \qquad I = R_o t_R$$

thus

$$I\alpha = \tfrac{1}{2} R_o t_R^2 = \tfrac{1}{2} I t_R$$

or

$$t_R = 2\alpha = 2(a + \bar{a})$$

$$R_o = \frac{I}{2\alpha} = \frac{I}{2(a + \bar{a})}$$

Now

$$ML' = \tfrac{1}{2} R_o a^2 = \tfrac{1}{2} \frac{I a^2}{2(a + \bar{a})} \qquad (L' = 48'')$$

so

$$4ML'(a + \bar{a}) = I a^2$$

or

$$I a^2 - 4ML'a - 4ML'\bar{a} = 0$$

So

$$a = \frac{2ML'}{I}\left\{1 + \sqrt{1 + \frac{I\bar{a}}{ML'}}\right\}^{\frac{1}{2}} = 0.370$$

and

$$\alpha = 0.377 \qquad t_R = 0.754 \qquad R = 2660$$

Finally, note $\dot{x} = -259$ in/sec when $t = a$.

EXAMPLE 2

Define $\dot{x} = -V = -259$ "/sec when $t = a$. Data is unchanged.

Now

$$-MV = -R_o a = -\frac{Ia}{2(a+\bar{\alpha})}$$

or

$$2MVa + 2MV\bar{\alpha} = Ia$$

so

$$a = \frac{2MV\bar{\alpha}}{I - 2MV} = 0.339$$

thus

$$\alpha = 0.346 \qquad t_R = 0.692 \qquad R = 2990$$

Note the sensitivity of this method. A change of 1 "/sec means a change of approximately 500 pounds. Or assume $V = +258$ "/sec and compute L. ($R = 2990$)

$$(ML + I\alpha) = \tfrac{1}{2} I t_R \qquad MV = R_o a$$

$$I = R_o t_R \qquad \alpha = a + \bar{\alpha}$$

Solving

$$t_R = 0.692 \qquad a = 0.328 \qquad \alpha = 0.335 \qquad L = 5.9''$$

CASE 3 $\quad a = b = 0 \qquad t_R < t_e$

Now, if $t_R < t_e$; then, the equations thus far developed do not hold and it is necessary to utilize a computer for solution.

EXAMPLE 1

Assume $R(t)$ to be constant, and

$M = 3.8$ #sec²/in $\quad L = 7''$ $\quad I = 2009$ #sec $\quad \alpha = 0.007$sec

then

$$I = R t_R \qquad ML + I\alpha = \tfrac{1}{2} R t_R^2$$

so

$$t_R = \frac{2(ML + I\alpha)}{I} = 0.0405$$

but $t_e = 0.088$, thus this solution is invalid. By computer

$t_R = 0.0393 \quad R = 50,700 \quad \alpha = 0.0063 \quad I = 1996$ (when $t = t_R$)

DYNAMIC ANALYSIS OF FIRING PLATFORM

1. Introduction

Figure 1 depicts a 105^{mm} Howitzer mounted on a floating raft. The stability of the structure is to be determined under firing conditions.

It will be assumed that the entire structure is rigid; also, the relative motion of the recoiling parts will be neglected. Hence the applied load, R(t) - rod pull, will be the only excitation force applied. Also, it is assumed that this load will be applied through the trunnions and parallel to the center line of the gun tube.

The primary resistance is offered by the buoyant forces of the displaced fluid. Translation is resisted by drag forces. Finally, springs and dashpots are introduced to simulate anchoring.

2. Coordinate Systems

Assume the body to have three degrees of freedom namely ξ, η, ϕ.

Define a coordinate system (O-XYZ) at the overall c.g. in the prefiring condition

Define a coordinate system (O′-X′Y′Z′) with its origin fixed in the raft (at the c.g.) and at all times parallel to O-XYZ.

Define a coordinate system (O′-X″Y″Z″) fixed in the raft.

Define a coordinate system (O″-ΞHZ) with its origin at the trunnion and at all times parallel to O′-X″Y″Z″.

Finally, define a coordinate system (O″-Ξ′H′Z′) such that O″-H′ makes a constant angle of $-\gamma$ with O″-H at all times at all times (in the O″-Ξ′H plane)

Define the coordinates of O′ in O-XYZ as $(\xi, \eta, 0)$.
Define the coordinates of O″ in O′-X″Y″Z″ as $(a, b, 0)$.

8

Dynamic Analysis of Firing Platform

Time-Motion of Base Considering Buoyant and Viscous Drag Forces on Platform

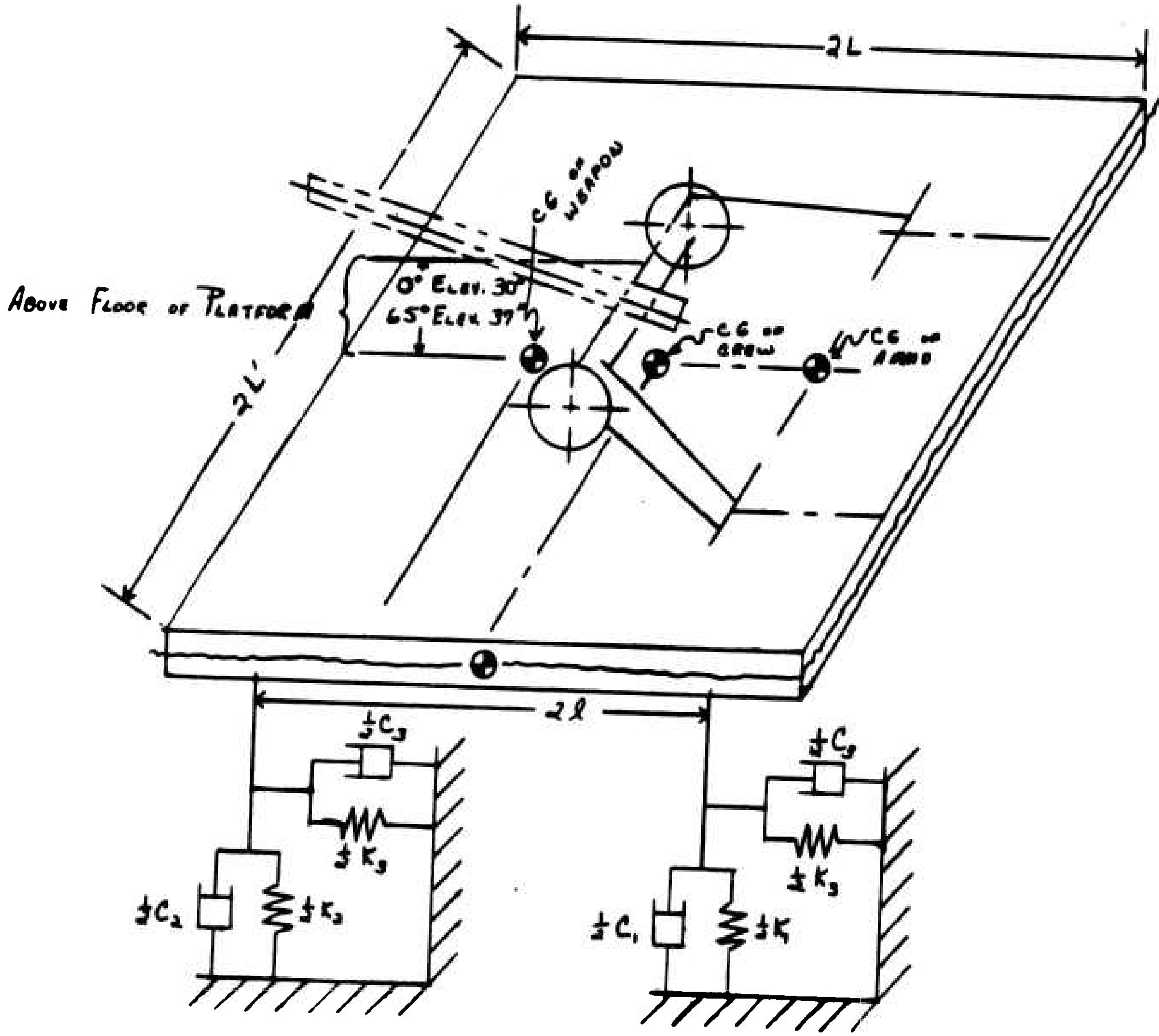


fig 1.

So

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} + \begin{bmatrix} \xi \\ \eta \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X'' \\ Y'' \\ Z'' \end{bmatrix}$$

$$\begin{bmatrix} X'' \\ Y'' \\ Z'' \end{bmatrix} = \begin{bmatrix} \Xi \\ H \\ Z \end{bmatrix} + \begin{bmatrix} a \\ b \\ 0 \end{bmatrix}$$

and

$$\begin{bmatrix} \Xi \\ H \\ Z \end{bmatrix} = \begin{bmatrix} \cos\bar{\gamma} & \sin\bar{\gamma} & 0 \\ -\sin\bar{\gamma} & \cos\bar{\gamma} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Xi' \\ H' \\ Z' \end{bmatrix} \quad \text{where } \bar{\gamma} = -\gamma$$

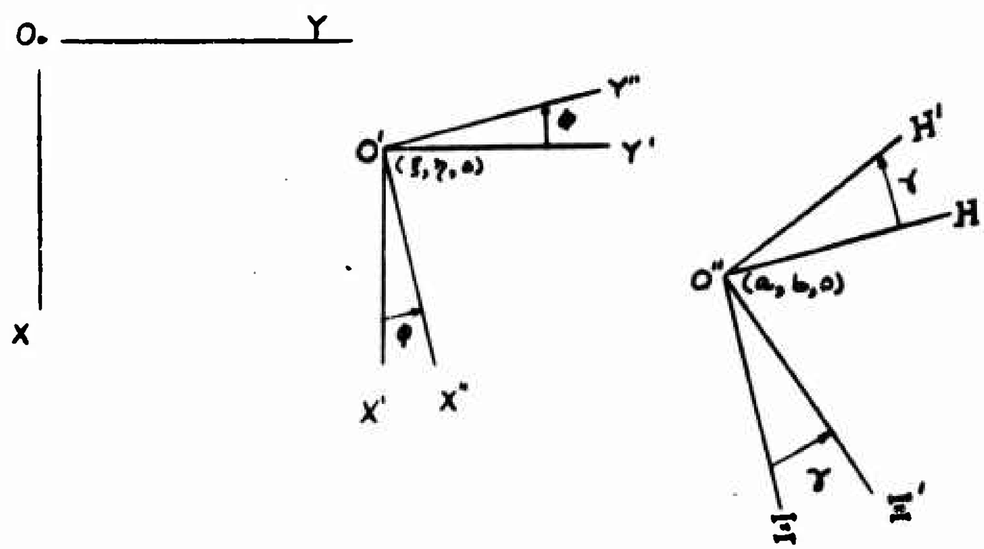


Fig. 2

3. Applied Load

Since $R(t)$, the applied load, acts in the direction of $O''H'$,

$$\begin{bmatrix} R_x \\ R_y \\ R_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\bar{\gamma} & \sin\bar{\gamma} & 0 \\ -\sin\bar{\gamma} & \cos\bar{\gamma} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ R(t) \\ 0 \end{bmatrix}$$

So

$$R_x = R(t)\,Sin(\bar{\gamma}-\phi) \qquad R_y = R(t)\,Cos(\bar{\gamma}-\phi)$$

where $\bar{\gamma}$ is the angle of elevation.

The moment about the mass center due to $R(t)$ is

$$M_\phi = R(t)[a\,Cos\bar{\gamma} - b\,Sin\bar{\gamma}]$$

4. Buoyant Forces

The buoyant force is equal to the weight of the displaced fluid and its line of action is vertical through the center of mass of the displaced fluid.

