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UNITED STATES NAVY

PROJECT SQUID

FIELD SURVEY REPORT

FLUID MECHANICS

Volume I, Part 4

DOWNGRADED 30 June 1947

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FLUID MECHANICS

FIELD SURVEY REPORT

Volume I: RESEARCH

Part 1. Combustion	R. C. Bryant and A. W. Sloan
Part 2. Fuels	A. W. Sloan
Part 3. Materials	R. C. Bryant
Part 4. Fluid Mechanics	J. H. Wakelin
Part 5. Heat Transfer and Cooling	George Vaux
Part 6. Instrumentation	J. W. Fitzgerald

Volume II: DEVELOPMENT

Part 1. Pulse Jet Engines	F. A. Parker
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PROJECT SQUID

FLUID MECHANICS

Field Survey Report

Volume I, Part 4

by

JAMES H. WAKELIN

Engineering Research Associates, Inc.

Washington, D. C.

30 June 1947



Princeton University, the central management organization of Project SQUID, arranged for the preparation of the *Field Survey Report* under Contract Number N6ori-105, Task Order III, with the Office of Naval Research, Navy Department.

This report was prepared by the Technical Survey Group of Project SQUID as a cooperative effort of Princeton University and Engineering Research Associates, Inc. Engineering Research Associates was given primary responsibility for the preparation of these reports in accordance with the provisions of Task Order II under Purchase Order Number 08451 with Princeton University.

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FOREWORD

The *Field Survey Report* on liquid propellant rockets and pulse jet engines was prepared at the suggestion of the Policy Committee, in order that the fundamental research in Project SQUID might be related to other projects and programs of research in this field, and to problems arising in the development of rocket and pulse jet engine equipment.

In order to fulfill this purpose the *Field Survey Report* had to be more than a brief outline of the work of each contractor, but time did not permit it to be prepared as a monograph in each branch of the field of propulsion. The choice of presentation of the work in each volume of the report was governed in part by the amount of available information and by its relation to the research now being sponsored by Project SQUID.

The Policy Committee will use the *Field Survey Report* as a basis for adjustments in the research program of Project SQUID, in order to ensure a more effective attack on the fundamental problems in the field of propulsion. The Policy Committee hopes that this report may also be useful to scientists conducting research and development in fields relating to propulsion, and to members of government organizations responsible for the planning and integration of research programs in propulsion.

HUGH S. TAYLOR, Chairman
Policy Committee, Project SQUID

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PREFACE

The *Field Survey Report* was prepared by the Technical Survey Group, Project SQUID, under the direction of Engineering Research Associates, Inc.

The assembly of the material and the preparation of each part of the report was undertaken as a group effort, to which the staffs of both Princeton University and Engineering Research Associates, Inc., have contributed. Mr. F. A. Parker, Project Organizer and Mr. W. C. House, Chief Technical Aide, of the central administrative staff of Project SQUID at Princeton served as members of the Technical Survey Group and prepared Volume II. In addition, Prof. J. V. Charyk of the Aeronautical Engineering Department at Princeton visited the California Institute of Technology and furnished basic information concerning the research program there. He also offered many helpful suggestions with regard to several parts of Volume I.

In the preparation of this report the members of the Technical Survey Group have received the assistance, counsel and cooperation of representatives of the War and Navy Departments and other Government agencies, and of representatives of academic and industrial laboratories who are under contract to the government for research and development in this field.

The authors are indebted to a number of scientists who have reviewed each part of the report and have offered much constructive criticism. The authors also wish to express their appreciation for the assistance which was so generously given by representatives of the Office of Naval Research and of the Bureau of Aeronautics.

THE TECHNICAL SURVEY GROUP

CONTENTS

Foreword	v
Preface	vii
I. Summary	1
II. Recommendations	2
III. Introduction	2
IV. Propagation and Interaction of Shock Waves	4
A. Introduction	4
B. Research in Progress	5
New York University	5
Princeton University	5
V. Boundary Layer Studies	7
A. Introduction	7
B. Research in Progress	8
Studies on Subsonic Flow	8
National Bureau of Standards	8
University of Washington	8
Supersonic Studies	9
Polytechnic Institute of Brooklyn	9
Princeton University	9
California Institute of Technology	9
Massachusetts Institute of Technology	11
Marquardt Aircraft Company	11
University of Virginia	11
VI. Flow Through Ducts: Diffusers and Nozzles	12
A. Introduction	12
United Aircraft Corporation	14
Bell Aircraft Corporation	14
North American Aviation, Inc.	14
B. Research in Progress	14
1. Subsonic Diffuser Studies	14
National Advisory Committee for Aeronautics	14
National Bureau of Standards	14
University of Washington	14
2. Supersonic Diffuser Studies	15
Cornell University	15
Marquardt Aircraft Company	15
Massachusetts Institute of Technology	15
Menasco Manufacturing Company	16
National Advisory Committee for Aeronautics (Flight Propulsion Research Laboratory)	16
(a) Spike Diffusers	17
(b) Perforated Diffusers	17
(c) Variable Throat Diffusers	18
(d) Other Diffusers	18
National Advisory Committee for Aeronautics (Langley Memorial Aeronautical Laboratory)	18
National Advisory Committee for Aeronautics (Ames Aeronautical Laboratory)	18

CONTENTS — Continued

New Mexico School of Mines	18
North American Aviation, Inc.	18
Polytechnic Institute of Brooklyn	19
3. Supersonic Nozzle Studies	19
Massachusetts Institute of Technology	19
(a) Schlieren Observations of Supersonic Discharge	19
(b) Investigation of the Condensation Shock in Air by Use of the Schlieren Method	19
(c) Experimental Study of the Flow in a Nozzle Designed to Provide a Parallel Jet at Supersonic Speed	19
Naval Ordnance Laboratory	20
VII. Hydraulic Analogy Studies	20
A. Introduction	20
B. Research in Progress	21
National Advisory Committee for Aeronautics	21
Massachusetts Institute of Technology	21
New York University	22
California Institute of Technology	22
Menasco Manufacturing Company	22
Cornell University	22
North American Aviation, Inc.	22
University of Iowa	23
VIII. Turbulence	24
A. Introduction	24
B. Research in Progress	30
Massachusetts Institute of Technology	30
University of Illinois	31
Applied Physics Laboratory (Forest Grove Burner Laboratory)	31
University of Virginia	31
Battelle Memorial Institute	31
Bureau of Mines	31
Experiment, Incorporated	31
University of Delaware	31
IX. Atomization and Mixing	32
A. Introduction	32
B. Research in Progress	33
1. Atomization and Liquid Mixing	33
University of California	33
University of Illinois	33
(a) Venturi Atomization	34
(b) Entrainment of Gases	34
Massachusetts Institute of Technology	34
University of Michigan	34
Marquardt Aircraft Company	35

CONTENTS — Continued

2. Mixing of Gas Streams	35
Massachusetts Institute of Technology	35
Northwestern University	36
X. Mechanics of Non-Uniform Gases	36
A. Introduction	36
B. Research in Progress	37
California Institute of Technology	37
University of California	37
Naval Ordnance Laboratory	38
New York University	38
University of Wisconsin	38
XI. Theoretical Studies Relating to the Pulse Jet Cycle	38
A. Introduction	38
B. Research in Progress	38
Cornell University	38
New York University	39
Polytechnic Institute of Brooklyn	39
C. Acoustic Radiation Pressure	39
XII. Special Facilities in Design or Under Construction	39
Applied Physics Laboratory (Johns Hopkins University)	40
Bell Aircraft Corporation	40
California Institute of Technology	40
Johns Hopkins University (Baltimore, Maryland)	40
Massachusetts Institute of Technology	40
University of Minnesota	40
National Advisory Committee for Aeronautics	40
A. Flight Propulsion Research Laboratory	40
B. Langley Memorial Aeronautical Laboratory	41
Naval Ordnance Laboratory	41
North American Aviation, Inc.	41
Princeton University	41
University of Washington	41
XIII. Survey Studies	41
Appendix	43
A. Wind Tunnel Facilities in Existence	44
1. Subsonic	44
2. Transonic	51
3. Supersonic	53
B. Wind Tunnel Facilities in Design or Under Construction	55
References	58
Abbreviations	64

ILLUSTRATIONS

Figure 1. Sketch of (a) circular arc airfoil and (b) boundary layer separation ahead of shock wave at rear of airfoil. (After Liepmann)	10
Figure 2. Test section for studying interaction of boundary layer and shock in transonic flow. (1) upper nozzle wall, (2) lower nozzle wall, (3) auxiliary plate, (4) minimum cross section, (5) total pressure tube or static pressure probe, (6) total pressure tube adjustment in the Z-direction, and (7) total pressure tube adjustment in X-direction. (After Ackerdt, Feldmann and Rott)	10
Figure 3. Apparatus for studying interaction of boundary layer and shock waves. (Massachusetts Institute of Technology)	11
Figure 4. Sketch showing position of compression shocks in conical diffusers. (After Oswatitsch)	12
Figure 5. Maximum total pressure recovery between Mach number 1.0 and 4.0 for a normal shock and several diagonal shocks. (a) normal shock, (b) normal shock and one diagonal shock, (c) normal shock and two diagonal shocks, and (d) normal shock and three diagonal shocks. (After Oswatitsch)	13
Figure 6. Model for testing the effectiveness of a conical diffuser. (After Oswatitsch)	13
Figure 7. Reversed de Laval nozzle, (a) with isentropic flow and (b) stable supersonic diffuser flow (for circular diffuser $b^2/c^2 = C_r$ where C_r is the contraction ratio.) (After Kantrowitz and Donaldson)	13
Figure 8. Sketch of adjustable spike diffuser for ramjet operation between Mach number 1.7 and 3.0 (McNasco Manufacturing Company)	16
Figure 9. Possible shock configurations for conical diffusers operating at high Mach numbers. (After Clauser)	17
Figure 10. Relation of normal shock position to perforated area in perforated diffusers. (a) normal shock ahead of inlet, (b) normal shock partly swallowed, and (c) normal shock near throat. (National Advisory Committee for Aeronautics)	17
Figure 11. Photograph of the intersection of two linear water waves to form a Mach reflection. (California Institute of Technology)	22
Figure 12. Photograph of model of smooth curved spike diffuser for hydraulic analogy studies. (McNasco Manufacturing Company)	23
Figure 13. Photograph of hydraulic analogue to shock pattern for conical diffuser for Mach number 3.56. (North American Aviation, Inc.)	23
Figure 14. Graph showing relation between correlation coefficient R_τ and y . (After Goldstein)	26
Figure 15. Velocity profiles for turbulent flow in a pipe. Comparison of experimental data with vorticity and momentum transfer theories where U is the velocity at a distance r from the axis of the pipe, U_c is the velocity at the center of the pipe, $U_\tau = (\tau_w/\rho)^{1/2}$ and τ_w is the intensity of the wall friction. (After Goldstein)	28
Figure 16. Velocity profiles for turbulent flow in channels. Comparison of experimental data with vorticity and momentum transfer theories where U is the velocity at the normal distance y from the solid boundary, U_c is the velocity at the center of the channel, $U_\tau = (\tau_w/\rho)^{1/2}$ and τ_w is the intensity of the wall friction; h is at half-width of the channel and y' is the distance measured from the central plane of the channel. (After Goldstein)	29
Figure 17. Temperature profile in the wake behind a heated body of revolution. Comparison of experimental results with those predicted by several turbulence theories. T is the temperature at a point r from the center of the wake, T_c is the temperature at center and T_o is the temperature outside the wake. R is the value of r for which the velocity is $1/2$ the maximum velocity. (1) Momentum transfer theory, (2) Vorticity transfer theory: symmetrical turbulence: $1/r^{-1}$, (3) Vorticity transfer theory: symmetrical turbulence: $1/r^{-2}$, (4) Modified vorticity transfer theory, and (x) experimental mean results. (After Goldstein)	30
Figure 18. Sketch of apparatus for studying the mixing of two subsonic air streams. (Massachusetts Institute of Technology)	36
Figure 19. Realms of flow in fluid mechanics. (After Tsien)	37

I. SUMMARY

This report is a survey of research projects sponsored by the government in the field of compressible fluid flow, with particular reference to the bearing of these projects on aircraft propulsion devices.

The fundamental research on compressible fluid flow phenomena appears to be concentrated at the moment in the transonic and supersonic velocity regions, and is mainly concerned with the study of shock waves and their interaction with fluid and solid boundaries. Theoretical studies on transonic and supersonic flow phenomena are greatly retarded by the intractability of the mathematical equations, when the effects of heat, viscosity, and compressibility are included in three dimensions. It is quite apparent that the physical facts relating to this domain will not be adequately explained, nor the mathematical equations easily solved, until mathematical computing mechanisms of very high speed and of highly advanced design are employed on these problems.

In the field of boundary layer investigations, the research in the subsonic domain is concerned with studies of boundary layer stability and the transition of a laminar to a turbulent boundary layer. The theory and mechanism of boundary layer stability appears to be well understood in the subsonic velocity region. In the transonic and supersonic region, the work on boundary layers concerns stability studies and the interaction of boundary layer and shock waves. Boundary layer theory for transonic and supersonic velocities is not well developed; this part of the fluid flow field is one of great importance to the control and maneuverability of air missiles.

The majority of the research on diffusers and nozzles is directed toward obtaining enough fundamental information for the improvement of diffuser and nozzle design, especially in the Mach number range from 1.5 to 3.5. The current development of missiles for this speed range demands fundamental information concerning compressible flow through ducts. The development and use of hydraulic analogy techniques has been concentrated on flow problems in relation to the improvement of diffuser or nozzle design. In only one project not connected with the design of channel shapes is the hydraulic technique used for investigating the fundamentals of wave interaction in compressible fluid flow.

The phenomenon of turbulence itself is very incompletely understood. As in the case of compressible

fluid flow in the transonic or supersonic regions, a generalized theory of turbulence can be explained only through the solution of the equations of motion in three dimensions. In the case of turbulent flow, for example, the elimination of a dimension in the analysis eliminates the phenomena which the analysis is attempting to describe. It is only by chance that the one- and two-dimensional theories of turbulence can give as good an agreement with experiment as has been observed. With one or two exceptions, the effort in turbulence is concerned with its study in connection with phenomena whose mechanism is as little understood as turbulence itself. There is much interest in studying the combination of turbulence with combustion by correlating the velocity of a chemical reaction with changes in the controlled turbulence level of the fluid entering a combustion zone. It is unfortunate, however, that so much turbulent flow is created by the combustion process that the controlled turbulence introduced before combustion takes place has little apparent effect on the mechanism of the chemical reaction.

Closely associated with the macroscopic phenomena of turbulence is the mixing of fluid streams. This includes liquid-liquid mixing, liquid-gas mixing and gas-gas mixing of streams flowing at nearly the same or at greatly varying velocities. The cases of primary interest are, as one would suspect, the most complicated. These cases include the condition in which a phase change takes place, such as in the atomization of liquid fuels, or the condition in which there exists a shock front between two fluid streams with no evaporation or change in phase. Atomization processes are not well understood at all. Current work on their relation to combustion is being conducted on a purely empirical basis, in order to obtain enough experimental data to enable, or to point the way toward a valid theory for the dependence of droplet size on viscosity, surface tension, droplet velocity and other parameters describing the fluid state.

The mechanics of non-uniform gases has just begun to receive widespread attention through the requirements of understanding flight conditions at high altitudes and under conditions where the mean free path is comparable with a linear dimension of the missile.

A relatively small effort is being exerted in the application of theory to the aerodynamic and to the

combustion phenomena in a system subjected to periodic forces, such as the pulse jet.

The wind tunnel facilities now in operation or in design for transonic and supersonic research provide

only for the undertaking of small scale studies of flow phenomena through or around airfoil sections. There are no facilities for full scale testing of body or wing sections of proposed guided missiles at supersonic speeds.

II. RECOMMENDATIONS

On the basis of the present survey of research projects sponsored by government agencies in the field of fluid mechanics, the following recommendations are made:

A. That a program be formulated to provide for a general study of compressible fluid flow problems on large scale computing machines. The solution of certain differential equations for three-dimensional flow is necessary to advance basic knowledge in fluid mechanics. In order to obtain solutions to certain problems in this field the work is so laborious as to preclude the possibility of manual computation. In the field of turbulence, for example, it would be helpful to undertake a three-dimensional analysis of one of the more successful two-dimensional theories. This should serve as a guide to the development of a general theory for large scale turbulence and ought to be undertaken in conjunction with Recommendation B below.

B. That additional sponsorship be provided for the formulation of a new theoretical approach to the theory of turbulence. There is only one such project planned; this will make use of modern advances in statistics aiming at a generalized theory for large scale turbulence. The effect of turbulent transport phenomena is of basic importance to the kinetics of combustion reactions.

C. That fundamental studies of atomization and mixing of liquid streams be continued. The influence of mixing and atomization on the combustion process is not clear. A program in this field must start with a study of the simple mixing of two fluid streams with-

out combustion reactions; at a later stage these studies should be augmented by introducing a combustion reaction.

D. That the hydraulic analogy technique be investigated for more general use as a simple tool for qualitative compressible flow studies and as an educational aid to instruction in high velocity flow phenomena.

E. That the application of the theory of acoustic radiation be studied further with reference to the pulse jet engine and to the formulation of a valid theory of the pulse jet cycle.

F. That continued support be given to studies of boundary layer phenomena in the transonic and supersonic velocity range. A much better understanding of drag forces, surface heat problems, shock wave formation and interaction is essential to the advancement of the guided missiles program.

G. That the effect of high Reynolds numbers at high velocities, particularly in the supersonic range, be studied carefully before design specifications are formalized for a large supersonic wind tunnel.

H. That the design of a large supersonic wind tunnel with test section of the order of $20' \times 20'$ for studies up to Mach number 3.0 be sponsored by the government, making use of the basic information in government sponsored and other research in this field for assistance in design. This design study should point out, if possible, what the capabilities are for such a wind tunnel facility and whether it appears feasible to test full scale missiles by this method as opposed to open range testing.

III. INTRODUCTION

Fluid mechanics is treated here only insofar as the fluid particle motion affects the performance of a propulsive device. This report therefore is limited to physical phenomena relating to compressible fluid flow into, through and out of the combustion chamber.

Specifically the subjects treated include diffusers and nozzles, shock waves, the interaction of shock waves and boundary layer, turbulence, atomization and mixing, the mechanics of non-uniform gases, and pulse jet theory.

For steady flow the field of fluid mechanics may be sub-divided into regions of subsonic, transonic, and supersonic flow. The motion of fluid particles at velocities well below the velocity of sound in the medium is fairly well understood and can be treated with classical mathematical techniques developed for an incompressible fluid. In the compressible or transonic range, where the fluid particle velocity is near the velocity of sound, the effect of compressibility becomes strong enough to prevent the description of fluid motion by incompressible techniques. The flow phenomena in this velocity range are not well understood. In the supersonic region where the fluid particles are moving at speeds faster than the local velocity of sound in the medium, the most interesting phenomenon is the production and behavior of shock waves. The knowledge concerning reflection, refraction and transmission of shock waves is still in an unsatisfactory state. This condition demands careful and precise experimentation with close association of the experimental results with the theoretical studies before significant advances can be made. For general reading on the problems relating to flow in the transonic and supersonic region, references 1-17 are suggested.

One of the fundamental problems in the design of a propulsive device which requires an external working fluid for combustion, is the partial conversion of the kinetic energy of the moving fluid to a static pressure sufficient to provide for proper combustion conditions. The conversion of kinetic energy to pressure takes place through a diffuser; this usually constitutes the nose of the missile or aircraft itself. For subsonic flight speeds, the problem of efficient conversion of velocity to pressure head is one which can be treated easily by classical analysis when the equation of state of the working fluid is known. Since no shock waves are formed in subsonic flow the subsonic diffuser may be treated simply as a reversed nozzle of the convergent-divergent, or de Laval type. In transonic and supersonic speeds, however, the conversion of velocity of the entering fluid to pressure in the combustion chamber takes place across the shock wave pattern, the control of which is most important both to the efficiency of the thermodynamic cycle and to the aerodynamic drag forces on the missile itself. It turns out, however, that it is possible in certain cases to use a convergent-divergent nozzle efficiently up to a Mach number 1.5; for speeds above Mach number 1.5 it is desirable to diffuse the fluid to a subsonic speed by one or several oblique shocks and one normal shock with further diffusion taking place by expansion in the subsonic speed range. The diffusers using this oblique shock pattern were first investigated by Oswatitsch at

Göttingen; this design has been improved by Ferri and others in this country.

The boundary layer studies cover the regions of subsonic, transonic, and supersonic flow. In the subsonic region, research is continuing on studies of the turbulence in separated boundary layers, on the transition from laminar to turbulent flow in a boundary layer and on the effects of viscosity on the action of both laminar and turbulent boundary layers. In general, the experimental work follows the subsonic boundary layer theories developed by Tollmien, Schlichting, Lin, and Lees. No suitable boundary layer theory has been developed for the transonic and supersonic region. Since the boundary layer theory depends in large measure on a successful theoretical approach to the problem of turbulence, the future advances in boundary layer theory, especially in the supersonic region, must be closely related to a successful treatment of turbulence as a three dimensional problem.

Qualitative research has already indicated that shock waves affect the character of a boundary layer and that a certain type of boundary layer modifies a shock wave in its vicinity. These effects have recently been treated quantitatively and further research is indicated before a fundamental understanding of this mechanism can be obtained. At present the interaction may be separated into two phases: the interaction of a shock wave with a laminar and with a turbulent boundary layer. The interaction of boundary layer and shock is related both to the Mach number of the flow and the Reynolds number. For this reason it is important to separate the effects of particle velocity from the effects of scale and to study the interaction of shock waves with boundary layer at several Mach numbers and one Reynolds number and at one Mach number and several Reynolds numbers.

The motion of fluid particles in a turbulent flow condition has a profound effect on aerodynamic drag, on heat transfer, on molecular diffusion, on the mixing processes of fuel and oxidizer, and on the efficiency of the combustion process. There have been many theories for describing the effect of this transient condition on the transfer of both intensive and extensive quantities. Most of these theories are inadequate since they have attempted to treat the three dimensional phenomena of turbulence with a two dimensional analysis. The reason for this approach to the problem is not altogether reprehensible since the equations describing turbulent fluid flow in three dimensions are, in most cases, intractable. It is perhaps nothing more than a coincidence that experimental results on velocity and temperature profiles, on heat transfer, and on drag coefficients are found to agree with the two di-

mensional theories of turbulence developed by Prandtl, Taylor, von Karman, and others. A new approach to the theory of turbulence is needed which will make use of the advanced statistical methods developed in quantum mechanical problems. It is doubtful whether significant results can be obtained by combining the present theories of turbulence with the kinetics of reaction in combustion, so that the effect of turbulence on the chemical process may be determined. Since turbulence and the chemical mechanisms are not themselves clearly understood, there is little reason to suppose that their combination will yield easily interpretable results. A large quantity of experimental data is being built up which may indicate the proper steps to be taken in evaluating the effects of non-linear transient fluid flow on the chemical reaction of a fuel and oxidizer.

The atomization of fluid particles in a stream and the mixing of gas streams has not until recently been studied as a fundamental problem removed from combustion. There is a great body of empirical data on the effect on droplet size of the various chemical and physical properties of the fluid. It is significant that there are few references in the literature and no text which treats atomization as a basic problem.

In view of the importance of the physics of high altitudes the recent theoretical and experimental research in this direction is encouraging. The theory of non-uniform gases has been applied to heat flux and stress analysis in the slip-flow region where the mean free path of the gaseous particles is between 10^{-2} and 10^{-1} times the smallest intrinsic dimension characterizing the flow. Recent calculations for high particle speeds in the slip-flow region, using a second order approximation to the Maxwell-Boltzman distribution function, indicate that the gas-dynamical value for

skin friction is reduced by only 10 per cent at Mach number 3.0 and an altitude of 250,000 feet.

Work of a less basic nature but of considerable importance to the application of research techniques and results to development problems is included in sections on hydraulic analogy studies, on theoretical studies applied to the pulse jet cycle, and on certain plans for future experimental facilities.

The division of the field of fluid mechanics, as it appears in this report, is one dictated by the projects outstanding in this field at the present time, through consideration for those engaged in research and for those responsible for organizing and directing a program of research in fluid mechanics.

Each section has an introduction which gives a brief account of the state of the art in that part of the field. It is most important to note that no attempt has been made to write a technical monograph on the field of fluid mechanics or the sub-divisions of it as they appear in this report. Rather, it was felt that a few introductory paragraphs would be helpful to those who are not experts in the field or who have not conducted original research on compressible fluid flow. The material for the introduction to each section has been gathered primarily from text books, from progress reports of contractors to the government, and from many discussions with research scientists, and government representatives concerned with this field.

The description of research projects under the heading "Research in Progress" was controlled in part by the availability of material and by the importance attached to the scientific implication of the work. References to the open scientific literature and to certain classified progress reports have been made in order that those especially interested in a particular phase may obtain further details.

IV. PROPAGATION AND INTERACTION OF SHOCK WAVES

A. Introduction

During the war a considerable effort was expended on the theory and experimental investigations relating to shock wave phenomena. The work on the interaction and reflection of shock waves was started in 1942 by von Neumann (28) under the sponsorship of the Bureau of Ordnance, Navy Department. In addition to this early work and to that conducted in Army and

Navy laboratories, the National Defense Research Committee initiated research in this field under Division 2 and under the Applied Mathematics Panel. As part of this Panel, the Applied Mathematics Group at New York University undertook theoretical studies of supersonic flow and shock wave phenomena (5). This work was directed by Dr. Richard Courant and Dr. Kurt O. Friedrichs. Under Division 2, Kirkwood and

Brinkley (21) at Cornell formulated a new theory of propagation of one-dimensional shock waves which is valid for air and water. This deals with the pressure-time dependence for plane, cylindrical and spherical shock waves; the problem is reduced to the solution of two ordinary differential equations relating the peak pressure and energy of the wave to its distance from the origin of the shock. Other recent contributions to the theory and analysis of shock interactions have been made by Polachek and Seeger (30) and by Keenan and Seeger (20). The status of the theoretical work on refraction, reflection and intersection has been reviewed by von Neumann (29).

The related experimental research on shock waves was conducted by Ladenburg and Bleakney at the Princeton Station, Division 2 and by E. B. Wilson and collaborators at the Woods Hole Station, Division 2. The work of Ladenburg involved the development and application of special interferometric techniques to study the refractive indices of gases and hence the density, pressure and temperature in free jets (22). Bleakney (18) undertook experimental research on the reflection and refraction of shock waves, on the pressure-distance relation for spherical shock waves and on optical methods for measuring shock wave velocity. The theory of Kirkwood and Brinkley has been confirmed in part by the experimental work of Bleakney and Wilson (34). Some of the experimental techniques relating to supersonic flow, with especial reference to the optical phenomena, are discussed in a recent Bureau of Ordnance publication (27).

B. Research in Progress

The Institute for Mathematics and Mechanics¹ of the New York University is continuing the theoretical work on gas dynamics started under the sponsorship of the National Defense Research Committee. A thorough review and amplification of the material previously collected in the *Shock Wave Manual* (5) has been completed. A major publication containing many new features will be published soon as a book of approximately 500 pages.

The research activities now being pursued include theoretical problems relating to the interaction of weak or medium-strong shock waves such as those occurring in jet propulsion or the flight of missiles at transonic and supersonic speeds. A new theory of such interaction has been developed which is ap-

plicable to problems of interior ballistics as well as to problems of supersonic airfoils. A first report will be available on this subject in the near future.

Compressible fluid flow through nozzles is being continued following the lines indicated in the *Shock Wave Manual*. Theoretical problems with regard to flame fronts and to the theory of turbulent burning are being conducted by members of the Institute in conjunction with the work of Project SQUID, under the general direction of Dr. J. K. L. MacDonald. Part of the theoretical work relating to detonation and explosion phenomena will be of importance in understanding the fundamentals of shock wave reflection and transmission.

The Institute also plans to devote some effort to the study of boundary layers and their interaction with non-viscous flow. In the realm of statistical mechanics a comprehensive study of the problems of gas dynamics, burning and detonation has been started. A measure of success in unifying the points of view of Boltzmann and Gibbs appears to be assured. Since most of the studies in fluid mechanics deal with non-linear problems an attack on the theory of systems of non-linear partial differential equations in two independent variables has been started and some progress has been made to date. These equations occur in gas dynamics with entropy-change and with vorticity.

Walker Bleakney² at Princeton University is continuing experimental studies on shock wave reflection and refraction on the pressure-distance relation for spherical shock waves, and on optical methods for measuring shock wave velocity. Theoretical calculations have been made on the ENIAC at the Moore School of Electrical Engineering for the case of a shock wave impinging on a partially reflecting barrier, such as a boundary between two different gases or of the same gas at different densities. Photographs of shock wave reflection³ obtained by L. G. Smith (31) have been enlarged for further study and have been remeasured. The object of the re-examination of this work was to increase, if possible, the precision of the measurements in order to test the theory of regular shock reflection and Mach reflection with the three shock theory. Although the data on measurement (19) seemed to be more accurate and consistent than the original work, the conclusions were not changed in any important respect. The discrepancies between experiment and theory appear to be as bad as before.

¹New York University, New York, New York; Richard Courant and K. O. Friedrichs; ONR Contract N6onr 201, T.O. 1; *Unclassified*.

²Princeton University, Princeton, New Jersey; Walker Bleakney; ONR Contract N6ori-105, T.O. 2; *Unclassified*.

³Work undertaken at Princeton University for Division 2 of the National Defense Research Committee.

A. H. Taub (32) while at Princeton, also conducted a numerical analysis of the refraction of plain shock waves, in which he obtained numerical solutions of the equation governing the position of the various relevant quantities in an assumed shock configuration for the refraction of plain shocks. The assumed shock configuration consists of an incident shock, a reflected shock and a transmitted shock. It was further assumed that the pressure is constant in the angular domains between these shocks, and that across the density discontinuity responsible for the refraction, the pressure is continuous as is the deflection of the flow. The derivation and analytic discussion of the shock wave equations is discussed in a paper recently published (33).

