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THE RADOME PROBLEM IN GUIDED MISSILES [U]

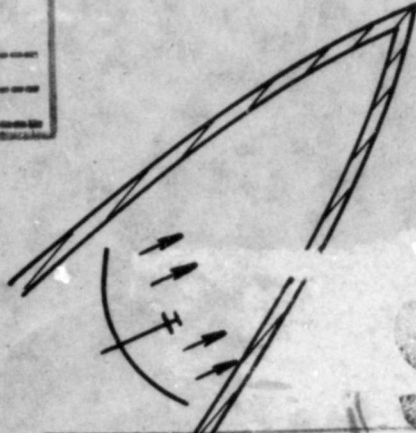
REPORT OF THE PRINCETON UNIVERSITY RADOME STUDY

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THE RADOME PROBLEM IN GUIDED MISSILES [U]
 REPORT OF THE PRINCETON UNIVERSITY RADOME STUDY

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CONTENTS

	<u>Page</u>
<u>Abstract</u>	iv
<u>Summary of Recommendations and Conclusions</u>	v
<u>THE PRINCETON UNIVERSITY RADOME STUDY</u>	1
<u>I Introduction</u>	2
<u>II Main results and conclusions</u>	4
<u>Appendices</u>	
<u>A Radome Error and the Missile Guidance System</u>	A1
I Influence of radome errors on navigation	A2
II The use of radomes with reproducible error, and the system compensation thereof	A10
III Radome error sensing	A15
IV The treatment of target noise	A19
V Relation between aerodynamic configuration and sensitivity to radome error	A21
<u>B An Analysis of Sources of Radome Error</u>	B1
I Introduction	B1
II The projected radome approximation	B2
III Shift of conical scan cross-over by radome error	B6
IV Theoretical analysis of first order effects	B8
V Application of theory of first order effects	B18
VI Analysis and discussion of radome errors of higher order than the first	B36
<u>C Reflection and Transmission of Parallel Sheets and Sandwiches</u>	C1
I Reflections	C1
II Insertion phase differences	C6
<u>D A Strong Material of Low Dielectric Constant for the Construction of Radomes</u>	D1
I Effect of dielectric absorption upon dielectric constant of medium	D3
II Methods of introducing absorption at low frequencies	D5
III Theory of the electric dipole lattice	D7
IV Properties of solenoidal resonators	D23
V Measurements	D28
<u>E Metal-Resonant Iris Construction of Radomes</u>	E1
<u>F Aerodynamics of Radomes</u>	F1
I Drag penalty from nose rounding	F1
II Surface roughness	F3
III Conical windshield	F4
IV Temperatures and pressures	F4
<u>G Materials for Radomes</u>	G1
<u>H Polarization Errors</u>	H1

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	<u>Page</u>
<u>I Test Equipment</u>	II
I Flat panel testing	II
II Fundamental radome tests	I2
III Equipment for systems design work	I4
IV Production test equipment	I6
<u>J Alternative to Dish Type Antennas</u>	J1
I Linear arrays A	J1
II Linear arrays B	J2
III Combination of dish and interferometer antennas	J3
<u>K Bibliography and Who's Who</u>	K1
<u>Distribution List</u>	

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AUTHORSHIP OF APPENDICES

Appendix A	Radome Error and the Missile Guidance System	Hamilton (Tukey, Dicke)*
Appendix B	An Analysis of Sources of Radome Error	Griffith, Hamilton, Dicke Smyth, Reynolds
Appendix C	Reflection and Transmission of Parallel Sheets and Sandwiches	Reynolds, Smyth
Appendix D	A Strong Material of Low Dielectric Constant for the Construction of Radomes	Dicke, (Mozley)
Appendix E	Metal-Resonant Iris Construction of Radomes	Teichmann, Dicke
Appendix F	Aerodynamics of Radomes	Griffith (Bleakney)
Appendix G	Materials for Radomes	Smyth
Appendix H	Polarization Errors	Silver, Mozley
Appendix I	Test Equipment	Mozley (White, Dicke)
Appendix J	Alternates to Dish Type Antennas	Mozley, Dicke

* Kibitzer's names are in parenthesis.

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ABSTRACT

This is the final report of a study of radomes for guided missiles carried out by the Physics Department of Princeton University, and made at the request of the Office of Naval Research. The study had two objects, the investigation of the effect of radome error on missile guidance, and the analysis and correction of causes of radome errors. The first part of the report studies the deleterious effects of radome error and shows how these harmful effects may be reduced by making certain changes in the remainder of the missile. The remainder of the report develops a simple theory of the causes of radome error, considers their relative importance, evaluates the various radome constructions now being used or developed, and suggests several new constructions.

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SUMMARY OF RECOMMENDATIONS AND CONCLUSIONS

It is strongly recommended;

1. That the polarization of the antenna system for the missile's guidance radar be designed, or be redesigned if already designed otherwise, so as to be symmetrical with respect to the missile axis.
2. That a moderately good reproducible radome be developed and used with an error correction scheme in the missile to serve as an interim solution to the radome problem. This solution is probably feasible only if the antenna polarization is symmetrical with respect to the missile axis.
3. That the loaded dielectric of the general type described in Appendix D be actively pushed as a long range solution to the radome problem, and that it be considered as a possible short range solution.
4. That a program of radome phase front measurements be instituted, the purpose of the measurements being to determine the causes and cures of errors in radomes rather than giving a mere figure of merit for the radome.

It is further recommended:

5. That the choice of aerodynamic configuration of the complete missile be made only after carefully considering its effect on the sensing-control system. In particular the advantages, from a radome point of view of movable wing control should be considered.
6. That more information be obtained on the nature of the target. In particular more angle noise and more polarization noise measurements are needed.
7. That sufficient work be done on the perforated metal construction of radomes, as discussed in more detail in Appendix E, to allow a conclusion as to its feasibility.
8. That the crossed linear array antenna system described in Appendix J be studied to determine its advantages and disadvantages.
9. That single layer construction of radome walls be studied, particularly using radome shape as a design criterion.

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Supplementary conclusions:

1. The requirements placed upon the radome and the remainder of the sensing-control means depends intimately upon the properties of the remainder of the missile. Consequently the missile should be designed as a coherent whole rather than allowing it to grow through a process of patchwork.
2. There is a possibility of continually sensing and correcting for the radome error. More angle noise data is necessary before the feasibility of this scheme can be predicted.
3. The most important single cause of radome error is probably phase distortion. There are many other contributing causes however.
4. There is a wide range of radome shapes that are aerodynamically permissible. The variation in shape is a convenient design parameter which should be exploited.
5. A single layer wall construction for a supersonic radome has some advantages not possessed by the various sandwich constructions. In particular it should prove to be a feasible construction for a moderately good reproducible radome to be used in conjunction with an error compensation scheme as an interim solution.
6. A ceramic construction offers some useful properties particularly in the design stage of a program which is investigating the radome shape as a design parameter.
7. It is highly desirable to achieve uniformity in the measuring equipments used by the separate laboratories.

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THE RADOME PROBLEM IN GUIDED MISSILES

REPORT OF THE PRINCETON UNIVERSITY RADOME STUDY

The Princeton Radome Study was initiated at the request of the Office of Naval Research to make a study of the radome as it relates to the operation of guided missiles employing radar type homing systems. The group's attention was directed to the basic questions of the effect of the radome upon the operation of the missile and the causes and cures of radome errors.

Because of the limited amount of time which the members of the study group could make available, no attempt was made to conduct a program in radome research or to formulate a solution to the radome problem. Rather, it was the purpose of this program to make a broad study of the radome problem with the goal of formulating as clearly as possible the nature of the problem and indicating reasonable lines of attack upon the problem.

With the exceptions of a member of Rutgers University, and in the early phases a member of the University of California, the group consisted of members of the Physics Department of Princeton University. In no case had a member of the group had previous experience with radomes. However, in every case the members had had experience in allied fields. This choice of personnel was deliberately made to obtain a completely fresh view of the problem.

The study was started in the last week of August 1951 and is concluding with the publication of this report. Unfortunately because of teaching duties it was only possible to devote half time to the project after the last week of September. The list of members of the group with an indication of the relevant part of their technical background follows;

R. H. Dicke (Project leader), Princeton University, Microwave circuits.
W. Bleakney, Princeton University, Supersonic flow, shockwaves.
W. Griffith, Princeton University, Supersonic flow, shockwaves.
D. R. Hamilton, Princeton University, Klystron vacuum tubes.
R. Mozley, Princeton University, Microwave radar
G. T. Reynolds, Princeton University, Upper atmosphere research.
S. Silver, University of California, Microwave radar antennas
H. T. Smyth, Rutgers University, Materials, Ceramic materials
T. Teichmann, Princeton University, Electromagnetic theory

Consultants

J. Tukey, Princeton University, Noise and systems performance analysis
M. J. White, Princeton University, Microwave radar, pulser design.

Lt. Carl Hartdegen, ONR Washington, served as scientific liason officer with the group. Because of his broad knowledge of the guided missile field, and his ready assistance the group was saved a great deal of time and effort in the early stages of the study.

THE PRINCETON UNIVERSITY RADOME STUDY

The month of September was spent by the group in being briefed on the nature of the radome problem by groups from a number of laboratories which are at present concerned with the radome as it relates to guided missiles. Among those who aided in this essential part of the program were:

I. H. McLaren	Bendix Aircraft
K. H. Fischbeck	Bendix Aircraft
L. F. Millet	Bendix Aircraft
R. Howe	Raytheon Mfg. Co.
T. Phillips	Raytheon Mfg. Co.
H. Schutz	Glenn L. Martin Co.
S. Stark	Glenn L. Martin Co.
E. B. McMillan	McMillan Laboratories
S. Mason	McMillan Laboratories
M. J. Madigan	NADC Johnsville, Pa.
M. H. Paiss	NADC Johnsville, Pa.
E. W. Schlieben	NADC Johnsville, Pa.
O. J. Snow	NADC Johnsville, Pa.
A. M. Winzemer	NADC Johnsville, Pa.
A. S. Locke	N.R.L.
P. Watermann	N.R.L.
J. B. Garrison	Applied Physics Laboratory
A. M. Stone	Applied Physics Laboratory

The Princeton group is indebted to the above people for the excellent orientation they provided.

I. INTRODUCTION. The study made by the Princeton group can be conveniently divided into two main sections. The first section was concerned with the radome as it related to the remainder of the missile. This part of the study had two main objects, to investigate and understand the deleterious effects of the radome error on the performance of the missile, and to investigate the possibility of introducing changes in the remainder of the missile in order that the effects of radome error would be less severe. The results of this part of the study may be found in Appendix A of the present report.

The second part of the study project consisted of a systematic investigation of the radome itself. In this connection a deliberate decision was made to devote little attention to the mechanical properties of the radome. As was brought out in the meeting of the radome section of the Electronics Panel of the R.D.B. at San Francisco, August 1951,

THE RADOME PROBLEM IN GUIDED MISSILES

in recent years the main effort in radome research has gone into the materials development program. It is important to have materials which will stand the temperatures and stresses encountered, but it seemed to the study group to be far better to have a radome design which is electromagnetically suitable before worrying in detail about how it can be made strong.

The various possible causes of radome error are considered in Appendix B of the report. In this section it is shown that although there are a number of different causes contributing to radome error, it is possible to classify them in a way which brings out their relative importance. It is further shown how by making certain simplifying assumptions it is possible to develop an elementary theory of the radome.

Appendix C discusses the relative merits of the various wall constructions now used for the construction of radomes or being developed for this purpose.

Appendix D is concerned with a suggested new wall construction for use in radomes. The dielectric material in this construction is loaded with a lattice array of small resonant structures in such a way as to reduce the dielectric constant of the material to nearly unity. This in one step reduces both the reflection coefficient and the transmission phase shift by very substantial amounts.

Appendix E is concerned with another new construction for radome walls. This construction consists of a thin metal wall perforated with a lattice array of small resonant irises.

Appendix F is concerned with aerodynamic questions. The discussion considers among other things the possible types of radome shapes which may be used without seriously increasing aerodynamic drag.

Appendix G discusses briefly materials available for experimental work on radomes.

Appendix H contains a discussion of some of the effects of polarization on radome errors.

Appendix I is a discussion of the problem of measuring equipment for a systematic research program on radomes. There is a serious need for measuring equipment which helps locate experimentally the source of beam distortion.

Appendix J is concerned with alternate antenna systems which do not have a radome problem. These systems compare favorably with dish type systems and are free of some of the difficulties.

Appendix K contains a bibliography of reports consulted by the study group. It also contains a list of people concerned with radomes for guided missiles. The list is incomplete but contains the names of

THE PRINCETON UNIVERSITY RADOME STUDY

people with whom the group has had direct dealings.

II. MAIN RESULTS AND CONCLUSIONS. This report has been organized in such a way that the details are all in the appendices. However, the chief results and conclusions are collected in this section of the report, which also serves as a summary of each of the appendices.

Before discussing in detail the main results and conclusions of the study program it should be emphasized that the group has no desire to claim originality for all of the ideas and conclusions. It is inevitable that a new group will come up with ideas that had occurred to earlier workers in the field. Furthermore, it is very difficult to acknowledge in an adequate way all the sources of information that have become incorporated into the conclusions of the group.

Appendix A. This appendix, as was pointed out in the introduction, is concerned with the radome as it relates to the performance of the missile. Part I considers the limits placed on the radome by the requirement of a satisfactory over-all performance of the missile. It is found that these requirements differ from one tactical situation to another, that they depend upon aerodynamic properties of the missile, the size of warhead, type of control system, and noise and maneuver characteristics of the target. It is clear from this that, insofar as possible, the missile should be considered as a single over-all device. Generally speaking it should be designed from the ground up rather than patched together.

The second section of this appendix deals with a recommended short range solution to the radome problem. This solution depends upon the fact that the errors produced by a radome can be compensated out in the sensing mechanism if the radome error is a known function of look angle. For an antenna system having a polarization symmetrical with respect to the axis of the radome, the radome in turn being symmetrical about its axis, the error to be compensated is a function only of a single parameter, namely the look angle of the beam. Consequently a simple cam or potentiometer can be used to compensate for the radome error. This seems to be quite easy to do, hence it is recommended that a program be initiated to produce moderately good but reproducible radomes and to compensate their errors in the sensing mechanism. It is felt that the symmetrical polarization should be provided by the antenna system and that changes should be made where necessary to provide it.

The third section of Appendix A deals with a scheme for automatically and continuously sensing the sign and magnitude of the radome error in order to introduce compensation. It was found to be impossible to analyze adequately this scheme in a theoretical way because of a lack of target noise data of the right type. It is recommended that the program of measurements of target characteristics be continued and expanded. Not only is more target noise information needed, particularly at short range, but information on depolarization of the signal by the target is needed.

THE RADOME PROBLEM IN GUIDED MISSILES

Part IV of appendix A considers the signal to noise problem in the performance of a guided missile of the dish-homing type. In particular the effect of the radome error in causing a poor response by the missile to the noise properties of the target is discussed.

Part V considers the advantages to be gained from a movable wing missile. With such a missile the control mechanism may be so arranged that the axis of the missile points more nearly in the direction of its motion than with a fixed wing missile. Lateral accelerations are produced by the wings rather than by the body angle of attack, consequently the look angle of the antenna is slowly varying and radome errors contribute low frequency spurious signals to change only the low frequency response characteristics of the missile. It is strongly recommended that the aerodynamic configuration of a missile be fixed only after considering and weighing its effect and importance on the homing operation of the missile. In particular the step-wise procedure of designing an air frame and then adding a homing system has certain perils of which designers should be aware.

Appendix B. This appendix is concerned with a classification and evaluation of the various causes of radome error. The various sources of error are classified in accordance with the number of reflections which are involved. It is shown in part VI that generally speaking the more reflections the less important is the cause of the error.

Part II shows that to a certain approximation, the central part of the antenna pattern is identical with that produced by replacing the radome by a flat sheet obtained from the radome by a certain projection procedure. The equivalence is very useful as it is much easier to calculate and to visualize in a physical way the beam distortion produced by a non-uniform plane sheet than that produced by the curved radome surface.

Part III considers the effect of beam distortion on the cross over point of a conical scanning antenna. The beam distortion is classified as a change in the height, width, and position of the beam. The contribution of each of these effects to the position of the conically scanning null is computed.

Part IV consists of a simplified theoretical treatment of first order radome effects. A first order effect is the type obtained where the radiated beam makes only a single transit of the radome without reflected waves being included. It is shown that in the approximation considered in Part II that beam deflection is not obtained unless there is phase distortion. Amplitude distortion in the form of reflection loss contributes to beam distortion but not deflection. Methods of computing beam deflection and distortion are developed for the cases of smooth variation over the radome and discontinuous breaks.

Part V considers the application of the theory developed in Part IV to a number of examples. The indication from these examples is that for a typical case beam deflection produced by phase distortion is

THE PRINCETON UNIVERSITY RADOME STUDY

more important than effects resulting from beam distortion.

Part VI considers higher order causes of radome error including the Lloyd's mirror effect and trapped waves. The conclusion is reached that although under extreme conditions one or more of these effects may be important, they are generally negligible in comparison with first order effects.

The chief conclusion to be drawn from Appendix B is that phase distortions are probably the single most important cause of the poor performance of a supersonic radome. As most of the attention has in the past been directed to the reduction of reflection by the radome wall it is now recommended that the emphasis be shifted to a balanced consideration of both phase and amplitude distortion with particular emphasis on designing walls with small phase distortion.

Appendix C considers the radome wall constructions currently being used or developed from the standpoint of the transmission phase shifts and reflection coefficients as a function of the angle of incidence. It is concluded from this study that, as is widely known, high dielectric materials in general have properties more nearly constant with incidence angle than low dielectric materials. The discussion points out the necessity of optimizing performance by selecting the proper design angle of incidence. It is a further conclusion of this study that a single layer construction has advantages not possessed by the sandwiches. In particular this would seem to be a suitable construction for the reproducible radome recommended as a short range solution in Appendix A. In particular a glass radome with a sufficiently close wall tolerance might be a suitable construction.

Appendix D. This appendix considers a new type of wall construction for radomes. The structure consists of a dielectric material loaded with a compact lattice array of small microwave resonators. Experiments have been made upon a material of this type in which the lattice resonators were in the form of small copper solenoids. These measurements showed that the reflection coefficient could be made almost 20 d.b. below that of the unloaded dielectric in the case of a slab 1/8" thick. Although quantitative transmission phase shift measurements were not made, qualitative checks showed a very substantial reduction in phase shift as a result of the coil loading. Both of these conclusions are in agreement with the theoretical expectation.

The effect of the resonators (when tuned to a resonant frequency lying below the operating frequency) is to reduce the dielectric constant of the material. With a proper choice of parameters the effective dielectric constant of the material can be made nearly unity. The appendix contains the theoretical and experimental details and in addition contains a rather specialized mathematical development which is not of general interest but should prove of value to any group engaged in a research program on materials of this type.

THE RADOME PROBLEM IN GUIDED MISSILES

It is the recommendation of the group that an active program of development of materials of this type be organized. It is suggested that a long range research program is necessary for the best exploitation of this idea but that the possibilities of a short range solution should not be overlooked.

Appendix E is concerned with a possible new radome construction in which a conical radome is fabricated from a thin metal sheet. The metal sheet is perforated with a lattice array of resonant irises. A few measurements were made on structures of this kind but insufficient work was done to reach a definite conclusion. It is believed that the structure holds sufficient promise to warrant a further careful examination. This appendix contains a rather large theoretical section which is not of general interest but which is included to make it available to a few specialists to whom it should prove valuable. It is the recommendation of the group that the experimental and theoretical work which was started by this group be continued until a definite answer can be given to the question of the feasibility of a structure of this type.

Appendix F is concerned with aerodynamic questions connected with the radome. One question considered is the variations in the shape of the radome that are compatible with a reasonable aerodynamic drag. As is pointed out in Appendix B, the shape of the radome is a convenient design parameter, consequently it is desirable to know what limitations are placed on the radome by aerodynamic considerations. The chief result is that a large range of shapes are allowable so long as the fineness ratio is sufficiently great.

Surface roughness and its effect is considered with the conclusion that considerable roughness can be tolerated.

A section of this appendix considers the effect of a "stinger" on missile drag and the conclusion is reached that the beneficial results from a stinger probably are not sufficiently great to balance its poor electromagnetic properties.

Appendix G. This appendix considers materials for an experimental program on radomes. It should be emphasized again that this study has largely ignored the mechanical questions which are undoubtedly important in the realization of a practical radome. This is not meant to minimize the importance of these questions but merely to put things in a reasonable perspective. In the past a very large effort has gone into the development of materials for particular types of radome constructions without any real reason for believing that the resulting structure would be electromagnetically suitable. In fact in many cases the structures seem to be completely unsuitable from an electromagnetic point of view.

This appendix considers the suitability of materials for a research program in the single layer construction of radomes. The materials

THE PRINCETON UNIVERSITY RADOME STUDY

are considered not so much for their suitability as a final practical material as their suitability in a design program where it may be desirable to change the shape of the radome readily.

Appendix H. This appendix considers some of the effects associated with antenna polarization. The conclusion is reached that in all cases it is highly desirable to have an antenna the polarization of which is symmetrical with respect to the missile axis. This polarization may be plane or circular. Furthermore there appear to be some advantages associated with circular polarization in the case of a semi-active system. It is recommended that antennas be designed with a polarization symmetric with respect to the axis of the missile. Furthermore missiles having unsymmetrical polarization should be modified to make the polarization symmetric.

Appendix I. This appendix considers test equipment for systematic research and development. In the past the measurements have been largely directed toward determining how well the antenna-radome system has been performing. These measurements have been successful in giving a measure of the performance of the radome but unfortunately they gave little or no indication as to why the radome was not working. This study group wishes to recommend strongly that these measurements be supplemented by measurements made at the radome and having the purpose of exposing the source of difficulty in the radome. The radome antenna system is complex in its behavior and there is a real need for a measurements program which will contribute to the understanding of the source of difficulties in the radome in addition to giving a measure of the perfection of the radome.

This appendix illustrates with examples the various types of test equipment which are believed essential to a well rounded program of research on radomes for guided missiles.

Appendix J. This appendix considers antenna systems alternate to the dish type system. The antennas considered are ones that have essentially no radome problem even if they require radomes. In this connection the question is sometimes asked whether the interferometer system is not a more satisfactory device than the dish type because of the absence of radome difficulties. The interferometer has some advantages over the dish type antenna system, but in the view of this group it has serious deficiencies. It is probably ineffective against planes flown in properly coordinated pairs. The flying of aircraft in coordinated pairs is a counter measure which an intelligent enemy would probably quickly learn.

Two of the antenna systems described involve the use of a crossed system of linear arrays. A sharp single beam is defined by using one set of arrays in transmission and the other set in reception. The beam is sharper

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THE RADOME PROBLEM IN GUIDED MISSILES

than that of the dish type antenna. The system is mechanically fairly simple, electrically simpler than the interferometer system, and the antenna gain is nearly as great as that of the paraboloid. Because of the advantages associated with the sharp beam (multiple target resolution) and the absence of radome error it is recommended that systems of this general type receive attention.

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APPENDIX A

RADOME ERROR AND THE MISSILE GUIDANCE SYSTEM

While this study has been concerned primarily with radomes as such, it has seemed necessary to look into, and to report here upon, the way in which radome errors influence missile navigation. There were several reasons for this: the natural desire of the component man to check up on the specifications laid down by the systems man; the necessity of understanding the component specifications in order to judge better any improvements suggested for the radome; and the hope that some new methods of alleviating the radome problem by systems changes might become apparent.

In what follows, section I is concerned with summarizing the way in which radome error affects navigation; this section represents a condensation of the present general state of knowledge on the subject. The ways in which radome error affects attitude stability, missile control time constant, navigation constant, flight path instability, and response to target noise are discussed and summarized. It is seen that the acceptable limits of radome error depend upon the details of the control system used and that the limits placed on the radome by different systems may differ with good reason. A good representative value of limiting acceptable radome error for current missile performance is seen to be a range of ± 0.05 degrees per degree not necessarily centered about zero; this is set primarily by acceptable range of fluctuation in navigation constant.

Subsequent sections are concerned with more specific suggestions made concerning systems with an eye to ameliorating the radome problem.

Section II discusses the question of how one would go about living with bad radome error if radomes could be made reproducible in their degree of error. From this it appears that the method of compensating in the system for reproducible error in the radome is straightforward. It is felt that the quickest way to get reproducible radomes is to use the monogeneous single sheet construction which is amenable to mass production; and by using the later results of Appendix B on the causes of radome error, it is concluded that quite feasible manufacturing tolerances would be required to make the radome reproducible within the system tolerances. This method of approach is therefore strongly recommended as a short range temporary solution to the radome problem.

The remaining sections are concerned with somewhat more controversial points and therefore lead primarily to suggestions for directions for further work. Thus Section III presents a method whereby the radome error which enters into system performance could be sensed continuously during flight, and hence continuously neutralized. Such an arrangement, once operative, would make radome error irrelevant. However, it is found that a definitive answer concerning the feasibility of this method cannot be given until there becomes available further information about target noise, in the frequency range of several tens of radians per second.

THE PRINCETON UNIVERSITY RADOME STUDY

The noise question also enters, more directly, into section IV. Here it is observed that noise is very relevant to radome questions in that radome error may make worse the system time lag which is introduced in the first place on account of target noise. It is pointed out that the sophisticated methods of dealing with noise which have been developed for anti-aircraft fire control have been applied hardly at all to missile "fire-control" (i.e. navigation); and it is recommended strongly that more data on target noise be obtained and that more sophistication be applied to the counteracting of target noise with minimum loss of sensitivity to target maneuver.

Finally, section V concerns the observation, already made by the British, the degree of difficulty with radome error is strongly influenced by the aerodynamic configuration of the missile; to be more specific, that a movable wing missile, in which less attitude change is required for a given maneuver than is the case for a fixed wing missile, may be much less sensitive to radome error. This conclusion is amplified and strengthened by a simple analysis of a movable wing system in which both body and wing lift contribute to lateral acceleration. Since many factors enter into the choice of aerodynamic configuration, the observations of this section are not used as the basis for any recommendation that all missiles have movable wings; but we feel strongly that the existence of this relation between radomes and aerodynamics should be recognized by the aerodynamicist.

It will be seen that several of these sections range into questions rather different from radomes as such. This serves simply to point up a feeling which has been strong in this project, namely that one should not expect to cure the radome problem by steps taken with reference only to radomes alone. Perhaps our strongest recommendation is that radomes be considered as an integral part of the system, not as something to be patched up after everything else is all designed and frozen.

I. THE INFLUENCE OF RADOME ERRORS ON NAVIGATION. It should be mentioned immediately that an excellent Hughes report on this subject¹ appeared after the Princeton Radome Study was practically completed. While the report to which reference is made is concerned with one particular missile, the Falcon, most of the material is rather generally applicable. The reader should refer to this Hughes report for many details which will not be discussed in the present report.

Notation. For purposes of our discussion we shall use the following notation which is a composite of those in general use.

Angles measured with respect to fixed space reference and specifying the indicated vectors:

¹Technical Memorandum No.258, "The Effect of Radome Error on Falcon Navigation," Hughes Aircraft Company.

γ	missile velocity
θ	missile axis
ψ	direction of antenna look (antenna scan axis for conical scan)
σ	true direction of target (true "line of sight")
σ_r	apparent direction of target, erroneous because of radome error (apparent line of sight)

Difference angles:

$$\alpha \equiv \theta - \gamma \quad \text{angle of attack}$$

$$\epsilon \equiv \sigma_r - \gamma \quad \text{error angle sensed by radar.}$$

Other quantities:

$$\lambda, \lambda_0 \quad \text{navigation constant in presence and absence, respectively, of radome error}$$

$$\tau \quad \text{time constants as specified}$$

$$K = \left(\frac{\partial \sigma_r}{\partial \theta} \right)_{\sigma} \quad \text{radome error slope; see below for sign convention.}$$

These angles are indicated graphically in Fig. A1. In Fig. A2 is indicated a ray diagram specifying the sign convention on K the radome error slope. This sign convention is the same as the Hughes convention, apposite to the Raytheon convention. According to this convention,

$$\sigma_r = \sigma - K(\sigma - \theta) \quad (\text{A1})$$

In this equation it is assumed that the radome error slope is constant over the range of θ which is of interest for the applications; and an irrelevant additive constant in the equation has been dropped.

Proportional Navigation. In most homing navigation systems at present under development, the basic piece of information about the target is $d\sigma_r/dt$, the rate at which the line of sight to the target appears to be moving with respect to a fixed direction in space. This is because these navigation systems aim to bring the missile onto a collision course with the target; for a given target position and velocity, the instantaneous "collision course" for the missile is defined

THE PRINCETON UNIVERSITY RADOME STUDY

as that direction of motion which would have the straight line constant velocity projections of the missile and target paths intersecting at a later time. Such a collision course is characterized by the fact the line of sight from missile to target has a constant direction in space, hence $d\sigma/dt$ measures deviation from the collision course.

The basic response of such a navigation system, therefore, is to turn the missile in such a direction as to reduce $d\sigma/dt$; or, more accurately in the presence of radome error, $d\sigma_r/dt$; this corresponds to producing a $d\delta/dt$ of the same sign as $d\sigma/dt$; and the simplest thing to do is to make the response proportional to the input.

$$\frac{d\delta}{dt} = \lambda \frac{d\sigma}{dt}$$

Here λ is called the "navigation constant" of the system. A more detailed analysis shows that λ must be greater than, say, 3, in order that the missile be able to catch the target within the limit of present speed differentials and with various tactics. But λ cannot be too large or the missile will respond to target maneuver (and to target noise, about which more later) with too large an acceleration, too much expenditure of servo power, and too much overshooting at a later stage of the flight. All told, a variation by a factor two in the navigation constant from missile to missile or within a given flight.

Other refinements of the navigation process, as for example by the inclusion of range data and closing-velocity data, are possible and if not too expensive, they are desirable, since they allow a helpful adjustment of the navigation characteristics to the characteristics of the pursuit at a given moment. Such refinements are relevant to the radome problem in that they ease up on the requirements of the basic proportional navigation system and hence on the radome. But they do not involve the radome as directly as does the direct measurement of angle data.

Further modifications of the navigation process in which knowledge of the absolute value of σ is used on a par with $d\sigma/dt$, are under development at Bendix (for Sparrow II) but the characteristics of this system have not yet been investigated in the same detail as the conventional proportional navigation system; and such systems will not be considered further here.

Radome Error in Proportional Navigation. According to Eq. (A1) a radome error falsifies the basic input data for a proportional navigation system. It should be noted, however, that for such a system a constant radome error (i.e. a constant value of $\sigma_r - \sigma$) is of no consequence, and has no effect on miss distance; hence to a first approximation one may represent radome presence by constant radome error slope K .

The most basic fact about the influence of radome error is that the dimensionless parameter which is important is not K , but the product $K\lambda$; and since λ is considerably larger than unity, this means that very small values of K can be important. The

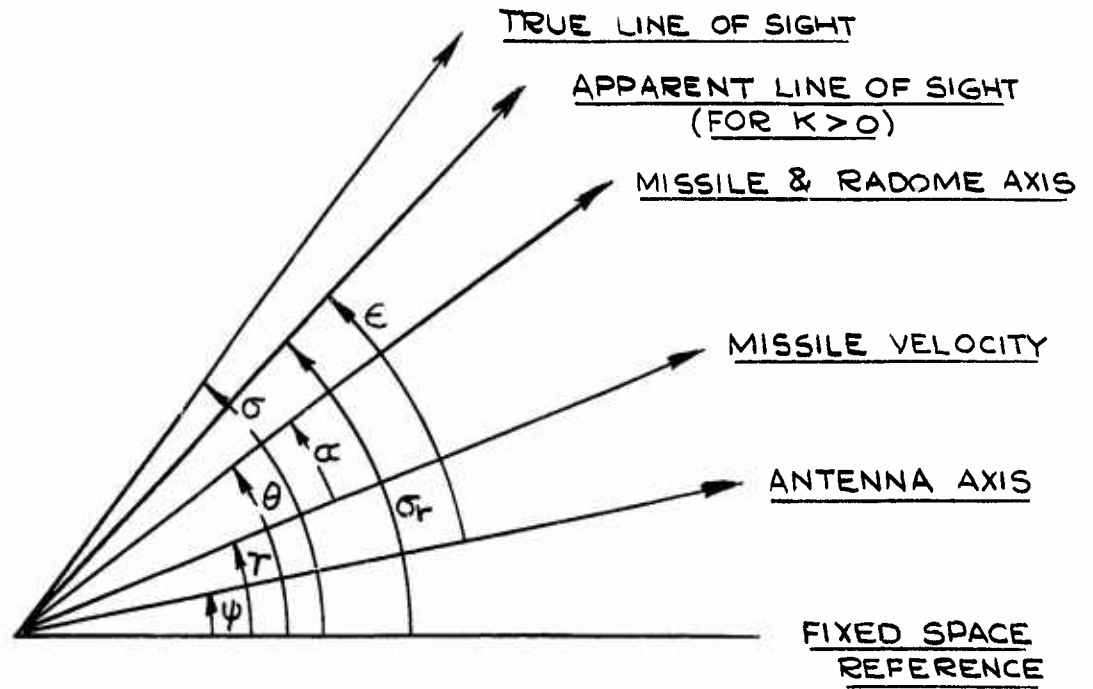


FIG. A-1
ANGLE NOTATION FOR NAVIGATION CONSIDERATIONS

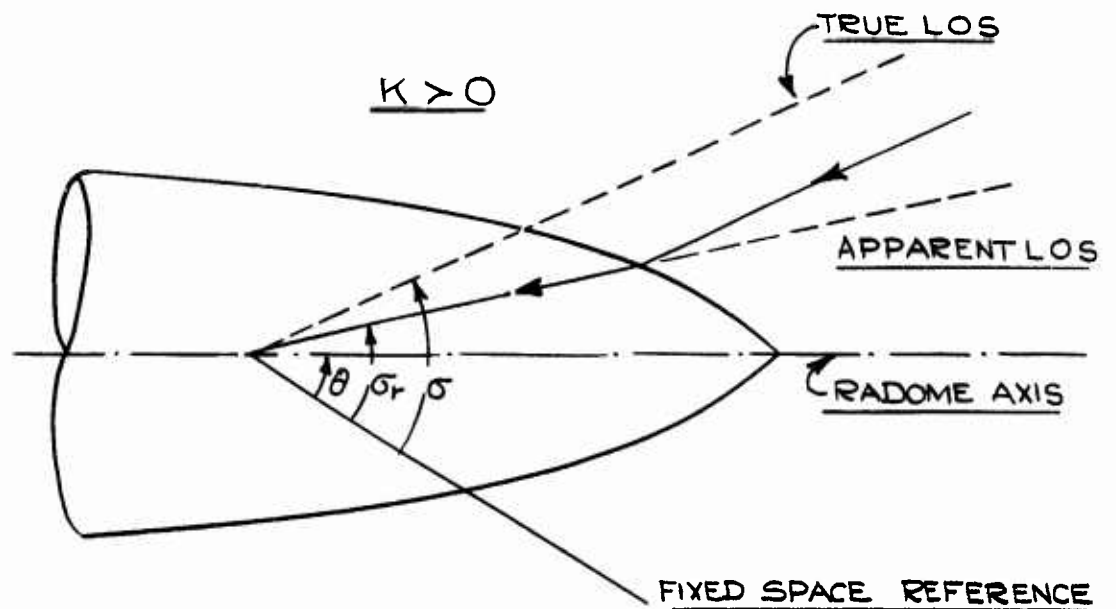


FIG. A-2

RAY DIAGRAM INDICATING SIGN CONVENTION
ON RADOME ERROR SLOPE K

easiest way to see why this is so is to consider the case when

$$\lambda = 5, \quad K = +0.2, \quad K\lambda = 1.$$

The value

$$K = +0.2$$

means that if the missile axis turns through 1° , there is an apparent target motion of 0.2° in the same direction, if in actual fact the target is fixed (i.e. true line of sight fixed). In consequence of this apparent radome-induced target motion, if one gives the missile axis a transient angular velocity the target will have an apparent angular velocity one-fifth as great, and because the navigation constant is five this will call forth a missile response which maintains in a steady state the original transient. We thus have a complete instability at

$$K\lambda = +1.$$

One will expect bad effects even at smaller values of $K\lambda$. To analyze this point in more detail one falls back on the methods of feedback analysis. As indicated in Fig. A3 the effect of the radome may be broken down formally into two parts: the radome changes the appearance of the target motion, and it creates a feedback loop whereby a change in the missile attitude θ causes a change in the target information which determines θ . The details of these effects will depend on the frequency dependence of the transfer functions of the three boxes in Fig. A3, none of which we shall give explicitly beyond noting the fact that the functions $f(p)$ and $g(p)$ (p being the operator d/dt of Laplace transform theory) vanish at low frequencies, i.e. as $p \rightarrow 0$.

Effect of Radome Error on Navigation Constant. Consider first the effect of radome error on the steady state, i.e. as $p \rightarrow 0$. For no radome error,

$$p\delta = \lambda_0 p\sigma \quad (\text{cf. Eq. (1)})$$

But for

$$\begin{aligned} K \neq 0, \quad p\delta &= \lambda_0 [(1-K)p\sigma + K_p\delta] \\ &= p\sigma \lambda_0 (1-K) / (1-K\lambda) \end{aligned}$$

that is, the effective steady state navigation constant on the presence of radome error, λ , is given by

$$\lambda = \lambda_0 (1-K) / (1-K\lambda) \quad (\text{A2})$$

THE PRINCETON UNIVERSITY RADOME STUDY

Thus in the example $K\lambda = +1$ which was discussed above, the non-zero steady state angular velocity set off by an initial transient with no target motion corresponded to $\lambda = \infty$. The increase of λ over λ_0 when $0 < K\lambda < 1$ may be understood on the same basis as the $K\lambda = +1$ instability—that is, the response to true target motion induces an apparent target motion in the same direction as the true motion, and hence an over-response to the true target motion. Similarly, in reverse sense, for $K < 0$.

One consequence of Eq. A2 is that a variation of K from point to point in the radome causes a variation of the effective navigation constant depending on what part of the radome the radar is looking through; for $\lambda_0 = 5$, $K = \pm .08$ gives λ the values 3.9 and 7.7 which leave no room for other variations in λ before the limiting variation of a factor 2 in λ is reached. On the other hand, if $\lambda_0 = 7.7$ then $K = 0$ and -0.18 give $\lambda = 7.7$ and 3.8, of a slightly larger range of K for the same range of λ .

Radome Error and Attitude Stability for $K < 0$. Another question which arises concerns the stability of the feedback loop in Fig. A3 which non-zero K brings into existence. Instability here would correspond to an attitude oscillation of the missile even if the larger loop were not closed—i.e. even if σ remain fixed.

The possibility of exciting such oscillation is readily apparent when $K < 0$. Here a motion of Θ (missile axis) in one direction causes an apparent motion of the target (σ_r) in the opposite direction, which calls forth a command to Θ to reverse its motion. Primarily because of the missile inertia, this can give rise to an oscillation situation at a relatively high frequency, comparable to the natural aerodynamic frequency of oscillation of the missile. Such an oscillation would cause excessive drag and consumption of servo power even if it did not cause large lateral displacements.

By building into the missile controls another feedback loop whereby the command to the controls is influenced by rate of change of attitude one can add damping to the natural aerodynamic oscillation and hence make it more difficult to excite this oscillation by means of negative radome error slope. In one form or another this is done in all missiles.

$d\sigma/dt$

All missile control systems now being developed measure/by measuring the lag with which an intentionally sluggish radar follows the target; this is simpler than making the radar follow the target tightly and then picking off the rate of turn of the radar head. But if the command to the radar head operates to move it with respect to the missile axis rather than with respect to an axis fixed in space, then a sudden change in missile attitude will cause a transient change in antenna axis in the same direction, a transient radar error signal such as would be produced by target motion in the opposite direction, and a transient

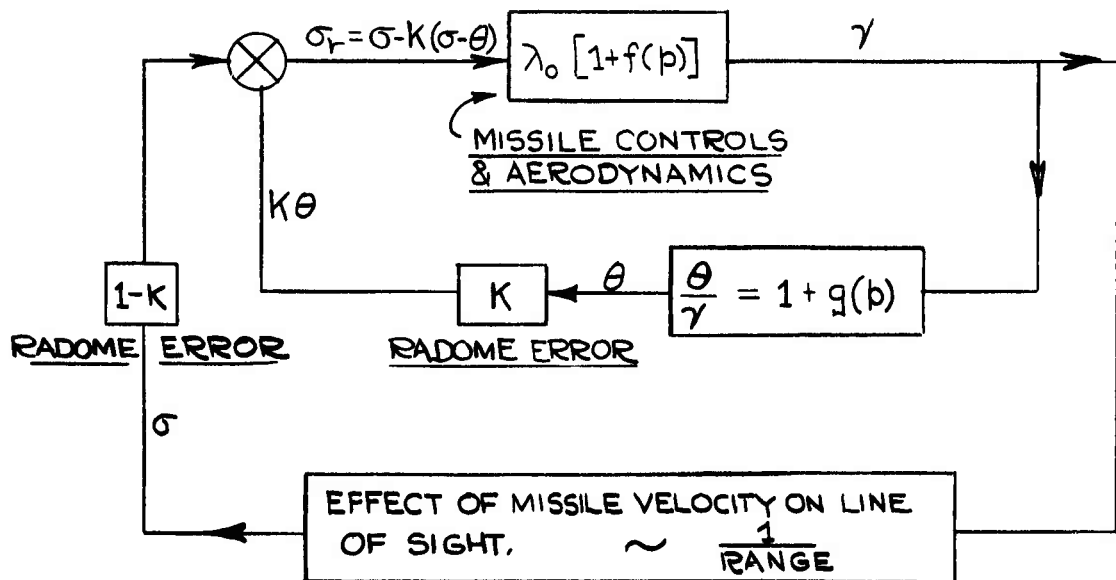


FIG. A 3
SCHEMATIC OF MISSILE FEEDBACK LOOPS
INVOLVING RADOME ERROR

SYSTEMS

command to the missile in opposition to the original attitude change; this situation can give rise to a high frequency attitude oscillation completely independent of the radome. This occurs in the Raytheon Sparrow III, and the cure is artificial damping at the natural aerodynamic or weathercock oscillation frequency, just as in the radome-induced case. In such a system any radome errors modify the oscillation conditions somewhat and hence the necessary damping.

The net result in all cases is that the $K < 0$ high frequency attitude oscillation effect can be damped out to where it is no longer an important factor in the requirements on the radome.

Radome Error and Attitude Instability for $K > 0$. The detailed analysis of the stability of the radome-error feedback loop in Fig. A3 also shows that attitude oscillation at a low frequency can occur for $K > 0$. An extreme example of this is provided by the "zero-frequency oscillation" at $K\lambda = +1$ as already discussed. When $K\lambda < 1$ oscillation at low but non-zero frequency can occur, particularly when the response of missile velocity to angle of attack is slow, as at high altitudes. More particularly oscillation can occur when $K\lambda > T/T_s$ where T is the control time constant of the missile and T_s is the time constant of response of missile velocity to angle of attack-- i.e. where $\rho\gamma = \alpha/T_s$. As an example of what this can mean we may note that for the Falcon missile of 1950, at very high altitudes and low velocities T/T_s went as low as 0.1.

The frequency of this oscillation is however so low as to have a period comparable with the time of flight of the missile, which means that it is not realistic to talk about the stability of the small loop in Fig. A3 by itself. In fact the question here is not one of stability but rather one of the overall missile response in the closing stages of flight when the closure of the larger loop in Fig. A3 becomes increasingly important.

Crosstalk Radome Error. By crosstalk is meant an apparent target motion in one plane caused by a missile attitude change in a perpendicular plane. In general, for any given orientation of missile and line of sight there will be four coefficients relating the two directions in which the nose of the radome can move and the two directions in which the target may appear to move because of radome error. Two of these coefficients represent "cross-talk" radome errors; and in general their geometric mean has a deleterious effect on stability in any given plane of motion comparable to the direct radome error slope in that plane; thus the presence of cross-talk will act to tighten radome error limits set by stability considerations. We are not aware of the carrying out of any application of cross-talk considerations to other than attitude stability questions, and as we have just seen, the attitude stability question has been solved for $K < 0$ and is not the right question to ask for $K > 0$.

THE PRINCETON UNIVERSITY RADOME STUDY

Effect of Radome Error on Missile Time Constant and Flight Path Instability. It is apparent, by considering the response of a missile to target maneuver when $K > 0$, that the time constant of this response is increased because $K > 0$. This is simply because, as already noted, the initial response to true target motion calls forth a further spurious target motion subsequent in time to the true motion, this spurious motion calling forth further delayed response, the process converging if $0 < K\lambda < 1$.

This increase in missile time constant becomes particularly important at the end of flight. At any time in flight, a given angular deviation from the collision course on the part of the missile produces a resulting $d\sigma/dt$ which is inversely proportional to the target range, as indicated in Fig. A3. Therefore at short range a deviation from collision course produces a large $d\sigma/dt$, a large response on the part of the controls, and a quick reversal of $d\sigma/dt$; the speed of the missile's response to this reversal is limited by the missile control time constant, during which time $d\sigma/dt$ is getting bigger; and a dangerous hunting and overshooting results. A more exact analysis shows that this inherent flight path instability or overshooting sets in when time away from target equals missile control time constant; hence any increase in time constant, as by positive radome error, makes the instability set in earlier and increases miss distance. This effect, since it involves the large feedback loop in Fig. A3 the feedback gain of which is varying with time, is not simply amenable to constant coefficient feedback analysis and is best handled with simulator studies.

It should be noted that anything which decreases the effect of this flight-path instability will make the radome's job easier.

Radar Target Noise. The existence of radar target noise is indirectly responsible in several ways for radome specifications.

The basic reason for building in a missile control time lag is the need for filtering out the target's amplitude and angle noise which would otherwise overload the controls and cause undue drag and acceleration. Without this noise and time lag the end-of-flight instability and its earlier onset from positive radome error would be unimportant.

Aside from the instability resulting from the time lag, the apparent random motion of the target due to target noise will in itself cause misses. The necessity of holding down the missile response to the low frequency components of this noise contributes to the upper bound on navigation constant and in turn, via Eq. A2 sets bounds on radome error. This is, however, somewhat of an oversimplification; simulator studies reported by Hughes indicate that in the presence of even a small amount of positive radome error the effects of scintillation noise on miss distance are markedly increased. This would indicate that the best radome error limits might well be asymmetrical because of the noise question.

Summary of Sources of Limitations on Radome Error. Before summarizing the limits which must be placed on radome error, we note that these limits may be expected to vary from system to system due to a number of factors, such as:

- 1) Lethal miss distance for a given missile, as dependent on war-head characteristics and target characteristics.
- 2) Degree of refinement in the control system, such as the inclusion of range and closing velocity data.
- 3) Degree of sophistication used in separating target information from target and other noise.
- 4) Degree to which large attitude changes are aerodynamically necessary in order to produce given maneuver; for the larger the attitude change for a given maneuver, the larger the apparent target motion resulting from radome error.
- 5) Altitudes, speeds and tactical situations for which the missile is intended.

Some comments will be made shortly on points 2, 3 and 4 in this list. Meanwhile certain generalizations may be made:

- a) Attitude stability considerations are not primary; the high frequency attitude oscillations with which one is concerned when $K < 0$ can be damped out by adding attitude rate feedback as has been done in all missiles as far as we are aware, and the "low frequency attitude instability" for $K > 0$ is better spoken of as an effect on missile time constant and navigation constant.
- b) Assuming that a variation of a factor 2 in effective navigation constant represents the leeway between ability to maneuver to catch the target and sufficient insensitivity to noise, then for $\lambda_0 = 5$, the limits $K = \pm 0.08$ give just this variation of a factor 2 (3.9 to 7.7 on λ); if one were to allow a 10 percent residual fluctuation in λ_0 in the system, as for example from component variations, then the above limits on K reduce to ± 0.05 .
- c) Simulator studies on the effect of radome error on miss due to target noise suggest that the range be shifted to lie mostly in negative K . This also allows a slightly larger range of K for given range of λ ; e.g. $K = 0$ to -0.18 and $\lambda_0 = 7.7$ gives $\lambda = 7.7$ to 3.8 , the same as $K = \pm 0.08$ at $\lambda_0 = 5$.

The net effect of these factors leads us to take the limits $K = +.03$ to $K = -.07$ as a good average range for the radome art to shoot at for present missiles. The quoted Hughes Falcon limits are slightly tighter ($+.02$, $-.05$); the limits for the Martin Oriole ($+.10$, $-.15$) and Raytheon Sparrow III (± 0.10) are somewhat looser. Factors 1, 2 and 4

THE PRINCETON UNIVERSITY RADOME STUDY

from the list at the beginning of this summary section may contribute appreciably to this difference, which we are inclined to think is to some degree a real and objective one.

II. THE USE OF RADOMES WITH REPRODUCIBLE ERROR, AND THE SYSTEM COMPENSATION THEREOF. It has been general experience to date that not only do radomes introduce radar errors, but also the errors fluctuate between various radomes of the same type. It has also been true in these cases that because these radomes represented a response to the great pressure to remove radome errors, these radomes were complex experimental structures from which it was hoped that the radome errors would disappear completely; the attempt to get rid of the errors has involved intentional inhomogeneities, e.g. sandwich constructions and, at least in the experimental stage, great skill in individual fabrication.

This line of attack, that of obtaining by some means a zero-error radome, must certainly be followed as a long-range solution. But it would seem to us that insufficient emphasis has been placed upon the short range solution of using a radome with errors which in themselves are intolerable but which are accurately reproducible. As will be discussed shortly, reproducible errors may be compensated in the system by straightforward means; therefore radome error is not nearly as bad as variation in radome error.

If reproducible radomes are to be sought as a short range solution, it would seem that the best structure is one for which the fabrication involves the fewest and the most commercially standardized processes, and hence the fewest chances for random fluctuation. This suggests a homogeneous radome of a single material that has already received extensive use in manufacturing processes.

An example of a promising possibility of this type, which was called to our attention at NADC Johnsville, was a glass radome fabricated by Corning Glass on the machinery used for television cathode ray tube manufacture. The manufacturing process used in this sample involved centrifugal casting and has already been worked on intensively with the aim of eliminating the bubbles, "seeds" and inhomogeneities which cause weakness in the end product. The changes involved in conversion of the manufacturing facilities from television tubes to radomes are minor; and capacity is in terms of thousands of pieces per day.

It should also be noted that glass is a satisfactory substance for a radome in regard to loss and temperature characteristics and strength. The same should be true of many ceramics. Furthermore the Corning Glass Company has already been experimenting with the type of precision grinding which would be used on a radome if modification after casting were required.

It is not our intention to recommend the glass radome as such; for example, ceramics such as those developed by Stupakoff should also

SYSTEMS

be considered; but we believe glass to be the most promising example that we have seen of a radome from which one might hope to get the best reproducibility the soonest. For some additional relevant comments on the materials question see Appendix G.

Some detailed discussion is necessary on two points which have been lightly passed over above: Given reproducible errors, how does one compensate for them in the system? To what tolerances must a homogeneous single-material radome be held to give sufficient reproducibility? These will now be considered.

System Compensation for "Standard" Radome Error. When the antenna polarization is symmetrical with respect to the plane formed by the missile axis and beam direction, the radome error is a function only of the look-angle of the beam. Then, given radomes all of which have some known standard look-angle dependence in their error, the simplest way to remove this error from the navigation process is to feed into the controls not only the radar "error signal" which says how far away the target appears to be from the antenna axis, but to feed in also a "compensating" error signal which neutralizes the radome error's unwanted contribution to the radar error signal. (Note the use of "error" in two different senses!)

The compensation will not be perfect for two reasons: the radomes won't be perfectly standard in their errors, and the compensating mechanism won't be perfect because of discontinuities in elements and variation of circuit constants from missile to missile.

Consider first the means of generating the compensating signal. This means will not have the ideally specified dependence on look angle which is required to match the radome characteristics. For example, this means might be a potentiometer with winding tapered to fit the radome characteristics, the motion of the movable contact being governed by the angle between antenna scan axis and missile axis. The lack of infinite smoothness in the potentiometer appears as an undesired variation on the compensating signal; the finer the winding the smaller the amplitude and period of these unwanted variations.

Entirely aside from imperfections in mechanism, the compensating "radome errors" include some contributions (e.g. from reflections of waves which leak around behind the dish) which are present even in the absence of a radome; these errors may be compensated for along with the rest, but variations will give rise to imperfections in the compensation.

Finally, in each actual radome there will be variations from the "standard" radome for which the compensation is designed.

THE PRINCETON UNIVERSITY RADOME STUDY

Tolerances in Radome Standardization and System Compensation.

All the effects just discussed combine to give some residual radome error after a "standard" radome has been decided upon and a means of compensating for the errors of the standard radome has been devised. How much residual radome error can we tolerate, how much of this tolerance can deviations from the "standard" radome be allowed to soak up, and to what sort of geometrical tolerances in the manufacture of the "standard" radome does this lead?

In Section I of this appendix it has been noted that for current missiles a fluctuation $\pm .05$ in the radome error slope K would be in most cases acceptable. One would certainly be doing remarkably well, and leaving plenty of room for the above discussed variations in the compensating mechanism, if one demanded of manufactured radomes that the radome error slope K deviate from that of the standard radome by not more than $\pm .01$.

This is however not a complete specification since one might tolerate a deviation rather larger than this, over a small enough range of look angle. This raises the whole question of what happens with K variable rather than constant. Nearly all the system analysis of radome error has been based on the assumption of constant K ; but a few qualitative comments may be made.

A few degrees of attitude variation, at a rapid rate related to the weathercock oscillation frequency, will occur during normal flight due to aerodynamic buffeting; except for the effect of radome error on attitude oscillations at frequencies of the order of the weathercock frequency (and these oscillations are intentionally damped out in most missiles anyway), the effective radome error will be the value averaged over a few degrees.

As discussed in Section I of this appendix, the most deleterious effect of radome error is the low frequency effect, for $K > 0$, of increase in navigation constant, with its two-fold consequence of increased response to noise and earlier occurrence of flight path instability at the end of flight. Simulator data on this point, i.e. on the influence of radome error on miss distance in the presence of noise, is presented in a recent Hughes report¹. Here there was assumed a radome error slope K with value greater than zero over a range of angle A centered on the collision path, zero elsewhere. The non-zero value of $(+.05)$ was half the value which would give low-frequency attitude instability, and the non-zero range A was varied. The results may be summarized thus: that for A equal to 20 times the r.m.s. apparent target motion (noise) due to amplitude scintillation, the miss was 1.3 times that due to the noise alone; for A infinite (that is, uniform radome error) the miss would have been 2.3 times that due to noise. In this instance, then, at least, the radome error became relevant only when the range of angle over which it applied was several tens larger than the r.m.s. target noise. (This refers to target motion due to amplitude noise and is hence a more stringent condition on the range of angle than would be obtained if one referred to the angle noise, which

¹Hughes Technical Memorandum No. 258, October 15, 1951, page 24.

has the larger r.m.s. value in close.) Taking for the order of magnitude of the r.m.s. noise 0.15° (2 cps bandwidth and using amplitude scintillation data from Hughes¹). This would say that in this test $K = .05$ was irrelevant if it extended only over a range of 3° . This data corresponds to conditions (40,000 ft, Mach 1.8) which place very extreme conditions upon radome error.