Assume a deflected position of the fluid mass as shown in Fig. 3. Now

$$x_A = r_1\,Cos(\alpha_1+\phi) \qquad y_A = r_1\,Sin(\alpha_1+\phi)$$

$$x_B = r_2\,Cos(\alpha_2+\phi) \qquad y_B = r_2\,Sin(\alpha_2+\phi)$$

$$x_C = r_3\,Cos(\alpha_3-\phi) \qquad y_C = r_3\,Sin(\alpha_3-\phi)$$

$$x_D = r_4\,Cos(\alpha_4-\phi) \qquad y_D = r_4\,Sin(\alpha_4-\phi)$$

Vertical depth of water line $= h-\xi$

$$FH = r_4\,Cos(\alpha_4-\phi) - (h-\xi) \qquad EI = r_1\,Cos(\alpha+\phi) - (h-\xi)$$

Also

$$x_E = h-\xi \qquad y_E = r_1\,Sin(\alpha_1+\phi) - [r_1\,Cos(\alpha_1+\phi) - (h-\xi)]\tan\phi$$

let

$$A = r_1\,Cos\,\alpha_1 = h+\xi_0 \qquad B = r_1\,Sin\,\alpha_1 = L+q$$

so

$$x_A = r_1\,Cos(\alpha_1+\phi) = r_1\,Cos\alpha_1\,Cos\phi - r_1\,Sin\alpha_1\,Sin\phi = A\,Cos\phi - B\,Sin\phi$$

$$x_E = h-\xi$$

$$y_A = r_1\,Sin(\alpha_1+\phi) = r_1\,Sin\alpha_1\,Cos\phi + r_1\,Sin\phi\,Cos\alpha_1 = A\,Sin\phi + B\,Cos\phi$$

$$y_E = r_1\,Sin(\alpha_1+\phi) - [r_1\,Cos(\alpha_1+\phi) - (h-\xi)]Tan\phi$$

$$= A\,Sin\phi + B\,Cos\phi - [A\,Cos\phi - B\,Sin\phi - (h-\xi)]\,Tan\phi$$

$$= A\,Sin\phi + B\,Cos\phi - A\,Sin\phi + B\,Sin\phi\,Tan\phi + (h-\xi)\,Tan\phi$$

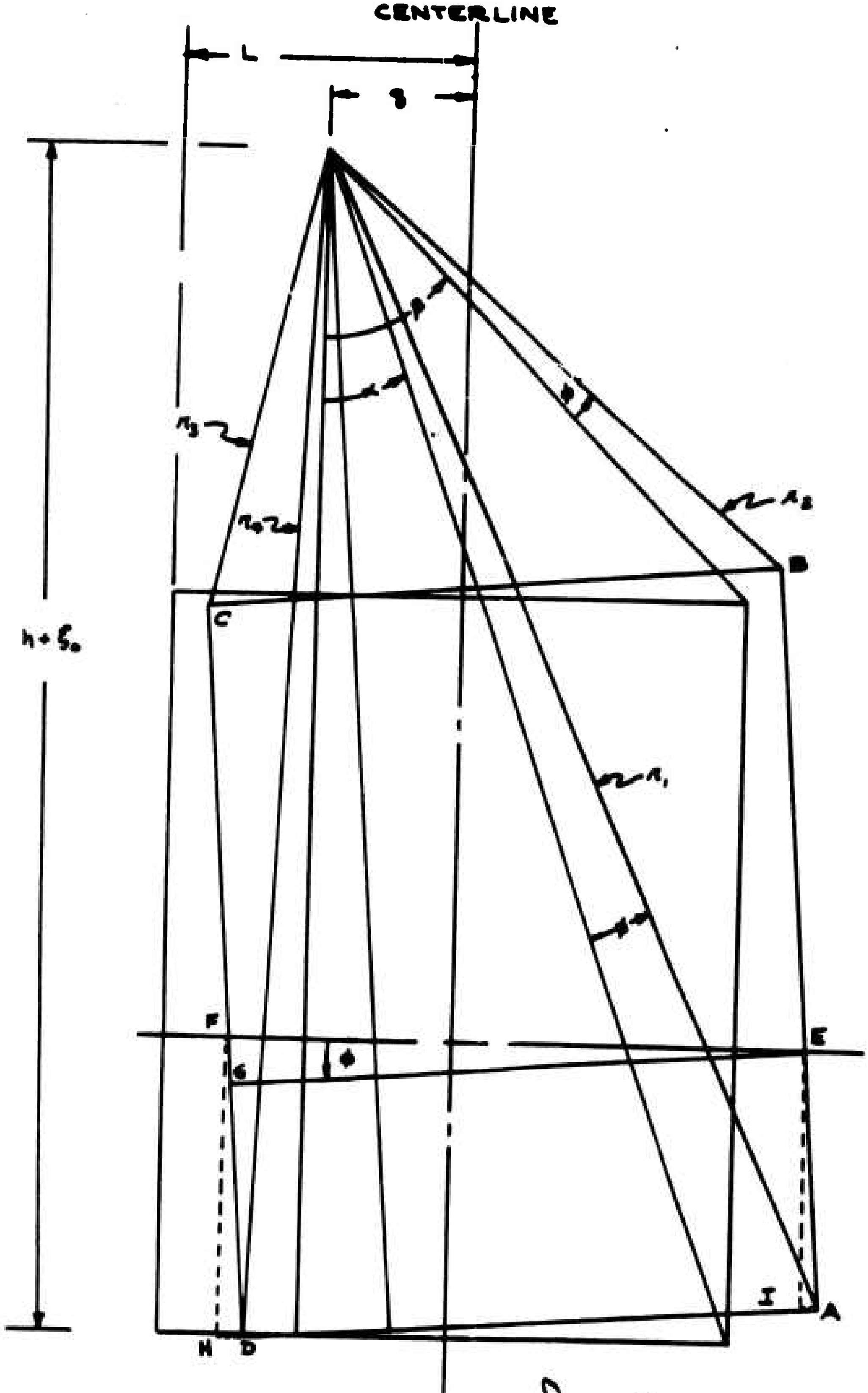


fig 3.

12

or

$$y_E = B\,\mathrm{Sec}\,\phi + (h-\xi)\,\mathrm{Tan}\,\phi$$

Now

$$DA = 2L \qquad EA = \{(x_A - x_E)^2 + (y_A - y_E)^2\}^{\frac{1}{2}} = A - B\tan\phi - (h-\xi)\,\mathrm{Sec}\,\phi$$

Define the coordinate system $E - x'y'$ by

$$x = x'\,\mathrm{Cos}\,\phi - y'\,\mathrm{Sin}\,\phi + (h-\xi)$$

$$y = x'\,\mathrm{Sin}\,\phi + y'\,\mathrm{Cos}\,\phi + y_E$$

so

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \mathrm{Cos}\,\phi & -\mathrm{Sin}\,\phi \\ \mathrm{Sin}\,\phi & \mathrm{Cos}\,\phi \end{bmatrix} \cdot \begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} h-\xi \\ y_E \end{bmatrix}$$

Now

$$x'_A = \overline{EA} \qquad y'_A = 0$$

$$x'_D = \overline{EA} \qquad y'_D = -2L$$

$$x'_E = 0 \qquad y'_E = 0$$

$$x'_F = -2L\,\mathrm{Tan}\,\phi \qquad y'_F = -2L$$

$$x'_G = 0 \qquad y'_G = -2L$$

thus

$$x'_\triangle = \tfrac{1}{3}\,\overline{FG} = \tfrac{1}{3}(-2L\,\mathrm{Tan}\,\phi)$$

$$= -\tfrac{2}{3}L\,\mathrm{Tan}\,\phi$$

$$y'_\triangle = \tfrac{2}{3}\,\overline{GE} = \tfrac{2}{3}(-2L)$$

$$= -\tfrac{4}{3}L$$

$$x'_\square = \tfrac{1}{2}\,\overline{EA}$$

$$y'_\square = \tfrac{1}{2}\,\overline{GE} = \tfrac{1}{2}(-2L)$$

$$= -L$$

So

$$y_\triangle = -\tfrac{2}{3}L\,\mathrm{Sin}\,\phi\,\mathrm{Tan}\,\phi - \tfrac{4}{3}L\,\mathrm{Cos}\,\phi + y_E$$

$$y_\square = \tfrac{1}{2}\,\overline{EA}\,\mathrm{Sin}\,\phi - L\,\mathrm{Cos}\,\phi + y_E$$

13

Also

$$\text{Area}_{\Delta} = \tfrac{1}{2}(\overline{GE}\cdot\overline{GF})$$

$$= \tfrac{1}{2}(-2L\,\mathrm{Tan}\,\phi)(-2L)$$

$$= 2L^2\,\mathrm{Tan}\,\phi$$

$$\text{Area}_{\square} = \overline{DA}\cdot\overline{EA}$$

$$= 2L\cdot\overline{EA}$$

$$= 2L(A - B\,\mathrm{Tan}\,\phi - [h-\xi]\,\mathrm{Sec}\,\phi)$$

and

$$C.G. = \frac{\text{Area}_{\Delta}\cdot y_{\Delta} + \text{Area}_{\square}\cdot y_{\square}}{\text{Area}_{\Delta} + \text{Area}_{\square}}$$

The buoyant force is

$$F(\phi) = 2L'(\text{Area}_{\Delta} + \text{Area}_{\square})\sigma$$

$$= 4LL'\left[A + (L-B)\,\mathrm{Tan}\,\phi - (h-\xi)\,\mathrm{Sec}\,\phi\right]\sigma$$

and the moment due to the force is

$$M(\phi) = F(\phi)\cdot(C.G) = 2L'\left[\text{Area}_{\Delta}\cdot y_{\Delta} + \text{Area}_{\square}\cdot y_{\square}\right]\sigma$$

$$= 2L'\sigma\Big\{2L^2\,\mathrm{Tan}\,\phi\left[-\tfrac{2}{3}L\,\mathrm{Sin}\,\phi\,\mathrm{Tan}\,\phi - \tfrac{4}{3}L\,\mathrm{Cos}\,\phi + B\,\mathrm{Sec}\,\phi + (h-\xi)\,\mathrm{Tan}\,\phi\right]$$

$$+ 2L(A - B\,\mathrm{Tan}\,\phi - [h-\xi]\,\mathrm{Sec}\,\phi)\Big(\tfrac{1}{2}[A - B\,\mathrm{Tan}\,\phi - (h-\xi)\,\mathrm{Sec}\,\phi]\,\mathrm{Sin}\,\phi$$

$$- L\,\mathrm{Cos}\,\phi + B\,\mathrm{Sec}\,\phi + (h-\xi)\,\mathrm{Tan}\,\phi\Big)\Big\}$$

$$M(\phi) = 4LL'\Big\{-\tfrac{2}{3}L^2\,\mathrm{Tan}^2\phi\,\mathrm{Sin}\,\phi - \tfrac{4}{3}L^2\,\mathrm{Cos}\,\phi\,\mathrm{Tan}\,\phi + BL\,\mathrm{Tan}\,\phi\,\mathrm{Sec}\,\phi$$

$$+ (h-\xi)L\,\mathrm{Tan}^2\phi + \tfrac{1}{2}[A - B\,\mathrm{Tan}\,\phi - (h-\xi)\,\mathrm{Sec}\,\phi]^2\,\mathrm{Sin}\,\phi$$

$$+ [A - B\,\mathrm{Tan}\,\phi - (h-\xi)\,\mathrm{Sec}\,\phi][-L\,\mathrm{Cos}\,\phi + B\,\mathrm{Sec}\,\phi + (h-\xi)\,\mathrm{Tan}\,\phi]\Big\}$$

Expanding and reducing by Trigonometric identities.

$$M(\phi) = 4LL'\Big\{\left(BL - \tfrac{2}{3}L^2 - \tfrac{B^2}{2}\right)(1+\sec^2\phi)\,\mathrm{Sin}\,\phi + (L-B)[(h-\xi)\,\mathrm{Sec}^2\phi - A\,\mathrm{Cos}\,\phi]$$

$$+ \tfrac{1}{2}\,\mathrm{Sin}\,\phi\left[A^2 - (h-\xi)^2\,\mathrm{Sec}^2\phi\right]\Big\}\sigma$$

Linearizing and substituting for A and B

$$F(\phi) = 4LL'(\xi + \xi_0 - g\phi)\sigma$$

$$M(\phi) = 4LL'\Big\{(\xi+\xi_0)(g + h\phi) + \tfrac{\phi}{6}\left[3(\xi_0^2 - \xi^2) - 2(L^2 + 3g^2)\right]\Big\}\sigma$$

5. Drag Forces

The drag force D is given by

$$D = \tfrac{1}{2} C_D \frac{\sigma}{g} 2L' [\xi + \xi_0 - (L+q)\phi]\dot{\eta}^2$$

or

$$D = \frac{\sigma L' C_D}{g} [\xi + \xi_0 - (L+q)\phi]\dot{\eta}^2 \quad (\text{where } C_D \sim 1.7)$$

6. Spring & Dashpot Forces

The forces resisting motion in the η-direction F_η are,

$$F_\eta = K_3 \eta + C_3 \dot{\eta}$$

The forces resisting motion in the ξ-direction F_ξ are

$$F_\xi = K_1 [\xi + \xi_0 - (\ell + q)\phi] + K_2 [\xi + \xi_0 + (\ell - q)\phi]$$
$$+ C_1 [\dot{\xi} - (\ell + q)\dot{\phi}] + C_2 [\dot{\xi} + (\ell - q)\dot{\phi}]$$

The moments resisting motion in the ϕ-direction F_ϕ are

$$F_\phi = - K_1 [\xi + \xi_0 - (\ell + q)\phi](\ell + q) + K_2 [\xi + \xi_0 + (\ell - q)\phi](\ell - q)$$
$$- C_1 [\dot{\xi} - (\ell + q)\dot{\phi}](\ell + q) + C_2 [\dot{\xi} + (\ell - q)\dot{\phi}](\ell - q)$$