The work of Professor Ladenburg at Princeton on the study of shock waves by interferometry was started in May, 1944 under Division 2 of the National Defense Research Committee. Experiments were conducted on an improved interferometer of the Jamin-Mach type (23, 35) on which interferometric pictures of gas jets under pressures up to 80 pounds per square inch were obtained, the gas escaping from a pressure tank of $1\frac{1}{2}$ cubic meters capacity through a special smooth valve of 1" diameter. The density, pressure, and temperature distribution inside a jet, including especially the shock region, were calculated from the pictures obtained. For this use, the interferometric method was reported to be superior to the Schlieren and the shadowgraph for providing accurate quantitative data of the density, pressure, and temperature changes in shock waves occurring in a free jet.

The work of Prof. Ladenburg at Princeton was transferred from the National Defense Research Committee to the sponsorship of the Bureau of Ordnance⁴ in 1945. A final report (26) summarizes the status of this work up to 30 May 1945. Under the sponsorship of the Bureau of Ordnance, the investigations were continued on jet studies, but at pressures up to 100 pounds p.s.i. gauge. Effort was also devoted to the construction of a small open supersonic "wind tunnel" providing a jet of homogeneous density and speed and to its use of determining, by interferometry, the complete air flow around models of various shapes supported in this jet. This homogeneous jet was obtained by letting the air from a pressure tank escape through a small de Laval nozzle of circular cross-section. This open "wind tunnel" has a diameter of $\frac{1}{2}$ " and supplies a jet of Mach number 1.7, constant within $1\frac{1}{2}\%$, over a conical volume of 0.55 " height. Density, pressure, and velocity distribution around a cone of 30° half-angle was computed, and the results were in

fair agreement with the theory of Taylor and Maccoll with satisfactory agreement.

During the fall of 1945, the main effort was devoted to the construction of a closed circuit for discharging the gas from a pressure tank into a second tank which could be evacuated. Two closed chambers, one using a three-dimensional jet as before, and the other a two-dimensional jet along with the necessary parts for connecting the two tanks, were constructed in the shops of the Palmer Physical Laboratory. Studies were initiated on the two-dimensional nozzle, particularly to obtain data on the two-dimensional inhomogeneous jets by shadowgrams and interferograms. A few experiments with transient shock waves from a small spark were tried in order to learn whether it would be possible to get interferometric pictures of this phenomenon and to evaluate the density and pressure distribution in such shock waves.

The Princeton contract⁵ for Prof. Ladenburg's work has recently been redefined to include:

1. A study of inhomogeneous jets (using straight and streamlined orifices of various diameters).
2. A study of inhomogeneous jets from a de Laval nozzle (open wind tunnel).
3. A study of gas flow around objects suspended in the wind tunnel (these objects include cones of half-angles of 10° , 30° , and 45° , yawing cones, and arrowhead projectiles).
4. The construction of a closed circuit from high to low pressures, through chambers with special glass windows for interferometric observation.
5. A study of two-dimensional jets and of three-dimensional cylindrical jets through closed chambers.
6. A study of the action of a conical attachment to a gun upon the flow of the propellant gases.
7. A study of transient spherical shock waves.
8. The construction of an interferometer with 5" plates and of the necessary optical accessories.
9. A study of the interaction of shock waves and boundary layers in supersonic flow.

Designs for an intermittent wind tunnel at Mach number 2.5, including those for the diffuser and for the connection to the vacuum tank, have been completed. The 5" plane parallel glass windows for the two-dimensional wind tunnel are in the final stages of preparation. Present indications resulting from a comparison between the experimental observations on free jets and the underlying theory appear to show that neither the experimental results on the homogene-

⁴Princeton University, Princeton, New Jersey; Rudolf Ladenburg; BuOrd Contract NOrd 9240; Confidential.

⁵Sponsorship for this work was transferred in March, 1947, to the Office of Naval Research, Navy Department, Washington, D. C.

ous jet nor on the inhomogeneous jet correspond with the three-shock theory. This lack of correspondence, according to Prof. Ladenburg, may be due to the fact that the gas flows from the reflected and from the Mach shock have different velocities and temperatures, and the ensuing slip stream between these two gas

flows is actually a boundary where viscosity and heat conductivity play important roles entirely neglected in the derivation of the theory.

A partial account of the Princeton work on the interferometric study of supersonic phenomena is available (24). Several parts are in preparation (25).

V. BOUNDARY LAYER STUDIES

A. Introduction

The concept of a boundary layer was introduced by Prandtl in 1904 (48), to explain flow phenomena near a boundary where the rate of change of velocity outward from the boundary is high. The boundary layer concept is closely associated with the Reynolds number of flow. Since the Reynolds number demands a characteristic length, the Reynolds number of the boundary layer will be characterized by the fluid velocity, kinematic viscosity and thickness of boundary layer outward from the surface. It is easily seen that for fluid flow over or along a surface the Reynolds number characteristic of the length of the flow increases as the fluid particles move across the surface. Osborne Reynolds (51) has shown that there is a characteristic number associated with the transition from laminar to turbulent flow. One would therefore expect that as the fluid particles move across the surface, the associated Reynolds number would eventually become large enough so that the transition from a laminar to turbulent flow condition would occur. Indeed the mechanism of this transition has been a subject of much controversy during the past twenty years, starting with the work of W. Tollmien in 1929 (55), who studied the stability of laminar flow in a boundary layer over a plate without pressure gradient. Tollmien predicted that small disturbances in velocity having wave lengths in a certain critical region would be amplified in the laminar boundary layer, whereas disturbances of shorter or longer wave lengths would be attenuated. Tollmien's calculations were repeated again in 1933 and 1935 by H. Schlichting (52). He verified Tollmien's original work which pointed to the fact that boundary layer disturbances in the critical wavelength region would grow in amplitude until the laminar boundary layer was no longer stable and transition to a turbulent boundary layer occurred. The oscillations predicted by Tollmien and Schlichting were not observed until 1940, when Schubauer and Skramstad, at the National Bureau of Standards (53), obtained records of the velocity fluctuations in the boundary layer of a flat plate by means of a hot wire

anemometer. Following the experimental verification of the Tollmien-Schlichting waves, C. C. Lin (46) revised the mathematical theory of stability of two-dimensional parallel flow making slight changes in those parts of the Tollmien-Schlichting theory which had been criticized adversely. The results on a flat plate, observed at the National Bureau of Standards, were repeated by H. W. Liepmann (42) at the Guggenheim Aeronautical Laboratory, California Institute of Technology,⁶ who at that time was studying the transition of a laminar to a turbulent boundary layer at the convex and concave boundaries of a curved plate. He observed Tollmien-Schlichting waves on the convex boundary using the oscillating ribbon technique of Skramstad (53) as well as an imposed regular roughness of variable wavelength and acoustic excitation for the disturbances. Boundary layer transition at a convex boundary was found to be the same as on the flat plate while the concave curvature was found to have a strong destabilizing effect due to centrifugal forces (43). Subsequent work on the control and stability of laminar boundary layer has shown that a negative pressure gradient in the direction of flow prevents transition whereas a positive pressure gradient in the flow direction causes early transition. The transition has been shown by Ackeret (50) to be controllable through the removal of part of the boundary layer by suction through a small number of slots in the surface over which the flow is taking place. Lees and Lin (41) have extended the boundary layer theory to investigate the stability of two-dimensional laminar compressible flow where (a) the surface adds or removes heat from the boundary layer, and (b) a boundary layer disturbance exchanges energy with the main stream by a vortex transport mechanism. They found that the temperature disturbances have only a small effect on those velocity solutions depending on the viscosity coefficient, for the case of ordinarily encountered Reynolds numbers. In the limit-

⁶California Institute of Technology, Guggenheim Aeronautical Laboratory; H. W. Liepmann; Research sponsored by the National Advisory Committee for Aeronautics.

ing case of infinite Reynolds numbers, however, a disturbance will gain energy from the main flow under the condition that the gradient of the mean density and mean vorticity near the surface has an algebraic sign opposite to that near the outer edge of the boundary layer. This has enabled a stability criterion to be established for the compressible case in terms of the gradient of the mean density and mean vorticity analogous to the Rayleigh-Tollmien criterion for incompressible flow. The boundary layer is found to be destabilized by the addition of heat; this is the result of the change in the distribution of the product of the density and vorticity. When heat is removed from the boundary layer the reverse obtains and a more stable condition of flow results.

Although it required twelve years for the experimental verification of the original Tollmien (55) critical wavelength theory of transition, it appears now that the boundary layer theory for subsonic flow is in a satisfactory state. Boundary layer theory for transonic and supersonic flow have not as yet been developed to a comparable degree. However, experimental investigations are being conducted on this problem at the Technische Hochschule, Zurich (36), at the California Institute of Technology (45), at the Massachusetts Institute of Technology (54), at the National Advisory Committee for Aeronautics (37, 38), at Princeton University (49) and at the Polytechnic Institute at Brooklyn (47). The effect of compressibility, of shock conditions in supersonic flow, of positive and negative pressure gradients, and of temperature gradients must be taken into account for a suitable boundary layer theory in the transonic and supersonic regions of flow.

H. L. Dryden (39) summarized the recent contributions to the study of transition and turbulent boundary layers in a paper⁷ presented at the Sixth International Congress for Applied Mechanics in Paris in September, 1946. A critical survey of the effects of viscosity in transonic and supersonic flow has been prepared by Kuethe and Epstein (40) at the University of Michigan⁸.

B. Research in Progress

Studies on Subsonic Flow. The National Bureau of Standards⁹ is continuing work on the study of turbu-

lent boundary layers, on wind tunnel turbulence and on instrumentation relating to the measurement of turbulence. Measurements are being conducted on double correlations in a separating boundary layer, for the purpose of exploring further the turbulence processes associated with the development and separation of a turbulent boundary layer in an adverse pressure gradient. Equipment for the current investigation has been installed in the Bureau's 10 foot wind tunnel, which is capable of supplying air at speeds approximating 100 mph in the test section. An airfoil partition extending from the floor to the ceiling of the tunnel has been installed and has been designed to supply two-dimensional flow over regions about 3 feet in width, along the horizontal center line of the airfoil. The leading edge, with the radius of curvature of 1 inch is joined longitudinally to cylindrical surfaces of 23 foot radius forming the nose. The trailing portion of this airfoil partition is flat on one side and has the form of a circular cylinder of 31 feet radius on the other side. The symmetrical shape was found necessary to secure separation of the boundary layer. A blister was added at the tunnel wall opposite the mid-section of the partition, in order to modify the pressure distribution to cause separation of the boundary layer to occur well upstream from the trailing edge.

Turbulent shearing stress in the boundary layer has already been measured. This involves one kind of double correlation. The double correlations referred to in this project, however, concern the scale of the turbulence, particularly the micro-scale connected with the rate of dissipation of the turbulence. Correlation between like components of the fluctuation will be measured as a function of the distance between the measuring points in several directions. The procedure will be similar to that for isotropic turbulence. In this project, however, the non-isotropic character of the turbulence involves more components and more complicated hot-wire arrangements than in the previous work. In the absence of any theory or previous work on non-isotropic turbulence where an adverse pressure gradient exerts a dominating influence, the results themselves must serve as the only guide as to what types of correlations will be suitable for revealing the processes which accompany the initiation of boundary layer separation.

Research at the University of Washington¹⁰ on the

⁷This summary contains references to the principal scientific articles on this subject; a bibliography is included under the following subject headings: Laminar Flow in Boundary Layers, Stability and Transition, Turbulent Flow, Boundary Layer Suction, Drag Calculations, and Flight Tests on Boundary Layers.

⁸University of Michigan, Department of Engineering Research, Ann Arbor, Michigan; Arnold M. Kuethe; BuOrd Contract NOrd 7924; *Restricted*.

⁹National Bureau of Standards, Washington, D. C.; G. B. Schubauer; Research Authorization R.A. S47-1; Sponsored by the National Advisory Committee for Aeronautics; *Unclassified*.

¹⁰University of Washington, Seattle, Washington; F. S. Eastman; ONR Contract N6ori-217, T.O. 1; *Unclassified*.

effect of boundary layers in connection with the diffuser problem will be discussed in the section of this report entitled "Flow Through Duets: Diffusers and Nozzles." The relation of this work at subsonic speeds to the general study of boundary layer effects stems primarily from the fact that this study is concerned with the influence of positive and negative pressure gradients on the boundary layer in a diffuser section. This work is expected to result in data which can be used in establishing the optimum configuration for the diffuser with the associated boundary layer control devices.

Supersonic Studies. The Polytechnic Institute of Brooklyn¹¹ is conducting research on the development of an approximate boundary layer theory for supersonic flows and on the application of methods of boundary layer control for the limitation of skin friction and for the possible limitation of shock pressure rise following the shock wave with its subsequent separation and drag. Since the existing theories do not explain adequately the phenomenon of boundary layer and shock wave interaction, a new physical model appears necessary. The requirement for a physical model demands that it relate the state of the boundary layer to the development of the shock wave. In order to treat the state of the fluid, including compressibility, viscosity and entropy effects at every point in the field, it is necessary to discover whether the medium may be treated classically as a continuum or whether it must be treated by statistical methods. In order to assist in the development of a proper technique for handling this problem, the mathematically simple fluid motion given by a vortex of an adiabatic, irrotational compressible fluid is also being investigated. Preliminary calculations indicate that this simple flow may embody some physical principles of importance in explaining phenomena of a more complex nature. The stability of vortex motion and the possibility of simulating boundary layer effects by heat are being considered.

Calculations in connection with the design of transonic and supersonic wind tunnels suitable for boundary layer-shock wave interaction experiments, together with the design of suitable wind tunnel nozzles have been made. A closed return, continuous flow type is presently considered the most adequate from the point of view of initial economy and flexibility of operation.

The interaction of shock waves and boundary layers for a series of Mach numbers varying from 1.5 to

3.0 and in a Reynolds number range from 2×10^6 to 40×10^6 is to be undertaken by the Princeton Aeronautical Laboratory, Princeton University¹². This wide range of conditions will be obtained in a supersonic "blow down" tunnel which is to be operated by a high pressure air supplying system, capable of providing up to 150,000 cubic feet of air at pressures ranging from approximately 30 psi to 500 psi (absolute) (49). Recent work has indicated the importance of boundary layer-shock wave interaction at transonic and supersonic compressible fluid speeds (44, 50). The oblique shock wave at or behind the edge of an airfoil communicates a continuous pressure rise through the subsonic portion of the boundary layer over the trailing edge. The consequent increase in boundary layer thickness along the airfoil surface producing a region of compression ahead of the main trailing-edge shock and the pressure distribution over the airfoil, differs from the distribution calculated on the assumption of non-viscous supersonic flow. The present study extends the work of Liepmann (44) and Ackeret (36) to high Reynolds numbers and Mach numbers for the purpose of obtaining fundamental information about boundary layer-shock wave interactions over a wide range of the parameters which control this phenomenon.

H. W. Liepmann at the Guggenheim Aeronautical Laboratory, California Institute of Technology¹³ is investigating the shock wave pattern and the pressure distribution over a circular arc airfoil as affected by the state of the boundary layer (44). The experiments of Liepmann were undertaken with a 12% thick, double circular arc section of 3 inch chord at zero angle of attack (Figure 1a). The investigations of transonic flow covered the range between the critical Mach number 0.79 for this airfoil to 0.91, where the wind tunnel was found to choke. The Reynolds number variations range from 8.3×10^5 to 1.75×10^6 . The change from a laminar to a turbulent boundary layer for a given Mach number changed the flow pattern considerably, which indicated that the Reynolds number effects can be appreciable and that the character of the boundary layer does influence the transonic flow pattern. Two types of measurements were made. Schlieren photographs were taken and found to give both clear qualitative and quantitative information. Pressure distribution measurements furnished valuable

¹¹Polytechnic Institute of Brooklyn, Brooklyn, New York; R. P. Harrington; ONR Contract N6ori-206, T.O. 1; *Unclassified*.

¹²Princeton University, Princeton Aeronautical Laboratory, Princeton, New Jersey; J. V. Charyk and Lester Lees; ONR Contract N6ori-105, T. O. 3; Project SQUID; *Unclassified*.

¹³California Institute of Technology, Guggenheim Aeronautical Laboratory, Pasadena, California; H. W. Liepmann; AAF Contract W33-038-ac-1717; *Unclassified*.

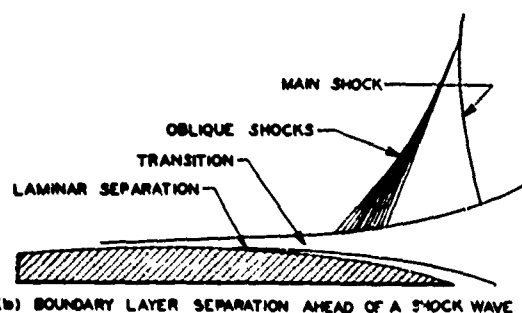
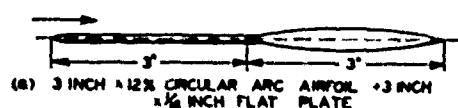


Figure 1. Sketch of (a) circular arc airfoil and (b) boundary layer separation ahead of shock wave at rear of airfoil. (After Liepmann)

information also, although the large time lag in the measurements on scale models prevented a one-to-one comparison with the Schlieren photographs, which are the results of nearly instantaneous observations. The conclusions from work conducted in 1945 indicate the importance of the boundary layer influence on the shock wave pattern and on the pressure distribution

in transonic flow. It is also concluded that shock waves can interact with the boundary layer in a manner similar to the reflection of a shock wave from a free surface. The shock wave in this case is followed by an expansion zone; the pressure gradient normal to the boundary layer is of the same order of magnitude as the pressure gradient parallel to the boundary layer. This effect finds no analogue in the behavior of boundary layers in subsonic flow and it appears to be an important factor in the understanding of supersonic flow near solid surfaces. Additionally the separation of the boundary layer from the surface does not necessarily take place near the base of a shock wave and can occur ahead of the base of the shock wave (Figure 1b). The classical boundary layer theory postulates that the separation of the boundary layer is due to a shock wave; this does not appear to apply in supersonic flow in the immediate neighborhood of shock wave base.

Mention at this point should be made of the work at the Technische Hochschule in Zurich (Ackeret, Feldmann and Rott) (36). This work was conducted during the war period and so information regarding the details and conclusions were not available until 1945. Essentially Ackeret conducted studies on the interaction of shock wave and boundary layer in a wind tunnel test section consisting of a nozzle with a

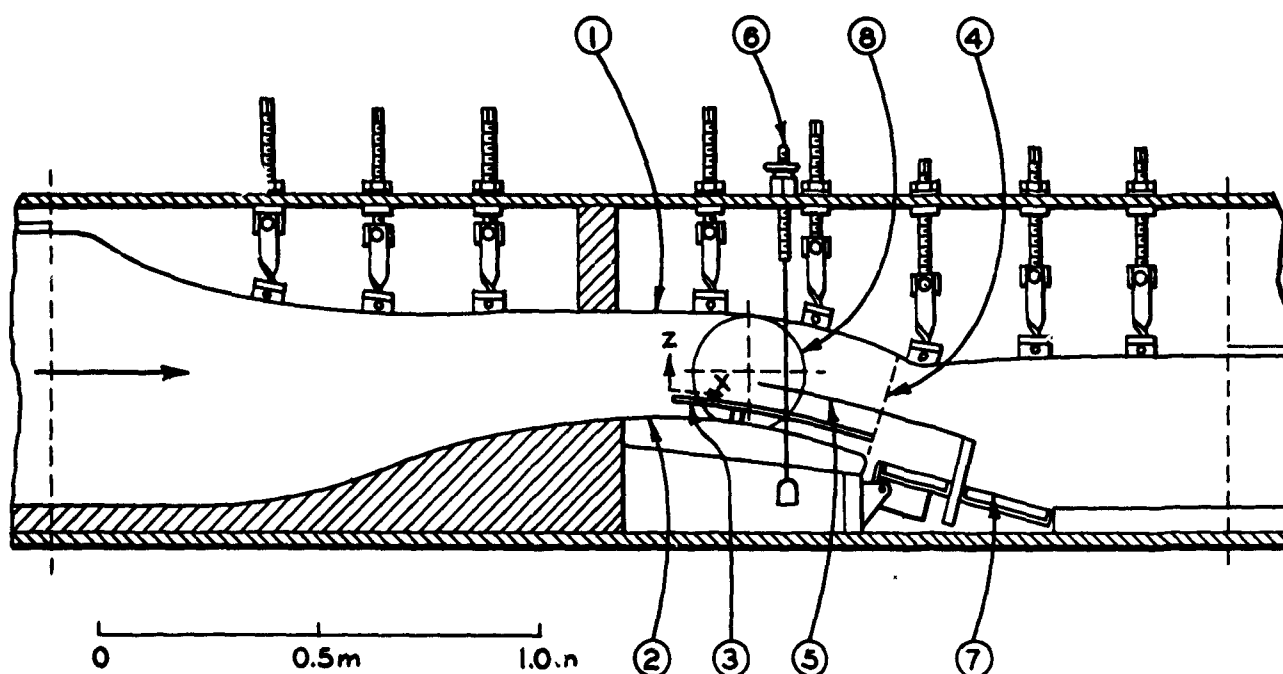


Figure 2. Test section for studying interaction of boundary layer and shock in transonic flow. (1) upper nozzle wall, (2) lower nozzle wall, (3) auxiliary plate, (4) minimum cross section, (5) total pressure tube or static pressure probe, (6) total pressure tube adjustment in the Z-direction, and (7) total pressure tube adjustment in the X-direction. (After Ackeret, Feldmann and Rott)

curved axis to provide two-dimensional flow (Figure 2). A portion of the lower wall of the nozzle was shaped like a wing surface, while the upper wall of the nozzle was arranged so that it corresponded approximately to a stream of the flow about the wing. The tests were carried out so that sonic velocity was not reached over the whole nozzle cross section which enabled a supersonic region to be localized and not to extend to the upper wall of the nozzle at any point. Schlieren pressure and humidity measurements were made in connection with these shock wave studies. The Mach number range varied from 1.1 to 1.4 while the Reynolds number was varied from 1.33×10^6 to 2.7×10^6 .

Quite independently of the conclusions arrived at by Liepmann, Ackeret and his co-workers concluded that the flow phenomena over the nozzle surface depends upon whether the boundary layer ahead of the shock wave is laminar or turbulent. For a laminar boundary layer and for Mach numbers not very much greater than 1, multiple shock waves occur which decrease in number as the Mach number is increased. Finally a simple λ -shock is formed which consists of a normal shock wave with a preceding oblique compression shock. Turbulent boundary layers produce only normal shocks independent of the Reynolds and Mach numbers. It does not matter whether the boundary layer becomes turbulent naturally or artificially.

The interaction of boundary layer with shock is also under investigation at the Massachusetts Institute of Technology,¹⁴ under the direction of Prof. E. S. Taylor and Prof. E. P. Neumann (54). Observations were made in a rectangular channel of cross section $\frac{1}{2} \times 1$ ", with the aid of Schlieren photographs. An adjustable wedge was employed for initiating oblique shock waves in the channel (Figure 3). The wedge could be placed anywhere inside the boundary layer on the channel walls or anywhere in the free stream, and was adjustable with reference to angle of attack. These observations indicated that the interaction of boundary layer and shock is very marked; that the existence of shock waves influences the behavior of the boundary layer and the modification of the boundary layer in turn alters the shock wave and flow patterns. An oblique shock wave generated near a surface with boundary layer attached was found to be initiated in general, by a smooth curvature in the boundary layer. The boundary layer is thickened by the pressure rise occurring across the shock wave and this boundary layer thickening tends to magnify the strength of the

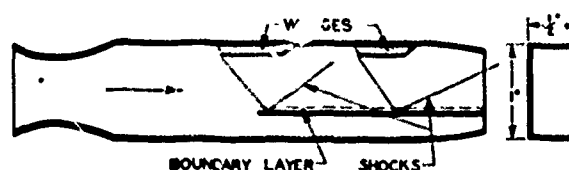


Figure 3. Apparatus for studying interaction of boundary layer and shock waves. (Massachusetts Institute of Technology)

shock. When the pressure rise across the shock is large, the boundary layer is found to detach from the channel wall at the point where the shock wave is formed.

The interaction of an oblique shock incident upon a surface with boundary layer attached may result in several modes of reflection, depending upon the strength of the shock and the thickness of the boundary layer. When the incident shock is weak it becomes curved as it penetrates the boundary layer and is normal to the stream at Mach number 1. The reflection of such a shock is of the mirror-image type and it therefore has little effect on the boundary layer itself. When the incident shock is strong or when the boundary layer is very thick, two reflections occur. One reflection is generated upstream of the point of incidence because of the thickening of the boundary layer; the other reflection is caused by the combination of the reflected compression waves which arise from the curvature of the incident shock as it enters the boundary layer.

A normal shock wave generated in the passage with boundary layer attached is not a simple normal compression wave, but consists of a series of oblique or forked shocks. When the boundary layer is thick these oblique shocks extend all the way to the center of the channel; this usually results in the separation of the boundary layer from the walls. If the boundary layer is thin or if the Mach number is low, the oblique shock waves extend part way into the stream and join a normal shock there. The lengths of the oblique shocks decrease as the boundary layer thickness is reduced.

In connection with a ramjet engine and ducting system, the Marquardt Aircraft Company¹⁵ is undertaking a design study, both subsonic and supersonic, which includes boundary layer investigations.

Boundary layer and shock wave studies are under investigation at the University of Virginia¹⁶ where a short duration spark photographic technique has been

¹⁴Massachusetts Institute of Technology, Cambridge, Massachusetts; E. S. Taylor and E. P. Neumann; BuOrd Contract NOrd 9661; Project METEOR; Confidential.

¹⁵Marquardt Aircraft Company, Venice, California; R. E. Fisher.

¹⁶University of Virginia, Charlottesville, Virginia; J. W. Beams; BuOrd Contract NOrd 7873; Project BUMBLEBEE (Report No. 56, March 1947); Confidential.

used to study these effects on high speed rifle bullets in flight. The short duration spark light source has also been employed for study of turbulence in supersonic air streams, and in their associated boundary layers. Preliminary investigations have also been undertaken of a projectile moving from air into CO_2 at faster than sound velocity. Air and CO_2 are ini-

tially contained in chambers separated by a thin partition which the projectile can penetrate without much loss in speed. Short duration spark photographs have provided excellent detail of the shock wave and boundary layer structure at the gaseous interface and a marked change in shock wave pattern was noted when the projectile passed from air to CO_2 .

VI. FLOW THROUGH DUCTS: DIFFUSERS AND NOZZLES

A. Introduction

The important problem in the intake of air to a propulsive device moving at supersonic speeds is the efficient conversion of the kinetic energy of the air to pressure in the combustion chamber of the missile. Before self-powered missiles requiring an external source of oxygen were contemplated for military application, the principal effort in supersonic diffusers related to their use in high speed rotary compressors, supersonic air intakes and supersonic wind tunnels. Crocco (61) has made a study and a review of supersonic diffusers bringing the available literature up to 1935. For diffusers starting with initial boundary layer, the experimental work of Crocco indicates that the transition of air from supersonic to subsonic speeds gives rise to such large total head losses as to impair seriously the efficiency of mechanisms using this type of diffusion.

The first attempt at a design analysis for highly efficient pressure recovery through a "supersonic" diffuser system was made by Oswatitsch (75). He concluded the use of a convergent-divergent (de Laval) nozzle was less efficient than a subsonic diffuser with sharp duct edges (divergent channel) on the basis of pressure recovery. However, in order to reduce the high drag of a compression shock across the intake throat, Oswatitsch proposed compressing the air at a cone-shaped point and leading the air through a ring shaped slit to the inside of the missile. During its passage across the cone the air is compressed by a number of diagonal shock waves which lead into a compression shock in the angular ring. A subsonic or divergent diffuser increases the pressure behind the normal shock. Experimental tests of this design have proved highly satisfactory especially in the Mach number range greater than 1.5.

Figure 4 is a sketch showing several arrangements of a normal shock with various oblique shock configurations. Figure 4a shows the arrangement with one diagonal and one normal shock. Figures 4b and 4c indi-

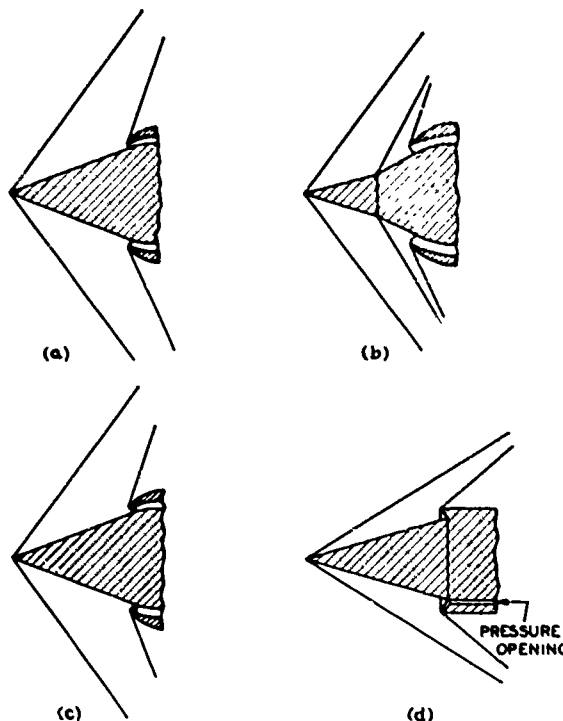


Figure 4. Sketch showing position of compression shocks in conical diffusers. (After Oswatitsch)

cate two principal arrangements for two oblique shocks, the second resulting from a break in the cone angle. Figure 4d is a sketch of the model used to study simple configurations for pressure recovery. Figure 5 shows the calculated total pressure recovery for a normal shock and several diagonal shocks; Figure 6 is a sketch of the actual model used by Oswatitsch for testing the effectiveness of the spike diffuser.