Similar simulator data (unpublished) at NADC also indicates that a radome error extending over several degrees has a minor effect compared to that occurring for a much wider extent.

These bits of evidence and argument all sound consistent and very reasonable, and lead one to the conclusion that it is a rather conservative statement to say that the radome error that really matters is the average value of the radome error slope, averaged at least over two or three degrees; if this average can be held to deviations $\Delta K = \pm .01$ from the standard radome, one will be doing very well.

Tolerances in Radome Geometry. Variations from the standard radome may arise either from variable radome geometry or from unintentional inhomogeneity of the material. A variation of dielectric constant or loss factor in the material of the radome may be expressed for many purposes as an equivalent change of wall thickness; and if one of the surfaces of the radome has a fixed shape (as, for example, the outer surface of a centrifugally cast glass radome) then thickness variation represents the only variation of geometry. Hence our tolerances for reproducibility of a single-material radome may be specified in terms of an equivalent single tolerance on wall thickness.

Let us then represent the deviation of any one radome from the "standard radome" by the addition to the standard radome of a thin sheet of material of variable thickness. This sheet has "negative" thickness if the radome is thinner than the standard radome, and so on. The total radome error of this thin sheet then represents the variation in total radome error from "standard radome" error.

The radome error resulting from this thin sheet is easily deduced from the discussion in Appendix B, of the radome error in a homogeneous sheet with smoothly varying taper and thickness; one will expect the deviation of an actual radome from the standard radome to be a smoothly varying function, even when the shape of the standard radome itself is doing unsmooth things. The deviations of radome geometry from standard are averaged over the beam width in determining the radome error; and in any case, it has just been shown that the relevant deviations are deviations in radome error slope which are averaged over several degrees of antenna look angle, a process which tends further to smooth things out.

¹Hughes Progress Report 40-22-P, p. 70, 15 July 1951.

THE PRINCETON UNIVERSITY RADOME STUDY

All told, therefore, the parameters α and D which, in the notation of the relevant section of Appendix B, specify the taper and thickness of the deviation from the standard radome, are to be averaged over somewhat more than the beam width in specifying radome tolerances.

Both α and D contribute to $\partial \delta / \partial \theta$, and hence to K hence tolerances for both α and D should be specified to hold $\partial \delta / \partial \theta$ (and hence approximately K) below $\pm .01$ from standard. Since conditions are worst at glancing incidence, we note the final result of section Vd Appendix B; putting in numbers, we find that for glass ($n = 1.5$) and for $\theta_i = 70^\circ$, $\partial \delta / \partial \theta = 12\alpha + 7D/R$. To hold the contributions of each of these terms below .01 we require approximately

$$|\alpha| < 10^{-3} \quad (\theta_i = 70^\circ)$$

$$D < 10^{-3} R$$

A beam 4 inches wide at angle of incidence $\theta_i = 70^\circ$ uses 12 inches of radome; this tolerance on α means for this beam that, averaged over 12 inches of radome one way and 4 inches the other, the average increase in deviation of radome thickness from standard shall not exceed one mil per inch.

As to the tolerance in D : we note that the tolerance $D < 10^{-3} R$ refers to the plane in which $\theta_i = 70^\circ$ and in which R is large, of the order of 10 inches; for this circumstance the tolerance in D is less than 10 mils. But our surface is not a simple surface with a radius of curvature in only one dimension. The radius of curvature just mentioned refers to the section of the radome cut by the plane which is determined by the antenna axis and the axis of the radome; for the section of the radome cut by a plane perpendicular to the radome axis the radius of curvature will be much smaller, of the order of two inches say. But because of the symmetry of the radome the curvature in this plane will not give rise to any beam deflection; and in any case the relevant angle in this plane corresponding to θ_i is approximately zero degrees, which fact would, according to the equation giving the final result of section IVd of Appendix B have eased up on the tolerances in α and D by a factor of at least ten. Hence the tolerance of 10 mils in D stands.

All told, it would seem that for $n = 1.5$, homogeneous radomes held to a thickness tolerance of several mils at all points would be sufficiently well standardized to have their radome errors satisfactorily compensated.

We have not discussed manufacturing problems with potential manufacturers of reproducible radomes, but we will be very much surprised if radomes to these tolerances cannot be built.

III. RADOME ERROR SENSING. The existence of radome error brings into being a feedback loop whereby a spurious radar error signal, $-K(\sigma - \theta)$ is generated and added to the true radar error signal. If it were possible for the missile, at any given moment in the flight, to sense the value of K at that moment, then it would be simple to generate a signal $+K(\sigma - \theta)$ to cancel the spurious signal; the radome error feedback loop would thereby be cut and its damaging effects removed.

There are at least three quantities, observable one way or another by a missile in flight, which vanish for an ideal, but not for a non-ideal, radome:

- a) derivative of radar error signal with respect to radome orientation, target and antenna held fixed,
- b) $1 - (\text{derivative of error signal with respect to antenna motion, target and radome held fixed}), \text{ i.e. } 1 - (\partial(\psi - \sigma_r) / \partial \psi)_{\sigma, \theta}$.
- c) derivative of antenna angle with respect to radome orientation, target and radar error signal held fixed.

Of these (a) is the quantity to which the radome error feedback in the system is proportional, and the quantity which must be sensed. To sense it we must actually move the radome and observe the resultant apparent target motion. To observe item (b) one must only move the antenna (i.e. with a conical scan system, move the scan axis); but one measures the difference between two quantities, only one of which involves the calibration of the radar error signal; to measure to $\pm .01$ the equivalent radome error constant associated with quantity (b), one would need to have the radar error signal calibration to better than 1 per cent. Perhaps more relevant is the fact that quantities (b) and (c) neither measure nor have any direct relation to the quantity which hurts the system, quantity (a), although it is difficult to see how quantities (a) and (c) could differ by an operationally significant amount.

Granted that significant sensing of the radome error involves observing the effects of an actual motion of the radome axis with respect to the line of sight, one can imagine various ways to do this. One way might be by continuously observing in some sophisticated manner the correlation between observed change of θ and observed (radome-induced plus time) change in σ . At sufficiently long range the spurious target motion caused by attitude changes associated, for example, with aerodynamic buffeting, might well outweigh the true target motion; but at closer ranges, where it becomes important to reduce radome error for the purpose of delaying flight path instability, it may no longer be possible to ascribe to radome errors the bulk of apparent target motion. Here some sort of filtering out of the significant tactical frequency range would be necessary before engaging in correlation. All told, this procedure does not appear too promising.

THE PRINCETON UNIVERSITY RADOME STUDY

A second method, and one which appears worthy of examination in more detail, would be to introduce deliberately, at a single frequency ω , a wobble of amplitude θ , say, in the missile attitude. This would cause an additional radar error signal, the phase and amplitude of which give the sign and magnitude of the radome error. For this scheme to be feasible, a number of conditions must be met; the most basic are listed below as a basis for further discussion:

1. The wobble amplitude θ , should be small enough not to add significantly to:
 - a) g's to which the missile is subjected
 - b) servo power consumption
 - c) aerodynamic drag
2. The wobble frequency ω , should not be near enough to any structural resonance to excite undue structural vibration.
3. The band width $\Delta\omega$ of the radome error sensing mechanism must be wide enough to give information on radome error in a time short enough to be significant in removing the effects of radome error on flight path.
4. This same band width $\Delta\omega$ must be small enough so that total target noise in this band width is less than the intentional wobble signal which exists in the presence of a significant radome error.

Let us consider first the conditions on band width of the sensor. The value of the radome error should be sensed within a time comparable to that within which a significant change in radome error occurs; and this would seem to be the overall time constant of the missile. Random buffetings of the missile can cause attitude changes in shorter time, of the order of $1/\omega_0$, where ω_0 is the natural aerodynamic oscillation frequency, and if one could sense the radome error within this time then one would remove the apparent target noise resulting from buffeting-plus-radome-error; but sensing within the system time constant $\tau \gg 1/\omega_0$ will serve to take care of the troubles associated with the radome error feedback loop. Thus for the $K < 0$ high frequency attitude oscillation the significant value of K is the average value over several cycles at the frequency ω_0 . For the low frequency $K > 0$ attitude oscillation one wants to know the value of K over a time less than the period of attitude oscillation; this period is generally larger than the control time constant of the missile. Similar comments hold for the $K > 0$ effect on effective time constant or on navigation constant. All told then, the radome error feedback loop may be effectively neutralized if the band width is such as to sense K in the system control time constant τ , i.e. if

$$\tau \Delta\omega / 2\pi = 1 \quad (A4)$$

One would not want to use any larger band width than necessary because the noise picked up in the sensor will be proportional to this band width.

For control time constants of 0.5 or 0.3 seconds, Eq.(1) gives $\Delta\omega = 12$ or 20 radians/sec. The center of this band, i.e. the frequency ω_1 , needs to be enough larger than $\Delta\omega$ to avoid the region $\omega \sim 0$ which has a great deal of target noise and maneuver in it; in consequence of the above conclusion concerning $\Delta\omega$, suppose we assume that

$$\omega_1 \geq 30 \text{ radians/sec} \quad (\text{A5})$$

The noise in a given $\Delta\omega$ diminishes as the wobble frequency ω_1 increases. But the resultant desire to raise ω_1 is inhibited by the certain of the considerations in items 1 and 2 of the tabulation given above. Item 2 (avoidance of structural resonances) is specific to a particular missile and presumably does not limit the general range of ω_1 . Hence it will not be discussed further, although it must be recognized as a serious problem as the number of resonances which have to be considered is increased. But items (a) and (c) (acceleration and drag due to the wobble) can be discussed more specifically.

The necessity of limiting transverse acceleration turn out to be the more serious of the two. Suppose we specify that the presence of the wobble should add not more than $2g$ to the transverse acceleration; since r.m.s. acceleration, in g 's, at a distance 2 meters from a center of oscillation, is $0.14 \omega_1^2 \theta_1$, we have the condition

$$\omega_1^2 \theta_1 \leq 14 \text{ radians}^3/\text{sec}^2 \quad (\text{A6})$$

Since we want as large a θ_1 as possible in order to improve the signal-noise ratio in the sensor, Eq. (A6) tells us to reduce ω_1 ; in our ignorance of the target noise spectrum we have no good grounds for raising ω_1 above the value of Eq. (A5) at the price of reducing the θ_1 from Eq. (A6), so suppose we take $\omega_1 = 30 \text{ rad/sec}$; this gives us $\theta_1 = 0.9^\circ$.

We need now to check this value for aerodynamic drag, which will depend only on θ_1 and not on ω_1 . The general magnitudes involved may be deduced from data on the Falcon missile¹ which may be fitted by the relation

$$\text{Fractional increase in drag coefficient} \approx .018 \alpha^2$$

where α is angle of attack in degrees. At the frequencies involved here, $\alpha = \theta_1$ and for $\theta_1 = 0.9^\circ$, the maximum increase in drag coefficient will be $.018 \times 0.9^2 = 1 \frac{1}{2}$ percent; this is completely negligible.

We have not been able to get as straightforward figures in servo power requirements. It should be noted, however, that if $\omega_1 \gg \omega_2$, then the flipper amplitude required for a given attitude amplitude is proportional to ω_1^2 , making the power input for given θ_1 proportional to ω_1^3 ; this provides powerful incentive for holding ω_1 down.

¹"Fast Supersonic Wind-tunnel Tests of a Model of the MX904 Missile," Report No. 40-104-A, Nov. 1948. P. 20. Hughes Aircraft Co.

THE PRINCETON UNIVERSITY RADOME STUDY

The principal remaining question concerns signal-noise ratio in the sensor. A radome error K causes an apparent target motion $K\theta$, and if K is to be sensed to an accuracy ΔK then we should have

$$(\theta, \Delta K)^2 \geq (\text{target noise power in } \Delta\omega \text{ at } \omega_1).$$

Let us take $\Delta K = .02$ and $\theta_1 = 0.9^\circ$; then we require

$$(\text{target noise in } \Delta\omega \text{ at } \omega_1) \leq (.02^\circ)^2.$$

Target noise arises both from amplitude and angle scintillation, and of course ordinary receiver noise or microphonics will also contribute to apparent target motion. But the time when it is most important to get rid of radome error (in view of the general success in damping out high-frequency attitude oscillations) is as one approaches the end of the flight, say one second from collision; here a positive radome error can cause an earlier onset of flight path instability. (At the very end of flight, this instability or the blind time if any make the radome error rather irrelevant.) In this region the most important noise is the angle noise.

Knowledge of this noise is meager. The total r.m.s. fluctuation of the equivalent radar center of the target is generally stated as some fraction of the target length. Suppose this is 20 ft; this corresponds to an r.m.s. target noise angle, 1 second from impact, of the order of 0.6° . Comparing this with the $.02^\circ$ radome-induced apparent target motion which the sensor has to work on, it is apparent that the sensor band width $\Delta\omega$ must contain not more than the fraction $(0.02/0.6)^2 = 10^{-3}$ of the total angle noise power.

Data in a Hughes Progress Report¹ gives the relative angle noise spectrum up to $\omega = 3$ radians/second, with no data in the range in which we are interested. The Hughes theory of this effect says that the spectrum should become effectively zero above frequencies of several radians/second; in any case, a very rough estimate from the trend of the experimental data suggests that at values of $\omega_1 \geq 10$ radians/sec., the angular scintillation noise power per cycle per second might well be less than 10^{-3} times the total angular noise power.

The conclusion of these considerations seems to be that the feasibility of radome error sensing depends primarily on the properties of the target angle noise spectrum which are at present unknown. An estimate of these properties indicates feasibility.

It should be emphasized that the numerical values used above will not fit any one particular system, but have been picked to be broadly typical and to indicate orders of magnitude.

¹Progress Report 40-21-P (First Quarter 1951) p. 61; Hughes Aircraft Co.

IV. THE TREATMENT OF TARGET NOISE. The existence of target noise is to a large degree responsible for one of the most basic characteristics of missile guidance systems; the inclusion of a means of filtering out the noise and the consequent existence of time lag in the missile controls.

Wiener Optimisation Technique. Noise is also a problem in many other related fields. The best related example is probably that of anti-aircraft fire control,¹ from which the missile guidance problem differs primarily in two ways: the fire control (missile control) is applied continuously throughout flight, and the apparatus for doing it must be smaller and simpler. But this does not negate one of the basic and successful fire-control design procedures, which is as follows: From a careful study of target characteristics and possible tactics, and from observations on target and system noise, the statistical characteristics of the noise and information are deduced. Given this data, there exists a mathematical technique, developed by Wiener², for determining uniquely the system frequency response for optimizing any desired performance parameter— for example, r.m.s. miss distance. To obtain this unique frequency response for the system the practically attainable system embodiments are combined with filters of properly shaped frequency dependence.

By contrast, the procedure which has been used in missile control system design to date and which is necessary to some extent but cannot go on forever, has been somewhat as follows: One starts with a missile which seems to be pretty good aerodynamically and for range and carrying power, and one decides to develop a control system to put in it. This control system turns out to have certain instabilities so one develops modifications to get rid of these. Target noise turns out to be bad so one gives the system the behaviour of an RC filter; if this is not enough one adds another RC filter. Radome errors turn out to exist and to amplify miss due to noise, so one tries to cut down on radome errors. The resolution of multiple targets late in flight turns out to give bad misses and one worries about some neat way of beating this.

This procedure is perhaps necessary, or at least perhaps gives as good results as the more sophisticated procedure, as long as basically new phenomena like the existence of radome error or the fact that attacking planes may not come singly are still being uncovered. At this stage, each patchwork modification may make a big reduction in miss distance. But when the dust of opening up a new field shows signs of settling, as it does now, then the control system should be optimized in the sense of the Wiener procedure. We are told by people who were concerned with anti-

¹"Data Smoothing and Prediction in Fire Control Systems," -excerpt from "Gun Fire Control," Summary Report of Division 7, NORC, Vol. 1, 15 August 1948; MGC 12/1.

²Wiener, Norbert, "Extrapolation, Interpolation and Smoothing of Stationary Time Series, with Engineering Applications." John Wiley and Sons, Inc., New York, 1949.

THE PRINCETON UNIVERSITY RADOME STUDY

aircraft fire in the last war that the use of this systematic optimization procedure after all the emergency patchwork has been done can be expected to reduce miss distances by from one to three factors of two, which may make all the difference.

Perhaps the strongest deterrent to this at present is the great ignorance as to the exact characteristics of target noise. This is no basic excuse, since one can apply a good procedure to bad data with more success than a bad procedure; but if the good procedure is unfamiliar then the fact that it would have to be applied to bad data may deter one from using it.

Good target noise data is hard to obtain. We are aware of good data taken at NRL^{1,2} and Hughes^{3,4} but much more extensive data is needed.

Application of the Wiener optimization procedure to missiles has been carried out by Project Meteor at MIT, but without including missile aerodynamics in the considerations.

Lack of information on target noise is the principal stumbling block to judging the feasibility of the radome-error sensing mechanism which is discussed in section III of this appendix.

In the light of all these comments, we wish to recommend strongly that studies of expected target tactics and observed target noise be pushed fast and hard with the aim of encouraging application to missile control of the general sophisticated design procedures which have already been worked out in other types of fire-control problems.

Possible Beneficial Use of Non-Linearity in Control System.

We do not know whether non-linearities can be put to good use in a control system. We do know that the handling of noise by linear filters introduces great difficulties attendant upon the resulting lags. One can imagine various non-linear ways of handling noise with the aid of clipping; and our principal comment on these other ways of handling noise is that they seem to us to have been ignored in present Navy developments with which we have been in contact in favor of things that are easier to think about and more closely justifiable in terms of meeting tomorrow's delivery date. Noise being the problem it is, we feel that non-linear ways of handling it are worthy of careful consideration.

¹"Amplitude Fluctuation on Radar Echo Pulses," Naval Research Laboratory. Report No. R-3487, June 20, 1949.

²"Noise in Tracking Radars," Naval Research Laboratory. Report No. 3759, November 15, 1950.

³Progress Report 40-21-P (First Quarter 1951) p. 61. Hughes Aircraft Co.

⁴Progress Report 40-22-P (Second Quarter 1951) p. 69. Hughes Aircraft Co.

Variation of Control Parameters. Another question which arises when one tries to look into some of the ramifications of noise is the question of whether the guidance system ought not to have its parameters adjusted as a function of range to the target--- although having variable parameters makes the application of linear optimization analysis difficult. Expressed in terms of the frequency response of the system, which is its basic characteristics from the optimization point of view, the question to which one needs the answer is this: how do the frequency characteristics of signal and noise and inherent missile response vary as functions of range? There may be range-correlated evasive maneuvers; target noise characteristics, or the relative importance of various types of noise, may be a function of range; sudden resolution of multiple targets late in missile flight constitutes the addition of high frequency signal components late in flight, and the later the higher; non-linearities late in flight, e.g. acceleration limiting, change the missile control characteristics as a function of range. All of these points suggest the programming of parameters as a function of sensed range and possibly range velocity.

Another and perhaps even more obvious reason for this variation of parameters is the fact that the gain of the geometry feedback loop--- i.e. the transfer function (target angle/missile velocity)--- is inversely proportional to range. It is in fact the increase of this gain at short ranges which makes this feedback loop unstable and gives rise to "flight path instability" when time to collision equals control time constant. An equivalent way of stating this would be that there is a drastic change, with range, of the frequency response of the missile to the target, and that at a certain range the frequency response is distorted to the point of giving instability. From the point of view of avoiding this end-of-flight phenomenon, or from the more general point of view of tailoring the system response to the range dependence of the signal and noise frequency characteristics, one might be led to navigation in the late stages of flight based more nearly upon "position control" (i.e. missile response proportional to target linear velocity) than upon "angle control."

It is beyond the scope of these comments in a radome report to say in what direction development along these lines should go. Furthermore it should be noted that programming of parameters is being energetically pushed in some systems, the best example to our knowledge being the Raytheon Sparrow III. Our primary purpose in these last few paragraphs has been to note that programming of parameters seems to us a very important part of any more searching attack upon the noise problems, which are so intimately tied up with the radome problem.

V. RELATION BETWEEN AERODYNAMIC CONFIGURATION AND SENSITIVITY TO RADOME ERROR. With radome error present, a change in missile attitude causes an apparent change in target position. The spurious target shift is proportional to the attitude change; it might therefore be expected that radome troubles could be minimized by demanding from the missile the minimum attitude change in response to a given true target maneuver. For a given navigation system (e.g. proportional navigation with given navigation constant), this attitude change depends upon the missile's aerodynamic

THE PRINCETON UNIVERSITY RADOME STUDY

configuration; and it is the purpose of this section to see what the best configuration is from this point of view.* (See footnote page A28)

One extreme of behaviour in respect to attitude change for a given velocity change is provided by the fixed-wing missile in which turn rate is achieved as a two-step process; first the rudder flippers turn the missile with respect to the air stream, generating angle of attack; the body lift resulting from non-zero angle of attack in turn generates lateral acceleration. The transient change in missile attitude in any given situation may be much larger than the actual change in missile heading which is being called for-- this is particularly true at high altitudes, where a high angle of attack develops turn rate only slowly.

At the other extreme, requiring the minimum attitude change for a given maneuver, there comes any movable-wing missile which is so adjusted that, as far as possible, turn rate is generated entirely by the wings and not at all by body lift; that is, so adjusted that the missile axis always points in the direction of motion. It seems reasonable, and will be shown shortly on the basis of quantitative consideration, that such a missile should be free from much of the radome-induced instability and sluggishness of other missiles.

The configuration just mentioned has compensatory disadvantages. It would presumably involve tail controls as well as main wing controls. It would involve more servo power and more storage space for the larger wings. It is not our purpose here to weigh all these factors and come up with a final recommendation applicable in all cases. We do wish, however, to emphasize very strongly to missile designers the fact that there is a strong interplay between aerodynamic configuration and radome troubles which cannot be ignored.

Illustrative Example. While the above argument that one should choose the aerodynamic configuration which requires the minimum attitude change seems straightforward enough, one cannot in practice attain--and perhaps for aerodynamic reasons would not want to obtain-- the ideal behaviour in which no angle of attack is generated. So it becomes relevant to ask how beneficial it is to go part way, but not all the way, in this direction. To obtain a quantitative answer to this question we consider as an illustrative example the system represented in Fig. A4.

In this system we consider only that part of the system which concerns the attitude feedback loop as it is closed by radome error. Here a single variable, δ , describes the state of the wing and rudder controls and indicates the wing angle of attack. This angle δ , in the absence of radome errors, is proportional to $\dot{\sigma}$, the rate of change of target bearing, with added noise filtering. However, δ influences the direction, γ , of target velocity, in two ways. The direct wing lift gives a contribution to lateral acceleration $\dot{\gamma} = \delta/\tau_1$; But at the same time the wing variable δ develops body angle of attack α which gives a contribution $\dot{\gamma} = \alpha/\tau_2$ to lateral acceleration. Thus the two parameters τ_1 , and τ_2 , the "partial turning-time constant", specify the relative influence of wing and body lift; the total acceleration is given by $\dot{\gamma} = \alpha/\tau_2 + \delta/\tau_1$

The relative influence of wing angle in developing body angle of attack is indicated by the constant C in the transfer function α/δ in Fig. A4.

The net relative importance of wing and body lift in this example is best specified in terms of their contribution to the steady state navigation constant λ in the absence of radome error. From Fig. A4 we have

$$\lambda = \left(\frac{1/p\delta}{1/p\sigma} \right)_{\text{Steady State}} = g\tau_A \left(\frac{1}{\tau_1} + \frac{C}{\tau_2} \right).$$

If we define the parameters μ_1 and μ_2 by the relations

$$\begin{aligned}\mu_1 \lambda &= g\tau_A / \tau_1 \\ \mu_2 \lambda &= g\tau_A C / \tau_2\end{aligned}$$

Then we have

$$\lambda = (\mu_1 + \mu_2) \lambda ; \quad \mu_1 + \mu_2 = 1$$

Thus μ_1 and μ_2 give the relative contributions of wing and body lift to navigation constant; and the problem which we have found ourselves in this illustrative example is to see how large μ_1 must be in order to reduce significantly the effects of radome error.

Limiting Values for Illustrative Example. The loop transfer function Y_ℓ for the feedback loop in Fig. A4 has the value

$$Y_\ell = \frac{K\lambda}{(1+p\tau_A)(1+p\tau_L)} \left[\mu_1 + \mu_2 \frac{1+p\tau_2}{1 + \frac{2p}{\omega_0^2 \tau_0} + \frac{p^2}{\omega_0^2}} \right] \quad A7$$

The attitude loop will break into oscillation when $Y_\ell = 1 + j0$. The frequency at which such oscillation will occur is therefore that for which $\text{Im}(Y_\ell) = 0$, and will be independent of K but not necessarily of μ_1 and μ_2 .

The case $\mu_2 = 1$, $\mu_1 = 0$ corresponds to a fixed wing missile with gyrostabilized head, as for example in the Hughes Falcon. The instabilities here, as is well known, are of three types:

1) Oscillation at $\omega = 0$ for $K\lambda \geq +1$. (This is a characteristic of any proportional navigation system; it occurs for the system of Fig. A1 for all values of μ_1 and μ_2 , and will not be discussed further.)

THE PRINCETON UNIVERSITY RADOME STUDY

2) Oscillation at $\omega \approx \omega_0$ for

$$K\lambda \approx -2T_A T_L / T_0 T_2 \quad \text{Eq. A8}$$

3) Oscillation at the low frequency

$$\omega^2 = [1 - (T_A + T_L) / T_2] / T_A T_L \quad \text{Eq. A9a}$$

provided that

$$T_2 > T_A + T_L \quad \text{Eq. A9b}$$

$$K\lambda > (T_A + T_L) / T_2 \quad \text{Eq. A9c}$$

However, since the frequency given by Eq. A9a may be very low, attitude stability criteria are not conclusive. In particular, an inherent flight path instability occurs near the end of flight at a time when time-to-collision equals system time constant; this time constant is increased by a positive radome error. This point will be discussed in more detail a little later.

For $\mu_2 = 0$ (no body angle of attack), there is no attitude instability other than at $\omega = 0$, $K\lambda = 1$. This bears out the intuitive comments made concerning the ideal movable wing missile. (If other time lags than those in the schematic of Fig. A4 are included-- e.g. wing servo lags-- those oscillations at $\omega > 0$ can occur, but at such a high frequency that there is no possibility of exciting them.)

Illustrative Example Continued for the General Case. The second and third types of instability listed above will now be discussed in more detail for the case of arbitrary μ_1 and μ_2 .

Oscillation at the aerodynamic oscillation frequency, $\omega \approx \omega_0$ for K negative. Investigation of the roots of the equation in $Y_e = 0$ shows that the root which is at $\omega = \omega_0$ for $\mu_2 = 1$ remains at $\omega \approx \omega_0$ until finally the root disappears, i.e. becomes imaginary, for

$$\left| \frac{\mu_2 \omega_0^3 T_2 T_0 \sqrt{T_A T_L}}{16} \right| \leq 1; \quad \text{Eq. A10}$$

In the derivation of this relation and of those that follow use has been made of the fact that normally $\omega_0 T_0, \omega_0 T_2, \omega_0 T_A, \omega_0 T_L \gg 1$. If Eq. A10 is satisfied, then the $\mu_2 = 0$ regime takes over-- i.e. no oscillation at $\omega \approx \omega_0$, no matter how bad the radome is.

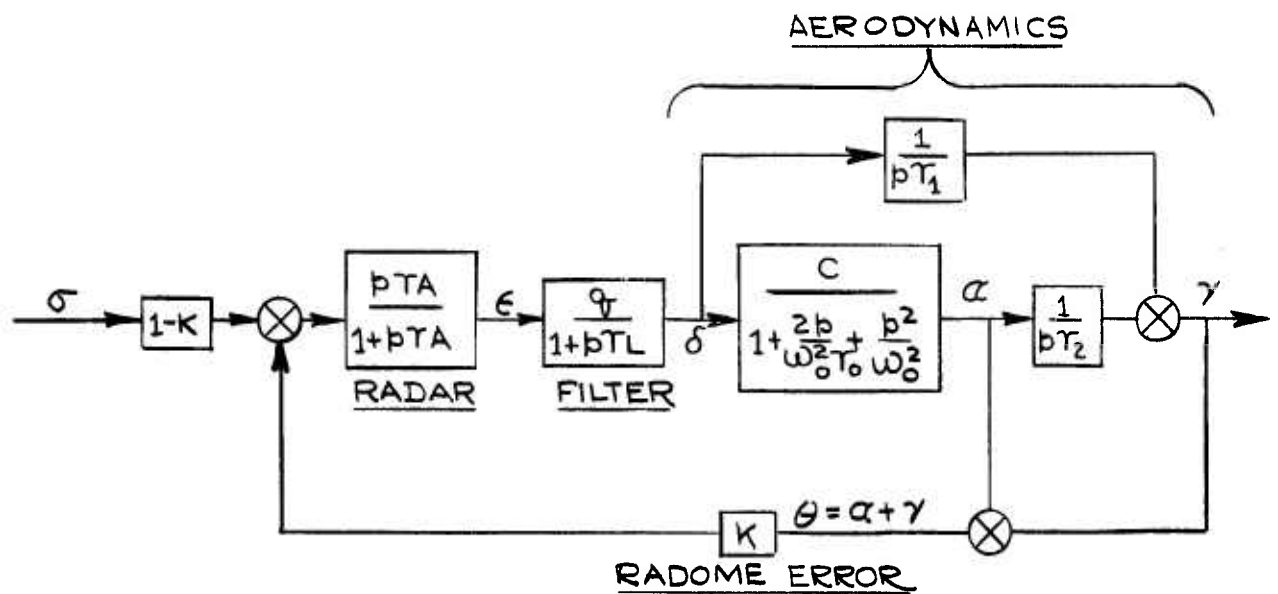


FIG. A 4

ATTITUDE FEEDBACK LOOP, CLOSED BY RADOME ERROR, IN MISSILE WITH ARBITRARY RELATIVE AMOUNTS OF WING AND BODY LIFT.

SYSTEMS

However, even if Eq. A10 is not satisfied, it may be impossible to excite oscillation at $\omega \sim \omega_0$ i.e. the requirement that $\text{Re}(Y_\ell) \geq 1$ may not be met. At $\omega = \omega_0$, from Eq. A7

$$Y_\ell \approx - \frac{K\lambda}{\omega_0^2 T_A T_L} \left(\mu_1 + \frac{1}{2} \mu_2 \omega_0^2 T_2 T_0 \right) \quad \text{Eq. A11}$$

The condition for no oscillation at $\omega = \omega_0$, i.e. that $Y_\ell < +1$ takes various forms depending on μ_2 :

If: Then Condition for no oscillation is:

$$\frac{\mu_2 \omega_0^2 T_2 T_0}{2} \gg 1: \quad \frac{K\lambda}{2} \mu_2 \frac{T_0 T_2}{T_A T_L} > -1 \quad \text{Eq. A12a}$$

$$= 1: \quad \frac{2K\lambda}{\omega_0^2 T_A T_L} > -1 \quad \text{Eq. A12b}$$

$$\ll 1: \quad \frac{K\lambda}{\omega_0^2 T_A T_L} > -1 \quad \text{Eq. A12c}$$

The parameter μ_1 does not appear in Eq. A12 because when

$$\frac{1}{2} \mu_2 \omega_0^2 T_2 T_0 \leq 1, \quad \mu_1 \approx 1$$

Numerical Values. It is instructive to see what these criteria amount to in a specific case, for which we shall take the Hughes A4A missile because data are readily at hand. In the first place, referring to Eq. A10, $\omega_0^3 T_2 T_0 \sqrt{T_A T_L}$ for this missile is of the order of 10^6 for a wide variety of altitudes and velocities; hence Eq. A10 is roughly equivalent to the condition $\mu_2 \leq .02$ for it to be impossible for oscillation to exist for any $K\lambda$; this condition probably cannot be met, nor is there much point to trying.

For this same missile, the quantity $\frac{1}{2} \omega_0^2 T_2 T_0$ which occurs in Eq. 12a ranges from 60 to 400, so that in all likelihood Eq. 12a is the relevant one. Under these conditions the effect of having $\mu_2 < 1$ is simply to replace $K\lambda$ by $K\lambda \mu_2$ -- i.e. if one has $\mu_2 = 0.2$, the allowable range of $K\lambda$ has been increased five-fold by this fact. For the worst combination of numbers with the Hughes A4A missile, at 50,000 ft. altitude and Mach 1.1, $T_0 T_2 / T_A T_L \sim 200$; here for stability one needs $K\lambda \mu_2 > -.01$. This means that even with $\mu_2 = 0.2$, stability cannot be obtained without electronic damping of the aerodynamic

THE PRINCETON UNIVERSITY RADOME STUDY

oscillations. One wants this electronic damping in any case, but its job is made much easier-- i.e. its range of utility is greatly increased-- by reducing μ_2 .

It should be noted that for $0 < \mu_2 \omega_0^2 T_0 T_2 < 1$, the condition $\Im(Y_e) = 0$ has an additional root for $\omega > \omega_0$; but this represents an oscillation which is even more difficult to excite than the one at $\omega = \omega_0$, and is hence unimportant.

Effects at $0 < \omega < \omega_0$ for $K > 0$: At sufficiently low frequencies the time taken to build up body angle of attack may be neglected, i.e. one may set $\alpha = c \delta$ and neglect the aerodynamic denominator. Here the loop transfer function given in Eq. A7 simplifies to

$$Y_e \cong \frac{K \lambda (1 + p \mu_2 T_2)}{(1 + p T_A)(1 + p T_L)} \quad \text{Eq. A13}$$

One gets oscillation, i.e. $Y_e = 1 + j0$ when both the following conditions are satisfied:

$$\mu_2 T_2 > T_A + T_L \quad \text{Eq. A14a}$$

$$K \lambda \mu_2 > (T_A + T_L) T_2 \quad \text{Eq. A14b}$$

The frequency of oscillation is given by

$$\omega^2 T_A T_L = 1 - (T_A + T_L) / \mu_2 T_2 \quad \text{Eq. A14c}$$

It will be seen that the general case represented by Eqs. A2 differs from the usual case, $\mu_2 = 1$ Eqs. A9, in that the oscillation is much more difficult to excite.

As already noted, however, the time constant of transient response is more important than attitude stability. For the description of low-frequency transient response, we note that when we make the same approximation $\omega < \omega_0$ as was used in Eq. A13, we may write

$$\begin{aligned} \frac{\gamma}{\sigma} &= \frac{(1-K)\lambda}{(1+pT_A)(1+pT_L) - K\lambda(1+p\mu_2T_2)} \\ &= \frac{(1-K)\lambda}{T_A T_L \left[p^2 + p \frac{T_A + T_L - K\lambda\mu_2T_2}{T_A T_L} + \frac{1-K\lambda}{T_A T_L} \right]} \end{aligned} \quad \text{Eq. A15}$$

SYSTEMS

This represents a transient response of δ to ϕ which if underdamped or critically damped is characterized by an effective time constant

$$T_{eff} = 2T_A T_L / (T_A + T_L - K\lambda \mu_2 T_2) ; \quad \text{Eq. A16}$$

the condition for underdamping is

$$(T_A + T_L - K\lambda \mu_2 T_2)^2 < 4T_A T_L (1 - K\lambda) \quad \text{Eq. A17}$$

When this inequality is reversed (overdamping), the transient comprises two terms with two different time constants, of which the damping one is the larger, which we shall continue to call T_{eff} .

$$T_{eff} = \frac{1}{2(1-K\lambda)} \left[T_A + T_L - K\lambda \mu_2 T_2 + \sqrt{(T_A + T_L - K\lambda \mu_2 T_2)^2 - 4T_A T_L (1-K\lambda)} \right] \quad \text{A18}$$

It will be noted that at critical damping, Eq. A18 gives

$$T_{eff} = \frac{T_A + T_L - K\lambda \mu_2 T_2}{2(1-K\lambda)} = \sqrt{\frac{T_A T_L}{1-K\lambda}} = \frac{2T_A T_L}{T_A + T_L - K\lambda \mu_2 T_2}$$

in agreement with Eq. A16.

It is apparent from Eq. A16 that when one has critical damping or underdamping for $\mu_2 = 1$, then reduction in μ_2 will be equivalent to a reduction in the radome error and in T_{eff} . Once this process has been carried to the point where overdamping sets in, however, a further decrease in μ_2 may cause T_{eff} to rise.

These points are best discussed with the aid of specific examples. For such an example, let us consider again the Hughes A4A missile, for which $T_A = T_L = 1/4$ second, while T_2 may range from 1/2 second, at 5000 ft. altitude and Mach 1.7, to 4.6 seconds, at 50000 ft. and Mach 1.1.

For the first of these circumstances, $T_2 = 1/2$ second, gives an underdamped but stable system for the range $0 < K\lambda < 1$ with $T_{eff} = 1/4(1-K\lambda)$. From Eq. A16 it is apparent that a decrease in μ_2 will decrease T_{eff} , which is advantageous; but if $\mu_2 = 1/2$, for which we have overdamping for $0 < K\lambda < 1$

$$T_{eff} = (1 + K\lambda/2) / 4(1-K\lambda)$$

and for $\mu_2 = 0$, $T_{eff} = (1 + \sqrt{K\lambda}) / 4(1-K\lambda)$

So all told, decreasing μ_2 too far does more harm than good here.

¹Progress Report 40-18-P (Second Quarter 1950). Hughes Aircraft Co.

THE PRINCETON UNIVERSITY RADOME STUDY

For the other extreme example, with $T_2 \sim 4.5$ seconds, Eq. A14 shows that small μ_2 is a big help in removing the attitude oscillation. If $\mu_2 = 0.2$, the attitude stability limit is $K\lambda < 0.5$, transients are underdamped for this range, and $T_{eff} = 4/(1 - 1.8K\lambda)$. For comparison, at $\mu_2 = 1$ the system is stable and underdamped for $0 < K\lambda < .11$ and $T_{eff} = 4/(1 - 9K\lambda)$. Here the decrease in μ_2 has been of great benefit.

Conclusions: The discussion which has just been given has for sake of definiteness been limited to one particular type of head stabilization, similar to that in the Falcon or Oriole missiles; and the specific numerical examples have been taken from data for the Hughes A4A Falcon. For this particular example, the addition of wing control and the consequent reduction in μ_2 , the fraction of lateral acceleration provided by body lift, appears to have the following effects:

- 1) For $K < 0$ the stability range for attitude oscillation at the natural aerodynamic frequency is set by the parameter $K\lambda/\mu_2$; that is, the stability range is proportional to $1/\mu_2$, all other things equal.
- 2) For $K > 0$ the same statement holds true for stability with respect to low frequency attitude oscillations.
- 3) For $K > 0$ when the transient response of the missile is underdamped the damping time constant is decreased by decreasing μ_2 . This is particularly beneficial at high altitudes. At lower altitudes, in cases where the missile response is already overdamped, a decrease in μ_2 may increase the time constant but not by very much.

Without further generalization to other types of missile control systems or other specific numerical examples, we feel that these results warrant careful future consideration of the relation between aerodynamic properties of the missile and the requirements placed upon the radome.

*(See page A22) Technical Note No.33, "Preliminary note on the effect of radome aberration on the stability of homing flight." Bristol Aeroplane Co. Ltd. January 1951.

APPENDIX B

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

I. INTRODUCTION. An exact calculation of the effect of the radome upon the antenna pattern would be very tedious requiring numerical methods and the use of a large computing machine. Furthermore it would be of dubious value giving the same sort of results that can be obtained more readily by making antenna pattern measurements. This should not be interpreted, however, as an argument against the use of theoretical methods in connection with the radome problem. Theoretical methods properly used can be very valuable in breaking down a complicated problem into a pattern of simpler sub-problems, in the separating of the various effects that occur, and in judging their relative importance.

In this connection though it is generally undesirable to strive for a great deal of meticulous detail in one's calculations. To do so generally involves calculations in which the essential physical ideas are easily obscured or lost. The thing that should be expected from one's calculations is an understanding of the basic causes of radome error, not a detailed knowledge of the electromagnetic fields surrounding a radome.

A convenient classification of causes of radome error can be made in terms of the number of reflections involved. A first order cause of radome error is one in which the radiation passes directly through the radome. In a second order cause the radiation appears in the far field after a single reflection from the radome, etc.

This type of classification is sensible for two reasons. First, the higher the order of the effect the higher the number of reflections from the radome, hence the more the beam is attenuated. Furthermore, in general the more reflections that have occurred, the more the beam has been thrown into random directions and the smaller the chance of a coherent strong reinforcement or interference with the original beam. Consequently one would expect that in general the lower the order of the effect, the more important it is.

1. In the case of first order effects there is a direct modification of the beam as it passes through the radome. The modification consists of a non-uniform removal of power from the beam and a non-uniform phase shift over the aperture of the beam. The removal of power reduces the height of the main beam. Any non-uniformity of the power removal and phase shift leads to a widening of the beam and a further reduction in height. Furthermore, any linear component of the change of phase across the aperture leads to a shift of the position of the beam. These features will be discussed in more detail later.

2. Second order effects concern the recombination, with the primary beam, of energy which has been abstracted from the primary beam and is immediately returned to the far radiation field, there to interfere with the primary beam. First order effects have been

THE PRINCETON UNIVERSITY RADOME STUDY

considered in detail in section IV of this appendix. It is the purpose of this section to consider second and higher order effects. The most clear cut example of this is provided by what we shall call the "Lloyd's mirror." In the Lloyd's mirror phenomenon a part of the primary beam is incident upon the radome at glancing incidence and is partially reflected, the reflected beam being nearly parallel to the primary beam; in the far field the diffraction patterns of the primary and reflected beams interfere and cause beam distortion.

In second and higher order effects it must be remembered that although the amount of power reflected from the radome may be small, it is the wave amplitudes which must be combined and not powers. For example, if 1% of the radiated power is reflected back into the antenna in such a way as to be reradiated by the antenna the height of the main lobe can on a power basis be increased or decreased by about 20%. In a similar way large changes in the width and position of the beam can occur.

The foregoing considerations lead one to believe that the higher the order of the effect in general the less important it is. There will be exceptions to this. For example a radome shaped in such a way as to cause the reflected energy to be focused, even after several reflections, onto the primary dipole of a parabolical antenna, will obviously cause the radiation to be reradiated as a focused beam. In fact there has been evidence obtained by the group at the Naval Air Development Center that in the case of a certain blunt radome this effect although of third order may be of considerable importance. Quantitative estimates of the relative importance of the various order radome errors will be obtained later.

In the case of a first order cause of radome error the radiation field from the antenna is presumed known, the physical characteristics of the radome in terms of wall thickness etc. are also presumed known and it is desired to calculate the effect of the radome on the radiation pattern of the antenna. It is not necessary to know all the details of the antenna pattern. As is discussed later the height, width, and position of the main lobe are sufficient. As will be shown shortly the field distribution over the outer surface of the radome is sufficient to calculate the distant fields, i.e. the antenna pattern in the presence of the radome. This outer surface field is to a good approximation simply related to the fields over the inner surface. These are in turn, neglecting internal reflections, just the fields radiated by the antenna and assumed known.

II. THE PROJECTED RADOME APPROXIMATION. The calculation of the antenna pattern from the fields over the outer surface of the radome is not very convenient as it involves an integration over a curved surface. It is the purpose of this section to show that the effect of the radome on the antenna pattern is, over a limited part of the pattern, essentially identical with that produced by an equivalent flat non-uniform transparent sheet derived from the radome in a manner to be described. In this case the integration is performed over a flat sheet and is consequently more easily done. In addition to simplifying the formalism, the effect of the radome is reduced to terms more readily visualized in a physical way so that it is possible to obtain a qualitative picture of the effect of a certain change in the radome without making a calculation.

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AN ANALYSIS OF THE SOURCES OF RADOME ERROR

The first problem to be considered is the calculation of the field on the outside of the radome in terms of that on the inside. The transmission properties of infinite flat sheets are easily calculated for incident plane electromagnetic waves. The curved radome is not a flat sheet and the incident field inside is not a plane wave. These complications are not as serious as they at first seem. A consideration of the internal reflection of radiation in the wall of the radome shows that in general (omitting from our considerations trapped waves and similar effects) the curved surface can be regarded as a locally flat surface in computing polarization, amplitude, and phase shifts produced by transmission through the wall of the radome. The reason for this result is the fact that a wave which enters at one point of the radome cannot wander far as a result of internal reflections before it is strongly attenuated, as a result of leakage of radiation from the surface. Consequently, the radiation leaving a particular point is derived from radiation which entered over a region sufficiently small that it can be regarded as flat.

If one imagines the exterior surface of the missile including radome as constituting a surface S over which two cartesian components of the electric field and their normal derivatives are known, then the external field can be everywhere calculated. A single cartesian component of the electric field obeys the scalar wave equation and consequently the Kirchhoff-Huygens diffraction theory can be used (see Stratton, Electromagnetic Theory, p. 460). For example, if the X component of electric field, E_x and its normal derivative are known over the surface, the same component of the electric field at a point external to the surface S is given by

$$E_x(P) = \frac{1}{4\pi} \int_S [\phi \text{ grad } E_x - E_x \text{ grad } \phi] \cdot d\mathbf{S}$$

where $\phi = \frac{e^{ikr}}{r}$, $k = \frac{\omega}{c}$, time dependence is of the form $e^{-i\omega t}$ and r is the distance from the point P to the point on the surface S . A vector analogue of this equation is commonly used for the vector wave equation. It is however, rather clumsy for the present problem.

In a similar way another cartesian component can be obtained at the point P and the third component from the divergence condition. The three components of $\mathbf{H}$ can be obtained from $\mathbf{E}$ using another of Maxwell's equations.

If the point P is very far from the radome, ϕ can be written as

$$\phi(r) = \frac{e^{-ik_m r}}{r}, \quad \text{grad } \phi = -ik_m \frac{e^{-ik_m r}}{r}$$

THE PRINCETON UNIVERSITY RADOME STUDY

where $\underline{k}$ is a vector directed from the radome toward the point P.

The wave leaving the antenna is approximately plane and may be written as the product of a plane wave by a slowly varying amplitude-phase function. For example

$$\underline{E}(\underline{r}) = \underline{A}(\underline{r}) e^{i \underline{k}_0 \cdot \underline{r}}$$

where $\underline{k}_0$ has the direction of the center of the antenna pattern. Subject to the simplifications outlined above the electric field over the outside surface of the radome may be written as

$$\underline{T}(\underline{r}) \underline{E}(\underline{r})$$

where the scalar $\underline{T}$ gives the phase shift and amplitude reduction produced by a traversal of the radome. If there is a shift in polarization upon traversal, $\underline{T}$ becomes a square matrix. In order to simplify the discussion $\underline{T}$ will be regarded as a scalar. However, the end results are easily extended to take into account the effect of modification of polarization by the radome.

The surface integral involves the gradient of the cartesian components of the field. If both $\underline{T}$ and $\underline{A}$ can be regarded as slowly varying functions, the gradient of for example the x -component becomes approximately

$$\text{grad } \underline{T} \underline{A}_x(\underline{r}) e^{i \underline{k}_0 \cdot \underline{r}} = i \underline{k}_0 \underline{T} \underline{A}_x e^{i \underline{k}_0 \cdot \underline{r}}$$

Applying the Kirchhoff formula one has for the x component of the field at the point P

$$\underline{E}_x(P) = \frac{1}{4\pi r} \int_S i(\underline{k}_0 + \underline{k}) \underline{T} \underline{A}_x e^{i(\underline{k}_0 + \underline{k}) \cdot \underline{r}} \cdot d\underline{S}$$

Because of the great distance from the point P to the radome the term in $1/r$ is nearly constant and may be taken outside of the integral.

The idea of the projected radome can now be introduced. In Fig. B1 there is shown the surface S together with the vectors $\underline{k}_0$ and $\underline{k}$. Introduce a plane surface S' perpendicular to $\underline{k}_0$. Project on it the values of $\underline{T}$ and $\underline{A}_x$ occurring at the corresponding point on the surface S. The vector connecting corresponding parts on the two surfaces is denoted by $\underline{D}$. If to a point $\underline{r}$ on S there corresponds a point $\underline{r}'$ on S', then

$$\underline{r}' = \underline{r} + \underline{D}$$

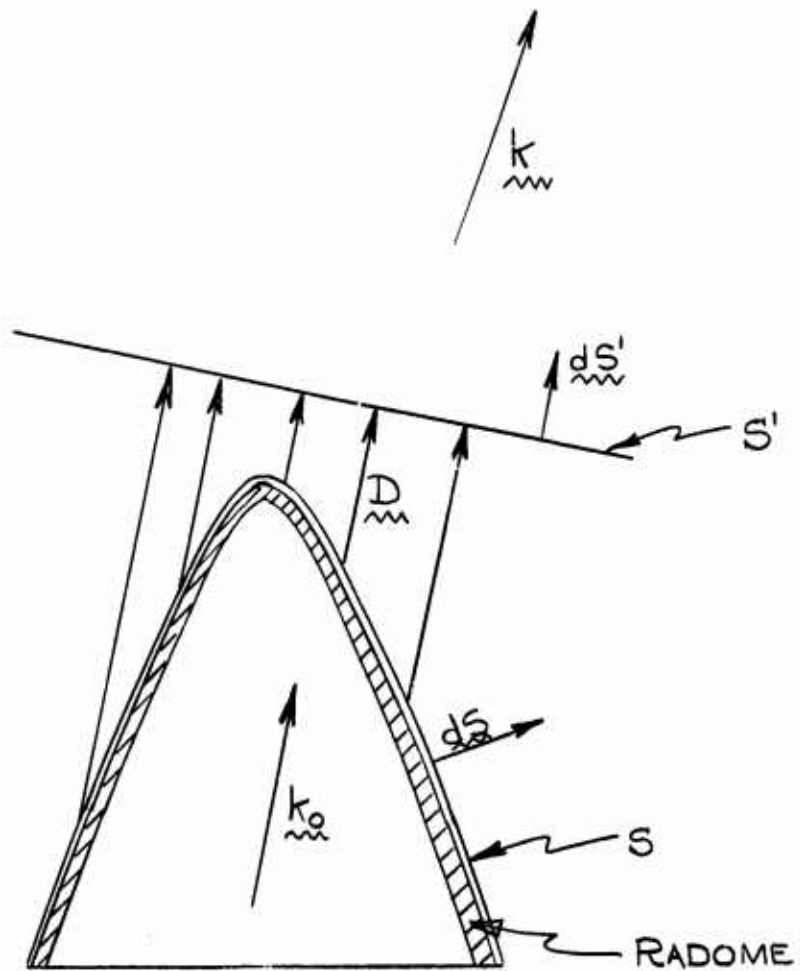


FIG. B1
PROJECTED RADOME

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

The integral for $E_x(P)$ evaluated at the point P can be written

$$E_x(P) = \frac{1}{4\pi r} \int_S i(\underline{k}_0 + \underline{k}) T(\underline{r}) A_x(\underline{r}) e^{i(\underline{k}_0 - \underline{k}) \cdot (\underline{r}' - D)} \cdot dS$$

Assuming that $\underline{k}_0$ and $\underline{k}$ point in nearly the same direction, $\underline{k}_0 - \underline{k}$ is almost perpendicular to D , also its magnitude is small compared with k . The integral becomes approximately

$$E_x(P) = \frac{1}{4\pi r} \int_S i(\underline{k}_0 + \underline{k}) T A_x e^{i(\underline{k}_0 - \underline{k}) \cdot \underline{r}'} \cdot dS$$

Since $\underline{k}$ is nearly the same direction as $\underline{k}_0$, the projected element of area is approximately

$$dS' = \frac{(\underline{k}_0 + \underline{k}) \cdot dS}{2k}$$

Making this substitution, the integral becomes

$$E_x(P) = \frac{2ki}{4\pi r} \int_{S'} T(\underline{r}') A(\underline{r}') e^{i(\underline{k}_0 - \underline{k}) \cdot \underline{r}'} dS'$$

where the integral is now over the surface S' . The approximation is only satisfactory for $\underline{k}_0$ and $\underline{k}$ lying in nearly the same directions, but this is the part of the antenna pattern in which there is the greatest interest. The above integral can be interpreted as the diffraction produced by a plane screen with an illumination distribution $T(\underline{r}') A_x(\underline{r}')$. Here $T(\underline{r}') A(\underline{r}')$ has the same value as occurs on the corresponding projected point on the radome. This can also be written as a Fourier transform

$$E_x(P) = \frac{2ki}{4\pi r} \int T(x, y) A_x(x, y) e^{-i(k_x x + k_y y)} dx dy$$

where the z axis is assumed perpendicular to S' . There is a similar expression for $E_y(P)$. Since $\underline{k}$ is nearly in the z direction, $E_x(P)$ and $E_y(P)$ are essentially the wave amplitudes for the two polarizations at the point P .

THE PRINCETON UNIVERSITY RADOME STUDY

III. SHIFT OF CONICAL SCAN CROSS-OVER BY RADOME ERROR. The most general effect of radome error on the antenna pattern which is observed, as one holds a radar antenna fixed and varies the target location, can be described in terms of three separate effects: diminution of maximum signal, distortion in relative shape, and actual shifting of the point of maximum intensity. This break-down is a rather arbitrary one which is made useful primarily by its convenience in describing the effect of radome error on apparent target position.

The apparent target direction for a conical scanning radar is that direction of the scan axis for which the signal received by the radar is the same at all parts of the scan, i.e. has no scan modulation. Looked at in two-dimensional cross-section, for which the scan reduces to an alternation between two antenna directions, the target appears to be at the cross-over of the two patterns composed of received-signal-versus-target-location for the two antenna directions. For a given antenna each of these patterns will depend on whether the radar is active or semi-active, the pattern in the former case being the square of the latter because the transmitted as well as the received signal is involved. But the cross-over point for two semi-active patterns will also be the cross-over point for the square of these patterns, i.e. for the active patterns. Thus to determine the error in target location introduced by radome error we can speak in terms of the pattern which is effective for the particular type of radar, without distinguishing at this stage between active or semi-active. The distinction must, however, be made if one is making a specific numerical calculation.

We shall discuss the effect of radome error upon cross-over in locating the target in one dimension, so that we are concerned with comparing two antenna patterns at scan points 180° out of phase. At each of these points we shall describe the effect of the radome on the antenna pattern (active or semi-active as the case may be) by the following parameters:

- α fractional decrease in maximum intensity
- β fractional increase in width at half power
- δ shift of maximum, in radians

We are now concerned with the cross-over of two patterns characterized by different values, say $\alpha_1, \beta_1, \delta_1$ and $\alpha_2, \beta_2, \delta_2$ of the three parameters; in the absence of radome error, let patterns 1 and 2 be centered at angles respectively $-\phi_{00}$ and ϕ_{00} with respect to the cross-over at $\phi = 0$. Then in the presence of radome error the new cross-over will be shifted by an amount Δ . The sign conventions involved are indicated in Fig. B2, where the undistorted and distorted patterns are indicated by dashed and solid lines respectively.