7. Equations of Motion

$$M\ddot{\eta} = R(t)\cos(\bar{\gamma} - \phi) - \frac{\sigma L' C_D}{g} [\xi + \xi_0 - (L+q)\phi]\dot{\eta}^2 - K_3 \eta - C_3 \dot{\eta}$$

$$M\ddot{\xi} = R(t)\sin(\bar{\gamma} - \phi) - 4LL'(\xi + \xi_0 - q\phi)\sigma - K_1 [\xi + \xi_0 - (\ell + q)\phi]$$
$$- K_2 [\xi + \xi_0 + (\ell - q)\phi] - C_1 [\dot{\xi} - (\ell + q)\dot{\phi}] - C_2 [\dot{\xi} + (\ell - q)\dot{\phi}] + W$$

$$I\ddot{\phi} = R(t)[a\cos\bar{\gamma} - b\sin\bar{\gamma}] + 4LL'\sigma\{(\xi + \xi_0)(q + h\phi) + \tfrac{\phi}{2}[3(\xi_0^2 - \xi^2) - (L^2 + 3q^2)]\}$$
$$+ K_1 [\xi + \xi_0 - (\ell + q)\phi](\ell + q) - K_2 [\xi + \xi_0 + (\ell - q)\phi](\ell - q)$$
$$+ C_1 [\dot{\xi} - (\ell + q)\dot{\phi}](\ell + q) - C_2 [\dot{\xi} + (\ell - q)\dot{\phi}](\ell - q)$$

with initial condition on ϕ, say ϕ_i

$$\phi_i = \frac{\xi_0 [K_1(\ell + q) + 4qLL'\sigma - K_2(\ell - q)]}{-4LL'\sigma\left(\xi_0 h + \frac{\xi_0^2}{2} - \frac{L^2 + 3q^2}{3}\right) + K_1(\ell + q)^2 + K_2(\ell - q)}$$

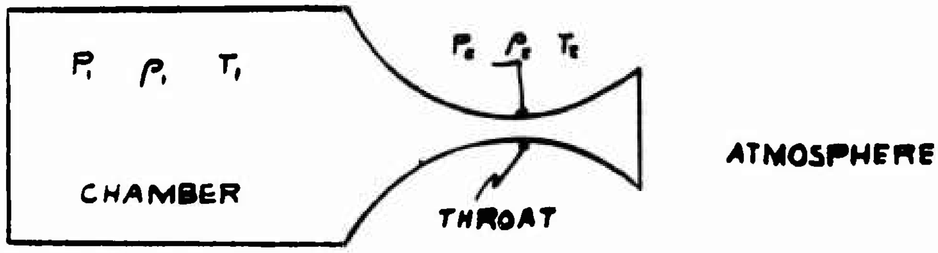
where

$$\xi_0 = \frac{W}{K_1 + K_2 + 4LL'\sigma}$$

GAS FLOW STUDIES

Problem 1:

Consider a reservoir of gas at pressure p_0, density ρ_0, and temperature T_0. Assume that the gas is to expand by an adiabatic process through a convergent-divergent nozzle to the atmosphere; and that the flow is "steady-state. Determine the mass rate of flow of the gas. Define the subscript t to refer to conditions at the throat and the subscript 1 to refer to conditions in the chamber.



The continuity equation for steady flow is given as

$$Q = \rho \mu a = \rho_t \mu_t a_t \qquad (1)$$

where Q is the mass rate of flow, a is the area of the nozzle, and μ is the velocity of the gases.

The energy equation for steady flow between the reservoir and any nozzle section is

$$C_p T + \tfrac{1}{2}\mu^2 = C_p T_1 \qquad (2)$$

where C_p is the specific heat at constant pressure.

Now, equations 3, 4, 5 hold for adiabatic flow of a perfect gas

$$C_p - C_v = R \qquad (3)$$

$$C_p / C_v = \gamma \qquad (4)$$

$$C_p = \frac{\gamma}{\gamma - 1} R \qquad (5)$$

where R is the gas constant and C_v is the specific heat at constant volume

Substituting C_p from eq. 5 into eq. 2 and solving for μ,

$$\mu = \sqrt{\frac{2\gamma}{\gamma-1} R (T_1 - T)} \qquad (6)$$

Now, the equation of state is given by

$$\frac{P}{\rho} = RT \quad (7)$$

and for adiabatic expansion this becomes

$$\frac{P}{\rho^{\gamma}} = \text{constant} \quad (8)$$

Differentiating eq. 8

$$\frac{1}{\rho^{\gamma}} dP - P\gamma \frac{1}{\rho^{\gamma+1}} d\rho = 0$$

or

$$\frac{dP}{d\rho} = \gamma \frac{P}{\rho} \quad (9)$$

and substituting from eq. 7

$$\frac{dP}{d\rho} = \gamma RT \quad (10)$$

The local velocity of sound is given by the equation

$$c = \left(\frac{dP}{d\rho}\right)^{\frac{1}{2}} \quad (11)$$

Substituting from equation 10

$$c = \sqrt{\gamma RT} \quad (12)$$

Now, equation 6 may be rewritten as

$$\frac{\gamma - 1}{2\gamma R} u^2 = T_1 - T$$

or as

$$\frac{T_1}{T} = 1 + \frac{\gamma - 1}{2\gamma RT} u^2$$

substituting from eq. 12

$$\frac{T_1}{T} = 1 + \frac{\gamma - 1}{2} \cdot \frac{u^2}{c^2} \quad (13)$$

Denote $\frac{u}{c}$ by M (Mach number) then eq. 13 becomes

$$\frac{T_1}{T} = 1 + \frac{\gamma - 1}{2} M^2 \quad (14)$$

Assume that there is no heat loss from this process (isentropic flow). For this type of flow

$$\frac{P}{P_1} = \left(\frac{\rho}{\rho_1}\right)^{\gamma} \quad (15)$$

$$\frac{T}{T_1} = \left(\frac{P}{P_1}\right)^{\frac{\gamma - 1}{\gamma}} \quad (16)$$

Substituting equations 15 and 16 into equation 14

$$\frac{P_1}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \tag{17}$$

$$\frac{\rho_1}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma-1}} \tag{18}$$

where P_1, ρ_1, and T_1 refer to conditions in the reservoir such that the flow is from P_1, ρ_1, T_1 to P, ρ, T.

Now, due to the convergent-divergent nozzle, assume that the velocity of the gases at the throat is equal to the local velocity of sound; thus, at the throat the Mach number (M) is unity. Hence equations 14, 17, and 18 become

$$\frac{T_1}{T_t} = \frac{\gamma + 1}{2} \tag{19}$$

$$\frac{P_1}{P_t} = \left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma}{\gamma-1}} \tag{20}$$

$$\frac{\rho_1}{\rho_t} = \left(\frac{\gamma + 1}{2}\right)^{\frac{1}{\gamma-1}} \tag{21}$$

From equation 6, evaluated at the throat

$$\mu_t^2 = \frac{2\gamma}{\gamma - 1} R (T_1 - T_t)$$

Now from equation 19

$$T_t = T_1 \left(\frac{2}{\gamma + 1}\right)$$

hence

$$\mu_t^2 = \frac{2\gamma}{\gamma - 1} R T_1 \frac{\gamma - 1}{\gamma + 1}$$

or

$$\mu_t^2 = \frac{2\gamma}{\gamma + 1} R T_1 \tag{22}$$

The flow equation may be derived from equation 1.

$$Q = \rho_t \mu_t a_t$$

Substituting from equation 7

$$Q = P_t (R T_t)^{-1} \mu_t a_t$$

Substituting from equations 19, 20 and 22

$$Q = \left[\gamma \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}} \frac{1}{R T_1}\right]^{\frac{1}{2}} P_1 a_t \tag{23}$$

Define U_1 = volume of chamber, then

$$Q = -U_1 \dot{\rho}_1 \tag{24}$$

From equation 8

$$P_0 \rho_0^{-\gamma} = P_1 \rho_1^{-\gamma}$$

so

$$\rho_1 = \rho_0 \left(\frac{P_1}{P_0}\right)^{\frac{1}{\gamma}} \tag{25}$$

also from equation 7 and equation 25

$$RT_1 = \frac{P_1}{\rho_1} = P_1 \rho_0^{-1} P_0^{1/\gamma} P_1^{-1/\gamma} = \rho_0^{-1} P_0^{1/\gamma} P_1^{\frac{\gamma-1}{\gamma}} \tag{26}$$

Differentiating equation 25

$$\dot{\rho}_1 = \rho_0 P_0^{-1/\gamma} \cdot \frac{1}{\gamma} P_1^{\frac{1-\gamma}{\gamma}} \dot{P}_1 \tag{27}$$

Substituting equation 27 into equation 24 and equation 26 into equation 23, and equating

$$-U_1 \rho_0 P_0^{-1/\gamma} \cdot \frac{1}{\gamma} P_1^{\frac{1-\gamma}{\gamma}} \dot{P}_1 = \left[\gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \rho_0 P_0^{-\frac{1}{\gamma}} P_1^{\frac{1-\gamma}{\gamma}}\right]^{\frac{1}{2}} P_1 a_\varepsilon$$

or

$$P_1^{\frac{1-3\gamma}{2\gamma}} dP = -\frac{a_\varepsilon}{U_1} \rho_0^{-\frac{1}{2}} P_0^{\frac{1}{2\gamma}} \gamma^{1/2} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} dt$$

Integrating

$$P_1^{\frac{1-\gamma}{2\gamma}} = \frac{\gamma-1}{2\gamma} \frac{a_\varepsilon}{U_1} \rho_0^{-\frac{1}{2}} P_0^{\frac{1}{2\gamma}} \gamma^{3/2} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} t + C$$

when $t = 0$, $P_1 = P_0$ thus

$$P_1 = \left\{P_0^{\frac{1-\gamma}{2\gamma}} + \frac{a_\varepsilon(\gamma-1)}{2U_1}\left[\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \frac{P_0}{\rho_0}\right]^{\frac{1}{2}} P_0^{\frac{1-\gamma}{2\gamma}} t\right\}^{\frac{2\gamma}{1-\gamma}}$$

or

$$P_1 = P_0 \left(1 + t/\phi\right)^{-\frac{2\gamma}{\gamma-1}} \tag{28}$$

where

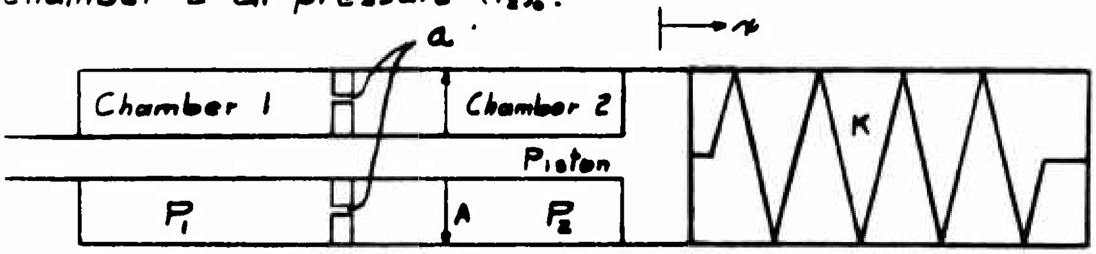
$$\phi = \frac{2U_1}{a_\varepsilon(\gamma-1)}\left[\frac{1}{\gamma}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{\gamma-1}} \frac{\rho_0}{P_0}\right]^{\frac{1}{2}} \tag{29}$$

also, it is useful to note

$$\rho_0 = P_0 (RT_0)^{-1} \tag{30}$$

Problem 2:

Define the flow of gas from Chamber 1 to Chamber 2 where Chamber 1 is initially under pressure $(P_1)_0$ and Chamber 2 at pressure $(P_2)_0$.



Define:

x = Translation of piston

U_1 = Volume available to gas in Chamber 1

$U_2 + Ax$ = Volume available to gas in Chamber 2

V = Velocity of gas flow thru a (per unit time)

Q = Mass rate of gas flow thru a (per unit time)

W = Initial weight of gas

ω = Specific weight of gas

v = Specific volume of gas

ρ = Density of gas

P = Pressure

k = Ratio of specific heats

M = Mass of piston

A = Effective gas pressure area of piston

a = Orifice Area

K = Spring Rate

R = Gas Constant divided by average molecular weight

T = Temperature

g = Gravitational Constant

Note:

The subscript 1 refers to conditions in Chamber 1

The subscript 2 refers to conditions in Chamber 2

At time t

$$\omega_1 = \frac{W_1 - \int_0^t gQ\,dt}{U_1} \qquad (1)$$

$$\omega_2 = \frac{W_2 + \int_0^t gQ\,dt}{U_2 + Ax} \qquad (2)$$

20

So

$$\rho_1 = \frac{w_1}{g} = \frac{\frac{W_1}{g} - \int_0^t Q\,dt}{U_1} \tag{3}$$

$$\rho_2 = \frac{w_2}{g} = \frac{\frac{W_2}{g} + \int_0^t Q\,dt}{U_2 + Ax} \tag{4}$$

For an adiabatic expansion

$$Pv^k = \text{Constant}$$

so for chamber 1,

$$(P_1)_0 (v_1)_0^k = P_1 v_1^k$$

or

$$\frac{P_1}{(P_1)_0} = \left(\frac{(v_1)_0}{v_1}\right)^k = \left(\frac{w_1}{(w_1)_0}\right)^k = \left(\frac{\rho_1}{(\rho_1)_0}\right)^k = \left\{\frac{W_1 - g\int_0^t Q\,dt}{W_1}\right\}^k \tag{5}$$

Similarly

$$\frac{P_2}{(P_2)_0} = \left\{\frac{\left[W_2 + g\int_0^t Q\,dt\right]U_2}{W_2(U_2 + Ax)}\right\}^k \tag{6}$$

Assuming that the length of the orifice is extremely short and no expansion of the gases occur within the orifice (incompressible flow); then,

$$V^2 = 2g\,c^2 (P_1 - P_2) v_1 \tag{7}$$

where c is the coefficient of discharge. Also

$$Q = aV\rho_1 . \tag{8}$$

The equation of motion is simply

$$M\ddot{x} = AP_2 - K(x + x_{sr}) \tag{9}$$

where $K x_{sr} = A(P_2)_0$.