Kantrowitz and Donaldson (68) have further treated the deceleration of air from supersonic to subsonic velocities in a reversed de Laval nozzle (Figure 7a). They report that a normal shock is apparently necessary in the diverging part of the diffuser to ensure stable flow and they consider methods of

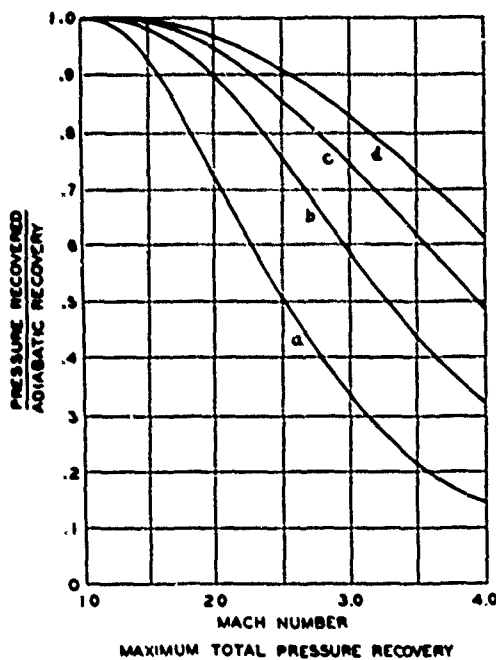


Figure 5. Maximum total pressure recovery between Mach number 1.0 and 4.0 for a normal shock and several diagonal shocks, (a) normal shock, (b) normal shock and one diagonal shock, (c) normal shock and two diagonal shocks, and (d) normal shock and three diagonal shocks. (After Oswatitsch)

minimizing the intensity of this shock through variations in the contraction ratio in the entrance region (Figure 7b). On the basis of their calculations, supersonic diffusers have been designed which, starting with no initial boundary layer, recovered over 90 per cent of the kinetic energy in supersonic air up to a Mach number of 1.85.

Ferri (63) has applied the method of characteristics to the diffuser problem to determine the physical properties of supersonic rotational flow in a two-dimensional field. He finds that the effect of rotation is not very important if there is a small variation of entropy but is important in internal flow where the entropy variation is usually large. With a circular conical

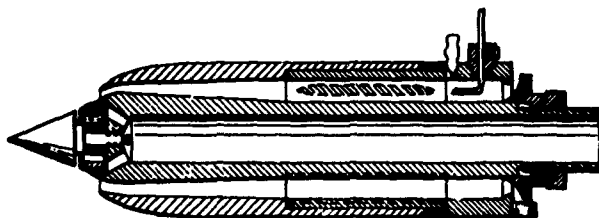
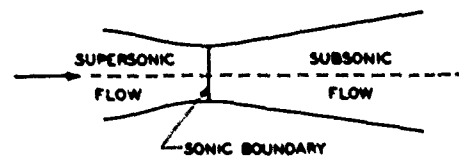
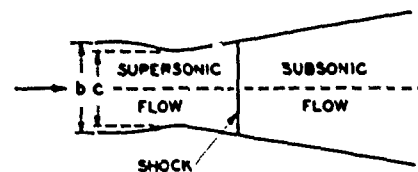


Figure 6. Model for testing the effectiveness of a conical diffuser. (After Oswatitsch)



(a) REVERSED de LAVAL NOZZLE WITH ISENTROPIC FLOW (UNSTABLE)



(b) STABLE SUPERSONIC DIFFUSER FLOW.

Figure 7. Reversed de Laval nozzle, (a) with isentropic flow and (b) stable supersonic diffuser flow (for circular diffuser $b^2/c^2 = C_r$ where C_r is the contraction ratio). (After Kantrowitz and Donaldson)

channel, the shock produced at the inlet lip becomes stronger toward the axis of the inlet and is a normal shock at the axis. The region across which the normal shock occurs increases with the angle of the internal cone. The method of characteristics permits the design of an internal-channel shape that results in a very efficient recompression at the inlet. Low shock drag for a body of revolution may be obtained if an effuser is connected with this diffuser. Ferri's paper makes reference to the method of characteristics used by Prandtl and Busemann, by Frankl and by Ferrari. A subsequent paper by Ferri and Nucci (64) considers further the problem of a new type of supersonic inlet making use of a variant of the oblique shock method for supersonic diffusers.

Kantrowitz (69) has examined the formation and stability of normal shock waves in channel flow in relation to the problem of obtaining shock-free deceleration of supersonic air through the speed of sound. He has extended Riemann's theory of the propagation of finite-amplitude disturbances in a homogeneous medium to consider the effect of upstream-moving pulses on the stability of the flow in the diffuser. If a pulse is generated in the subsonic region behind the minimum throat area and if the resulting shock wave formed is small, the shocks neither consume nor amplify the pulse. The results of these calculations have been applied to a de Laval nozzle. As the back pressure is lowered it is shown that the shocks are formed

when the back pressure goes below the point where sonic velocities first appear in the throat. A pulse starting from the rear of the channel will be permanently trapped in the sonic region where the resulting shock waves accompanying the pulse are small. If the weak shock wave restriction is removed it is found that trapped expansion pulses are consumed by the shock motion and trapped compression pulses are amplified. The smooth transition from supersonic to subsonic air speeds is therefore unstable to compression pulses arising in the rear of the channel. The theory developed by Kantrowitz shows that supersonic diffusers with long throats may allow a lower shock intensity and therefore a higher efficiency when the pulse disturbance level is relatively high.

A comprehensive theoretical treatment of the flow through diffusers and nozzles has been prepared by the New York University Group (56, 57) of the Applied Mathematical Panel of the National Defense Research Committee. A review of the general considerations regarding air intake phenomena has been treated by Troller (82) and by Zirkind (83); these reports contain numerous references to the literature and to classified reports. Coffin (59) has presented an analytical investigation of the flow through nozzles and diffusers¹⁷, in which he has developed a simple graphical method for obtaining the exit gas temperature, velocity and pressure when these parameters at the entrance to a nozzle or diffuser are known. Calculations enable this graphical method to be used for isentropic, adiabatic flow in the subsonic, transonic and supersonic regions. The theory of ideal one-dimensional flow has been reviewed by Hall (66) of the Research Division, United Aircraft Corporation¹⁸, and a set of basic charts for the theoretical performance of this type of nozzle has been developed. The flow through conical shock diffusers has been analyzed recently by Riley and Rinehardt (77) of Bell Aircraft Corporation.¹⁹

F. H. Clauser (58) has presented a general review of the important factors in the aerodynamics of ducted bodies. A review of supersonic diffusers is under preparation by the Staff of the Aerophysics Laboratory (76) of the North American Aviation, Inc.²⁰

B. Research in Progress

1. *Subsonic Diffuser Studies.* In the Langley Induction Aerodynamics Laboratory, the National Advisory Committee for Aeronautics²¹ is investigating circular and annular diffusers (72) designed for high speed subsonic entrance flow. These data are applicable both to subsonic diffusers and to subsonic diffusion downstream of a supersonic air intake. Preliminary results indicate that, for an area expansion of 1.75 to 1 and an equivalent cone angle of 12°, diffuser efficiencies of over 90% can be maintained up to an entrance Mach number 0.95.

The National Bureau of Standards²² is conducting research in the subsonic field concerning the flow in wide-angle diffusers containing screens. This is the second part of a study on wind tunnel turbulence conducted by the National Bureau of Standards for the National Advisory Committee for Aeronautics. The first part of this research concerned the use of damping screens for reduction of wind tunnel turbulence (62). The research was conducted in a 4½ foot closed wind tunnel at the Bureau and the final report is now in preparation.

There are few fundamental data available to enable the proper design of diffuser-screen combinations for reduction of wind tunnel turbulence. With the proper design information, it will be possible to effect savings in structural costs, if the tunnel cross section could be increased rapidly without the separation of the flow just upstream from the diffuser-resistance combination. Results on part 1 of this investigation indicated that a stream might be expanded rapidly ahead of a screen by means of a wide angle diffuser. In order to study this phenomena, a duct system was designed to allow exploring the possibilities of diffuser-screen combinations suitable for wind tunnel application. Velocities and pressures could be measured in any part of the system by means of the traversing apparatus carrying small pitot and static tubes. By means of this equipment, the boundary layer flow has been determined in detail and the efficiency of certain diffuser-screen combinations has been evaluated.

The University of Washington²³ is conducting studies on the effect of the boundary layer in connec-

¹⁷Kaiser Cargo, Inc., Fleetwings Division, Bristol, Pennsylvania; Carl de Ganahl; BuAer Contract NOa(s) 8274; *Confidential*.

¹⁸United Aircraft Corporation, East Hartford, Connecticut; Mr. John G. Lee; BuOrd Contract NOrd 9845; *Confidential*.

¹⁹Bell Aircraft Corporation, Buffalo 5, New York; Mr. Robert Stanley.

²⁰North American Aviation, Inc., Inglewood, California; William Bollay; BuOrd Contract NOrd 9784; *Confidential*.

²¹The National Advisory Committee for Aeronautics, Langley Memorial Aeronautical Laboratory, Langley, Virginia; Full Scale Research Division; (Research sponsored by NACA); *Confidential*.

²²National Bureau of Standards, Washington, D. C.; G. B. Schubauer; National Advisory Committee for Aeronautics Research Authorization; RA Contract S 46-3; *Unclassified*.

²³University of Washington, Seattle, Washington; F. S. Eastman; ONR Contract N6ori-217, T.O. 1; *Unclassified*.

tion with diffuser design, in order to determine the boundary layer properties under the influence of positive and negative pressure gradients through slots in the diffuser section. A free flight tunnel, having a 4 foot by 4 foot test section and powered by a 10 h.p. electric motor will be modified for this research. This tunnel contains a 1 foot by 4 foot throat capable of expanding to a 4 foot by 4 foot diffuser section, in which the included angle between the diffuser walls can be varied. A test model containing a single full-sized boundary-layer control unit has been used to determine design parameters. A 3 h.p. electric motor, driving a B-11 supercharger will provide the pressure or suction air for the single control unit. Preliminary planning of the test equipment needed to measure the "merit factors" is under consideration. A mathematical analysis is now in progress, in which the power consumption of the boundary layer control is compared with the power consumption of the primary air. These data will be used in establishing the optimum configuration of the diffuser and boundary layer control device. The objectives of the preliminary test program are to study the velocity distribution in the wall slot, the velocity distribution in the bell mouth, which serves to measure the quantity of air passing through the system, the proper location for the wall slot pressure-differential points, and the amount of air, and its adjustment, permitted to enter or blow out through the wall slots by means of tapered throttling cones. This investigation will attempt generally to correlate and evaluate presently known facts concerning boundary layer control in connection with the subsonic diffuser problem.

2. *Supersonic Diffuser Studies.* There is active research in the field of supersonic diffusers in order to improve the efficiency of pressure conversion for ramjet applications and to obtain fundamental data on fluid flow in ducts at velocities faster than the velocity of sound.

Cornell Aeronautical Laboratory, Cornell University²⁴ is undertaking experimental research on two dimensional diffuser models in the Cornell Aeronautical Laboratory supersonic wind tunnel at Mach number 1.7. Schlieren photographs, static and stagnation pressure measurements have been obtained at various points in the duct to study the effect of a pulsating back pressure on the flow pattern. A two-dimensional Kantrowitz duct is under test for the study of the

relationship between frequency of pressure fluctuation and the travelling of the normal shock. High speed motion pictures will be available in the near future to give more complete information concerning the fluctuating conditions within the diffuser. Investigations are also in process to determine the effect of boundary layer on the characteristics of this diffuser and on the shock wave positions, under the influence of periodic disturbances. The latter is in connection with the theory developed by Kantrowitz (69) with regard to the pressure and stability of shock waves in diffusers.

Cornell Aeronautical Laboratory²⁵ is also studying the design of a supersonic diffuser for shock position control for intake air to a ramjet. A study of the Oswatitsch diffuser at off-design conditions is in progress. There is no basic or design data in the region below design Mach number conditions; considerable basic research is needed to obtain this information.

The Marquardt Aircraft Company²⁶ is undertaking theoretical studies to calculate the net power output per unit frontal area of a ramjet with a spike or Oswatitsch supersonic diffuser operating at Mach number 2.0. The drag of the cone has been calculated by the Taylor-Maccoll (81) method and the lip drag by the method of Prandtl-Meyer²⁷. The lip drag will also be calculated by the method of characteristics²⁸.

A report on the design studies of this ramjet power plant will be available in the near future.

Massachusetts Institute of Technology²⁹ is conducting a research project on supersonic diffusers (71, 80) as part of an experimental study of shock waves in a circular pipe. The experimental equipment for this work includes a nozzle to provide supersonic flow at a Mach number 4.3 at the entrance to a 1 inch diameter tube 54 diameters long. Air is supplied to the nozzle from the outside atmosphere. A steam ejector of the Elliott type at the exit end of the tube provides the necessary pressure gradient for flow. The pressure at the tube wall is measured at intervals along the tube length; from these measurements, together with knowledge of the flow rate through the tube, the state of the fluid at any point along the tube may be predicted on basis of a one-dimensional analysis involving momen-

²⁴Cornell Aeronautical Laboratory, Cornell University, Buffalo, New York; J. V. Foa; ONR Contract N6ori-119, T.O. 1; Project SQUID; *Unclassified*. Semi Annual Progress Report, Project SQUID, 1 January 1947; Quarterly Project Report, Project SQUID, 1 April 1947; *Unclassified*.

²⁵Cornell Aeronautical Laboratory, Buffalo, New York; C. C. Furnas; BuOrd Contract NOrd 8993, T.O's. 1 BB 14, 1-C and 1 EE 1; *Confidential*.

²⁶Marquardt Aircraft Company, Venice, California; Roy Marquardt; BuAer Contract NOa(s) 8271; *Confidential*.

²⁷Reference 10, pp. 189-97.

²⁸Ibid — Chapter 13.

²⁹Massachusetts Institute of Technology, Cambridge 39, Massachusetts; E. S. Taylor, E. P. Neumann; BuOrd Contract NOrd 9661, T.O. A and D; *Confidential*.

tum, continuity and energy considerations. In order to observe the effects of the shock on the pressure measured at the tube wall, the pressure at the exit end of the tube is raised until the shock is introduced. These pressure measurements combined with Schlieren photographs indicate that the stream separates from the tube wall where the shock is created.

In order to predict the efficiency possible with simple supersonic diffusers for wind tunnel use, tests were conducted on five different diffusers at an entrance Mach number 2.32. Each diffuser was placed in a 1-inch tube 30.5 diameters from the open end; each diffuser had an exit diameter of 1.8-inches. The following results were obtained for the efficiency of each diffuser.

- A divergent cone with included angle of $2^{\circ}40'$ with an entrance diameter of 1-inch, —67% efficient.
- A 1-inch diameter tube, 10.5 inches long, followed by a divergent cone with an included angle of 6° , —70.5% efficient.
- A cone with included angle of 15° converging from 1-inch diameter to 0.9-inch diameter followed by a divergent cone with an included angle of 3° , —70.5% efficient.
- A cone with an included angle of 15° converging from 1-inch diameter to 0.883-inch diameter (minimum of diameter for starting) followed by a divergent cone with an included angle of 3° , —71.5% efficient.
- A cone with an included angle of 15° converging from 1-inch diameter to 9-inch diameter followed by a diverging cone with a 6° included angle, —75% efficient.

A diffuser of the Oswatitsch type is under design to supplement the studies of supersonic diffuser of the circular tube with boundary layer at the diffuser entrance.

The Menasco Manufacturing Company³⁰ has designed a ramjet which, it is believed, will operate more efficiently than the conventional ramjet. The improved performance is expected to result from the type of supersonic diffuser designed for this ramjet, and the control of the chamber pressure and normal shock wave by the adjustment of the area of the exit nozzle. The oblique shock pattern resulting from the diffuser, designed by the Menasco Manufacturing Company, was based on specifications supplied by the Langley Memorial Aeronautical Laboratory of the National Ad-

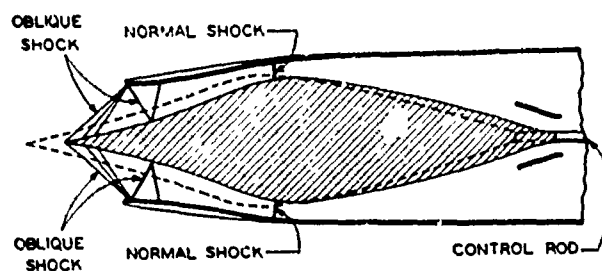


Figure 8. Sketch of adjustable spike diffuser for ramjet operation between Mach number 1.7 and 3.0. (Menasco Manufacturing Company)

visory Committee for Aeronautics, and is intended to provide efficient pressure conversion in a Mach number range of 1.7 to 3.0. The diffuser designed by Ferri (64) and used by Menasco, consists of a smooth curve, instead of a series of straight conical surfaces as in the original Oswatitsch design. The oblique shock pattern of such a diffuser is shown in Figures 8 and 9. In order to produce several oblique shock waves just inside the diffuser lip, the Menasco spike diffuser is designed to operate at Mach numbers from 1.7 to 3.0 through a fore and aft adjustment, which preserves the oblique shock wave interaction with the diffuser lip at the same position for various Mach numbers. The diffuser spike is automatically controlled (Figure 8) and actuated by pressure from the diffuser section acting on a piston contained in the aft of the central body. A tension-type control rod, through the center of the combustion chamber, serves also to control the opening of the variable exhaust nozzle. This control regulates the position of the normal shock wave just inside the diffuser lip; the spike position controls the movement of the diffuser spike and controls the oblique shock position just within the diffuser lip. A test proposal for a 1/15 scale model of this inlet diffuser provides background information, concerning its intended use and control (67). Qualitative design information for this type of diffuser was obtained through the use of water analogy studies at the Menasco plant. (See Section on Hydraulic Analogy Studies of this report).

The Flight Propulsion Research Laboratory of the National Advisory Committee for Aeronautics³¹ is conducting research on supersonic diffusers; this research includes perforated diffusers, spike diffusers, variable throat diffusers, and certain other diffusers combining the principles of the perforated and the shock diffusers.

³⁰Menasco Manufacturing Company, Burbank, California; Nathan C. Price; AAF Contract W33-038-ac-14759; Confidential.

³¹National Advisory Committee for Aeronautics, Flight Propulsion Research Laboratory, Cleveland, Ohio; Carlton Kemper; Sponsored by NACA; Confidential. (See also Annual Report of the NACA for 1946).

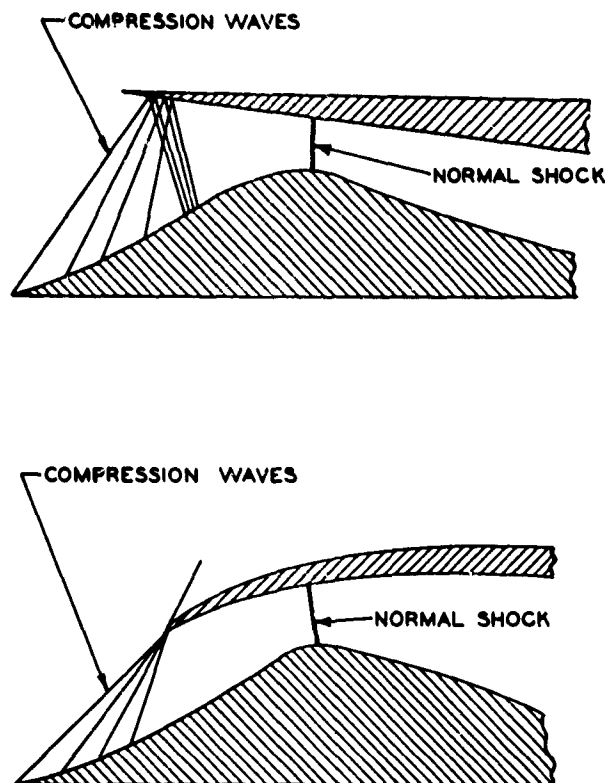


Figure 9. Possible shock configurations for conical diffusers operating at high Mach numbers. (After Clauser)

a. *Spike Diffusers.* A smoothly curved spike diffuser (Figure 9) designed by the method of characteristic, as outlined by Ferri (64), gave a total pressure recovery of 0.966 at the Mach number 1.85. Previous tests of a 30° total angle conical shock diffuser, in conjunction with a straight inlet, gave a pressure recovery point of 0.879. However, when the inlet to this diffuser was perforated, the conical shock was contained within the diffuser and the pressure recovery increased to 0.954. Additional experiments on oblique shock diffusers were continued to determine the pressure recoveries which may be obtained when the conical shock from the tip is confined within an unperforated inlet. Both of these pressures are higher than on the normal convergent-divergent diffuser, but they do not compare with the recoveries of the normal spike diffuser and the perforated diffuser.

b. *Perforated Diffusers.* The essentials of this type of diffuser have already been reported on by the National Advisory Committee for Aeronautics (74); a schematic drawing of a perforated diffuser is shown in Figure 10. Through the perforations in the inlet,

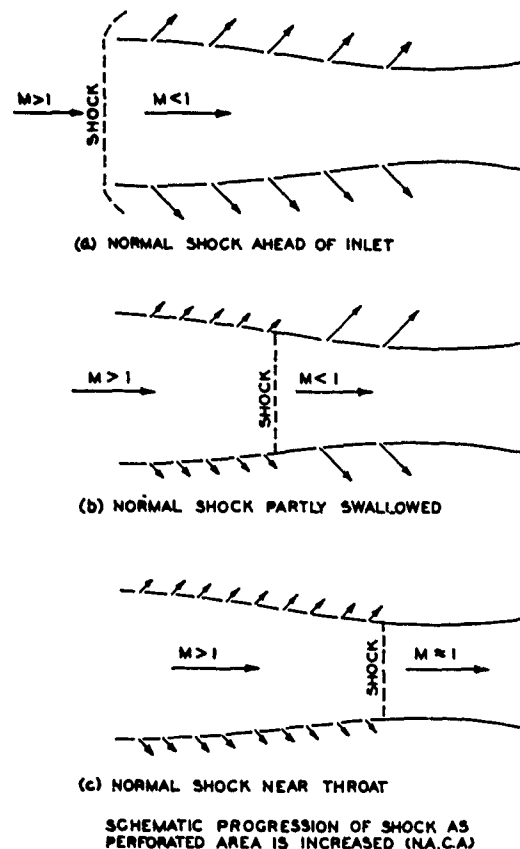


Figure 10. Relation of normal shock position to perforated area in perforated diffusers. (a) normal shock ahead of inlet, (b) normal shock partly swallowed, and (c) normal shock near throat. (National Advisory Committee for Aeronautics)

enough flow is conducted away through the wall of the diffuser to allow the detached shock to enter. When supersonic flow in the inlet has been established, the pressures, densities and velocities, before and after the shock wave is swallowed, indicate a marked decrease in the mass flow through the perforations. A preliminary model of the perforated diffuser, with an area ratio of 1.49 to 1, was tested at a Mach number 1.85. The pressure recoveries at angles of attack of 0° , 3° and 5° were 0.931, 0.920 and 0.908 respectively, compared with the highest recovery (0.838) obtained on the usual convergent-divergent diffuser of the same Mach number. Since the flow does not choke at the throat on the perforated diffuser, there is no essential discontinuity of mass flow as in the normal convergent-divergent diffuser. This allows the shock to be positioned at any place on the convergent portion of the diffuser.

c. *Variable Throat Diffusers.* A diffuser has recently been designed with a variable throat, to give a larger operating internal contraction ratio. This diffuser had unstable characteristics and the highest pressure point obtained was 0.891, as compared with 0.838 for the convergent-divergent diffuser and 0.966 for the best spike or external compressible diffuser.

d. *Other Diffusers.* A diffuser is being designed which combines the principles of the perforated and the shock diffuser in a multiple diffuser unit. The emphasis here is to obtain the shortest feasible unit for a given entrance area and Mach number. The multiple unit diffuser may possibly give a more satisfactory exit Mach number distribution for some purposes.

In addition to the foregoing work on diffusers, a program has been outlined for studying the drag of the various types of diffusers experimentally. A two-dimensional diffuser has also been designed for application to ramjets installed in wing position.

At the Langley Memorial Aeronautical Laboratory, Langley Field, the National Advisory Committee for Aeronautics³² is conducting a supersonic diffuser research program, in which preliminary experimental results have been obtained for a three-dimensional variable-geometry supersonic diffuser designed to operate at a Mach number range of 2.0 to 3.5. The configuration considered consists of a movable axially symmetrical central body and an annular circular inlet. A pressure recovery was obtained of the order of 0.8 for a Mach number 2.8, compared with the Oswatitsch fixed geometrical diffuser recovery of 0.65 for the same Mach number. This type of diffuser was developed by Ferri (63, 64) and enables low drag and high pressure recovery for a wide range of Mach numbers by varying the position of the central body or spike.

The Ames Aeronautical Laboratory³³, National Advisory Committee for Aeronautics, is undertaking studies on the effect of appreciable boundary layer on the pressure recovery in supersonic duct inlets. Preliminary tests through the Mach number range 1.36 and 2.01 have been completed in the 8 inch by 8 inch supersonic wind tunnel, and a report is in preparation. The annular duct inlets of the models received all of the boundary layer from the flow over comparatively long bodies. The effects of a variable entrance con-

traction ratio and of the shape of the bodies upon the character of this boundary layer flow were studied. Measurements of the total pressure recovery and the static pressure distribution, along the subsonic diffuser, together with Schlieren photographs of the flow about the entrance, were made at an angle of attack of 0° for a variation of the outlet-inlet area ratio of the models.

The results indicate a serious loss of total pressures arising from the interaction of the normal shock wave in the diffuser and the boundary layer from the body. The normal shock wave causes thickening and separation of the boundary layer ahead of the duct entrance. As a result, the total pressure recoveries are reduced to about 2/3 of that through a normal shock wave occurring at the same free-stream Mach number. Changing the boundary layer from laminar to turbulent, maintaining a favorable pressure gradient, or reducing the length of the boundary layer run by 50%, produced only a small change in the pressure recovery and the character of the flow about the entrance. Improvement can apparently be achieved by eliminating the interaction between the boundary layer and the normal shock; this means removing or diverting the boundary layer. Tests are being conducted upon models to achieve these ends.

In connection with the improvement of the Oswatitsch type diffuser, the New Mexico School of Mines³⁴ has conducted a series of spark shadowgraph tests on various diffuser noses for the North American Aviation, Inc. The results of these tests and a description of the spark shadowgraph techniques for obtaining flow patterns have been published (60).

The North American Aviation, Inc.³⁵ is preparing an article on supersonic diffusers for the Handbook of Supersonic Aerodynamics, published by the Applied Physics Laboratory of the Johns Hopkins University.

Analytical and theoretical studies of ramjet units have been undertaken during the last six months of 1946, and the preliminary design of a supersonic (intermittent) wind tunnel has been completed. Detailed theoretical design studies of ramjets, particularly in the range of Mach numbers 2.0 to 4.0, have been made on the basis of a spike diffuser. A more elegant generalized method has been developed for calculating the performance of any supersonic diffuser and using only a few generally permissible approximations for combustion pressure drop and nozzle efficiency (75).

³²National Advisory Committee for Aeronautics, Langley Memorial Aeronautical Laboratory, Langley Field; Research sponsored by the Compressibility Research Division, NACA; *Confidential*.

³³National Advisory Committee for Aeronautics, Ames Aeronautical Laboratory, Moffett Field, California; Research sponsored by NACA; *Unclassified*.

³⁴New Mexico School of Mines, Albuquerque, New Mexico; E. J. Workman; BuOrd Contract NOrd 9817, T.O. C; *Restricted*.

³⁵North American Aviation, Inc., Inglewood, California; W. Bollay; BuOrd Contract NOrd 9784; *Confidential*.

The Polytechnic Institute of Brooklyn³⁶ has started an investigation of the fundamentals of supersonic flow in and around propulsion devices. This phase is particularly pointed toward research on the mechanics, thermodynamics, and molecular kinetics of shock waves in laminar and turbulent flow. The general three-dimensional dynamical equations of flow, the continuity equation, the equation of state, and the equation of heat balance in a steady flow have been set up for a perfect gas, including viscosity, conduction, and convection. The one-dimensional case is under investigation at present.

3. *Supersonic Nozzle Studies.* The Massachusetts Institute of Technology³⁷ is sponsoring research on the flow through supersonic nozzles and on supersonic nozzle design under the direction of Prof. A. H. Shapiro. In connection with the supersonic wind tunnel, a study has been made of some of the problems bearing on the design of two-dimensional supersonic nozzles for producing uniform, parallel streams of air. Several of these nozzles for use at Mach numbers 1.5 and 2.0 were designed after the method developed by Foelsch (65). This has been reported by Shapiro, Edelmann, and Snyder (78); the contours of supersonic nozzles are given up to a Mach number of about 5.0 in intervals of Mach number 0.1 and for two maximum half-angles of divergence, namely 5° and 10°. Tables have been prepared giving the functions commonly employed in conjunction with the method of characteristics. Other fundamental studies on the flow through supersonic nozzles at MIT are treated by Shapiro (79), and by Keenan and Neumann (70).

The thesis studies undertaken at MIT on the phenomena of supersonic flow through nozzles include:

a. *Schlieren Observations of Supersonic Discharge.* This was initiated to study the effect of Mach number variation and the change in boundary layer thickness at one Mach number on the discharge of a supersonic air stream. Schlieren photographs, together with pressure measurements, were made at the discharge of a passageway of rectangular cross section for both supersonic and subsonic discharge velocities. The experimental results showed that the pressure of the region into which a supersonic jet discharges can affect conditions upstream of the nozzle exit by action through the boundary layer.

b. *Investigation of the Condensation Shock in Air by Use of the Schlieren Method.* This was a study of the effect of variation of moisture content in the air on the formation of a condensation shock in a supersonic stream. The flow through a single convergent-divergent nozzle of rectangular cross section was studied; the humidity of the inlet air, varied by means of a dehumidifier, was measured with a wet and dry bulb thermometer. The position of the condensation shock in the nozzle was determined by means of Schlieren observations. Pressures were measured at eight stages along the wall of the nozzle.

For values of the specific humidity varying from 0.00581 to 0.0159, with a given nozzle inlet temperature, the distance of the condensation shock downstream of the throat varied inversely as the specific humidity of the air. The measurement of pressure rise across the condensation shock is only approximate, due to the relatively large spacing between the pressure taps. There is an indication, however, that this pressure rise varies directly as the specific humidity for a given inlet pressure. Analytical work to predict the size of water drops, formed during condensation, indicated that the drop size for these experiments was between 2×10^{-9} to 5×10^{-9} feet. These results appear to be questionable since the surface tension may have no significance for drops containing only a relatively few molecules.

c. *Experimental Study of the Flow in a Nozzle Designed to Provide a Parallel Jet at Supersonic Speed.* This research was undertaken for the purpose of designing a passageway, based on the Prandtl-Meyer analysis³⁸ for two-dimensional expansion of a gas flowing around a corner at supersonic velocities. A nozzle of rectangular cross section with glass side walls, was built on the basis of the theory of Prandtl and Meyer. The flow was bounded by two streamlines somewhat removed from the actual corner. Schlieren photographs and pressure distribution along the walls, which are defined by the theory as streamlines, were obtained.