Since the scan angle ϕ_{00} is usually picked so as to make the cross-over occur relatively near the maximum power point (for example, the 80% power point), we may with sufficient accuracy describe the undistorted beams by the relative intensities

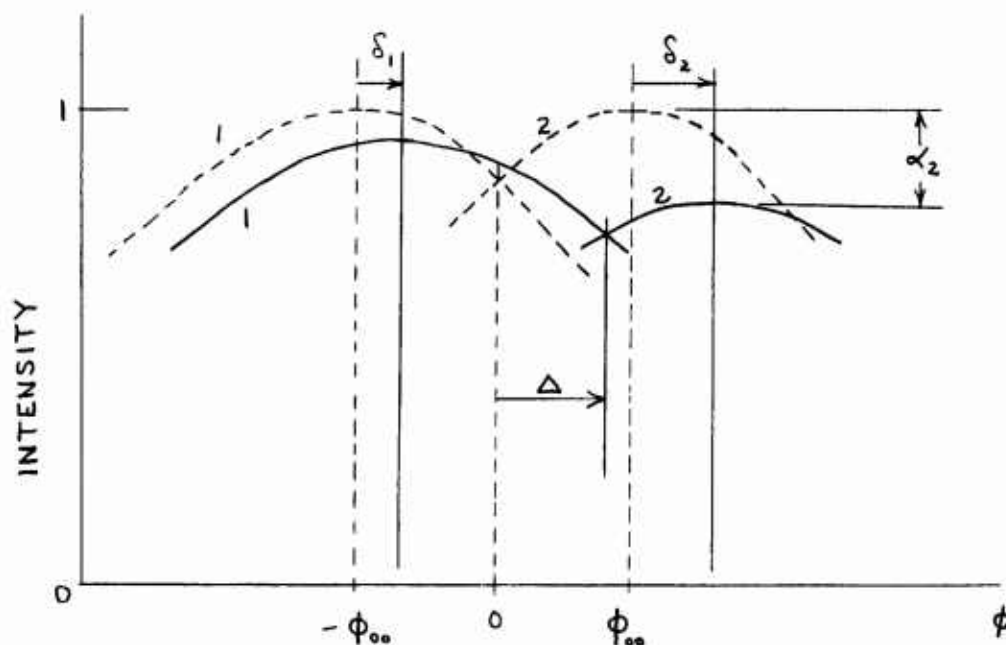


Fig. B2 Sign conventions and partial notation for calculation of shift of cross-over.

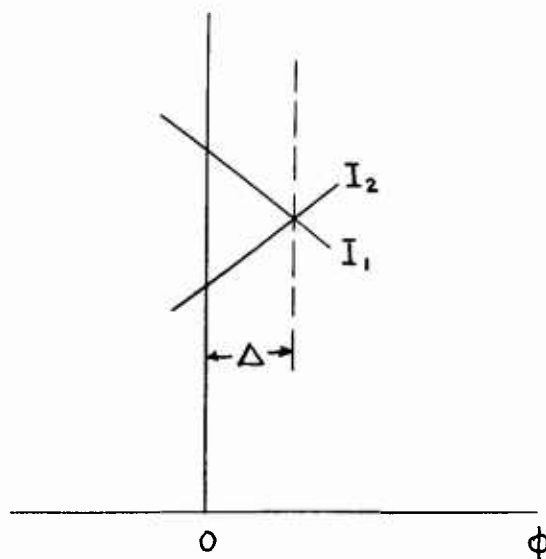


Fig. B3 Relations involved in calculation of Δ .

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

$$I(\phi) = 1 - \frac{1}{2} [(\phi \pm \phi_{00})/\phi_0]^2 \quad \text{Eq. B1}$$

Here ϕ is angle measured across the beam, and $2\phi_0$ is the full width of the beam at half-maximum power.

In terms of the parameters α, β, δ for the two patterns, the two distorted beams are given by

$$I_1(\phi) = (1 - \alpha_1) \left\{ 1 - \frac{1}{2} [(\phi + \phi_{00} - \delta_1)/\phi_0(1 + \beta_1)]^2 \right\} \quad \text{Eq. B2a}$$

$$I_2(\phi) = (1 - \alpha_2) \left\{ 1 - \frac{1}{2} [(\phi + \phi_{00} - \delta_2)/\phi_0(1 + \beta_2)]^2 \right\} \quad \text{Eq. B2b}$$

This statement involves the assumption, which seems to be a good one, that the change in width represented by β is a symmetrical one.

For small distortions we will want to find the cross-over shift, Δ , only to an accuracy of first order in the small parameters. To this accuracy the shift Δ is given by

$$\Delta = \frac{I_1(0) - I_2(0)}{2} \bigg/ \left(\frac{dI_2}{d\phi} \right)_{\phi=0} ; \quad \text{Eq. B3}$$

this is indicated diagrammatically in Fig. B3.

By Eqs. B1, B2, and B3 to first order in α, β, δ we have

$$\Delta = (\alpha_2 - \alpha_1) \left(\phi_{00}^2 / 2\phi_{00} \right) + (\beta_1 - \beta_2) (\phi_{00}/2) + \frac{\delta_1 + \delta_2}{2} \quad \text{Eq. B4}$$

The term in $(\alpha_2 - \alpha_1)$ may be given a more explicit form if we use the information about the cross-over point for which the radar was designed. Suppose that with no distortion the cross-over is at the 80% power point (a common value); then $(\phi_{00}/\phi_0)^2 = 0.40$ or $\phi_{00} = .63 \phi_0$, and we have

$$\Delta = 0.80 \phi_0 (\alpha_2 - \alpha_1) + 0.32 \phi_0 (\beta_1 - \beta_2) + \frac{\delta_1 + \delta_2}{2} \quad \text{Eq. B5}$$

Certain orders of magnitude immediately become apparent from Eq. B5. Thus for a beam 10° wide ($\phi_0 = 5^\circ$), a one percent difference in intensities, $(\alpha_2 - \alpha_1) = 0.01$, gives $\Delta = 0.04^\circ = 0.7$ mils; a one percent difference in beam widths, $\beta_2 - \beta_1 = 0.01$, gives $\Delta = 0.016^\circ = 0.3$ mils.

THE PRINCETON UNIVERSITY RADOME STUDY

IV. THEORETICAL ANALYSIS OF FIRST ORDER EFFECTS. As indicated in section I of this appendix, a first order cause of radome error has its origin in amplitude loss and phase shift produced by a traversal of the radome wall.

Several generalizations concerning such first order effects, which will be deduced shortly, are summarized in this paragraph. A uniform phase shift over the aperture obviously has no effect upon the radiation pattern. A non-uniform distribution of phase shift results in a modification of the beam height, width, and position. Reduction in the transmitted amplitude whether uniform or non-uniform results in a reduction in the height of the main lobe. A non-uniform distribution in the amplitude results also in a variation in the beam width. It is shown below, however, that amplitude variation produces no shift in the beam position; a phase distortion is necessary to produce a beam shift. It must be remembered, though, (see part III of this appendix) that a change in beam height and width may lead to a shift in the cross-over position. But in this connection, we note that several numerical examples are considered later which indicate that generally speaking one should expect the beam deviation to be a more important source of error than the cross-over shift resulting from variations in beam strength or width. Consequently one would expect a phase distortion in percent of a radian to be more serious than a percentage amplitude distortion which is comparable.

It will become apparent to the reader of this section that we have been concerned here with a double purpose: developing methods for analyzing first order radome errors, and carrying out enough analyses to allow a few broad generalizations. The basic theory and the examples which are worked out later thus give results which, in the case of the examples, may be compared with experiment to provide criteria for the accuracy of the method; and results which, since they agree with experiment, can be used to obtain certain generalizations like those summarized in the preceding paragraphs. But we wish to emphasize, before proceeding with details, that the methods developed are as important as the generalizations and should be applied by other workers in much more detail than has been possible in the time available to us.

Fourier Transform Theory of the Antenna Pattern. We now proceed to recapitulate the well-known methods of calculating the antenna pattern in terms of the Fourier transform of the illumination of an aperture which corresponds to the source of radiation. For this analysis we refer to the radome projection theorem of section II, this appendix. Thus the central part of the antenna pattern is given correctly by

$$G(k_x, k_y) = \frac{1}{2\pi} \iint S(x, y) e^{-i(k_x x + k_y y)} dx dy$$

Here $S(x, y)$ is the complex amplitude representing the illumination

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

over the aperture of the "projected radome." The coordinates x, y locate a point on the projection surface; k_x and k_y are the components of the propagation constant of the plane wave component whose amplitude is given by $G(k_x, k_y)$. The notation can be simplified by writing the above formula in terms of the two dimensional vectors $\underline{k}$ and $\underline{r}$ where $d\underline{r}$ refers to an element of area.

$$G(\underline{k}) = \frac{1}{2\pi} \int S(\underline{r}) e^{-i \underline{k} \cdot \underline{r}} d\underline{r}$$

If there is amplitude distortion but no phase distortion, without loss in generality S can be taken as purely real. Then

$$G(-\underline{k}) = \frac{1}{2\pi} \int S(\underline{r}) e^{i \underline{k} \cdot \underline{r}} d\underline{r} = G(\underline{k})$$

$$|G(-\underline{k})| = |G(\underline{k})|$$

The antenna pattern is symmetric about $\underline{k} = 0$. Consequently $\underline{k} = 0$ is the peak of the main lobe. Thus there is no shift in the position of the beam. This is a very important generalization to which we shall make repeated reference.

Further generalizations depend upon making hypotheses as to the dependence, of the phase and amplitude of illumination, on position in the equivalent aperture. Various methods suggest themselves for various circumstances.

Thus when the antenna is looking out through the side of the radome (and in particular, when the beam does not pass too near the nose) the phase and amplitude of illumination may be expected to vary with position quite smoothly, like the leading terms in a power series expansion; and good results may be obtained in terms of a power series expansion of the phase and amplitude of aperture illumination.

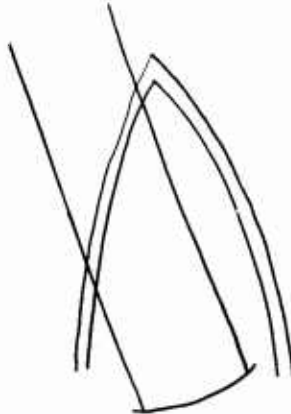
On the other hand, when the beam passes through a pointed nose this entails sudden discontinuities or rapid variations in angle of incidence, insertion phase shift, and reflection, hence a power series procedure only becomes accurate if an uneconomical number of terms are used. One may hope that in this case perhaps it may be possible to replace the single aperture with its discontinuities of illumination by several discrete apertures, within each one of which illumination varies smoothly, but with these several apertures so chosen that the discontinuities of illumination occur at points in their common boundaries.

Lastly, of course, in handling the Fourier integrals one always can have recourse to numerical integration to get results which are difficult otherwise or to check on the more approximate methods.

THE PRINCETON UNIVERSITY RADOME STUDY

In what follows, details of all these procedures except the last will be presented by precept or example, in approximately the above order.

Power Series Expansion Method. First we consider an analytic method for determining the direction of the maximum of a beam resulting from a non-uniformly illuminated aperture, where the arbitrary variation in phase and amplitude is sufficiently smooth and continuous so that it can be represented to high accuracy by a power series expansion with only a few leading terms. It is evident that a combination of the two methods can be applied to a case involving a semi-infinite wedge illustrated in the figure .



Consider a slit, $-a < x < a$, the illumination of which is characterized by an arbitrary amplitude $S(x)$ and arbitrary phase $\phi(x)$. Then at large distances the amplitude is proportional to $G(k)$ where

$$G(k) = \int_{-a}^a e^{-ikx} S(x) e^{-\phi(x)} dx$$

and where

$$k = \frac{2\pi}{\lambda} \sin \theta$$

λ is the wave length;

θ is the angle that the line of observation makes with the normal to the plane of the slit.

Suppose the variations in phase and amplitude are such that it is possible to expand them in power series as follows:

$$\phi(x) = \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \dots$$

$$S(x) = 1 + \beta_1 x + \beta_2 x^2 + \dots$$

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

Then

$$G = \int_{-a}^a e^{ikx} \left[1 + \beta_1 x + \beta_2 x^2 + \dots \right] \left[1 + i\alpha_1 x + i\alpha_2 x^2 + i\alpha_3 x^3 - \frac{\alpha_1^2 x^2}{2} - \frac{\alpha_2^2 x^4}{2} - \alpha_1 \alpha_2 x^3 + \dots \right]$$

Integrating, we find

$$\begin{aligned} \frac{G}{2a} &= \frac{\sin ka}{ka} + \frac{(-\alpha_1 + i\beta_1)}{k} \left[\frac{\sin ka}{ka} - \cos ka \right] \\ &\quad - \frac{1}{k^2} \left(\beta_2 + i\beta_1 \alpha_1 + i\alpha_2 - \frac{\alpha_1^2}{2} \right) \left[(2 - k^2 a^2) \frac{\sin ka}{ka} - 2 \cos ka \right] \end{aligned}$$

Expanding $\sin ka$ and $\cos ka$ we get

$$\begin{aligned} \frac{G}{2a} &= 1 + \frac{a^2}{3} \left(\beta_2 - \frac{\alpha_1^2}{2} \right) - \frac{k\alpha_1}{3} a^2 - \frac{k^2 a^2}{6} - \frac{k^2 \beta_2 a^4}{10} + \frac{k^2 \alpha_1^2 a^4}{20} \\ &\quad + i \left[\frac{k\beta_1}{3} a^2 + \frac{\alpha_2}{3} a^2 + \frac{\beta_1 \alpha_1}{3} a^2 - \frac{k^2 \beta_1 \alpha_1}{10} a^4 \right] \end{aligned}$$

from which

$$\left| \frac{G}{2a} \right|^2 = 1 + \frac{2}{3} a^2 \left(\beta_2 - \frac{\alpha_1^2}{2} \right) - \frac{2}{3} k \alpha_1 a^2 - 2 \frac{k^2 a^2}{6} + \dots$$

To find the beam deviation δ , i.e. the angle of the maximum amplitude, we find the value of k at which $|G|^2$ is a maximum. This occurs when

$$\frac{d}{dk} \left| \frac{G}{2a} \right|^2 = -\frac{2}{3} \alpha_1 a^2 - \frac{2}{3} k a^2 = 0, \text{ i.e., for } k = -\alpha_1.$$

THE PRINCETON UNIVERSITY PADOME STUDY

If we recall $k = \frac{2\pi}{\lambda} \sin \theta = k_0 \sin \theta \doteq k_0 \theta$ then it follows that $\delta = -\alpha_1/k_0$ i.e. the beam deviation is seen to depend, to a first approximation, only on the linear term of the phase expansion.

We note furthermore that in case of uniform phase ($\alpha_1 = \alpha_2 = \dots = 0$) then the parameters $\beta_1, \beta_2, \dots$ which describe the variation of amplitude of illumination do not enter in the coefficients of odd powers of k . This corresponds to the general theorem previously derived at the beginning of this section (Section IV), to the effect that amplitude variations cannot cause beam shifts in the absence of phase variations across the equivalent aperture.

If we wish a somewhat more exact approximation for the beam shift we may obtain an expression accurate to second order terms in α and k as follows:

$$\delta = \frac{\alpha_1}{k_0} \left[1 + \frac{1}{3} \beta_2 a^2 - \frac{1}{2} (\alpha_1 a)^2 - \frac{1}{3} (\beta_1 a)^2 \right]$$

We observe that all the terms in the bracket are of second order, there being no first order terms; hence the approximation $\delta = -\alpha_1/k_0$ is an excellent one. Also note that the parameter α_2 does not enter, that is, that to this order the curvature of the wave front is irrelevant. We also observe that the inclusion, in finding δ , of terms up to second order in k corresponds to ignoring the cubic terms in k which correspond to an asymmetry in the pattern $|G(k)|^2$.

Probably one should use with circumspection the above more complicated expression for δ for the following reason. If the slit is uniformly illuminated by a plane wave, then we have $\beta_1 = \beta_2 = \dots = 0$ and $\delta = -(\alpha_1/k_0) [1 - \frac{1}{2} (\alpha_1 a)^2]$. The leading term here corresponds to the center of the diffraction pattern lying in the direction in which the plane wave was going when it was incident upon the slit; the second order correction represents a deviation of the plane wave from this direction because the plane slit is not quite parallel to the plane wave fronts. The leading term one may expect to hold true for any slit and plane wave; but the higher order term depends markedly upon the assumption of a particular plane slit and would be different for a less simple slit. For our later purposes, for which the "slit" may for example be an antenna dish, or a radome discontinuity, the exact form of the second order terms above cannot be expected to hold true.

The principal relevant conclusions of the foregoing may therefore be summed up:

1) The center of the diffraction pattern of an aperture of any sort is the normal to the plane which most closely approximates the surface of constant phase near the slit.

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

2) Small variations in the aperture illumination, or small curvature in the constant phase fronts, modify the first order beam deflection only in terms of the third order in small quantities. These terms depend too strongly on the particular aperture to be useful.

3) Hence, to terms in second order of small quantities, our results are stated by item 1 above.

"Center of Gravity" Beam Shift Analysis. There will be many cases in which the phase and amplitude of illumination vary fairly continuously over the transmitted beam, but perhaps in a not sufficiently simple manner to make one confident if the power series expansion method when only leading terms in the expansion are considered. In such cases another method is available which will be satisfactory when the shape of the beam is sufficiently near normal that the shift in the "center of gravity" of the beam provides a good measure of beam shift. Clearly this condition is not met in situations in which sizable phase changes occur over small regions in the interior of the projected dish outline; for such localized distortions throw radiation into the side lobes of the antenna pattern and hence affect the average in a disproportionate way.

To find the center of gravity of the antenna pattern we proceed as follows. As in the previous sample calculation the phase shifts will be projected onto a plane perpendicular to the dish axis and the radiation pattern computed from this. Let the illumination pattern be a complex function such that

$$S(\underline{x}, y) = 0 \quad \text{for} \quad r \geq R.$$

As was indicated at the beginning of this section, the antenna pattern is given by the Fourier transform

$$G(\underline{k}) = \frac{1}{2\pi} \int S(\underline{r}) e^{-i \underline{k} \cdot \underline{r}} d\underline{r}$$

The "center of gravity" of the antenna pattern is

$$\underline{\bar{k}} = \frac{\int |\underline{G}|^2 \underline{k} d\underline{k}}{\int |\underline{G}|^2 d\underline{k}}$$

But

$$\begin{aligned} S(\underline{r}) &= \frac{1}{2\pi} \int G(\underline{k}) e^{i \underline{k} \cdot \underline{r}} d\underline{k} \\ \underline{\text{grad}} S &= \frac{1}{2\pi} \int i \underline{k} G(\underline{k}) e^{i \underline{k} \cdot \underline{r}} d\underline{k} \\ \int \bar{S} \underline{\text{grad}} S d\underline{r} &= i \int \underline{k} |\underline{G}|^2 d\underline{k} \end{aligned}$$

THE PRINCETON UNIVERSITY RADOME STUDY

Also
$$\int |S|^2 d\underline{r} = \int |G|^2 d\underline{k}$$

$$\underline{k} = \frac{-i \int \bar{S} \text{grad } S d\underline{r}}{\int |S|^2 d\underline{r}}$$

$\underline{k}$ is purely real.
Writing $S(\underline{r})$ as a product of a real amplitude and a phase factor

$$S(\underline{r}) = S_0(\underline{r}) e^{i\gamma(\underline{r})}$$

$$\bar{S} \text{grad } S = \frac{1}{2} \text{grad } S_0^2 + i S_0^2 \text{grad } \gamma$$

The first term is real and does not contribute a real part to $\underline{k}$.

$$\therefore \underline{k} = - \frac{i \int S_0^2 \text{grad } \gamma d\underline{r}}{\int S_0^2 d\underline{r}} = \frac{\int S_0^2 \text{grad } \gamma d\underline{r}}{\int S_0^2 d\underline{r}}$$

This is the weighted mean of $\text{grad } \gamma$ with S_0^2 as a weighting factor. It should be noted that if there no phase distortion $\text{grad } \gamma = 0$ and $\underline{k} = 0$. Integrating the numerator by parts we find

$$\int S_0^2 \text{grad } \gamma d\underline{r} = - \int \gamma \text{grad } S_0^2 d\underline{r}$$

When $|S_0|^2 = 1 \quad r < R$
 $= 0 \quad r \geq R$

that is, the aperture is uniformly illuminated only phase distortion occurs in the radome, no reflection or absorption, we may write

$$\text{grad } S_0^2 = -\frac{R}{r} \delta(r-R).$$

No contribution is made to the integral except on the periphery of the circle.

$$\underline{k} = \frac{\int \gamma \frac{R}{r} \delta(r-R) d\underline{r}}{\pi R^2} = \frac{\int \gamma(R, \theta) \frac{R}{r} R d\theta}{\pi R^2}$$

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

The x and y components of $\underline{k}$ become

$$\underline{k}_x = \frac{\int \gamma(R, \theta) \cos \theta d\theta}{\pi R}, \quad \underline{k}_y = \frac{\int \gamma(R, \theta) \sin \theta d\theta}{\pi R}$$

Expressed in radians the beam deviation is

$$\delta_x = \frac{\underline{k}_x}{k} = \frac{\int_0^{2\pi} \gamma(R, \theta) \cos \theta d\theta}{\pi R k}$$

$$\delta_y = \frac{\underline{k}_y}{k} = \frac{\int_0^{2\pi} \gamma(R, \theta) \sin \theta d\theta}{\pi R k}$$

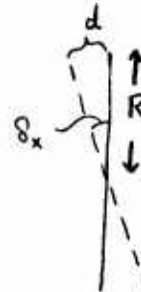
This is a very simple result since only the phase shift at the periphery is needed. This greatly simplifies the labor of detailed numerical computations for a specific radome. To see that the formulas are reasonable when phase shifts across the radome are monotonic a simple example is given.

Example: $\gamma(R, \theta) = a_0 \cos \theta$

$$a_0 = \frac{2\pi d}{\lambda}$$

$$\delta_x = \frac{\int_0^{2\pi} \frac{a_0 \cos^2 \theta d\theta}{\pi R k}}{\pi R k} = \frac{a_0 \pi}{\pi R k} = \frac{d}{R}$$

$$\delta_y = 0$$



This agrees with the result one would expect from swinging the phase fronts through an angle d/R .

Other Methods. The other methods which have been listed, including the replacement of a single aperture with discontinuities by several apertures each one of which individually has no discontinuities, and including a detailed numerical integration of the Fourier integral, require no detailed discussion of principles and will be left to a subsequent working out of detailed examples in the first instance, and will be left as an exercise for the reader in the second case.

The Effect of Phase Distortion on Antenna Gain, Beam Width, and Through These on Cross-Over in Conical Scan. We have already noted that numerical examples to follow will indicate that the displacement of the individual beams seems to be the most important cause of radome error, and all the foregoing theoretical development has been based upon the need to calculate this shift in a single beam. However, as noted in section III, a change in beam shape or height can cause a radome

THE PRINCETON UNIVERSITY RADOME STUDY

error via the mechanism of conical scan, in which the relevant quantity is the location of the cross-over of two adjacent beams. We therefore develop now the general theory of the way in which phase distortion affects beam height and shape. This will be done in a manner easily adaptable to numerical integration for a specific radome (See section Vc) and is therefore supplementary to the treatment of the same question by the power series expansion method in section IV.

The effect of phase distortion on antenna gain and beam width give additional contributions to the radome error in conical scan. Approximation formulas will now be derived which may be used for calculations of the change in these two beam parameters.

For small distortion one can neglect product effects and assume that the effect on the antenna is linear in the distortion. The antenna pattern is

$$G(\underline{k}) = \frac{1}{2\pi} \int S(\underline{r}) e^{-i \underline{k} \cdot \underline{r}} d\underline{r}$$

When no phase distortion occurs $G(0)$ is seen to be a stationary point at the peak of the gain curve and is given by

$$G(0) = \frac{1}{2\pi} \int S(\underline{r}) d\underline{r}$$

Since $\underline{k} = 0$ is a stationary point for the undisturbed pattern the antenna gain is approximately $G(0)$ with an arbitrary small distortion since the peak gain will not move far. A better approximation is $G(\underline{k})$ where $\underline{k}$ is the first moment center of the new pattern as found in the previous section.

$$G(\underline{k}) = \frac{1}{2\pi} \int S(\underline{r}) e^{-i \underline{k} \cdot \underline{r}} d\underline{r}$$

Taking the gradient of G

$$\text{grad } G = 0 = \frac{1}{2\pi} \int (-i \underline{r}) S(\underline{r}) e^{-i \underline{k} \cdot \underline{r}} d\underline{r}$$

Expanding in a power series

$$0 = \frac{1}{2\pi} \int S(\underline{r}) \underline{r} \left(1 - i \underline{k} \cdot \underline{r} - \frac{|\underline{k} \cdot \underline{r}|^2}{2!} + \dots \right) d\underline{r}$$

$$0 = \frac{1}{2\pi} \int S(\underline{r}) i \underline{k} \cdot \underline{r} \left(1 - i (\underline{k} \cdot \underline{r}) - \dots \right) d\underline{r}$$

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

Expanding $G(\underline{k})$,

$$G(\underline{k}) = \frac{1}{2\pi} \int S(\underline{r}) \left(1 - i \underline{k} \cdot \underline{r} - \frac{(\underline{k} \cdot \underline{r})^2}{2!} + \dots \right) d\underline{r}$$

Adding the last two equations gives for $G(\underline{k})$ finally

$$G(\underline{k}) = \frac{1}{2\pi} \int S(\underline{r}) \left(1 + \frac{1}{2} |\underline{k} \cdot \underline{r}|^2 + \dots \right) d\underline{r}$$

A numerical summation can be carried out to evaluate the beam height and from this the relative decrease in height computed.

The width of the main lobe is measured by the two second derivatives since these give the coefficients of the second power term in an expansion about the peak gain. Assume that the symmetry axes of the antenna pattern are k_x and k_y .

$$G_{k_x k_x}(\underline{k}) = -\frac{1}{2\pi} \int S(\underline{r}) x^2 e^{-i \underline{k} \cdot \underline{r}} d\underline{r}$$

$$G_{k_y k_y}(\underline{k}) = -\frac{1}{2\pi} \int S(\underline{r}) y^2 e^{-i \underline{k} \cdot \underline{r}} d\underline{r}$$

Again let x be measured along the direction of antenna offset so that

$$x = \frac{\underline{k} \cdot \underline{r}}{k}$$

The term important to determining beam deviation is

$$G_{k_x k_x}(\underline{k}) = -\frac{1}{2\pi k^2} \int S(\underline{r}) (\underline{k} \cdot \underline{r})^2 e^{-i \underline{k} \cdot \underline{r}} d\underline{r}$$

To a good approximation this may be taken as

$$G_{k_x k_x}(0) = -\frac{1}{2\pi k^2} \int S(\underline{r}) (\underline{k} \cdot \underline{r})^2 d\underline{r}$$

The height of the antenna pattern is

$$G(\underline{k}) = \frac{1}{2\pi} \int |S| \left[1 + \frac{1}{2} (\underline{k} \cdot \underline{r})^2 + \dots \right] [\cos \delta + i \sin \delta] d\underline{r}$$

$$\doteq \frac{1}{2\pi} \sum \cos \delta \Delta x \Delta y + \frac{1}{2\pi} \sum \frac{1}{2} (\underline{k} \cdot \underline{r})^2 \cos \delta \Delta x \Delta y$$

$$\alpha = \frac{G(0) - G(\underline{k})}{G(0)}$$

and the width

$$G_{k_x k_x}(0) \doteq -\frac{1}{2\pi k^2} \sum |S| \frac{1}{2} (\underline{k} \cdot \underline{r})^2 \cos \delta \Delta x \Delta y$$

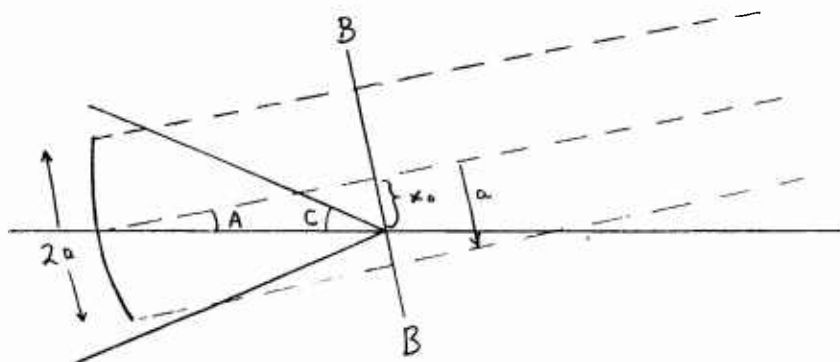
$$\rho = 1 - \frac{1}{k^2} \frac{\sum |S| \frac{1}{2} (\underline{k} \cdot \underline{r})^2 \cos \delta \Delta x \Delta y}{\sum |S| \frac{1}{2} x^2 \Delta x \Delta y}$$

THE PRINCETON UNIVERSITY RADOME STUDY

V. APPLICATIONS OF THE THEORY OF FIRST ORDER EFFECTS. The theoretical development of section IV will now be applied to a series of examples which will serve to illustrate the principles involved; the examples will furthermore give us a better sense of proportion and perspective as to which are the important effects.

The simplest case to treat is any one to which the power series expansion method is applicable; but the really big radome errors occur when the beam is passing through the radome nose, a case to which the power series method is not applicable. Hence we shall treat first several idealized problems which have to do with a beam passing through the nose. To get a feeling for the relative importance of various beam parameters in causing errors in conical scan a two-dimensional problem is tackled first. Although this may seem at first glance to be quite unrealistic the simplicity of the calculations offers a good opportunity to visualize clearly the role played by phase shift alone in producing a shift in the cross-over point. In section Vb radiation through a conical radome is considered for angles of look where the tip of the radome is quite near one side of the beam so that only a small perturbation is added to the undisturbed pattern. An actual radome shape (the Lark) is used in section Vc in applying the "center of gravity" theory of section IV for large angles of look where the beam is free of the nose. Finally analytic methods for handling the problem under special conditions are presented in sections Vd and Ve.

a. Two Dimensional Plane Wedge Radome. To simplify calculations of radome error to the point where the underlying processes become apparent the ideas presented in sections II, III and IV are used to find the beam distortion and resulting shift in crossover point produced by an infinite wedge of dielectric material arranged as shown in the figure below.



The antenna is tipped at an angle of look A such that the upper part of the beam strikes the wedge (of angle $2C$) at an angle of incidence $90^\circ - (C + A)$ and the lower part at an angle of incidence $90^\circ - (C - A)$. The vertex of the wedge is a fraction x_0/a from the beam axis to the edge. According to section II the resulting beam pattern may be found by projecting the insertion phase shifts for the upper and lower portions onto the plane BB and calculating the pattern from there.

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

Let the illumination amplitude $S(x)$ of the upper region $-a < x < x_0$ be given by $T_1 = 1$ and that from the lower region $x_0 < x < a$ by $T_2 = e^{-i\phi}$. This is equivalent to assuming that the illumination of the upper and lower regions are equal in amplitude but different in phase by the angle ϕ . Clearly ϕ is the difference of insertion phase shifts for the two angles of incidence. And clearly this is an example to which the power series expansion method cannot be applied, at least initially, with confidence, because of the abrupt phase discontinuity at the nose of the radome. The procedure, therefore, will be to calculate the diffraction pattern (amplitude) resulting from each sheet individually and then add them up (as amplitudes).

The resulting beam amplitude $G(k)$ is the sum of the amplitudes from the two parts and is given by the Fourier transform of

$$\begin{aligned} G(k) &= \int_{-a}^a S(x) e^{-ikx} dx = T_1 \int_{-a}^{x_0} e^{-ikx} dx + T_2 \int_{x_0}^a e^{-ikx} dx \\ &= T_1 \frac{e^{-ikx_0} - e^{-ika}}{-ik} + T_2 \frac{e^{-ika} - e^{-ikx_0}}{-ik} \\ &= \frac{2T_1}{k} e^{i\frac{k}{2}(a-x_0)} \sin \frac{k}{2}(a+x_0) + \frac{2T_2}{k} e^{-i\frac{k}{2}(a+x_0)} \sin \frac{k}{2}(a-x_0) \end{aligned}$$

The beam intensity I is

$$\begin{aligned} I = |G|^2 &= \frac{4}{k^2} \left[\sin^2 \frac{k}{2}(a+x_0) + \sin^2 \frac{k}{2}(a-x_0) \right. \\ &\quad + T_1 \bar{T}_2 e^{ika} \sin \frac{k}{2}(a+x_0) \sin \frac{k}{2}(a-x_0) \\ &\quad \left. + \bar{T}_1 T_2 e^{-ika} \sin \frac{k}{2}(a+x_0) \sin \frac{k}{2}(a-x_0) \right] \end{aligned}$$

Using the fact that $T_1 \bar{T}_2 = e^{i\phi}$ and $\bar{T}_1 T_2 = e^{-i\phi}$ this becomes

$$I = \frac{4}{k^2} \left[\sin^2 \frac{k}{2}(a+x_0) + \sin^2 \frac{k}{2}(a-x_0) + 2 \sin \frac{k}{2}(a+x_0) \sin \frac{k}{2}(a-x_0) \cos(\phi + ka) \right]$$

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THE PRINCETON UNIVERSITY RADOME STUDY

When this is expanded in a power series in k the resulting expression for I is, to the fourth order in ka ,

$$\frac{I}{4a} = I' = \frac{1 + \cos \phi}{2} + \frac{x_o^2}{2a^2} (1 - \cos \phi) - \frac{ka}{2} \left(1 - \frac{x_o^2}{a^2}\right) \sin \phi \\ - \frac{k^2 a^2}{24} \left[1 + 7 \cos \phi + \left(\frac{x_o^4}{a^4} + 6 \frac{x_o^2}{a^2} \right) (1 - \cos \phi) \right].$$

As a check this is seen to give the familiar Fraunhofer diffraction pattern for $\phi = 0$:

$$I' = \frac{\sin^2 ka}{ka^2} = 1 - \frac{k^2 a^2}{3} + \dots$$

The equation for the intensity is of the form

$$I' = A - \frac{B}{2} (k - \underline{k})^2$$

where the parameters A , B and $\underline{k}$ are given by

$$B = \frac{a^2}{12} \left[1 + 7 \cos \phi + \left(\frac{x_o^4}{a^4} + 6 \frac{x_o^2}{a^2} \right) (1 - \cos \phi) \right]$$

$$\underline{k} = -\frac{a}{2B} \left(1 - \frac{x_o^2}{a^2} \right) \sin \phi$$

$$A = \frac{B}{2} \underline{k}^2 + \frac{1 + \cos \phi}{2} + \frac{x_o^2}{2a^2} (1 - \cos \phi)$$

These are related to the quantities α , β , δ used in section III for describing the error in conical scan in the following way

$$\alpha = \frac{A_o - A}{A_o}, \quad \beta = \frac{B_o - B}{B_o}, \quad \delta = \frac{k}{\underline{k}}$$

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

where the subscript ϕ_0 denotes the value when the relative phase shift ϕ is zero. The half power point ϕ_0 is found to be

$$\frac{1}{2} = \frac{B_0}{2} k^2 = \frac{a^2}{3} \frac{\phi_0^2}{\lambda^2}, \therefore \phi_0 = \sqrt{3} \frac{\lambda}{a}$$

A numerical solution has been carried through for a 40° wedge with a dish of width $2a = 9.75''$ radiating 3.2 cm. power. Insertion phase shifts are taken for a single dielectric sheet designed to give low reflection up to high angles of incidence. For seven angles of look values are given in the table for the angles of incidence on the upper and lower surfaces, the relative insertion phase shift corresponding to these angles, the location of the wedge vertex and the computed values of $\alpha \beta \delta$

Angle of look, degrees	0	2.5	5	7.5	10	12.5	15
Angle of incidence on upper surface, degrees	70	67.5	65	62.5	60	57.5	55
Angle of incidence lower surface, degrees	70	72.5	75	77.5	80	82.5	85
Relative phase shift degrees	0	6.70	13.6	18.8	22.6	25.6	-
λ_0/a	0	.171	.342	.503	.684	.837	1
α percent	0	0.10	1.00	1.50	1.20	1.12	0
β percent	0	0.60	2.50	3.67	4.05	3.30	0
δ radians $\times 10^3$	0	-6.60	-7.19	-9.77	-12.5	-7.82	0

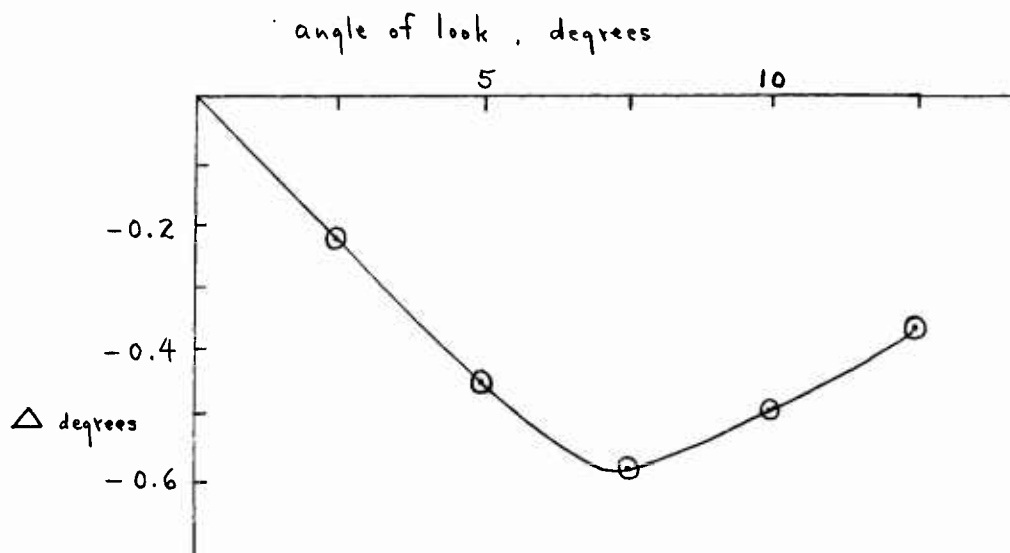
Errors in conical scan can be computed by taking the differences of $\alpha \beta \delta$ for two of the angles of look in the table. For example, if the squint angle $2\phi_0$ is assumed to be 5° , the formula giving the radome error Δ is

$$\Delta = 0.074(\alpha_{n+1} - \alpha_{n-1}) - 0.0218(\beta_{n+1} - \beta_{n-1}) + \frac{\delta_{n+1} + \delta_{n-1}}{2}$$

THE PRINCETON UNIVERSITY RADOME STUDY

The contributions of each of the three terms at five angles of look are given in the table below and the resulting shift in cross-over point plotted in the graph.

Angle of look degrees	2.5	5	7.5	10	12.5
$.0724(\alpha_{m+1} - \alpha_{m-1}) \times 10^3$	.724	1.013	.145	-.275	-.869
$-.0218(\beta_{m+1} - \beta_{m-1}) \times 10^3$	-.574	-.670	-.338	.081	.883
$\frac{\delta_{m+1} + \delta_{m-1}}{2} \times 10^3$	-3.60	-8.18	-9.84	-8.84	-6.25
$\Delta \times 10^3, \text{ radians}$	-3.43	-7.84	-10.03	-8.60	-6.24



In this table the deviation of the cross-over Δ expressed in radians is given as a function of look angle. The 2nd row from the bottom of the table represents the contribution of beam bending to the radome error Δ . It is evident that it is by far the most important part.

It is concluded that insofar as the above example is relevant, beam displacement is more important than beam distortion. Furthermore, although the calculation was made for a phase distortion, and amplitude distortion of the same magnitude in percent would produce a comparable degree of beam distortion but no beam displacement. Hence to the extent that the above example is typical, phase distortion is considerably worse than a comparable amount of amplitude distortion.

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

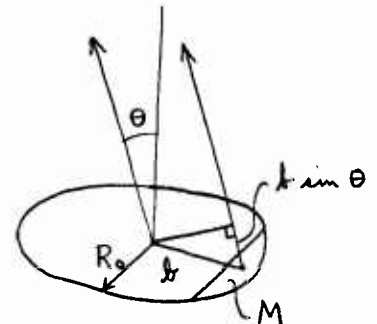
b. Radiation From A Circular Aperture With Two Regions Of Differing Phase. Methods similar to those used in the previous section will now be applied to the problem of a circular dish looking through a radome at such an angle that the tip is near one edge of the beam. For this case the angles of incidence are not as easy to find as in the two-dimensional cases just discussed but may be developed by projective geometry. An example of the method is given in the next part. The result for a 43.5° conical radome is shown in Fig. B4. It may be seen that angles of incidence from 52° to 84° are encountered in this case.

It should be noted that strictly speaking one should consider the two principle polarizations at each point on the radome, phase shift each one and then recombine after passing through the radome. It seems to be generally true, however, that the difference between the phase shifts of the two principle polarizations is fairly small compared with the phase shifts variations themselves. Hence phase shift differences may be neglected in a first approximation. A single phase shift is assumed for both polarizations.

After projecting the corresponding phase shifts as before, an approximate calculation of the beam deflection may be made by considering the total radiation as coming from two sources, that from the circular aperture and that in a different phase β from the small area M where the angles of incidence are high. For purposes of this calculation we are thus replacing the conical radome by two equivalent sources, just as in the preceding discussion of the two dimensional wedge; but of course the accuracy will not be expected to be as good as in the previous case. Nevertheless it will be interesting to see how closely this crude model gives results in recognizable accord with experiment.

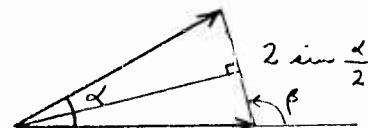
The Fraunhofer pattern for a circular aperture is given by

$$G_1(\theta) = \frac{2\lambda}{R \sin \theta} J_1\left(\frac{R \sin \theta}{\lambda}\right)$$



where J_1 is the Bessel function of the first kind. To this must be added a contribution from M which will have nearly constant amplitude over the region of interest as long as $M \ll \pi R_0^2$. This amplitude is

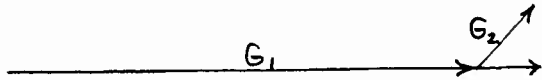
$$G_2 = 2 \frac{M}{\pi R_0^2} \left| \sin \frac{\alpha}{2} \right|$$



where α is the insertion phase difference between the two regions,

THE PRINCETON UNIVERSITY RADOME STUDY

and its phase is seen to be $\beta = \pi/2 + \alpha/2$ plus the change
in phase due to a longer path length: $\frac{b \sin \theta}{\lambda}$. In adding
this to G_1 only the component in the direction of G_1 need be
considered because $|G_2| \ll |G_1|$;



The total amplitude thus becomes

$$G = G_1(\theta) + \frac{2M}{\pi R_0^2} \left| \sin \frac{\alpha}{2} \right| \cos \left(\beta + \frac{b}{\lambda} \sin \theta \right).$$

with an 8.75" dish and x-band radiation $\frac{R_0}{\lambda} = 21.8$

and for small values of θ , $G_1(\theta) \doteq 10.5(1 - \theta^2)$

Then

$$G = 10.5(1 - \theta^2) + \frac{2M}{\pi R_0^2} \left| \sin \frac{\alpha}{2} \right| \cos \left(\beta + \frac{b}{\lambda} \theta \right)$$

The shift in the peak obtained by setting $\frac{\partial G}{\partial \theta} = 0$
and solving for θ is

$$\Delta \theta = \frac{\frac{M}{\pi R_0^2} \left| \sin \frac{\alpha}{2} \right| \frac{b}{\lambda} \sin \beta}{10.5 + \frac{M}{\pi R_0^2} \left| \sin \frac{\alpha}{2} \right| \left(\frac{b}{\lambda} \right)^2 \cos \beta}$$

The calculation was carried out taking 58° and 83° as average angles of incidence for two regions of Fig. B4 and for two other angles of look using guess-values based on this same figure. Phase shifts were taken from parallel polarization curves for an inverted sandwich, $\epsilon_{\text{skin}} = 3$, $\epsilon_{\text{core}} = 9$. * The results are shown in the table and plotted in the figure.

Angle of look	11.1°	15°	17.3°
Relative phase shift	34.9°	41.3°	41.8°
Deflection	-5.7°	-3.2°	-1.4°

* Private communication from E. O. Hartig, Goodyear Aircraft Corporation

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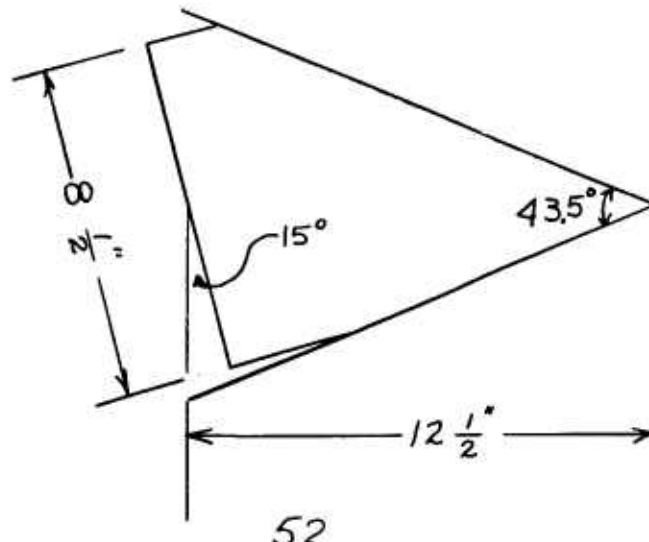
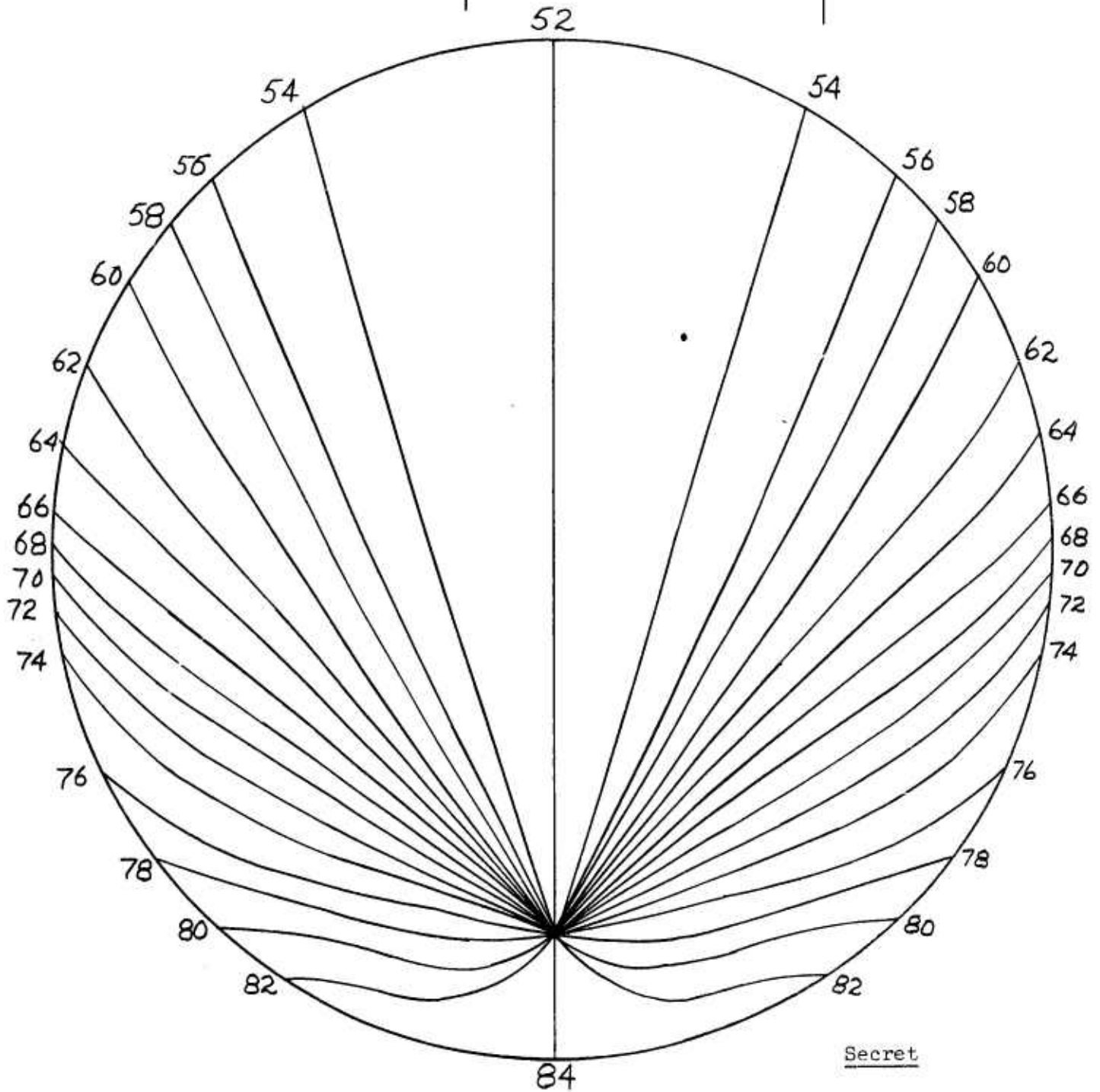


FIG. B4

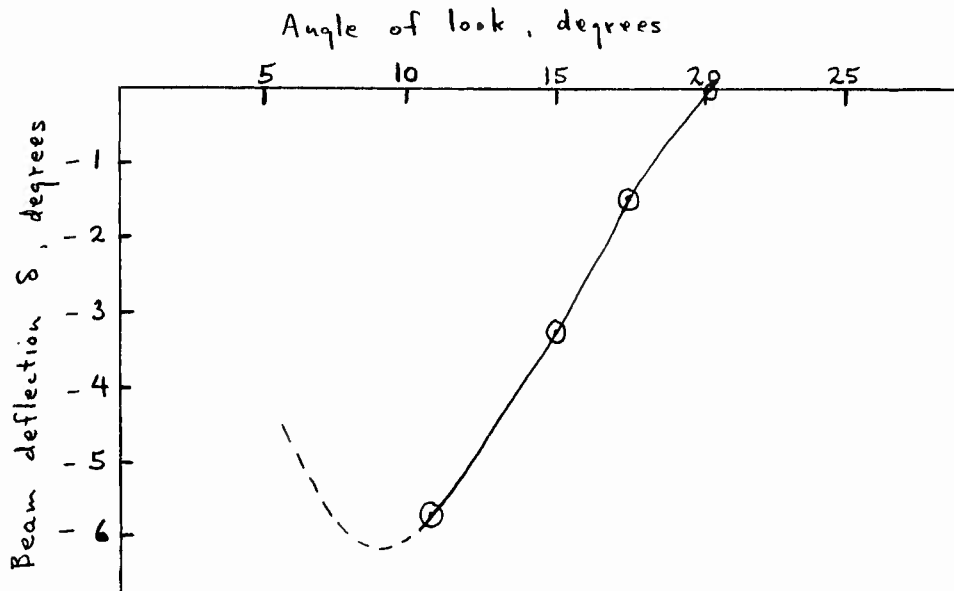
Angles of
Incidence on
Conical Radome



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AN ANALYSIS OF THE SOURCES OF RADOME ERROR

Deflection of Center of Beam due to Phase Shift



The salient feature of the result is that radome errors as large or larger than those observed experimentally can arise solely through the mechanism of phase shifts. Thus a radome that is well-designed so far as low power absorption and reflection are concerned may still give very poor performance if phase relations are poor.

c. Application of Center-of-Gravity Theory to Detailed Analysis of a Specific Radome (Lark Radome). The preceding two examples have served to emphasize the importance of phase effects. The first example concerned an accurate calculation for an unrealistic (two-dimensional) radome, while the second involved a crude approximation applied to an actual radome. In the present section we apply to an actual radome the center-of-gravity theory of beam displacement, using detailed numerical data concerning insertion phase shift in order to obtain predicted radome error. The effects of phase distortion alone are included.

The center of gravity theory is certainly suitable for angles of look great enough for the beam to be free of the nose. In the case of Lark this should apply to all angles of look greater than about 17° . Numerical values are computed for one angle of look, 17° . As in the

THE PRINCETON UNIVERSITY RADOME STUDY

previous calculation for a conical shape the wall is assumed to be an inverted sandwich construction with $\epsilon_{\text{skin}} = 3$ and $\epsilon_{\text{core}} = 9$. As was indicated previously, shifts for such a sandwich differ for parallel and perpendicular polarization and a complete solution would require knowing the polarization of the radiation incident on the radome. Because of the generally small difference in insertion phase for the two polarizations, it is again assumed for simplicity that the radome properties are independent of polarization.

To determine phase shifts at any point on the projected radome one must first determine the angle of incidence and the polarizing angle at the corresponding point on the real radome.

The angle of incidence is defined as the angle between the direction of the ray and normal to the radome surface at the point where the ray meets the surface.