Differentiating equations 3 and 4

$$Q = -U_1 \dot{\rho}_1 \tag{10}$$

$$Q = \frac{d}{dt}\left[\rho_2 (U_2 + Ax)\right] \tag{11}$$

thus

$$-U_1\, d\rho_1 = d\left[\rho_2 (U_2 + Ax)\right]$$

integrating

$$-U_1 \rho_1 = \rho_2 (U_2 + Ax) + C$$

When $x = 0$

$$\rho_1 = (\rho_1)_0 = \frac{W_1}{gU_1} \quad \text{and} \quad \rho_2 = (\rho_2)_0 = \frac{W_2}{gU_2}$$

thus

$$-\frac{W_1}{g} = \frac{W_2}{g} + C$$

so

$$C = -\frac{W_1 + W_2}{g}$$

thus

$$\rho_1 = \frac{W_1 + W_2}{gU_1} - \rho_2 \frac{U_2 + Ax}{U_1} \qquad (12)$$

Substituting equation 10 into equation 8

$$-U_1 \dot{\rho}_1 = a V \rho_1$$

or by equation 7,

$$-U_1 \dot{\rho}_1 = a \rho_1 c \left[2g v_1 (P_1 - P_2) \right]^{\frac{1}{2}}$$

and by definition of v_1 $(g v_1 = 1/\rho_1)$,

$$-U_1 \dot{\rho}_1 = a c \left[\frac{2}{\rho_1} (P_1 - P_2) \right]^{\frac{1}{2}} \rho_1$$

or

$$-U_1 \dot{\rho}_1 = a c \left[2 \rho_1 (P_1 - P_2) \right]^{\frac{1}{2}} \qquad (13)$$

but

$$P_1 = B_1 \rho_1^{k} \qquad P_2 = B_2 \rho_2^{k} \qquad (14)$$

where (eq. 5)

$$B_1 = (P_1)_0 \left(\frac{gU_1}{W_1} \right)^{k} \qquad B_2 = (P_2)_0 \left(\frac{gU_2}{W_2} \right)^{k} \qquad (15)$$

Substituting eq's 14 into eq's 9 and 13 and rewriting eq. 12,

$$M \ddot{x} = A B_2 \rho_2^{k} - K (x + x_{sr}) \qquad (16)$$

$$-U_1 \dot{\rho}_1 = a c \left[2 \rho_1 (B_1 \rho_1^{k} - B_2 \rho_2^{k}) \right]^{\frac{1}{2}} \qquad (17)$$

$$\rho_1 = \frac{W_1 + W_2}{gU_1} - \rho_2 \frac{U_2 + Ax}{U_1} \qquad (18)$$

where

$$B_1 = (P_1)_0 \left(\frac{gU_1}{W_1} \right)^{k} \qquad B_2 = (P_2)_0 \left(\frac{gU_2}{W_2} \right)^{k}$$

and P_1 and P_2 may be computed from equations 14. Equations 16, 17, 18 are the desired equations for the system.

Assume expansion of the gases exists in the orifice. Now, it is necessary to redefine the flow of gas through the orifice, a. The energy equation for an element of gas in the orifice is:

$$\frac{V^2}{2} = \frac{kRT_1}{k-1}\left(1 - \frac{T}{T_1}\right) \tag{19}$$

and the equation of state of the gas is

$$P = \rho R T \tag{20}$$

Thus one can write equation 19 as

$$V^2 = \frac{2k}{k-1}\left(\frac{P_1}{\rho_1} - \frac{P_2}{\rho_2}\right) \tag{21}$$

Now at the exit of the orifice and by use of eq.'s 14

$$V_2 = \left\{\frac{2k}{k-1}\left(B_1\rho_1^{k-1} - B_2\rho_2^{k-1}\right)\right\}^{\frac{1}{2}} \tag{22}$$

and since

$$Q = a V_2 \rho_2 \tag{23}$$

then by eq. 10

$$-U_1\dot{\rho}_1 = a\rho_2\left\{\frac{2k}{k-1}\left(B_1\rho_1^{k-1} - B_2\rho_2^{k-1}\right)\right\}^{\frac{1}{2}} \tag{24}$$

Now the desired equations are

$$M\ddot{x} = AB_2\rho_2^{k} - K(x + x_{st}) \tag{25}$$

$$-U_1\dot{\rho}_1 = a\rho_2\left\{\frac{2k}{k-1}\left(B_1\rho_1^{k-1} - B_2\rho_2^{k-1}\right)\right\}^{\frac{1}{2}} \tag{26}$$

$$\rho_1 = \frac{W_1 + W_2}{gU_1} - \rho_2\frac{U_2 + Ax}{U_1} \tag{27}$$

PROBLEM

Determine the relationship between momentum index (B) and intrinsic efficiency (I.E.)

SOLUTION

Define

W_o	= Weight of recoiling parts without brake
W_B	= Weight of muzzle brake
$W_o + W_B$	= Weight of recoiling parts with brake
V_1	= Maximum free recoil velocity (occurring at end of gas ejection period) of $W_o + W_B$
V_2	= Maximum free recoil velocity (occurring at end of gas ejection period) of W_o
v_1	= Free recoil velocity at end of in-bore period of $W_o + W_B$
v_2	= Free recoil velocity at end of in-bore period of W_o
I_1	= Impulse of in-bore period acting on recoiling parts
I	= Total impulse acting on recoiling parts without brake.

By definition

$$B = 1 - \frac{(W_o + W_B)(V_1 - v_1)}{W_o(V_2 - v_2)} \qquad 1$$

$$I.E. = 1 - \frac{W_o + W_B}{W_o} \cdot \frac{(W_o + W_B)V_1^2}{W_o V_2^2} = 1 - \left[\frac{(W_o + W_B)V_1}{W_o V_2}\right]^2 \qquad 2$$

but

$$(W_o + W_B)v_1 = W_o v_2$$

thus

$$B = 1 - \frac{(W_o + W_B)(V_1 - v_1)}{W_o V_2 - (W_o + W_B)v_1}$$

or

$$B = 1 - \frac{(W_o + W_B)V_1 - C}{W_o V_2 - C} \qquad 3$$

where

$$C = (W_o + W_B)v_1 \qquad 4$$

From eq. 2

$$(W_0 + W_B)V_1 = W_0 V_2 \sqrt{1 - I.E.}$$

Substituting this equation into eq. 3

$$B = 1 - \frac{W_0 V_2 \sqrt{1 - I.E.} - C}{W_0 V_2 - C}$$

or

$$(1 - B)(W_0 V_2 - C) = W_0 V_2 \sqrt{1 - I.E.} - C$$

expanding

$$W_0 V_2 - C - B W_0 V_2 + BC = W_0 V_2 \sqrt{1 - I.E.} - C$$

or

$$W_0 V_2 + B(C - W_0 V_2) = W_0 V_2 \sqrt{1 - I.E.}$$

Define

$$D = \frac{C}{W_0 V_2} = \frac{(W_0 + W_B)V_1}{W_0 V_2}$$

then

$$1 + B(D - 1) = \sqrt{1 - I.E.}$$

Squaring both sides

$$1 + 2B(D-1) + B^2(D-1)^2 = 1 - I.E.$$

thus

$$I.E. = -2B(D-1) - B^2(D-1)^2$$

or

$$I.E. = 2B(1-D) - B^2(1-D)^2$$

Let

$$K = (1-D)B$$

then

$$I.E. = K(2-K) \qquad 5$$

where

$$K = \left[1 - \frac{(W_0 + W_B)V_1}{W_0 V_2}\right]B = \left(1 - \frac{W_0 V_3}{W_0 V_2}\right)B = \left(1 - \frac{I_1}{I}\right)B \qquad 6$$

APPENDIX A

PROPELLANT FORM FUNCTION

A1. Introduction

It is possible to express the volume of grain burned in several different ways. If we define N/C as the fraction of charge burned and f as the fraction of charge remaining then N/C is approximated by such expressions as $N/C = 1 - f$ or $N/C = K_0 + K_1 f + K_2 f^2$. The exact equation is a cubic of the form $N/C = k_0 + k_1 f + k_2 f^2 + k_3 f^3$. The cubic arises since the change in volume is proportional to a length cubed. Since it is desirable to develop a ballistic system that will utilize a computer, the cubic equation presents no serious problem and undoubtedly adds a considerable amount of precision to the final result. Hence the propellant shapes to be considered here will be in the form of a cubic.

We will use the symbolism of Ref. 1, for this Appendix. Thus, we let

N/C	=	fraction of charge burned
N	=	amount of charge burned
C	=	initial amount of charge
V_0	=	initial volume of grain
V_x	=	volume remaining at any linear distance (x)
S_0	=	initial surface of grain
S_x	=	surface of grain remaining at any linear distance (x)
V_x/V_0	=	fraction of propellant remaining ($N/C = 1 - V_x/V_0$)
x	=	linear distance burned to flame front, perpendicular to burning surface
f	=	fraction of web remaining
S_x/S_0	=	progressivity
R, r	=	larger and smaller radii, respectively, of grain geometry
W		web, or thickness
L		length
D		width

Other symbols will be defined as needed.

A2 Cord Propellant

From Figure A1 we can determine

$$S_o = 2\pi RL + 2\pi R^2 = 2\pi R(L+R)$$

$$S_x = 2\pi(R-x)(L-2x) + 2\pi(R-x)^2$$

$$= 2\pi(RL - xL - 2Rx + 2x^2 + R^2 - 2Rx + x^2)$$

$$= 2\pi[R(L+R) - (L+4R)x + 3x^2]$$

thus

$$\frac{S_x}{S_o} = \frac{2\pi[R(L+R) - (L+4R)x + 3x^2]}{2\pi R(L+R)}$$

$$\frac{S_x}{S_o} = 1 - \frac{4R+L}{R^2+RL}x + \frac{3}{R^2+RL}x^2 \qquad (A1)$$

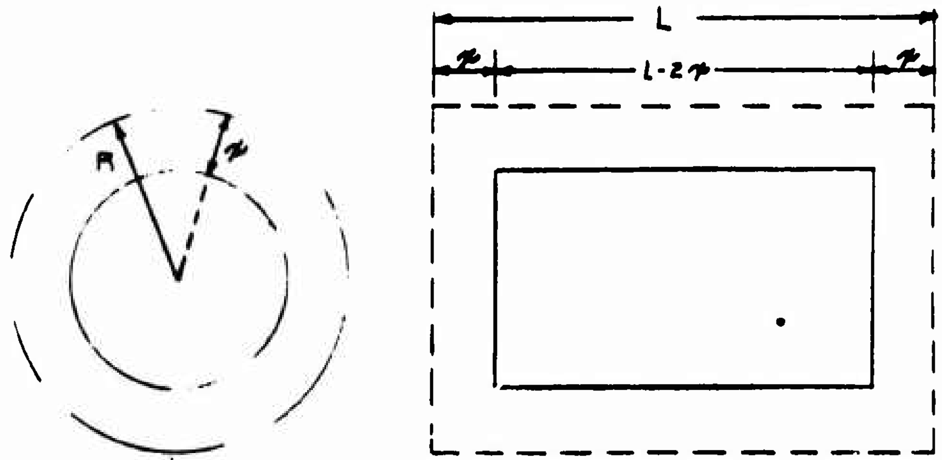


Fig. A1

also

$$V_o = \pi R^2 L$$

$$V_x = \pi(R-x)^2(L-2x) = \pi(R^2 - 2Rx + x^2)(L-2x)$$

$$= \pi(R^2L - 2R^2x - 2RxL + 4Rx^2 + Lx^2 - 2x^3)$$

$$= \pi[R^2L - (2R^2 + 2RL)x + (4R+L)x^2 - 2x^3]$$

so that

$$\frac{V_x}{V_o} = \frac{\pi[R^2L - (2R^2 + 2RL)x + (4R+L)x^2 - 2x^3]}{\pi R^2 L}$$

$$\frac{V_x}{V_o} = 1 - \frac{2(R^2+RL)}{R^2L}x + \frac{4R+L}{R^2L}x^2 - \frac{2}{R^2L}x^3 \qquad (A2)$$

Now

$$\frac{N}{C}(x) = 1 - \frac{V_x}{V_0} = 1 - 1 + \frac{2(R^2+RL)}{R^2L}x - \frac{4R+L}{R^2L}x^2 + \frac{2}{R^2L}x^3$$

$$\frac{N}{C}(x) = \frac{2(R^2+RL)}{R^2L}x - \frac{4R+L}{R^2L}x^2 + \frac{2}{R^2L}x^3 \qquad \text{(A3)}$$

and since

$$x = R(1-\xi)$$

write eq. A3 as:

$$\frac{N}{C}(\xi) = \frac{2(R^2+RL)}{R^2L}R(1-\xi) - \frac{4R+L}{R^2L}R^2(1-\xi)^2 + \frac{2}{R^2L}R^3(1-\xi)^3$$

$$\frac{N}{C}(\xi) = \frac{2}{L}(R+L)(1-\xi) - \frac{4R+L}{L}(1-2\xi+\xi^2) + \frac{2R}{L}(1-3\xi+3\xi^2-\xi^3)$$

$$\frac{N}{C}(\xi) = \frac{2R}{L} + 2 - \frac{2R}{L}\xi - 2\xi - \frac{4R+L}{L} + \frac{8R+2L}{L}\xi - \frac{4R+L}{L}\xi^2 + \frac{2R}{L}$$
$$- \frac{6R}{L}\xi + \frac{6R}{L}\xi^2 - \frac{2R}{L}\xi^3$$

$$\frac{N}{C}(\xi) = \left(\frac{2R}{L} + 2 - \frac{4R}{L} - 1 + \frac{2R}{L}\right) + \left(-\frac{2R}{L} - 2 + \frac{8R}{L} + 2 - \frac{6R}{L}\right)\xi$$
$$+ \left(\frac{6R}{L} - \frac{4R}{L} - 1\right)\xi^2 - \frac{2R}{L}\xi^3$$

or

$$\frac{N}{C}(\xi) = 1 + \frac{2R-L}{L}\xi^2 - \frac{2R}{L}\xi^3 \qquad \text{(A4)}$$