The only shock observed in the flow was that caused by the condensation of water in the nozzle. An effort was made to determine whether the flow in this passage behaved as though it were flowing around a corner. The corner was to be determined by drawing a straight line through points of equal pressure, at the top and bottom wall. It was difficult to form dependable conclusions regarding the nature of the flow, owing to the presence of the condensation shock, and the fact

³⁶Polytechnic Institute of Brooklyn, Brooklyn, New York; R. P. Harrington; ONR Contract N6ori-98, T.O. 2; Project SQUID; Unclassified.

³⁷Massachusetts Institute of Technology, Cambridge, Massachusetts; Thesis Investigations in Gas Turbine Laboratory.

³⁸Reference 10, page 189.

that the subsonic portion of the nozzle probably did not provide a Mach number 1 line which was straight and located at the position assumed in the design. For the case studied, however, the lines drawn through equal points of pressure did not intersect at a point.

The Naval Ordnance Laboratory³⁰ has a program to study rocket blast characteristics and is carrying on applied experimental and theoretical research on the exhaust blast characteristics of liquid fuel and double-base solid propellant rocket motors. It is planned to assemble, verify, and compile data for use in the design of aviation ordnance rocket conveying, supporting, launching, and controlling equipment.

The immediate objectives include certain flow studies on the velocity of the exhaust gases in ducts and in free air, the investigation of shock wave phenomena, the study of turbulence and temperature distribution as affected by the use of rough wall exhaust tubes of a flexible type, and the study of pressure and blast surrounding nozzles and breeches of recoilless guns. Attention is being given to the problem of oblique reflection of shock waves, in connection with a study of the interaction of oblique shock waves with a boundary layer. Schlieren photographs are being studied to determine the correct interpretation of reflection phenomena under the existing non-linear conditions.

VII. HYDRAULIC ANALOGY STUDIES

A. Introduction

The hydraulic analogue to two-dimensional air flow has become a useful and attractive experimental tool for studying both subsonic and supersonic flow problems. The water analogy technique is a simple and inexpensive method for making field surveys and for studying flow conditions where turbulence, boundary layer, shock waves, and vortex motion are present. The technique is admittedly not well perfected. Much work has to be undertaken to allow quantitative comparison of the effects observed in liquid flow, with the two-dimensional flow within or around similarly shaped bodies. The question of model scale factor in the water flow experiments demands careful attention, in order that the comparisons drawn with compressible fluid flow are valid. The equipment exists essentially in two forms: one in which the water actually flows through a channel simulating the flow of a compressible fluid through a duct of varying cross sectional area; the other is a water table in which surface waves are generated and their interactions simulate the intersection of sound or shock waves in air. The theory for the water analogy to air flow through a convergent-divergent nozzle was developed by Riabouchinsky (93); Preiswerk (92) showed that the analogy holds for water with a free surface. Recently Orlin, Lindner and Bitterly (90) have reviewed the entire water analogy technique and have described the water analogy studies at the Langley Memorial Aeronautical Laboratory of the National Advisory Committee for Aeronautics at Langley Field, Virginia. The presentation here follows that in this NACA publication

(90). Additional treatments of the analogy may be found in textbooks on fluid mechanics (84). An extensive exposition of the shallow water theory is given by Stoker (95) in a paper about to be published.

For water flow under gravity, the energy equation in the form of Bernoulli's theorem determines the velocity of flow:

$$v^2 = 2g (d_0 - d) \quad (1)$$

The maximum velocity of flow is therefore:

$$v_m^2 = 2gd_0 \quad (2)$$

A similar equation for the velocity of flow of a compressible gas,

$$v^2 = 2gC_p (T_0 - T) \quad (3)$$

with

$$v_m^2 = 2gC_p T_0 \quad (4)$$

allows a comparison of the ratio of (v/v_m) for water and for a gas. By equating these ratios, it is seen that

$$\frac{d}{d_0} = \frac{T}{T_0} \quad (5)$$

where the depth ratio from Torricelli's law is analogous to a temperature ratio in compressible flow. The equation of continuity for both water and gas flow

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (6)$$

where $\rho \neq f(t)$ shows the analogy between gas density and depth.

$$\frac{d}{d_0} = \frac{\rho}{\rho_0} = \frac{T}{T_0} \quad (7)$$

For adiabatic conditions,

$$pV^\gamma = \text{constant} \quad (8)$$

and from the general gas equation

$$pV = RT \quad (9)$$

³⁰Naval Ordnance Laboratory, U. S. Naval Gun Factory, Washington 25, D. C.; Dr. Ralph Bennett; Aviation Ordnance Project 408; Confidential.

it is easily shown that

$$\frac{p}{p_0} = \left(\frac{T}{T_0} \right)^{\frac{1}{\gamma-1}} \quad (10)$$

where $\gamma = C_p/C_v$, the ratio of the specific heats at constant pressure and constant volume. From Equations (7) and (10) it is obvious that the specific heat ratio, γ , must be 2 for equivalence of incompressible and compressible systems. With $\gamma = 2$ the pressures and depths are related by

$$\frac{p}{p_0} = \left(\frac{d}{d_0} \right)^2 \quad (11)$$

By comparing the velocity potential relations for water and for a gas, the velocity $v = \sqrt{gd}$ in the liquid is found to correspond to the velocity of sound in the gas, so that the ratio of $v/\sqrt{gd}$ is the liquid analogue of the Mach number. For water flow where $v < \sqrt{gd}$ the condition of flow is analogous to subsonic gas flow ($M < 1$); for water flow where $v > \sqrt{gd}$ ($M > 1$) the condition is analogous to supersonic flow. The special condition where $v = \sqrt{gd}$, the case of a hydraulic jump, corresponds to a shock wave in compressible gas.

From the theory of the hydrodynamics of perfect fluids (91), the relation for the velocity of gravity waves in a fluid

$$v = \left[\frac{g\lambda}{2\pi} + \frac{2\pi\sigma}{\rho\lambda} \tanh \frac{2\pi d}{\lambda} \right]^{\frac{1}{2}} \quad (12)$$

where λ is wavelength, σ the surface tension, and d the depth of the channel. The first term in parentheses is due to gravity waves when the wavelength of the disturbance is long in comparison with the depth;

$$\text{for } d \ll \lambda, \tanh \frac{2\pi d}{\lambda} \approx \frac{2\pi d}{\lambda} \text{ and} \\ v = \sqrt{gd} \quad (13)$$

The second term in brackets is the contribution to the velocity of the surface tension σ of the fluid. This effect is important only for the short waves while the gravity contribution, on the other hand, controls the velocity of the long waves.

Since there is no condition in the flow of gases analogous to a surface tension, which is a condensed phase phenomenon, the surface waves arising in fluid flow analogy studies are influences perturbing to the gravity wave studies and to precise measurements on the wavelength and depth of the phenomena below the

surface. The effect of ripples may be suppressed by introducing into the water chemical agents which decrease the surface tension; in practice the ripple effects do not appear to be a serious handicap and, in some cases, are found to be quite useful in the interpretation of the results.

B. Research in Progress

The National Advisory Committee for Aeronautics is conducting studies at the Langley Memorial Aeronautical Laboratory on the water analogy to two-dimensional compressible gas flow. A water channel was designed and constructed in 1940 in the 8 foot highspeed wind tunnel at Langley Field. Subsonic and supersonic velocities about aerodynamic bodies can be studied in this channel by the analogy method. The apparatus and the techniques used, together with applications of this method to flow through nozzles and about circular cylinders, have recently been reported in some detail (90). Tests were made with these shapes at various depths, Mach numbers, and with several model sizes. Some trouble was encountered with surface waves (ripples) arising through the influence of surface tension. Although the ripple perturbation was not serious, it decreased the accuracy of measuring the depth at which the flow phenomena were studied. The vertical component of accelerations produced by flow around or along bodies in the stream distort the flow from that due to gravity and create conditions which depart from the analogy. It is pointed out that the slope of the free fluid surface is a measure of these accelerations. Boundary layer development can also be a serious factor in disturbing the analogy flow conditions because of the boundary layer displacement thickness (0.1") which is appreciable compared with the total depth of fluid in the channel. Other effects such as the allowance for the fact that the analogy holds for gas with a specific heat ratio of 2 rather than 1.4, for the choking effect on the channel of various sized models, and for pressure gradients developed by boundary layer formation have been discussed (90).

Hydraulic analogy studies are being conducted in the Mechanical Engineering Department at the Massachusetts Institute of Technology under the direction of Prof. Ascher H. Shapiro.⁴⁰ A water channel with a test section 12 inches wide and 2 inches deep has been fitted with a convergent-divergent nozzle for a Mach number of two.⁴¹ This equipment is being used

⁴⁰Massachusetts Institute of Technology, Cambridge, Massachusetts; A. H. Shapiro; BuOrd Contract 9661; Project METEOR; Confidential.

⁴¹Reference 71, pages 20 and 21.

to undertake qualitative analogy studies to two-dimensional flow of compressible fluid moving faster than the velocity of sound. The present work has been of an exploratory nature in determining experimental procedures for handling the flow in the channel and in making the flow patterns visible. The apparatus to be used in making these flow patterns visible will include shadowgraph and polarization techniques, shear patterns and the photographing of dye filaments. Depth measuring equipment will be installed so that quantitative analogy measurements may be obtained.

The New York University⁴² contemplates the use of hydraulic analogues in conjunction with the study of gas motions in pulsating jets and rockets in order to determine the characteristics of simple theoretical models. These analogy studies relate particularly to research in flame and particle motion (89).

R. T. Knapp and H. A. Einstein⁴³ are conducting water analogy studies in the Ripple Tank of the Hydrodynamics Laboratory, California Institute of Technology. This is a study of the analogy between shock waves on the surface of liquids and density shocks in compressible gases (88). Optical methods have been developed to study the two-dimensional wave problems including the interaction, reflection, diffraction, spreading, and dissipation. An optical method has been developed for determining the height of the shock wave. Preliminary photographic results indicate that the surface wave interactions are of the Mach type. Figure 11 shows the Mach wave produced by two intersecting "shocks". Note the turbulent region behind the Mach wave.

A new method has been developed under the second part of this project, for determining the height of any point in the surface of a wave in relation to the time after initiation of the wave. This method makes use of the recording oscillograph; with this equipment the measurements on waves of medium height indicate an error of approximately $\pm 3\%$. The height measurements can be used to check wave velocities and to enable contour maps to be prepared giving the surface elevations for any type of wave. A set of equations for the regular and the Mach type of wave interactions has been developed. A general solution has not been found for the Mach wave interactions and the experimental work of Einstein indicates that it is most important that a suitable theory for this case be developed.



Figure 11. Photograph of the intersection of two linear water waves to form a Mach reflection. (California Institute of Technology)

The Menasco Manufacturing Company⁴⁴ has made use of water flow analogy technique to study spike (Oswatitsch) diffusers for a ramjet development (86). Preliminary experiments were conducted using this analogy to determine shock wave positions and characteristics. Water was pumped past bodies whose dimensions simulated those of the ramjet (Figure 12). The water model test, however, revealed discrepancies between the results indicated from the action of the water on the model and a practical consideration of the flow of air through such a model. The discrepancies could be explained in part as due to the difference between the two-dimensional character of the flow in water and the three-dimensional character of the flow in air. Menasco plans to make a quantitative comparison of the two-dimensional flow data in water with similar data obtained in wind tunnel tests of a 1/15th-scale model of the diffuser (87).

The Cornell Aeronautical Laboratory⁴⁵ is planning to investigate water analogy techniques for application to pulse jet systems with particular regard to a study of the system along lines suggested by Schultz-Grunow method (94).

North American Aviation, Inc.⁴⁶ has made preliminary studies of flow patterns in and around diffusers, using water analogy techniques (85). The

⁴²New York University, New York, New York; J. K. L. MacDonald; ONR Contract N6ori-11, T.O. 2; Project SQUID, Unclassified.

⁴³California Institute of Technology, Pasadena, California; R. T. Knapp; BuOrd Contract NOrd 9612; Restricted.

⁴⁴Menasco Manufacturing Company, 805 South San Francisco Boulevard, Burbank, California; Nathan C. Price; AAF Contract AFP 377231-W-33-038-ac-14759; Confidential.

⁴⁵Cornell University, Cornell Aeronautical Laboratory, Buffalo, New York; J. V. Foa, ONR Contract N6ori-119, T.O. 1; Project SQUID; Unclassified.

⁴⁶North American Aviation, Inc., Inglewood, California; William Bollay; Aerophysics Laboratory.



Figure 12. Photograph of model of smooth curved spike diffuser for hydraulic analogy studies. (Menasco Manufacturing Company)

water analogy equipment is a shallow basin, about 4 feet wide by 20 feet long traversed by a light carriage, on which models are mounted. The preliminary studies with this equipment indicate that it is very useful for an informal test program for quantitative study of supersonic flow, that since high supersonic Mach numbers are simulated at only a few feet per second, visual observation of these transient phenomena may be made, and that observations are also possible for fluid flow under condition of changing speed. The equipment will be used extensively for quantitative observations which may assist in the understanding of supersonic phenomena. One problem already being investigated is the conditions under which an oblique shock becomes detached. Preliminary observations have indicated the possibility of "attached normal shocks" which are not predicted by the theory of oblique shocks. Information of this kind will be used in particular with reference to the design of supersonic inlet diffusers.⁴⁷ Figure 13, taken from reference 85, shows the type of photograph representative of



Figure 13. Photograph of hydraulic analogue to shock pattern for conical diffuser for Mach number 3.56. (North American Aviation, Inc.)

shock wave analogy obtained in a reflected wave diffuser operating at a simulated Mach number 3.56. Results of the research to determine the optimum conditions of water depth and model size are also presented.

The contribution of the hydraulic research at the University of Iowa to the certain ballistic problems should be mentioned here as an illustration of the usefulness of the analogy concept. R. N. Thomas of the Ballistics Research Laboratory at the Aberdeen Proving Grounds, Maryland, devised a drag coefficient⁴⁸ which is applicable to a broad class of projectiles in the supersonic region. The discovery of this function was stimulated by some hydraulic research conducted by Prof. Hunter Rouse⁴⁹ at the Iowa Institute of Hydraulics Research.

Rouse sent some results to Thomas on studies of the

⁴⁷See Reference 76 for reports by NAA on Supersonic Inlet Diffusers.

⁴⁸Reference 4, page 157.

⁴⁹The Iowa Institute of Hydraulics Research, University of Iowa, Iowa City, Iowa.

characteristics of flow past an object in an open water channel. On analyzing these measurements Thomas found that the square root of the pressure acting on the front of the object changed linearly with

a hydraulic parameter $(v/\sqrt{gd})$ corresponding to the Mach number in gas flow. The analogy allowed a drag function to be built up taking into account the nose pressure variation with Mach number.

VIII. TURBULENCE

A. Introduction

By the term *turbulence* is meant the departure of the motion of a fluid from a flow pattern which can be described by a scalar or vector function of position in space. The velocity of fluid particles in stream-line or laminar flow can be derived from a scalar potential function which identifies the velocity of a particle with a position in space. For every coordinate of the space in which fluid flow exists, there is associated with that position a definite particle velocity which is not time-dependent. This flow is usually described as stream-line flow; stream-lines are curves in a fluid which are everywhere parallel to the particle velocity. If the fluid particle velocity has components u , v , and w along directions x , y , and z , the differential equations describing the stream-lines are:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \quad (14)$$

If the fluid particles have an angular velocity ω , the flow is described as vortex motion. A vortex line is a curve which has everywhere the direction of the axis of rotation of the fluid particles. The differential equations describing the stream-lines are:

$$\frac{dx}{\xi} = \frac{dy}{\eta} = \frac{dz}{\zeta} \quad (15)$$

where ξ , η , ζ are the components of ω . For ideal fluids in which the density is a function only of the pressure, and the external forces acting on the fluid elements are conservative, Lord Kelvin⁵⁰ has shown that there can be no angular velocity in a fluid originally irrotational, and that if an initial circulation is present, it will never diminish in intensity. If, however, the density is not a function simply of the pressure, if the external force is non-conservative, and if there are non-conservative forces in the fluid due to viscosity, vortex motion may be set up in a fluid which was originally irrotational.

Turbulent flow differs from laminar flow in that it refers to fluid particle motion which is non-linear and

transient. Turbulent flow should not be construed to mean simply vortex motion since, as we have seen above, it is possible for steady rotational motion to exist in a fluid where the velocity and rotation of the fluid elements may be described by stream or current functions of position in space alone. We are therefore interested in conditions which depart from steady linear or angular velocity in a fluid. Those departures may arise in volume elements of a fluid remote from its geometrical boundaries. This is frequently referred to as "free stream turbulence." The steady flow of a fluid passing across or near to a geometrical boundary may be altered from stream-line flow to vortex motion in an irregular manner. This gives rise to a phenomenon known as boundary layer turbulence. The conditions under which the transition from potential to turbulent flow takes place were first investigated by Osborne Reynolds⁵¹ and have been the subject of many scientific investigations during the past 65 years.

Since turbulence relates to a non-linear transient fluid flow, the particle motion is difficult to treat analytically. The rapid fluctuations of velocity occlude the exchange of both intensive and extensive quantities between fluid elements in fluctuation and fluid elements moving with the mean velocity of the stream. The kinetic theory of gases explains the nature of the exchange of these quantities in terms of the mean motion of molecules, equilibrium conditions of pressure, density, and temperature. The collision frequency between molecules, the distance traversed by a molecule between collisions, the coefficients of viscosity, diffusion, and thermal conductivity are all based on the principle of equipartition of energy and can be described in terms of molecular motions only under equilibrium conditions. The transient departure of fluid elements from equilibrium conditions and from flow derived from a potential require modifications to be made in the classical theory to explain the exchange of quantities between fluid elements in turbulent flow. In this report we shall be interested primarily in the

⁵⁰Reference 91, pp. 229-232.

⁵¹Reference 9, pp. 664-666 and reference 51.

effect of turbulent motion on the transfer of kinetic energy, momentum, and stress between fluid elements of a turbulent flow.

To express the departure from steady flow due to fluctuations in particle velocity, it will be necessary to examine briefly the dynamical relations which obtain for the steady motion of a viscous fluid. These include the force equation, the equation of continuity and the equation of state of the fluid. The force equation for a unit volume of the fluid may be written down as follows:

$$\rho \frac{d\mathbf{V}}{dt} = \rho \mathbf{F} - \nabla p + \frac{1}{3} \mu \nabla \nabla \cdot \mathbf{V} + \mu \nabla \cdot \nabla \mathbf{V} \quad (16)$$

where $\mathbf{V}$ is the velocity vector, ρ the density, $\mathbf{F}$ the external force per unit mass, p the pressure, μ the viscosity, and ∇ the differential operator

$$\nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z}. \quad (17)$$

$\nabla \mathbf{V}$ is the rate of strain dyadic.

The remaining two relations which must be satisfied to allow a determination of $\mathbf{V}$ are the equation of continuity,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \quad (18)$$

and the equation of state,

$$f(p, \rho, T) = 0 \quad (19)$$

where T is the temperature.

To the steady fluid velocity $\mathbf{V}$, let us add a fluctuating velocity $\mathbf{V}'$ with components u' , v' , and w' ; assume that $\mathbf{V}'$ satisfies the continuity equation. We shall average the fluctuating velocity $\mathbf{V}'$ over times large compared with the period and wavelength of the fluctuations but small in comparison with times and distances appropriate to the mean motion $\mathbf{V}$, after the method of averaging these fluctuations developed by Reynolds.

Recalling in equation (16) that

$$\frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \quad (20)$$

the force equation with the added fluctuating velocity $\mathbf{V}'$ becomes

$$\frac{\partial(\mathbf{V} + \mathbf{V}')}{\partial t} + \rho(\mathbf{V} + \mathbf{V}') \cdot \nabla(\mathbf{V} + \mathbf{V}') = \rho \mathbf{F} - \nabla p + \frac{1}{3} \mu \nabla \nabla \cdot (\mathbf{V} + \mathbf{V}') + \mu \nabla \cdot \nabla (\mathbf{V} + \mathbf{V}') \quad (21)$$

In the method of averaging according to Reynolds, the only terms remaining are those quadratic in the fluctuating velocity, since $\overline{\mathbf{V}'}$ is by definition zero; the cross product of $\overline{\mathbf{V}'}$ with the mean velocity $\overline{\mathbf{V}}$ is also zero. (The bar over the quantity denotes this average.)

Therefore (21) becomes

$$\rho \overline{\mathbf{V}' \cdot \nabla \mathbf{V}} + \rho \overline{\mathbf{V}' \cdot \nabla \mathbf{V}'} = \rho \mathbf{F} - \nabla p + \frac{1}{3} \mu \nabla \nabla \cdot \overline{\mathbf{V}} + \mu \nabla \cdot \nabla \overline{\mathbf{V}} \quad (22)$$

and the effect of a velocity fluctuation $\mathbf{V}'$ is to add a force per unit volume equal to $-\rho \overline{\mathbf{V}' \cdot \nabla \mathbf{V}'}$. In scalar form the components u' , v' , and w' contribute normal stresses $-\rho \overline{u'u'}$, $-\rho \overline{v'v'}$ and $-\rho \overline{w'w'}$ and shearing stresses $-\rho \overline{u'v'}$, $-\rho \overline{u'w'}$ and $-\rho \overline{v'w'}$. These are referred to as *Reynolds stresses* and are due to the turbulent fluid motion.

From their similarity to the stresses due to molecular viscosity, Reynolds, G. I. Taylor, and others have treated these contributions as due to an "eddy" viscosity ϵ which is assumed to satisfy six scalar equations of the form:

$$\begin{aligned} -\rho \overline{u'u'} &= 2\epsilon \frac{\partial \overline{u}}{\partial x} - \frac{2}{3}\epsilon \left(\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} \right) \\ -\rho \overline{u'v'} &= \epsilon \left(\frac{\partial \overline{u}}{\partial y} + \frac{\partial \overline{v}}{\partial x} \right) \end{aligned} \quad (23)$$

If these relations are used in conjunction with the force equation, it is possible to describe turbulence by treating the sum of the molecular and eddy viscosity coefficients as an effective viscosity coefficient $(\mu + \epsilon)$.

It is natural that the early theories of turbulence drew heavily upon concepts developed in the kinetic theory of gases. Since the transport of extensive and intensive quantities between fluid elements must arise through the motion and the interaction of particles, the turbulence theories have a formal similarity to the kinetic theory to which, of course, the equations of motion must reduce as the fluctuations go to zero. The fluctuations with which we are concerned here stand in the same relation to the mean fluid motion as the molecular fluctuations to the root mean square speed of molecules in the kinetic theory of gases. To be observable, however, the velocity fluctuations in turbulence must include large numbers of molecules which are moving together in translation or in rotation. This implies that turbulence is, in part, a cooperative phenomenon and that departures from the mean fluid flow involving large numbers of molecules moving together are to be treated in a manner analytically different from the molecular fluctuations on the kinetic theory of gases.

Some of the analytical methods for treating these fluctuations have been named with reference to the quantities transported by turbulent flow, such as the mixture length theories, the momentum transfer theory, and the general vorticity and modified vorticity transfer theories; the remaining methods in-

clude the similarity theory and the statistical development of turbulence theory, which are more general in character.

Turbulence is described by two parameters, the intensity of turbulence and the scale of turbulence. In isotropic turbulence, where $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{w'^2}$ are equal, the intensity is defined by $\overline{u'^2}$ or the intensity level $\sqrt{\overline{u'^2}}/U$ where U is the mean velocity of the fluid in the x direction. The scale of turbulence has the units of length and is a measure of the average size of the eddies giving rise to dissipation of energy if a space average of the fluctuating velocity is taken; the scale is analogous to a mixture length if a time average of the fluctuating velocity is taken at one point. Since the scale depends upon the relation between the components of the fluctuating velocities, their correlation is an important parameter in defining the scale quantitatively.

The coefficient of correlation between u' and v' , for example may be defined as

$$R_{u'v'} = \frac{\overline{u'v'}}{\overline{u'_r v'_r}} \quad (24)$$

where u'_r and v'_r are the root mean square fluctuating velocity components along the x and y directions respectively. It is also convenient to obtain correlation between the fluctuating components at position (1) and at position (2) in the fluid at the same time. This is the basis of one statistical representation of turbulent flow. Let u' , v' , and w' , be the components of the fluctuating velocity at position (1) and u'_2 , v'_2 , and w'_2 be those at position (2) at the same instant. Then the coefficient of correlation may be written as

$$R_y = \frac{\overline{u'_1 u'_2}}{\overline{u'_{1,r} u'_{2,r}}} \quad (25)$$

where $u'_{1,r}$ and $u'_{2,r}$ are the root mean square fluctuating velocity components along x at points (1) and (2) respectively; y is the distance between points (1) and (2) and its axis lies along the direction connecting the points. If points (1) and (2) are close together the velocities u'_1 and u'_2 are closely correlated; the degree of correlation is expected to decrease with increasing. The dependence of R_y on y may be seen on the curve of Figure 14 which represents data for turbulence produced in an airstream of velocity 15 ft./sec. passing over a grid of square meshes 3 inches x 3 inches. R_y is zero at 2.3 inches. For similar grids of different sizes the scale of the turbulence may be assumed to be proportional to the sizes of these grids and the scale of the coefficient of correlation a measure of the scale of turbulence. The scale of R_y may therefore be taken to

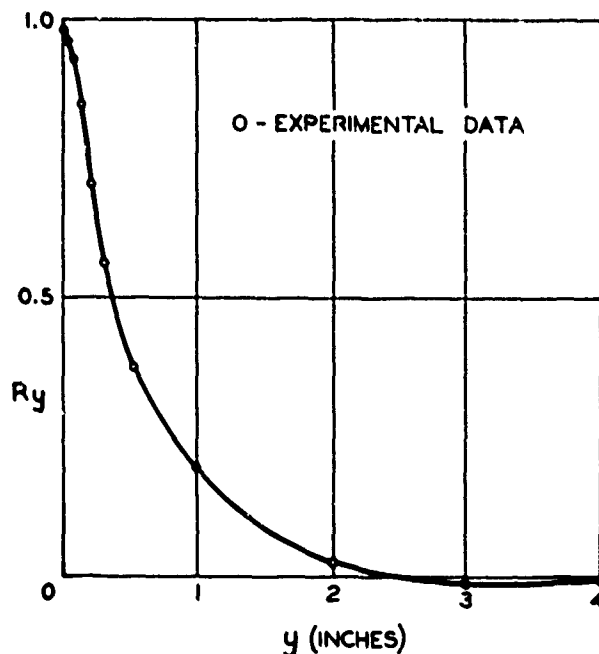


Figure 14. Graph showing relation between correlation coefficient R_y and y . (After Goldstein)

represent a length, the scale of turbulence, where

$$1 = \int_0^y R_y dy \quad (26)$$

y is the length at which R_y is zero, or where there is no correlation between u'_1 and u'_2 . In the dissipation of isotropic turbulence l may be considered to represent the average size of the eddies.

There is a close connection between the coefficient of correlation $R_{u'v'}$ (Equation 24) and the Reynolds stresses of the form $-\rho \overline{u'v'}$, $-\rho \overline{v'w'}$, and $-\rho \overline{u'w'}$ since, in the case of u' and v' ,

$$-\rho \overline{u'v'} = -\rho u'_{1,r} v'_{1,r} R_{u'v'} \quad (27)$$

The stresses contributed by the turbulent flow provide a mechanism by which one element of the fluid may transmit to a neighboring element normal and shearing forces in addition to those due only to molecular velocity fluctuations. Also the relation of the coefficient of correlation to a length, such as l in Equation 26, makes it attractive to connect the range of these turbulent stresses to a characteristic length in the fluid. In moving through a distance l , for example, the perturbed fluid element is considered to transport quantities such as vortex motion, momentum or energy; at the end of this distance this element is then in equilibrium with the mean motion, having dissipated the excess vorticity momentum or energy along the path l .

In the momentum transfer theory of turbulent flow the early theoretical work found some measure of success in carrying into the field of turbulence the classical relationship between shearing stress and the gradient of the mean velocity, namely

$$-\rho \bar{u'v'} = (\mu + \epsilon) \frac{dU}{dy} \quad (28)$$

where μ and ϵ are the molecular and "eddy" viscosities, respectively, and U is the mean fluid motion along x . From Equation 28, the mean rate of transfer of momentum per unit volume may be derived; this is related to a characteristic length and to the coefficient of correlation between the fluctuating velocities involved. The dependence of the mean velocity on the virtual viscosity led Prandtl to assume an ad hoc relationship between the Reynolds stresses and certain derivatives of the mean motion. For momentum transfer other assumed relationships have been introduced

connecting the virtual stresses to $\frac{dU}{dy}$ and $\frac{d^2U}{dy^2}$. It

must be emphasized, however, that such relationships are not firmly founded, that they are based primarily on phenomenological arguments and that turbulent theories using them do not produce generally satisfactory agreement with experiment.

In the two-dimensional vorticity transfer theory, where vortex motion is considered to be conserved, the theory for the rate of momentum transfer per unit volume leans heavily on connecting the vorticity with the gradient of the mean flow. The characteristic length l in this theory differs from that in the momentum transfer theory by a factor $\sqrt{2}$. Again, however, the relation of shearing stress to the derivatives of the mean flow does not appear to be firmly established.

Experimental measurements on velocity profiles in flow through channels and pipes provide an opportunity to compare the momentum and vorticity transfer theories. For a pipe the vorticity transfer theory predicts velocities more nearly in accord with the measurements than does the momentum transfer theory; for channels the two theories give approximately equivalent agreement. (See Figures 15 and 16.) The sensitivity of agreement to the geometrical configuration suggests again that these theories are not general in nature.