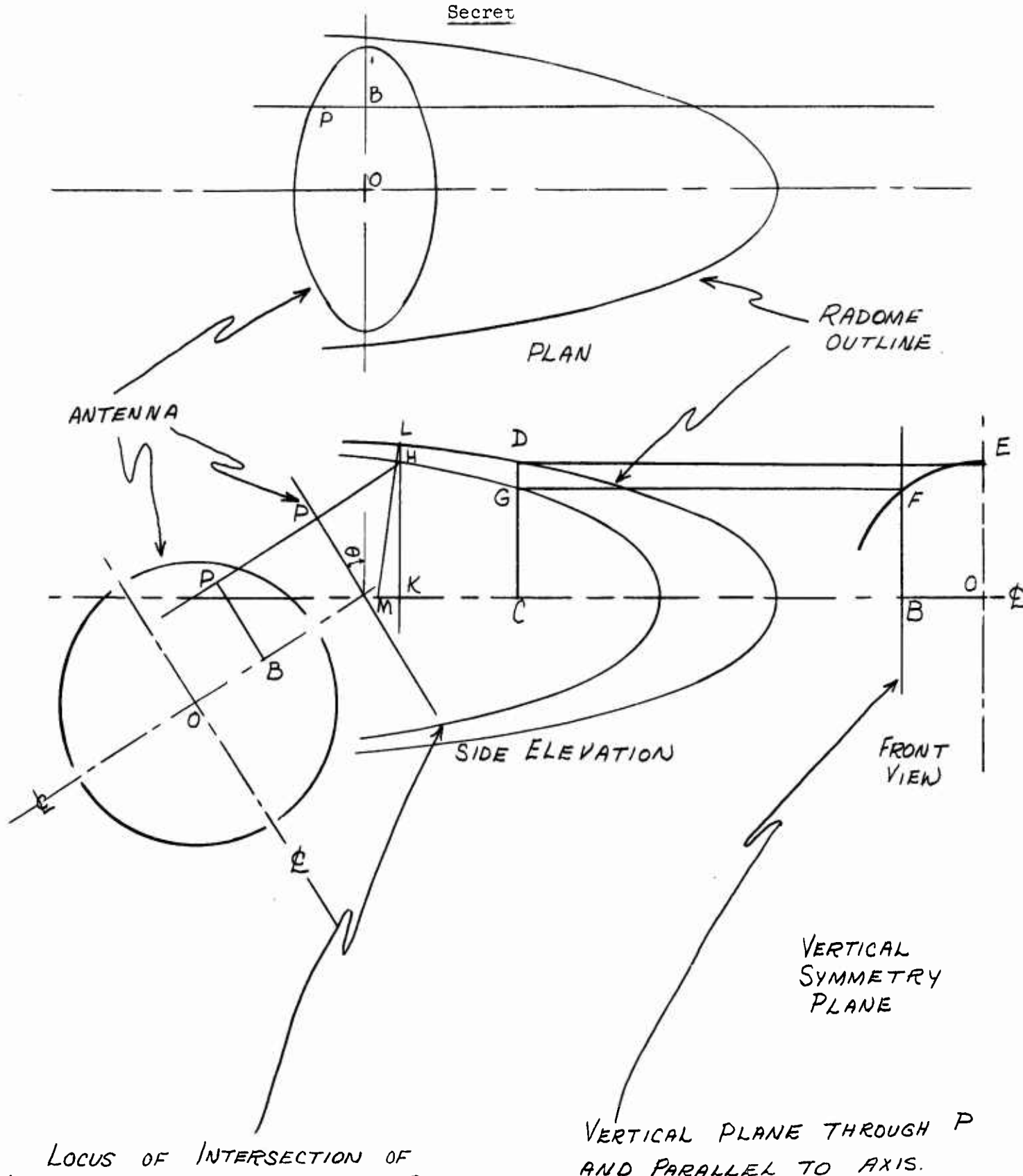
The polarizing angle is the angle between the plane of incidence (containing the ray and the normal) and the plane containing the $\mathbf{E}$ -vector.

The angles in a specific case may be determined as follows. It is assumed that the radome surface is a surface of revolution around the missile axis.

In Fig. B5 there is shown in the upper part of the drawing a plan view of the radome. The lower part shows a side elevation of the radome with a front view on the right hand side and a normal view of the antenna on the left hand side.

To find where a ray from a point P on the antenna strikes the radome we determine first the locus of intersection of a vertical plane through P parallel to the radome axis with the radome surface. To do this take any point C on the radome axis and erect a perpendicular CD meeting the radome at D . Draw DE parallel to the radome axis to meet the vertical plane of symmetry at E in the front view. With O as centre and OE radius draw an arc intersecting the vertical plane through P at F . Draw FG parallel to the axis to meet DC at G . G is a point on the required intersection as seen in the side elevation. The complete curve may be plotted by taking a series of points C along the axis.

A ray from a point P on the antenna meets this curve (and the radome surface) at H as shown. Draw LHK perpendicular to the radome axis meeting the axis at K and the outline of the surface at L . Draw a normal to the outline at L to meet the axis at M . Then M is the point at which all normals to the surface in the plane of KHL intersect the axis, and LM in the figure is the true distance from H to M .



LOCUS OF INTERSECTION OF
VERTICAL PLANE THROUGH P
PARALLEL WITH AXIS WITH
THE RADOME SURFACE.

VERTICAL PLANE THROUGH P
AND PARALLEL TO AXIS.

FIG. B5 DIAGRAM SHOWING HOW TO DETERMINE ANGLE
OF INCIDENCE OF ANY RAY.

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AN ANALYSIS OF THE SOURCES OF RADOME ERROR .

The direction cosines of the normal at H are then $\left(\frac{MK}{ML}, \frac{OB}{ML}, \frac{KH}{ML} \right)$ where the x direction is along the radome axis, the y direction is into the paper and the z direction lies in a vertical plane through the axis. Let these be denoted by (l, m, n) .

The direction cosines of all rays from the antenna are $(\cos \theta, 0, \sin \theta)$ so that the cosine of the angle of incidence is $l \cos \theta + n \sin \theta$. Similarly the angle of incidence can be determined for all other points on the antenna.

The direction cosines (l', m', n') of the normal to the plane of incidence are given by

$$ll' + mm' + nn' = 0$$

$$l l' \cos \theta + n n' \sin \theta = 0$$

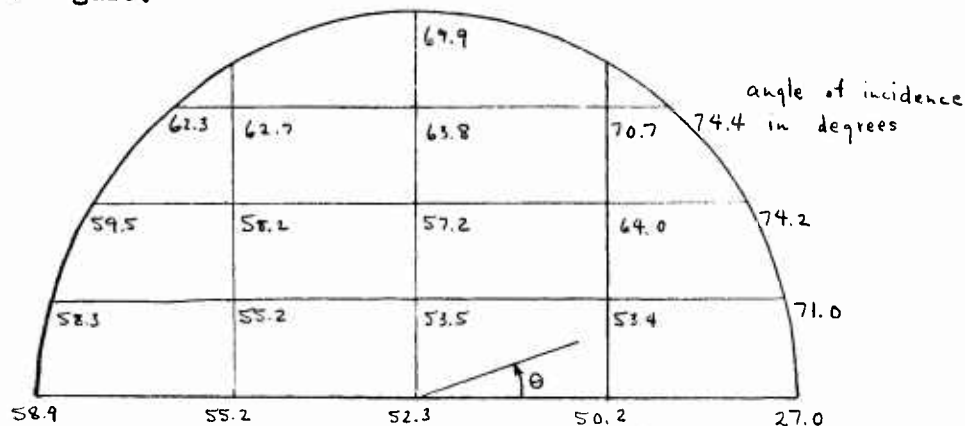
$$l'^2 + m'^2 + n'^2 = 1$$

from which l', m', n' can be easily computed.

If the E vector lies in the plane of the paper the direction cosines of a normal to the plane are $0, 1, 0$ and the cosine of the polarizing angle is m' . For more general cases where the plane of polarization may not lie in this plane the polarizing angle can be almost as easily computed.

Calculations similar to those above have been carried out for a conical radome in a report entitled "Design of Radome for P/A-VII Pilotless Aircraft." by the U.S. Naval Air Development Centre at Johnsville, Pennsylvania. The conical form allows use of the equation for the surface and the graphical method described above is not necessary.

For a Lark radome at an angle of look of 17° the angles of incidence were found by construction at a number of points. These are shown in the figure:



For this case the electromagnetic theory developed earlier in this section is applicable since the phase shift changes monotonically across the aperture. Interpolated values at 15° intervals around the edge were found and the insertion phase shifts taken from the graph for parallel polarization

Secret

THE PRINCETON UNIVERSITY RADOME STUDY

and the inverted sandwich already mentioned. Since an arbitrary phase angle is present anyway the same value was subtracted from all phase shifts to make the calculation a bit simpler. The resulting values are given in the table below along with the product $\gamma \cos \theta$ needed for calculating the shift in center of gravity of the beam.

θ Degrees	Relative Phase Shift Radians	$\cos$
7.5	.03	.030
22.5	.06	.055
37.5	.12	.095
52.5	.19	.116
67.5	.29	.111
82.5	.38	.050
97.5	.47	-.062
112.5	.56	-.214
127.5	.64	-.390
142.5	.64	-.508
157.5	.58	-.537
172.5	.09	-.089

For a dish having a radius of 11 cm. and 3.2 cm. waves the beam deviation is

$$\delta = \frac{k_r}{k} = \frac{2 \frac{\pi}{12} \sum \gamma \cos \theta}{\pi R k} = \frac{2 \times \frac{\pi}{12} \times 1.543}{\pi \times 11 \times 1.985} = 0.68^\circ$$

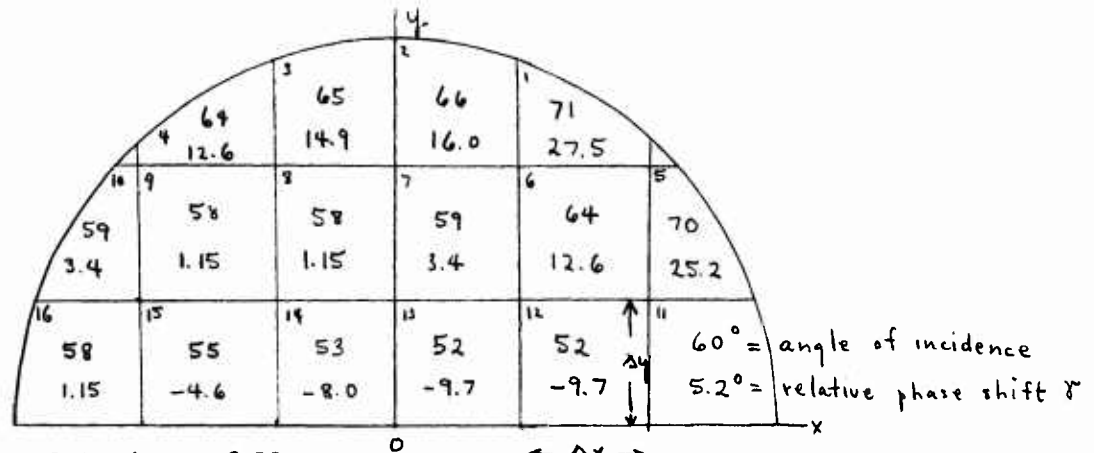
This is the same direction and magnitude as errors actually measured on well designed radomes of the Lark type.

The foregoing analysis of the bending of a single beam by this radome may be supplemented by applying the results of section IV on the effect of phase distortion in beam height and width and hence in conical scan crossover.

The required integrations over the circular aperture were done by dividing the circle into (nearly) equal sections and summing the contributions from all of these, neglecting the slight variation in area and completely omitting two small corners. As in the calculation for beam shift, angles of incidence were interpolated for the center of each block. The resulting angles and relative phase shifts are given on the figure:

Secret

AN ANALYSIS OF THE SOURCES OF RADOME ERROR



Numerical data is as follows:

Area	$\cos Y$	Center of Gravity, x cm.	$\frac{1}{2}(kx)^2 \times 10^3$	$\frac{1}{2}(kx)^2 \cos Y \cdot 10^3$
1	.976	5.50	8.32	8.12
2	.966	1.83	0.926	0.88
3	.961	-1.83	0.926	0.88
4	.887	-5.50	8.32	7.40
5	.998	9.17	23.16	23.12
6	1.000	5.50	8.32	8.32
7	1.000	1.83	.926	0.92
8	.998	-1.83	.926	0.92
9	.976	-5.50	8.32	8.12
10	.905	-9.17	23.16	20.96
11	1.000	9.17	23.16	23.16
12	.997	5.50	8.32	8.32
13	.990	1.83	.926	0.92
14	.986	-1.83	.92	0.88
15	.986	-5.50	8.32	8.80
16	.996	-9.17	23.16	23.80
15.622			145.6	

These numbers give, when used in the last equation of section IV, the following results. For the fractional decrease, α , in the beam height:

$$\alpha = \frac{G(0) - G(\frac{k}{2})}{G(0)} = 1 - \frac{15.622 + .146}{16} = .0145 = 1.45\%.$$

And for the fractional increase in width, $\beta = 1.1\%$.

Since the values of α and β are the order of a percent their contributions to radome error are small compared to that from the beam deviation δ . This same conclusion was reached in part a) for a two-dimensional radome.

THE PRINCETON UNIVERSITY RADOME STUDY

In conclusion it may be said that the sample calculations carried out in parts a), b), and c) are illustrative of an approach to the origin of radome error whereby parts of the total radiation problem are studied individually. Enough simplifying assumptions are made to clarify the contribution of recognizable factors to the total beam pattern and also to make calculations relatively simple.

The importance of phase shifts in accounting for radome errors has been amply illustrated by these results and should serve as a guide in forming a compromise between the goals of complete elimination of power reflection and complete matching of phase shifts over the radome. For this purpose beam deflection seems to be the most important effect and phase distortion is needed to provide beam deflection.

d. Radome Error Due to Smooth Taper and Curvature. Smooth taper and curvature are encountered when the beam looks out the side of the radome. This is not a region of quite as much interest as the region near the nose, since observed errors here are smaller. However, this whole question of smooth taper and curvature becomes important in the discussion in Appendix A of the possibility of a short range solution to the radome problem by the use of radomes with reproducible and hence compensatable error; for if such a solution is attempted, the small deviations from standard radome thickness produce small deviations from standard radome error, and these small deviations from the standard radome will usually be smooth and unlocalized. Hence their effects are both important and amenable to treatment by the power series expansion method of section IV.

For this purpose we therefore consider the effect on the beam of a thin sheet having both taper and curvature. Some initial comments on the relation of the accurate physical optics diffraction picture of this process and the geometric ray tracings picture are in order.

A closer look at the influence of smooth taper and curvature on transmitted radiation should be taken since such variations are encountered by the beam at all angles of look.

A bounded plane wave, such as that issuing from the antenna dish, does not change direction on passage through a sheet of homogeneous dielectric which is bounded on two sides by plane parallel surfaces. If, however, the sheet is bounded by two non-parallel planes it constitutes a simple prism and deflection will result. Similarly, if the sheet has uniform thickness but is curved as a whole, then from the geometric optics point of view the normals to the boundaries of the sheet are not in the same direction for a given ray when it enters and emerges from the sheet; for these rays the sheet also acts like a prism and gives a deflection. But we must always beware of using without due circumspection this geometric optics ray picture. As is discussed in section BIV, the important thing is the behaviour of the diffracted beam at infinity; in determining this behaviour all parts of the beam take part in a way best expressed not in terms of the emergent directions of various rays, but rather in terms of the insertion phase shift and beam

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

attenuation caused by the radome's presence.

In the case of smooth variation of the insertion phase shift- which is the case with which we are concerned here- it has been shown that the first approximation to the beam deflection is obtained by considering only the linear term in the variation of the phase shift across the beam - that is, by representing the radome influence in terms of the average value, across the beam, of the first derivative of phase shift with respect to position across the beam. To the same degree of accuracy as that to which the presence of the radome is represented by this first derivative of phase, the wave front on the emergent side of the radome will consist of plane surfaces of constant phase; the normal to the emergent phase planes will have been wheeled around with respect to that of the incident wave by an angle equal to the total radome insertion phase shift, (measured in units of length) across the beam width, divided by the beam width.

To this same degree of accuracy of representing the radome effect, an incident bundle of parallel rays gives a parallel bundle of emergent rays; on both sides of the radome the rays are perpendicular to the (assumed) plane surfaces of constant wave phase. Therefore to this degree of accuracy - linear variation of insertion phase shift across the beam - the geometric optics ray picture will give the same answer as the diffraction method. By "smooth" taper and curvature in this section we mean "smooth enough so that only the linear variation of phase need be considered" - hence we may go ahead and use the ray picture if we wish. For any specific values of the various derivatives of insertion phase shift across the beam, we may refer to the more general discussion in sections BIV and BV to see the magnitude of the errors which our linear approximation has introduced.

For sufficiently small tapers and thicknesses the beam deflection introduced will be linear in the taper and the thickness, with cross-product terms being of higher order, hence we shall calculate each of these two effects independently and then add them up.

Let us consider first, therefore, the situation indicated by Fig. B6 where we have a dielectric bounded by 2 planes making an angle with each other. Let us consider this first from the ray point of view. A beam of light enters the material (assume to have index of refraction, n) at the angle of incidence θ_1 and leaves in a direction which makes an angle θ_4 with that normal with respect to which θ_1 is measured. Because the two faces of the slab are not parallel, $\theta_3 \neq \theta_1$; and in fact the deflection of the beam in passing through the material, call it δ , is given simply by $\delta = \theta_3 + \alpha - \theta_1$. This deviation δ corresponds to the radome error introduced by the sheet of material.

By Snell's law,

$$\sin \theta_1 = n \sin \theta_2$$

$$\sin \theta_3 = n \sin (\theta_2 - \alpha)$$

THE PRINCETON UNIVERSITY RADOME STUDY

from this there follows, after some manipulation:

$$\delta = \alpha \left[1 - \sqrt{(n^2 - \sin^2 \theta_1) / (1 - \sin^2 \theta_1)} \right] \quad \text{Eq. B6}$$

$$\left(\frac{\partial \delta}{\partial \theta_1} \right)_{\alpha} = -n \alpha \sin \theta_1 / \cos^3 \theta_1 \quad (n^2 \gg 1) \quad \text{Eq. B7}$$

In both these expressions only linear terms in α have been included since we shall be interested in the region $\alpha \approx 0$; and in Eq. B7 a further simplification of a complicated expression has been introduced by assuming $n^2 \gg 1$. This same assumption converts Eq. B6 to read

$$\delta = -\alpha n / \cos \theta_1 \quad (n^2 \gg 1) \quad \text{Eq. B8}$$

For the situation of Fig B6 the insertion phase shift is seen to be given by

$$\phi = \frac{D}{\cos \theta_2} (n - \cos(\theta_1 - \theta_2)) \quad \text{Eq. B9}$$

where D is the local thickness of the sheet; ϕ is dependent on α only through the derivatives of ϕ . In particular, since $dD/ds = \alpha$ where s is distance measured along the radome surface, then for plane surfaces

$$\frac{\partial \phi}{\partial s} = \alpha \frac{d\phi}{dD} = \frac{\alpha (n - \cos(\theta_1 - \theta_2))}{\cos \theta_2}$$

For $n \gg 1$ this becomes $\partial \phi / \partial s \approx \alpha n$. For a beam of width l , the total phase shift across the beam is given by

$$\Delta \phi = \frac{\partial \phi}{\partial s} \frac{l}{\cos \theta_1} = \frac{\alpha n l}{\cos \theta_1}$$

and the resultant beam deflection δ is given by

$$\delta = \frac{-\Delta \phi}{l} = -\frac{\alpha n}{\cos \theta_1}$$

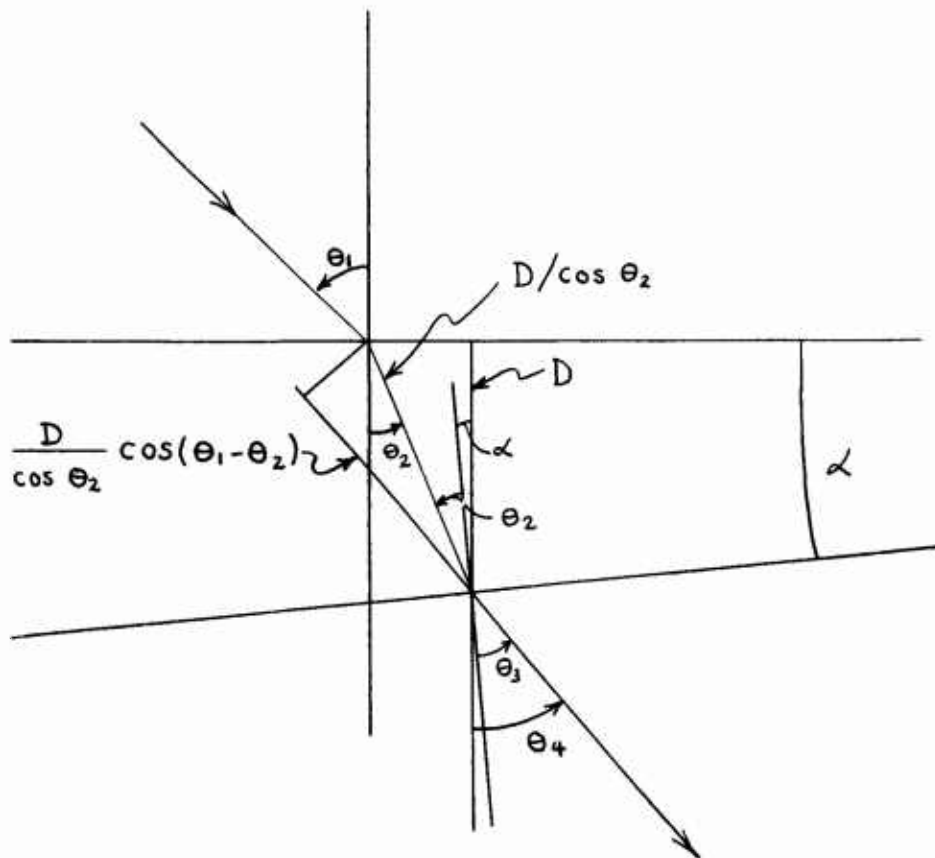


Fig. B6

Geometry for Tapered Sheet of Dielectric

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

which agrees with the result of Eq. B8. By our general considerations we know that to the present approximation it should agree; the exhibition of the agreement has been given merely as an illustrative example.

Effect of Curvature with Non-Zero Thickness. Let us consider now the question of beam deflection introduced by the curvature of a slab of uniform thickness. To the degree that the curvature is small enough for a parallel bundle of incident rays to give a parallel bundle of emergent rays we may use the ray picture, this time not bothering to reproduce the same result with the phase-shift method.

In Fig. B7 is shown a sheet of thickness D and radius of curvature R . It is apparent that the indicated ray makes a different angle with the first and second surfaces, just as if it were passing through a sheet bounded by two plane surfaces making an angle

$$\alpha = \frac{D \tan \theta_2}{R}$$

The resulting beam deflection when $n^2 \gg 1$ is given by

$$\delta = -\frac{D}{R} \tan \theta_1 \quad (n^2 \gg 1) \quad \text{Eq. B10}$$

The final result, giving the combined effects of taper and curvature, is obtained by combining the results of Eqs. B8 and B10:

$$\delta = -\frac{1}{\cos \theta_1} \left(\alpha n + \frac{D}{R} \sin \theta_1 \right) \quad (n^2 \gg 1) \quad \text{Eq. B11}$$

The above considerations have referred to a cylindrical (i.e. two-dimensional) radome rather than the radomes of axial symmetry which are actually used. To the degree that the radome has axial symmetry, the insertion phase shift will have a linear variation in only one direction and these considerations will still give a good representation of the facts.

It is interesting to note that if $\alpha < 0$, that is, if the geometry is as indicated in Fig. B8, then over a range of θ_1 one may make the effects of taper and curvature cancel each other. This may have some possibilities in radome design; to check further on this point one would need to investigate, via the phase shift and diffraction method, the influence of the higher order terms which have been neglected when in Eq. B11 we write $\sin \theta_1$, measuring the value of $\sin \theta_1$ at the center of the beam. The linear variation of $\sin \theta_1$ across the beam, which corresponds to a quadratic variation of ϕ , will not affect ϵ ; but higher terms will.

THE PRINCETON UNIVERSITY RADOME STUDY

A perhaps more relevant point concerning this discussion is the fact that when the beam is pointing through the nose of the radome, the angle of incidence θ_1 has very marked variation and the "smooth" approximation breaks down.

One further comment; the important thing in radome error is not the absolute error but rather the error slope K , that is, the derivative of ϵ with respect to angle between antenna axis and radome axis. This angle does not appear in any of the immediately foregoing equations, but when it changes θ_1 will change by the same order of magnitude and the parameters α and D may change. Some indication of the size of K may therefore be obtained by calculating $\partial \delta / \partial \theta_1$ from Eq. B11:

$$\frac{\partial \delta}{\partial \theta_1} = \frac{1}{\cos^2 \theta_1} \left[\alpha n \sin \theta_1 - \frac{D}{R} \right] \quad \text{Eq. B12}$$

Thus $\partial \delta / \partial \theta_1$ increases as θ_1 approaches $\pi/2$. At $\theta_1 = 70^\circ$, $\partial \delta / \partial \theta_1 \approx 9 (\alpha n - D/R)$, if $D/R = .02$, $n = 1.5$ and $\alpha = 10^{-2}$, $\partial \delta / \partial \theta_1 \approx .13-.18$. Thus whether or not one tries to bring about at all points the partial cancellation apparent here, it is clear that one should at least attend to having opposite signs in the two contributions to $\partial \delta / \partial \theta_1$ and hence to K .

Effect of Radome Shape. The remainder of section V of this appendix considers briefly some of the effects connected with the shape of the radome. As was mentioned earlier and will be seen in detail in Appendix H, when the antenna polarization is symmetric with respect to the missile axis, radome error is a function of only two parameters, the look angle and the polarization of the incoming radiation. Furthermore in an active system, assuming negligible target depolarization (an assumption which may not be borne out in fact) the radome error is a function of the look angle only, also under certain circumstances even for the semi-active system. The radome error may be an insensitive function of polarization. In these cases the variation of but a single parameter as a function of look angle may be sufficient to eliminate the radome error. One such parameter might be taken to be the radome diameter as a function of its length. (These parameters are related to the look angle.) Conceivably, wall thickness or some other wall parameter as a function of distance from the nose might be suitable.

In this connection reference should be made to the numerical examples given earlier in this section in which it was noted that in the case of the accurate calculation of radome error for the cylindrical equivalent of the conical radome, and for the rough calculation in the case of the conical radome, the effect of phase distortion was a bending of the beam toward the nose. However, an ogive shape when the beam cleared the nose caused a deviation away from the nose. It might be concluded from these calculations that by changing the shape of the radome, radome errors might be changed, conceivably even to the point of reversing the sign of part of the error curve.

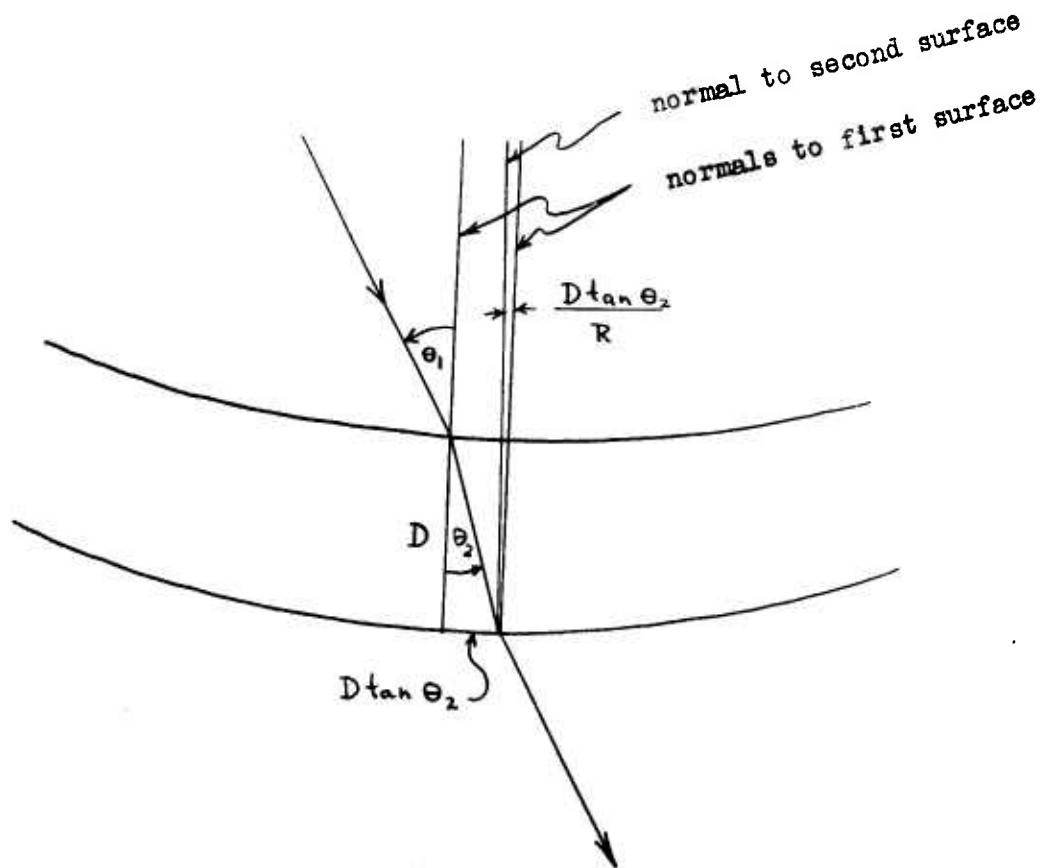


Fig. B7

Geometry for Curved Dielectric Sheet

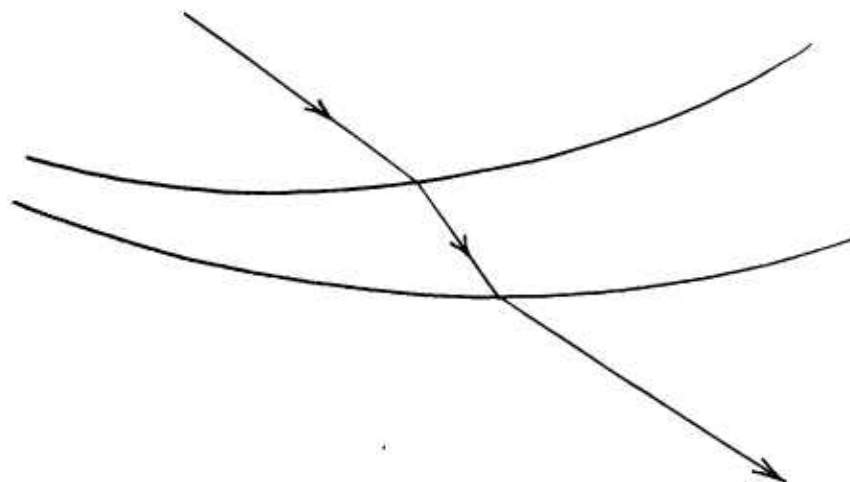


Fig. B8

Sheet with both Curvature and Taper

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

In any program for attempting to optimize the shape of the radome it would be important to know the ranges of angles of incidence which occur at various parts of the radome. The section considers these angles and indicates how they can be varied and how their ranges of angles of incidence can be varied with limits.

In Fig. B9 is shown a radome shape assumed to be a surface of revolution around an axis ON . The antenna dish AB is shown rotated to extreme position, and for simplicity assumed to rotate around the point O . AD and BC are rays from the dish in its extreme position meeting the radome at D and C respectively.

The circle $AEBF$ is the intersection of the plane of the drawing with a sphere with centre at O and radius equal to the dish radius. This sphere will be referred to as the antenna sphere. When the centre of rotation lies behind the antenna this sphere will have its centre at the centre of rotation and have a radius equal to the antenna radius.

Any point in the radome surface between C and the nose will therefore, for various positions of the antenna, receive rays which may lie anywhere within the tangent cone drawn from P to the antenna sphere.

If PG is a normal to the radome surface at P the greatest angle a ray can make with the normal will be the angle EPG and the least angle will be the angle FPG .

If P' is a point further from the nose than C but still such that the normal $P'G'$ at P' does not lie within the antenna sphere, then the greatest angle of incidence at P' will be the angle $E'P'G'$ where $P'E'$ is tangent to the circle $AEBF$, and the least angle will be the angle $HP'G'$ where $P'H$ is perpendicular to the dish in its extreme position.

For a point P'' so close to the nose that the normal at P'' meets the antenna sphere the range of angles of incidence encountered will be from normal incidence to whichever is the greater of the two angles $E''P''G''$ or $F''P''G''$.

Some general points may be noted; The further a point on the radome surface is from the antenna (as for example a point near the nose of a very long radome) the smaller the range of angles of incidence of rays going through this point.

As we move back along the radome the range of angles of incidence will increase but will be limited ultimately by the limited angle of rotation of the dish.

A point close to a rounded nose will have a small range of angles of incidence but this small range will be in a very different region (i.e. close to normal incidence) from the range of most of the other points on the radome.

THE PRINCETON UNIVERSITY RADOME STUDY

This suggests some points on choice of shape if we are interested in holding reflections down and also in having minimum phase variations at any one position of the dish and also minimum phase variation as the dish is turn from one position to another.

The logarithmic spiral so chosen that a line from the centre of the dish meets the radome always at the same angle means that for all positions in the radome the angle of incidence will vary on both sides of this angle, with a somewhat greater variation as we move back from the point.

This logarithmic spiral should be modified as we move back so that each position in the dome will keep the same approximate median value for the angles of incidence.

A rounded dome means a sudden jump into a new region of angles of incidence near the nose and if phase changes greatly with angle of incidence we might expect errors because of this rounding.

It is concluded that radome shape is a suitable design parameter and that for a symmetric antenna polarization and sufficient insensitivity of random error with incoming polarization it may be a sufficient design parameter to reduce radome error to negligible proportions. It is shown that the ranges of angles and angle of incidence may be adjusted within limits by properly choosing the radome shape.

VI. ANALYSIS AND DISCUSSION OF RADOME ERRORS OF HIGHER ORDER THAN FIRST. In section I of this appendix the method of breaking down the sources of radome error into successively higher order effects has been discussed; and in section IV and V the treatment of first order errors has been treated in some detail. The present section extends this discussion to the treatment of the higher order errors.

By "higher order" effects we mean any process by which energy is removed from the primary beam by some process such as reflection, and then immediately or eventually recombined with the beam. Thus higher order effects will include trapped waves; the effect of waves which leak out to the radiation field after one or more reflections; the effect of power being reflected back to the antenna and feed and being re-radiated; the effect of energy diffracting around to the metal parts behind the dish and being reflected back to the dish or the radiation field; the effect of power being reflected back to the dish and then pulling the magnetron or local oscillator. In the trapped wave phenomenon the radome material acts as a dielectric wave guide which can transmit energy in a direction parallel to the surface, once the corresponding waves have been excited at a point where the metal of the antenna dish is close to the radome; the trapped waves travel along the radome until they reach a point of high curvature and leak out. All these effects will usually be small compared to first order effects, and are discussed in more detail shortly.

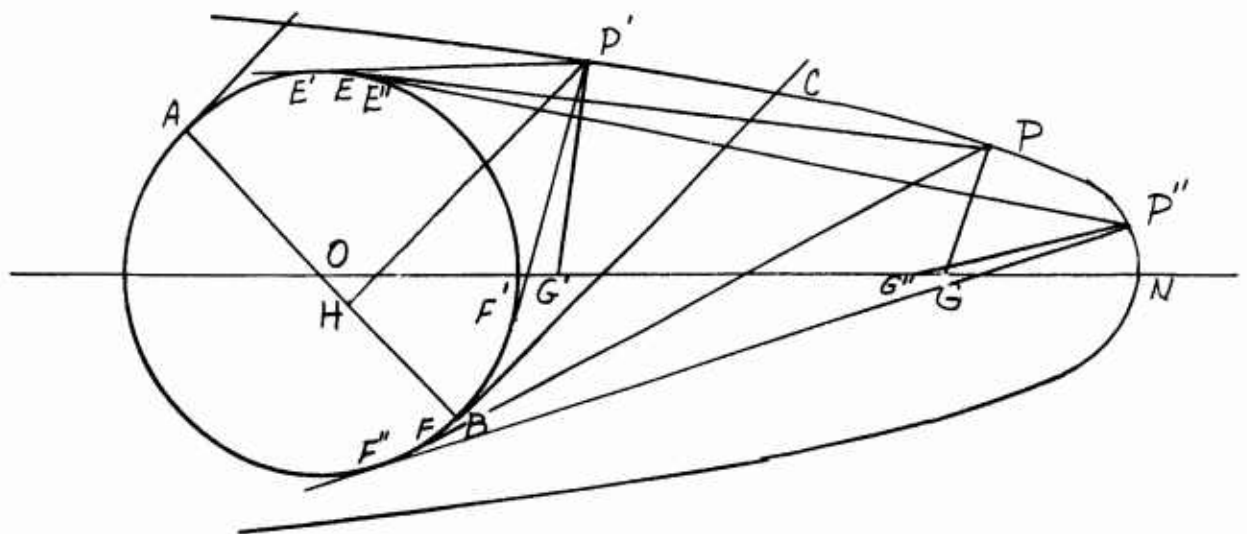
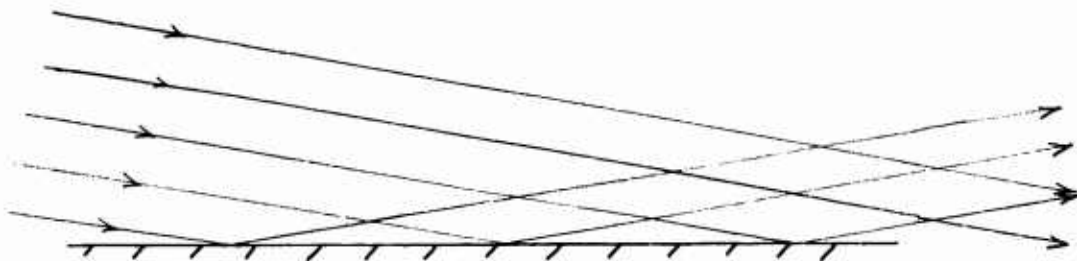


FIG. B9. DIAGRAM TO SHOW THE RANGE OF ANGLES OF INCIDENCE
WITH POSITION IN RADOME.

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

This procedure of analysis into first order and higher order processes is analogous to calculating the transmission coefficient of a plane parallel sheet of dielectric by tracing in detail each successive reflection at an interface, carrying the process ad infinitum and then adding up all the individual waves thus calculated. For the plane parallel sheet, which can easily be handled as a simple Maxwell's equation boundary value problem, the analysis in terms of successively higher order processes is cumbersome; but for the radome problem it seems the only feasible method.

Lloyd's Mirror. This term derives from the Lloyd's mirror of physical optics, a sketch of which is indicated below:



Here an incident and a reflected beam give interference fringes at some distance from the reflector. Farther away the incident and reflected beams, considered as rectilinear, will become separate; however, at sufficiently large distances the diffraction patterns of these two separate "beams" will overlap and give a combined pattern like neither pattern individually. It is this last phenomenon, the interference of two Fraunhofer diffraction patterns at large distances, in which we are interested.

In Fig. B10 is indicated an oversimplified two-dimensional radome and the primary and reflected beams concerned. From this it is apparent that the reflected beam which is narrower in the near zone is the more important in the Lloyd's mirror phenomenon, i.e. overlaps the primary beam more in the distant radiation zone, both because it is deflected at a smaller angle from the primary beam and because diffraction converts its near-zone narrowness into far-zone width.

When the look angle of the antenna, Θ in Fig. B10 is zero, the situation is symmetric and no beam shift exists. For Θ only slightly larger than zero, the reflected beam has a large deflection and is narrow in the distant zone; both effects keep the beam shift small. When Θ is large enough so that the beam clears the nose, the small reflected beam is absent. Some place between these two limits - say at look angles of 5 or 10 degrees for most radomes - the beam distortion

THE PRINCETON UNIVERSITY RADOME STUDY

caused by the Lloyd's mirror effect will be maximum.

In Fig. B11 is indicated, as a cross-hatched area, that part of the radome of Fig. B10, reflections from which will contribute to the Lloyd's mirror beam shift. And in Fig. B12 is another view of the radome optics, indicating with cross hatching the part of the beam the reflection of which is going to contribute to the Lloyd's mirror. It looks as if the look angle has been taken a little large here to give the maximum effect. After meditating upon these figures, it seems a reasonable estimate to say that

- 1) power reflected from, at most, an area equivalent to that of a circle having one quarter of the primary beam diameter, can contribute to this effect;
- 2) the maximum effect will occur at look angles 5° or 10° away from straight ahead;
- 3) considering the ogive shape of actual radomes, the angle of incidence on the radome, for that part of the primary beam with which we are concerned, will be say 77° ;
- 4) the angle between the reflected beam and the primary beam will be of the order of magnitude of 25° .

For a beam 10° wide at half power, the reflected beam would thus be, say, 40° wide, giving the reflected beam better than half its maximum amplitude when it has diffracted to the center of the primary beam. This was of course one of the considerations governing the above estimate of the look angle at which the maximum effect occurs.

From the discussion of details which has just been gone through, we may describe the situation in more general language like this: We have two approximately circular beams of radiation diverging at an angle comparable to their diffraction width, so that they overlap, at a distance, in the Fraunhofer diffraction region; and we are interested in the interference of the two Fraunhofer diffraction patterns near the maximum of the narrow intense pattern. Let us consider this general problem before specializing it further.

In such a case, where one has a diffraction pattern caused by the coherent illumination of one or more apertures, one calculates the diffraction pattern by finding the Fourier transform, in terms of plane waves, of the complex slit illuminations. That amplitude of the Fourier transform which corresponds to a plane wave running in a particular direction gives the amplitude of the total pattern in that direction. Now if we have diffraction from two apertures, the respective positions and phases of the two slits will influence the Fourier transform and the diffraction pattern.

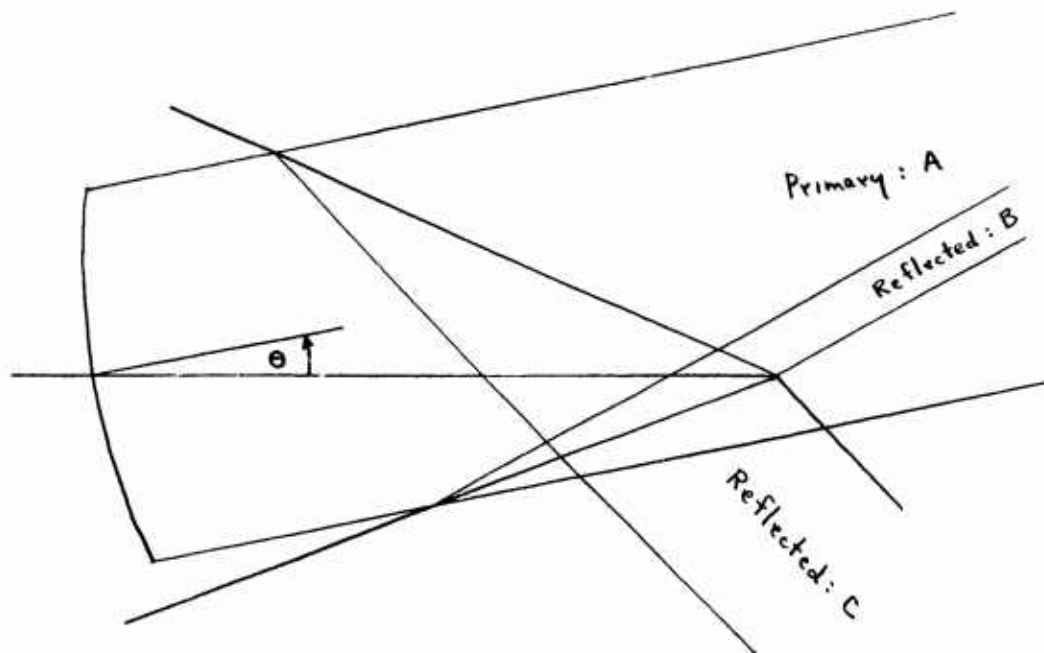


FIG. B 10 Ray Optics in Schematic Radome.

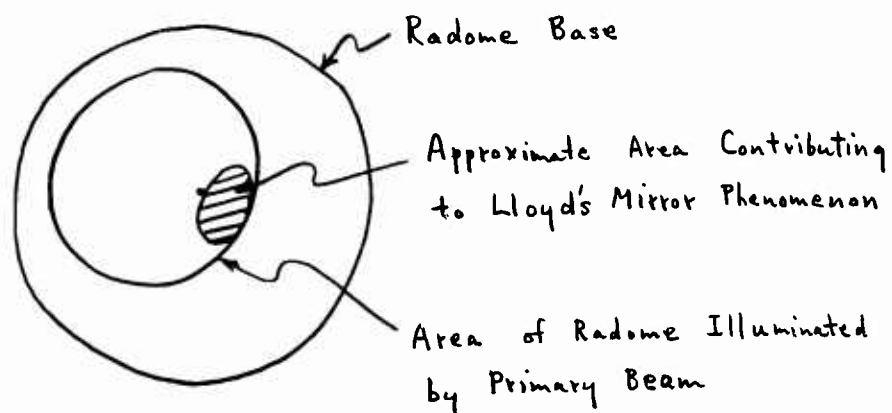


FIG. B 11 Radome Viewed from Base Showing Area Contributing to Lloyd's Mirror Phenomenon.

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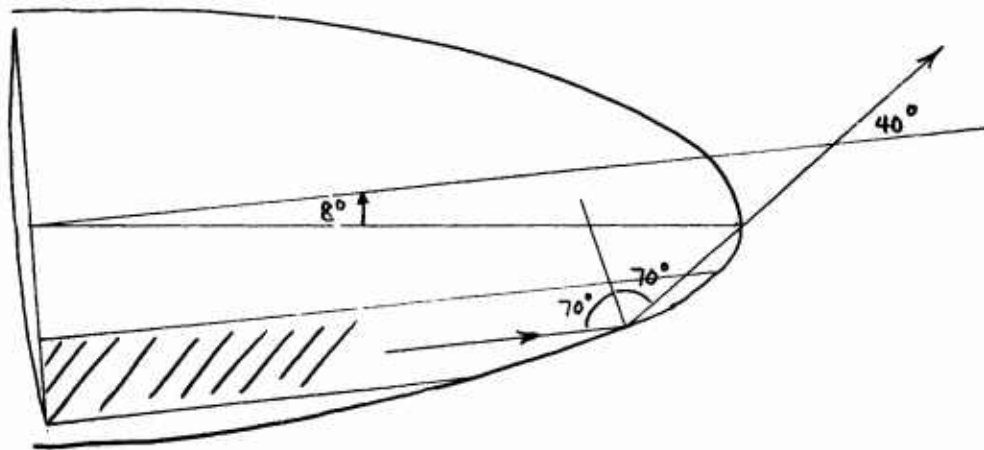


FIG. B 12 Radome Optics

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AN ANALYSIS OF THE SOURCES OF RADOME ERROR

In the present case, the defining aperture for the reflected beam is not uniquely defined, to the degree of our approximation. The defining aperture for the primary beam may be taken simply as the antenna dish. But for the reflected beam, in one sense one might say that the narrower reflected beam in Fig. B10 has its lower edge defined by the apex of the radome and its upper edge by the image, in the radome, of the lower edge of the dish. However, just as well one could take this beam to be limited by that part of the radome which does the reflecting, or by any transverse slit coming to the two edges of the narrow reflected beam in Fig. B10. These various nominal apertures will all give a diffraction pattern for the reflected beam of the same shape, but with differing phase relations to the diffraction pattern of the primary beam.

If the two defining apertures are taken to lie in the same plane and to have the same centers, which is a choice as good as any, then the two Fraunhofer diffraction patterns will have the same relative phase at all points; this relative phase will depend on the path length to the apertures from the dish via the two beams, and will also depend on the phase change at reflection from the radome. Since the kind of approximation we are using has not allowed us to pin down this phase difference or its variation in more detail, subsequent discussion will be based upon the assumption of the worst possible constant phase difference; this will be zero. Thus we assume that in the two Fraunhofer diffraction patterns the fields are in phase, giving the reflected beam the maximum effect in the primary beam. Thus the results will constitute an upper limit for beam distortion by the Lloyd's mirror phenomenon.

As a simple model for calculation, let us consider the interference of two beams, A and B as in Fig. B10 which diverge from each other at an angle ϕ_1 . The radius of the beam B is taken as the fraction $\beta \ll 1$ of the radius of beam A. If the half-power diffraction half-width of beam A is ϕ_0 , the corresponding width of beam B will be ϕ_0/β . If beam B is formed by reflection from beam A by a surface of power reflection coefficient $\alpha \ll 1$ then the diffraction pattern of beam A will not be sensibly affected by this process, and the relative total powers in beams A and B will be 1 and $\alpha \beta^2$. Since the diffraction pattern of beam B has $1/\beta^2$ times the solid angle of beam A, the relative power densities in the diffraction fields of A and B are 1 and $\alpha \beta^4$; and the relative field strengths are 1 and $\sqrt{\alpha} \beta^2$.

The relative variation of the field strengths of the beams A and B will be approximately as

$$1 - \frac{1}{4} (\phi / \phi_0)^2 \quad (A)$$

$$1 - \frac{1}{4} [(\phi - \phi_1) \beta / \phi_0]^2 \quad (B)$$

Secret

THE PRINCETON UNIVERSITY RADOME STUDY

where ϕ is angle measured from the center of the intense beam along the line joining the centers of the two beams. Thus the total relative field strength in the diffraction field, measured at points along this same line, will be

$$1 - \frac{1}{4}(\phi/\phi_0)^2 + \alpha^{\frac{1}{2}}\beta^2 \left\{ 1 - \frac{1}{4}[(\phi - \phi_1)\beta/\phi_0]^2 \right\}$$

If $\alpha = 0$ (no beam B) this has a maximum at $\phi = 0$. Near $\phi = 0$, (i.e. $\phi \ll \phi_1$), the above expression for the field strength becomes, to leading terms in the various powers of ϕ ,

$$1 + \alpha^{\frac{1}{2}}\beta^2 + \alpha^{\frac{1}{2}}\beta^4 \phi \phi_1 / 2\phi_0^2 - (\phi/2\phi_0)^2.$$

This has a maximum at a displacement $\delta\phi$ from $\phi = 0$ given by

$$\delta\phi = \alpha^{\frac{1}{2}}\beta^4 \phi_1$$

The sign here is of no significance since it could be reversed by reversal of the arbitrary phase choice made earlier. The magnitude depends upon the assumption that the maxima of the two diffraction patterns are close enough so that the half power point of beam B overlaps the maximum of beam A — i.e. that the quadratic approximation to the amplitude of field B, as it was used above, is good.

If this approximation is not satisfied — i.e. if $\phi_1 > \phi_0/\beta$ then the slope of pattern B at the maximum of pattern A will be reduced and the shift will be reduced with it. Thus finally we have

$$|\delta\phi| \leq \alpha^{\frac{1}{2}}\beta^4 \phi_1 \quad \text{for } \phi_1 \leq \phi_0/\beta$$

and for an absolute maximum shift

$$|\delta\phi|_{\max.} = \alpha^{\frac{1}{2}}\beta^3 \phi_0 \quad \text{at } \phi_1 = \phi_0/\beta$$

Of course, this maximum value may not be realized, since ϕ_1 may always be greater than ϕ_0/β . But for purposes of orientation concerning orders of magnitude, we will take the above value of $|\delta\phi|_{\max.}$ as representing an estimate which is pessimistically large on several counts.

From the earlier numerical discussion it appears that $\beta = \frac{1}{4}$ is a good representative value to be expected at the point of maximum effect. If $\phi_0 = 5^\circ$ (primary beam 10° wide) then $\phi_0/\beta = 20^\circ$, which is approximately equal to the expected value $\phi_1 = 25^\circ$. If we take $\alpha = 0.2$, a rather large value for angles of incidence in the range $75-80^\circ$, we then obtain

$$|\delta\phi|_{\max.} = 0.45 \cdot 5^\circ / 64 = 0.035^\circ = 0.6 \text{ mils.}$$

Secret

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

Similarly, the fractional increase in the peak intensity will be $\Delta \rho^2 = 3$ percent. According to the analysis in section III of the effect of intensity change on the cross over in conical scan an intensity change of this sort occurring in one side of conical scan but not the other could give a 2 mil crossover shift or radome error.

As will be seen, other sources of radome error appear to be present which can give errors larger by an order of magnitude than those estimated here; furthermore, an error of several mils which requires 5 or 10° look angle (80-160 mils) to develop is within current system tolerances on radome error slope. Hence the Lloyd's mirror phenomena does not seem to make an important contribution to radome error; but its existence adds some incentive for cutting down the reflection coefficient at the high angles of incidence which are involved here.

Higher Order Effects. Energy from the primary beam might, after several reflections, get back to the antenna dish and from there be focused into the antenna feed, whence it could reradiated. A somewhat similar phenomenon seems to have been observed at NADC. This group observed the radome error introduced by a plane dielectric sheet placed in front of the radar, nearly normal to the beam. The resulting "radome" error showed a periodicity in the angle of tilt of the sheet which was consistent with the following hypothesis as to its origin:

The sheet will serve simply to reflect energy back to the dish. This energy will come back in the form of a plane wave making an angle with the axis of the dish equal to twice the angle which the plane sheet makes with the dish axis. Because the wave returns not quite parallel to the dish axis, it will not be focused exactly on the antenna feed. Furthermore, since it is being refocused by a dish of finite size, and since it started out from the finite-width dish in the first place, it will refocus not to a point but to a point surrounded by a series of diffraction rings. As the reflecting panel is now tilted away from normalcy to the dish axis, these diffraction rings will move across the antenna feed giving alternately weak and strong absorption and re-radiation of energy by the feed. This analysis predicts the observed periodicity of radome error with tilt of the panel. No attempt has been made to follow up with a more detailed analysis.

An analogous effect has also been observed at NADC in testing blunt-nosed subsonic radomes for radome error. Here it was observed that the radome error was very sensitive to slight mechanical malpositioning of the radome; axial or transverse shifts of the radome by a fraction of a wave length made changes in the radome error pattern of the order of 100 percent.

In both these cases one was dealing with shapes (flat panel, blunt subsonic radome) which could reflect energy directly back into the antenna on the first bounce. For the sharper-nosed supersonic radomes at least two bounces, and perhaps more, would be needed. In

THE PRINCETON UNIVERSITY RADOME STUDY

particular, one can design the radome shape with the conscious intent of requiring the maximum number of bounces before energy returns to the feed. If this is done, we would expect higher order effects of the type just discussed to be negligible.

Another type of "higher order effect" which has been observed in several places is the "radome error" which is present in the absence of the radome and which is caused by the leakage of power to the back of the dish and its reflection, by the metal parts behind the dish, back into the radiation field. The cure for this is obvious and has been tried with success at Hughes, i.e. the placing of absorbent ("Harp") material on structures behind the dish so as to soak up any energy which gets back into this region. This suggests also that this stratagem might be used with success on the previously discussed higher order effects-- that is, that they might be alleviated, if they are found to exist (and we are not aware of any evidence that they do exist with supersonic radomes), by placing Harp material at all points in front of the dish where it will not interfere with the primary beam. As examples we may note the outside of the feed, and the empty space beyond the feed and coaxial with it.

Trapped Waves. Trapped waves constitute a rather special case of a higher order effect. A sheet of dielectric may serve as a wave guide for the propagation of energy in a direction parallel to the surface of the sheet¹. In a plane or gently curved sheet there is no energy loss by radiation, but the fields extend out beyond the dielectric falling off exponentially with distance from the dielectric. The phenomenon involved is essentially repeated complete internal reflection, and for it to exist the wall (if homogeneous) needs to be at least a half-wave length thick (as measured in the dielectric).

In regions of high curvature of the surface, a wave which was completely reflected in the previous bounce may be incident at a new angle which allows the wave to escape. Since except in such regions of high curvature the electromagnetic fields fall off exponentially with distance from the dielectric, the trapped waves will be excited only when some part of the antenna dish is close to the radome wall. The energy taken out of the primary beam in this manner will then propagate along the interior of the radome in all directions until at some point near the nose the increasing curvature will let it leak out, i.e. be re-radiated.

The problem then becomes very similar to that of the Lloyd's mirror, i.e. the problem of calculating the interference, in the Fraunhofer diffraction zone, of the re-radiated wave with the primary beam. We have not made any detailed analysis of this problem. The basic questions are two: How much energy is taken out of the primary beam by this process, and in what pattern is it re-radiated?