A3 Single-Perforated Propellant

From Fig. A2, we find

$$S_0 = 2\pi(R^2 - r^2) + 2\pi(R+r)L$$

$$S_x = 2\pi\left[(R-x)^2 - (r+x)^2\right] + 2\pi\left[(R-x)(L-2x) + (r+x)(L-2x)\right]$$

$$= 2\pi\left[R^2 - 2Rx + x^2 - r^2 - 2rx - x^2 + RL - xL - 2Rx + 2x^2 + rL + xL - 2rx - 2x^2\right]$$

$$= 2\pi\left[R^2 - r^2 + L(R+r)\right] - 8\pi(R+r)x$$

thus

$$\frac{S_x}{S_0} = \frac{2\pi\left[R^2 - r^2 + L(R+r) - 4(R+r)x\right]}{2\pi\left[R^2 - r^2 + L(R+r)\right]} = 1 - \frac{4(R+r)x}{R^2 - r^2 + L(R+r)}$$

$$\frac{S_x}{S_0} = 1 - \frac{4}{R - r + L}x \qquad \text{(A5)}$$

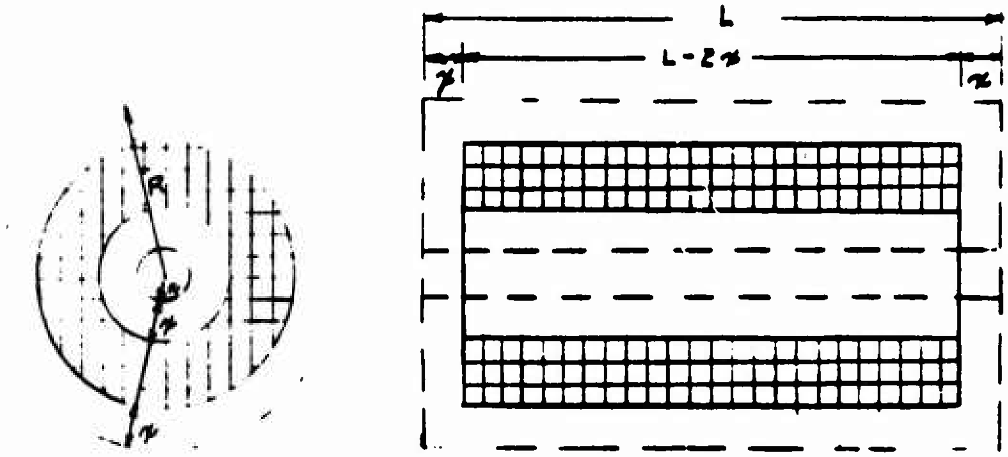


Fig. A2

also

$$V_0 = \pi(R^2 - r^2)L$$

$$V_x = \pi[(R-x)^2 - (r+x)^2](L-2x)$$

$$= \pi(R^2 - 2Rx + x^2 - r^2 - 2rx - x^2)(L-2x)$$

$$= \pi(R^2L - 2R^2x - 2RLx + 4Rx^2 - Lr^2 + 2r^2x - 2rLx + 4rx^2)$$

$$= \pi L(R^2 - r^2) + 2\pi(r^2 - R^2 - RL - rL)x + 4\pi(R+r)x^2$$

so that

$$\frac{V_x}{V_0} = \frac{\pi L(R^2 - r^2) + 2\pi[(R+r)(r-R-L)]x + 4\pi(R+r)x^2}{\pi(R^2 - r^2)L}$$

$$\frac{V_x}{V_0} = 1 - \frac{2(R-r+L)x}{(R-r)L} + \frac{4x^2}{(R-r)L} \qquad \text{(A6)}$$

Now,

$$\frac{N}{C}(x) = 1 - \frac{V_x}{V_0}$$

$$\frac{N}{C}(x) = \frac{2(R-r+L)x}{(R-r)L} - \frac{4x^2}{(R-r)L} \qquad \text{(A7)}$$

and since

$$R - x - (r+x) = (R-r)\xi$$

$$R - r - 2x = (R - r)f$$

so

$$x = \tfrac{1}{2}(R - r)(1 - f)$$

Substituting this expression into eq. A7

$$\frac{N}{C}(f) = \frac{2(R - r + L)}{(R - r)L} \cdot \frac{1}{2}(R - r)(1 - f) - \frac{4}{(R - r)L} \cdot \frac{1}{4}(R - r)^2 (1 - f)^2$$

$$\frac{N}{C}(f) = \frac{R - r + L}{L}(1 - f) - \frac{R - r}{L}(1 - 2f + f^2)$$

$$\frac{N}{C}(f) = \frac{R - r + L}{L} - \frac{R - r + L}{L}f - \frac{R - r}{L} + \frac{2(R - r)}{L}f - \frac{R - r}{L}f^2$$

$$\frac{N}{C}(f) = \frac{1}{L}(R - r + L - R + r) - \frac{1}{L}(R - r + L - 2R + 2r)f - \frac{1}{L}(R - r)f^2$$

$$\frac{N}{C}(f) = 1 + \frac{R - r - L}{L}f - \frac{R - r}{L}f^2 \qquad \text{(A8)}$$

A4 Seven Perforated Propellant

From Fig. A3

$$S_0 = 2\pi(R + 7r)L + 2\pi(R^2 - 7r^2)$$

$$S_x = 2\pi\left[(R - x) + 7(r + x)\right](L - 2x) + (R - x)^2 - 7(r + x)^2\right]$$

$$S_x = 2\pi\left[RL - 2Rx - Lx + 2x^2 + 7rL + 7xL - 14rx - 14x^2 + R^2 - 2Rx + x^2 - 7r^2 - 14rx - 7x^2\right]$$

$$S_x = 2\pi\left[(RL + 7rL + R^2 - 7r^2) + (-2R - L + 7L - 14r - 2R - 14r)x + (2 - 14 + 1 - 7)x^2\right]$$

$$S_x = 2\pi\left\{\left[(R^2 - 7r^2) + L(R + 7r)\right] - \left[4(R + 7r) - 6L\right]x - 18x^2\right\}$$

thus

$$\frac{S_x}{S_0} = \frac{2\pi\left\{\left[(R^2 - 7r^2) + L(R + 7r)\right] - \left[4(R + 7r) - 6L\right]x - 18x^2\right\}}{2\pi\left[(R + 7r)L + (R^2 - 7r^2)\right]}$$

$$\frac{S_x}{S_0} = 1 - \frac{4(R + 7r) - 6L}{(R^2 - 7r^2) + L(R + 7r)}x - \frac{18x^2}{(R^2 - 7r^2) + L(R + 7r)} \qquad \text{(A9)}$$

also

$$V_0 = \pi(R^2 - 7r^2)L$$

$$V_x = \pi\left[(R - x)^2 - 7(r + x)^2\right](L - 2x)$$

or

$$V_x = \pi[R^2 - 2Rx + x^2 - 7r^2 - 14rx - 7x^2](L - 2x)$$

$$V_x = \pi[R^2L - 2RxL + x^2L - 7r^2L - 14rxL - 7x^2L - 2R^2x + 4Rx^2 - 2x^3 + 14xr^2 + 28rx^2 + 14x^3]$$

$$V_x = \pi[(R^2L - 7r^2L) + (-2RL - 14rL - 2R^2 + 14r^2)x + (L - 7L + 4R + 28r)x^2 + 12x^3$$

$$V_x = \pi\{(R^2 - 7r^2)L - 2[(R + 7r)L + (R^2 - 7r^2)]x + [4(R + 7r) - 6L]x^2 + 12x^3\}$$

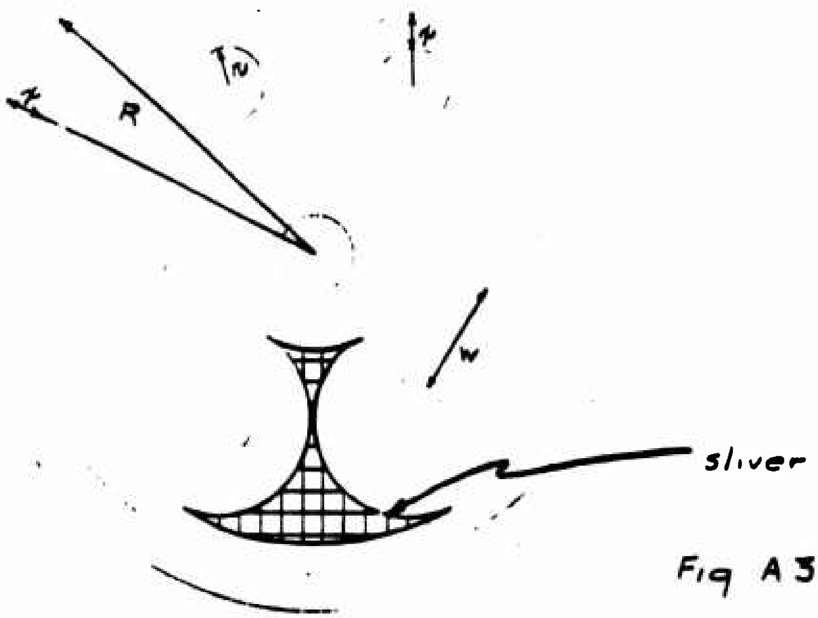


Fig A3

$$\frac{V_x}{V_0} = \frac{\pi\{(R^2 - 7r^2)L - 2[(R + 7r)L + (R^2 - 7r^2)]x + [4(R + 7r) - 6L]x^2 + 12x^3\}}{\pi(R^2 - 7r^2)L}$$

$$\frac{V_x}{V_0} = 1 - \frac{2[(R + 7r)L + (R^2 - 7r^2)]}{(R^2 - 7r^2)L}x + \frac{4(R + 7r) - 6L}{(R^2 - 7r^2)L}x^2 + \frac{12}{(R^2 - 7r^2)L}x^3 \qquad (A10)$$

Now

$$\frac{N}{C}(x) = 1 - \frac{V_x}{V_0}$$

$$\frac{N}{C}(x) = \frac{2[(R + 7r)L + (R^2 - 7r^2)]}{(R^2 - 7r^2)L}x - \frac{4(R + 7r) - 6L}{(R^2 - 7r^2)L}x^2 - \frac{12}{(R^2 - 7r^2)L}x^3 \qquad (A11)$$

A

From Fig. A3, we note that before burning

$$R = 2W + 2r + r$$

or

$$W = \frac{1}{2}(R - 3r)$$

After burning of a distance, x

$$W\xi = \frac{1}{2}[(R-x) - 3(r+x)]$$

or

$$\xi = \frac{\frac{1}{2}[(R-x)-3(r+x)]}{\frac{1}{2}(R-3r)} = \frac{R-3r-4x}{R-3r}$$

or

$$\xi = 1 - \frac{4x}{R-3r}$$

hence

$$x = \frac{1}{4}(R-3r)(1-\xi)$$

Substituting this value of x into eq. A11,

$$\frac{N}{C}(\xi) = \frac{2[(R+7r)L + (R^2-7r^2)]}{(R^2-7r^2)L}\cdot\frac{1}{4}(R-3r)(1-\xi) - \frac{4(R+7r)-6L}{(R^2-7r^2)L}\cdot\frac{1}{16}(R-3r)^2(1-\xi)^2$$

$$- \frac{12}{(R^2-7r^2)L}\cdot\frac{1}{64}(R-3r)^3(1-\xi)^3$$

thus

$$(R^2-7r^2)L\cdot\frac{N}{C}(\xi) = \frac{1}{2}(RL+7rL+R^2-7r^2)(R-3r)(1-\xi) - \frac{1}{16}(4R+28r-6L)(R^2-6Rr+9r^2)(1-\xi)^2$$

$$- \frac{3}{16}(R^3-3R^2\cdot 3r+3R\cdot 9r^2-27r^3)(1-\xi)^3$$

$$(R^2-7r^2)L\cdot\frac{N}{C}(\xi) = \frac{1}{2}(RL+7rL+R^2-7r^2)(R-3r-R\xi+3r\xi)$$

$$- \frac{1}{16}(4R+28r-6L)(R^2-6Rr+9r^2)(1-2\xi+\xi^2)$$

$$- \frac{3}{16}(R^3-3R^2\cdot 3r+27Rr^2-27r^3)(1-3\xi+3\xi^2-\xi^3)$$

$$(R^2-7r^2)L\cdot\frac{N}{C}(\xi) = \frac{1}{2}(RL+7rL+R^2-7r^2)(R-3r-R\xi+3r\xi)$$