Theodore von Karman has approached the problem of the transfer of quantities in turbulent flow by as-

suming that the eddy flow for regions at the same coordinate y , for example, are similar and differ only by scale and velocity with reference to a coordinate system moving with the mean velocity of the fluid. This theory is known as the similarity theory; it has been applied so far only in two-dimensional form. The eddy stress calculated by von Karman is the same as in the momentum transfer theory based on the local value of the mean motion. It has been stated recently (39) that the turbulent shearing stress may not be related to the local mean motion in the form suggested by Prandtl and by von Karman. Furthermore, it may turn out that the stress cannot be derived at all from the mean motion. The success of the similarity theory in agreement with some of the experimental work shows, however, that the concept of similarity is a useful one.

It has been seen that the effect of a fluctuating velocity was to add a virtual viscosity coefficient to the molecular viscosity coefficient in the shearing stress between elements of the fluid. In heat transfer the turbulent velocity also adds an eddy heat transfer term to the molecular conductivity so that turbulent motion promotes a higher rate of heat transfer between fluid elements themselves or fluid elements and the boundary of the flow than the molecular conductivity under laminar flow conditions. Explicitly consider the effect of adding a fluctuating V' to V in connection with the basic temperature relation

$$\rho C \frac{dT}{dt} = k \nabla \cdot \nabla T \quad (29)$$

and the equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0 \quad (30)$$

where C is the specific heat, ρ the density, k the molecular thermal conductivity and T the temperature. We recall again that the operator

$$\frac{d}{dt} = \frac{\partial}{\partial t} + V \cdot \nabla \quad (31)$$

and hence

$$\rho C \frac{dT}{dt} = \rho C \left\{ \frac{\partial T}{\partial t} + V \cdot \nabla T \right\} \quad (32)$$

Let us add to V the fluctuating velocity V' and to T the fluctuating temperature T' . Then the temperature equation becomes

$$\rho C \left\{ \frac{\partial (T + T')}{\partial t} + (V + V') \cdot \nabla (T + T') \right\} = k \nabla \cdot \nabla (T + T') \quad (33)$$

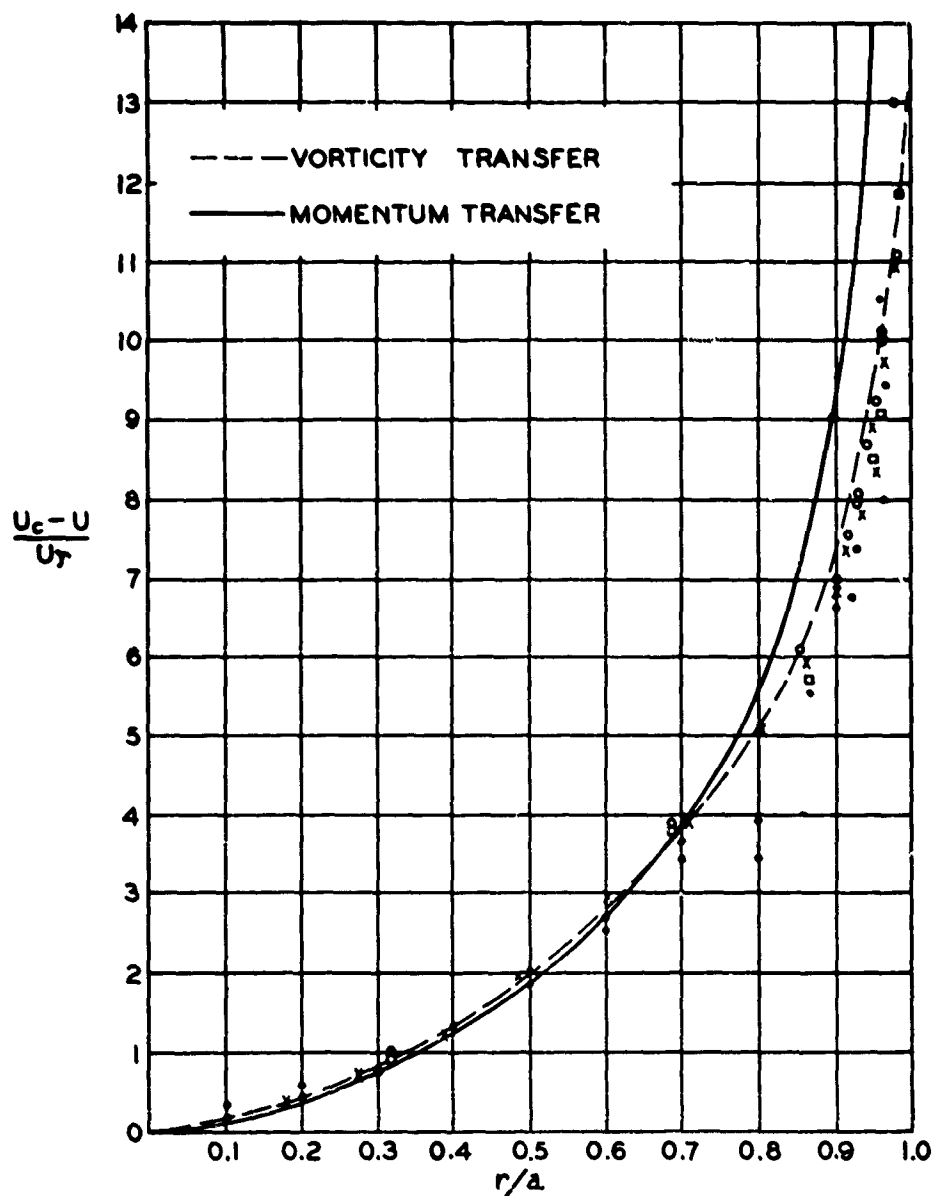


Figure 15. Velocity profiles for turbulent flow in a pipe. Comparison to experimental data with vorticity and momentum transfer theories where U is the velocity at a distance r from the axis of the pipe, U_c is the velocity at the center of the pipe and $U_r = (\tau_o/\rho)^{1/2}$ and τ_o is the intensity of the wall friction. (After Goldstein)

The average of T' and V' is zero over a time long compared with the frequency of the fluctuations so that on averaging, according to the method proposed by Reynolds, the terms which remain are:

$$\rho C \left\{ \frac{\partial \bar{T}}{\partial t} + \overline{V \cdot \nabla T} + \overline{V' \cdot \nabla T'} \right\} = k \overline{\nabla \cdot \nabla T} \quad (34)$$

leaving an excess over the laminar flow condition of $-\rho C \overline{V' \cdot \nabla T'}$ (35)

Therefore

$$\rho C \left(\frac{\partial \bar{T}}{\partial t} + \overline{V \cdot \nabla T} \right) = k \overline{\nabla \cdot \nabla T} - \rho C \overline{V' \cdot \nabla T'} \quad (36)$$

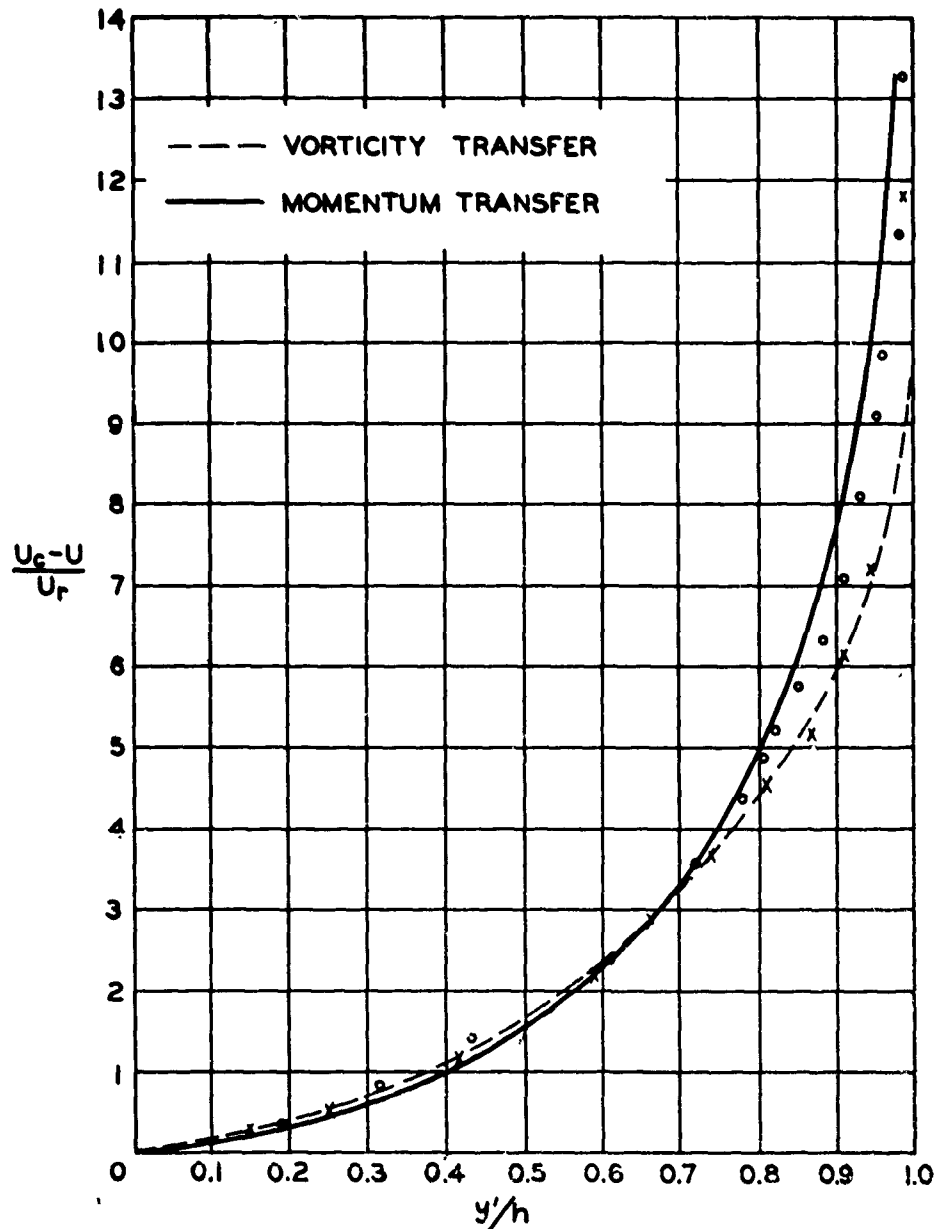


Figure 16. Velocity profiles for turbulent flow in channels. Comparison of experimental data with vorticity and momentum transfer theories where U is the velocity at the normal distance y from the solid boundary, U_c is the velocity at the center of the channel, and $U_r = (\tau_0/\rho)^{1/2}$ and τ_0 is the intensity of the wall friction; h is at half-width of the channel and y' is the distance measured from the central plane of the channel. (After Goldstein)

or

$$\rho C \frac{d\bar{T}}{dt} = \nabla \cdot (k \nabla \bar{T} - \rho C \bar{V}' T') \quad (37)$$

where the differential operator is understood to operate only on the temperature but takes the component designation through its scalar product with the velocity V' .

Since the molecular heat transfer rate is $k \nabla T$ with components along the axes appropriate to the operator ∇ , the terms $-\rho C \bar{u}' T'$, $-\rho C \bar{v}' T'$, and $-\rho C \bar{w}' T'$ are heat transfer rates contributed by the turbulent velocity components u' , v' , and w' and the turbulent or fluctuating temperature T' . The turbulent heat transfer contribution is usually large compared with that from molecular motion.

Since temperature is a transferable property in the same sense as momentum and vorticity, there is a mixture length associated with the transfer of temperature.

There are many refinements relating to the treatment of the eddy heat transfer terms which are not appropriate for this brief discussion. It is interesting, however, to indicate the failures and successes of the various theories of turbulence transfer in predicting the temperature variation in a wake behind a heated body of revolution. Figure 17 is a comparison of calculations of the temperature profile $\frac{T - T_o}{T_c - T_o}$ in the wake with the experimental results. It is seen that the momentum and modified vorticity theories predict close agreement with the measured values of the temperature profile while the vorticity theory, with assumptions concerning the dependence of the mixing length on the inverse first and three halves power of the radial distance, and with symmetrical turbulence, is badly in disagreement.

The brief outline of several of the theories of turbulence given above is intended to indicate only the

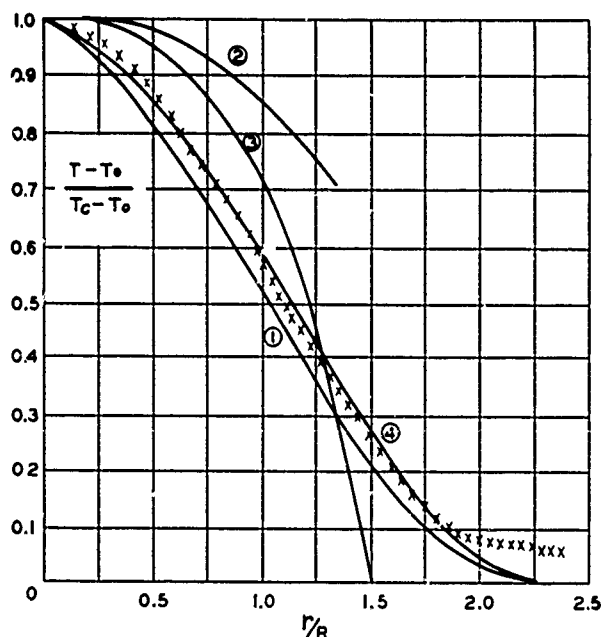


Figure 17. Temperature profile in the wake behind a heated body of revolution. Comparison of experimental results with those predicted by several turbulence theories. T is the temperature at a point r from the center of the wake, T_c is the temperature at center and T_o is the temperature outside the wake. R is the value of r for which the velocity is $1/2$ the maximum velocity. (1) Momentum transfer theory, (2) Vorticity transfer theory: symmetrical turbulence: $1/r$, (3) Vorticity transfer theory: symmetrical turbulence: $1/r^{3/2}$, (4) Modified vorticity transfer theory, and x experimental mean results. (After Goldstein)

general nature of the mathematical background and to present and to expose some of the concepts developed in the analysis of turbulent flow. More extensive treatments of the subject are present by Prandtl (109), Dryden (101) and Goldstein⁵² where further details of the various theories may be found. A critical discussion of turbulent flows in channels and circular tubes is given by Millikan (107); Goldstein (103) has discussed the similarity theory and flow between parallel planes and through pipes. The statistical theory of turbulence has been developed principally by G. I. Taylor (110); reviews of the statistical theories may be found in papers by von Karman (105), Dryden (102) and Taylor (111). Other contributions to the statistical theory of turbulence have been made by Kolmogoroff (105a), by Obukhoff (107a) and by Heisenberg (103a). Weiszacker (112a) has treated the problem of the spectrum of turbulence at high Reynolds numbers; he has attempted to relate the frequency distribution of the turbulent oscillations to the distribution of energy among Planck oscillators in black body radiation.

Damkoehler (100), Chambre and Lin (99) and Levine, May and Scurlock (106) have considered the effect of turbulence on combustion. Barr (97) has recently conducted a survey of turbulence with reference particularly to its influence on the combustion process. Dryden⁵³ has reviewed the fundamental advances in turbulence from 1938 to 1946 in a paper relating to the mechanism of transition from a laminar to a turbulent boundary layer.

B. Research in Progress

The Department of Chemical Engineering at the Massachusetts Institute of Technology⁵⁴ is conducting studies on the effect of turbulence on flame stabilization. The experimental equipment consists of a vertical channel 16 inches long, with a combustion chamber of rectangular cross section 3 inches by 1 inch. The air and fuel mixtures flow through a calming section and a nozzle, with a 25 to 1 contraction ratio, and enter the combustion chamber at velocities between 20 feet and 350 feet per second. Rods of various diameters are used as flame holders and the turbulence in the combustion chamber is varied by inserting Tyler screens of various wire diameters and mesh sizes at the entrance to the combustion chamber. The experi-

⁵²Reference 8, Vol. I, pp. 191-233; Vol. II, pp. 331-399 and pp. 646-673.

⁵³Reference 39.

⁵⁴Massachusetts Institute of Technology, Cambridge 39, Massachusetts; H. C. Hottel and G. C. Williams; Sponsored jointly by the Bureau of Ordnance Contract NOrd 9661; Confidential, and the Bureau of Aeronautics, Contract NOn(s) 8632; Restricted.

mental results (98) from preliminary runs indicate that there is no change in flame angle with change in the controlled turbulence from substantially zero turbulence to a turbulence level of 2.7%. It is tentatively concluded therefore that since the observed flame velocities are found to be much greater than laminar flame velocities, and since these apparent flame velocities increase with the velocity of the gases entering the combustion chamber, the high flame velocity is due to turbulence. It is also concluded that since the Tyler screens upstream of the flame holders apparently have little or no effect on the observed flame velocity, the turbulence causing the large increase in flame velocity must be produced by the flame holder, or must result through the instability of the flame front itself. High speed motion pictures indicate that periodic perturbations are produced in the gas stream by the flame stabilizer. The stabilization process probably depends upon eddy formation. It is interesting to note that the eddies created by a rod flame holder are shed simultaneously, rather than alternately, as in the von Karman trail (108). This seems to indicate that the flame has a significant influence on the eddy formation by the flame holder.

The University of Illinois⁵⁵ is investigating turbulence produced in free jets, with special attention to the effects on turbulence of: (a) the geometric configuration of the nozzle, (b) the velocity of gas flowing through the jet, and (c) the density, viscosity, temperature, and pressure of the entrained gas and the gas flowing from the jet. With this apparatus, it is expected that data will be provided to form a qualitative representation of the turbulence produced in free jets. For quantitative measurements, however, the Department of Chemical Engineering, University of Illinois, hopes to construct or obtain a hot-wire anemometer for a more complete study of the scale and intensity of turbulence in jets (112).

The effect of turbulence on flame propagation is being studied (96) at the Forest Grove Burner Laboratory.⁵⁶ Turbulence is introduced into the flow by the insertion of a grid or perforated plate which completely fills the channel. Turbulence intensities up to 5% have been obtained in this manner. A hot-wire anemometer has been used to make an extensive point-by-point record of turbulence intensities in regions of the channel which are of particular interest; from these records it will be possible to calculate the scale of turbulence although extremely long and laborious

methods are required. Following the preliminary phase of the study, which was concerned almost entirely with calibration, the effect of turbulence on flame propagation will be studied in an attempt to relate turbulence conditions in the channel with and without a flame present, in order that the effect of turbulence on flame propagation may be determined.

Studies of turbulence in supersonic air streams and in their boundary layers have been undertaken at the University of Virginia,⁵⁷ by two independent methods: the first technique involves the use of shadowgraphs of the air stream and the boundary layer, and the second technique is a method developed at the University of Virginia for observing the amount of light scattered due to the fluctuations in the density of the air stream. In the shadowgraph technique, photographs were taken with a spark light-source of duration 10^{-7} seconds; this short exposure time made possible the observation of turbulence effects across the entire stream. The turbulence in the boundary layer and throughout the supersonic stream of Mach number 1.83 was studied by this technique.

The light scattering method gave a photographic record of the difference in air stream density due to turbulent fluctuations, an increased amount of light being scattered from regions where turbulence existed. The magnitude of the turbulence can be calculated from the scattering data. Observation and photographs of the scattered light indicated that the supersonic air stream study was turbulent throughout, and that the turbulence increased near the boundary layer, a result which agreed with information derived from short duration spark shadowgraphs.

Although not primarily concerned with research on the mechanism of turbulence itself, mention should be made here of the studies of turbulence in connection with other combustion research. The combustion aspects of this work are discussed at some length in Vol. I, Part 1 of this Survey, where a brief description is given of the turbulence and the combustion research at the Battelle Memorial Institute,⁵⁸ at the Bureau of Mines,⁵⁹ at Experiment, Incorporated,⁶⁰ and at the University of Delaware.⁶¹

⁵⁷University of Virginia, Charlottesville, Virginia; J. W. Beams; BuOrd Contract NOrd 7873; *Confidential*.

⁵⁸Battelle Memorial Institute, Columbus, Ohio; J. E. Foster; AAF Contract W33-038-ac-14202; *Confidential*.

⁵⁹Bureau of Mines, 4800 Forbes Street, Pittsburgh, Pennsylvania; Bernard Lewis; ONR Contract NAnr 25-47; *Unclassified*.

⁶⁰Experiment, Incorporated, Post Office Box 1-T, Richmond, Virginia; J. W. Mullen; BuOrd Contract NOrd 9756; *Confidential*.

⁶¹University of Delaware, Newark, Delaware; Kurt Wohl; Subcontract under United Aircraft Corporation, BuOrd Contract NOrd 9845; *Confidential*.

⁵⁵University of Illinois, Urbana, Illinois; H. F. Johnstone; ONR Contract N6ori-71, T.O. 11, *Unclassified*.

⁵⁶Applied Physics Laboratory, Johns Hopkins University, Silver Spring, Maryland; L. R. Hafstad; BuOrd Contract NOrd 7386; *Confidential*.

IX. ATOMIZATION AND MIXING

A. Introduction

This Section is a continuation of the material treated under Turbulence insofar as it connects the mixing of fluid streams with the turbulence process. Atomization and mixing, however, are so closely related through their influence on combustion that the two processes have been treated together here to emphasize the propulsion application and because most of the current research on one includes the other.

There has been little fundamental research on the mechanism of atomization although it has been studied over a long period of time in connection with the development and the improvement of internal combustion engines. This effort has been largely empirical with special attention being devoted to the influence of atomization, spray characteristics and mixing on the efficiency of combustion reactions of hydrocarbons and atmospheric oxygen. However, when new fuels and oxidizers were studied, in the development of rocket motors, the atomization and mixing techniques could not be predicted from the data on the hydrocarbons and air. Each fuel and oxidizer combination required a trial and error method to discover the correct nozzle design and spray characteristics for optimum burning conditions. The conditions for the injection of liquid fuel and oxidizer into a rocket motor, for example, vary greatly with the type of fuel and the type of oxidizer. This variation depends upon the physical and chemical constants of the compounds, such as vapor pressure, specific heat, surface tension, etc., on the velocity of injection, on the turbulent state of the flow, and on the chemical reaction rate of the fuel and oxidizer. Studies of atomization have shown that the droplet size in fuel sprays depends upon the viscosity, density, and surface tension of the fuel, on the injection pressure, and on the Reynolds number associated with the flow in the injector. It has been estimated that the best conditions for combustion were those in which the smallest droplets and the most uniform size distribution were obtained. This has not been confirmed.

Roesch and Rose (117) have recently made a survey of the literature on atomization. They reference 74 papers on atomization, including studies on spray intensity and distribution, on the dispersion, penetration and cone angle of the spray, on the mean velocity; attention is given to a number of theoretical papers on spray and droplet formation. The droplet size studies reported include the work of Longwell (115); he used the frozen drop method to determine droplet size and

was able to correlate the mean droplet size with cone angle, injection pressure, orifice diameter and viscosity of the fuel. Other correlations have been made by Scheubel (118), by Nukiyama and Tanasawa (116) and by Vogt.⁶² It is interesting to compare these empirical correlations.⁶³ The functional disparities of droplet size dependence on the physical constants and on the conditions of flow show clearly that research is required to understand more about the physics of atomization and liquid sprays.

Barr⁶⁴ has made a survey of the literature on turbulent mixing. He has reviewed the relation of turbulent transport theories to the case of the mixing of fluids flowing unrestricted in a pipe at high Reynolds numbers, the mixing in the turbulent wake behind an obstacle in a stream and the mixing of the fluid in a jet with the medium into which it is flowing. It is apparent that the mixing of fluids must be studied in a fundamental way before the influence of this process on combustion can be evaluated. Especial preference ought to be given to the mixing of two

⁶²University of California, Berkeley, California; Carl J. Vogt, Mechanical Engineering Division.

⁶³(a) Nukiyama and Tanasawa

$$D = \frac{585}{V} \left(\frac{\sigma}{\rho_f} \right)^{1/2} + 597 \left(\frac{\mu}{\sqrt{\sigma \rho_f}} \right)^{0.45} \left(\frac{1000 \varphi_1}{\varphi_a} \right)^{1.5}$$

(b) Longwell

$$D = A \frac{B \gamma}{(\Delta p)^c \sin(\varphi/2)}$$

(c) Scheubel

$$D = \frac{\sigma}{\rho_f V^2} \cdot f \left(\frac{\sigma}{\gamma v} \right)$$

(d) Vogt

$$D = \frac{2 (Re)^y \sigma \rho_f}{\Delta p \cdot \gamma \cdot \rho_a}$$

Where: D is the mean droplet diameter (arbitrary units), A , B , and c are constants, r , the radius of the orifice, (φ_1/φ_a) , the ratio of the volume flow rate of liquid to the volume flow rate of air at the vena contracta, V the rel. velocity air and fuel, γ the coefficient of friction of the fuel, μ the viscosity, ν the kinematic viscosity, σ the surface tension, ρ_f and ρ_a the density of the fuel and of the atmosphere, respectively, $f(\sigma/\gamma v)$ a function to be determined experimentally, (Re) the Reynolds number of flow, y a number to be determined, Δp the pressure drop in the injection process and φ the cone angle of the spray.

⁶⁴Reference 97, pp. 12-19.

liquid jets under conditions where the flow, including atomization, can be carefully controlled.

B. Research in Progress

1. *Atomization and Liquid Mixing.* Under the direction of Prof. Carl J. Vogt⁶⁵ at the University of California, research on spray formation, droplet size and distribution from a nozzle is being conducted. This is essentially a study of the pre-combustion phases in a burner. The work involves the measurement of particle size and velocity with the aid of a strong light source and equipment for taking simultaneous photographs through a set of microscopes, mounted at right angles to the direction of the spray. The microscopes can be focused at various distances from the center of the spray cloud and at various distances from the jet nozzle. A shield or deflector is used to remove successive portions of the outer parts of the spray before the spray reaches the microscope. This permits microphotographs of the interior of the spray to be made.

For the present investigations, Diesel oil is being studied. Prof. Vogt has developed an ingenious device for starting and stopping the flow of oil through a hypodermic needle used for the fluid injection. The valve is a plunger seating against the rear of the hypodermic needle and is magnetically operated; the weight of fuel injected can be very accurately controlled by a timing mechanism on the field coil current. The stream of oil from the needle travels for a distance as a liquid rod and then breaks into a spray. Simultaneous high speed photographs indicate the rate of evaporation, as well as the droplet size. The use of carefully controlled fuel weight and fuel velocity in the injection mechanism has made it possible to express the droplet size as a function of Reynolds number, the surface tension, density of the fuel and coefficient of the friction of the fuel, the atmospheric density and the pressure difference before and after release through the needle. The following equation relates these quantities:

$$r = \frac{(R_e)^{\gamma} \sigma \rho_f}{\gamma \rho_a \Delta p} \quad (38)$$

where:

r = droplet size (radius),

R_e = Reynolds number,

γ = an exponent, the value of which is not yet known,

σ = the surface tension of the fuel,

γ = the coefficient of friction of the fuel (values of 0.04 and 0.28 have been used for γ , but a more correct value appears to be 0.55 for Diesel fuels),

ρ_f = density of the fuel,

ρ_a = density of the atmosphere,

Δp = Pressure difference of fuel before and after release through the needle.

Additional data will be obtained on a variety of fuels; further work is required to refine the technique and to determine the general applicability of the relation for droplet size. An attempt will be made to determine the physical basis of the exponent γ to which the Reynolds number is raised. It is assumed that the smallest droplet size and most uniform size distribution produces evaporation in the least time, and therefore provides the best conditions for combustion.

The mechanism of mixing in fluid streams is under study at the University of Illinois.⁶⁶ This includes considerations for the kinetics of transfer of momentum, mass and heat in the process of fluid mixing (112). This is intended to provide basic information to enable design data to be assembled for fuel injection systems, atomizers, carburetors, and combustion chambers. The tasks assigned to the University of Illinois include a critical evaluation of certain theoretical phases of hydrodynamics as related to jets, the design and construction of plexiglass venturis and related equipment for the study of kinetics of atomization, and the design of apparatus for investigation of entrainer performance and design. An extensive survey of literature, directed along the lines which the experimental work will follow, is under way; an investigation will be made into the factors which are important in entrainer and ejector operation and design. These factors will include momentum and temperature distribution, as well as velocity distribution in isothermal and non-isothermal jets, composition distribution when the jet and the entrained fluid are of different composition, the composition of entrained gases as a function of the total flow downstream from the entrainer and the effect of varying the entrainer design. The influence and effects of turbulence in any one of the above phenomena will be investigated by means of a hot-wire thermocouple device for exploring the wake, in an effort to obtain information concerning the scale and intensity of turbulence as related to the other variables in the entrainment and atomization studies.

The research on atomization and mixing of fuel streams at the University of Illinois has been described by E. W. Comings (113). The relationship of experimental data at the University of Illinois to the experimental results on atomization of liquid by Nukiyama

⁶⁵Research supported by the University of California.

⁶⁶University of Illinois, Urbana, Illinois; H. F. Johnstone; ONR Contract N6ori-71, T.O. 11; *Unclassified*.

and Tanasawa is discussed briefly. The results at Illinois have been expressed in terms of a relation developed by Nukiyama and Tanasawa (116) for the mean droplet diameter in terms of the liquid density, viscosity and surface tension, the relative velocity of the liquid and air streams, and a parameter describing the flow near the orifice.

The assignments for study of atomization and entrainment at the University of Illinois are described briefly.

a. *Venturi Atomization.* Spark photographic techniques are being used to study the action of water droplets and water injection phenomena in the plexiglass venturis. Five plexiglass venturis have already been constructed to compare the effect of introducing liquid into the center of the stream with liquid introduced along the walls of the flow channel. Other venturis will be used to study the effects of the angle of the divergent sections, and to study the effects of the increasing length of the throat to obtain a longer section of high speed air. Some consideration will be given to the use of double venturis, similar to those now in use to supply reduced air pressure for the operation of aircraft flight instruments. A change in the design of the pressure taps, in which the copper tubing extends only halfway to the wall of the plexiglass and the plexiglass then drilled to the inside diameter of the tubing will be attempted, in an effort to obtain more consistent manometer readings.

b. *Entrainment of Gases.* The composition of gas samples withdrawn from the entrainer, in which the ratio of air entrained is a criterion against which other variables are compared, is determined for variations in several conditions. The distances between the nozzle and the entrainer will be varied, nozzles of different diameter will be used, and the discharge resistance will be varied by means of the entrainer exit valve. This same analysis will be performed for variations in primary gas velocity and entrainer mouth diameter. Different gases including nitrogen, carbon dioxide, ammonia and hydrogen or helium will be studied. These investigations will be carried on in a series of experiments, where the variation of downstream flow versus entrained gas will be determined for various settings of the throttle valve.