¹Ramo and Whinnery, "Fields and Waves in Modern Radio," John Wiley and Sons Inc., New York, 1944; pp. 349 et seq.

AN ANALYSIS OF THE SOURCES OF RADOME ERROR

The energy taken out of the beam will depend sharply on distance from the antenna to the radome wall, in a manner dependent on the decay constant for the exponential decay of the trapped field away from the dielectric; and this in turn depends on ratio of wavelength to wall thickness. Once the energy is abstracted from the beam and trapped in the wall, it has a considerable distance to go before reaching the nose and will go by a variety of routes, setting up a rather detailed standing wave pattern. The radiation pattern to which this energy gives rise as it leaks out in the region of high curvature (i.e. the nose) will depend to a considerable degree on how sharp the nose is. A very pointed nose gives a concentrated region of leakage and hence, as with the case of a small antenna, a broad beam; while a blunt nose will radiate trapped waves over a wider area and they could conceivably give a beam comparable in width to the primary beam. All told there seems little reason, when one has a non-blunt nose and a dish not too near the radome wall, to believe that this phenomenon could give rise to larger effects than the Lloyd's mirror effect.

At a recent meeting⁽¹⁾ devoted to an informal theoretical discussion of radome problems very interesting data on a trapped-wave like phenomenon were presented by Dr. L. C. Van Atta of Hughes Aircraft Co.; we are taking the liberty of making some interpretive comments on this data before seeing it in private. This data was taken under conditions which might be expected to emphasize trapped-wave phenomena in that the antenna-to-wall spacing was unusually small (.150") and the radome had a very wide and blunt nose (hemispherical). A radome error curve with error slopes up to 0.4 degrees per degree was observed, with a periodicity in look angle which corresponded exactly to what one would expect if there were a standing wave pattern in the wall with the free-space wavelength. The fact that the error pattern could be damped out to some extent by placing a damping strip of aquadag on the base edge of the radome also suggested an identification of the error pattern with some sort of standing wave pattern in the wall.

Experiments of this type, in which conditions are so arranged as to emphasize one particular phenomenon to the point where it may be empirically identified and its characteristics studied, are in keeping with the philosophy of analysis which this report recommends and are highly to be encouraged. In this particular instance it would be most useful to see now to what extent the trapped wave phenomenon can be removed by increasing the wall-antenna separation and the sharpness of the nose, and to see how it is affected by wall thickness. (This last comment is made because the walls involved, .030" and .060" thick, do not seem thick enough to support the type of trapped wave discussed in Ramo and Whinnery's book.) In any case, the existence of an apparent trapped-wave phenomenon here does not seem to negate our overall qualitative conclusion that trapped waves, Lloyd's mirror and other higher order effects are expected to be negligible unless attempts are made to emphasize them for experimental purposes or unless designs which obviously are prey to them are used.

(1) Composed of representatives from McMillan Laboratories, Hughes Aircraft, Goodyear Aircraft, NADC, Bureau of Aeronautics, Air Force Cambridge Research Laboratory, and Princeton Radome Study Group.

APPENDIX C

REFLECTION AND TRANSMISSION BY PARALLEL SHEETS AND SANDWICHES

I. REFLECTIONS. Some theoretical considerations have been made concerning the problem of reducing the power reflection of electromagnetic radiation from suitable radome sections. Some considerations of phase changes have also been made and are discussed in part II below. In this section emphasis has been placed upon the determination of suitable design criteria to secure optimum conditions for low reflection over a range of angles as large as possible. In this phase of the work, attention has been restricted to single sheets or B type sandwiches of suitable plastic or ceramic materials. It is not the purpose of the present report to exhaust the theoretical possibilities of the procedures suggested, nor to present results of laborious calculations. Rather, the purpose is to indicate the desirability of the approach discussed by means of some results that have been obtained for a few selected configurations, indicating that a systematic approach along these lines may lead to significant reductions of reflections.

It is pointed out in Appendix G that dielectric constants of 10 are reasonable to consider. Therefore, consideration was given single sheets and B sandwich cores with this value of the dielectric constant.

a. Single Sheets. One of the design criteria to be considered can be illustrated with reference to reflections from single sheets. The curves for selected reflected powers as functions of sheet thickness and angle of incidence are plotted in Fig. C1. The notation employed is as follows:

d_s is the thickness of the sheet

n_s is the index of refraction of the sheet

λ_0 is the free space wave length of the incident radiation

r_{ar} is the reflection coefficient at the air-dielectric interface.

also, let
$$\phi_s = \frac{2\pi d_s}{\lambda_0} \sqrt{n_s^2 - \sin^2 \theta_0}$$

It can then be shown, that the fractional reflected power $|R|^2$ from the sheet is expressed as

$$|R|^2 = \frac{4 r_{ar}^2 \sin^2 \phi_s}{(1 - r_{ar}^2)^2 + 4 r_{ar}^2 \sin^2 \phi_s}$$

Further, for perpendicular polarization (electric vector perpendicular to the plane of incidence)

$$r_{ar\perp} = - \frac{n_s \cos \theta_1 - \cos \theta_0}{n_s \cos \theta_1 + \cos \theta_0}$$

THE PRINCETON UNIVERSITY RADOME STUDY

for parallel polarization

$$r_{\parallel} = \frac{n_3 \cos \theta_0 - \cos \theta_1}{n_3 \cos \theta_0 + \cos \theta_1}$$

In Fig. C1, d_s/λ is plotted as ordinate and θ_0 as abscissa. The curves correspond to reflected powers of zero and 5%, for the case of perpendicular polarization incident on a single sheet of dielectric constant = 10. Fig. C1 illustrates the importance of the design criterion of selecting the thickness so that it corresponds to zero reflection at a large angle of incidence rather than at a small angle of incidence. This is important in the present problem since significant portions of the radiated energy impinge on the radome at angles greater than 60°. At these angles, high reflected powers are encountered if the thickness has been chosen to give zero reflection at normal incidence, whereas Fig. C1 shows that when the thickness is determined by the 90° incidence angle point, uniformly low reflected powers are maintained over the whole range of incident angles. The possibility of a tapered wall thickness based on local range of angles of incidence is discussed in Appendix B and referred to again in section II concerned with phase.

The major difficulty encountered in the single plates is that of severe thickness tolerances due to the behavior of curves for perpendicular polarization at near glancing angles. For example, at 80° the thickness tolerance for $|R|^2 = \pm 5\%$ is seen to be about 1%. On the other hand it is well to point out that the corresponding curves for parallel polarization, affected as they are at the Brewster angle, show much less severe tolerances as is apparent in Fig. C2. It is also notable that the thickness required by parallel polarization for reflection at $\theta_0 = 90^\circ$ is the same as that required by perpendicular polarization.

b. Sandwiches. B sandwiches were considered with a view toward relaxing the thickness tolerances mentioned above in the case of perpendicular polarization reflected from single sheets at near glancing incidence. Results are shown in Figs. C3 and C4, and the notation is as follows:

d_s is the thickness of the skin

n_s is the index of refraction of the skin

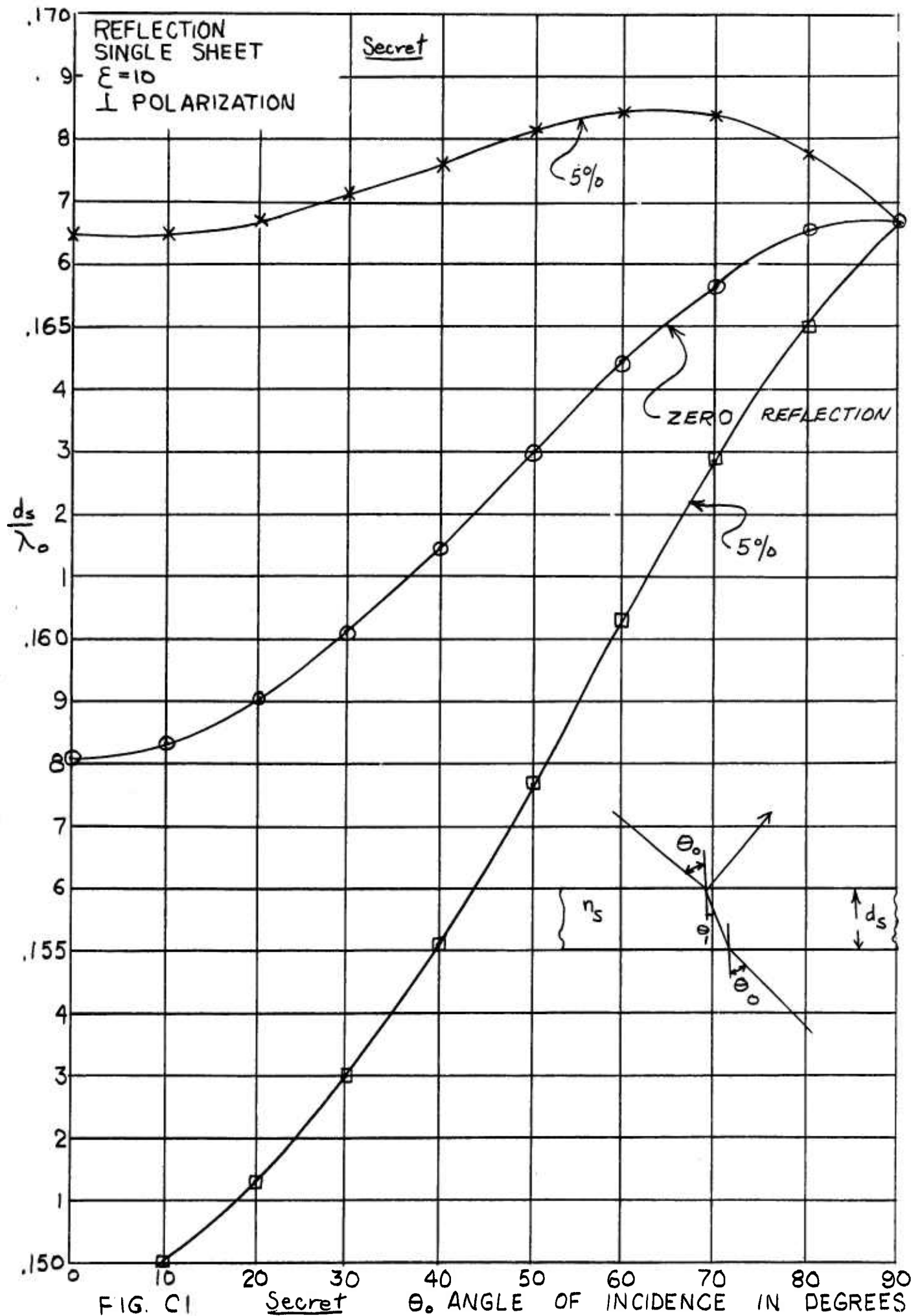
λ_0 is the free space wave length of the incident radiation

r_{as} is the reflection coefficient at the air-skin interface

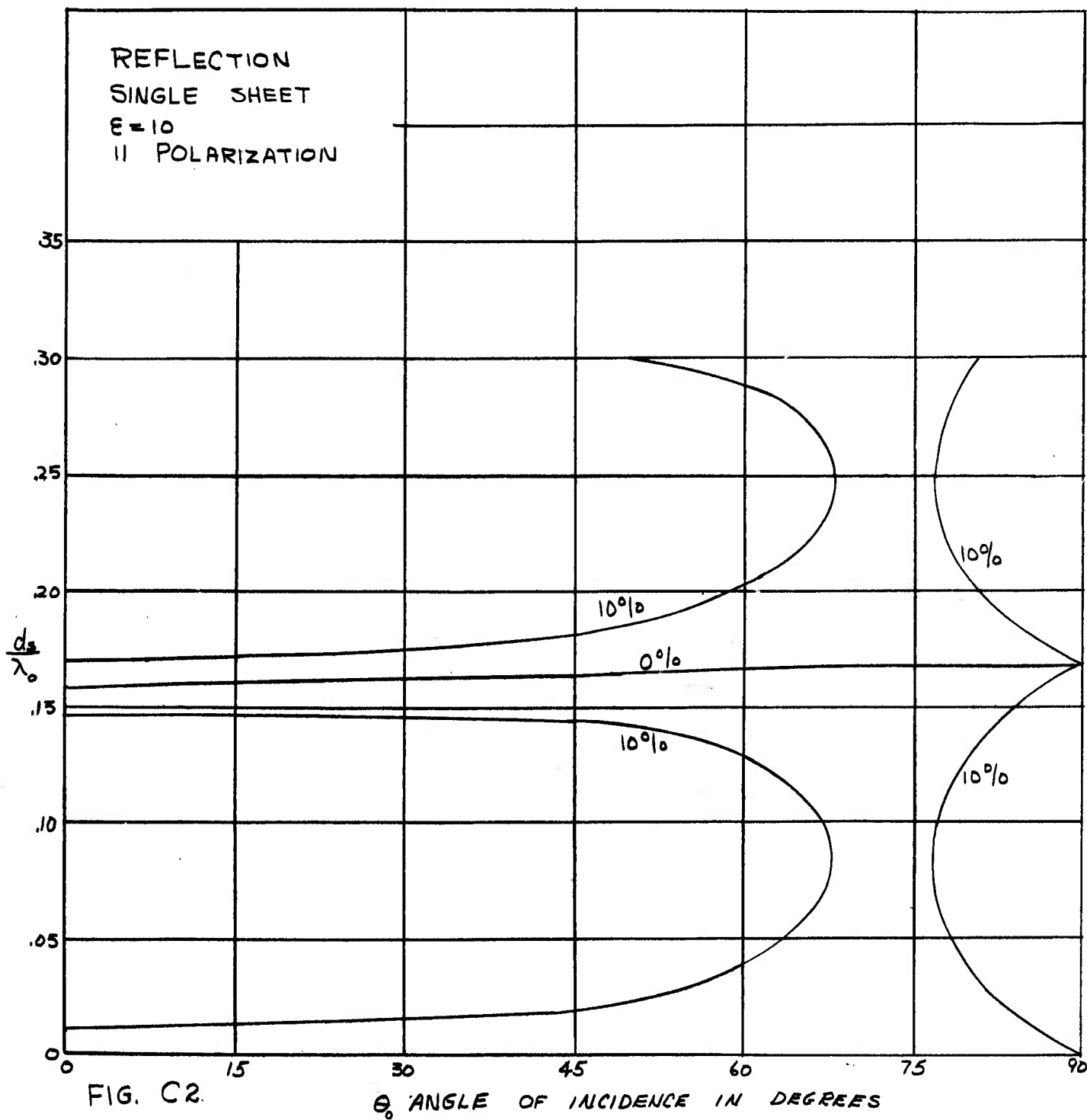
d_c is the thickness of the core

n_c is the index of refraction of the core

r_{sc} is the reflection coefficient at the skin core boundary



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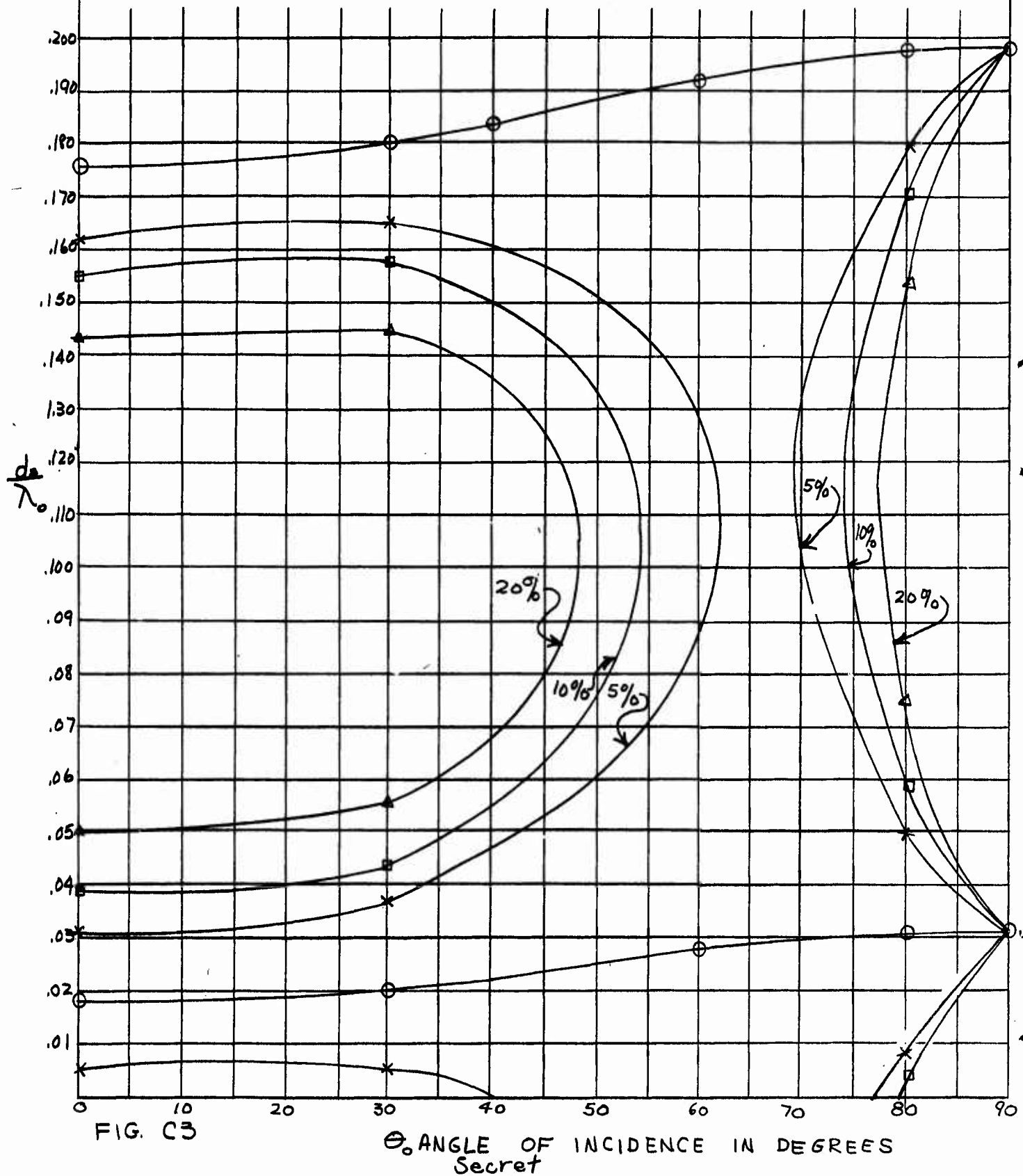
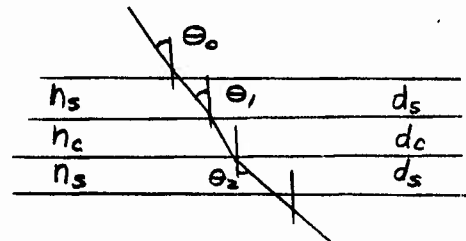
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REFLECTION
B SANDWICH
⊥ POLARIZATION

$$\epsilon_c = 10, \epsilon_s = 1.90$$

$$\phi_s = \frac{2\pi}{4.04} \sqrt{1.900 - \sin^2 \theta_0}$$

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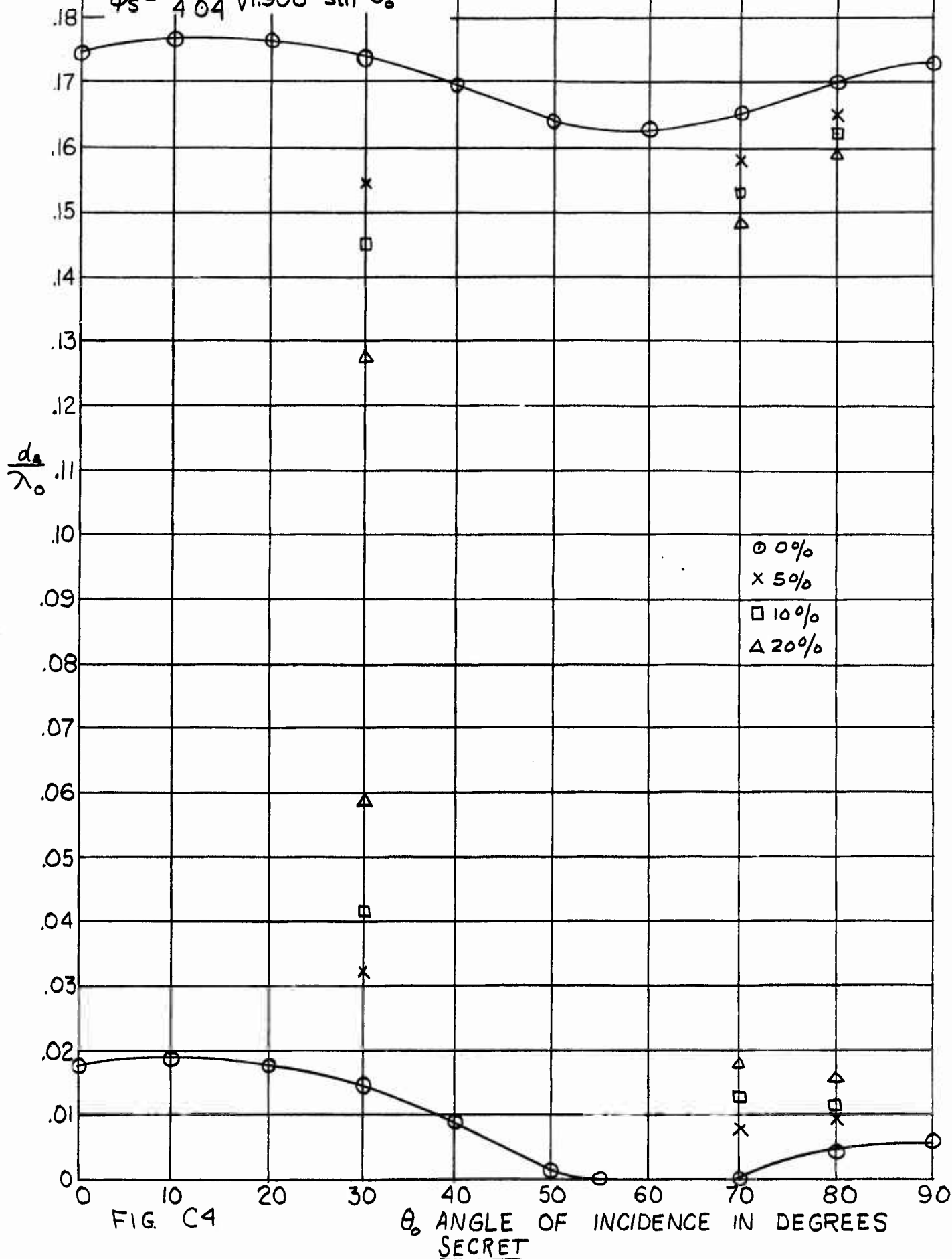


REFLECTION, B SANDWICH
 II POLARIZATION

$$\epsilon_c = 10, \epsilon_s = 1.90$$

$$\phi_s = \frac{2\pi}{4.04} \sqrt{1.900 - \sin^2 \theta_0}$$

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REFLECTION AND TRANSMISSION BY PARALLEL SHEETS AND SANDWICHES

Let $\phi_s = \frac{2\pi d_s}{\lambda_0} \sqrt{n_s^2 - \sin^2 \theta_0}$

$$\phi_c = \frac{2\pi d_c}{\lambda_0} \sqrt{n_c^2 - \sin^2 \theta_0}$$

ρ be the effective reflection coefficient of the core skin interface from within the core. It can then be shown that the reflected power is given by

$$|R|^2 = \frac{4\rho^2 \sin^2(\phi_c + \psi)}{(1-\rho^2)^2 + 4\rho^2 \sin^2(\phi_c + \psi)} \quad (01)$$

and that

$$\rho^2 = \frac{r_{ar}^2 + r_{rc}^2 + 2r_{ar}r_{rc} \cos 2\phi_s}{1 + r_{ar}^2 r_{rc}^2 + 2r_{ar}r_{rc} \cos 2\phi_s}$$

$$\psi = -\pi + \tan^{-1} \frac{(1-r_{rc}^2)r_{ar} \sin 2\phi_s}{r_{rc}(1+r_{ar}^2) + r_{ar}(1+r_{rc}^2) \cos 2\phi_s} \quad (02)$$

where

$$r_{ar\perp} = - \frac{n_s \cos \theta_1 - \cos \theta_0}{n_s \cos \theta_1 + \cos \theta_0}$$

$$r_{rc\perp} = - \frac{n_c \cos \theta_2 - n_s \cos \theta_1}{n_c \cos \theta_2 + n_s \cos \theta_1}$$

$$r_{ar\parallel} = \frac{n_s \cos \theta_0 - \cos \theta_1}{n_s \cos \theta_0 + \cos \theta_1}$$

$$r_{rc\parallel} = \frac{n_c \cos \theta_1 - n_s \cos \theta_2}{n_c \cos \theta_1 + n_s \cos \theta_2}$$

It is now possible to choose the dielectric constants and skin thickness so that the reflection factor $\rho = 0$ at some selected angle, and then determine the core thickness required to give zero reflection at other angles of incidence. One common procedure is to make $\rho = 0$ at $\theta_0 = 0$. This can be done by making

$$n_c = n_s^2$$

THE PRINCETON UNIVERSITY RADOME STUDY

and $\frac{d_3}{\lambda_0} = \frac{1}{4n_3}$ (ordinary "impedance matching")

If this is done with a core of dielectric constant 10, then

$$n_c = 3.163$$

$$n_3 = 1.778$$

$$d_3/\lambda = .141$$

The core thicknesses required for zero reflection at various values of θ_0 are easily determined from equations (C1) and (C2). To determine the deviation $\Delta d_c/\lambda_0$ from the zero line required to give a reflected power $|R|^2$ or to determine the maximum $|R|^2$ for a given angle θ_0 , the following relation is useful:

$$\frac{\Delta d_c}{\lambda_0} = \frac{1}{2\pi \sqrt{n_c^2 - \sin^2 \theta_0}} \sin^{-1} \pm \frac{1 - \rho^2}{2\rho \sqrt{\frac{1}{\rho^2} - 1}}$$

It can be seen that the tolerance is somewhat improved (over the single sheet) at 80° , markedly improved at 70° and so much improved at lower angles, that there is no reflection greater than 5% at angles less than 40° .

Although the improvement by this type of B sandwich over single sheet for perpendicular polarization is marked at low angles it is not spectacular at angles near 80° . It is possible to make such an improvement however by a different choice of n_3 and d_3 . Selecting $\theta_0 = 70^\circ$ as the angle at which ρ is to equal zero requires (if the index of refraction of the core is to stay fixed at $\sqrt{10}$) that $n_3 = \sqrt{1.90} = 1.38$. For $n_3 = \sqrt{1.90} = 1.38$ and $\rho = 0$ at $\theta_0 = 70^\circ$, then $d_3/\lambda_0 = .248$. If now the reflection curves for this combination of values are plotted, Fig. C3 results. It is seen that the tolerances are now decidedly improved.

It is necessary however to consider the corresponding curves for parallel polarization. These are given in Fig. C4. A difficulty is immediately apparent due to the fact that at near glancing angles, the core thickness required by parallel polarization for zero reflection is very different from that required by perpendicular polarization. (These two thickness requirements were the same in the case of the single sheet.) Several ways out of the difficulty may be investigated. These include 1) the possibility of optimizing skin thickness and dielectric constant for the parallel polarization, with a corresponding compromise in the matter of tolerances for the

REFLECTION AND TRANSMISSION BY PARALLEL SHEETS AND SANDWICHES

perpendicular polarization; 2) designing a radome that avoids angles of incidence greater than, say, 75° . In view of the aerodynamic considerations discussed in Appendix F this seems to be a distinct possibility.

It is interesting to note that the same sort of difficulty arises when perpendicular and parallel polarizations are considered for the case of the "double sandwich." To illustrate this point, Fig. 05 and 06 are given⁽¹⁾.

If a particular application does not allow the elimination of values of $\theta_0 > 75^\circ$, then serious consideration should be given to the advantages of the single sheet in the matter of accommodating both polarizations.

¹⁾ Goodyear Aircraft Corporation Report GER-2731, "The Theory of the Lossless Double Symmetrical Sandwich Radome."

THE PRINCETON UNIVERSITY RADOME STUDY

II. INSERTION PHASE DIFFERENCES. As mentioned in the discussion concerning causes of radome errors in Appendix B, it appears that a major source of beam deflection, and error due to cross over point, is in the phase difference introduced by the radome. It is likely that this source of error is significantly more important than the reflections discussed in part I above. It appears from the reports and discussions available that considerably less attention has been paid to phase distortion than to reflection, although workers associated with McMillan Laboratories and Goodyear are considering the problem.

The purpose of the present discussion is to indicate the magnitude of the phase difference caused by the insertion of a dielectric structure in the beam of an antenna. This difference will be seen to depend on the type of structure, of course, and for a given structure upon the angle of incidence and the polarization. Calculations have been made for typical radome structures, for perpendicular and parallel polarization. Some of the theory is reproduced in order to give a feeling for the quantities influencing the difference. The results are given for situations that can be used in the discussion of sources of radome error (Appendix B).

a. Single Sheets. Fig. C7 gives the terminology used in describing the phase difference introduced by the presence of a single sheet radome structure. θ_0 is the angle of incidence, i_j is the normal, d_s the thickness of the sheet, and λ_0 the free space wavelength. Let δ_{ij} be the phase difference between the points i and j at opposite ends of the normal. (We assume lossless sheets.) Let the "insertion phase difference" measure the phase difference in the direction of propagation due to the presence of the sheet. This is then

$$\delta_{ij} = \frac{2\pi d_s \cos \theta_0}{\lambda_0}$$

There is an evident extension of these ideas to more complex sheets, i.e. to sandwiches.

For the single sheet it is possible to show that the transmission coefficient representing the wave at j corresponding to a wave incident at i is given by

$$\tau_{ij} = \frac{1 - r_{ar}^2}{\sqrt{1 - r_{ar}^4 - 2r_{ar}^2 \cos 2\phi_s}} e^{-j(\phi_s + \delta)}$$

where r_a is assumed ≈ 1 and r_{ar} is the reflection coefficient at the ar interface, for a wave incident from a to r .

ENERGY REFLECTION CURVES,
K_u- BAND DOUBLE SANDWICH;
SKIN THICKNESS, 0.025 INCH;
PERPENDICULAR POLARIZATION

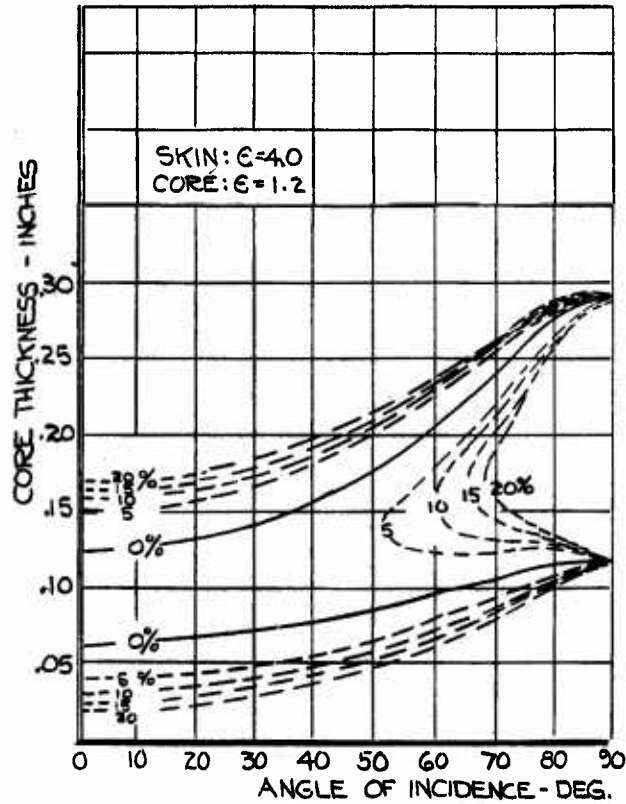


FIGURE C5

ENERGY REFLECTION CURVES,
K_u- BAND DOUBLE SANDWICH;
SKIN THICKNESS, 0.025 INCH;
PARALLEL POLARIZATION

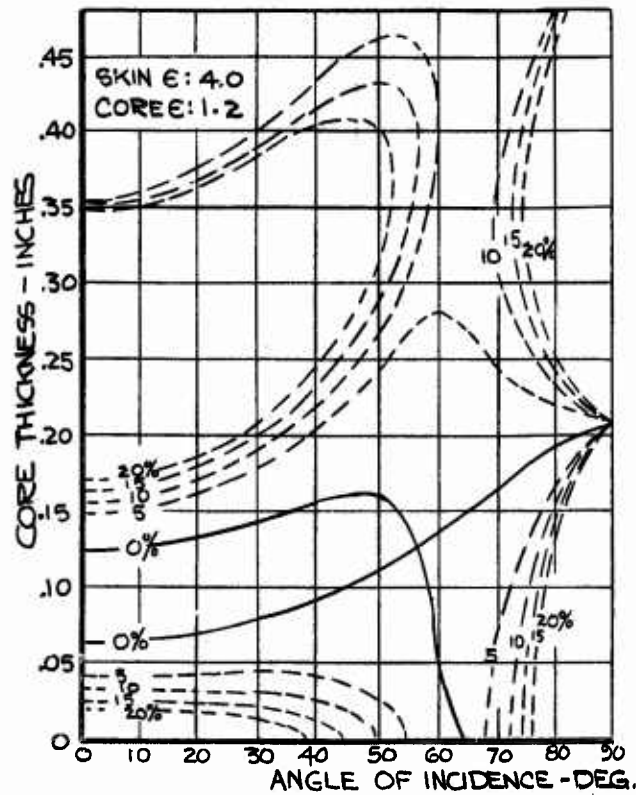


FIGURE C6

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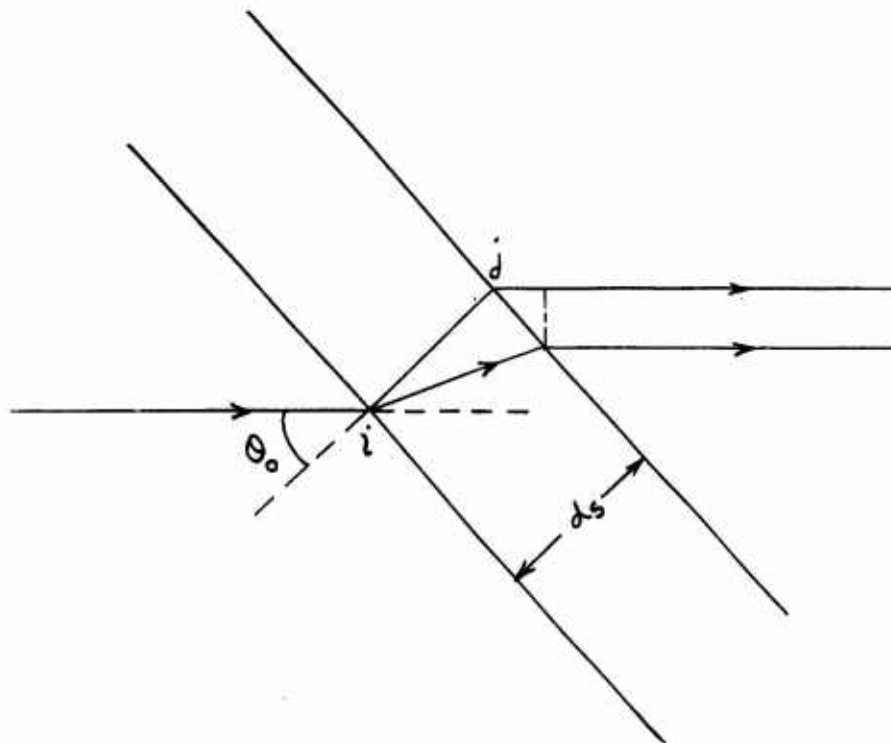


FIG. C7

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REFLECTION AND TRANSMISSION BY PARALLEL SHEETS AND SANDWICHES

$$\phi_s = \frac{2\pi d_s}{\lambda_0} \sqrt{n_s^2 - \sin^2 \theta_0}$$

and

$$\tan \gamma = \frac{r_{ar}^2 \sin 2\phi_s}{1 - r_{ar}^2 \cos 2\phi_s}$$

Thus

$$\delta_{ij} = \frac{2\pi d_s}{\lambda_0} \sqrt{n_s^2 - \sin^2 \theta_0} + \tan^{-1} \frac{r_{ar}^2 \sin 2\phi_s}{1 - r_{ar}^2 \cos 2\phi_s}$$

and the insertion phase difference is given by

$$\frac{2\pi d_s}{\lambda_0} \left[\sqrt{n_s^2 - \sin^2 \theta_0} - \sqrt{1 - \sin^2 \theta_0} \right] + \gamma$$

Figs. G8 - G13 are from the work of the Goodyear group and show the insertion phase difference as a function of angle for cases of $n_s^2 = 1.5$ and $n_s^2 = 4.0$. Curves corresponding to perpendicular and parallel polarization are given, as well as curves for the difference in phase difference for the two polarizations. Fig. G11 gives in addition corresponding results for $n_s^2 = 10$ with $\perp$ polarization, with a thickness chosen to give no reflection at 50° in order to compare with the Goodyear results. It can be seen that increasing n_s , with thicknesses chosen for zero reflection, show a greater maximum excursion of phase differences over all angles of incidence, but that if just angles between 0° and 82° are considered (and this is a reasonable limitation) then increasing the dielectric constant from 4 to 10 helps to reduce the phase difference, as far as $\perp$ polarization is concerned, by a factor of nearly 35 % $\therefore (\epsilon = n^2)$

b. Sandwiches. The calculations of the preceding paragraph can be extended to B type sandwiches. Here the insertion phase difference is found to be:

$$2\phi_s + \phi_c - \frac{2\pi}{\lambda_0} (2d_s + d_c) \cos \theta_0 + 2\delta + 2\eta$$

THE PRINCETON UNIVERSITY RADOME STUDY

1. $d_s =$ thickness of skin
 $n_s =$ index of refraction of skin

$$\phi_s = \frac{2\pi d_s}{\lambda_0} \sqrt{n_s^2 - \sin^2 \theta_0}$$

$$\phi_c = \frac{2\pi d_c}{\lambda_0} \sqrt{n_c^2 - \sin^2 \theta_0}$$

$d_c =$ thickness of core

$n_c =$ index of refraction of core

2.
$$\tan 2\delta = \frac{\rho^2 \sin 2(\psi + \phi_c)}{1 - \rho^2 \cos 2(\psi + \phi_c)}$$

$$\rho^2 = \frac{r_{ar}^2 + r_{rc}^2 + 2r_{ar}r_{rc} \cos 2\phi_s}{r_{ar}^2 r_{rc}^2 + 1 + 2r_{ar}r_{rc} \cos 2\phi_s}$$

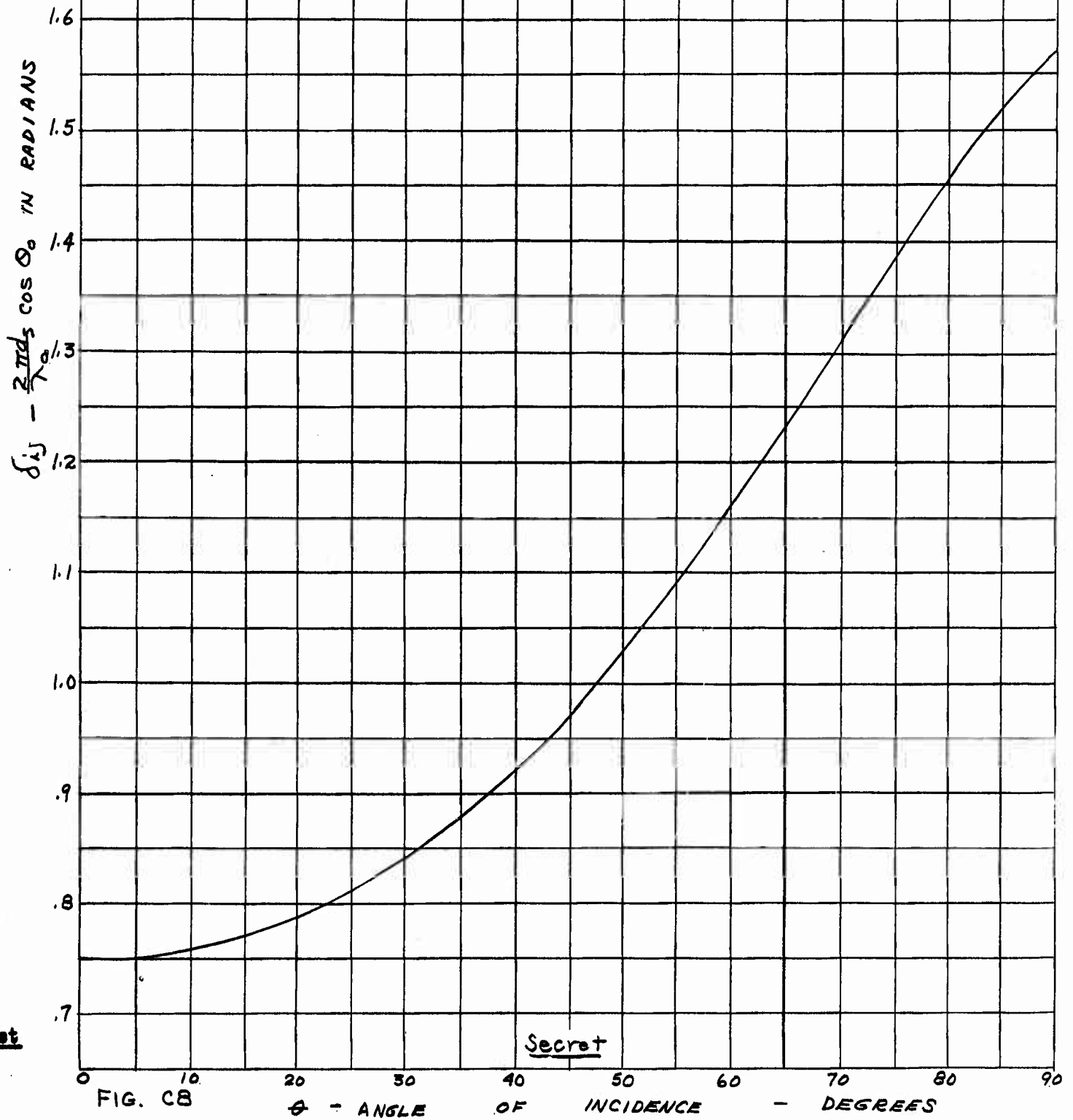
$$\tan \psi = \frac{(1 - r_{rc}^2) r_{ar} \sin 2\phi_s}{r_{rc}(1 + r_{ar}^2) + r_{ar}(1 + r_{rc}^2) \cos 2\phi_s}$$

3.
$$\tan \eta = \frac{r_{ar} r_{rc} \sin 2\phi_s}{1 + r_{ar} r_{rc} \cos 2\phi_s}$$

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SINGLE DIELECTRIC SHEET
 $\epsilon = 1.5$

PERPENDICULAR POLARIZATION
 $\phi_s = \pi$

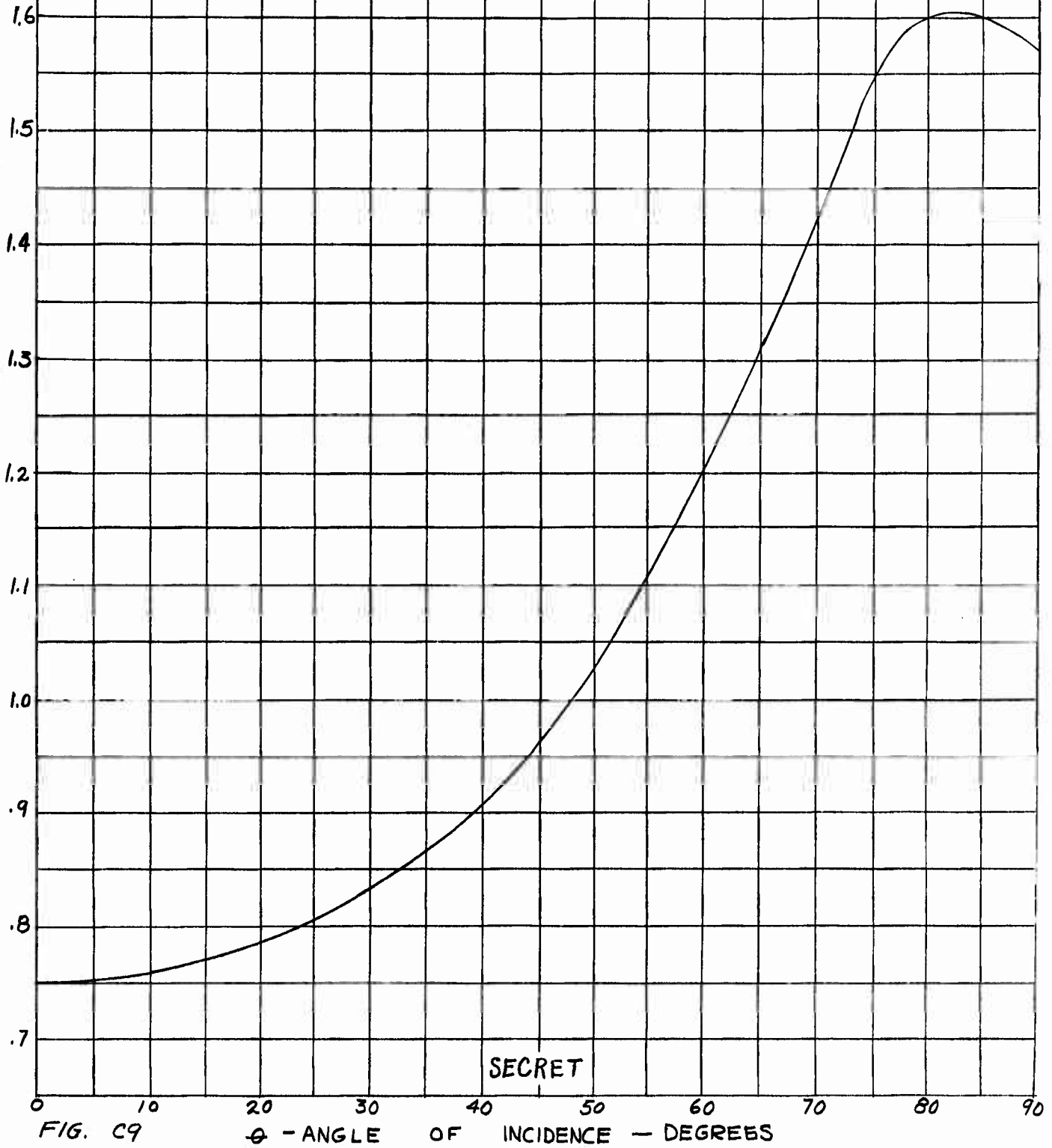


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SINGLE DIELECTRIC SHEET
 $\epsilon = 1.5$

PARALLEL POLARIZATION
 $\phi = \pi$ for $\theta = 50^\circ$

$\delta_{ij} - \frac{2\pi d}{\lambda_0} \cos \theta$ IN RADIANS



SECRET

FIG. C9

θ - ANGLE OF INCIDENCE - DEGREES

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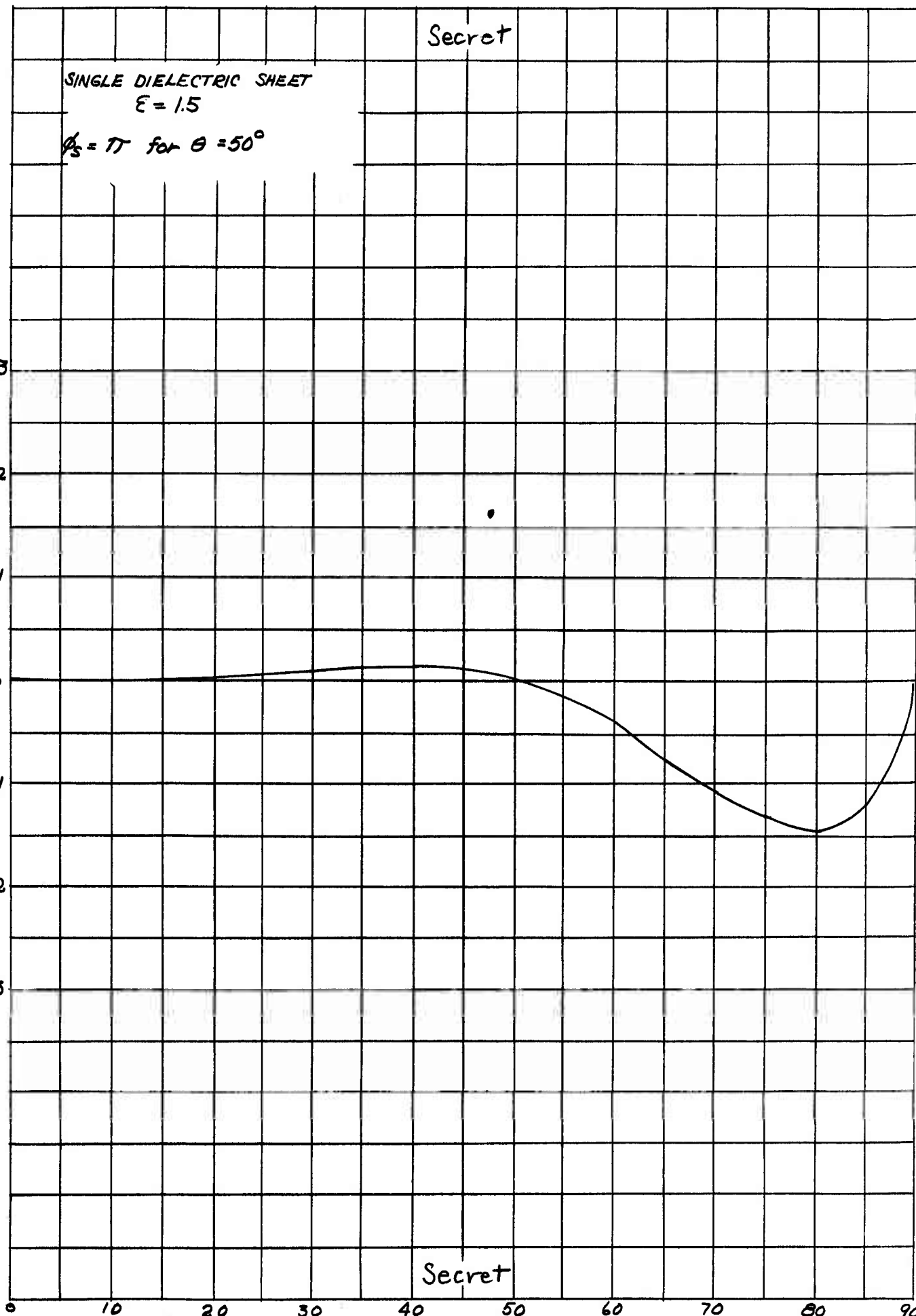
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SINGLE DIELECTRIC SHEET
 $\epsilon = 1.5$

$\phi_s = \pi$ for $\theta = 50^\circ$

$\delta_{ij\perp} - \delta_{ij\parallel}$
 (RADIAN)