$$-\frac{1}{16}(4R^3-24R^2r+36Rr^2+28rR^2-168Rr^2+252r^3$$

$$-6LR^2+36RrL-54r^2L)(1-2\xi+\xi^2)$$

$$-\frac{3}{16}(R^3-9R^2r+27Rr^2-27r^3)(1-3\xi+3\xi^2-\xi^3)$$

AT

Expanding

$$(R^2-7r^2)L\cdot\frac{N}{C}(\xi) = \frac{1}{2}(R^2L-3RrL-R^2L\xi+3RrL\xi+7RrL-21r^2L-7rRL\xi$$

$$+21r^2L\xi+R^3-3rR^2-R^3\xi+3R^2r\xi-7Rr^2+21r^3+7r^2R\xi-21r^3\xi)$$

$$-\frac{1}{16}(4R^3-24R^2r+36Rr^2+28rR^2-168Rr^2+252r^3-6LR^2+36RrL$$

$$-54r^2L-8R^3\xi+48R^2r\xi-72Rr^2\xi-56rR^2\xi+336Rr^2\xi-504r^3\xi$$

$$+12LR^2\xi-72RrL\xi+108r^2L\xi+4R^3\xi^2-24R^2r\xi^2+36Rr^2\xi^2+28rR^2\xi^2$$

$$-168Rr^2\xi^2+252r^3\xi^2-6LR^2\xi^2+36RrL\xi^2-54r^2L\xi^2)-\frac{3}{16}(R^3-9R^2r$$

$$+27Rr^2-27r^3-3R^3\xi+27R^2r\xi-81Rr^2\xi+81r^3\xi+3R^3\xi^2-27R^2r\xi^2$$

$$+81Rr^2\xi^2-81r^3\xi^2-R^3\xi^3+9R^2r\xi^3-27Rr^2\xi^3+27r^3\xi^3)$$

So

$$16(R^2-7r^2)L\cdot\frac{N}{C}(\xi) = (8R^2L-24RrL+56RrL-168r^2L+8R^3-24rR^2$$

$$-56Rr^2+168r^3-4R^3+24R^2r-36Rr^2-28rR^2+168Rr^2-252r^3$$

$$+6LR^2-36RrL+54r^2L-3R^3+27R^2r-81Rr^2+81r^3)+(-8R^2L$$

$$+24RrL-56rRL+168r^2L-8R^3+24R^2r+56r^2R-168r^3+8R^3$$

$$-48R^2r+72Rr^2+56rR^2-336Rr^2+504r^3-12LR^2+72RrL$$

$$-108r^2L+9R^3-81R^2r+243Rr^2-243r^3)\xi+(-4R^3+24R^2r$$

$$-36Rr^2-28rR^2+168Rr^2-252r^3+6LR^2-36RrL+54r^2L-9R^3$$

$$+81R^2r-243Rr^2+243r^3)\xi^2+(+3R^3-27R^2r+81Rr^2-81r^3)\xi^3$$

thus

$$\frac{N}{C}(\xi) = \frac{R^3-R^2r-5Rr^2-3r^3+14R^2L-4RrL-114r^2L}{16(R^2-7r^2)L}$$

$$+\frac{9R^3-49R^2r+35Rr^2+93r^3-20R^2L+40RrL+60r^2L}{16(R^2-7r^2)L}\xi$$

$$+\frac{-13R^3+77R^2r-111Rr^2-9r^3+6R^2L-36RrL+54r^2L}{16(R^2-7r^2)L}\xi^2$$

$$+\frac{3R^3-27R^2r+81Rr^2-81r^3}{16(R^2-7r^2)L}\xi^3 \qquad (A12)$$

A5 Spherical Grain

Without the benefit of a sketch, we can write

$$S_o = 4\pi R^2$$

$$S_x = 4\pi (R-x)^2 = 4\pi (R^2 - 2Rx + x^2)$$

thus

$$\frac{S_x}{S_o} = \frac{4\pi(R^2-2Rx+x^2)}{4\pi R^2}$$

$$\frac{S_x}{S_o} = 1 - \frac{2}{R}x + \frac{1}{R^2}x^2 \qquad \text{(A13)}$$

also

$$V_o = \frac{4}{3}\pi R^3$$

$$V_x = \frac{4}{3}\pi (R-x)^3 = \frac{4}{3}\pi(R^3 - 3R^2x + 3Rx^2 - x^3)$$

so

$$\frac{V_x}{V_o} = \frac{\frac{4}{3}\pi(R^3-3R^2x+3Rx^2-x^3)}{\frac{4}{3}\pi R^3}$$

$$\frac{V_x}{V_o} = 1 - \frac{3}{R}x + \frac{3}{R^2}x^2 - \frac{1}{R^3}x^3 \qquad \text{(A14)}$$

Now

$$\frac{N}{C}(x) = 1 - \frac{V_x}{V_o} = 1 - 1 + \frac{3}{R}x - \frac{3}{R^2}x^2 + \frac{1}{R^3}x^3$$

$$\frac{N}{C}(x) = \frac{3}{R}x - \frac{3}{R^2}x^2 + \frac{1}{R^3}x^3 \qquad \text{(A15)}$$

Now

$$x = R(1-\xi)$$

so, write eq. A15 as

$$\frac{N}{C}(\xi) = \frac{3}{R}\cdot R(1-\xi) - \frac{3}{R^2}R^2(1-\xi)^2 + \frac{1}{R^3}R^3(1-\xi)^3$$

$$\frac{N}{C}(\xi) = 3(1-\xi) - 3(1-2\xi+\xi^2) + 1 - 3\xi + 3\xi^2 - \xi^3$$

$$= 3 - 3\xi - 3 + 6\xi - 3\xi^2 + 1 - 3\xi + 3\xi^2 - \xi^3$$

$$\frac{N}{C}(\xi) = 1 - \xi^3 \qquad \text{(A16)}$$

A6 Strip Propellant

From Fig A4

$$S_0 = 2DL + 2LW + 2DW$$

$$S_0 = 2(DL + LW + DW)$$

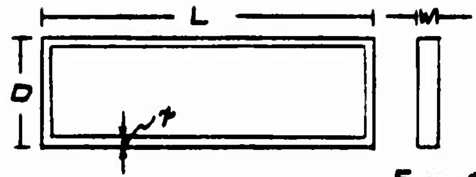


Fig A4

and

$$S_x = 2[(L-2x)(D-2x) + (L-2x)(W-2x) + (D-2x)(W-2x)]$$

$$S_x = 2[(LD + LW + DW) - 2x(D+L+L+W+D+W) + 12x^2]$$

$$S_x = 2(LD + LW + DW) - 8x(D+L+W) + 24x^2$$

so

$$\frac{S_x}{S_0} = \frac{2(LD+LW+DW) - 8x(D+L+W) + 24x^2}{2(DL+LW+DW)}$$

$$\frac{S_x}{S_0} = 1 - \frac{4(D+L+W)}{DL+LW+DW}x + \frac{12}{DL+LW+DW}x^2 \qquad \text{(A17)}$$

Now,

$$V_0 = DLW$$

$$V_x = (D-2x)(L-2x)(W-2x) = (D-2x)(LW - 2xW - 2xL + 4x^2)$$

$$= DLW - 2xWD - 2xLD + 4x^2D - 2xLW + 4x^2W + 4x^2L - 8x^3$$

$$= DLW - 2x(WD + LD + LW) + 4x^2(D+W+L) - 8x^3$$

so

$$\frac{V_x}{V_0} = \frac{DLW - 2x(WD+LD+LW) + 4x^2(D+W+L) - 8x^3}{DLW}$$

$$\frac{V_x}{V_0} = 1 - \frac{2(WD+LD+LW)}{DLW}x + \frac{4(D+W+L)}{DLW}x^2 - \frac{8}{DLW}x^3 \qquad \text{(A18)}$$

Now

$$\frac{N}{C}(x) = 1 - \frac{V_x}{V_0} = \frac{2(WD+LD+LW)}{DLW}x - \frac{4(D+W+L)}{DLW}x^2 + \frac{8}{DLW}x^3$$

$$\frac{N}{C}(x) = \frac{2(WD+LD+LW)}{DLW}x - \frac{4(D+W+L)}{DLW}x^2 + \frac{8}{DLW}x^3 \qquad \text{(A19)}$$

but

$$W - 2x = fW$$

So

$$x = \tfrac{1}{2} W(1-\xi)$$

Substituting into A19

$$\frac{N}{C}(\xi) = \frac{2(WD+LD+LW)}{DLW} \cdot \frac{W}{2}(1-\xi) - \frac{4(D+W+L)}{DLW} \cdot \frac{W^2}{4}(1-\xi)^2 + \frac{8}{DLW}\frac{W^3}{8}(1-\xi)^3$$

$$= \frac{WD+LD+LW}{DL}(1-\xi) - \frac{W(D+W+L)}{DL}(1-2\xi+\xi^2) + \frac{W^2}{DL}(1-3\xi+3\xi^2-\xi^3)$$

$$= \frac{WD+LD+LW}{DL} - \frac{WD+LD+LW}{DL}\xi - \frac{WD+W^2+WL}{DL} + \frac{2WD+2W^2+2WL}{DL}\xi$$

$$- \frac{WD+W^2+WL}{DL}\xi^2 + \frac{W^2}{DL} - \frac{3W^2}{DL}\xi + \frac{3W^2}{DL}\xi^2 - \frac{W^2}{DL}\xi^3$$

$$\frac{N}{C}(\xi) = \frac{1}{DL}\left[WD+LD+LW-WD-W^2-WL+W^2\right]$$

$$+ \frac{1}{DL}\left[-WD-LD-LW+2WD+2W^2+2WL-3W^2\right]\xi$$

$$+ \frac{1}{DL}\left[-WD-W^2-WL+3W^2\right]\xi^2 - \frac{W^2}{DL}\xi^3$$

$$\frac{N}{C}(\xi) = 1 + \frac{LW+WD-LD-W^2}{DL}\xi + \frac{2W^2-WL-WD}{DL}\xi^2 - \frac{W^2}{DL}\xi^3 \qquad (A20)$$

A7. Square Propellant

We may determine the desired equations by substituting D = L into equations A17, A18, A19, and A20. Thus:

$$\frac{S_x}{S_o} = 1 - \frac{4(2L+W)}{L^2+2LW}x + \frac{12}{L^2+2LW}x^2 \qquad (A21)$$

$$\frac{V_x}{V_o} = 1 - \frac{2(2WL+L^2)}{L^2W}x + \frac{4(2L+W)}{L^2W}x^2 - \frac{8}{L^2W}x^3 \qquad (A22)$$

$$\frac{N}{C}(x) = \frac{2(2WL+L^2)}{L^2W}x - \frac{4(2L+W)}{L^2W}x^2 + \frac{8}{L^2W}x^3 \qquad (A23)$$

$$\frac{N}{C}(\xi) = 1 + \frac{2LW-L^2-W^2}{L^2}\xi + \frac{2(W^2-WL)}{L^2}\xi^2 - \frac{W^2}{L^2}\xi^3 \qquad (A24)$$

A8. Cube Propellant

Here the desired equations may be determined from eq.s A21, A22, A23, and A24 by setting W = L. Thus:

$$\frac{S_x}{S_0} = 1 - \frac{4}{L}x + \frac{4}{L^2}x^2 \tag{A25}$$

$$\frac{V_x}{V_0} = 1 - \frac{6}{L}x + \frac{12}{L^2}x^2 - \frac{8}{L^3}x^3 \tag{A26}$$

$$\frac{N}{C}(x) = \frac{6}{L}x - \frac{12}{L^2}x^2 + \frac{8}{L^3}x^3 \tag{A27}$$

$$\frac{N}{C}(f) = 1 - f^3 \tag{A28}$$

A9. Inhibitors

The geometrical form factor for a given shape may be altered by use of an inhibitor. The inhibitor prevents burning of the surface which it covers. For example, consider an inhibitor placed on the outside surface of a single-perforated propellant cylinder but, not over the ends. Thus, the burning surfaces are the two ends and the inside surface. See Fig. A5.

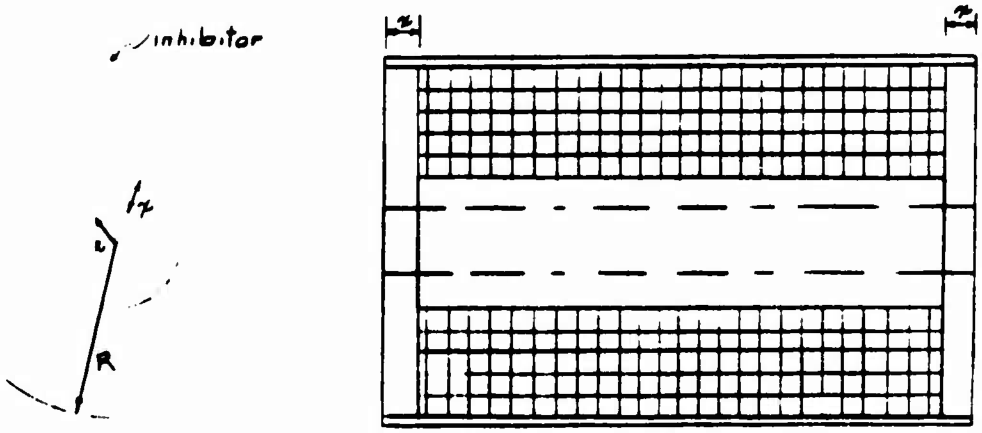


Fig. A5

The crosshatched section indicates the unburned propellant.