Air atomization of liquids is also under investigation at the Massachusetts Institute of Technology⁶⁷ using an apparatus with a single high speed spark as a light source for obtaining two shadow photographs simul-

taneously. The photographs are taken at right angles to each other, and to the direction of the air flow. The test section, in which the atomization takes place, is a glass-walled duct, having a 1 inch square cross section, into which the air flows from a convergent nozzle to produce a stream of uniform velocity.⁶⁸

A qualitative interpretation of the role of "viscosity and surface tension of a liquid on its atomization was obtained by using a 80% glycerine-water mixture, whose surface tension was 66 dynes/cm at 18°C and whose viscosity was 164 centipoise at 25°C, and a 50% ethyl alcohol-water mixture of surface tension equal to 27.5 dynes/cm at 30°C and viscosity equal to 2.4 centipoise at 25°C." The effect of high liquid viscosity and surface tension was in general found to be adverse to atomization.

In connection with studies on hydrogen peroxide⁶⁹ at the Massachusetts Institute of Technology⁷⁰ a fundamental program will be undertaken on the mixing of liquids by impinging jets to obtain further information on liquid-liquid and liquid-solid contacting. These investigations will include work on the effect of the two jet velocities, their diameter, liquid densities, surface tension and viscosity on the spray distribution pattern and on the distribution of composition among the droplets. It is planned to follow this work with corresponding study of a geometrically similar system in which chemical reaction takes place.

The University of Michigan⁷¹ is investigating rates of evaporation of fuel sprays in a wind tunnel at the Research Willow Run Laboratories. A spray nozzle will be set up inside a venturi and this apparatus will be operated in the wind tunnel at an air speed of 100 mph. The first test will be made with a nozzle spraying fuel downstream. The angles of the cone spray will be measured for various air speeds and different degrees of turbulence. The intensity and scale of the turbulence will be determined for each of these test runs. A hot-wire anemometer is being developed and calibrated for use in connection with the turbulence measurements. Light will be directed through the spray and the amount absorbed by it will be measured by a photometer to attempt to find a quantity significant for the determination of evaporation rates.

⁶⁸Reference 104, page 20.

⁶⁹See Volume I, Part 2 of this Survey.

⁷⁰Massachusetts Institute of Technology; C. N. Satterfield; BuOrd Contract NOrd-9107, Task C; *Confidential*; To be transferred to the Office of Naval Research, 30 June 1947.

⁷¹University of Michigan, Ann Arbor, Michigan; Arnold M. Kuethe; BuOrd Contract NOrd 7924; *Restricted*.

⁶⁷Massachusetts Institute of Technology, Cambridge, Massachusetts; H. C. Hottel and G. C. Williams; BuOrd Contract NOrd 9661, Project METEOR; *Confidential*.

The Marquardt Aircraft Company⁷² is conducting work on spray patterns and droplet size of fuel for fuel systems in connection with ramjet development. Spray patterns will be recorded by tracing on vellum, or similar paper, or plastic material. The shadow cast by the droplet size of the fluid particles in the stream will be measured by two methods. In the first method photographs of the actual droplet will be taken, using a very short exposure time. In the second method the mean droplet size will be measured by the light intensity decrease when a fuel spray is placed between a light source and a light intensity measuring instrument. A polaroid set will be placed between the light source and the fuel spray, and also between the measuring instrument and the fuel spray. By this method light reflected from the fluid droplets and other surface will not be measured by the instruments. Measurements obtained by this method will give only comparative results. It is hoped, however, by the use of both methods that an actual value for the droplet size can be obtained. Airflow, fuel pressure, air temperature, fuel flow and fuel temperature will be measured by standard methods.

The problem of the proper location for a fuel nozzle will be investigated from a theoretical viewpoint. Pressure losses for various types of nozzles located in different positions in a diffuser will be calculated. This data will be coordinated with the spraying characteristics determined by tests on three different nozzle designs. These nozzles will be tested at sea level, and at simulated altitudes of 15,000 and 30,000 feet for a number of angles of attack for fuel pressures of 5, 30 and 60 psi, and air velocities of 50, 150, 300 and 500 feet per second at the fuel nozzle.

A two-dimensional flow diffuser of the flared type⁷³ of transparent plastic will be fabricated to study fuel spraying characteristics and the action of fuel nozzles at the diffuser. It has been found that an extremely large pressure range resulted with a fixed orifice type of fuel nozzle, and that this pressure variation caused the fuel flow rate to vary over a wide range. A spring-loaded type of spray nozzle is being developed by Marquardt at the present time; it is hoped that the fuel flow in this device will vary linearly with the fuel pressure. Spray studies of this nozzle will be made. A new type of fuel nozzle, in which the fuel is intro-

duced into the air-stream by means of a jet pump principle, will be developed and tested. It is felt that, in addition to the atomization achieved during the jet pump action, a more nearly linear relationship may be achieved between the rate of fuel flow and the pressure.

In connection with subsonic ramjet development, the Marquardt Aircraft Company⁷⁴ will conduct fuel distribution studies in a 48 inch ramjet (cold flow) test tunnel, now under design. This tunnel will have a 5 foot x 7 foot test section which will enable air speeds of Mach number 0.5 with a 48 inch throat and Mach number 0.9 with a 30 inch throat. In the cold flow test, water instead of fuel will be used for the fuel distribution study. The water will leave the fuel nozzle in an atomized and partially vaporized state and travel through the flame holder and combustion chamber in approximately the time that will obtain when gasoline is used for the burner test.

The first test of the internal flow engine will be made on a small model, through which air will be drawn by means of the Allison-Stratos supercharger unit. A sampling tube will be used to explore the combustion chamber and tail pipe area, to determine the ratio of water to air in the various test stations. The sampling tube will have a method of controlling the inlet flow so that the velocity in the tube is the same as the particle free stream velocity. The matching of the two flows is important in preventing droplet or stream-line separation. Immediately after the water and air mixture enters the tube, it will impinge on a heating unit which completely vaporizes the water. Excess heat will be added to ensure complete evaporation and to bring the mixture a reasonable distance beyond the saturation point. The relative humidity and temperature will then be measured at the exhaust outlet of the sampler tube.

2. *Mixing of Gas Streams.* The Massachusetts Institute of Technology⁷⁵ is investigating the mechanism of mixing of two gas streams, which are concentric and are rotationally-symmetric and flow parallel to each other. The inner gas stream is a mixture of 90% air and 10% helium, while the outer stream is air. Measurements on velocity, concentration and temperature profiles at various cross sections downstream of the point where mixing begins, will be used to study the mixing processes (Figure 18). Tests will be initiated to discover the effects of velocity ratio, area

⁷²Marquardt Aircraft Company, 4221 Lincoln Blvd., Venice, California; J. W. Wiggins; Sub-contract under University of Southern California; Contract NOa(s) 8257, Item 13; *Restricted*.

⁷³Description of the flared type diffuser may be found in the report of Marquardt Aircraft Company entitled "Development of a 30" Diameter Ramjet Engine"—AAF Contract W33-038-ac-14152, *Restricted*.

⁷⁴Marquardt Aircraft Company, 4221 Lincoln Blvd., Venice, California; Karl Schakel; AAF Contract Project Order (W33-038) 47-934-E; *Restricted*.

⁷⁵Massachusetts Institute of Technology, Cambridge 39, Massachusetts; A. H. Shapiro; BuOrd Contract NOrd 9661, Project METEOR; *Confidential*.

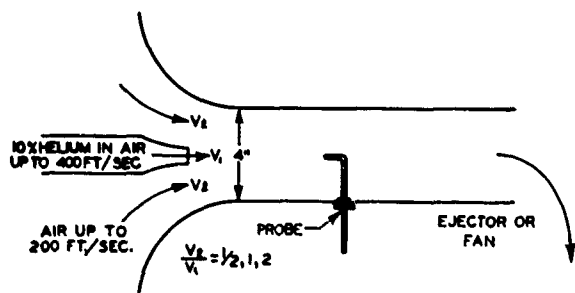


Figure 18. Sketch of apparatus for studying the mixing of two subsonic air streams. (Massachusetts Institute of Technology)

ratio and degree of turbulence. These measurements will serve as a test for various theories of mixing, principally the Prandtl momentum transfer theory and the Taylor vorticity transfer theory. Preliminary studies indicate that the Prandtl mixing-length theory serves to predict with fair accuracy the boundaries of the mixing region.

A program has also been initiated at the Massachusetts Institute of Technology⁷⁶, to study experimentally and analytically the mixing of a supersonic and a subsonic stream to determine the conditions which govern the degree of reversibility in the mixing processes. This work is a continuation of the ejector investigation, undertaken in the Mechanical Engineering Department, for the Elliott Company.

Dr. L. F. Stutzman of the Northwestern Univer-

sity⁷⁷ is studying fluid flow theories, in particular those relating to mixing-lengths, Taylor's scale of turbulence and the momentum and vorticity transfer theories relating to the flow of gases. The application of these theories to problems in the absorption of gases is to be investigated. In his previous work, Dr. Stutzman studied the process of diffusion of particles in a liquid stream. He injected a dye into a stream of water and determined the distance at which there existed a particular concentration gradient. In addition, he studied the eddy shape and volume as a function of velocity. The experimental results which he obtained agreed with the theory developed on the basis of molecular diffusion.

Dr. Stutzman plans to extend this work to gaseous flow. He will use a heat source capable of supplying a large amount of energy. The heat so generated will be introduced into a gas stream and the distribution of temperature downstream of the source will be measured with thermocouples. It is planned to study a number of gases and to determine the concentration downstream by means of infrared techniques. Mixing length will be related to molecular structure and to parameters descriptive of the state of flow, particularly pressure, temperature, and velocity. Experimental results will be compared with existing theories in an attempt to understand more fully the fundamental mechanisms involved in turbulent diffusion and mixing.

X. MECHANICS OF NON-UNIFORM GASES

A. Introduction

There has been considerable interest recently in the field of non-uniform gases, particularly rarefied gases, in view of the importance attached to understanding the fundamentals of flow at high altitudes, and in low density systems generally. As the density of a gas is decreased, the transmission of viscous forces and of heat departs from the classical relationship developed by treating the gas as a continuum. The mechanics of non-uniform and rarefied gases treats these departures of transport phenomena from the classical kinetic theory of gases. A review of the development of the theory of non-uniform gases has been presented by Chapman and Cowling (121).

Gas dynamics may be divided into several realms whose upper and lower bounds depend upon the ratio between the mean free path, l , of a gas particle and a linear dimension L characteristic of the gas flow. Tsien (125) has used this ratio to divide the realms of gases into: (1) that of a continuum where the mean free path is less than $10^{-2}L$, (2) a "slip-flow" regime where the mean free path lies between $10^{-2}L$ and L , (3) an "intermediate" domain in which the mean free path lies in the region between L and $10L$, and (4) a region of fully developed molecular flow where the mean free path is greater than $10L$ and where intermolecular collisions are negligible in comparison to the collisions of gas molecules with themselves, or with

⁷⁶Massachusetts Institute of Technology, Cambridge 39, Massachusetts; J. H. Keenan; BuOrd Contract NOrd 9661, Project METEOR; Confidential.

⁷⁷Northwestern University, Evanston, Illinois; Dr. L. F. Stutzman; ONR Contract N6ori-158, T.O. 3; Unclassified.

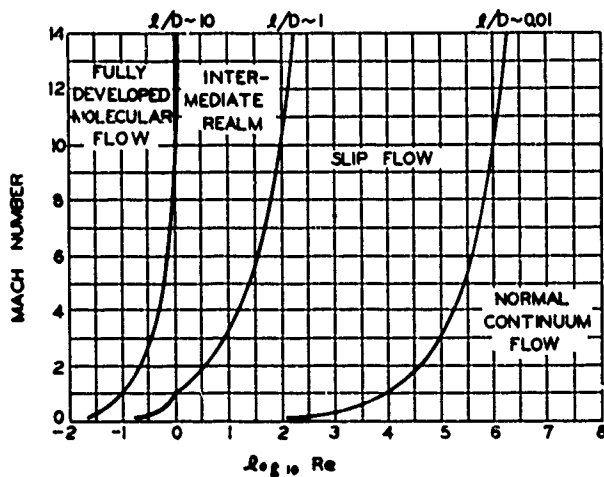


Figure 19. Realms of flow in fluid mechanics. (After Tsien)

solid surfaces (Figure 19). The principal concern in the region intermediate between fully developed molecular flow and the continuum realm is in the extension of higher order terms in the classical statistics to account for the transport of stress and heat.

Applications of the theory of non-uniform gases have been made by Brillouin (119), Lennard-Jones (122), Burnett (120) and others. An interesting application of non-uniform theory to the propagation of sound waves in rarefied gases has been made by Tsien and Schamberg (125). These authors have used the second approximation to the viscous stresses and heat flux as given by Chapman and Cowling to calculate the propagation of plane sound waves in rarefied gases or the propagation of plane sound waves of very small wavelengths in gases of normal density. These investigations indicate that even under extreme conditions the increase in the propagation velocity of sound, from the normal value at average gas densities, is less than 2%. The application of non-linear theory to the mechanics of rarefied gases has recently been reviewed by Tsien (125).

B. Research in Progress

Schamberg (124) at the California Institute of Technology has recently completed a thesis investigation whose primary objective was the derivation and application of the mathematical boundary conditions required for the determination of particular flow solutions from the general differential equations of motion for slip-flow⁷⁸. A unified treatment of the derivation of the complete distribution function from the Boltz-

mann equation is presented in this work; a method of solution discovered by Hilbert and adapted to a particular computation by Burnett are explained. In the region of slip-flow, the macroscopic differential equation of motion is applied to the propagation of plane sound waves in rarefied gas, to the solution of a problem for the Couette flow of a rarefied gas, and to the flow of a rarefied gas confined between two concentric circular cylinders, one of which is rotating at high speed while the other is held stationary. In these three applications Schamberg's calculations indicate that the high speed slip-flow of a rarefied gas does not differ materially from the slip-flow at lower speeds. The second order slip-flow theory results in flow patterns which are intermediate to those calculated from the continuum theory and the first order slip-flow theories. In the case of the Couette flow calculation, for example, the slip of the air over a plate of chord 1 foot, moving at Mach number 3.0 and at an altitude 250,000 feet, lowers the continuum value of the friction coefficient by only about 10%.

Since these calculations were conducted on the basis of a second order approximation to the velocity distribution of a non-uniform gas, the conclusion seems to indicate that the results to be expected from a third order approximation to the velocity distribution function do not warrant the effort required for calculation.

The University of California⁷⁹ is conducting research on aerodynamic heating and heat transfer at supersonic velocities and at low pressures. A small wind tunnel of rectangular test section $\frac{3}{8}$ inch $\times$ 1 inch high is being built for use up to Mach number 3.0 and at a pressure of about 1 inch of mercury absolute. Three additional tunnels have been proposed for construction in order that a pressure range of 0.01 mm of Hg up to 3 inches — 4 inches of Hg may be obtained. Tests sections for these tunnels are expected to be approximately 3 inches $\times$ 4 inches, with an air velocity up to Mach number 4.0; the choice of the lower limit of pressure (0.01 mm Hg) was made on the basis that the mean free path of the air molecules should be comparable with one dimension of the test model. Air will enter the equipment at atmospheric pressure and will be expanded to a pressure of 0.01 mm Hg with a 5-stage steam ejector of the Elliott type.

The small wind tunnel about to be completed, and mentioned above, will be used to study the problem of relative humidity, in order to find out what percent

⁷⁸The author is indebted to Dr. Schamberg for the supply of a copy of Part 1 of his thesis.

⁷⁹University of California, Berkeley, California; R. G. Folsom and E. T. Kane; ONR Contract No. N7onr-295, T.O. 3; Unclassified.

super-saturation of air is permissible without the formation of ice crystals due to rapid expansion. Pressure recovery will be studied with the larger equipment to determine diffuser efficiency when operating at very low pressures and at supersonic velocities. Heat transfer and drag data will also be obtained. With the aid of a small test section, it is hoped that pressures can be reduced to about 10^{-5} mm at Hg, where the mean free path will be so large that the air will be moving under conditions of "molecular flow".

The Naval Ordnance Laboratory⁸⁰ is planning to undertake a theoretical investigation of the mechanics of rarefied gases. No progress has been made in this regard as yet.

The New York University⁸¹ is investigating the applicability of the theory of non-uniform gases to systems with chemical reactions and to systems with-

out chemical reactions (123). The study is concerned with the applicability of these techniques to drag coefficients experienced by a missile moving in a low density or rarefied atmosphere, where the mean free path of the molecules is large compared to a dimension of the missile. Another example of a field of applicability is to the statistical theory of the formation and structure of "shock" waves. In systems where chemical reactions occur, the theory of non-uniform gases will be helpful in shedding some light on the fundamental principles of the combustion processes in moving fluid systems.

The University of Wisconsin⁸² has been investigating the applicability of the theory of non-uniform gases to the study of certain combustion phenomena. There has been no report of the status of this work as yet.

XI. THEORETICAL STUDIES RELATING TO THE PULSE JET CYCLE

A. Introduction

The development of a propulsion unit of the pulsating-tube type has required theoretical research on the analysis of the pressure cycle in this system. The principal theoretical contributions to pulse jet operation have been made by Bailey and Wilson (126), by MacDonald (129), by Schultz-Grunow (94) and by Tsien (133). The background and development of the pulse jet engine has been reviewed by Edelman (128). In addition, Volume II of this Survey contains an extensive bibliography on the research and development relating to pulse jet engines.

The mechanism of operation of the jet pulse engine is not clearly understood. In order to enable a better understanding of this type of propulsion system, it will be necessary to obtain much more basic information about the mechanism of chemical reactions in highly disturbed non-linear fluid flow. The intermittent character of the pressure cycle makes this system a more difficult one to analyze than the ramjet. The research on the pulse jet engines is concerned at pres-

ent with the investigation of several factors such as turbulence, shock effects, etc., which seem to be of paramount importance to its operation.

B. Research In Progress

Cornell Aeronautical Laboratory, Cornell University⁸³ is undertaking analogy studies in connection with the mechanism of a pulse jet, to investigate simple systems approximating the behavior of a pulse jet engine (127). Electrical analogy was first studied where the gas flow in pulse jet pipe is compared with the flow of electricity in transmission lines. While it was found possible to represent the acoustical case by a linear electrical system, no way has been found to represent the gas oscillations of large amplitudes occurring in the jet by a corresponding non-linear electrical system. A water channel analogy is being considered to study pressure waves in the gas flow of a pulse jet.

With relation to the MacDonald (129) pressure cycle for the pulse jet, a mathematical investigation was undertaken to determine the frequency response

⁸⁰Naval Ordnance Laboratory, White Oak, Maryland; R. J. Seeger, Mechanics Division; Research sponsored by the Office of Naval Research.

⁸¹New York University, New York, New York; J. K. L. MacDonald; ONR Contract N6ori-11, T.O. 2, Project SQUID; *Unclassified*.

⁸²University of Wisconsin, Madison, Wisconsin; J. O. Hirschfelder; BuOrd Contract NOrd 9938; *Restricted*.

⁸³Cornell University, Cornell Aeronautical Laboratory, Buffalo, New York; J. V. Foa; ONR Contract N6ori-119, T.O. 1, Project SQUID; *Unclassified*.

required for a pressure pick-up, in order to obtain satisfactory records. It was concluded that a satisfactory pressure pick-up should have uniform response up to about six times the frequency of the cycle.

New York University⁸⁴ is undertaking a mathematical study of the skin drag of bodies of revolution when small oscillation are superimposed on the steady fluid flow of the external surface (130). The theoretical estimates of skin drag are considered from the use of an idealized model, consisting of a long torpedo shaped solid of revolution placed in a slightly pulsed subsonic air stream flowing parallel to the axis. Outside a thin boundary layer around the solid the pressure P and the tangential velocity U of the air are regarded as determined by non-viscous incompressible potential flow. The velocity and pressure functions are regarded as the sum of steady and oscillating component with the time. The contributions due to the oscillating term are expected to affect the air flow oscillations over the surface of the jet tube, due to the pulsating intake in exhaust; no estimate of the magnitude of this effect has been obtained to date. The effect of variations in temperature due to heat transfer and boundary layer phenomena will also be investigated.

Polytechnic Institute of Brooklyn⁸⁵ is undertaking a theoretical study of the aerodynamic forces exerted in periodic compressible flow on moving valve surfaces, and on the dynamics of the valve mechanism itself, under the action of these periodic aerodynamic forces (131). Theoretical investigations of two-dimensional compressible flow are being conducted for hinging and clamped-edge reed valves. Particular solutions of the differential flow equations have been investigated to give simple shapes of the reed valves.

C. Acoustic Radiation Pressure

In the usual approach to the explanation of the pulse-jet operation, the thrust is considered to arise from the mean outflow of the exhaust gases. There is, however, another thrust producing mechanism associated with the propagation of a wave-like disturbance in a medium. This is known as "radiation pressure" and its theoretical explanation lies in the want of linearity in the wave equation when account is taken of the quadratic terms. The result is that sound waves, impinging on an obstacle, exert on it an unidirectional pressure in addition to the usual alternating pressure. Similarly a sound source experiences such an indirectional pressure when radiating into a medium.

According to Rayleigh (132) the acoustic radiation pressure for a plane wave is given by:

$$P_r = \frac{1}{2} (\gamma + 1) \frac{p^2}{\rho c^2} \quad (39)$$

where:

- P_r — acoustic radiation pressure
- γ — ratio of specific heats
- p — pressure amplitude of the acoustic wave
- c — sound velocity in the medium
- ρ — density of the medium.

It is interesting to compute the radiation pressure for a pulse jet, considering it simply as an acoustic source. Assuming a value of one atmosphere for p and using $\gamma = 1.4$, $c = 331 \text{ m/sec}$, and $\rho = 0.0012$, the acoustic radiation pressure is

$$P_r = 13.6 \text{ lbs/in}^2 \quad (40)$$

This is considerably higher than now realized in pulse-jet operation.

XII. SPECIAL FACILITIES IN DESIGN OR UNDER CONSTRUCTION

In the Appendix of this report there is presented a tabulation of wind tunnel facilities in existence in this country for subsonic, transonic and supersonic investigations. There is also a table of wind tunnels in design or under construction.⁸⁶ It seems appropriate here, however, to include additional information

with regard to plans for future wind tunnel facilities, principally for examining the transonic and supersonic flow phenomena. Where relevant and where possible, reference is made to published reports giving a more detailed description of these facilities than possible in this review. The following is a brief outline of the wind tunnel facilities now in design or under construction.

⁸⁴New York University, New York, New York; J. K. L. MacDonald; ONR Contract N6ori-11, T.O. 2, Project SQUID; *Unclassified*.

⁸⁵Polytechnic Institute of Brooklyn, Brooklyn, New York; R. P. Harrington; ONR Contract N6ori-98, T.O. 2, Project SQUID; *Unclassified*.

⁸⁶Not all of the facilities for the proposed wind tunnels listed in the Appendix are described in this Section.

*Applied Physics Laboratory, Johns Hopkins University.*⁸⁷ Designs have been completed for an intermittent small scale hypersonic wind tunnel (134) of maximum test section 1 inch x 2.2 inches. The tunnel is in the process of construction at the Forest Grove Laboratory, Forest Grove, Maryland, and will allow low density aerodynamic studies to be conducted at Mach numbers up to approximately 7 with air, and 15 with helium.

*Bell Aircraft Corporation, Buffalo, New York.*⁸⁸ A supersonic combustion channel of the open-circuit type using large steam jets exhaust is under design (135). The test sections will vary considerably with the operating conditions; for a cold tunnel the maximum test section area will be 12 inches x 12 inches at a Mach number 2.5. The system of 5-steam jets provide single stage operation to $M = 3.6$, double stage to $M = 7.1$ and triple stage to at least to $M = 10$. The Mach number range is limited only by the limitations of construction of good supersonic nozzles, since four and five stage steam jets which are capable of operating a tunnel up to Mach numbers in excess of 15 or 20 appear to be available.

The studies planned for this tunnel include simulated altitude conditions up to a Mach number 4.0 and an altitude of approximately 110,000 feet; low pressure and high Mach number studies for slip-flow phenomena and studies of aerodynamic heating up to extreme Mach numbers with model temperatures approaching red heat.

California Institute of Technology, Pasadena, California. A hypersonic tunnel with test section 5 inches x 5 inches is under construction in the Guggenheim Aeronautical Laboratory (139). A supersonic wind tunnel with test section 12 inches x 12 inches and for a Mach number 2.5 and a tunnel of test section 20 inches x 20 inches for a Mach number 4.0 are under construction at the Jet Propulsion Laboratory. All of these tunnels are designed for continuous operation.

*Johns Hopkins University, Baltimore, Maryland.*⁸⁹ The Office of Naval Research is sponsoring a design study of a supersonic wind tunnel of test section approximately 4 inches x 4 inches. This wind tunnel will be similar to the NACA Compressible Flow Unit; the precise design parameters will be subject to further study by the Johns Hopkins University.

*Massachusetts Institute of Technology, Cambridge, Massachusetts.*⁹⁰ The Department of Aeronautical Engineering (136) is designing a wind tunnel of test section 18 inches x 24 inches for work at Mach numbers 1.5, 2.0 and 2.5.

University of Minnesota, Minneapolis, Minnesota. The University has recently acquired facilities at the former Gopher Ordnance Works in Rosemount, Minnesota. These facilities can be suitably converted for use in aerodynamic and propulsion experimentation. They include reciprocating compressors aggregating 6300 hp, two 38 foot steel spheres, a ballistics building containing six 300 foot ranges, a steam plant capable of producing 1,000,000 lb/hr and extensive shop facilities.

A high speed wind tunnel of the induction type has been designed and built under the supervision of the University of Minnesota Aeronautical Laboratory. The tunnel has a 16 inch x 16 inch test section, in which velocities from low subsonic to low supersonic can be sustained continuously. A three component balance system, Schlieren apparatus and spark photography equipment is under fabrication for this tunnel.

The construction of a supersonic tunnel of the closed circuit type is expected to be completed this summer, in which it will be possible to undertake supersonic research up to Mach numbers of 4 and through a range of Reynolds numbers from 2.45×10^6 at Mach number 2 to 18.8×10^6 at Mach number 3.0. Additional supersonic research, using the two 38 foot steel spheres, will be conducted in the test section producing supersonic velocities in the Mach number region of 2.5. The spheres can be pressurized to four atmospheres with dry air; a Mach number 2.5 can be maintained for 106 seconds in a test section approximately 10 inches in diameter or for about 26 seconds for a test section 20 inches in diameter.

The steam plant can be utilized to power an induction-type wind tunnel for an estimated Mach number of 2.5 with a 16 square foot cross section. The power plant is fitted with adequate combustion controls and has sufficient water supply to enable a wide range of operation of such a tunnel without regard for water recovery. The factors concerned with operating such a power plant on an intermittent basis are being studied.

National Advisory Committee for Aeronautics.

A. At the Flight Propulsion Research Laboratory in Cleveland, Ohio, a supersonic wind tunnel of test section 6 feet x 8½ feet, for Mach number of 2.0 is under construction.

⁸⁷Applied Physics Laboratory, Johns Hopkins University, Silver Spring, Maryland; L. R. Hafstad; BuOrd Contract NOrd 7386; Confidential.

⁸⁸Bell Aircraft Corporation, Buffalo, New York; Robert Stanley; BuOrd Contract NOrd 9876, Project METEOR; Confidential.

⁸⁹Johns Hopkins University, Baltimore, Maryland; F. H. Clauser; ONR Contract N6onr-243, T.O. 5; Unclassified.

⁹⁰Massachusetts Institute of Technology, Cambridge, Massachusetts; J. R. Markham; BuOrd Contract NOrd 9661; Confidential.

B. At the Langley Memorial Aeronautical Laboratory, Langley, Virginia, a wind tunnel for transonic and supersonic research is under development. This equipment is known as the NACA Compressible Flow Unit (137); the development of this unit is under the joint sponsorship of the National Advisory Committee for Aeronautics⁹¹ and the Office of Naval Research, Navy Department. The transonic tunnel and the supersonic tunnel operate from the same compression system. The transonic tunnel will have a rectangular test section 4 inches x 16 inches and is designed to provide speeds in the Mach number range from 0.3 to 1.4. The supersonic tunnel with a square test section of 4 inches x 4 inches is designed to provide speeds in the Mach number range from 1.4 to 4.0.

Naval Ordnance Laboratory, White Oak, Maryland. The Kochel wind tunnel, transported from Germany, is being set up for fundamental research on supersonic flow phenomena. There are seven test sections planned at present: Four sections 12 cm x 12 cm, one section 18 cm x 18 cm, one section 40 cm x 40 cm, and one section 80 cm x 80 cm. The air will be taken from

the atmosphere and exhausted into a large vacuum sphere for intermittent operation.

*North American Aviation, Inc.*⁹² The design for a 16 inch x 16 inch test section tunnel for Mach number 4.5 for intermittent operation of 15-20 seconds duration has been completed (140).

*Princeton University.*⁹³ The Department of Aeronautical Engineering has under construction a supersonic wind tunnel of the "blow down" type (138). The tunnel will be capable of supplying air speeds from Mach numbers 1.5 to 5.0 in a Reynolds number range from 2×10^6 to 40×10^6 . This tunnel will have two test sections, one 4 inches x 8 inches for Mach numbers greater than 3.0 and the other 4 inches x 5 inches for Mach numbers 1.5 and 2.0. Models up to 3 inch chord can be tested at the two lower Mach numbers while models from 3 inches to 6 inch chord can be tested at Mach numbers 3 or greater.

University of Washington, Seattle, Washington. A design study, under the direction of Prof. F. S. Eastman, has been initiated for a simple small supersonic wind tunnel for fundamental studies in the Mach number range from 4.0 to 8.0. No details are as yet available concerning this study.