0.3
0.2
0.1
0
-0.1
-0.2
-0.3

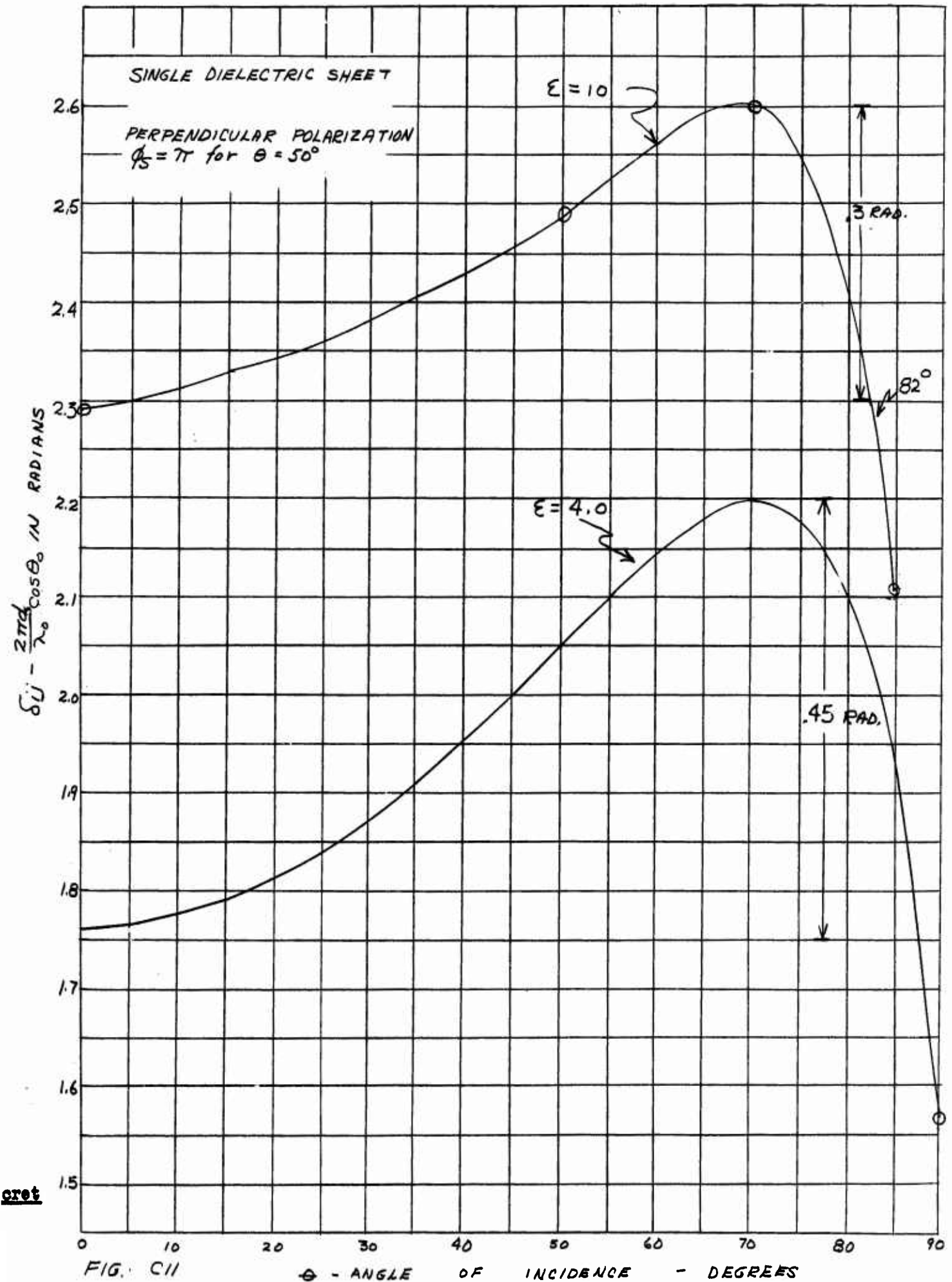


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FIG. C10

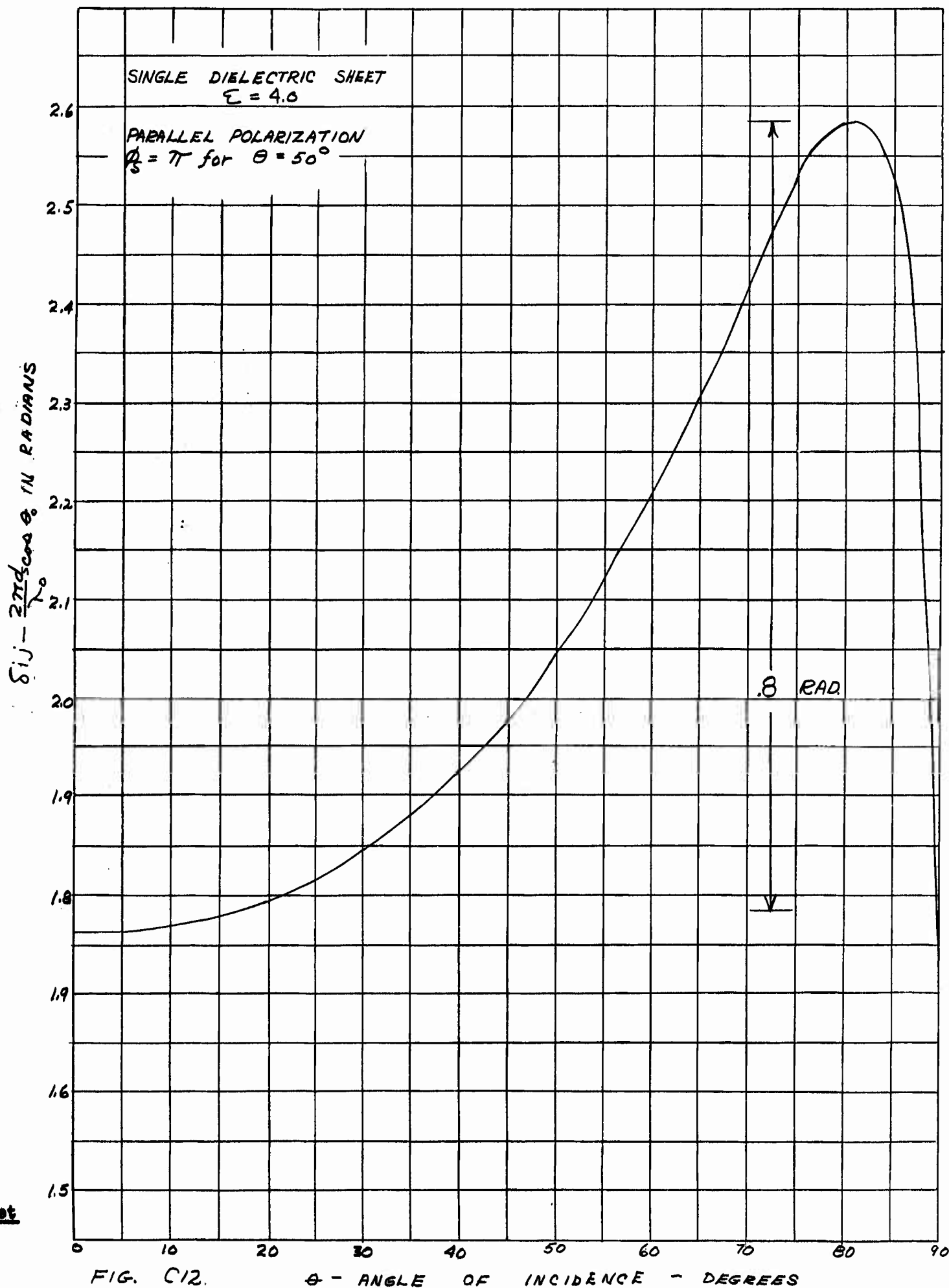
θ - ANGLE OF INCIDENCE - DEGREES

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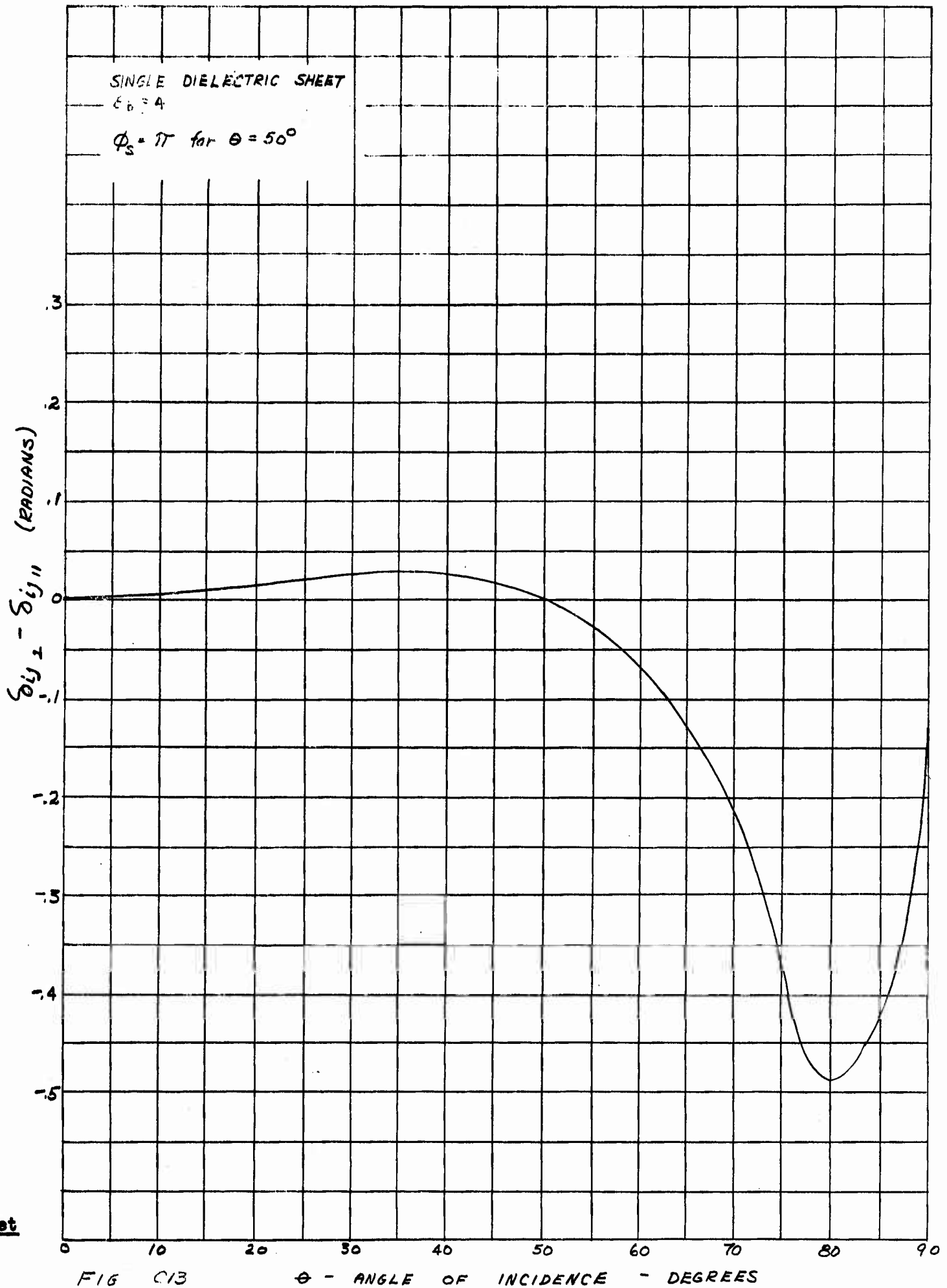


FIG C13

θ - ANGLE OF INCIDENCE - DEGREES

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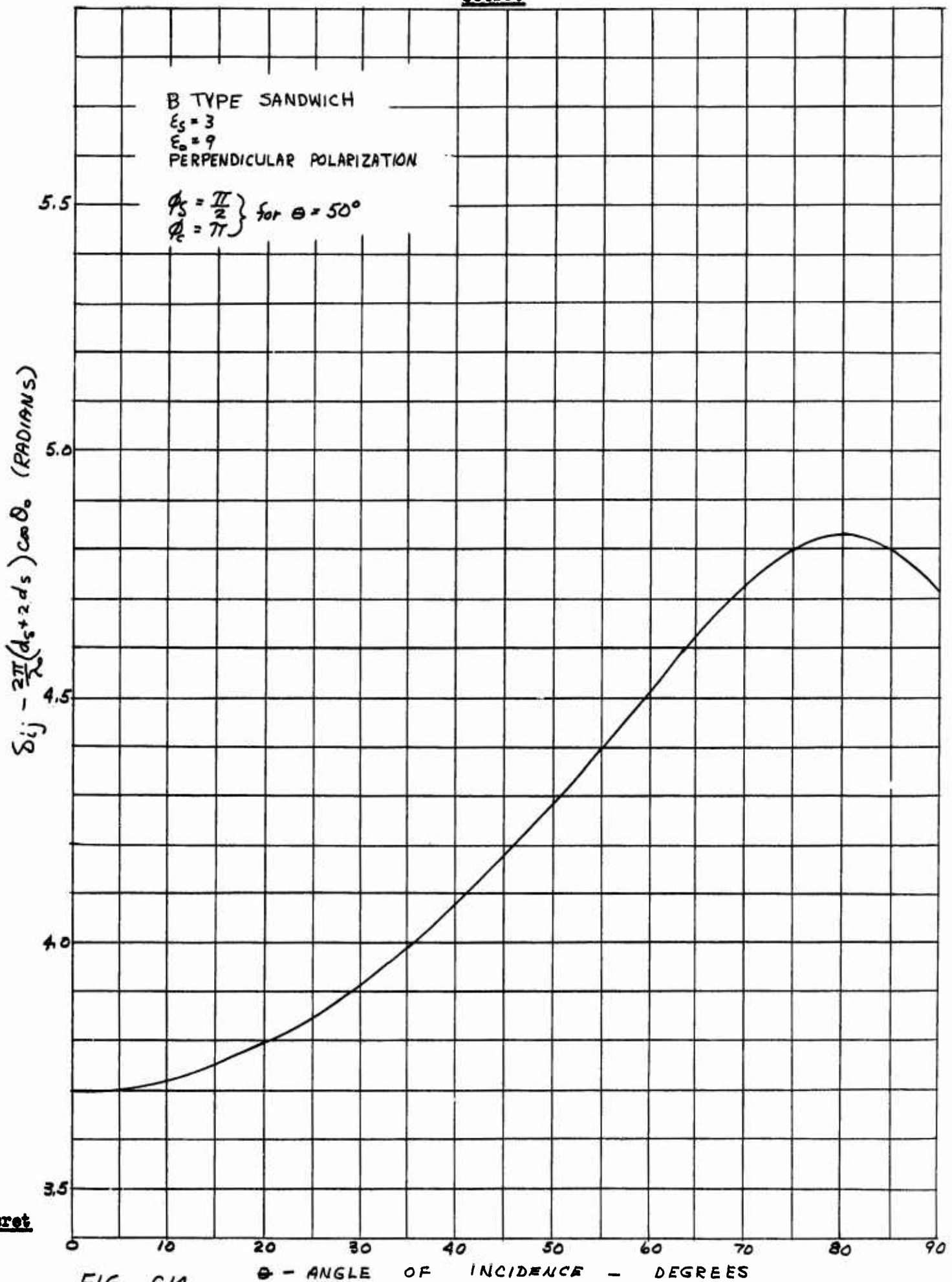
REFLECTION AND TRANSMISSION BY PARALLEL SHEETS AND SANDWICHES

Results are shown in Figs. C14 - C16 for insertion phase differences for a B type sandwich with $n_1^2 = 3$, $n_2^2 = 9$, $\phi_1 = \pi/2$ and $\phi_2 = \pi$ for $\theta_0 = 50^\circ$, parallel and perpendicular polarization and the difference between the two polarizations. It can be seen by comparing these results with those for a single sheet, that the excursion in the phase difference is greater for the B sandwich than for the single sheet, but the difference in phase between the 2 polarizations is similar to that for the single sheet in the examples shown.

Similar results obtained by the Goodyear group⁽¹⁾ for their double sandwich are shown in Figs. C17 and C18. Here it is seen that this construction is good for minimizing the phase differences due to one polarization, but further consideration is necessary to secure a compromise when both polarizations are considered. Recent correspondence from McMillan Laboratories indicates they are now approaching the problem from this direction, using B type sandwiches.

As discussed in Appendix B, the range of angles of incidence experienced at a given point on the radome as the antenna rotates is such that a relatively small range of phase differences is encountered at a fixed point. On the other hand, this range shifts over the phase curve (e.g. Fig. C11) as the point on the radome shifts. This suggests that tapered walls designed to minimize phase variations would materially improve distortions due to insertion phase differences.

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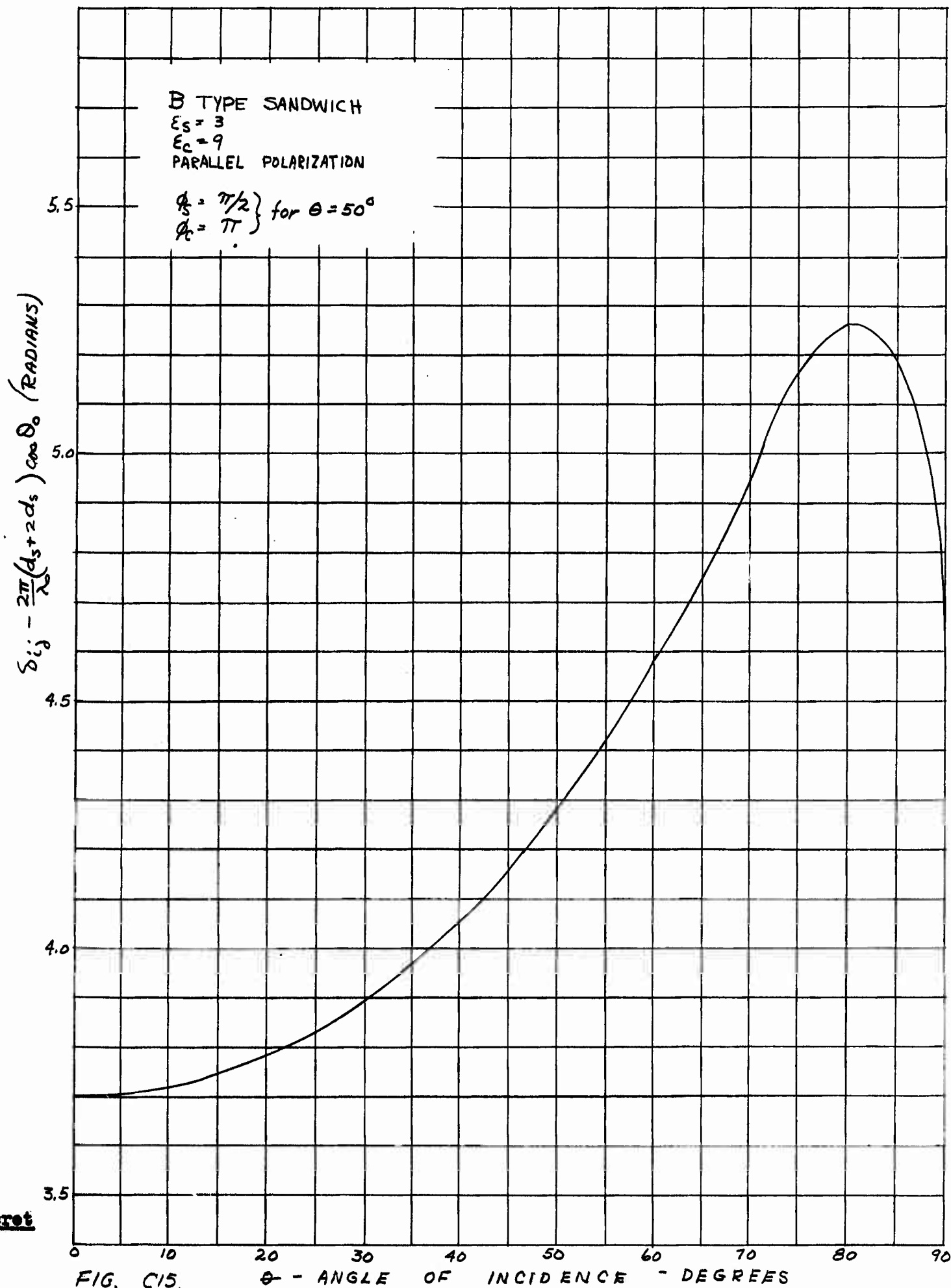
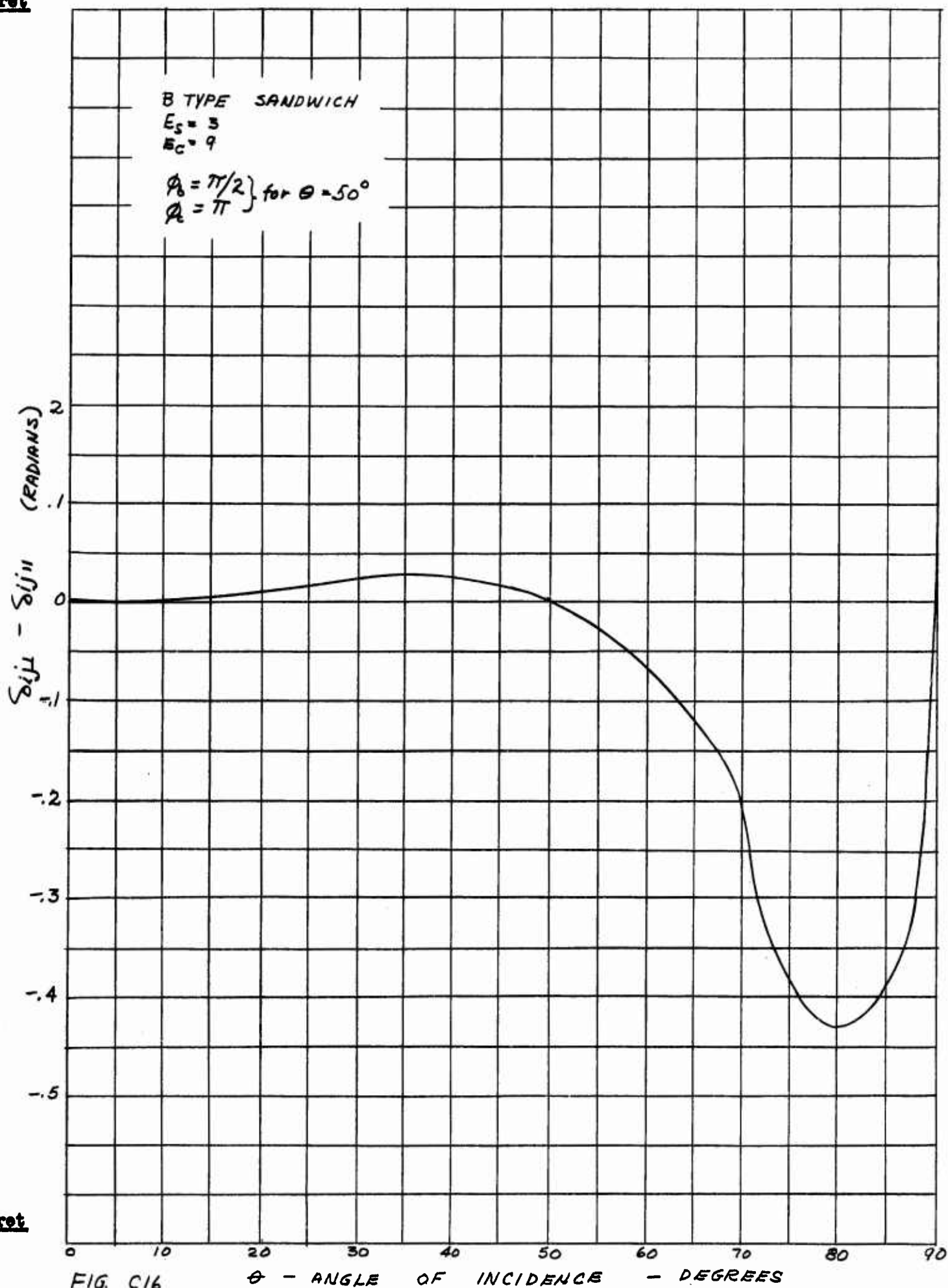


FIG. C15.

θ - ANGLE OF INCIDENCE - DEGREES

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$$\delta_{ij} - \frac{2\pi}{\lambda_0} (A_{ds} + 2d_c) \cos \theta \quad (\text{RADIAN})$$

DOUBLE SANDWICH

$$\epsilon_b = 4$$

$$\epsilon_c = 1.2$$

$$d_s = .020 \text{ IN.}$$

$$d_c = .066 \text{ IN.}$$

PARALLEL POLARIZATION

2.1

2.0

1.9

1.8

0 10 20 30 40 50 60 70 80 90

FIG. C17 θ ANGLE OF INCIDENCE - DEGREES

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DOUBLE SANDWICH

$$\epsilon_s = 4$$

$$\epsilon_c = 1.2$$

$$d_s = .020 \text{ IN.}$$

$$d_c = .066 \text{ IN.}$$

PERPENDICULAR POLARIZATION

$$S_{ij} - \frac{2\pi}{\lambda_0} (4d_s + 2d_c) \cos \theta_0 \quad (\text{RADIANS})$$

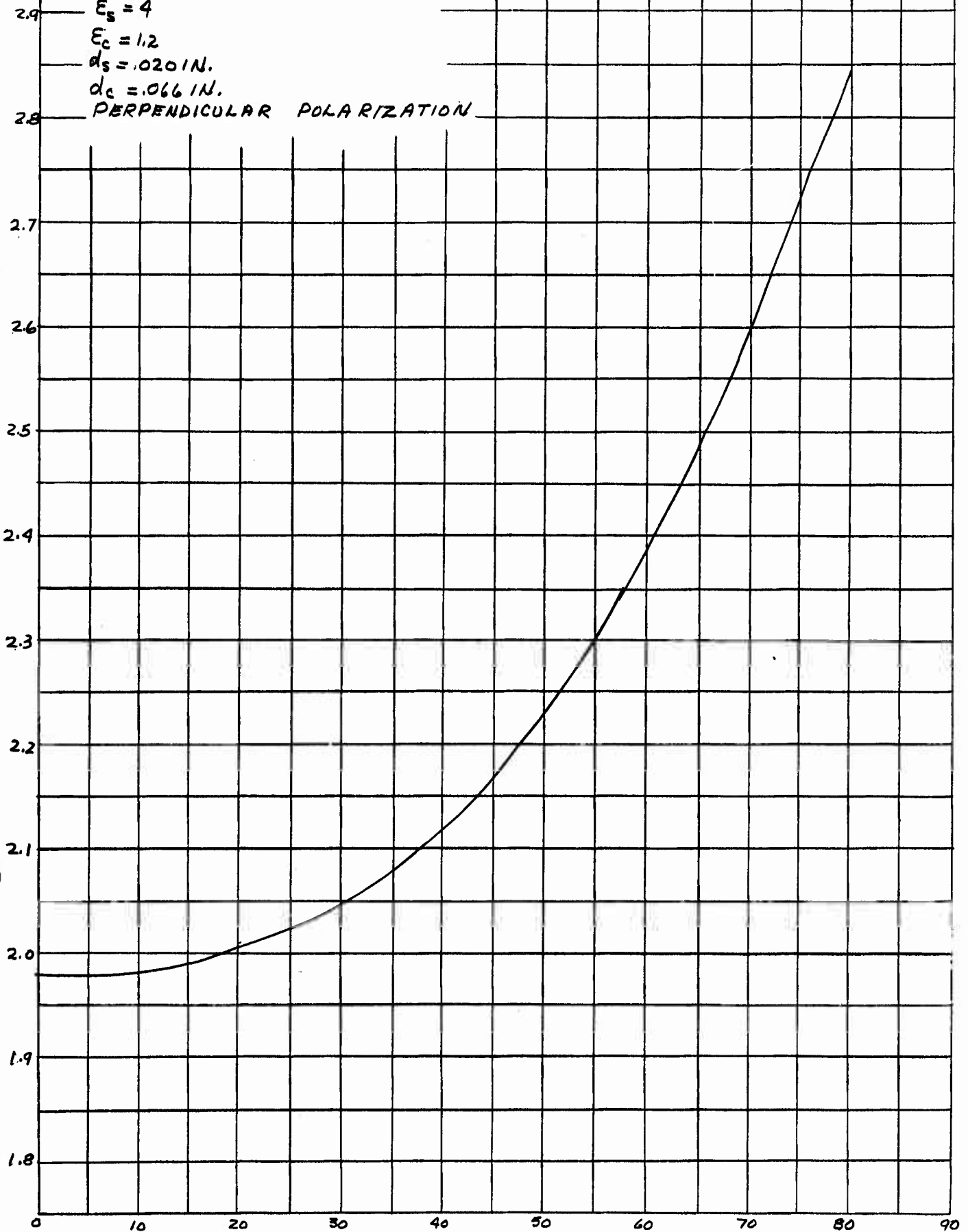


FIG. C18

θ - ANGLE OF INCIDENCE - DEGREES

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APPENDIX D

A STRONG MATERIAL OF LOW DIELECTRIC CONSTANT
FOR THE CONSTRUCTION OF RADOMES

If a material were available which was mechanically strong but which, at the microwave frequency employed, had dielectric properties and magnetic properties of a vacuum, a radome which from an electrical point of view was perfect could be built. Although such a material is obviously an idealization, preliminary calculations and measurements have indicated that a material having the mechanical properties of a solid plastic but with a dielectric constant and permeability differing only slightly from unity can be constructed. For example the reflected power from a slab .100" thick of a sample of a material of this type was at normal incidence approximately 20 d.b. below that from polystyrene of the same thickness. This corresponds to a power reflection coefficient of .0014. This is roughly equivalent to an dielectric constant of 1.2 for the medium.

An easy way of understanding the principle of such a material is to picture an ordinary dielectric as a collection of charged particles which surge back and forth under the influence of the impressed electric field. The electromagnetic radiation having its origin in these moving charges constitutes the sole effect of the dielectric upon the electromagnetic field. If one could add to the material a system of charges that moved in the opposite direction to those of the dielectric, they would radiate a system of waves which tended to cancel those radiated by the dielectric. By adjusting the intensity correctly the second set of waves can be made to cancel the waves radiated by the dielectric. This is true not only of waves external to the dielectric but also of internal waves. Therefore not only is the reflection from the material corrected but transmission phase shifts are also eliminated.

From another but equivalent point of view, the dielectric constant of a material at a given frequency is given by

$$\epsilon = 1 + 4\pi\delta$$

Here δ is the dipole moment induced in a cm^3 of the material by a unit electric field strength. If a system of charges is added to the dielectric, such that the charges move in the opposite direction from that of the dielectric charges, they contribute to the dipole moment per unit volume but with the opposite sign, hence they reduce δ and reduce the dielectric constant. A proper mixture of the two different types of dipoles can be made to give $\delta = 0$ and a unity dielectric constant for the material.

A system of electric dipole resonators tuned to a frequency below the operating frequency contributes a negative susceptibility to the medium. To see this it should be remembered that the displacement of a simple harmonic oscillator is in phase with the impressed force at frequencies below

THE PRINCETON UNIVERSITY RADOME STUDY

the resonant frequency, but it is 180° out of phase at frequencies lying above the resonance frequency. Consequently, it is only necessary to imbed in the dielectric a lattice array of resonators tuned to a frequency below the operating frequency to reduce the dielectric constant of the medium.

Still another and more fundamental way of viewing such a medium is to regard it as a collection of scattering centers. It is the superposition of all the wavelets scattered by individual atoms of the medium that results in the reflected and refracted waves. By imbedding in a standard plastic scattering elements which are tuned to low frequency, the superposition waves scattered by the elements can be made to largely cancel the wave scattered from the plastic.

If there existed a suitable dye which could be used to make a plastic such as polystyrene strongly absorbing at a frequency of say 1000 m.c. but which would be transparent in the microwave region, it should be possible to construct a medium of unity dielectric constant. Lacking such a dye one attempts to construct a "dye molecule" in the form of a small microwave resonator. These artificial "molecules" are then imbedded in a plastic causing it to be absorbing at frequencies lower than the operating frequency.

There are a few basic principles which should be followed in constructing such an artificially "dyed plastic". These principles will be listed but discussed separately.

1. The microwave resonators should be physically small.
2. They should radiate, hence scatter, only as an electric dipole. In particular, magnetic dipole and electric quadrupole moments should be small.
3. Losses should be small, i.e., the unloaded Q of the resonator should be high.
4. The resonator should have preferably one mode of oscillation lying below the operating frequency. All other modes should be at quite high frequencies and should radiate weakly.
5. The resonators should be preferably arranged in some type of regular crystal lattice.
6. To produce material in thin sheets the resonators should be in the form of a two dimensional lattice.
7. The resonators should be as closely packed as their size will allow.

The condition #1 listed above is to enable conditions 2 and 7 to be satisfied. In general the smaller a radiator, the less important are the higher electric and magnetic moments. A number of resonators of various types were tried. It was found that the resonator must be quite small to perform satisfactorily. If magnetic dipoles are present they contribute to the permeability of the medium. The reason for eliminating electric quadrupoles, etc., is the fact that for uniform polarization it is only the dipole moments which contribute to the polarization of the medium.

MATERIAL OF LOW DIELECTRIC CONSTANT

Regarding item 3, the lower the loss in a resonator the more nearly is the phase of its oscillator such that complete cancellation of reflection can be achieved.

With respect to item 4, absorption at frequencies above the operating frequency tends to increase the dielectric constant whereas absorption at frequencies below the operating frequency tends to decrease the dielectric constant.

The reason for the regular lattice required in item 5 is the fact that with a regular lattice there is no diffuse scattering. Furthermore with a lattice constant sufficiently small there are no diffracted waves. It is advantageous to eliminate diffuse scattering to reduce attenuation.

The two dimensional lattice of item 6 would represent a single crystal layer of a 3 dimensional crystal.

Regarding item 7, the closer the packing of a lattice the smaller the scattered amplitude required from any one center. Consequently, the greater the required detuning of the resonators and the greater the frequency band in which the material will have low index of refraction. Also the smaller the lattice spacing the more nearly the material behaves like an isotropic medium.

I. EFFECT OF DIELECTRIC ABSORPTION UPON DIELECTRIC CONSTANT OF MEDIUM. It is shown in a subsequent section that a dielectric loaded with regularly spaced electric dipoles acts like an isotropic dielectric medium provided the lattice spacing is sufficiently small. It is the purpose of this section to show that an isotropic dielectric medium can be made to have essentially zero loss and unity dielectric constant over a small frequency band, and that the dielectric constant is intimately connected with the dielectric absorption of the medium. It is also the purpose of this section to consider the frequency dependence of the dielectric constant near this point of unity dielectric constant. In particular it is desired to find those conditions which are imposed upon frequency dependence by the fundamental properties of electromagnetism (independent of any model).

The following theory is due in large part to Kramers and Kroenig. The basic ideas upon which the theory rests are four in number.

1. The Fourier transforms of the electric field and the dielectric polarization are proportional to each other

$$\underline{E}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \underline{E}(t) e^{i\omega t} dt$$

$$\underline{P}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \underline{P}(t) e^{i\omega t} dt$$

$$\underline{P}(\omega) = \underline{S}(\omega) \underline{E}(\omega)$$

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The proportionality constant $\delta(\omega)$ is a complex number, a function only of ω and is called the susceptibility by analogy with the corresponding static quantities. $\delta(\omega)$ can be used to define a dielectric constant as $\epsilon(\omega) = 1 + 4\pi\delta(\omega)$.

2. The dielectric can in the mean only absorb electrical energy, never give it up. This yields the condition

$$-i\omega(\delta - \bar{\delta}) > 0$$

Namely for positive ω the imaginary part of $\delta(\omega)$ is positive. In this connection it should be noted that absorption in the medium is given completely by the imaginary part of $\delta(\omega)$. The real part of δ gives the polarization amplitude in phase with the applied electric field.

3. If an electric field is suddenly applied to a dielectric the resulting polarization cannot occur before the field is applied. This self-evident statement leads to the condition that $\delta(\omega)$ which is an analytic function of ω has all its singularities in the bottom half of the ω plane.

4. As $\omega \rightarrow \infty$, $\delta(\omega) \sim \frac{1}{\omega^2}$ At sufficiently high frequency the electrons act as though free.

Because of conditions 3 and 4, a contour integral taken about the top half of the ω plane yields for real ω

$$\delta(\omega) = \frac{1}{\pi i} \int \frac{\delta(\alpha)}{\alpha - \omega} d\alpha$$

or since $\delta(\omega) = \bar{\delta}(-\omega)$

$$[\delta(\omega)]_{r.p.} = \frac{1}{\pi} \int \frac{[\delta(\alpha)]_{i.p.} \alpha}{\alpha^2 - \omega^2} d\alpha$$

Because of 4, this can be written

$$[\delta(\omega)]_{r.p.} = \frac{2}{\pi} \int_0^{\infty} \frac{[\delta(\alpha)]_{i.p.} \alpha}{\alpha^2 - \omega^2} d\alpha$$

From this if the dielectric constant be broken into real and imaginary parts,

$$\epsilon = \epsilon_1 + i\epsilon_2$$

MATERIAL OF LOW DIELECTRIC CONSTANT

One obtains

$$\epsilon_1(\omega) = 1 + \frac{2}{\pi} \int_0^{\infty} \frac{\epsilon_2(\alpha) \alpha d\alpha}{\alpha^2 - \omega^2}$$

Since ϵ_2 measures the absorption in the medium at a given frequency, it is clear that this formula expresses the real part of the dielectric constant as an integral over the imaginary part or the absorption. It should be noted that absorption at high frequencies contributes a positive quantity to ϵ_1 , and at low frequencies a negative quantity.

Note that since ϵ_2 must in the integral be positive or zero, there is no possibility of making $\epsilon_1 = 1$ over a wide frequency band. The best that can be done in this respect is to introduce absorption in the dielectric at frequencies small compared with the operating frequency. In this case assuming the remainder of the absorption in the dielectric is at very high frequencies, the formula becomes

$$\epsilon_1 = \epsilon - \frac{2}{\pi \omega^2} \int_{\alpha < \omega} \epsilon_2(\alpha) \alpha d\alpha$$

Here ϵ is the contribution to the dielectric constant resulting from high frequency absorption. It is independent of frequency. Assume

$$\epsilon_1(\omega) = 1 = \epsilon - \frac{2}{\pi \omega^2} \int_{\alpha < \omega} \epsilon_2 \alpha d\alpha$$

The rate of change of ϵ_1 with respect to frequency is

$$\epsilon_1'(\omega) = \frac{2}{\omega} (\epsilon - 1)$$

Thus if a dielectric with a constant ϵ is loaded to make it absorbing at low frequencies only, in such a way that its dielectric constant is unity at a frequency ω , its dielectric constant changes with frequency at a rate $2/\omega (\epsilon - 1)$. This is the best that one can do. In practice the change with frequency is likely to be worse.

II. METHODS FOR INTRODUCING ABSORPTION AT LOW FREQUENCIES. It is the purpose of this section to consider several means of producing low frequency absorption in a dielectric. It should be noted that a thorough survey of possible methods has not been made so that this list does not preclude other important methods.

The first possibility to be examined is a chemical dying or what is equivalent the introduction of atomic scale absorption at low frequencies. A great deal of thought has not been given to this possibility but at the moment it doesn't look too promising. The rather

THE PRINCETON UNIVERSITY RADOME STUDY

basic difficulty is the short relaxation time or broad resonance associated with an electric dipole in a solid or liquid. One material which would be ideal would be a semi-conductor or metal if the time for a collision of the electron with the lattice could be made long enough. This collision time is about 10^{-13} seconds in metal whereas 10^{-9} seconds would be needed. The effect of the short time between collisions is to make the absorption band extend up through the operating frequency.

Another possibility, which through lack of time has not been considered in detail is believed to hold some promise. This is a construction in which a regular cubic lattice of small metal objects (eg. balls) is embedded in dielectric. Between each pair of nearest metal balls there is connected a small coil. The lattice spacing should be rather small compared with a wave length. The lattice is so designed that internal resonances of both the inductive coils and metal balls are at frequencies much higher than the operating frequency whereas all the network modes are at lower frequencies.

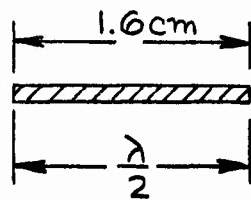
While an analysis of this system has not been made, it is believed that it will be found to act like an isotropic dielectric medium and that the dielectric constant can for suitable design be made nearly unity.

The third and only method considered in any detail for introducing absorption in the dielectric at low frequencies is to introduce into the dielectric small electric dipole resonators in a regular crystal lattice. This produces a material which is a scaled up version of a normal dielectric crystal. The lattice type used is cubic and it is known that for a lattice constant very small compared with a wave length a cubic lattice has the properties of an isotropic medium.

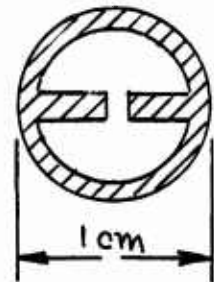
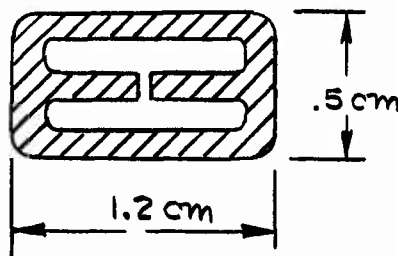
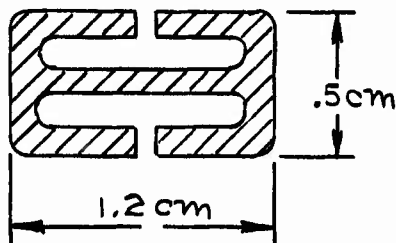
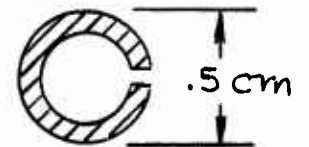
There are several reasons for making the resonators quite small compared with a wavelength. First this is necessary if the lattice spacing is to be much smaller than a wavelength and as will be pointed out later it is desirable to keep the lattice constant of the medium under a quarter of a wave length. The second reason is that this is necessary if the electric dipole moment is to be the important moment of the resonator. The occurrence of a magnetic dipole moment contributes to the permeability of the medium and higher moments either electric or magnetic give the medium properties not those of an isotropic medium. In particular a dielectric polarized with a constant electric quadrupole moment per unit volume is equivalent to a surface polarization of a normal dielectric containing only electric dipoles.

A number of different resonators were investigated before one was found which was satisfactory. The first thing tried was a straight wire in the form of a half wave dipole. This of course suffered from being long and requiring a large lattice spacing. It also has modes uniformly separated in frequency and furthermore the higher electric multipole moments are large. The next thing tried was a series of flat metal resonators of different types. See Fig. D1. Measurements were

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HALF WAVE DIPOLE



MADE OUT OF .005" COPPER SHEET $\lambda = 3.2$ cm.

FIG. DI. FLAT METAL RESONATORS

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MATERIAL OF LOW DIELECTRIC CONSTANT

made at 3.2 cm wave length to test the relative importance of the electric quadrupole moments. Although these resonators were smaller than the half wave dipoles the higher moments were still too large.

The final resonator tried which was found to be satisfactory was a self resonant solenoidal coil. These had (for operation at X band and embedded in dielectric an outer diameter of .060", about the same length, and 2 to 3 turns of wire. More exact dimensions will be given later. The dipoles are arranged in the form of three interpenetrating cubic lattices, one for each of three mutually perpendicular orientation of the coils.

III. THEORY OF THE ELECTRIC DIPOLE LATTICE. The theory of a regular lattice of electric dipole resonators consists of two parts. First the operation of an individual isolated resonator must be considered then the mutual interaction between resonators in the lattice. This latter question is the more difficult and is the only one considered in this section.

No attempt is being made to analyze theoretically all the questions which require answering. However, one of the questions will be analyzed in order to illustrate a mathematical method believed applicable to all the questions.

When the lattice spacing of a cubic lattice array of electric dipolar resonators is small compared with a wave length, the theory is well developed and quite old. (See for example Fowler "Statistical Mechanics", p. 439 MacMillan) The result is that the array acts in every way like an isotropic dielectric medium with a dielectric constant ϵ given by

$$\frac{\epsilon - 1}{\epsilon + 2} = \frac{4\pi}{3} n \mu$$

Here n is the number of resonators per unit volume and μ is the polarizability of each resonator.

If the electric field strength acting on a single isolated dipole is

$$[E e^{-i\omega t}]_{r.p.}$$

then its dipole moment is

$$[P e^{-i\omega t}]_{r.p.}$$

where

$$P = \mu E$$

THE PRINCETON UNIVERSITY RADOME STUDY

If each resonator resonates only with a single polarization, then a lattice composed of three interpenetrating cubic lattices consisting of three mutually perpendicular orientations of the dipoles is also equivalent to an isotropic medium and the above formula holds. Here η is $1/3$ of the number of resonators per unit volume.

If the lattice is embedded in a dielectric the modification to be made in the theory is the following: The connection between $\underline{P}$ and $\underline{E}$ is the same as before

$$\underline{P} = \mu \underline{E}$$

However $\underline{P}$ now represents the dipole moment induced in the resonator while in the dielectric medium. The connection between the polarizability and the dielectric constant now becomes

$$\frac{\epsilon - \epsilon_0}{\epsilon - \epsilon_0 + 3} = \frac{4\pi}{3} \eta \mu$$

where ϵ_0 is the dielectric constant of the medium without resonators. If $\epsilon = 1$ the expression becomes

$$\frac{4\pi}{3} \eta \mu = - \frac{\epsilon_0 - 1}{4 - \epsilon_0}$$

Note that if $\epsilon_0 = 4$ μ becomes infinite and for $\epsilon_0 > 4$ μ has a positive sign. The positive sign implies that the resonances frequency is higher than the operating frequency and it might be thought that this represents a violation of the general connection between absorption and the dielectric constant. This, however, is incorrect. The resonance frequency of an isolated resonator is indeed higher than the operating frequency. However, what is important is the resonance frequency of the resonator while interacting with neighboring resonators. This resonance occurs for $\epsilon = \infty$ or

$$1 = \frac{4\pi}{3} \eta \mu$$

The frequency at which this resonance occurs is below the operating frequency.

MATERIAL OF LOW DIELECTRIC CONSTANT

It is clear from the above arguments that there are two ways in which an $\epsilon = 1$ can be achieved. One is to start with a low dielectric constant material and to load it with resonators tuned to a frequency lying below the operating frequency. The other way is to start with a dielectric constant which is fairly large and to load with resonators tuned to frequencies above the operating frequency. There are advantages as well as disadvantages to this latter approach. First, the capacity loading of the resonators by the dielectric reduces their size for a given resonance frequency. Second, as the resonance frequency lies above the operating frequency there is a further decrease in size. Consequently there is a substantial reduction in the size of the resonator if the original medium has a high dielectric constant. This means that a much larger number of resonators may be used. On the other hand as is shown in section I of this appendix, starting with a high dielectric constant invariably means a narrow useful frequency band.

It should be noted that although the lattice has symmetry axes these axes do not appear in the electromagnetic properties of the medium.

The above theory rests upon the assumption that the lattice spacing is small compared with a wave length. The question to be answered then is whether these results are still substantially correct for a lattice spacing .1 to .2 of a wave length. In order to answer this question it is necessary to calculate the interactions between the dipoles including the radiation retardation effects. It might be assumed that it is only necessary to include interactions from near neighbors. This, however, is incorrect. Even with as high a power as an inverse cube interaction the distant resonators are for static interactions as important as the near ones and there are inverse cube, square, and first power terms present in the interaction.

Insofar as is known there is no previous adequate treatment of this problem. Consequently, it seemed desirable to find a mathematical method applicable to the problem. A straight forward summation of the interactions from the various dipoles seems hopelessly complicated. Consequently, if a theory of these interactions is to be had some sort of trick method of summation must be used.

The problem to be considered is the following: It is assumed that the "medium" consists of a cubic lattice of isotropic electric dipoles in a vacuum. It is desired to find for a given frequency the wavelength of a wave through the medium and the way this depends upon direction of propagation. As will be shown this problem may be reduced to the problem of finding the electric field strength acting on any dipole as a result of radiation being emitted by all the other dipoles.

To avoid the complications associated with the vector wave equation, the problem will be solved first for a scalar wave and then this solution used to generate a solution to Maxwell's equations. To illustrate how this may be done note that a scalar wave radiated by a source at the origin has the form

THE PRINCETON UNIVERSITY RADOME STUDY

$$\left[\frac{1}{k_0 r} e^{i(k_0 r - \omega t)} \right]_{\text{real part}}$$

The amplitude of the $-\omega_0$ component is

$$\frac{e^{ik_0 r}}{k_0 r}$$

This scalar wave can be used to generate a vector wave

$$\underline{H} = -i k_0^2 \underline{\text{Curl}} \left[\underline{p} \frac{e^{ik_0 r}}{k_0 r} \right]$$

$$\underline{E} = \frac{1}{k_0} \underline{\text{Curl}} \underline{H}$$

It is found upon examination that $(\underline{H}, \underline{E})$ have the form of the magnetic and electric fields radiated by an electric dipole of moment $\underline{p}$. (Gaussian form of Maxwell equations).

Scalar Problem Assume radiating sources situated at the points $\underline{r}_{lmn}$ the cartesian components of which are la, ma , and na . Here l, m, n are positive and negative integers including 0 and a is the lattice constant. A disturbance with a propagation constant k' is assumed propagating through the lattice. The wave radiated by the source at the point (l, m, n) has the form

$$e^{ik'_0 \cdot \underline{r}_{lmn}} \frac{e^{ik_0 |\underline{r} - \underline{r}_{lmn}|}}{k_0 |\underline{r} - \underline{r}_{lmn}|}$$

The factor $e^{ik'_0 \cdot \underline{r}_{lmn}}$ represents the phase retardation of the source at the point $\underline{r}_{lmn}$ with respect to that at the origin. That is to say the sources radiate with different phases as though a plane wave k_0 were passing through medium. The sum of all the waves radiated by all the sources is

$$F(\underline{r}) = \sum_{lmn} \frac{e^{ik'_0 \cdot \underline{r}_{lmn}} e^{ik_0 |\underline{r} - \underline{r}_{lmn}|}}{k_0 |\underline{r} - \underline{r}_{lmn}|}$$

MATERIAL OF LOW DIELECTRIC CONSTANT

In order to calculate the wave amplitude falling on the source at the origin, its own field must be subtracted. Denote the resultant field by

$$G(\underline{r}) = F(\underline{r}) - \frac{e^{ik_0 r}}{k_0 r}$$

The quantity desired, namely the field acting upon the source at the origin is

$$G(0)$$

In order to facilitate the sum in $G(\underline{r})$, G will be expressed as a Fourier integral and the summation performed in the transform. It may be easily shown that

$$\frac{e^{ik_0 r}}{k_0 r} = - \int \left[\frac{1}{2\pi^2} \frac{1}{k_0(k_0^2 - k^2)} - \frac{i}{4\pi k_0^2} \delta(k - k_0) \right] e^{i\mathbf{k} \cdot \mathbf{r}} d\mathbf{k}$$

Here $d\mathbf{k}$ is an element of volume in three dimensional k space. $\delta(k)$ is a function that vanishes everywhere but $k=0$ and such that

$$\int_{-\infty}^{\infty} \delta(k) dk = 1$$

An analogous function needed later is

$$\delta(\mathbf{k}) = \delta(k_x) \delta(k_y) \delta(k_z)$$

Note that

$$\int \delta(\mathbf{k}) d\mathbf{k} = 1 \quad d\mathbf{k} = dk_x dk_y dk_z$$

Using the above representation of $\frac{e^{ik_0 r}}{k_0 r}$ as a Fourier integral, the integral representation of $G(\underline{r})$ can be generated.

$$G(\underline{r}) = \int g(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}} d\mathbf{k}$$

$$g(\mathbf{k}) = - \sum_{lmn} \left[\frac{1}{2\pi^2} \frac{1}{k_0(k_0^2 - k^2)} - i \frac{1}{4\pi k_0^2} \delta(k - k_0) \right] \left[e^{i(\mathbf{k}_0 - \mathbf{k}) \cdot \mathbf{r}_{lmn}} - 1 \right]$$

Making use of a Fourier representation of $\delta(\underline{r})$ one has the equality

THE PRINCETON UNIVERSITY RADOME STUDY

$$\sum_{lmn} e^{i(k'_0 - k) \cdot r_{lmn}} = \left(\frac{2\pi}{a}\right)^3 \sum_{lmn} \delta(k - k'_0 - k_{lmn})$$

where k_{lmn} has the cartesian components $\frac{2\pi}{a}l, \frac{2\pi}{a}m, \frac{2\pi}{a}n$
If this be substituted one has

$$g(k) = - \sum_{lmn} \left[\frac{1}{2\pi^2} \frac{1}{k_0(k_0^2 - k^2)} - \frac{1}{4\pi k_0^2} \delta(k - k_0) \right] \left[\left(\frac{2\pi}{a}\right)^3 \delta(k - k'_0 - k_{lmn}) - 1 \right]$$

From this one obtains

$$G(0) = \int g(k) dk$$

The advantages of summing in the transform can now be seen. Since all singularities are at least a distance a away from the origin it is evident that the contributions from large values of k are unimportant in the integral. This can also be seen by examining $g(k)$. Unfortunately it is necessary to carry the integral far enough to include a fairly large number of terms in the sum. The reason for this is the fact that $G(r)$ is the difference between two functions both of which are singular at the origin. This sum is difficult to handle in a systematic way and would be desirable to reduce the range of integration if possible. This can be done if instead of calculating $G(0)$, a mean value of $G(r)$ be calculated the mean being taken inside of a sphere of radius a . This mean value is completely determined by $G(0)$. To see this, note that inside the sphere, of radius a there are no singularities and $G(r)$ satisfies the wave equation

$$\Delta G + k_0^2 G = 0 \quad r < a$$

Therefore $G(r)$ can be expanded in non-singular spherical waves for $r < a$

$$G(r) = \sum_{lm} A_{lm} Y_{lm}(\theta, \phi) \frac{J_{l+1/2}(k_0 r)}{2\sqrt{2}k_0 r}$$

Consequently if $G(r)$ is multiplied by any function of r only and integrated over the interior of the sphere of radius a all the terms on the right will be zero except the one with $l=0$

$$\text{Consequently } G(0) = \frac{\int G(r) f(r) dr}{4\pi \int_0^a \frac{\sin k_0 r}{k_0 r} f(r) r^2 dr}$$

MATERIAL OF LOW DIELECTRIC CONSTANT

providing

$$f(r) = 0 \quad r > a$$

The choice of $f(r)$ is somewhat arbitrary. However, a function is desired such that its Fourier transform is small for large K . This means that $f(r)$ should fall smoothly to zero at $r = a$. Such a function is

$$f(r) = \frac{\sin Kr}{Kr} \quad r < a \quad \text{where } Ka = \pi$$

$$= 0 \quad r \geq a$$

The Fourier transform of $f(r)$ is needed.

$$\int f(r) e^{ik \cdot r} d\mathbf{r} = 4\pi \frac{\sin ka}{k(K^2 - k^2)}$$

It should be noted that this goes rapidly to zero for large k .
Let

$$G_{ave} = \int G(\mathbf{r}_m) f(r) d\mathbf{r}_m = 4\pi \int \sum_{l, m, n} \left[\left(\frac{2\pi}{a} \right)^3 \delta(\mathbf{k} - \mathbf{k}_0' - \mathbf{k}_{l+m+n}) - 1 \right] \cdot$$

$$\left[-\frac{1}{2\pi^2} \frac{1}{k_0(k_0^2 - k^2)} + i \frac{1}{4\pi k_0^2} \delta(k - k_0) \right] \frac{\sin ka}{k(K^2 - k^2)} dk$$

The function $\frac{\sin ka}{k(K^2 - k^2)}$ falls smoothly to zero at $k = \frac{2\pi}{a}$

It then stays very small for larger values of k . Consequently the integral for G_{ave} can be cut off to a good approximation at

$$k = \frac{2\pi}{a} = 2K.$$

THE PRINCETON UNIVERSITY RADOME STUDY

$G(0)$ is then given by

$$G(0) = \frac{G_{ave}}{\frac{4\pi \sin k_0 a}{k_0(K^2 - k_0^2)}}$$

Here

$$G_{ave} = -\frac{16\pi^2}{a^3} \frac{1}{k_0 k_0' (k_0'^2 - k_0^2) (K^2 - k_0'^2)} \sin k_0' a$$

$$+ 4\pi \int_0^{2K} \left[\frac{1}{2\pi^2} \frac{1}{k_0 (k_0^2 - k^2)} - \frac{i}{4\pi k_0^2} \delta(k - k_0) \right] \frac{\sin ka}{k(K^2 - k^2)} dk$$

The integral in this expression can be evaluated approximately most readily by extending the limit to infinity and transforming to an integral over position space.

This second term then becomes

$$-\int_0^a \frac{e^{ik_0 r}}{k_0 r} \frac{\sin Kr}{Kr} 4\pi r^2 dr = -\frac{4\pi (e^{ik_0 a} + 1)}{k_0 (K^2 - k_0^2)}$$

Then

$$G_{ave} = \frac{16\pi^2}{a^3} \frac{1}{k_0 k_0' (k_0'^2 - k_0^2) (K^2 - k_0'^2)} \sin k_0' a - \frac{4\pi (e^{ik_0 a} + 1)}{k_0 (K^2 - k_0^2)}$$

It should be noted that this expression for G_{ave} is independent of the direction of k_0' . This field strength $G(0)$ is the driving force that causes the source to radiate. The magnitude of the propagation constant of the wave in the medium k_0' must be so chosen that $G(0)$ has the proper value to cause the source at the origin to radiate the unity amplitude wave $e^{ik_0 r}/k_0 r$. What this required amplitude

depends upon the nature of the source. Since $G(0)$ is independent of the direction of k_0' the medium acts like an isotropic medium.

Vector Field A vector field which is a solution to Maxwell's equation can be generated from the scalar field $F(k)$ as

$$\underline{H}' = -i k_0^2 \underline{\text{Curl}} \left[\underline{p} F(\underline{r}) \right]$$

$$\underline{E}' = k_0 \underline{\text{Curl}} \underline{\text{Curl}} \left[\underline{p} F(\underline{r}) \right]$$

This is the field radiated by a lattice array of electric dipoles with dipole moment amplitude of $\underline{p}$. It is desired to calculate the electric field acting upon the electric dipole at the origin. This is the electric field omitting the source at the origin.

$$\underline{E} = k_0 \underline{\text{Curl}} \underline{\text{Curl}} \left[\underline{p} G(\underline{r}) \right]$$

It must be evaluated at the origin.