From Fig. A5, we find

$$S_0 = 2\pi(R^2 - r^2) + 2\pi r L$$

$$S_x = 2\pi[R^2 - (r+x)^2] + 2\pi(r+x)(L-2x)$$

$$= 2\pi[R^2 - r^2 - 2rx - x^2 + rL - 2rx + Lx - 2x^2]$$

$$= 2\pi[R^2 - r^2 + rL + x(L-4r) - 3x^2]$$

thus

$$\frac{S_x}{S_0} = \frac{2\pi[R^2 - r^2 + rL + x(L-4r) - 3x^2]}{2\pi(R^2 - r^2 + rL)}$$

$$\frac{S_x}{S_0} = 1 + \frac{L - 4r}{R^2 - r^2 + rL}x - \frac{3}{R^2 - r^2 + rL}x^2 \qquad \text{(A29)}$$

also

$$V_0 = \pi(R^2 - r^2)L$$

$$V_x = \pi[R^2 - (r+x)^2](L-2x)$$

$$= \pi[R^2L - r^2L - 2rxL - x^2L - 2R^2x + 2r^2x + 4rx^2 + 2x^3]$$

$$= \pi[(R^2-r^2)L + x(2r^2 - 2R^2 - 2rL) + (4r-L)x^2 + 2x^3]$$

so

$$\frac{V_x}{V_0} = \frac{\pi[(R^2-r^2)L - 2(R^2+rL-r^2)x + (4r-L)x^2 + 2x^3]}{\pi(R^2-r^2)L}$$

$$\frac{V_x}{V_0} = 1 - \frac{2(R^2+rL-r^2)}{(R^2-r^2)L}x + \frac{4r - L}{(R^2-r^2)L}x^2 + \frac{2}{(R^2-r^2)L}x^3$$

Now (A30)

$$\frac{N}{C} = 1 - V_x/V_0$$

$$\frac{N}{C}(x) = \frac{2(R^2+rL-r^2)}{(R^2-r^2)L}x - \frac{4r-L}{(R^2-r^2)L}x^2 - \frac{2}{(R^2-r^2)L}x^3 \qquad \text{(A31)}$$

Now

$$R - (r+x) = (R-r)f$$

$$x = (R-r)(1-f)$$

Substituting into A31

$$\frac{N}{C}(f) = \frac{2(R^2+rL-r^2)}{(R^2-r^2)L}(R-r)(1-f) - \frac{4r-L}{(R^2-r^2)L}\cdot(R-r)^2(1-f)^2 - \frac{2}{(R^2-r^2)L}(R-r)^3(1-f)^3$$

or

$$\frac{N}{C}(\xi) = \frac{2(R^2+rL-r^2)}{(R+r)L}(1-\xi) - \frac{(4r-L)(R-r)}{(R+r)L}(1-\xi)^2 - \frac{2(R-r)^2}{(R+r)L}(1-\xi)^3$$

so

$$\begin{aligned}(R+r)L\cdot\frac{N}{C}(\xi) &= 2R^2+2rL-2r^2-(2R^2+2rL-2r^2)\xi-(4Rr-4r^2-LR+Lr)\\ &\quad+2(4Rr-4r^2-LR+Lr)\xi-(4Rr-4r^2-LR+Lr)\xi^2-(2R^2-4Rr+2r^2)\\ &\quad+3(2R^2-4Rr+2r^2)\xi-3(2R^2-4Rr+2r^2)\xi^2+(2R^2-4Rr+2r^2)\xi^3\end{aligned}$$

Combining terms

$$\begin{aligned}(R+r)L\cdot\frac{N}{C}(\xi) &= (2R^2+2rL-2r^2-4Rr+4r^2+LR-Lr-2R^2+4Rr-2r^2)\\ &\quad+(-2R^2-2rL+2r^2+8Rr-8r^2-2LR+2Lr+6R^2-12Rr+6r^2)\xi\\ &\quad+(-4Rr+4r^2+LR-Lr-6R^2+12Rr-6r^2)\xi^2+2(R-r)^2\xi^3\end{aligned}$$

or

$$\begin{aligned}(R+r)L\cdot\frac{N}{C}(\xi) &= (R+r)L+(4R^2-2LR-4Rr)\xi\\ &\quad+(LR-Lr-6R^2+8Rr-2r^2)\xi^2+2(R-r)^2\xi^3\end{aligned}$$

thus

$$\frac{N}{C}(\xi) = 1+\frac{4R^2-2LR-4Rr}{(R+r)L}\xi+\frac{LR-Lr-6R^2+8Rr-2r^2}{(R+r)L}\xi^2+\frac{2(R-r)^2}{(R+r)L}\xi^3 \tag{A32}$$

APPENDIX B
THERMOCHEMISTRY OF PROPELLANTS

B1. The Atomic Composition of Propellant Constituents

We here compute the atomic composition (in gram atoms per gram of propellant) for the various chemical compounds of which the propellant is composed. Thus, we define:

$\{C\}$ = number of gram atoms of carbon per gram of propellant
$\{H\}$ = number of gram atoms of hydrogen per gram of propellant
$\{N\}$ = number of gram atoms of nitrogen per gram of propellant
$\{O\}$ = number of gram atoms of oxygen per gram of propellant

We will use M1 propellant as an example. Now, M1 propellant has constituents, by weight, of 85 parts Nitrocellulose (of which 13.15% is Nitrogen), 10 parts Dinitrotoluene, 5 parts Dibutylphthalate, 1 part Diphenylamine, 0.75 parts Ethyl Alcohol, and 0.50 parts Water. Thus:

M1 Propellant

Nitrocellulose (13.15% N)	85.00	83.13%
Dinitrotoluene	10.00	9.78%
Dibutylphthalate	5.00	4.89%
Diphenylamine	1.00	0.98%
Ethyl Alcohol	0.75	0.73%
Water	0.50	0.49%

The following atomic weights will be used:

C = 12.010 H = 1.008 N = 14.008 O = 16.000

First consider one gram molecule of water (H_2O) which consists of:

2 gram atoms of hydrogen
1 gram atom of oxygen

Thus

One Gram Molecule of H_2O = $(2)(1.008) + (1)(16.000)$
= 18.008 grams

So

$$\{H\} = \frac{2}{18.008} = 11106 \times 10^{-5} \qquad (B1)$$

$$\{O\} = \frac{1}{18.008} = 5553 \times 10^{-5} \qquad (B2)$$

For one gram molecule of ethyl alcohol (C_2H_5OH)

2 gram atoms of carbon

6 gram atoms of hydrogen

1 gram atom of oxygen

we have

One Gram Molecule of $C_2H_5OH = (2)(12.010) + (6)(1.008) + (1)(16.000)$

$= 46.068$ grams

thus

$$\{C\} = \frac{2}{46.068} = 4341 \cdot 10^{-5} \quad \text{(B3)}$$

$$\{H\} = \frac{6}{46.068} = 13024 \cdot 10^{-5} \quad \text{(B4)}$$

$$\{O\} = \frac{1}{46.068} = 2171 \cdot 10^{-5} \quad \text{(B5)}$$

For one gram molecule of diphenylamine ($C_{12}H_{11}N$)

12 gram atoms of carbon

11 gram atoms of hydrogen

1 gram atom of nitrogen

we have

One Gram Molecule of $C_{12}H_{11}N = (12)(12.010) + (11)(1.008) + (1)(14.008)$

$= 169.216$ grams

thus

$$\{C\} = \frac{12}{169.216} = 7092 \cdot 10^{-5} \quad \text{(B6)}$$

$$\{H\} = \frac{11}{169.216} = 6501 \cdot 10^{-5} \quad \text{(B7)}$$

$$\{N\} = \frac{1}{169.216} = 591 \cdot 10^{-5} \quad \text{(B8)}$$

For one gram molecule of dibutylphthalate ($C_{16}H_{22}O_4$)

16 gram atoms of carbon

22 gram atoms of hydrogen

4 gram atoms of oxygen

we have

One Gram Molecule of $C_{16}H_{22}O_4 = (16)(12.010) + (22)(1.008) + (4)(16.000)$

$= 278.336$ grams

thus

$$\{C\} = \frac{16}{278.336} = 5748 \cdot 10^{-5} \quad \text{(B9)}$$

$$\{H\} = \frac{22}{278.336} = 7904 \cdot 10^{-5} \quad \text{(B10)}$$

$$\{O\} = \frac{4}{278.336} = 1437 \cdot 10^{-5} \quad \text{(B11)}$$

Now, consider one gram molecule of dinitrotoluene ($C_7H_6N_2O_4$) which consists of

7 gram atoms of carbon
6 gram atoms of hydrogen
2 gram atoms of nitrogen
4 gram atoms of oxygen

thus

One Gram Molecule of $C_7H_6N_2O_4$ $= (7)(12.010) + (6)(1.008) + (2)(14.008) + (4)(16.000)$
$= 182.134$ grams

So

$$\{C\} = \frac{7}{182.134} = 3843 \cdot 10^{-5} \qquad (B12)$$

$$\{H\} = \frac{6}{182.134} = 3294 \cdot 10^{-5} \qquad (B13)$$

$$\{N\} = \frac{2}{182.134} = 1098 \cdot 10^{-5} \qquad (B14)$$

$$\{O\} = \frac{4}{182.134} = 2196 \cdot 10^{-5} \qquad (B15)$$

Finally, we consider the more complicated problem involving the nitrocellulose. The cellulose molecule is written as $C_6H_7O_2(OH)_3$. On nitration, some of the (OH) groups are replaced by (ONO_2) groups. The resulting compound is $C_6H_7O_2(OH)_{3-\varphi}(ONO_2)_\varphi$ where φ is the number of (OH) groups replaced by the (ONO_2) groups. Thus we have in one gram molecule of nitrocellulose $C_6H_7O_2(OH)_{3-\varphi}(ONO_2)_\varphi$:

6 gram atoms of carbon
$10-\varphi$ gram atoms of hydrogen
φ gram atoms of nitrogen
$5+2\varphi$ gram atoms of oxygen

Thus:

One Gram Molecule of $C_6H_7O_2(OH)_{3-\varphi}(ONO_2)_\varphi$
$= (6)(12.010) + (10-\varphi)(1.008) + (5+2\varphi)(16.000) + (\varphi)(14.008)$
$= 162.14 + 45\varphi$

So

$$\{C\} = \frac{6}{162.14 + 45\varphi}$$

$$\{H\} = \frac{10-\varphi}{162.14 + 45\varphi}$$

$$\{N\} = \frac{\varphi}{162.14 + 45\varphi}$$

$$\{O\} = \frac{5+2\varphi}{162.14 + 45\varphi}$$

Define

$$y = \text{per cent of nitrogen}$$

then since we have x gram atoms of nitrogen

$$y = \frac{14.008\,x}{162.14 + 45x} \cdot 100$$

or

$$y = \frac{1400.8\,x}{162.14 + 45x}$$

Now solving for x

$$162.14\,y + 45xy = 1400.8\,x$$

or

$$x = \frac{162.14\,y}{1400.8 - 45y}$$

Thus for $y = 13.15$ we have

$$x = 2.635$$

So

$$\{C\} = \frac{6}{162.14 + 45x} = 2137 \cdot 10^{-5} \qquad (B16)$$

$$\{H\} = \frac{10 - x}{162.14 + 45x} = 2624 \cdot 10^{-5} \qquad (B17)$$

$$\{N\} = \frac{x}{162.14 + 45x} = 939 \cdot 10^{-5} \qquad (B18)$$

$$\{O\} = \frac{5 + 2x}{162.14 + 45x} = 3659 \cdot 10^{-5} \qquad (B19)$$

Now, for M1 Propellant (eq.'s B1 thru B19)

$$\{C\} = [(.8313)(2137) + (.0978)(3843) + (.0489)(5748) + (.0098)(7092) + (.0073)(4541)] \cdot 10^{-5}$$

$$\{C\} = 2535 \cdot 10^{-5} \qquad (B20)$$

$$\{H\} = [(.8313)(2624) + (.0978)(3294) + (.0489)(7904) + (.0098)(6501) + (.0073)([illegible]) + (.0049)(1106)] \cdot 10^{-5}$$

$$\{H\} = 3103 \cdot 10^{-5} \qquad (B21)$$

$$\{N\} = [(.8313)(939) + (.0978)(1098) + (.0098)(581)] \cdot 10^{-5}$$

$$\{N\} = 894 \cdot 10^{-5} \qquad (B22)$$

$$\{O\} = [(.8313)(3659) + (.0978)(2196) + (.0489)(4437) + (.0073)(2171) + (.0049)(5553)] \cdot 10^{-5}$$

$$\{O\} = 3370 \cdot 10^{-5} \qquad (B23)$$

B2 Heat of Formation of the Propellant

The heats of formation of the products of combustion from graphite, hydrogen gas, nitrogen gas, and oxygen have been experimentally determined and are given by Table BI for conditions at 300 °K