XIII. SURVEY STUDIES

The Graduate Division of Applied Mathematics of Brown University is engaged in a long-range research program in the analogous fields of compressibility and plasticity. Theoretical work in these fields is currently in progress; corresponding experimental investigations will be initiated shortly. With reference to the field of compressibility, beginning July 1946 the U.S. Army Air Forces sponsored a survey of all available literature on this subject.⁹⁴ One object of this survey is the preparation of summaries and translations of foreign documents on compressibility. After screening, the available articles are reviewed initially in the original language, and if the article is considered of sufficient interest a translation of the article is made into English. If a translation of the article already exists, information in the summary indicates by whom this translation was made and where the original document may be physically located.

Another objective of this project is the preparation of a series of monographs in the field of compressibility

to fill the gap in this field that now exists in the Durand series. The ultimate objective of this compressibility project is to prepare an up-to-date treatise on the aerodynamics of compressible flow which is written on such a level that it may be readily understood and applied by aeronautical engineers. This work will cover both theoretical and experimental results.

At the present time approximately one hundred English Language Summaries of German documents have been prepared and five monographs are in preparation. Two of these monographs entitled "Method of Characteristics as Applied to Compressible Fluids" and "Aerodynamics of Oscillating Airfoils in Compressible Flow" have been completed. Three others entitled "Basic Theory of Combustion," "Methods of Linearization in Compressible Flow," and "Boundary Layers at High Speeds," respectively, are in progress. The monograph on the method of characteristics covers the application of this method to two- and three-dimen-

⁹¹National Advisory Committee for Aeronautics, Langley Memorial Aeronautical Laboratory, Langley, Virginia; John Stack; ONR Contract NAonr-14-47; *Unclassified*.

⁹²North American Aviation, Inc., Inglewood, California; W. Bollay; BuOrd Contract NOrd 9784; *Confidential*.

⁹³Princeton University, Princeton, New Jersey, Department of Aeronautical Engineering; Lester Lees and J. V. Charyk; ONR Contract N6ori-105, T.O. 3; *Unclassified*.

⁹⁴Brown University, Providence 12, Rhode Island; Dr. John Marchant, AAF Contract W33-038-ac-15004 (16351); *Restricted*.

FLUID MECHANICS

sional steady flow and one-dimensional non-steady flow. It will include charts, tables and numerical examples. The monograph on oscillating airfoils is a digest of all theoretical work pertaining to flutter problems in compressible fields of flow.

The work on plasticity is carried out under the sponsorship of the Office of Naval Research and its objectives are practically those of the compressibility project. To date a survey is being made of all fundamental work in the field of plasticity.

APPENDIX

Wind Tunnel Facilities

In collecting technical information relating to the field of Fluid Mechanics by visits to academic, industrial and government laboratories, it was found that the Technical Survey Group had obtained information about many wind tunnels in existence, in design, or under construction. These data on wind tunnel facilities, augmented by information made available through the assistance of the Aeronautical Board, the Office of Naval Research and the Bureau of Aeronautics, are presented here because of their general interest to the field of Fluid Mechanics.

A. WIND TUNNEL FACILITIES IN EXISTENCE

1. Subsonic

Organization — Location	Maximum Speed	Balance System	Size of Section	Throat; Type	Horse- power	Remarks
Akron, University of, Akron, Ohio	124 mph		6½ ft. dia.	Open; return		
Alabama, University of, Tuscaloosa, Alabama	120 mph		2 ft. 5 in. x 3 ft.	Open or closed; return		
Allied Aviation, Baltimore, Maryland	110 mph		7½ ft. dia.			
California Institute of Technology, Pasadena, California	200 mph		10 ft. dia.	Closed; return		¼ — 4 atmospheres vari- able pressure water cooled
California, University of, Berkeley, California	90 mph		3 x 3 ft.	Open; return		
	350 mph		7 in.	Non-return		Vacuum operated
Carnegie Institute of Technology, Pittsburgh, Pennsylvania	100 mph		4½ ft. dia.	Open		
Case School of Applied Science, Cleveland, Ohio	100 mph		3 x 3 ft.	Closed; return	75	Atmospheric
Catholic University, Washington, D. C.	40 mph	3 Component balance	6 x 6 ft.		120	Model stationary air twist
	40 mph		8 x 8 ft.	Closed; non-return		At Cabin John, Maryland
	70 mph		3 x 3 ft.		80	
Consolidated Vultee Aircraft Corporation, Downey, California	100 mph		15 ft. dia.			
	275 mph		4 ft. dia.		300	Atmospheric
Consolidated Vultee Aircraft Corporation, San Diego, California	300 mph	6 component Baldwin Southwark	8 x 12 ft. Octagonal	Closed; return	2250	Atmospheric
Curtiss-Wright Corporation, Caldwell, New Jersey	90 mph	U.S. Army modified NPF		Open		
Detroit, University of, Detroit, Michigan	104 mph		7 x 10 ft.	Open or closed		Atmospheric
Douglas Aircraft, El Segundo, California	175 mph		30 x 45 in. 48 in. long		75	Atmospheric

A. WIND TUNNEL FACILITIES IN EXISTENCE

1. Subsonic

Organization — Location	Maximum Speed	Balance System	Size of Section	Throat; Type	Horse- power	Remarks
Georgia Institute of Technology, Atlanta, Georgia	125 mph		9 ft. dia.	Closed; return		
G. M. Giannini, Pasadena, California	150 mph		1 x 1 ft.	Closed; return		
B. F. Goodrich Company, Akron, Ohio	80 mph	None	18 x 36 in. Elliptical 36 in. long	Open; return	20	Icing Tests — Elliptical Test Section
Guggenheim Airship Laboratory, Akron, Ohio	24 mph		15 x 16 ft.		35	Whirling arm
	34 mph		12 x 12 ft.		1000	Vertical gusts tunnel
	125 mph		6½ ft. dia.	Open	225	Lift, drag, yaw, moments and pressure distribution
Harvard University Cambridge, Massachusetts	175 mph		30 x 40 in.		80	Flight stability of incen- diary bombs
Illinois, University of, Urbana, Illinois	130 mph		30 x 48 in.	Closed	50	
Kansas, University of, Lawrence, Kansas	90 mph		5 ft. dia.	Closed; return		
Lockheed Aircraft Corporation, Burbank, California	300 mph		8 x 12 ft.	Closed; return	1250	Atmospheric
	180 mph		12 in. circular	Closed; return	2000	
Louisiana State University, University, Louisiana	100 mph		4 ft. dia.	Closed		
Maryland, University of, College Park, Maryland	100 mph		3 ft. dia.	Open		
	350 mph	6 component balance	7¼ x 11 ft.	Closed; return	2800	Force measurements and powered model tests
Massachusetts Institute of Technology, Cambridge, Massachusetts	80 mph		7½ ft. dia.	Closed; return		
	95 mph	NPL Balance	5 ft. dia.	Closed; return	95	
	120 mph	NPL Balance	5 x 7½ ft.	Closed	100	Atmospheric
	130 mph at 4 atmospheres 404 mph at ¼ atmospheres	6 component "Truncated pyramid" wire balance	7½ x 10 ft. 18 ft. long	Closed; Elliptical	2000	¼ to 4 atmospheres vari- able pressure Water cooled

A. WIND TUNNEL FACILITIES IN EXISTENCE

1. Subsonic

Organization — Location	Maximum Speed	Balance System	Size of Section	Throat; Type	Horse- power	Remarks
Michigan, University of, Ann Arbor, Michigan	100 mph	3 component wire	8 ft. octagonal	Grocco Closed; double- return		Under modification
Minnesota, University of Minneapolis, Minnesota	100 mph	3 component wire	4 x 4 ft.	Open; return		
	160 mph	Electric self-balancing 6 component beam balance	7 x 10 ft.	Closed; return	450	Atmospheric
National Advisory Committee for Aeronautics, Flight Propulsion Laboratory, Cleveland, Ohio	54 mph	Structural ring	5 ft. dia.	Closed; return		
	90 mph	Wire and modified NPL	5 ft. dia.	Closed; non-return		
	435 mph		6 x 9 ft.	Single; return	4160	Icing tunnel to -20 C
	500 mph		20 ft. dia.	Closed; return	18000	0.17 to 1 atmospheres vari- able pressure. (To 50,000 ft.) Air interchange up to 6000 pounds per minute
National Advisory Committee for Aeronautics, Langley Field, Virginia	60 mph		12 ft.— 12-sided polygon	Open; return	280	Free flight tunnel. Freon 12 gas used
	62 mph		20 ft.— 12-sided polygon	Closed; vertical, annular return	400	Free spinning tunnel
	100 mph (Model) Air gusts to 25 ft. per second	Catapult and arresting gear	8 x 14 ft.	Open; return	75	Model catapulted across vertical air jet of return flow tunnel
	118 mph		Full-scale	Open; double- return		
	120 mph		30 x 60 ft. elliptical 56 ft. jet length	Open	8000	Drag tests
	165 mph		3 x 7½ x 7 ft.	Closed; return	195	Low turbulence tunnel
	(220 mph) (220 mph) (360 mph)		(6 x 6 ft.) (6.3 ft. dia.) (6 x 2½ ft.) 22 ft. jet length	Closed; return	600	Adjustable working sec- tion size

A. WIND TUNNEL FACILITIES IN EXISTENCE

1. Subsonic

Organization — Location	Maximum Speed	Balance System	Size of Section	Throat; Type	Horse- power	Remarks
National Advisory Committee for Aeronautics, Langley Field, Virginia	250 mph 2½ atmospheres		19 ft. dia. 28.5 jet length	Closed	8000	1 to 2½ atmospheres vari- able pressure
	200 mph		7 x 10 ft. 15 ft. jet length	Closed; return	1600	Stability and control tun- nel
	300 mph at 1 atmosphere		3 x 7½ ft.	Closed; return	2000	1 to 10 atmospheres vari- able pressure Air cooled
National Advisory Committee for Aeronautics, Moffett Field, California	255 mph	3 component balance	40 x 80 ft. oval	Closed; return		
	300 mph	6 component Taller and Cooper Half Yoke	7 x 10 ft. 14.72 ft. long	Closed; return	1600	Atmospheric
	300 mph	6 component Taller and Cooper Half Yoke	7 x 10 ft. 14.72 ft. long	Closed; return	1600	Atmospheric
National Bureau of Standards, Washington, D. C.	100 mph	NPL — wire balance	10 ft. dia. 40 ft. jet length	Closed; non-return	700	Boundary layer studies
	100 mph	NPL	4½ ft. dia. 19 ft. long	Closed; non-return	75	5 screen turbulence re- ducer. Air cooled
	205 mph		6 ft. dia. 12 ft. 8 in. long	Closed; non-return	750	5 screen turbulence re- ducer. Air cooled
Navy Department, #1 David W. Taylor Model Basin, Washington, D. C.	180 mph	6 component Toledo	8 x 10 ft. 14.25 ft. jet length	Closed	710	Coarse screen turbulence correctors water-cooled; atmospheric
Navy Department, #2 David W. Taylor Model Basin, Washington, D. C.	160 mph	6 component Toledo	8 x 10 ft. 14 ft. long	Closed	750 max.	Coarse screen turbulence correctors water-cooled
Navy Department, Navy Yard, Washington, D. C.	75 mph	Zahn 6 com- ponent	8 x 8 ft. 33 ft. long	Closed; return	500	Atmospheric
	107 mph	Cross-arm type	6.3 ft. dia. 6 ft. 7 in. jet length	NPL Open; return	200	Atmospheric
New York University, New York, New York	48 mph	Cross-arm Zahn	4 x 4 ft.	Closed		
	140 mph		9 ft. Octagonal	Closed; return		

A. WIND TUNNEL FACILITIES IN EXISTENCE

1. Subsonic

Organization — Location	Maximum Speed	Balance System	Size of Section	Throat; Type	Horse- power	Remarks
North American Aviation Corporation, Inglewood, California	300 mph	6 component magnetic balance	7 $\frac{3}{4}$ x 11 ft.	Closed; return	2800	Atmospheric
Northrop Aircraft Corporation, Hawthorne, California	160 mph		10 ft. dia. 10 ft. long		1000	Tailless airplane tests — atmospheric
Northeastern University, Boston, Massachusetts	115 mph		3 ft. dia. hexagonal	Closed		
Notre Dame, University of, Notre Dame, Indiana	75 mph		38 x 38 in.			
Ohio State University, Columbus, Ohio	105 mph		3 ft. octagonal	Closed; return		
Oklahoma, University of Norman, Oklahoma	300 mph		4 x 6 ft. elliptical	Closed; return		
Pennsylvania State College, State College, Pennsylvania	130 mph		3 x 4 ft.	Open or closed; return	125	Atmospheric
	280 mph		2 x 3 ft. 5 ft. long	Open or closed; return	250	Atmospheric
Pittsburgh, University of Pittsburgh, Pennsylvania	90 mph		8 x 12 ft.	Open or closed; return		
	120 mph		4 ft. dia.	Open		
Polytechnic Institute of Brooklyn, Brooklyn, New York	100 mph		30 x 40 in.			Low turbulence (also small associated tunnels for in- struction purposes)
	135 mph		3 ft. 5 in. x 3 ft. 5 in.	Open or closed		
Princeton University Princeton, New Jersey	200 mph	Wire balance	4 x 5 ft.	Closed; return	250	Educational
Reaction Motors, Incorporated, Dover, New Jersey	150 mph		2 sq. ft.	Open; non- return		
Rensselaer Polytechnic Institute, Troy, New York	90 mph		8 x 12 ft.	Open or closed		
	130 mph		4 x 6 ft.	Closed; return	150	

A. WIND TUNNEL FACILITIES IN EXISTENCE

1. Subsonic

Organization — Location	Maximum Speed	Balance System	Size of Section	Throat; Type	Horse- power	Remarks
Southern California, University of, at Fontana, California	105,000 cu ft/min at 35 psig		17 x 20 in.			
Southern California, University of, at Los Angeles, California	100 mph 400 mph		3 x 5 ft. 2 ft. dia.	Closed; return Closed; return		
Stanford University, Palo Alto, California	100 mph	Hanging Wire	7.5 ft. dia.	Eiffel		
State College, Pullman, Washington	150 mph		3 ft. dia. hexagonal	Closed; return		
Texas A & M, College Station, Texas	150 mph		7 x 10 ft.	Closed; non-return	800	
Tri State College, Angola, Indiana	100 mph		51 x 30 in.	Open		
United Aircraft Corporation, Research Division, East Hartford, Connecticut	100 mph 170 mph 550 mph (200 mph) (600 mph)		4 x 6 ft. octagonal 15 x 60 in. 8 in. dia. (18 ft. dia.) (8 ft. dia.)		75 5000	Pilot tunnel Laminar flow channel Model of 18 ft. wind tun- nel (1 tunnel with interchange- able working sections). Exchange cooling. Effec- tive altitude 16,000 feet
United Aircraft Corporation, Sikorsky Division, Stratford, Connecticut	55 mph		5 ft. dia.		20	
Virginia, University of Charlottesville, Virginia	120 mph		30 x 50 in.	Open or closed; return		
War Department, Army Air Forces, Wright Field, Dayton, Ohio	150 mph 300 mph 350 mph open 450 mph closed		12 ft. 16 sided polygon 5 ft. dia. 18 ft. jet length 20 ft. dia. 20 ft. jet length	Vertical; closed Closed; return Open or closed; return	1000 900 30000	Spin tunnel, atmospheric Up to 2 atmospheres vari- able pressure. Power sup- plied for supersonic run- ning. Brine radiator cool- ing

A. WIND TUNNEL FACILITIES IN EXISTENCE

1. Subsonic

Organization — Location	Maximum Speed	Balance System	Size of Section	Throat; Type	Horse- power	Remarks
Washington, University of, Seattle, Washington	55 mph	NPL Balance auxiliary	4 x 4 ft.	Closed		Boundary layer control apparatus, free flight
	90 mph		3 ft. hexagonal			
	250 mph		8 x 12 ft.	Closed; return	1000	Flutter model and general tests
Wichita, Municipal University of, Wichita, Kansas	112 mph		4 ft. dia.	Closed; return		

A. WIND TUNNEL FACILITIES IN EXISTENCE

2. Transonic

Organization — Location	Maximum Speed	Balance System	Size of Section	Throat; Type	Horse-power	Remarks
Boeing Aircraft Company, Seattle, Washington	Mach No. 0.95	Magnetic	8 x 12 ft. Octagonal 15 ft. jet length	Closed; return	15000	Slanting arch cover over working section Exchange and spray vane cooling
	Approx. Mach No. 1		Approx. 7 x 7 ft.	Closed		
California Institute of Technology, Pasadena, California	Mach No. 0.95	6 component balance with 1, 2, or 3 strut support	8½ x 12 ft.	Closed; return	12000	¼ — 4 atmospheres variable pressure water cooled (Cooperative wind tunnel)
Guggenheim Aeronautical Laboratory	Mach No. 0.8 to 1.1		2 x 20 in.			Two-dimensional tunnel
	Mach No. 0.8 to 0.9		1 x 10 in.	Induction type		Two dimensional high speed subsonic tunnel
Cornell Aeronautical Laboratory, Buffalo, New York	Mach No. 0.4 at 4 atmos.		8½ x 12 ft.	Closed	9000 normal, 15000 max.	Exchange cooling ¼ — 4 atmospheres variable pressure
	Mach No. 0.6 at 1 atmos.					
	Mach No. 0.9 at ¼ atmos.					
	Mach No. 0.95.		1.63 x 2.35 in.		300	Atmospheric
	Mach No. 0.95		3 x 16 in.	Closed circuit	600	Atmospheric
Lone Star Laboratory, Daingerfield, Texas (Bureau of Ordnance, Navy Department)	Low subsonic to near supersonic	3 component balance; inside sting	19 x 27½ in.	Closed; non-return	16000	⅔ to 3 Atmos.
National Advisory Committee for Aeronautics, Langley Field, Virginia	Mach No. 0 to 1.0	Ring framed Toledo	8 ft. dia. 14.4 ft. jet length	Closed; return	16000	11 screen turbulence reducer. Air cooled
	Mach No. 0.4 to 1.0		12 ft. dia.	Closed	11000	
	Mach No. 1.0		7 x 10 ft. 15 ft. jet length	Closed; return	10000	Atmospheric tunnel
	Mach No. 1.0		24 in. dia. 16 in. jet length	Closed; non-return		
	Mach No. 0.2 to 1.4	3 component mechanical balance beams	4 x 18 in. 10 in. jet length	Closed; non-return		Atmospheric tunnel

A. WIND TUNNEL FACILITIES IN EXISTENCE

2. Transonic

Organization — Location	Maximum Speed	Balance System	Size of Section	Throat; Type	Horse- power	Remarks
National Advisory Committee for Aeronautics, Langley Field, Virginia	Mach No. 0.812 to 1.0		4½ ft. dia. 9 ft. jet length	Induction type; Closed throat	1000	0 to 1.3 atmospheres Uses Freon 12 — for flutter research
National Advisory Committee for Aeronautics, Moffett Field, California	Mach No. 1.0		1 x 3½ ft.	Closed; return	2000	Low turbulence tunnel
	Mach No. 0.05 to 0.90		16 ft. dia.	Closed; return	27000	Air exchanger
	Mach No. 0.4 to 0.98		12 ft. dia.	Closed		¼ to 6 atmospheres vari- able pressure. Screen in- let. Water cooled
War Department, Army Air Forces, Wright Field, Dayton, Ohio	Mach No. 1.0		10 ft. dia. 16 ft. 7 in. long	Closed	40000	⅛ to 2 atmospheres vari- able pressure
War Department, Army Ordnance, Aberdeen Proving Ground, Aberdeen, Maryland	Mach No. 0.1 to 0.9	Tate-Emory hydraulic cells	15 x 20 in. 20 in. long	Closed	6000 normal 9000 max.	0.4 to 1.7 atmospheres (Also listed in supersonic group)
	Mach No. 0.1 to 0.9	Tate-Emory hydraulic cells	15 x 20 in. 36 in. long	Closed	13000	0.03 to 0.3 atmospheres (Also listed in supersonic group)

A. WIND TUNNEL FACILITIES IN EXISTENCE3. Supersonic

Organization -- Location	Maximum Speed	Balance System	Size of Section	Throat; Type	Horse- power	Remarks
California Institute of Technology, Pasadena	Mach No. 3.2		2½ x 2½ in.	Exhaust operated		Pilot to Aberdeen tunnel
California, Guggenheim Aeronautical Laboratory	Mach No. 4		2½ x 2½ in.			Continuous operation
California, University of, Berkeley, California	Mach No. 3		¾ x 1 in.	Closed; non-return		Vacuum operated steam ejector powered, at 1 in. of Hg.
Cornell Aeronautical Laboratory, Buffalo, New York	Mach No. 1.7		2¼ x 6¾ in.	Closed; non-return	600	Vacuum operated — Atmospheric
Lockheed Aircraft Corporation, Burbank, California	Mach No. 1.4		3 x 16 in.	Open; non- return		
Lone Star Laboratory, Daingerfield, Texas (Bureau of Ordnance, Navy Department)	Mach No. 1.25 Mach No. 1.50 Mach No. 1.75 Mach No. 2.00 Mach No. 2.25 Mach No. 2.50	3 component balance; inside sting	19 x 27½ in.	Closed throat	16000	Speeds varied by changing nozzle sizes. ½ to 3 atmos. Arranged for combustion chamber tests
Michigan, University of Ann Arbor, Michigan	Mach No. 1 to 4.5	3 component wire balance	8 x 13 in.	Closed; non-return		Vacuum operated
National Advisory Committee for Aeronautics, Flight Propulsion Research Laboratory, Cleveland, Ohio	Mach No. 2.2 Mach No. 1.85		18 x 18 in. 20 in. dia.	Closed; non return Closed; non-return		Exhaust from large tunnel Exhaust from large tunnel
	Mach No. 2.0		3½ x 3½ in.			
National Advisory Committee for Aeronautics, Langley Field, Virginia	Mach No. 1.35 to 2.0 Mach No. 2.2	3 component mechanical balance beams	7½ x 7½ in. to 7½ x 9 in. 11 in. jet length 4 x 4 in.	Closed; non-return variable Closed; non-return	1000	½ to ½ atmospheres Sub atmospheric tunnel, flutter research

A. WIND TUNNEL FACILITIES IN EXISTENCE

3. Supersonic

Organization — Location	Maximum Speed	Balance System	Size of Section	Throat; Type	Horse-power	Remarks
National Advisory Committee for Aeronautics, Moffett Field, California	Mach No. 2.3	Internal 3 component balance in sting	3 x 1 ft.	Closed; return		Pressure fed; variable density
	Mach No. 2.6	Internal 3 component balance in sting	8 x 8 in.	Closed; return		
	Mach No. 3.4	Internal 3 component balance in sting	3 x 1 ft.	Closed; return		Pressure fed; variable density
North American Aviation, Inc., Inglewood, California	Mach No. 1.25 to 3.25		1 3/4 to 4 1/2 in.			Variable speed obtained by altering inlet diffuser
Northwestern University, Evanston, Illinois	Mach No. 1.0 at 25 psi gauge		2 1/4 in. dia.			
Southern California, University of, at Fontana, California	Mach No. 2.5		17 x 20 in.			A smaller throat is planned for Mach No. 3.0
Southern California, University of, at Los Angeles, California	Mach No. 2.0		4 x 4 in.	Non-return		
United Aircraft Corporation, East Hartford, Connecticut	Mach No. 1.4 Mach No. 1.5 Mach No. 1.6	6 component wire balance (under construction)	55 sq. in.	Closed; partial return		
War Department, Army Air Forces, Wright Field, Dayton, Ohio	Mach No. 2.5				5000	1/9 to 2 atmos. Pressure fed; brine radiator cooled
War Department, Army Ordnance, Aberdeen Proving Ground, Aberdeen, Maryland	Mach No. 1.3 to 1.7				6000 to 9000 max.	0.4 to 1.7 atmos. Pressure operated bomb tunnel. (Also listed in Transonic group)
	Mach No. 1.1 to 4.4	Tate-Emory hydraulic cells	15 x 20 in. 36 in. long	Closed; return	13000	0.03 to 3 atmos. Pressure operated ballistics tunnel. (Also listed in Transonic group)

B. WIND TUNNELS IN DESIGN OR UNDER CONSTRUCTION

Organization — Location	Maximum Speed	Balance System	Size of Section	Throat; Type	Horse- power	Remarks
Applied Physics Laboratory, Forest Grove Burner Laboratory, Forest Grove Station, Maryland	Mach No. to 7 with air Mach No. to 15 with helium		1 x 2.2 in.			Hypersonic tunnel inter- mittent flow
Bell Aircraft Corporation, Buffalo 5, New York	Mach No. 2.5 to 4		12 x 12 in. to 24 x 24 in.			Proposed supersonic chan- nel for aerodynamic and combustion tests
California, University of, Berkeley, California	Mach No. 1.5 to 4.0		In design	Open		Continuous low pressure tunnel
California, Institute of Technology, Guggenheim Aeronautical Laboratory, Pasadena, California	Mach No. 10.0		5 x 5 in.			In design
California Institute of Technology, Jet Propulsion Laboratory, Pasadena, California	Mach No. 3.0 Mach No. 4.8		15 x 15 in. 15 x 20 in.	Proposed closed throat	9000	
Johns Hopkins University, Baltimore, Maryland	Mach No. 4.0		2 x 2 ft.			
Johns Hopkins University, Laurel, Maryland	Mach No. 10		1½ x 2.2 in.			
Marquardt Aircraft Company, Venice, California	Mach No. 0.5 to 0.9		5 x 7 ft.	48 in. 30 in.		Ramjet cold flow test tun- nel for fuel distribution studies
Maryland, University of, College Park, Maryland	Mach No. 0.42		7¾ x 11 ft.		2800	
Massachusetts Institute of Technology, Cambridge, Massachusetts	Mach No. 1.5 Mach No. 2.0 Mach No. 2.5		18 x 24 in.	Closed throat	10000 to 12500 estimated	Speed varied by changing nozzle size. Due for opera- tion latter part 1948
Michigan, University of, Ann Arbor, Michigan	Mach No. 5.0		9 x 13 in.			