$\underline{E}$ may be transformed to

$$\underline{E} = k_0 \underline{\text{grad}} \underline{\text{div}} \left[\underline{p} G(\underline{r}) \right] - k_0 \underline{p} \Delta G(\underline{r})$$

In the interior of a sphere of radius a one has

$$\Delta G(\underline{r}) + k_0^2 G(\underline{r}) = 0$$

$$\underline{E} = k_0 \underline{\text{grad}} \left[\underline{p} \cdot \underline{\text{grad}} G \right] + k_0^3 \underline{p} G(\underline{r})$$

This may be written as a Fourier transform

$$\begin{aligned} \underline{E} &= k_0 \int \underline{g}(\underline{k}) \left[k_0^2 \underline{p} - \underline{k} (\underline{p} \cdot \underline{k}) \right] e^{i \underline{k} \cdot \underline{r}} d\underline{k} \\ &= -k_0 \sum_{\underline{m}, \underline{n}} \int \left[k_0^2 \underline{p} - \underline{k} (\underline{p} \cdot \underline{k}) \right] \cdot \left[\left(\frac{2\pi}{a} \right)^3 \delta(\underline{k} - \underline{k}' - \underline{k}_{\underline{m}, \underline{n}}) - 1 \right] \cdot \\ &\quad \left[\frac{1}{2\pi^2 k_0 (k_0^2 - k^2)} - \frac{i}{4\pi k_0^2} \delta(k - k_0) \right] e^{i \underline{k} \cdot \underline{r}} d\underline{k} \end{aligned}$$

THE PRINCETON UNIVERSITY RADOME STUDY

Here also it is desirable to form a space average of E inside the sphere of radius a .

$$E_{ave} = \int E(r) f(r) dr = -4\pi k_0 \sum_{lmn} \int [k_0^2 p - k(p \cdot k)] \cdot \left[\left(\frac{2\pi}{a} \right)^3 \delta(k - k'_0 - k_{lmn}) - 1 \right] \left[\frac{1}{2\pi^2 k_0 (k_0^2 - k^2)} - \frac{i}{4\pi k_0^2} \delta(k - k_0) \right] \cdot \frac{\sin ka}{k(k^2 - k_0^2)} dk$$

In the case of the scalar field this integral was cut off at $k = \frac{2\pi}{a}$. Here, however, because of the term $k(p \cdot k)$, the convergence is less rapid and this cannot be done. This integral can, however, be divided into three parts, one of which can be cut off and one of which can be easily evaluated.

$$E_{ave} = E_{1ave} + E_{2ave} + E_{3ave}$$

$$E_{1ave} = -4\pi k_0^3 p \sum_{lmn} \int \left(\frac{2\pi}{a} \right)^3 \delta(k - k'_0 - k_{lmn}) \frac{\sin ka}{2\pi^2 k_0 (k_0^2 - k^2) k(k^2 - k_0^2)} dk$$

$$E_{2ave} = 4\pi k_0 \int [k_0^2 p - k(p \cdot k)] \cdot \left[\frac{1}{2\pi^2 k_0 (k_0^2 - k^2)} - \frac{i}{4\pi k_0^2} \delta(k - k_0) \right] \cdot \frac{\sin ka}{k(k^2 - k_0^2)} dk$$

$$E_{3ave} = 4\pi k_0 \sum_{lmn} \int k(p \cdot k) \left(\frac{2\pi}{a} \right)^3 \delta(k - k'_0 - k_{lmn}) \frac{\sin ka}{2\pi^2 k_0 (k_0^2 - k^2) k(k^2 - k_0^2)} dk$$

E_{1ave} converges rapidly and may be cut off at $k = \frac{2\pi}{a}$ to give

$$E_1 = -16\pi^2 \frac{k_0^2 p \sin k_0' a}{a^3 k_0' (k_0^2 - k_0'^2) (k^2 - k_0'^2)}$$

MATERIAL OF LOW DIELECTRIC CONSTANT

The vector part of $\underline{E}_{m2 \text{ ave}}$ may be transformed as follows:

$$k_0^2 \underline{p} - \underline{k} (\underline{p} \cdot \underline{k}) = \text{grad}_{\underline{k}} \frac{1}{2} [k_0^2 p^2 - (\underline{p} \cdot \underline{k})^2]$$

where the gradient is taken with respect to the coordinate $\underline{k}$. As the direction of $\underline{k}$ occurs in the integral only in this one term, it may be averaged over all directions of $\underline{k}$ to give

$$\begin{aligned} [k_0^2 \underline{p} - \underline{k} (\underline{p} \cdot \underline{k})]_{\text{ave}} &= \text{grad}_{\underline{k}} \frac{1}{2} [k_0^2 p^2 - \frac{1}{3} k^2 p^2] \\ &= \underline{p} (k_0^2 - \frac{1}{3} k^2) \end{aligned}$$

$\underline{E}_{m2 \text{ ave}}$ becomes

$$\begin{aligned} \underline{E}_{m2 \text{ ave}} &= 4\pi k_0 \underline{p} \int [k_0^2 - \frac{1}{3} k^2] \cdot \left[\frac{1}{2\pi^2 k_0 (k_0^2 - k^2)} - \frac{i}{4\pi k_0^2} \delta(k - k_0) \right] \\ &\quad \frac{\sin ka}{k(K^2 - k^2)} dk \end{aligned}$$

If the integral is transformed to position space it becomes

$$\underline{E}_{m2} = -k_0 \underline{p} \int f(r) [k_0^2 + \frac{1}{3} \Delta] \frac{e^{ik_0 r}}{k_0 r} dr$$

where as before

$$\begin{aligned} f(r) &= \frac{\sin Kr}{Kr} & r < a \\ &= 0 & r \geq a \end{aligned}$$

By substituting from the wave equation

$$\Delta \left[\frac{e^{ik_0 r}}{k_0 r} \right] + k_0^2 \left[\frac{e^{ik_0 r}}{k_0 r} \right] = -\frac{4\pi}{k_0} \delta(r)$$

THE PRINCETON UNIVERSITY RADOME STUDY

The integral becomes

$$E_{2\text{ave}} = -k_0 \oint_m \int f(r) \left[\frac{2}{3} k_0^2 \right] \frac{e^{ik_0 r}}{k_0 r} dr + k_0 \oint_m \int f(r) \frac{4\pi}{3k_0} \delta(r) dr$$

$$= -\frac{8\pi}{3} \oint_m \frac{k_0^2 (e^{ik_0 a} + 1)}{K^2 - k_0^2} + \frac{4\pi}{3} \oint_m$$

The term $\frac{4\pi}{3} \oint_m$ is produced by the singularity at the origin. This term will be cancelled by a corresponding term in $E_{3\text{ave}}$ inasmuch as the total field is singularity free at the origin.

The final integral can be written as

$$E_{3\text{ave}} = \frac{16\pi^2}{a^3} \sum_{l,m,n} \int k (p \cdot k) \delta(k - k'_0 - k_{l,m,n}) Q dk$$

where

$$Q(k) = \frac{\sin ka}{(k^2 - k_0^2)k(k^2 - K^2)}$$

Then

$$E_{3\text{ave}} = \frac{16\pi^2}{a^3} \sum_{l,m,n} (k'_0 + k_{l,m,n}) [p \cdot (k'_0 + k_{l,m,n})] Q(|k'_0 + k_{l,m,n}|)$$

In Q the direction of k'_0 appears as a dot product under the radical in

$$|k'_0 + k_{l,m,n}| = \sqrt{k_0'^2 + k_{l,m,n}^2 + 2k'_0 \cdot k_{l,m,n}}$$

As it is assumed that $k_{l,m,n} \gg k'_0$ it would appear appropriate to expand in a power series in $k'_0 \cdot k_{l,m,n}$

MATERIAL OF LOW DIELECTRIC CONSTANT

$$Q(|\underline{k}_0' + \underline{k}_{lmn}|) = Q(\sqrt{k_0'^2 + k_{lmn}^2}) + \frac{Q'(\sqrt{k_0'^2 + k_{lmn}^2})}{\sqrt{k_0'^2 + k_{lmn}^2}} \underline{k}_0' \cdot \underline{k}_{lmn} \\ + \frac{1}{2} \left[\frac{Q''(\sqrt{k_0'^2 + k_{lmn}^2})}{k_0'^2 + k_{lmn}^2} - \frac{Q'(\sqrt{k_0'^2 + k_{lmn}^2})}{(k_0'^2 + k_{lmn}^2)^{3/2}} \right] (\underline{k}_0' \cdot \underline{k}_{lmn})^2 + \dots$$

If $\underline{p}$ is assumed perpendicular to $\underline{k}_0'$ the vector factors can be written

$$(\underline{k}_0' + \underline{k}_{lmn}) [\underline{p} \cdot (\underline{k}_0' + \underline{k}_{lmn})] = \frac{1}{2} \text{grad}_{\underline{p}} (\underline{p} \cdot \underline{k}_{lmn})^2 + \underline{k}_0' (\underline{p} \cdot \underline{k}_{lmn})$$

In $E_{3 \text{ ave}}$ the sum can be broken up into a sum of sub-sums each of which is composed of a given $l, m, \text{ and } n$ and all their permutations and sign changes. Thus for each sub-sum $\underline{k}_{lmn}$ is a constant over the sum. The following results are needed in evaluating the sum. Denote a sub-sum by Σ'

$$\Sigma' (\underline{A} \cdot \underline{k}_{lmn}) (\underline{B} \cdot \underline{k}_{lmn}) (\underline{C} \cdot \underline{k}_{lmn}) \dots = 0$$

for odd number of factors

$$\Sigma' (\underline{A} \cdot \underline{k}_{lmn})^2 = \frac{1}{3} \Sigma' A^2 k_{lmn}^2$$

$$\Sigma' (\underline{A} \cdot \underline{k}_{lmn}) (\underline{B} \cdot \underline{k}_{lmn}) = 0 \quad \text{providing} \quad \underline{A} \cdot \underline{B} = 0$$

The first result follows from $\underline{k}_{lmn} = -\underline{k}_{-l, -m, -n}$.

The second result follows because of the cubic lattice. The third result follows from the second. Using these results $E_{3 \text{ ave}}$ can be written

$$E_{3 \text{ ave}} = \frac{16\pi^2}{a^3} \sum_{lmn} \left[\frac{1}{2} \text{grad}_{\underline{p}} (\underline{p} \cdot \underline{k}_{lmn})^2 + \underline{k}_0' (\underline{p} \cdot \underline{k}_{lmn}) \right]$$

$$\left[Q + \frac{Q'}{\sqrt{k_0'^2 + k_{lmn}^2}} \underline{k}_0' \cdot \underline{k}_{lmn} + \frac{1}{2} \left(\frac{Q''}{k_0'^2 + k_{lmn}^2} - \frac{Q'}{(k_0'^2 + k_{lmn}^2)^{3/2}} \right) (\underline{k}_0' \cdot \underline{k}_{lmn})^2 \right. \\ \left. + \dots \right]$$

THE PRINCETON UNIVERSITY RADOME STUDY

The Q 's are to be evaluated at the point $\sqrt{k_0'^2 + k_{emn}^2}$.

$$E_{m3 \text{ ave}} = \frac{16\pi^2}{a^3} \sum_{l_{mn}} \frac{1}{6} g_{mn}^{\text{rad}} p^2 k_{emn}^2 Q$$

$$+ \frac{16\pi^2}{a^3} \sum_{l_{mn}} \frac{1}{4} g_{mn}^{\text{rad}} (p \cdot k_{mn})^2 (k_0' \cdot k_{emn})^2 \left[\frac{Q''}{k_0'^2 + k_{emn}^2} - \frac{Q'}{(k_0'^2 + k_{emn}^2)^{3/2}} \right]$$

The first term in $E_{m3 \text{ ave}}$ may be transformed to make it more strongly convergent. It may first be written

$$\frac{16\pi^2}{3a^3} p \int \sum_{l_{mn}} \delta(k - k_0' - k_{emn}) (k^2 - k_0'^2) Q(\sqrt{k_0'^2 + |k - k_0'|^2}) dk$$

Making use of the fact that E_3 (i.e. the part of E which when averaged over space gives $E_{3 \text{ ave}}$) satisfies the inhomogeneous wave equation this first term of $E_{m3 \text{ ave}}$ becomes after using a method similar to that used on E_2

$$= \frac{16\pi^2}{3a^3} p \int \sum_{l_{mn}} \delta(k - k_{emn} - k_0') (k_0^2 - k_0'^2) Q dk - \frac{4\pi}{3} p$$

Here as in the case of $E_{m2 \text{ ave}}$ the term $-\frac{4\pi}{3} p$ has its origin in the source function at the origin introduced by the wave equation. This term is cancelled by the corresponding term in $E_{2 \text{ ave}}$. The sum is now so strongly convergent that it is only necessary to take the first term in which case it becomes,

$$= \frac{16\pi^2}{3a^2} p (k_0^2 - k_0'^2) Q(k_0') - \frac{4\pi}{3} p$$

The second term in $E_{m3 \text{ ave}}$ is of second order in $k_0' \cdot k_{emn}$. Consequently the term is small and can be evaluated by using an approximation. The Approximation used is to replace the sum by an integral.

$E_{m3 \text{ ave}}$ then becomes

$$E_{m3 \text{ ave}} = -\frac{16\pi^2}{3a^3} p (k_0'^2 - k_0^2) Q(k_0') - \frac{4\pi}{3} p$$

$$+ \frac{p}{\pi} \int_0^{2\pi} \int_0^\pi \int_0^\infty \cos^2 \theta \sin^2 \theta \cos^2 \phi k_0'^2 k^2 \left[\frac{Q''}{k^2} - \frac{Q'}{k^3} \right] \sin \theta d\theta d\phi k^2 dk$$

This integral can be reduced to

$$\frac{4}{15} \int_0^\infty k_0'^2 \int_0^\infty \frac{d}{dk} \left(\frac{Q}{k} \right) k^5 dk$$

which after integrating by parts twice becomes

$$4 \int_0^\infty k_0'^2 \int_0^\infty Q k^2 dk$$

This integral is very similar to $E_{m2 \text{ ave}}$ and can be evaluated in a similar fashion to give

$$-2\pi \int_0^\infty k_0'^2 \frac{\cos k_0 a + 1}{K^2 - k_0^2}$$

Combining the first and second term $E_{m3 \text{ ave}}$ becomes

$$\begin{aligned} E_{m3 \text{ ave}} &= -\frac{16\pi^2}{3a^3} \int_0^\infty (k_0'^2 - k_0^2) Q(k_0') - \frac{4\pi}{3} \int_0^\infty \\ &\quad -2\pi \int_0^\infty k_0'^2 \frac{\cos k_0 a + 1}{K^2 - k_0^2} \\ &= -\frac{16\pi^2}{3a^3} \int_0^\infty \frac{\sin k_0' a}{k_0' (k_0'^2 - K^2)} - \frac{4\pi}{3} \int_0^\infty -2\pi \int_0^\infty k_0'^2 \frac{\cos k_0 a + 1}{K^2 - k_0^2} \end{aligned}$$

Combining all these results E_{ave} becomes

$$\begin{aligned} E_{\text{ave}} &= E_{m1 \text{ ave}} + E_{m2 \text{ ave}} + E_{m3 \text{ ave}} \\ &= \frac{16\pi^2}{3} \frac{(2k_0^2 + k_0'^2) \int_0^\infty \sin k_0' a}{a^3 k_0' (k_0'^2 - k_0^2) (K^2 - k_0'^2)} - \frac{8\pi}{3} \int_0^\infty \frac{k_0^2 (e^{ik_0 a} + 1)}{K^2 - k_0^2} \\ &\quad -2\pi \int_0^\infty k_0'^2 \frac{\cos k_0 a + 1}{K^2 - k_0^2} \end{aligned}$$

THE PRINCETON UNIVERSITY RADOME STUDY

From this the electric field acting on the dipole at the origin is

$$E_o = \frac{E_{ave}}{4\pi \sin k_o a} k_o (K^2 - k_o^2)$$

The dipole moment is proportional to E_o .

$$p_o = \mu E_o$$

here μ is the polarizability of the resonator. If this equation is used to eliminate E_o , $\frac{p_o}{\mu}$ may be divided out and the resulting equation can be solved for $\frac{1}{\mu}$ in terms of k_o' , k_o and a .

$$\frac{1}{\mu} = \frac{k_o (K^2 - k_o^2)}{4\pi \sin k_o a} \left[\frac{16\pi^2 (2k_o'^2 + k_o'^2) \sin k_o' a}{3 a^3 k_o' (k_o'^2 - k_o^2) (K^2 - k_o'^2)} - \frac{8\pi}{3} \frac{k_o^2 (e^{ik_o a} + 1)}{K^2 - k_o^2} - 2\pi k_o'^2 \frac{\cos k_o a + 1}{K^2 - k_o^2} \right]$$

In this formula μ is the polarizability of the resonator, a is the lattice constant of the regular cubic lattice of resonators. $K = \pi/a$ k_o is $2\pi/\lambda_o$ where λ_o is the free space wave length of the radiation. k_o' is the corresponding quantity for the wave length in the lattice.

This rather complicated formula should reduce to the classical result for $K \gg k_o, k_o'$. If small terms are neglected the above formula may be written after some rearrangement:

$$\frac{\epsilon - 1}{\epsilon + 2} = \frac{4}{3} \pi \frac{\mu}{a^3}$$

where

$$\epsilon = \left(\frac{k_o'}{k_o} \right)^2$$

MATERIAL OF LOW DIELECTRIC CONSTANT

Note that a^{-3} is the number of resonators per unit volume, and this result is identical with the classical formula. This classical formula is a good approximation for $a < .2\lambda$

In conclusion one can say that for $a < .2\lambda$ the classical Clausius-Mossotti formula is a good approximation. Furthermore even for lattice spacings so large that this formula is not valid the lattice acts like an isotropic medium in the sense that the wave length in the medium is independent of the direction of propagation of the wave.

There are several other problems which are of interest and may be handled using modifications of the above outlined theoretical method. One of these is to make the same calculation for a lattice consisting of polarized resonators. The lattice structures to be considered are forms of interpenetrating cubic lattice. Another problem of considerable importance is the problem of the semi infinite lattice and surface reflections. Finally there is the problem of the slab of material composed of resonators embedded in a dielectric medium. It is believed that all of these problems can be handled using modifications of this method.

Summary of Section III. The purpose of this section is the investigation of wave propagation in a medium consisting of a cubic lattice of isotropic dipole resonators. It is assumed that magnetic dipole as well as higher electric and magnetic moments are absent. Thus the resonators are regarded as infinitesimal in size. It is known from the classical work of Lorentz and others that such a lattice array of dipoles acts like an isotropic dielectric medium if the lattice spacing is very small compared with a wave length. The purpose of Section III is the consideration of the same problem for the intermediate case of a lattice spacing comparable with a wave length. The result of this calculation shows that for a lattice constant less than a fifth of a wave length the classical result for the wave propagation through the medium are essentially correct. Furthermore, for a somewhat larger spacing the lattice acts like an isotropic medium even though wave length in the medium is not correctly given by the classical result.

IV. PROPERTIES OF SOLENOIDAL RESONATORS. As was pointed out in the earlier parts of this appendix, the resonators used to load the dielectric should be fairly low loss and should contribute only electric dipole moments to the material and should have their strong resonance or resonances below the operating frequency and should have the remainder of their resonances weakly coupling and at quite high frequencies. The extent to which a solenoidal resonator satisfies these requirements will now be considered.

Consider a helix of length L having n turns per unit length placed in a coordinate system such that the X axis coincides with the axis of the coil and the origin of the coordinate system comes at the center of the coil. The capacity between the ends of the coil resonates with the inductance of the coil to give a low frequency

THE PRINCETON UNIVERSITY RADOME STUDY

resonance in which the current flow varies from zero at the ends of the coil to a maximum in the middle part.

If the current flow a distance X from the center of the coil be denoted by the complex $I(X)$, the magnetic dipole moment of the coil can be designated by

$$P_M = \frac{A}{c} n \int_{-\frac{L}{2}}^{\frac{L}{2}} I(X) dX$$

In this expression A is the area of the coil; and c is the velocity of light. On the other hand the electric dipole moment may be written as

$$P_E = \int_{-\frac{L}{2}}^{\frac{L}{2}} X \rho(X) dX$$

Here $\rho(X)$ is the charge (complex) per unit length of the coil. The other two components of the electric dipole are negligibly small. This can be transformed using the relation

$$i\omega\rho = \frac{dI(X)}{dX}$$

$$P_E = \frac{1}{i\omega} \int_{-\frac{L}{2}}^{\frac{L}{2}} X \frac{dI}{dX} dX = -\frac{1}{i\omega} \int_{-\frac{L}{2}}^{\frac{L}{2}} I dX = \frac{P_M c}{A n} \frac{i}{\omega}$$

or

$$P_M = -i \frac{2\pi A n}{\lambda} P_E$$

Thus if the magnetic dipole moment is to be small compared with the electric dipole moment one has

$$\frac{2\pi A n}{\lambda} \ll 1$$

Regarding the electric quadrupole moment, this may be written as a symmetric 3×3 matrix. If a coordinate system is chosen with the axis of the coil and the two fold symmetry axis of the coil as two of the coordinate axes, the diagonal elements and one of the off-diagonal elements of the matrix are all zero.

MATERIAL OF LOW DIELECTRIC CONSTANT

The off-diagonal elements are very small. It might be expected that they would be approximately the coil dimension times the dipole moment. Actually because of the periodic character of the coil they are much smaller. Difficulties from quadrupole moments should be negligible.

The magnetic dipole moment is another matter. The coil form is desirable to produce a small resonator but the coil form carries with it an objectionable magnetic dipole moment. To avoid these troubles the coil diameter should be kept small in such a way as to make

$$An \ll \lambda$$

However one is not at liberty to vary these quantities at will. The coil must be self resonant at the correct frequency.

The condition for resonance is that the stray capacity of the coil should be resonant with its inductance. This condition for resonance is most conveniently expressed in terms of the length of wire necessary to wind the coil. For a coil of 5 turns and a length equal to twice the coil diameter one finds empirically that the length required is about $.7\lambda$

For a larger spacing between turns the wire length is smaller not being less than $\frac{\lambda}{2}$. This may be written approximately

$$.5\lambda < 2n\pi r < .7\lambda$$

where r is the radius of the coil. This may be written

$$.25\lambda \frac{r}{L} < nA < .35\lambda \frac{r}{L}$$

Consequently the coil should be long compared with its diameter if the magnetic dipole moment is to be negligible.

A resonator having the desirable properties of the solenoid but with a smaller magnetic dipole moment would be desirable. One type worthy of consideration is shown in Fig. D2. It consists of a corrugated strip of copper. This type of resonator is also easily fabricated in the form of two crossed resonators by starting with a cross made from a single piece of copper. (Fig. D3) Although somewhat more difficult, an isotropic resonator of this general type could be made by starting with a thin piece of copper in the form of three crossed strips (Fig. D4) all making equal angles with each other. By corrugating the six arms and bending them into mutually orthogonal positions it should be possible to cause this resonator to resonate in three mutually perpendicular directions. In other words this is a possible form of an isotropic resonator.

THE PRINCETON UNIVERSITY RADOME STUDY

The chief point to be made in connection with the subject of resonators is that although a dielectric loaded with small coils shows on the basis of measurements made considerable promise, this is not to be taken as evidence that a small coil is the best type of resonator.

A rough calculation of the loss in a solenoidal resonator of the type used in the experiments to be described gives a loss Q of about 1000. As the operating frequency differs from the resonance frequency by about 10% in these experiments, it is clear that the phase and amplitude of the current flowing in the coils is determined essentially by the reactance of the coil rather than its resistance.

A more useful way to express this result is to calculate the imaginary part of the dielectric constant under the conditions that the loss of the medium has its origin in coils and that the dielectric constant of the dielectric medium has its value reduced to unity by the contribution from the real part of the polarizability of the coils.

Energy is dissipated in the medium at a rate per unit volume of

$$U = \frac{1}{2} [\underline{\underline{E}} \cdot \underline{\underline{J}}]_{\text{real part}}$$

Here $\underline{\underline{J}}$ is the polarization current of the medium (loaded with coils) This may be written

$$U = \frac{\omega \epsilon_2}{8\pi} |\underline{\underline{E}}|^2$$

where ϵ_2 is the imaginary part of the dielectric constant of the loaded medium. The dissipation power can also be explicitly calculated from the properties of the coils

$$U = \frac{1}{2} \frac{n N R \rho}{8 r} \int_{-\frac{L}{2}}^{\frac{L}{2}} I^2 dx$$

Here N is the number of coils per unit volume, ρ is the resistivity of the metal, R is the radius of the coil, r is the radius of the wire, n is the number of turns per unit length of the coil, L is the length of the coil, δ is the skin depth of the surface current, and $I(x)$ is the current flow at distance x along the length of the coil measure from the center.

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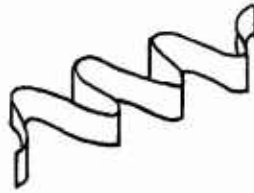


FIG. D-2
CORRUGATED STRIP RESONATOR

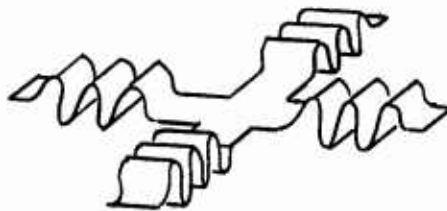


FIG. D-3
CROSSED CORRUGATED RESONATOR

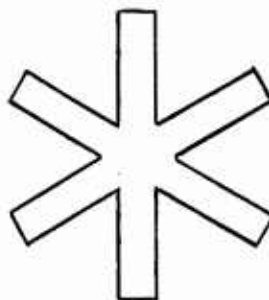


FIG. D-4
CROSS BEFORE CORRUGATING

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MATERIAL OF LOW DIELECTRIC CONSTANT

The contribution of the coils to the polarization of the material is

$$P_c = N \frac{i}{\omega} \int_{-\frac{L}{2}}^{\frac{L}{2}} I dx \quad \text{also} \quad P_c = \frac{1 - \epsilon_0}{4\pi} E$$

since this polarization is opposite to that of the unloaded dielectric.

Here ϵ_0 is the dielectric constant of the medium not loaded by the coils. Also

$$|P_c|^2 = \frac{N^2}{\omega^2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} I(x) I(x') dx dx' \leq \frac{N^2}{\omega^2} L \int_{-\frac{L}{2}}^{\frac{L}{2}} |I(x)|^2 dx$$

For a smoothly varying function the inequality may be taken approximately as an equality. The imaginary part of the dielectric constant may be written after combining these equations

$$\epsilon_2 = \frac{1}{4\pi} \frac{\eta R \rho \omega}{S r N L} (\epsilon_0 - 1)^2$$

For the coils used in the experiments to be described these constants have the following values.

$$\eta = 19 \text{ cm}^{-1}$$

$$N = 4 \text{ cm}^{-3}$$

$$R = .063 \text{ cm}$$

$$r = .01 \text{ cm}$$

$$L = .12 \text{ cm}$$

$$\omega = 6 \times 10^{10} \text{ sec}^{-1}$$

$$S = 7 \times 10^{-5} \text{ cm}$$

$$\rho = 2 \times 10^{-18} \text{ sec}$$

$$\epsilon_0 = 2.5$$

If these values are substituted one obtains the result

$$\epsilon_2 = .07$$

This is a satisfactorily small value considering the thickness of radome

THE PRINCETON UNIVERSITY RADOME STUDY

needed. For ϵ_2 of this value the dielectric absorption at X band is about 1 d.b. per inch of thickness.

Other resonance modes in the coils should be considered briefly. These modes occur at frequencies lying above the resonance frequency consequently they contribute an absorption at high frequencies hence increase the dielectric constant. It is desirable that these effects be small. The only mode about which anything very simple can be said is the mode of frequency next higher than the lowest mode. If the coil is symmetrical this mode has no dipole moment and does not contribute. Higher odd modes do contribute. However, for a given current flow the dipole moment is smaller the higher the mode. The reason for this is a partial cancellation of the dipole moment by currents flowing in opposite directions in various parts of the coil. It is evident from the above that there is considerable reason for making the solenoidal coils symmetrical.

V. MEASUREMENTS. The coils used in the loaded dielectric experiments were wound by us on a coil winding jig made for the purpose. Their shape and dimensions are shown in Fig. D5. It should be noted that they are unsymmetrical, having a straight section at one end used in winding the coil. It has been pointed out that such a lack of symmetry is undesirable in that it produces sizable electric quadrupole moments and that the even as well as odd modes radiate. It would be desirable therefore to use symmetrical coils in any future work.

The coils produced by the jig were found to be quite reproducible. The wire was cut accurately to length before the coil was wound. The outer diameter of the wire was constant to $\pm .001''$ and the spacing between turns to $\pm .0005''$. It is important that coil dimensions, particularly wire length be held carefully as these determine the resonance frequency.

In Fig. D6 there is plotted the reflected amplitude from single coils with air dielectric as a function of their wire length. The amplitude is normalized to read 10 for the reflection from a comparison piece of polystyrene in the form of a square .015" thick and weighing .19 gms.

The measurements and most of the others to be described were made by radiating at about 3.2 cms. with a horn having an opening of 2" x 3". The reflected wave entering back into the horn is fed to a crystal rectifier via a directional complex. The phase of this wave is varied by moving the reflector back and forth to obtain constructive and destructive interference with the normal amplitude resulting from other reflections. The crystal rectified currents under these conditions are combined to give the reflected amplitude. The amplitude is normalized by measuring the reflection from a piece of polystyrene of known shape.

In order that the coils could be embedded in a dielectric medium it is necessary that they be tuned correctly in the dielectric. Consequently coils made from wires of different lengths were in one series of experiments embedded in small blocks of polystyrene. There appears now for the first time the compensation by the coil of the re-

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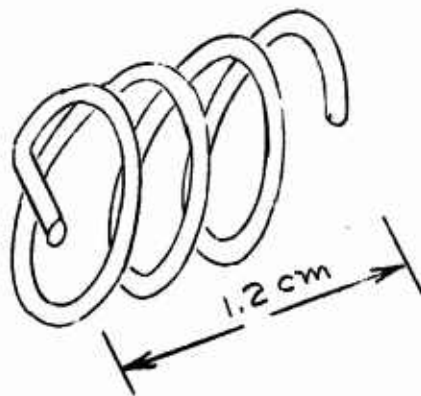
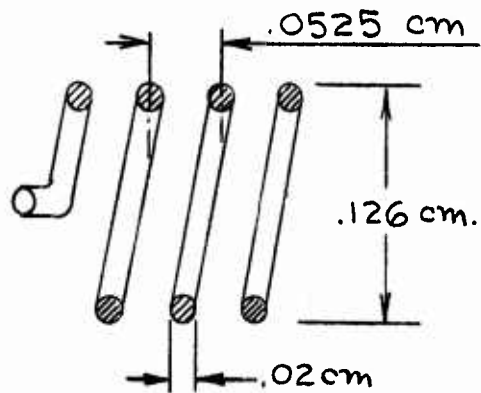


FIG. D 5

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REFLECTION FROM BLOCK OF POLYSTYRENE CONTAINING IMBEDED COIL

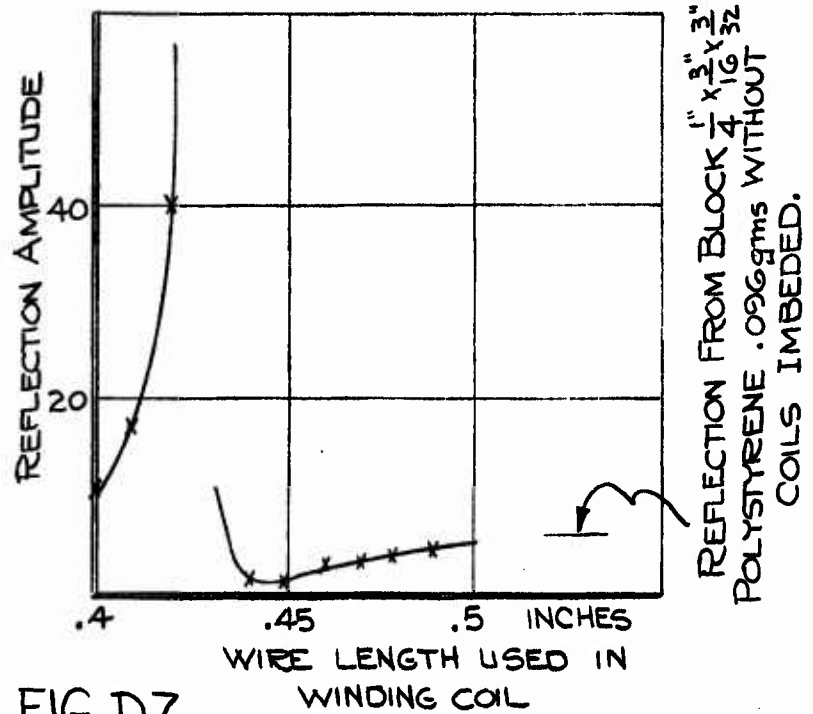


FIG. D7

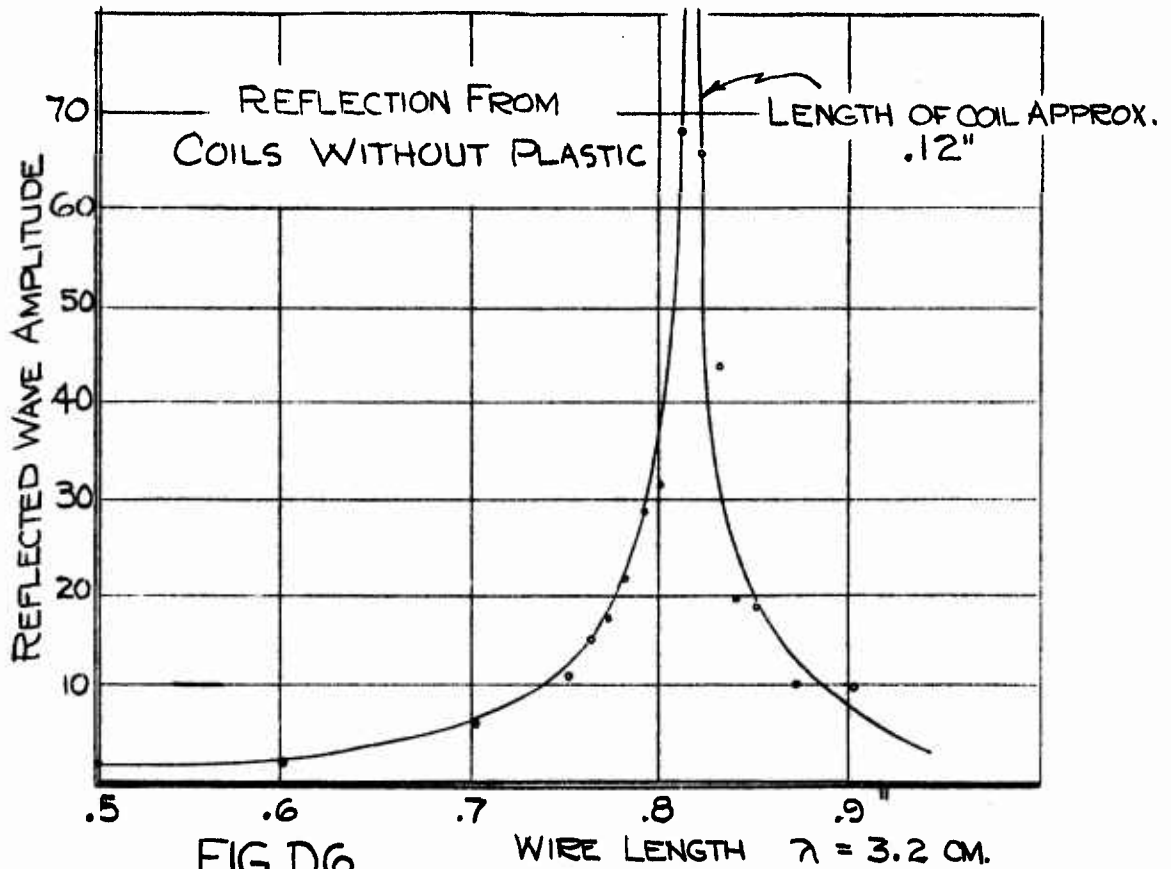


FIG. D6

MATERIAL OF LOW DIELECTRIC CONSTANT

flection from the polystyrene block. In Fig. D7 the reflected amplitude is plotted against wire length for coils embedded in the polystyrene blocks. The reflection from a block of the same shape and size without a coil is also shown. The blocks were roughly $1/4'' \times 3/16'' \times 3/32''$ in dimension. They were cut to have a weight of about .096 gms. Note that for a wire length greater than .42" the reflection falls to a value well under that from the polystyrene block alone.

A similar set of measurements was made with the isolated coil embedded in paraffin giving similar results. In this series of measurements readings were also taken with the coil rotated 90° so that it would be excited by the magnetic dipole interaction. These magnetic dipole measurements did not show a resonance curve thus indicating that the magnetic dipole interaction is small.

The next experiment performed was a measurement of the reflected power from a square of polystyrene loaded with many coils in a lattice array. The polystyrene was $1\ 1/2'' \times 1\ 3/8'' \times 5/32''$. It was composed of three sheets bonded together, the middle sheet of which was perforated with 41 oblong holes to retain the coils. The holes were not filled with a dielectric so the coils were tuned essentially as in free space. The coils had 5 complete turns and were oriented with their axes parallel. Few measurements were made with this sample. The chief result of interest is that at normal incidence and optimum frequency the reflected power was 20 d.b. below that of the sample with the coils removed.

In order to check the operation of the loaded dielectric with the coils actually embedded in the dielectric, another model was made in which the coils were placed in three mutually orthogonal positions, in wells milled in a polystyrene block $1'' \times 1'' \times .095''$. These holes were then filled with paraffin after the coils had been placed in them. The pattern of holes milled in the polystyrene block is shown in Fig. D8. It is to be noted that each of the three coil orientations has a face centered cubic lattice structure. This leads to a close packing of the coils.

In Fig. D9 there is plotted the amplitude and phase of the wave reflected by the sample at normal incidence against frequency in m.c. The amplitude scale is chosen so that the unloaded dielectric of the same dimensions has a reflection amplitude 1. It may be noted that at optimum the reflected power is about 20 d.b. below that from the unloaded dielectric. The phase of the reflected wave is also plotted and has the characteristic resonance shape.

Measurements were also made at normal incidence for other orientations of the electric vector relative to the sample. The frequency at which the lowest reflection occurred varied perhaps a few tenths of a percent among the various positions. The reflected power at optimum was about the same, however.

Secret

THE PRINCETON UNIVERSITY RADOME STUDY

Measurements of reflected power at an angle of incidence of 45° were made both with the electric vector in the plane of incidence and perpendicular to the plane of incidence. With the frequency at about 9370 m.c., the reflected power was for the electric vector perpendicular to the plane of incidence roughly 15 d.b. below that of the unloaded sheet. The reflected power for the other polarization was about 75% of that from the unloaded sheet. The reason for the poor performance in this case is not hard to find. For this polarization the longitudinal coils play a role and these are no doubt differently tuned because of the difference in the surrounding plastic and difference in the configuration of neighboring coils. It would be necessary to tune these especially for a given configuration. These measurements made at an angle of incidence of 45° are not particularly reliable. The reflector was so small that edge effects are important. In order to study reflection coefficients at large angles of incidence it would be desirable to have substantially larger samples of the material.

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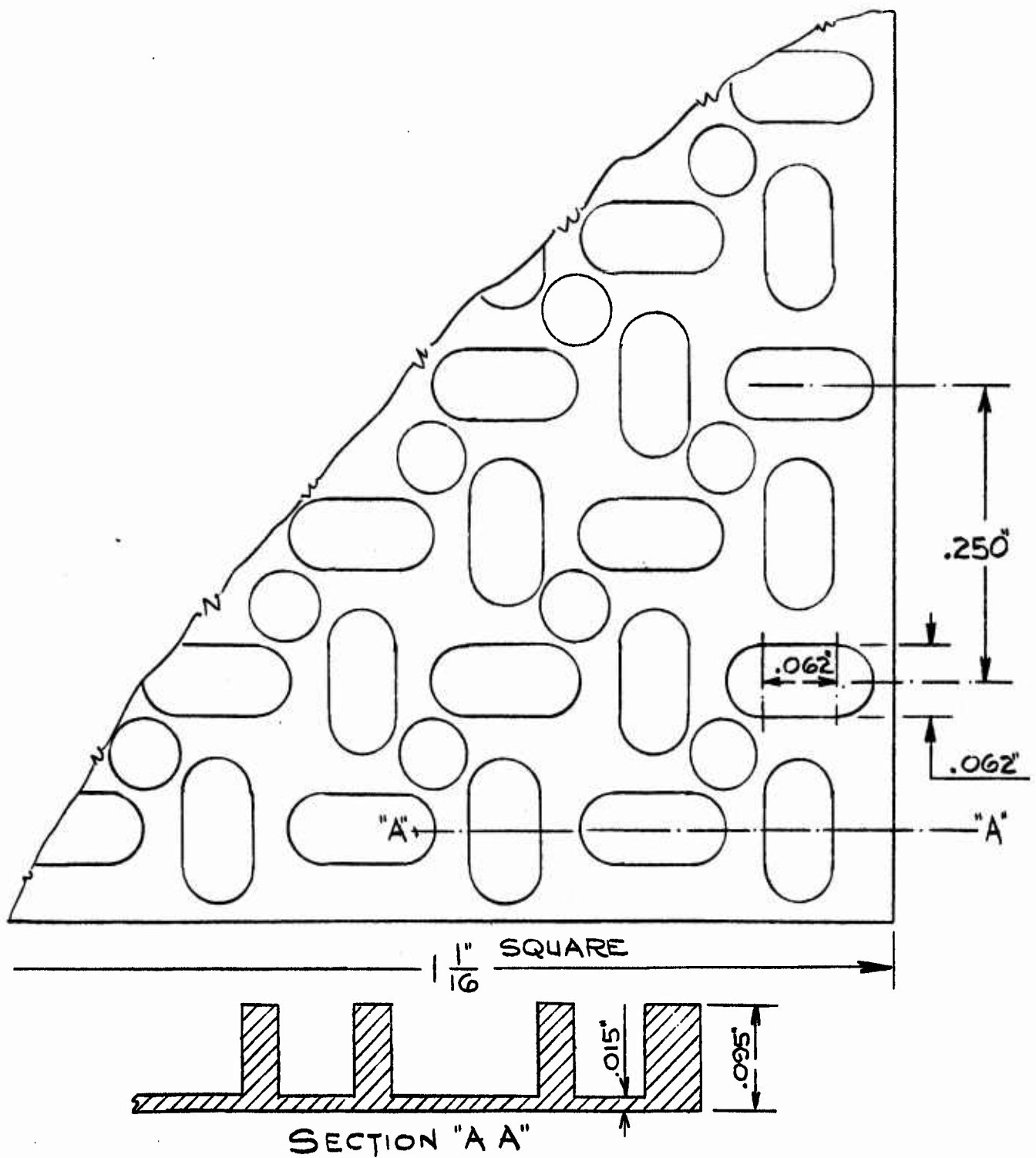


FIG. D8

POLYSTYRENE FORM FOR COIL LOADING

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REFLECTED AMPLITUDE FROM COIL LOADED DIELECTRIC NORMAL INCIDENCE

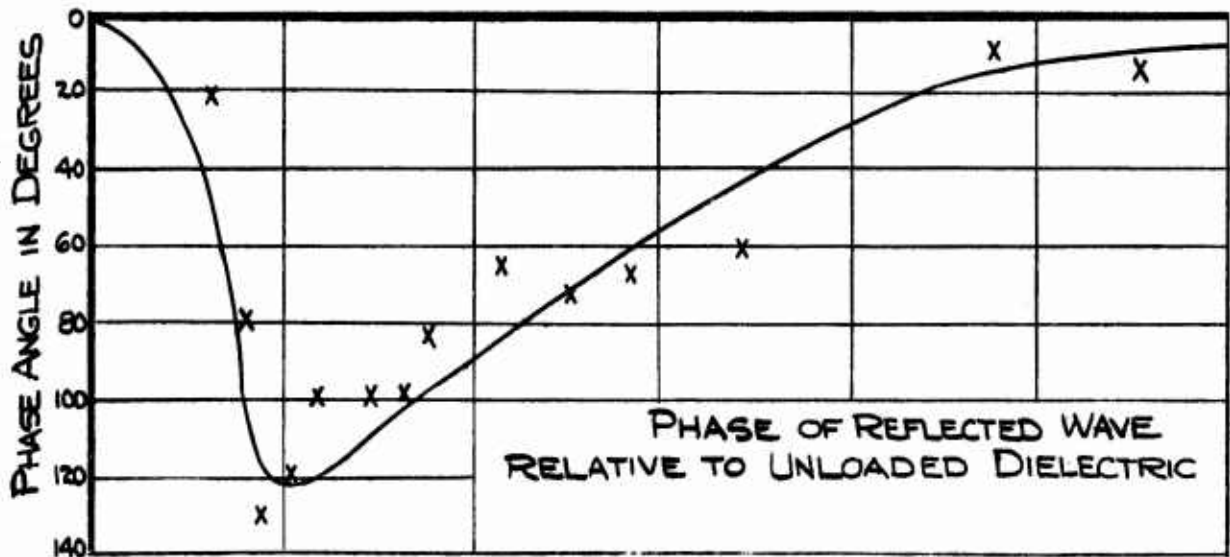
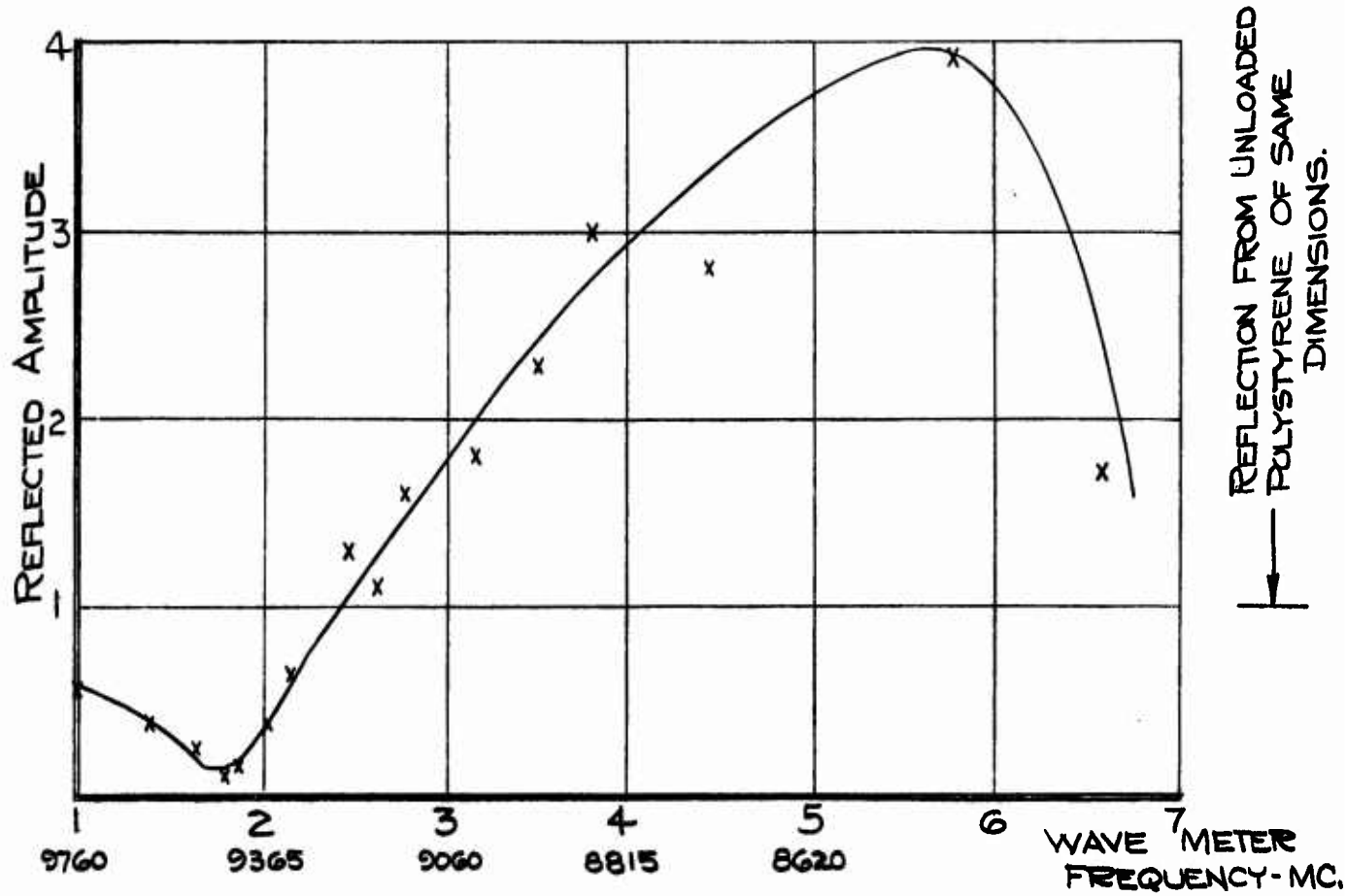


FIG. D9

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APPENDIX E

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

It is the purpose of this appendix to consider in a qualitative way the possible use of a metal screen containing a lattice array of resonant apertures as a radome wall material. Such a screen could be formed in a conical or ogive shape to serve as a radome. It could be used alone, or with the resonant apertures closed by a thin plastic skin, or conceivably it could be used as a middle element in a plastic wall to serve as an element for tuning out reflections.

It is a further purpose of this appendix to present the basic mathematical methods needed for theoretical considerations of screens of this type. This section was prepared by Dr. T. Teichmann. It represents the first collection in one place of the relevant theoretical methods required for a reasonably complete theoretical approach to the problem of diffraction by a thin metal screen containing an array of resonant apertures.

Fig. E1 shows one particular form which a screen of this type might take. In this case the resonant apertures are in the form of slots approximately $1/2$ wave length long. Fig. E2 shows a modification of E1 in which both polarizations are transmitted by the screen. Fig. E3, E4 and E5 show other forms which the screen may take. Some experimental work was done on the types of screen shown in Fig. E1 and E5. These results will be presented shortly.

The guiding physical principle which suggests the use of a thin conducting wall perforated by resonant irises as a suitable wall materials for the construction of radomes is found in "Babinet's principle." Just why it is given this name is not clear as it is a quite different result from the well known principle of physical optics.

Stated in the form most useful for the present purposes, "Babinet's principle" states that if a certain thin plane perfectly conducting metal screen reflects completely a certain incident plane electromagnetic wave, then the same wave but with the orthogonal polarization will be completely transmitted by the complementary screen. By complementary screen is here meant the thin metal screen obtained by replacing metal by air and air by metal in the original screen.

To illustrate this principle note that the screen complementary to Fig. E1 would be a set of parallel metal bars each $1/2$ wave long. Such a set of resonant antennas is known to strongly reflect a wave having its electric vector parallel to the bars. Consequently Fig. E1 (the complementary screen) strongly transmits the polarization with the electric field perpendicular to the slots.

As another example consider a metal wall without holes. It reflects all the radiation falling upon it. The complementary screen of course transmits everything.

THE PRINCETON UNIVERSITY RADOME STUDY

The chief usefulness of Babinet's principle is that it enables one to think of obstacles that are strongly reflecting and then to translate them into transmitting screens. It somehow seems easier to think qualitatively of a way to build a reflecting wall than a transmitting wall.

In the considerations of loaded dielectrics it was found desirable to employ a small resonator in a closely packed lattice. Some of these considerations also apply in the case of the metal screens. Fig. E3 and E4 illustrate a type of resonant iris which because of its snake-like shape can be caused to have small overall dimensions, hence for which a small lattice constant is possible. There are undoubtedly other shapes of resonator which can be constructed in a compact way.

The screen shown in Fig. E5 was found experimentally to have a very low reflection at normal incidence at the proper frequency. The power reflection coefficient was about .06% with a reflection coefficient under .15% over a 2% frequency band. Both of these measurements were for normal incidence. The reflection increased rather badly if for the same frequency the angle of incidence were increased to 45°.

Measurements on the screen shown in Fig. E1 gave comparable results at normal incidence. However, bandwidth measurements were not made. This screen continued to transmit essentially 100% for all angles of incidence lying in a plane such that the slots were perpendicular to the plane. The electric vector was of course in the plane of incidence. The same screen performed rather badly if the plane of incidence was parallel to the slots and the electric vector was perpendicular to the plane of incidence.

These experimental results are understandable in a qualitative way. Some of the difficulties are clearly related to the large size of the resonator and large spacing constant. As very little work has been done on screens of this type it is the opinion of this group that a modest effort should be made to explore the possibilities of this line of approach. In particular it is suggested that some work be done to ascertain the usefulness of the constructions shown in Figs. E3 and E4 as well as related structures having a small lattice spacing.

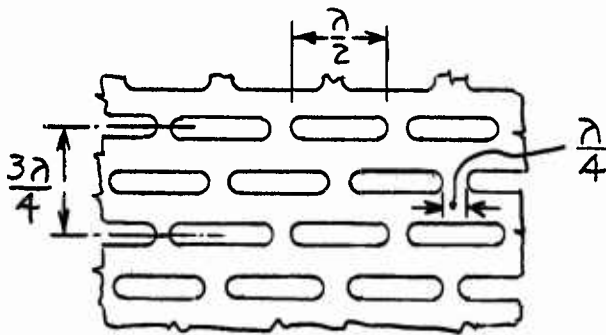


FIG. E1

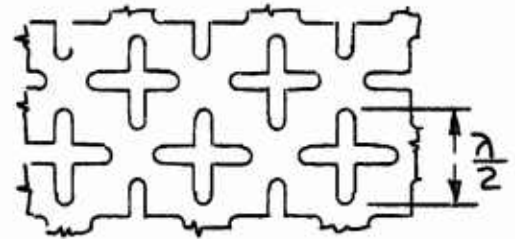


FIG. E2

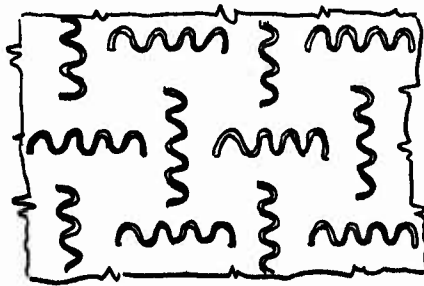


FIG. E3

COPPER .005" THICK
 $\lambda = 3.2$ cm.