TABLE BI

HEATS OF FORMATION OF THE PRODUCTS OF EXPLOSION

In kilocalories per gram molecule for formation from graphite, gaseous oxygen, hydrogen, and nitrogen

Substance	Molecular Weight	Heats of Formation (T=300 °K) At constant volume
CO	28.010	26.69
CO_2	44.010	94.03
H_2O liquid	18.016	67.43
H_2O gas	18.016	57.50
CH_4	16.042	17.27
NH_3	17.032	10.38
NO	30.008	-21.53
OH	17.008	- 9.31
N	14.008	-85.25
O	16.000	-58.75
H	1.008	-51.79

The technique of computing the heat of formation of the products of complete combustion for each propellant constituent (from graphite, hydrogen, nitrogen, oxygen) is illustrated below for MI Propellant. Note: complete combustion implies the products CO_2 and H_2O only, no CO, OH, etc. Thus

$$(CO_2) = \{C\} \qquad (H_2O) = \tfrac{1}{2}\{H\} \qquad (B24)$$

Define

HF_{PROD} = Heat of Formation of the Products of Complete Combustion of a Propellant Constituent

From Table BI we have 94030 cal./gm. as the heat of

58

TABLE B2

Constituent	Mol. Wt.	Atomic Composition in gm atoms/gm C	H	N	O	Heat of Combustion cals/gm.	HF of Products cals/gm.	HF of Substance cals/gm.
Nitrocellulose								
12.0 %N		0.02274	0.02933	0.00856	0.03608	2471	3127	656
12.2		0.02250	0.02879	0.00871	0.03617	2445	3087	642
12.4		0.02226	0.02826	0.00885	0.03626	2419	3046	627
12.6		0.02203	0.02772	0.00899	0.03635	2393	3005	612
12.8		0.02179	0.02718	0.00914	0.03643	2366	2965	599
13.0		0.02155	0.02664	0.00928	0.03652	2340	2925	585
13.15		0.02138	0.02624	0.00939	0.03658	2320	2894	574
13.2		0.02131	0.02610	0.00942	0.03661	2314	2884	570
13.25		0.02125	0.02596	0.00946	0.03663	2308	2873	565
13.4		0.02108	0.02556	0.00957	0.03669	2294	2843	549
Nitroglycerin $C_3H_5N_3O_9$	227.094	0.01321	0.02201	0.01321	0.03963	1635	1985	350
Nitroguanidine $CH_4N_4O_2$	104.074	0.00961	0.03843	0.03843	0.01921	2030	2199	169
Dinitrotoluene $C_7H_6N_2O_4$	182.134	0.03843	0.03294	0.01098	0.02196	4709	4724	15
Dibutylphthalate $C_{16}H_{22}O_4$	278.336	0.05748	0.07904	–	0.01437	7389	8070	681
Diethylphthalate $C_{12}H_{14}O_4$	222.232	0.05399	0.06299	–	0.01799	5144	7200	2056
Diphenylamine $C_{12}H_{11}N$	169.216	0.07092	0.06501	0.00591	–	9068	8860	-208
Ethyl Centralite $C_{17}H_{20}N_2O$	268.346	0.06529	0.07453	0.00745	0.03726	8418	8652	234
Ethyl Alcohol C_2H_5OH	46.068	0.04341	0.13024	–	0.02171	7111	8473	1362
Water H_2O	18.008	–	0.11106	–	0.05553	–	3744	3744
2 Nitrodiphenylamine	214.216	0.05601	0.04668	0.09336	0.09336	6976	6840	-136
Graphite C	12.010	0.8326	–	–	–	–	–	0
Carbon Black C	12.010	0.8326	–	–	–	–	–	0

formation of CO_2 and 67,430 cal/gm for H_2O (liquid). Thus

$$HF_{PROD} = 94030(CO_2) + 67430(H_2O) \qquad (B25)$$

Water (H_2O) eq. B1

$\{H\} = 0.11106$

by B24

$(H_2O) = 0.05553$

by B25

$HF_{PROD} = 3744$ cal/gm.

Ethyl Alcohol eq.'s B3 & B4

$\{C\} = 0.04341$. $\{H\} = 0.13024$

by B24

$(CO_2) = 0.04341$ $(H_2O) = 0.06512$

by B25

$HF_{PROD} = 8473$ cal/gm

Diphenylamine eq.'s B6 & B7

$\{C\} = 0.07092$ $\{H\} = 0.06501$

by B24

$(CO_2) = 0.07092$ $(H_2O) = 0.03250$

by B25

$HF_{PROD} = 8860$ cal/gm

Dibutylphthalate eq.'s B9 & B10

$\{C\} = 0.05748$ $\{H\} = 0.07804$

by B24

$(CO_2) = 0.05748$ $(H_2O) = 0.03952$

by B25

$HF_{PROD} = 8070$ cal/gm

Dinitrotoluene eq.'s B12 & B13

$\{C\} = 0.03843$ $\{H\} = 0.03294$

by B24

$(CO_2) = 0.03843$ $(H_2O) = 0.01647$

by B25

$HF_{PROD} = 4724$ cal/gm

88

Nitrocellulose (13.15% N)   eq.'s B16 & B17

$\{C\} = 0.02137$ $\{H\} = 0.02624$

by B24

$(CO_2) = 0.02137$ $(H_2O) = 0.01312$

by B25

$HF_{PROD} = 2894$ cal/gm

The heats of complete combustion of the propellant constituents (HC) have been experimentally determined. For M1 Propellant:

Water	HC = 0	
Ethyl Alcohol	HC = 7111	cal/gm
Diphenylamine	HC = 9068	cal/gm
Dibutylphthalate	HC = 7389	cal/gm
Dinitrotoluene	HC = 4709	cal/gm
Nitrocellulose (13.15% N)	HC = 2320	cal/gm

The heat of formation of a propellant constituent (HF_{CONST}) plus the heat of complete combustion of the propellant constituent (HC) is equal to the heat of formation of the products of complete combustion of the propellant constituent (HF_{PROD}). So:

$$HF_{CONST} = HF_{PROD} - HC$$

Thus we have

Water	HF_{CONST}	= 3744 - 0	= 3744
Ethyl Alcohol	HF_{CONST}	= 8473 - 7111	= 1362
Diphenylamine	HF_{CONST}	= 8860 - 9068	= - 208
Dibutylphthalate	HF_{CONST}	= 8070 - 7389	= 681
Dinitrotoluene	HF_{CONST}	= 4724 - 4709	= 15
Nitrocellulose (13.15% N)	HF_{CONST}	= 2894 - 2320	= 574

Now, we can determine the heat of formation of M1 Propellant (HF_{M1}) as:

$$HF_{M1} = (.8313)(574) + (.0978)(15) + (.0489)(681) - (.0098)(208) + (.0073)(1362) + (.0049)(3744)$$

$$HF_{M1} = 539.2 \text{ cal./gm.} \qquad (B26)$$

B3 Gas Pressure Due to the Products of Explosion

Define the equation of state of the gas as:

$$p = \frac{\eta R T}{V} \qquad \text{(B27)}$$

where p is the gas pressure (atm), V the volume per unit mass of the gas (cc/g), η the number of gram molecules in unit mass of gas, R the gas constant (82.06) and T the absolute temperature (°K).

For the conservation of atomic types

$$(N_2) = \tfrac{1}{2}\{N\} \qquad \text{(B28)}$$

$$(CO) + (CO_2) = \{C\} \qquad \text{(B29)}$$

$$(H_2) + (H_2O) = \tfrac{1}{2}\{H\} \qquad \text{(B30)}$$

$$(CO) + 2(CO_2) + (H_2O) = \{O\} \qquad \text{(B31)}$$

from which

$$\eta = \{C\} + \tfrac{1}{2}\{H\} + \tfrac{1}{2}\{N\} \qquad \text{(B32)}$$

The equilibrium constant for the water gas reaction is

$$K = \frac{(CO)(H_2O)}{(CO_2)(H_2)} \qquad \text{(B33)}$$

where K is given by the table

TABLE B3

T °K	K
1000	0.7185
1200	1.406
1400	2.212
1600	3.043
1700	3.438
1800	3.832
1900	4.206
2000	4.574
2100	4.909
2200	5.235
2300	5.533
2400	5.821
2500	6.081
2600	6.331
2700	6.532
2800	6.764
2900	6.949
3000	7.127
3100	7.281
3200	7.428
3300	7.562
3400	7.670
3500	7.764
3600	7.854
3700	7.923
3800	7.989
3900	8.037
4000	8.082

For M1 propellant

$$\{C\} = 2535 \times 10^{-5} \quad \{H\} = 3103 \times 10^{-5} \quad \{N\} = 894 \times 10^{-5} \quad \{O\} = 3370 \times 10^{-5}$$

$$\eta = 4534 \times 10^{-8}$$

Heat of Formation of Propellant = 538.2 cal./gm

Define

HF_{M1} = Heat of Formation of Propellant M1
HF_{GAS} = Heat of Formation of Gases
IE = Internal Energy of Gases
HR = Heat of Reaction = $HF_{GAS} - HF_{M1}$

It is necessary to determine T so that

$$HR - IE = 0$$

For M1 Propellant, Assume T = 2300 °K. Now, from Table B3 and equations B28, B29, B30, B31, and B33.

$$[CO_2] = 0.00250 \quad [CO] = 0.00285 \quad [H_2O] = 0.00585$$
$$[H_2] = 0.00966 \quad [N_2] = 0.00447$$

From Table B1

$$HR = (94{,}030)(0.00250) + (26{,}690)(0.02285) + (57{,}500)(0.00585) - 538.2$$
$$= 643.1$$

From Table B4

$$IE = [(11.157)(0.00250) + (6.086)(0.02285) + (8.484)(0.00585)$$
$$+ (5.[illegible]77)(0.00966) + (6.020)(0.00447)](T - 300)$$
$$= 594.8$$

So

$$HR - IE = 48.3$$

Hence the temperature (T) is incorrect.

T	$[CO_2]$	$[CO]$	$[H_2O]$	$[H_2]$	$[N_2]$	HR	I.E.	HR-IE
2300	.00250	.02285	.00585	.00966	.00447	643.1	594.8	48.3
2400	.00243	.02292	.00592	.00959	.00447	642.4	629.0	13.4
2500	.00237	.02298	.00598	.00953	.00447	641.8	663.1	-21.3
2600	.00232	.02303	.00603	.00948	.00447	641.3	697.3	-56.0

10

and by straight line interpolation

$$T = 2440 \ ^\circ K$$

which is the adiabatic flame temperature.

Also the force constant F is given by

$$F = \eta R T_0$$

$$F = 307,000 \ ft \cdot \#/\#$$

TABLE B4

Mean Molecular Heats over the Temperature Range 300 °K to T°K

In calories per gram molecule per degree at constant volume

T °K	CO_2	H_2O	CO	H_2	N_2	OH	NO	O_2
1000	9.409	6.823	5.403	5.055	5.326	5.136	5.592	5.751
1200	9.824	7.107	5.553	5.115	5.468	5.212	5.744	5.907
1400	10.165	7.388	5.684	5.189	5.597	5.298	5.869	6.037
1600	10.449	7.661	5.799	5.272	5.712	5.390	5.975	6.147
1700	10.577	7.797	5.852	5.318	5.766	5.439	6.024	6.198
1800	10.690	7.918	5.899	5.359	5.814	5.482	6.067	6.244
1900	10.798	8.044	5.945	5.405	5.861	5.529	6.110	6.290
2000	10.896	8.157	5.986	5.447	5.904	5.572	6.149	6.331
2100	10.990	8.273	6.012	5.492	5.945	5.617	6.186	6.373
2200	11.075	8.378	6.036	5.533	5.983	5.657	6.220	6.412
2300	11.157	8.484	6.086	5.577	6.020	5.700	6.252	6.452
2400	11.233	8.581	6.131	5.617	6.053	5.739	6.282	6.488
2500	11.306	8.678	6.162	5.659	6.086	5.780	6.310	6.525
2600	11.373	8.768	6.191	5.697	6.116	5.818	6.336	6.559
2700	11.438	8.857	6.218	5.736	6.146	5.856	6.361	6.594
2800	11.498	8.940	6.244	5.773	6.173	5.891	6.384	6.626
2900	11.556	9.022	6.269	5.811	6.199	5.927	6.406	6.659
3000	11.611	9.099	6.293	5.846	6.224	5.960	6.427	6.690
3100	11.664	9.174	6.315	5.882	6.248	5.993	6.448	6.722
3200	11.714	9.245	6.336	5.915	6.270	6.024	6.467	6.752
3300	11.762	9.315	6.357	5.948	6.292	6.055	6.486	6.782
3400	11.808	9.380	6.376	5.980	6.312	6.085	6.504	6.811
3500	11.853	9.444	6.395	6.012	6.332	6.115	6.521	6.840
3600	11.895	9.505	6.412	6.042	6.350	6.143	6.538	6.868
3700	11.936	9.565	6.429	6.073	6.368	6.171	6.554	6.895
3800	11.975	9.621	6.446	6.102	6.385	6.198	6.569	6.921
3900	12.013	9.677	6.462	6.131	6.402	6.225	6.585	6.947
4000	12.050	9.730	6.477	6.158	6.418	6.251	6.600	6.972

For monatomic gases, the mean molecular heats are 2.980 throughout