B. WIND TUNNELS IN DESIGN OR UNDER CONSTRUCTION

Organization — Location	Maximum Speed	Balance System	Size of Section	Throat; Type	Horse-power	Remarks
Minnesota, University of, Gopher Ordnance Works, Rosemont, Minnesota	Mach No. 1.0		42.6 sq. in.	Closed; induction type		Designed specifications of removable test sections and throats for transonic tunnel. (Sec. A, Part 2)
	Mach No. 2.0		58.1 sq. in.			
	Mach No. 2.5		64.3 sq. in.			
	Mach No. 3.0		67.7 sq. in.			
	Transonic	3 component balance	16 x 16 in.			
	Mach No. 2.5		16 sq. ft.	Closed; induction type		Under construction
National Advisory Committee for Aeronautics, Flight Propulsion Research Laboratory, Cleveland, Ohio	Mach No. 2.0		6 x 8 ft.	Closed throat		Under construction
	Mach No. 4.5		2 x 2 ft.			
National Advisory Committee for Aeronautics, Langley Field, Virginia	Mach No. 0.3 to 1.4	}	4 x 16 in.	}		Experimental tunnel under construction for ONR
	Mach No. 1.4 to 4.6		4 x 4 in.			
	Mach No. 0.9		16 ft. dia. circular			(Modification of present tunnel)
National Advisory Committee for Aeronautics, Moffett Field, California	Mach No. 1.8		6 x 6 ft.			Continuous operation
	Mach No. 2.3			Closed throat		
	Mach No. 3.4		1 ft. x 3 ft. rectangular	Closed throat		Intermittent operation
Navy Department, Naval Ordnance Laboratory, White Oak, Maryland	Mach No. 1.22 to 5.18	3 component spring type magnetic pick-up remotely mounted (40 x 40 on tunnel only)	40 x 40 cm	Vacuum operated; non-return		Kochel tunnels "Blow down" type
	Mach No. 1.3 to 5.2		18 x 18 cm			
	Mach No. 2.5 to 5.2		80 x 80 cm			
			12 x 12 cm			
Navy Department, David W. Taylor Model Basin, Carderock, Maryland	Mach No. 0.90		9.84 ft. dia.	LFM	16000	Under construction
	Mach No. 3.2		8 x 8 in.	Sonthofen		German tunnel

B. WIND TUNNELS IN DESIGN OR UNDER CONSTRUCTION

Organization — Location	Maximum Speed	Balance System	Size of Section	Throat; Type	Horse- power	Remarks
North American Aviation, Inc., Inglewood, California	Mach No. 4.5	6 component wire "truncated pyramid" type (strain gauge)	15% x 15% in.			Intermittent flow 15-20 seconds duration
Princeton University, Princeton, New Jersey	Mach No. 1.5 to 5.0		4 x 5 in. to 4 x 8 in.	Closed; exhaust operated		Under construction
	Mach No. 1.5 to 5.0		3 x 1½ in. to 3 x 3 in.	Closed; exhaust operated		Under construction
War Department, Army Air Forces, Wright Field, Dayton, Ohio	Mach No. 2.5		1.31 x 1.31 ft.		4500	Ottobrun tunnel
Washington, University of, Seattle, Washington	Mach No. 4.0 to 8.0					Free flow tunnel (Design stage)

REFERENCES

General

1. J. Ackeret, "Gasdynamik," *Handbuch der Physik*, Vol. 7, Springer, Berlin, 1927.
 2. A. Busemann, "Gasdynamik" in *Handbuch der Experimentalphysik*, Vol. 4, Akademische Verlagsgesellschaft, Leipzig, 1931.
 3. California Institute of Technology, *Jet Propulsion*. A text prepared by the Staffs of the Guggenheim Aeronautical Laboratory and the Jet Propulsion Laboratory, 1946. *Restricted*.
 4. A. C. Charters, "Some Ballistic Contributions to Aerodynamics" *J. Aero. Sci.* 14, 155-166 (1947).
 5. Richard Courant and K. O. Friedrichs, "Supersonic Flow and Shock Waves." A Manual on the Mathematical Theory of Non-Linear Wave Motion, Applied Mathematics Panel of the National Defense Research Committee, AMP Report 38.2R, August, 1944. *Unclassified*.
 6. H. L. Dryden, F. D. Murnaghan and H. Bateman, *Report of the Committee on Hydrodynamics*, Bulletin of the National Research Council, Number 84, February, 1932, Washington, D. C.
 7. H. W. Emmons, "Shock Waves in Aerodynamics," *J. Aero. Sci.* 12, No. 2, 188-194 (1945).
 8. S. Goldstein, *Modern Developments in Fluid Dynamics*, (Clarendon Press, Oxford, 1938) Volumes I and II
 9. H. Lamb, *Hydrodynamics* (Cambridge University Press, Cambridge, England, 1932).
 10. H. W. Liepmann and A. E. Puckett, *Introduction to Aerodynamics of a Compressible Fluid* (John Wiley & Sons, Inc., New York, 1947).
 11. L. R. Michel, *Bibliography on Flow of Compressible Fluids*, Department of Mechanical Engineering, Massachusetts Institute of Technology, 1942-1943.
 12. L. M. Milne-Thomson, *Theoretical Hydrodynamics* (Macmillan, London, 1938).
 13. N. Minorsky, *Introduction to Non-Linear Mechanics*, Part I, "Topological Methods of Non-Linear Mechanics," (Report 534, December, 1944); Part II, "Analytical Methods of Non-Linear Mechanics," (Report 546, September, 1945); Part III, "Non-Linear Resonance," (Report 558, May, 1946); Part IV, "Relaxation Oscillations," (Report 564); David W. Taylor Model Basin, Navy Department. September, 1946. *Unclassified*.
 14. Richard von Mises and K. O. Friedrichs, *Fluid Dynamics*, Advanced Instruction and Research in Mechanics, Summer, 1941, Brown University, Providence, Rhode Island. Contains supplementary notes on "The Hodograph Method in the Theory of Compressible Fluids" by Stefan Bergman, Summer, 1942.
 15. R. Sauer, *Theoretische Einführung in die Gasdynamik*, (Springer, Berlin, 1943, reprinted by Edwards Brothers, Inc., Ann Arbor, Michigan, 1945).
 16. John Stack, "Compressible Flows in Aeronautics," *J. Aero. Sci.* 12, No. 2, 127-148 (1945).
 17. H. S. Tsien, W. R. Sears, I. L. Ashkenas, C. N. Hsart, N. M. Newmark; *Aerodynamics and Aircraft Design*. (Report of the AAF Scientific Advisory Group, Published by Headquarters Air Materiel Command, Publications Branch, Intelligence T-2, Wright Field, Dayton, Ohio, May, 1946). *Restricted*.
- Propagation and Interaction of Shock Waves*
18. Walker Bleakney, Progress Reports, Division 2, Princeton University Station, NDRC. *Confidential*.
 19. F. B. Harrison and Walker Bleakney, "Re-measurement of Reflection Angles in Regular and Mach Reflection of Shock Waves," Princeton University, March, 1947.
 20. P. C. Keenan and R. J. Seeger, "Analysis of Shock Intersections," Bureau of Ordnance Confidential Explosives Report #15 (1944). *Confidential*.
 21. J. G. Kirkwood and S. R. Brinkley, Jr., O.S.R.D. Report 4814 (1945). *Confidential*.
 22. Rudolf Ladenburg, "Study of Shock Waves by Interferometry," Report on Project 208, May 15, 1944, Princeton University Station, Division 2, N.D.R.C. *Confidential*.
 23. Rudolf Ladenburg and G. Wolfsohn, "Dispersion of Gases and Vapours and Representation by the Dispersion Theory," Part III "Dispersion of Oxygen between 6000 Å and 1920 Å," *Zeits. f. Physik*, 79, 1-2, pp. 42-61, Nov. 4, 1932.
 24. R. Ladenburg, C. C. Van Voorhis and J. Winckler, "Interferometric Study of Supersonic Phenomena" Part I. "A Supersonic Air Jet at 60 lb/in² Tank Pressure," NAVORD Report 69-46, 17 April 1946; Part II "The Gas Flow Around

- Various Objects in a Free Homogeneous Supersonic Air Stream," NAVORD Report 93-46, 2 September 1946; and Part III "Interaction of Boundary Layers and Shock Waves," NAVORD Report 7-47, 19 February 1947. *Unclassified*.
25. R. Ladenburg, C. C. Van Voorhis and J. Winckler (Reference 24) Part IV "Interferometric Study of Inhomogeneous Jets Using Low and High Pressure Ratios" and Part V "Interferometric Study of Supersonic Flow and of Boundary Layers in a Two-Dimensional Divergent Channel." *Unclassified*. In preparation.
 26. National Defense Research Committee, Report No. A-332 (O.S.R.D. Report No. 5204) "Study of Shock Waves by Interferometry," Report by Division 2, 30 June 1945. *Confidential*.
 27. NAVORD Report No. 74-46 "Technical Conference on Optical Phenomena in Supersonic Flow," Bureau of Ordnance Publication, 30 May 1946. *Confidential*.
 28. J. von Neumann, "Oblique Reflection of Shocks" Bureau of Ordnance Confidential Explosives Research Report #12 (1943). *Confidential*.
 29. J. von Neumann, "Refraction, Intersection and Reflection of Shock Waves". Paper presented at Conference on Shock Waves and Supersonic Flow, 19 March 1945. NAVORD Report 203-45, pp. 4-12, 16 July 1945. *Confidential*.
 30. H. Polachek and R. J. Seeger, "Regular Reflection of Shocks in Ideal Gases," Bureau of Ordnance Confidential Explosives Research Report #13 (1944). *Confidential*.
 31. L. G. Smith "Photographic Investigation of the Reflection of Plane Shock Waves in Air," O.S.R.D. Report No. 6271, November, 1945.
 32. A. H. Taub, "Some Numerical Results on Refraction of Plane Shocks," University of Washington. *Unclassified*.
 33. A. H. Taub, "Refraction of Plane Shock Waves," Physical Review, **72**, 51-60 (1947).
 34. E. B. Wilson, Jr., W. D. Kennedy, A. H. Taub, R. D. Huntoon, R. J. Seeger, L. G. Smith, R. R. Halverson, "Air Burst for Blast Bombs," O.S. R.D. Confidential Report #4943, (1945). *Confidential*.
 35. G. Wolfsohn, "Dispersion of Gases and Vapours," Part V "Interferometer for Schumann Region," Zeits. f. Physik **85**, 5-6, pp. 366-372, Sept. 14, 1933.
 36. J. Ackeret, F. Feldmann and N. Rott, "Investigations of Compression Shocks and Boundary Layers in Gases Moving at High Speeds." NACA Technical Memorandum No. 1113, January, 1947.
 37. H. J. Allen, M. A. Heaslet, and G. E. Nitzberg, "The Interaction of Boundary Layer and Compression Shock and Its Effect upon Airfoil Pressure Distributions," N.A.C.A. Research Memorandum RM A7A 02, 10 April 1947. *Confidential*.
 38. Donaldson, Coleman du P., "Effects of Interaction between Normal Shocks and Boundary Layer," N.A.C.A. CB No. 4A27, 1944. *Confidential*.
 39. Hugh L. Dryden, "Some Recent Contributions to the Study of Transition and Turbulent Boundary Layers," Paper presented at the Sixth International Congress for Applied Mechanics, Paris, September 22-29, 1946. Reprinted as N.A.C.A. Technical Note No. 1168, April, 1947.
 40. A. M. Kuethe and H. T. Epstein, "Viscosity Effects in Transonic and Supersonic Flow," BUMBLEBEE Report No. 48 (1947). *Confidential*.
 41. Lester Lees and C. C. Lin, "Investigation of the Stability of the Laminar Boundary Layer in a Compressible Fluid," N.A.C.A. Technical Note No. 1115, September, 1946.
 42. H. W. Liepmann, "Investigations on Laminar Boundary Layer Stability and Transition on Curved Boundaries," N.A.C.A. Advance Confidential Report 31130 (1943). (Now *Unclassified*.)
 43. H. W. Liepmann, "Investigation of Boundary Layer Transition on Concave Walls," N.A.C.A. Advance Confidential Report 4J28, February, 1945. (Now *Unclassified*.)
 44. H. W. Liepmann, "The Interaction Between Boundary Layer and Shock Waves in Transonic Flow," J. Aero. Sc. **13** No. 12, 623-638 (1946).
 45. H. W. Liepmann and Gertrude H. Fila, "Investigations of Effects of Surface Temperature and Single Roughness Elements on Boundary Layer Transition," N.A.C.A. Technical Note No. 1196, April, 1947. *Unclassified*.
 46. C. C. Lin, "On the Stability of Two-Dimensional Parallel Flows," Quar. Appl. Math. **3** (No. 2) 117-142, July (1945); **3** (No. 3) 218-234, October (1945); **3** (No. 4) 277-301, January (1946).
 47. Polytechnic Institute of Brooklyn Quarterly Progress Report, October 1st to December 31st, 1946, to Office of Naval Research, Navy Department, Washington 25, D. C. *Unclassified*.

Boundary Layers

36. J. Ackeret, F. Feldmann and N. Rott, "Investigations of Compression Shocks and Boundary

48. L. Prandtl, *Verhandlungen des dritten internationalen Mathematiker-Kongresses, Heidelberg, 1904* (Leipzig, 1905), pp. 484-491.
 49. Princeton University, Semi Annual Progress Report, Project SQUID 1 January 1947. *Unclassified*. See also Reference 24, Part III.
 50. M. Ras and J. Ackeret, "Über Verhinderung der Grenzschicht-Turbulenz durch Absägung," *Helvetica Physica Acta*, 14 No. 5-6, 232 (1941).
 51. Osborne Reynolds, *Phil. Trans. Roy Soc.*, 1883; *Collected Papers II*, p. 51.
 52. H. Schlichting, "Zur Entstehung der Turbulenz bei der Plattenströmung." *Nach. Gesell. d. Wiss. z. Gött.*, M.P.K. 181-208 (1933) and "Amplitudenverteilung und Energiebilanz der kleinen Störungen bei der Plattenströmung." *Nach. Gesell. d. Wiss. z. Gött.*, M.P.K. 1 47-78 (1935).
 53. G. B. Schubauer and H. K. Skramstad "Laminar Boundary Layer Oscillations and Transition on a Flat Plate," N.A.C.A.-A.C.R. 1943. Reprinted in the *Journal of Research of the National Bureau of Standards*, 38, 251-292 (1947) *Research Paper 1772*.
 54. E. S. Taylor and E. P. Neumann, "Interaction of Shock Waves and Boundary Layer," in *METEOR Report No. 8 Interim Progress Report of the METEOR Project*, March, 1947, Page 20. *Confidential*.
 55. W. Tollmien, "Zur Entstehung der Turbulenz," *Nach. Gesell. d. Wiss. z. Göttingen*, M.P.K. 21-44 (1929); N.A.C.A. Technical Memorandum No. 609 (1931). See also "General Instability Criterion of Laminar Velocity Distributions," N.A.C.A. Technical Memorandum No. 792, April, 1936.
- Diffusers and Nozzles*
56. Applied Mathematics Panel, National Defense Research Committee "Theoretical Studies on the Flow through Nozzles and Related Problems." N.D.R.C. Report No. 82.1R, New York University Group, April, 1944, (OSRD No. 3591). Now *Unclassified*.
 57. Applied Mathematics Panel, National Defense Research Committee, "On Supersonic Compressors and Nozzles," N.D.R.C. Report No. 82.2R, New York University Group, October, 1944 (OSRD No. 4416). Now *Unclassified*.
 58. Applied Physics Laboratory, Johns Hopkins University, *Symposium on Aerodynamics*, BUMBLEBEE Report No. 29, pp. 29-38, December, 1945. *Confidential*.
 59. J. G. Coffin, "Subsonic and Supersonic Flow in Nozzles and Diffusers," Kaiser Cargo, Inc., Fleetwings Division, Bristol, Pennsylvania. Report No. PA-TR-1-1, November, 1946. *Confidential*.
 60. B. L. Crawford and R. J. Knapp, "Comparison of Spark Photographs with Theoretical Flow Patterns at Supersonic Speeds." North American Aviation, Inc., Report No. CF-531-AL 90. *Restricted*.
 61. Luigi Crocco, "Gallerie aerodinamiche per alta velocita," *L'Aerotecnica*, Vol. XV, fasc. 3, 237-275, March 1935 and Vol. XV, fasc. 7 and 8, 735-778, July and August 1935.
 62. Hugh L. Dryden and G. B. Schubauer, "The Use of Damping Screens for the Reduction of Wind-Tunnel Turbulence," *J. Aero. Sci.* 14, 221-228 (1947).
 63. Antonio Ferri, "Application of the Method of Characteristics to Supersonic Rotational Flow." N.A.C.A. Technical Note No. 1135, September, 1946.
 64. Antonio Ferri and Louis M. Nucci, "Preliminary Investigation of a New Type of Supersonic Inlet." N.A.C.A. Research Memorandum RM No. L 6J31, 27 November 1946.
 65. Kuno Foelsch, "A New Method of Designing Two-Dimensional Laval Nozzles for a Uniform and Parallel Jet," North American Aviation, Inc., Report No. NA-46-235 (1946) and No. NA-46-235-1 (1946), and No. NA-46-235-2 (1946). *Unclassified*.
 66. N. A. Hall, "Theoretical Performance of Convergent-Divergent Nozzles," *METEOR Report UAC-1*, United Aircraft Corporation, December, 1946. *Confidential*.
 67. I. M. Hirschfeld, "Test Proposal — $\frac{1}{15}$ Scale Model of Inlet Diffuser — XRJ 37 — MN-1," Menasco Manufacturing Company, Burbank, California. Report No. P-127, 26 August, 1946. *Confidential*.
 68. Arthur Kantrowitz and Coleman du P. Donaldson, "Preliminary Investigation of Supersonic Diffusers," N.A.C.A. Advance Confidential Report, ACR No. 15D20, May, 1945. (Now *Unclassified*.)
 69. Arthur Kantrowitz, "The Formation and Stability of Normal Shock Waves in Channel Flow," N.A.C.A. Technical Note No. 1225, March, 1947.
 70. J. H. Keenan and E. P. Neumann, "Measurements of Friction in a Pipe for Subsonic and Supersonic Flow of Air," *Journal of Applied*

- Mechanics, Trans. A.S.M.E. 13 No. 2, A-91—A-100, June, 1946.
71. Massachusetts Institute of Technology — METEOR Report No. 8, Interim Progress Report of the METEOR Project, March, 1947. *Confidential*.
 72. National Advisory Committee for Aeronautics, Progress Report of the Langley Memorial Aeronautical Laboratory, April 29, 1947. *Confidential*.
 73. National Advisory Committee for Aeronautics, *Thirty-Second Annual Report*, 1946.
 74. National Advisory Committee for Aeronautics, "The Use of Perforated Inlets for Efficient Supersonic Diffusion," Research Memo. No. E 7C26. *Confidential*.
 75. Klaus Oswatitsch, "Pressure Recovery in Missiles with Reaction Propulsion at High Supersonic Speeds" (The Efficiency of Shock-Diffusers). Reprint of Ordnance Department Report No. 1005, AAF translation, Report No. F-TS 609-RE, dated 13 May, 1946. (Available at Air Materiel Command, Army Air Force, Wright Field, Ohio.) *Unclassified*.
 76. W. C. Randels and B. L. Crawford, "Supersonic Inlet Diffusers," North American Aviation, Inc., Report No. CF 686-AL-199. *Confidential*. See also these North American Aviation reports: W. C. Randels and R. B. Pearce, "Supersonic Diffuser Development Program," No. CF-444-AL 62. *Confidential*. B. L. Crawford, "Design Data for Spike Diffusers," Report No. CF 637-AL 147. *Confidential*.
 77. C. F. Riley and T. F. Reinhardt, "Analysis of the Conical Shock Diffuser," Bell Aircraft Corporation, Buffalo, New York. Report No. 5G-982-004, 7 January, 1947. *Confidential*.
 78. A. H. Shapiro, G. M. Edelmann and E. S. Snyder, "Contours for Two Families of Two-Dimensional Nozzles," Report No. 2 of the Gas Turbine Laboratory, Massachusetts Institute of Technology.
 79. A. H. Shapiro, "Nozzles for Supersonic Flow without Shock Fronts," Mech. Trans. A.S.M.E. 66 A-93 (1944).
 80. E. S. Taylor and E. P. Neumann, "Progress Report on the Gas Turbine Laboratory," November 1, 1946. Massachusetts Institute of Technology.
 81. G. I. Taylor and J. W. Maccoll, "The Air Pressure on a Cone Moving at High Speeds," Proc. Roy. Soc. A, 139, 278-311 (1933).
 82. T. H. Troller, "Note on the Aerodynamics of Jet-Propelled Missiles." Jet-Propelled Missiles Panel of the Office of Field Service of the Office of Scientific Research and Development, May, 1945. *Confidential*.
 83. Ralph Zirkind, "A Summary of Ramjet Design Considerations," Bureau of Aeronautics, Navy Department, ADR Report No. 1003, April, 1946. *Confidential*.
- ### Hydraulic Analogy
84. R. C. Binder, *Fluid Mechanics* (Prentice-Hall, Inc., New York, 1947), pp. 211-216. Hunter Rouse, *Elementary Mechanics of Fluids* (John Wiley & Sons, Inc., New York, 1946), pp. 321-327.
 85. J. R. Bruman, "Application of the Water Channel-Compressible Gas Analogy," Report No. NA-47-87 of the North American Aviation, Inc., dated 3 March, 1947. *Unclassified*.
 86. R. E. Friedman, "Water Model Test of Diffuser Entrance Conditions of Ramjet," Report No. P-128 (XRJ-37-MN-1) of the Menasco Manufacturing Company, dated 30 September, 1946. *Confidential*.
 87. I. M. Hirschfeld, "Test Proposal $\frac{1}{15}$ Scale Model of Inlet Diffuser, XRJ 37-MN-1," Report No. P-127 of the Menasco Manufacturing Company, dated 26 August, 1946. *Confidential*.
 88. R. T. Knapp, "The Analogy between Surface Shock Waves on Liquids and Shocks in Compressible Gases," Report under Contract NORD, 9612, September, 1946. *Restricted*.
 89. New York University, New York, New York, Semi-Annual Progress Report, Project SQUID, 1 January, 1947. *Unclassified*.
 90. W. J. Orlin, N. J. Lindner and J. G. Bitterly, "Application of the Analogy between Water Flow with a Free Surface and Two-Dimensional Compressible Gas Flow," N.A.C.A. Technical Note No. 1185, Feb., 1947.
 91. Leigh Page, *Introduction to Theoretical Physics*, (D. van Nostrand Company, Inc., New York, 2nd Edition, 1935), pp. 246-250.
 92. Ernst Preiswerk, "Applications of the Method of Gas Dynamics to Water Flows with Free Surface," Parts I and II, N.A.C.A. Technical Memoranda 934 and 935, March, 1940.
 93. D. Riabouchinsky, "Mecanique des Fluides," Comptes rendus, 195, No. 22, 998-999 (1932).
 94. F. Schultz-Grunow, "Gas Dynamical Investigation of the Pulse Jet Tube," N.A.C.A. Technical Memorandum 1131, February, 1947.
 95. J. J. Stoker, "The Formation of Bores and Breakers." This will appear in September, 1947,

in the first number of the *Communications on Applied Mathematics* to be issued by the Institute for Mathematics and Mechanics, New York University, New York, New York.

Turbulence

96. Applied Physics Laboratory, Johns Hopkins University, BUMBLEBEE Report No. 43, pp. 10-11 (September, 1946, Survey) and No. 56, pp. 3-5 (March Survey). *Confidential*.
97. W. J. Barr, "Survey Report on Turbulence," Technical Publication No. 27, APL-JHU-CM373, December 31, 1946. *Restricted*. Bibliography contains 169 references to theoretical and experimental research on turbulence.
98. L. F. Blinman and R. S. Bevans, Progress Report for the month of March, 1947, Jet Propulsion Combustion Chamber Research — Contract NOa(s)8632 with the Bureau of Aeronautics, Navy Department. *Restricted*.
99. P. L. Chambre and C. C. Lin, "The Effect of Turbulence on Flame Propagation," California Institute of Technology, Report R3-5, 28 November, 1945.
100. Gerhard Damköhler, "The Effect of Turbulence on the Flame Velocity of Gas Mixtures," N.A. C.A. Tech. Memo. No. 1112, April, 1947.
101. H. L. Dryden in *Aerodynamic Theory*, Springer, Berlin, 1934. Edited by W. F. Durand, Division T, Volume VI, pp. 228-281 (reprinted by Durand Reprinting Committee, Calif. Inst. of Tech., Pasadena, 1943).
102. H. L. Dryden, "A Review of the Statistical Theory of Turbulence," *Quart. App. Math.* 1, 7 (1943).
103. S. Goldstein, "The Similarity Theory of Turbulence, and Flow Between Parallel Planes and through Pipes," *Proc. Roy. Soc.* 159, 473-496 (1937).
- 103a. Werner Heisenberg "Zur Statistischen Theorie der Turbulence." Unpublished paper. Reference to this work was provided by Dr. C. C. Lin, Brown Univ.
104. H. C. Hottel and G. C. Williams, Interim Progress Report of the METEOR Project, Report No. 8, p. 20, March, 1947. *Confidential*.
105. Theodore von Karman, "The Fundamentals of the Statistical Theory of Turbulence," *J. Aero. Sci.* 4, 131-138 (1937); "On the Statistical Theory of Turbulence," *Proc. Nat. Acad. Sci.* 23, 98 (1937); "Some Remarks on the Statistical Theory of Turbulence," *Proc. 5th Int. Cong. App. Mech.*, Cambridge, 1938.
- 105a. A. N. Kolmogoroff, "The Local Structure of Turbulence in Incompressible Viscous Fluid for very large Reynolds Numbers." *Comptes rendus (Doklady) de l'Academie des Sciences de l'U.R.S.S.* 30 No. 4, 301-305 (1941); "On Degeneration of Isotropic Turbulence in an Incompressible Viscous Liquid." *Ibid.* 31 No. 6, 538-540 (1941); "Dissipation of Energy in the locally Isotropic Turbulence." *Ibid.* 32 No. 1, 16-18 (1941).
103. R. S. Levine, R. G. May and A. C. Seurlock, "Note on Turbulence Measurement," METEOR Internal Report No. 1, Fuels Research Lab. M.I.T., October, 1946. *Unclassified*.
107. C. B. Millikan, "A Critical Discussion of Turbulent Flows in Channels and Circular Tubes," *Proc. 5th Int. Cong. App. Mech.*, Cambridge, Mass., 386-392 (1938).
- 107a. A. Obukhoff, "On the Energy Distribution in the Spectrum of a Turbulent Flow." *Comptes rendus (Doklady) de l'Academie des Sciences de l'U.R.S.S.* 32 No. 1, 19-21 (1941).
108. L. Prandtl and O. G. Tietjens, *Applied Hydro- and Aeromechanics*, (McGraw Hill Book Co., New York, New York, 1934), pp. 130-136.
109. L. Prandtl in *Aerodynamic Theory*, Reference 101, Division G, Volume III, pp. 34-208.
110. G. I. Taylor, "Statistical Theory of Turbulence I-IV," *Proc. Roy. Soc., A* 151, 421-454 (1935). "Statistical Theory of Turbulence V" *Ibid.* A156, 307 (1936).
111. G. I. Taylor, "Some Recent Developments in the Study of Turbulence," *Proc. 5th Int. Cong. App. Mech.*, Cambridge, 294 (1938).
112. University of Illinois, Quarterly Report No. 1, June 1 to August 31, 1946 and Quarterly Report No. 2, Sept. 2 to Nov. 30, 1946, to the Office of Naval Research, Navy Dept., Washington. D. C. *Unclassified*.
- 112a. C. F. Weiszaecker, "Das Spektrum der Turbulenz bei grossen Reynolds' sehen Zahlen." Unpublished paper — Reference to this work was provided by Dr. C. C. Lin, Brown University.

Atomization and Mixing

113. E. W. Comings, "Atomization and Mixing of Fluid Streams," Paper Presented at Meetings on Fundamentals of Combustion, Applied Physics Laboratory, Johns Hopkins University, March 17-18, 1947. *Unclassified*.
114. A. S. Lennard, "A Theoretical Investigation of Fuel Vaporizers for the Subsonic Ramjet," Jet Propulsion Laboratory, GALCIT, Report No. 3-10, Oct. 18, 1946. *Restricted*.

115. J. P. Longwell, "Fuel Oil Atomization," Thesis, Mass. Inst. Tech. (1943).
116. S. Nukiyama and Y. Tanasawa, Trans. Soc. Mech. Engrs. (Japan) **6**, No. 22, 11-7 (1940); **6** No. 23, 11-8 (1940) and earlier papers.
117. W. C. Roesch and R. F. Rose, "A Survey of the Literature on the Subject of Atomization," Calif. Inst. of Tech., Report PR-1-46 from the Jet Propulsion Laboratory, Feb. 28, 1946. *Unclassified*.
118. F. N. Scheubel, "On Atomization in Carburetors," N.A.C.A. Technical Memorandum, No. 644 (1931).
128. L. B. Edelman, "The Evaluation of the Pulsating Jet Engine and its Future Prospects," Project SQUID, Princeton University. Paper presented at the S.A.E. National Aeronautics Meeting, Los Angeles, October 3-5, 1946.
129. J. K. L. MacDonald, "A Gas Dynamical Formulation for Waves and Combustion in Pulse-Jets," Report No. AMG-NYU No. 151, prepared under Navy Contract NOn(s) 7370. New York University, June, 1946. *Unclassified*.
130. New York University, "Semi-Annual Progress Report, Project SQUID," January 1947 and Quarterly Report, Project SQUID, 1 April, 1947. *Unclassified*.

Mechanics of Non-Uniform Gases

119. M. Brillouin, "Kinetic Theory of a Gas in the Neighborhood of a Wall," Comptes rendus (Paris) **186**, 553-558 (1928).
120. D. Burnett, "The Distribution of Velocities in a Slightly Non-Uniform Gas," Proc. Lond. Math. Soc. **40**, 382 (1935).
121. S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases* (Cambridge University Press, Cambridge, England, 1939), pp. 380-388.
122. J. E. Lennard-Jones, "The Velocity Distribution Function and the Stresses in a Non-Uniform Rarefied Monatomic Gas," Trans. Roy. Soc. London **A223**, 1-33 (1922).
123. New York University, Semi-Annual Progress Report, Project SQUID, 1 January, 1947. *Unclassified*.
124. Richard Schamberg, "Differential Equations and Boundary Conditions for High Speed Slip-Flow," Thesis, California Institute of Technology, 1947.
125. Hsue-Shen Tsien, "Superaerodynamics, Mechanics of Rarefied Gases," J. Aero. Sci., **13**, 653-664 (1946); Hsue-Shen Tsien and Richard Schamberg, "Propagation of Plane Sound Waves in Rarefied Gases," Journal of the Acoustical Society of America, **18**, No. 2, 334-341, October, 1946.
131. Polytechnic Institute of Brooklyn, "Quarterly Report, Project SQUID" 1 April, 1947. *Unclassified*.
132. Lord Rayleigh, Phil. Mag. **3**, 338 (1902) and **10**, 364 (1905).
133. Hsue-Shen Tsien, in *Jet Propulsion*, a text prepared by the Staffs of the Guggenheim Aeronautical Laboratory and Jet Propulsion Laboratory, California Institute of Technology, 1946, Chapter XII. *Restricted*.

Facilities in Design or Under Construction

134. Applied Physics Laboratory, Johns Hopkins University, BUMBLEBEE Report No. 45, pp. 9-10 (October, 1946, Survey). *Confidential*.
135. Bell Aircraft Corporation, Description of Facilities in Interim Progress Report of the METEOR Project, Report No. 8, pp. 58-59, March, 1947. *Confidential*.
136. Massachusetts Institute of Technology, Design study description in Interim Progress Report of the METEOR Project, Report No. 5, pp. 26-27, December, 1946. *Confidential*.
137. "NACA Compressible Flow Unit," Descriptive Brochure published by the National Advisory Committee for Aeronautics, Washington, D. C. (1947).
138. Princeton University, Semi-Annual Progress Report, Project SQUID, 1 January, 1946. *Unclassified*.
139. Allen E. Puckett, "Hypersonic Wind Tunnel" Progress Reports PR-1 to 10, dating from 10 April, 1946, to present. Design studies and progress under Contract W-04-200, Ord-1463. California Institute of Technology. *Restricted*.
140. S. W. Stanwick, "Preliminary Specifications — Proposed 16" x 16" Supersonic Wind Tunnel (Closed return System), North American Aviation, Inc., Report No. CF 549-NA 46-1120, December 21, 1946. *Unclassified*.

Theoretical Studies Relating to the Pulse Jet Cycle

126. N. P. Bailey and H. A. Wilson, "The Intermittent Jet Engine," Report of the Jet-Propelled Missiles Panel of the Office of Field Service, Office of Scientific Research and Development. May, 1945. *Restricted*.
127. Cornell Aeronautical Laboratory, Cornell University. *Semi-Annual Progress Report*, Project SQUID, January 1947, and Quarterly Report, Project SQUID, 1 April, 1947. *Unclassified*.

ABBREVIATIONS OF GOVERNMENT AGENCIES

AAF	Army Air Forces, War Department, Washington 25, D. C.
AAF-AMC	Army Air Forces, Air Materiel Command, Wright Field, Dayton, Ohio
BuAuer	Bureau of Aeronautics, Navy Department, Washington 25, D. C.
BuOrd	Bureau of Ordnance, Navy Department, Washington 25, D. C.
BuShips	Bureau of Ships, Navy Department, Washington 25, D. C.
NACA	National Advisory Committee for Aeronautics, 1724 F Street, N.W., Washington, D. C.
NDRC	National Defense Research Committee, 1424 16th Street, N.W., Washington, D. C.
NBS	National Bureau of Standards, Washington 25, D. C.
NOL	Naval Ordnance Laboratory, White Oak, Maryland
ONR	Office of Naval Research, Navy Department, Washington 25, D. C.
OSRD	Office of Scientific Research and Development, 1424 16th Street, N.W., Washington, D. C.