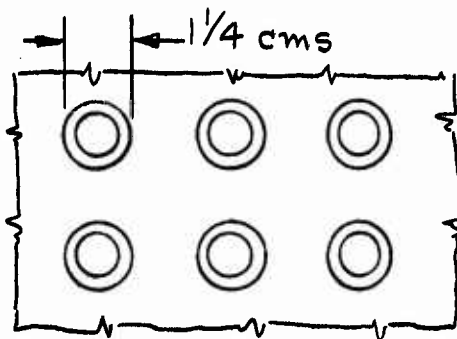


FIG. E5

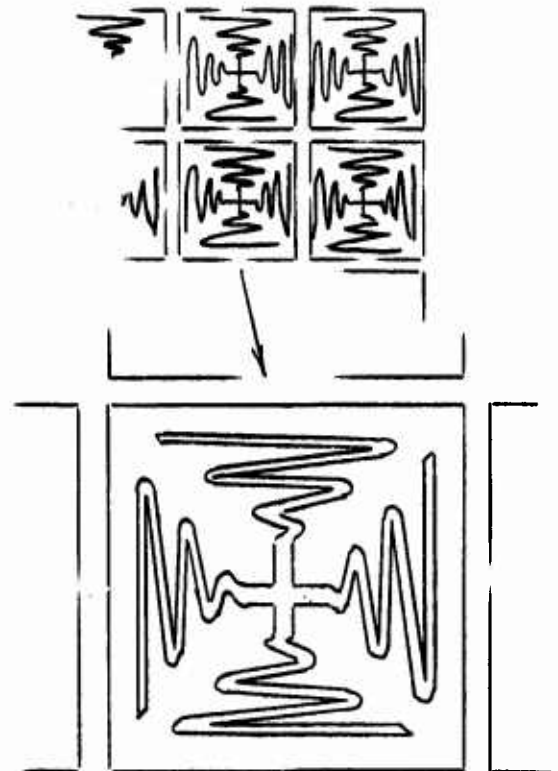


FIG. E4

RESONANT IRIS LATTICE STRUCTURES

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

Report On Plane Electromagnetic Diffraction

<u>Contents</u>	<u>Page</u>
I Introduction	E 4
II Scalar Diffraction	E 4
III Electromagnetic Diffraction; Basic Formulae	E 10
IV The Electromagnetic Babinet Principle	E 14
V Consequences of Screen Symmetry	E 22
VI The Far Field	E 29
VII Reformulation of the Variational Principle	E 34
VIII Concluding Remarks	E 40
IX References	E 40

THE PRINCETON UNIVERSITY RADOME STUDY

I. INTRODUCTION This report presents some general results on the scattering of electromagnetic waves by plane, perfectly conducting screens. Though many of these results are known to some extent, they have not previously been presented together, despite their common origin (mainly) in various symmetry properties.

In the first place, the symmetry of the problem with respect to the plane of the screen leads to equivalence relations for complementary screens, which are, of course, nothing more than the rigorous statements of Babinet's principle. The acoustic and electromagnetic "Babinet" principles are proved, by the methods of Bouwkamp⁽¹⁾ and Meixner⁽²⁾ respectively, and the latter is extended to the case of a plane screen symmetrically embedded in a dielectric sheet. It is also shown just why such a relation cannot hold in complete generality for a curved screen. (symmetry I)

Secondly, the symmetry of the screen itself, i.e. of the individual aperture(s), or of the array of apertures leads to certain symmetry properties of the aperture field, and hence of the scattered field. These properties are derived here from the integral equation formulation of the problem, taken in the form given by Schwinger and Levine⁽³⁾, which is concise, and most convenient for the present purpose, though it is not as manifestly rigorous as some of the other equivalent formulations. (eg. von Maue⁽⁴⁾, Severin⁽⁵⁾, Copson⁽⁶⁾, and various others)

II. SCALAR DIFFRACTION The subject is rendered rather more perspicuous by a preliminary consideration of some aspects of the scalar wave diffraction problem, which shows some of the difficulties which occur in the scalar case, and are present, a fortiori, in the case of vector waves.

Since the temporal variation of the disturbance to be considered may be expressed as a Fourier integral, it is sufficient to consider only a single frequency ω , and to write

$$K = \omega/c$$

C = velocity in the medium considered. The problem is thus reduced to finding solutions $u(x,y,z)$ of the equation

$$\nabla^2 u + K^2 u = 0 \quad (1)$$

satisfying the following conditions:

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

- (i) u satisfies the boundary condition

$$u = 0$$

on the screen S which lies in the plane
 $z = 0$

- (i)' $\frac{\partial u}{\partial n}$ satisfies the boundary condition

$$\frac{\partial u}{\partial n} = 0$$

on S

- (ii) $u, \frac{\partial u}{\partial n}$ are continuous across the aperture A (consisting of the remainder of the plane $z = 0$;

$$\Sigma = S + A \quad \text{being} \quad (3)$$

the entire z plane).

- (iii) u consists of a prescribed incident wave u_i , coming from the side $z < 0$, say) plus a scattered wave u_s which satisfies the radiation condition at infinity

$$\lim_{r \rightarrow \infty} r \left(\frac{\partial u_s}{\partial r} + i k u_s \right) = 0 \quad (4)$$

Applying Green's theorem to an infinite half space bounded by a surface Σ one knows that

$$u(P) = - \int_{\Sigma} \left(u(Q) \frac{\partial G(P, Q)}{\partial n_Q} - G(P, Q) \frac{\partial u(Q)}{\partial n_Q} \right) d\Sigma_Q \quad (5)$$

where the Green's function $G(P, Q)$ is given by

$$G(P, Q) = G(r_{PQ}) = e^{i k r_{PQ}} / 4\pi r_{PQ} \quad (6)$$

with

$$r_{PQ} = [(x_P - x_Q)^2 + (y_P - y_Q)^2 + (z_P - z_Q)^2]^{1/2}$$

THE PRINCETON UNIVERSITY RADOME STUDY

$u(P)$ is thus determined for $z_P \neq 0$ by the values of

$$u(Q), \quad \frac{\partial u(Q)}{\partial n_Q} \quad \text{for} \quad z_Q = 0,$$

i.e. by the value of u and its normal derivative on the screen S and over the aperture A . It is, however, possible to determine $u(P)$ in terms of either u or $\partial u / \partial n$ on Σ if Green's functions can be found satisfying respectively either

$$G^{(1)}(P, Q) = 0 \quad z_Q = 0 \quad \text{i.e. } Q \text{ on } \Sigma \quad (7)$$

or

$$\frac{\partial G^{(2)}(P, Q)}{\partial n} = 0 \quad z_Q = 0 \quad " \quad " \quad (8)$$

and having a singularity of the form

$$G^{(\nu)}(P, Q) \sim \frac{1}{4\pi r_{PQ}} \quad \text{as } r_{PQ} \rightarrow 0$$

Such functions do exist, but it is generally impossible to find them if Σ is not a plane. If Σ is a plane, however, they may be found on the grounds of symmetry of the problem with respect to

$$z = 0$$

(i.e. using the so-called "reflection principle"). To this end one introduces the operator Z , which reflects vectors in the plane

$$z = 0$$

i.e. if

$$P = (x_P, y_P, z_P)$$

then

$$ZP = (x_P, y_P, -z_P)$$

Then clearly

$$G^{(1)}(P, Q) = \frac{1}{2} [G(P, Q) - G(ZP, Q)] \quad (9)$$

$$G^{(2)}(P, Q) = \frac{1}{2} [G(P, Q) + G(ZP, Q)] \quad (10)$$

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

are the requisite function, and

$$u^{(1)}(P) = - \int_{\Sigma} u(Q) \frac{\partial G^{(1)}(P, Q)}{\partial n_Q} d\Sigma_Q \quad (11)$$

$$u^{(2)}(P) = \int_{\Sigma} \frac{\partial u(Q)}{\partial n_Q} G^{(2)}(P, Q) d\Sigma_Q \quad (12)$$

are the corresponding expressions for u in terms of its values and derivatives on Σ . In general one has

$$u(P) = u^{(1)}(P) + u^{(2)}(P) \quad (13)$$

The problem is thus reduced to the determination of the values of u and/or $\partial u / \partial n$ on Σ . It is clear from the above expressions that these quantities satisfy certain integral equations whose solution therefore gives that of the whole problem. It is then possible to determine these values of $u(Q)$, $\frac{\partial u(Q)}{\partial n_Q}$, Q on Σ

by setting up a suitable variational principle. It is not interesting to pursue this aspect of the matter further for the scalar case, since the vector case, which is the one of importance here, can be treated by analogous methods, and will be discussed in the next section.

On the other hand, despite certain formal similarities, it is worth while to give the scalar version of Babinet's principle in a more extended form, because it illuminates the idea of "complementary" screens.

One thus considers the two following scalar diffraction problems:
A disturbance u , incident from the left of the plane is diffracted
(a), by the aperture A in the screen S , with the boundary conditions

$$u = 0$$

on S ; u , $u' = \frac{\partial u}{\partial z}$ continuous across A ($A + S$ comprises the entire plane Σ , $z = 0$); and

(b) by the obstacle A , with the boundary condition $\bar{u}' = 0$ on A ; $\bar{u}$, $\bar{u}'$ continuous across S

[The bar denotes that $\bar{u}$ is a solution of the complementary problem (b). In the following only the z dependence will be indicated. In all equations where question may arise the values of (x, y) must be understood to be the same on both sides of the equation.]

(a) In the absence of any aperture, i.e. for

$$A = 0$$

THE PRINCETON UNIVERSITY RADOME STUDY

the total disturbance $u_o(z)$ may be written

$$u_o(z) = u_i(z) + u_r(z)$$

where $u_r(z)$ is the additional field radiated from Σ .
For $z > 0$

$$u_o(z) = 0$$

$$\therefore u_r(z) = -u_i(z) \quad \text{for } z > 0$$

Hence $u_o(0) = 0$ as required.
Furthermore,

$$u_r(z) = u_r(-z) \quad \text{by symmetry}$$

$$\therefore u_r(z) = -u_i(-z) \quad \text{for } z > 0$$

and hence

$$u_o(z) = u_i(z) - u_i(-z) \quad \text{for } z > 0 \quad (14)$$

$$\therefore u_o'(-0) = 2u_i'(0)$$

$$u_o'(+0) = 0$$

In the case of a non-vanishing aperture, $A \neq 0$ one may write for the total disturbance

$$u(z) = u_o(z) + u_p(z) \quad (15)$$

where $u_p(z)$ is the so-called perturbation disturbance.

$u_p(z)$ is a radiated field, and satisfies the following conditions:

- (i) The wave equation (1);
- (ii) The radiation condition at infinity (4);
- (iii) The boundary conditions

$$(a) \quad u_p(0) = 0 \quad \text{on } S \quad \therefore u(0) = u_o(0) = 0 \quad \text{on } S$$

$$(b) \quad u_p'(\pm 0) = \mp u_i'(0) \quad \text{on } A \quad \therefore u'(z) \text{ is continuous across } A.$$

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

$u_p(z)$ is thus completely determined in either halfspace, and satisfies the symmetry relation

$$u_p(z) = u_p(-z) \quad (16)$$

(b) In this case the total disturbance $\bar{u}(z)$ may be written

$$\bar{u}(z) = u_i(z) + \bar{u}_p(z) \quad (17)$$

where $\bar{u}_p(z)$ the radiated perturbation field in this case, satisfies the following conditions:

- (i) The wave equation (1);
- (ii) The radiation condition at infinity (4);
- (iii) The boundary conditions

$$(\alpha) \quad \bar{u}_p'(0) = -u_i'(0) \quad \text{on } A \quad \because \bar{u}'(0) = 0 \quad \text{on } A$$

$$(\beta) \quad \bar{u}_p(0) = 0 \quad \text{on } S \quad \because \bar{u}(z) \text{ is continuous across } S, \text{ and } \bar{u}_p(z) \text{ satisfies the symmetry condition}$$

$$\bar{u}_p(z) = -\bar{u}_p(-z) \quad (18)$$

$\bar{u}_p(z)$ is therefore also completely determined in either half space. In particular it is identical with $u_p(z)$ for $z < 0$

$$\text{i.e.} \quad \bar{u}_p(z) = u_p(z) \quad z \leq 0 \quad (19)$$

Hence

$$\bar{u}_p(z) = -u_p(z) \quad z \geq 0 \quad (20)$$

i.e.

$$\boxed{\bar{u}_p(z) + u_p(z) = 0} \quad \text{for } z \geq 0 \quad (21)$$

i.e. for the same excitation, the sum of the perturbation disturbances of the two complementary problems is zero for $z \geq 0$. Schematically the state of affairs may be represented as is shown in Fig. E6 a and b.

THE PRINCETON UNIVERSITY RADOME STUDY

III. ELECTROMAGNETIC DIFFRACTION: BASIC FORMULAE In the case of electromagnetic diffraction by a plane perfectly conducting screen, one is led to the following problem. An incident field $\underline{E}_i, \underline{H}_i$ is directed on to the screen S (lying in the plane $z=0$), from the half space $z < 0$, and it is required to find the total field $\underline{E}, \underline{H}$. As in the scalar case it is possible to restrict the discussion to a single frequency ω , and it is convenient to choose units so that the field equations become

$$\left. \begin{aligned} \nabla \times \underline{H} &= i k \underline{E} \\ \nabla \times \underline{E} &= -i k \underline{H} \end{aligned} \right\} \quad \begin{aligned} \nabla \cdot \underline{H} &= 0 \\ \nabla \cdot \underline{E} &= 0 \end{aligned} \quad (1)$$

where

$$k = \omega \sqrt{\frac{\epsilon \mu}{c^2}}$$

the various symbols having the usual significance. It is convenient to refer the solutions of the problem to the simple (trivial) solutions of two directly soluble cases, viz. the case of no screen at all

$$S = 0$$

and the case of no aperture

$$A = 0$$

In the first case, one has simply $\underline{E} = \underline{E}_i, \underline{H} = \underline{H}_i$ and one is therefore led to write more generally

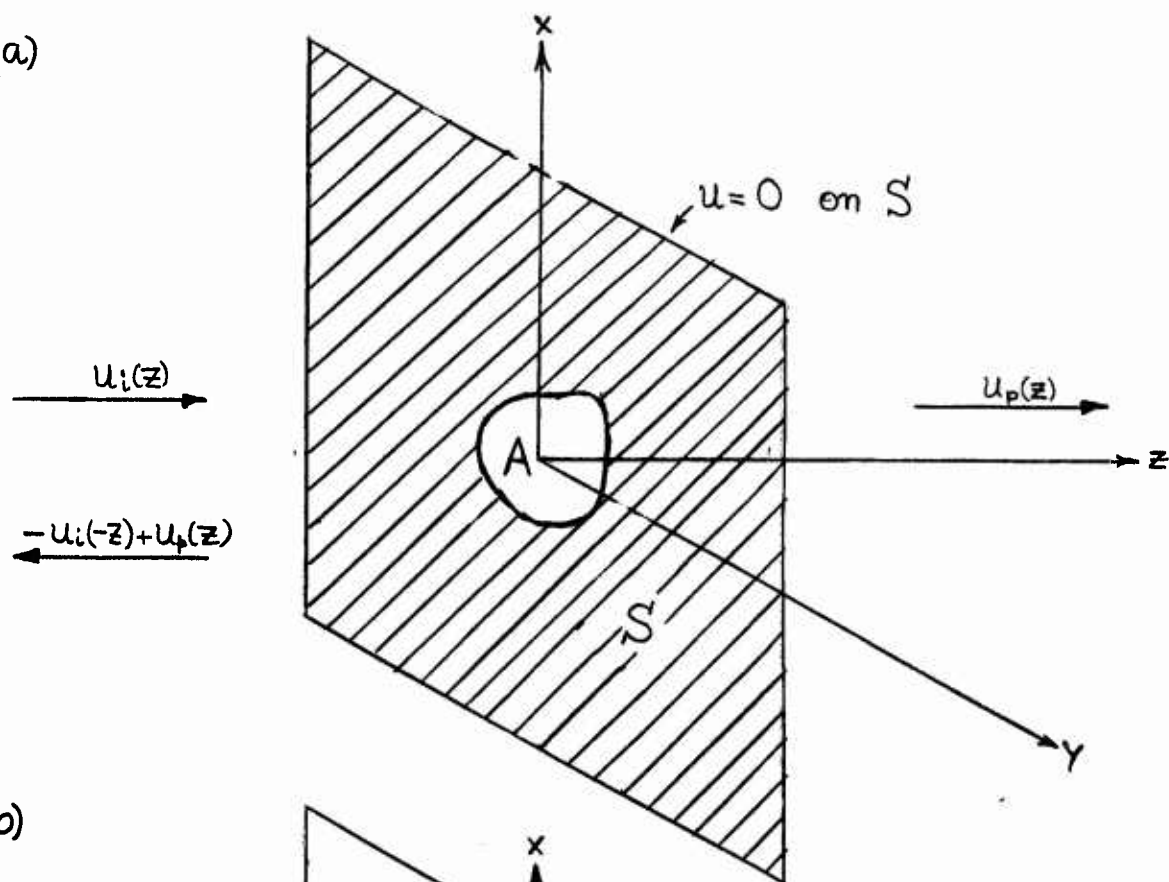
$$\left. \begin{aligned} \underline{E} &= \underline{E}_i + \underline{E}_p \\ \underline{H} &= \underline{H}_i + \underline{H}_p \end{aligned} \right\} \quad (2)$$

in the case when the screen S is finite, $S < \infty$. The second configuration corresponds to reflection by an infinite perfectly conducting plane, for which one has

$$\begin{aligned} \underline{E} &= \underline{E}_0 \\ &= \underline{E}_i + \underline{E}_r \\ \underline{H} &= \underline{H}_0 \\ &= \underline{H}_i + \underline{H}_r \end{aligned}$$

where $\underline{E}_r, \underline{H}_r$ are easily calculable from the condition that $\underline{E}, \underline{H}$

(a)



(b)

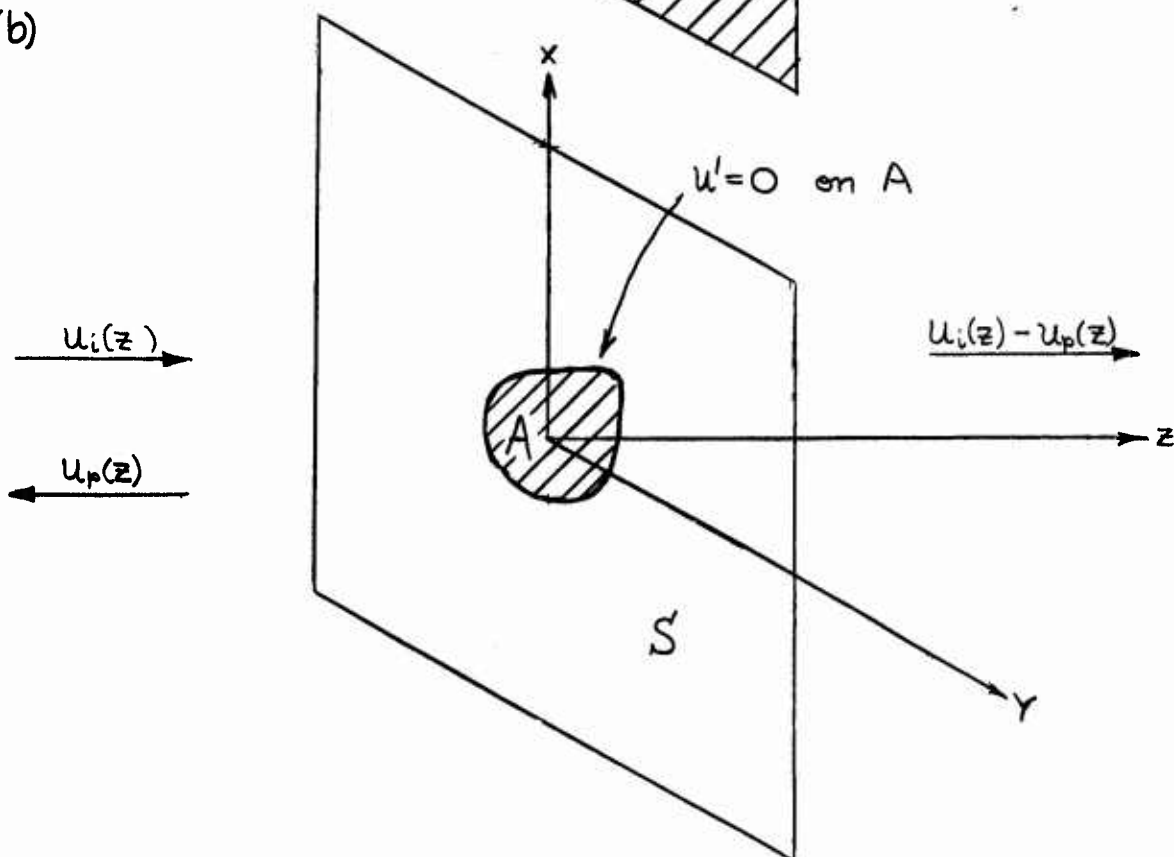


FIGURE E6.

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

must vanish for $z > 0$, and from the symmetry. (See section IV). In the case of a finite aperture $0 < A < \infty$ it is therefore natural to write

$$\left. \begin{aligned} \underline{\underline{E}} &= \underline{\underline{E}}_0 + \underline{\underline{E}}_P \\ \underline{\underline{H}} &= \underline{\underline{H}}_0 + \underline{\underline{H}}_P \end{aligned} \right\} \quad (3)$$

The $\underline{\underline{E}}_P, \underline{\underline{H}}_P$ thus defined by (2) and (3) are therefore the "natural" perturbation fields introduced by a finite obstacle and a finite aperture respectively. These two definitions are, of course, nothing more than two different points of view of the same situation. It may even turn out convenient, for mathematical purposes to use (2) for an infinite screen, or (3) for a finite obstacle, though the perturbation fields thus introduced cannot then have much physical significance. In the case when both the screen and the aperture are infinite, both points of view are equally admissible physically, and as before there is nothing to choose between them mathematically, except possible convenience.

The problem to be solved is then the following: It is required to find solutions $\underline{\underline{E}}, \underline{\underline{H}}$ of equations (1), having the form (2) or (3), and satisfying the conditions

$$(i) \quad \underline{\underline{E}} \times \underline{\underline{k}} = 0 \quad \text{on } S, \quad \underline{\underline{k}} \text{ being the normal to } S; \quad (4)$$

$$(ii) \quad \underline{\underline{E}}, \underline{\underline{H}} \quad \text{are continuous across } A; \quad (5)$$

$$(iii) \quad \underline{\underline{E}}, \underline{\underline{H}} \quad \text{satisfy integrability condition at the edge of } A$$

$$\text{so that the electromagnetic field energy in any finite volume remains finite;} \quad (6)$$

$$(iv) \quad \text{The perturbation fields } \underline{\underline{E}}_P, \underline{\underline{H}}_P \text{ of } \underline{\underline{E}}, \underline{\underline{H}} \text{ satisfy a radiation condition at infinity, to insure that there are only outgoing waves there (apart from } \underline{\underline{E}}_i, \underline{\underline{H}}_i) \left. \vphantom{\begin{aligned} \underline{\underline{E}}_P, \underline{\underline{H}}_P \end{aligned}} \right\} \quad (7)$$

Since $\underline{\underline{E}}, \underline{\underline{H}}$ satisfy the vector wave equations

$$\left. \begin{aligned} \nabla^2 \underline{\underline{E}} + \kappa^2 \underline{\underline{E}} &= 0 \\ \nabla^2 \underline{\underline{H}} + \kappa^2 \underline{\underline{H}} &= 0 \end{aligned} \right\} \quad (8)$$

it is natural to look for the analogue of the Green's function

$$G(\underline{\underline{r}}_P, \underline{\underline{r}}_Q) = G(P, Q) = G(\underline{\underline{r}}_{PQ})$$

THE PRINCETON UNIVERSITY RADOME STUDY

which solves the scalar problem. Since one now has unknown vector functions to deal with, it is clear that a Green's dyadic, rather than a scalar function, is required. Using the method of Weyl⁽⁷⁾, or Schwinger⁽⁸⁾, one finds that a suitable free space Green's dyadic for the equations (8) is given by

$$\begin{aligned}\Gamma(\underline{r}_P, \underline{r}_Q) &= \Gamma(P, Q) \\ &= (I - \frac{1}{k^2} \nabla_P \nabla_Q) G(P, Q)\end{aligned}\quad (9)$$

where I is the unit dyadic

$$I = \underline{i}\underline{i} + \underline{j}\underline{j} + \underline{k}\underline{k}$$

$\underline{i}, \underline{j}, \underline{k}$ being unit vectors in the x, y and z directions respectively. Rather as in the scalar case, one can, in principle construct electric and magnetic Green's dyadics, $\Gamma^{(1)}(P, Q)$ and $\Gamma^{(2)}(P, Q)$ which enable one to express the solutions $\underline{E}(H)$ in terms of the transverse electric (magnetic) or transverse magnetic (electric) fields respectively over the bounding surface Σ . $\Gamma^{(1)}, \Gamma^{(2)}$ have the same type of singularity as Γ , and satisfy the boundary conditions

$$\underline{n}_Q \times \Gamma^{(1)}(Q, P) = 0 \quad Q \text{ on } \Sigma \quad (10)$$

$$\underline{n}_Q \times (\nabla_Q \times \Gamma^{(2)}(Q, P)) = 0 \quad Q \text{ on } \Sigma \quad (11)$$

$\underline{n}_Q$ being the normal to Σ at the point Q . One then finds that

$$\left. \begin{aligned}\underline{E}(P) &= - \int_{\Sigma} d\Sigma_Q (\underline{n}_Q \times \underline{E}(Q)) \cdot (\nabla_Q \times \Gamma^{(1)}(Q, P)) \\ \underline{H}(P) &= i k \int_{\Sigma} d\Sigma_Q \Gamma^{(2)}(P, Q) \cdot (\underline{n}_Q \times \underline{E}(Q))\end{aligned} \right\} \quad (12)$$

or

$$\left. \begin{aligned}\underline{E}(P) &= -i k \int_{\Sigma} d\Sigma_Q (\underline{n}_Q \times \underline{H}(Q)) \cdot \Gamma^{(2)}(Q, P) \\ \underline{H}(P) &= - \int_{\Sigma} d\Sigma_Q (\underline{n}_Q \times \underline{H}(Q)) \cdot (\nabla_Q \times \Gamma^{(1)}(Q, P))\end{aligned} \right\} \quad (13)$$

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

It is generally not possible to find simple forms for $\Gamma^{(1)}$, $\Gamma^{(2)}$ if Σ is an arbitrary surface, but in the case where Σ is the plane

$$z = 0$$

one can apply the reflection principle, as for the scalar Green's function to obtain

$$\Gamma^{(1)}(P, Q) = \Gamma(P, Q) - \Gamma(P, ZQ) \cdot ZI \quad (14)$$

$$\Gamma^{(2)}(P, Q) = \Gamma(P, Q) + \Gamma(P, ZQ) \cdot ZI \quad (15)$$

for $z_P, z_Q \geq 0$ while for $z_P, z_Q \leq 0$

$$\Gamma^{(v)}(P, Q) = \Gamma^{(v)}(ZP, ZQ) \quad (16)$$

Using the expressions (12) to (16), and the boundary conditions to be satisfied on Σ viz. (4) and (5), (i.e. on the screen S and the aperture A), one may obtain integral equations for the aperture fields

$$\begin{aligned} \underline{\underline{F}}(Q) &= \underline{\underline{m}}_Q \times \underline{\underline{E}}(Q) \\ &= \underline{\underline{k}} \times \underline{\underline{E}}(Q) \end{aligned}$$

in the case of a plane, (17)

and

$$\underline{\underline{K}}(Q) = \underline{\underline{k}} \times \underline{\underline{H}}(Q) \quad (18)$$

and hence a variational principle for $\underline{\underline{F}}(Q)$, $\underline{\underline{K}}(Q)$ respectively, and expressions for the total field. These results will be stated in section V, in dealing with the consequences of symmetry in the plane Σ .

To conclude this general section three important theorems will be stated regarding symmetry with respect to the plane Σ , which follow almost immediately from Maxwell's equations (1) and the integral expressions above, and which are used in section IV:

THE PRINCETON UNIVERSITY RADOME STUDY

(i) If $\underline{E}, \underline{H}$ is a solution of Maxwell's equations, so is $\underline{E}', \underline{H}'$ where

$$\begin{aligned}\underline{E}'_t(P) &= \underline{E}_t(ZP) & \underline{H}'_t(P) &= -\underline{H}_t(ZP) \\ \underline{E}'_z(P) &= -\underline{E}_z(ZP) & \underline{H}'_z(P) &= \underline{H}_z(ZP)\end{aligned}\quad (20)$$

Here $\underline{E}_t$ denotes the transverse components $(E_x, E_y, 0)$ of $\underline{E}$, etc.

(ii) The outgoing wave from a plane screen S' lying in the plane $z = 0$

has the symmetry character

$$\begin{aligned}\underline{E}_t(ZP) &= \underline{E}_t(P) & \underline{H}_t(ZP) &= -\underline{H}_t(P) \\ \underline{E}_z(ZP) &= -\underline{E}_z(P) & \underline{H}_z(ZP) &= \underline{H}_z(P)\end{aligned}\quad (21)$$

This also follows from the scalar case by noting the boundary conditions

$$\underline{E}_t = 0, \quad \underline{H}_z = 0, \quad \frac{\partial \underline{E}_z}{\partial z} = 0, \quad \frac{\partial \underline{H}_t}{\partial z} = 0$$

on the conducting screen.

(iii) If $\underline{E}, \underline{H}$ is a solution of Maxwell's equations in a medium with constants ϵ, μ , then $\underline{E}', \underline{H}'$ is a solution of the equations in a medium $\epsilon' = \mu, \mu' = \epsilon$ where

$$\begin{aligned}\underline{E}' &= \pm \underline{H} \\ \underline{H}' &= \mp \underline{E}\end{aligned}\quad (22)$$

IV. THE ELECTROMAGNETIC BABINET PRINCIPLE. In analogy with the scalar problem, one would now like to compare the diffraction of a field $\underline{E}_i, \underline{H}_i$ incident from the left (say) by an aperture A in a perfectly conducting screen S (both lying in the plane Σ

$$\Sigma : z = 0$$

and that of the same incident field by the complementary obstacle A . Since $\underline{E}, \underline{H}$ play the roles of $u, \partial u / \partial n$ respectively in the vector case, this means that the complementary obstacle would have to have infinite magnetic permeability, since one requires

$$\underline{H}_t = 0$$

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

on the obstacle

$$\therefore \underline{E}_t = 0$$

on the perfectly conducting screen. Since magnetic screens are not even remotely realizable physically, it is more interesting to replace the second problem by one in which the complementary electromagnetic field $\underline{E}_i = \underline{H}_i$, $\underline{H}_i = -\underline{E}_i$ is scattered by the perfectly conducting electric screen (obstacle) A i.e. instead of scattering the same incident field by a completely (i.e. geometrically and electromagnetically) complementary screen, one scatters an electromagnetically complementary field by the geometrically complementary screen.

In order to consider the diffraction of the incident field $\underline{E}_i, \underline{H}_i$ by a finite aperture A in an infinite screen, one starts from the total field obtained if

$$A = 0$$

(i.e. total reflection). This field, $\underline{E}_0, \underline{H}_0$ is given by

$$\underline{E}_0 = \underline{E}_i + \underline{E}_r \tag{1}$$

$$\underline{H}_0 = \underline{H}_i + \underline{H}_r$$

(cf. notation in the scalar case, section II).
Since

$$(\underline{E}_0, \underline{H}_0) = (0, 0)$$

for $z \geq 0$, the incoming field being incident from the left, one has

$$\underline{E}_r(z) = -\underline{E}_i(z) \tag{2}$$

$$\underline{H}_r(z) = -\underline{H}_i(z)$$

for $z \geq 0$. Hence for $z \leq 0$

$$\underline{E}_{rt}(z) = -\underline{E}_{it}(z) \tag{3}$$

$$\underline{H}_{rt}(z) = \underline{H}_{it}(z) \tag{4}$$

by III(21), and in particular, for $z = 0$ (i.e. on Σ)

$$\underline{E}_{rt}(0) = -\underline{E}_{it}(0) \tag{5}$$

$$\underline{H}_{rt}(\pm 0) = \mp \underline{H}_{it}(0) \tag{6}$$

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THE PRINCETON UNIVERSITY RADOME STUDY

where ± 0 merely denotes that Σ is being approached from the right (+) or left (-) respectively. It is only necessary to consider the transverse components, because the longitudinal components can be obtained from the former using Maxwell's equations.

For a finite aperture $A \neq 0$, the field becomes

$$\begin{aligned}\underline{E} &= \underline{E}_0 + \underline{E}_p \\ \underline{H} &= \underline{H}_0 + \underline{H}_p\end{aligned}\tag{7}$$

$\underline{E}_p, \underline{H}_p$ being a perturbation field radiated by the screen, and hence satisfying the same symmetry conditions as $\underline{E}_r, \underline{H}_r$ with respect to reflection in the plane Σ ($z=0$). The boundary conditions then require

$$\underline{E}_{pt}(0) = 0 \quad \text{on } S\tag{8}$$

$$\underline{H}_{pt}(\pm 0) = \pm \underline{H}_{it}(0) \quad \text{on } A\tag{9}$$

In addition $\underline{E}_p, \underline{H}_p$ must satisfy Maxwell's equations, and integrability and radiation conditions. These requirements determine $\underline{E}_p, \underline{H}_p$ uniquely.

Now consider the complementary problem i.e. an incident field

$$\underline{E}_i' = \underline{H}_i, \quad \underline{H}_i' = -\underline{E}_i \quad (\text{from } z < 0)$$

diffracted by the (conducting) obstacle A . The total field $\underline{E}', \underline{H}'$ can be written

$$\begin{aligned}\underline{E}' &= \underline{E}_i' + \underline{E}_p' \\ \underline{H}' &= \underline{H}_i' + \underline{H}_p'\end{aligned}\tag{10}$$

where $\underline{E}_p', \underline{H}_p'$ is the perturbation field for this case. This field too satisfies the symmetry conditions for a radiated field.

The boundary conditions require that

$$\underline{E}'_{pt}(0) = -\underline{E}'_{it}(0) = -\underline{H}_{it}(0) \quad \text{on } A\tag{11}$$

$$\underline{H}'_{pt}(0) = 0 \quad \text{on } S\tag{12}$$

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

This latter condition follows from the continuity across S and the fact that $\underline{H}_{pt}$ is antisymmetric with respect to reflection in Σ . This latter condition, in turn, follows from the fact that $\underline{H}_p$ is the magnetic field radiated from a plane current distribution. It is therefore clear that condition (11) will generally not hold for curved surfaces Σ . It is not known whether there exist curved surfaces such that (11) holds for an arbitrary incident field, though it seems very unlikely. It is conceivable however, that certain combinations of curved surfaces Σ and incident fields $\underline{E}_i, \underline{H}_i$ may lead to the validity of (11), and such cases one will, of course, have a sort of restricted Babinet's principle, (the restriction is then on the incident fields!), by the same conclusion as that below, which obtains for Σ plane. In this case, too, since $\underline{E}'_p, \underline{H}'_p$ also must satisfy Maxwell's equations, and integrability and radiation conditions, the requirements determine $\underline{E}'_p, \underline{H}'_p$ uniquely everywhere.

Comparison of (8) (9) and (11) (12) now shows that for $z \gg 0$ $\underline{E}'_p, \underline{H}'_p$ satisfy exactly the same requirements as $-\underline{H}_p, \underline{E}_p$. Hence for $z \gg 0$, one has

$$\underline{E}'_p + \underline{H}_p = 0 \quad (13)$$

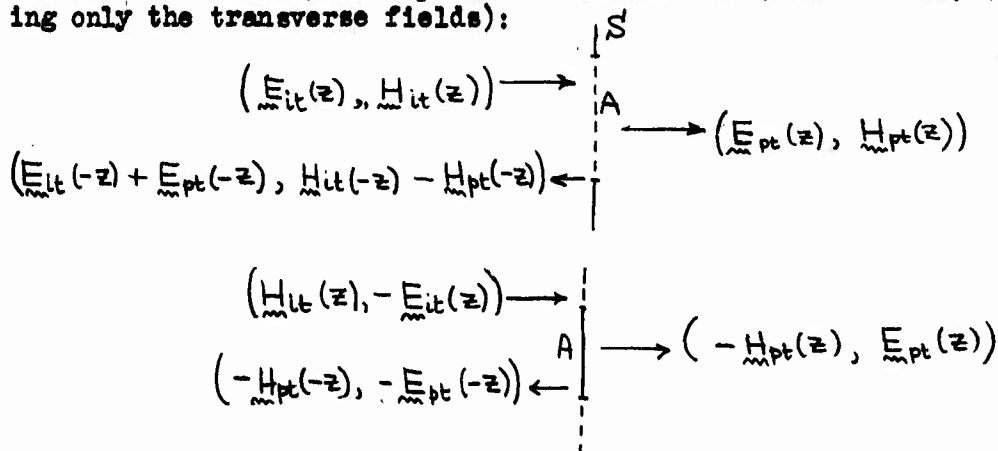
$$\underline{H}'_p - \underline{E}_p = 0 \quad (14)$$

or in terms of the total fields

$\underline{E}' + \underline{H} = \underline{H}_i$	(15)
$\underline{E} - \underline{H}' = \underline{E}_i$	(16)

The field is then given for $z \leq 0$ by the symmetry conditions.

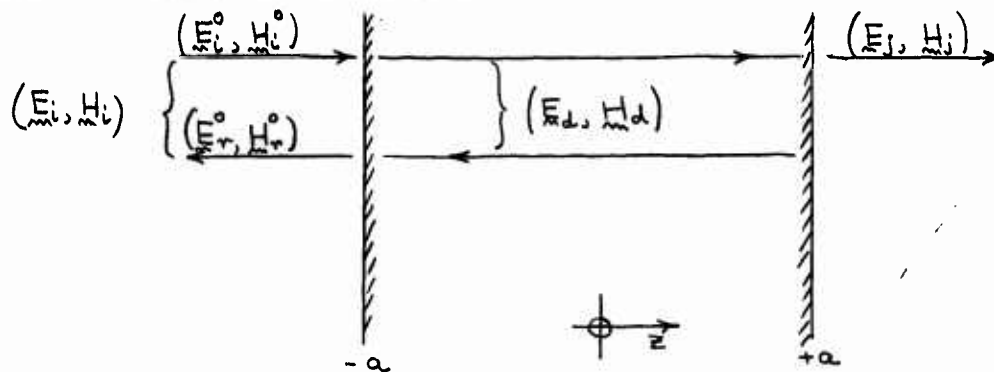
The relations (13), (14) or (15), (16) are the rigorous formulations of Babinet's principle for electromagnetic diffraction. The state of affairs may be represented schematically as follows, (considering only the transverse fields):



THE PRINCETON UNIVERSITY RADOME STUDY

It is possible to extend this result (in a manner conjectured by R. H. Dicke) by considering the screen embedded in the middle of an infinite parallel slab of dielectric, with constants (ϵ, μ) .

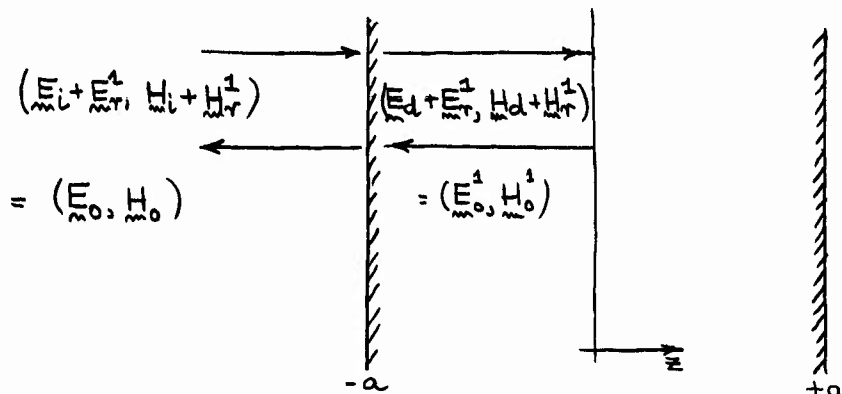
- (i) In order to do this, one starts by examining the effect of the dielectric slab alone. If $\underline{E}_i^0, \underline{H}_i^0$ is the incident field, one has the following state of affairs:



All the quantities illustrated satisfy Maxwell's equations, and in addition, one has the following conditions at $x = \pm a$

$$\begin{aligned}
 \underline{E}_{it}(-a-) &= \underline{E}_{dt}(-a+) & \underline{H}_{it}(-a-) &= \underline{H}_{dt}(-a+) \\
 \underline{E}_{iz}(-a-) &= \epsilon \underline{E}_{dz}(-a+) & \underline{H}_{iz}(-a-) &= \underline{H}_{dz}(-a+) \\
 \underline{E}_{jt}(a+) &= \underline{E}_{dt}(a-) & \underline{H}_{jt}(a+) &= \underline{H}_{dt}(a-) \\
 \underline{E}_{jz}(a+) &= \epsilon \underline{E}_{dz}(a-) & \underline{H}_{jz}(a+) &= \underline{H}_{dz}(a-)
 \end{aligned} \tag{17}$$

- (ii) Next one takes the case of the same slab with a solid conducting screen at its centre. The schematic state of affairs is then:



METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

The addition of the solid screen means an additional disturbance radiated from the screen $(\underline{E}_r^1, \underline{H}_r^1)$ which has the usual symmetry properties, and must satisfy the conditions

$$\left. \begin{aligned} \underline{E}_r^1 &= -\underline{E}_i \\ \underline{H}_r^1 &= -\underline{H}_i \end{aligned} \right\} \quad \text{for } z > a$$

$$\left. \begin{aligned} \underline{E}_r^1 &= -\underline{E}_d \\ \underline{H}_r^1 &= -\underline{H}_d \end{aligned} \right\} \quad \text{for } 0 \leq z < a$$

(18)

$\underline{E}_r^1, \underline{H}_r^1$ thus satisfies the right continuity conditions at $z = \pm a$

Introducing

$$\begin{aligned} \underline{E}_o^1 &= \underline{E}_d + \underline{E}_r^1 \\ \underline{H}_o^1 &= \underline{H}_d + \underline{H}_r^1 \\ \underline{E}_o &= \underline{E}_i + \underline{E}_r^1 \\ \underline{H}_o &= \underline{H}_i + \underline{H}_r^1 \end{aligned}$$

one finds for the transverse components

$$\left. \begin{aligned} \underline{E}_{ot}^1(z) &= \underline{E}_{dt}(z) - \underline{E}_{dt}(-z) \\ \underline{H}_{ot}^1(z) &= \underline{H}_{dt}(z) + \underline{H}_{dt}(-z) \end{aligned} \right\} \quad \text{for } 0 > z > -a$$

$$\left. \begin{aligned} \underline{E}_{ot}(z) &= \underline{E}_{it}(z) - \underline{E}_{jt}(-z) \\ \underline{H}_{ot}(z) &= \underline{H}_{it}(z) + \underline{H}_{jt}(-z) \end{aligned} \right\} \quad \text{for } -a > z > -\infty$$

(19)

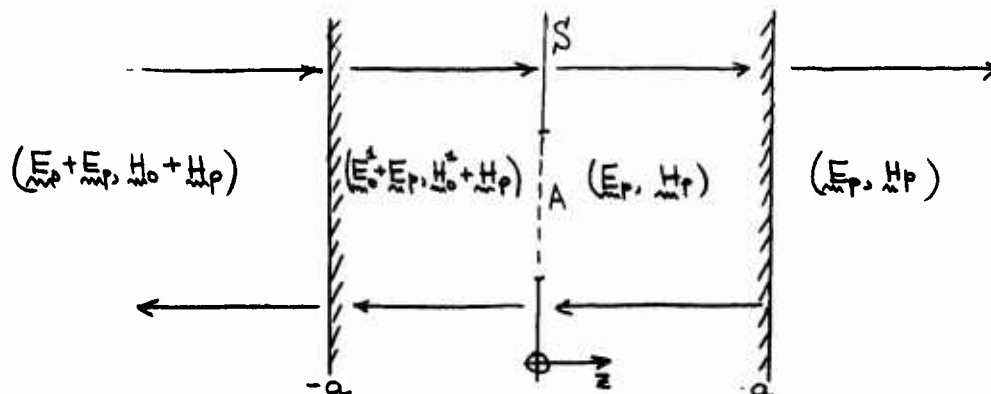
In particular, for $z = 0$

$$\begin{aligned} \underline{E}_{ot}^1(0) &= 0 \\ \underline{H}_{ot}^1(-0) &= 2\underline{H}_{dt}(0) \\ \underline{H}_{ot}^1(+0) &= 0 \end{aligned}$$

(20)

THE PRINCETON UNIVERSITY RADOME STUDY

(iii) One now comes to the original problem of a screen S with an aperture A embedded in the dielectric, which is again illustrated below:



There is now an additional perturbation field E_p, H_p as shown satisfying the usual symmetry conditions, and the following boundary conditions:

$$\underline{E}_{pt}(-a-) = \underline{E}_{pt}(-a+)$$

$$\underline{H}_{pt}(-a-) = \underline{H}_{pt}(-a+)$$

$$\epsilon E_{pz}(-a-) = \epsilon E_{pz}(-a+)$$

$$H_{pz}(-a-) = H_{pz}(-a+)$$

$$\underline{E}_{pt}(0) = 0$$

on S

(21)

$$\underline{H}_{pt}(\pm 0) = \pm H_{dt}(0)$$

on A

$$\underline{E}_{pt}(a-) = \underline{E}_{pt}(a+)$$

$$\underline{H}_{pt}(a-) = \underline{H}_{pt}(a+)$$

$$\epsilon E_{pz}(a-) = E_{pz}(a+)$$

$$H_{pz}(a-) = H_{pz}(a+)$$

Since E_p, H_p must also satisfy Maxwell's equations, and radiation and integrability conditions, they are uniquely determined everywhere.

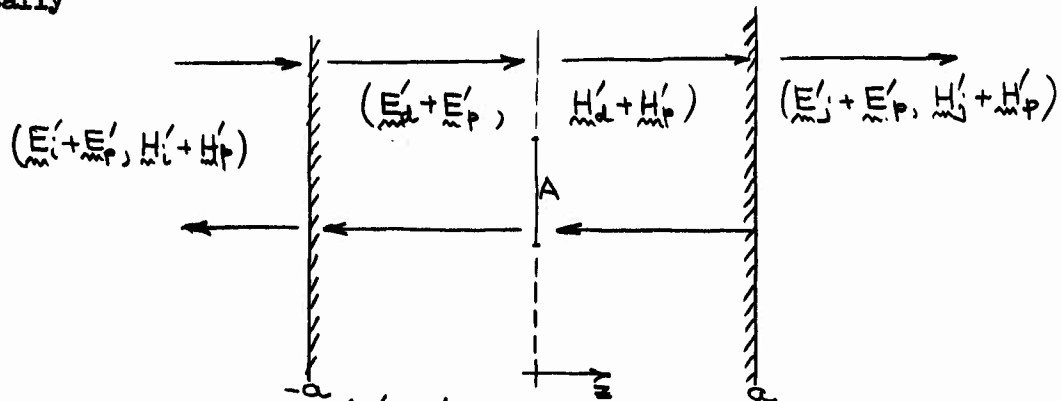
(iv) One may now consider the complementary problem, which is that of the diffraction of an incident field

$$(\underline{E}_i^{\circ'}, \underline{H}_i^{\circ'}) = (\underline{H}_i^{\circ}, -\underline{E}_i^{\circ})$$

by an obstacle A (perfectly conducting), embedded in the middle of a slab of magnetic material of constants μ, ϵ . In the absence of any obstacle one would, of course, have fields $(\underline{E}_1^{\circ}, \underline{H}_1^{\circ}), (\underline{E}_2^{\circ}, \underline{H}_2^{\circ}), (\underline{E}_3^{\circ}, \underline{H}_3^{\circ})$ in the three portions of space, which would be obtained from the corresponding fields $(\underline{E}_i, \underline{H}_i), (\underline{E}_d, \underline{H}_d), (\underline{E}_j, \underline{H}_j)$ for the

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

the dielectric slab by substitution $\underline{E} \rightarrow \underline{H}$, $\underline{H} \rightarrow -\underline{E}$. In the case of a non-vanishing obstacle one has the following state of affairs schematically



The perturbation field $(\underline{E}'_p, \underline{H}'_p)$ again satisfies the usual symmetry conditions, and the following boundary conditions:

$$\begin{aligned}
 \underline{H}'_{pt}(-a-) &= \underline{H}'_{pt}(-a+) & \underline{E}'_{pt}(-a-) &= \underline{E}'_{pt}(-a+) \\
 \underline{H}'_{pz}(-a-) &= \epsilon \underline{H}'_{pz}(-a+) & \underline{E}'_{pz}(-a-) &= \underline{E}'_{pz}(-a+) \\
 \underline{H}'_{pt}(0) &= 0 & \text{on } S \\
 \underline{E}'_{pt}(0) &= -\underline{E}'_{dt}(0) & \text{on } A & (22) \\
 \underline{H}'_{pt}(a-) &= \underline{H}'_{pt}(a+) & \underline{E}'_{pt}(a-) &= \underline{E}'_{pt}(a+) \\
 \epsilon \underline{H}'_{pz}(a-) &= \underline{H}'_{pz}(a+) & \underline{E}'_{pz}(a-) &= \underline{E}'_{pz}(a+)
 \end{aligned}$$

$(\underline{E}'_p, \underline{H}'_p)$ also satisfy Maxwell's equations and radiation and integrability conditions, and hence they too are uniquely determined everywhere. In particular, comparison of the above conditions with those of paragraph (iii) shows that $(\underline{E}'_p, \underline{H}'_p)$ satisfies the same boundary value problem in the right half plane as

$$(-\underline{H}_p, \underline{E}_p)$$

Therefore

$$\begin{aligned}
 \underline{E}'_p + \underline{H}_p &= 0 \\
 \underline{H}'_p - \underline{E}_p &= 0
 \end{aligned} \tag{23}$$

as in the case of no dielectric slab, while for the total fields the situation reads

THE PRINCETON UNIVERSITY RADOME STUDY

$$\begin{array}{l} \underline{H} + \underline{E}' = \underline{H}_j \\ \underline{E} - \underline{H}' = \underline{E}_j \end{array} \quad (24)$$

Note that it is the initial transmitted field which now appears on the right.

V. CONSEQUENCES OF SCREEN SYMMETRY Before proceeding to derive the consequences of screen symmetry, it is necessary to write down "explicitly" the solution of the diffraction problem, using the results of section III.

To begin with, there is no loss of generality in considering an incident plane wave coming from the left (since any incident field may be written as a superposition of such waves). One therefore writes

$$\left. \begin{array}{l} \underline{E}_i(\underline{r}; \underline{n}_0, \underline{e}_0) = \underline{e}_0 e^{ik\underline{n}_0 \cdot \underline{r}} \\ \underline{H}_i(\underline{r}; \underline{n}_0, \underline{h}_0) = \underline{h}_0 e^{ik\underline{n}_0 \cdot \underline{r}} \end{array} \right\} \quad (1)$$

where

$$\underline{h}_0 = \underline{n}_0 \times \underline{e}_0 \quad (2)$$

In the absence of any aperture the field for $z \leq 0$ is given by

$$\left. \begin{array}{l} \underline{E}_0(\underline{r}; \underline{n}_0, \underline{e}_0) = \underline{e}_0 e^{ik\underline{n}_0 \cdot \underline{r}} - \underline{e}_0 \cdot \underline{Z} \underline{I} e^{ik\underline{n}_0 \cdot \underline{Z} \underline{r}} \\ \underline{H}_0(\underline{r}; \underline{n}_0, \underline{h}_0) = \underline{h}_0 e^{ik\underline{n}_0 \cdot \underline{r}} + \underline{h}_0 \cdot \underline{Z} \underline{I} e^{ik\underline{n}_0 \cdot \underline{Z} \underline{r}} \end{array} \right\} \quad (3)$$

For $z = 0$

$$\left. \begin{array}{l} \underline{k} \times \underline{E}_0 = 0 \\ \underline{k} \cdot \underline{H}_0 = 0 \end{array} \right\} \quad (4)$$

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

It follows from section III and the boundary conditions derived in section IV that the total fields are

$$\underline{E}(\underline{r}; \underline{n}_0, \underline{e}_0) = \int_A dA(\underline{s}) \underline{E}(\underline{s}; \underline{n}_0, \underline{h}_0) \cdot (\nabla_{\underline{s}} \times \Gamma^{(1)}(\underline{s}, \underline{r})) \quad (5)$$

for $z \geq 0$

$$\underline{H}(\underline{r}; \underline{n}_0, \underline{h}_0) = i\kappa \int_A dA(\underline{s}) \Gamma^{(2)}(\underline{r}, \underline{s}) \cdot \underline{E}(\underline{s}; \underline{n}_0, \underline{h}_0) \quad (6)$$

and

$$\underline{E}(\underline{r}; \underline{n}_0, \underline{e}_0) = \underline{E}_0(\underline{r}; \underline{n}_0, \underline{e}_0) - \int_A dA(\underline{s}) \underline{E}(\underline{s}; \underline{n}_0, \underline{h}_0) \cdot (\nabla_{\underline{s}} \times \Gamma^{(1)}(\underline{s}, \underline{r})) \quad (7)$$

for $z \leq 0$

$$\underline{H}(\underline{r}; \underline{n}_0, \underline{h}_0) = \underline{H}_0(\underline{r}; \underline{n}_0, \underline{h}_0) + i\kappa \int_A dA(\underline{s}) \Gamma^{(2)}(\underline{r}, \underline{s}) \cdot \underline{E}(\underline{s}; \underline{n}_0, \underline{h}_0) \quad (8)$$

$\underline{E}(\underline{s}; \underline{n}_0, \underline{h}_0)$ being the aperture field

$$\underline{E}(\underline{s}; \underline{n}_0, \underline{h}_0) = \underline{k} \times \underline{E}(\underline{s}; \underline{n}_0, \underline{e}_0) \quad (9)$$

($\underline{s}$ denotes the current point on A).

$\underline{E}(\underline{s}; \underline{n}_0, \underline{h}_0)$ is determined by the integral equation

$$(\underline{k} \times \underline{h}_0) e^{i\kappa \underline{n}_0 \cdot \underline{s}'} = -2i\kappa \underline{k} \times \int_A dA(\underline{s}) \Gamma(\underline{s}', \underline{s}) \cdot \underline{E}(\underline{s}; \underline{n}_0, \underline{h}_0) \quad (10)$$

and the condition

$$\underline{E}(\underline{s}; \underline{n}_0, \underline{h}_0) = 0 \quad \text{on } S \quad (11)$$

Now suppose that the screen and aperture are invariant under certain rotations R (in the plane Σ),
i.e.

$$|R\underline{s}| = |\underline{s}|, \quad (12)$$

$$R\underline{k} = \underline{k},$$

$$R0 = 0,$$

THE PRINCETON UNIVERSITY RADOME STUDY

and under some translations T (also in the plane Σ)

$$T_{\underline{s}} = \underline{s} + \underline{t} \quad \text{say} \quad (13)$$

$$\underline{k} \cdot \underline{t} = 0$$

The translational invariance is only possible if the aperture is of infinite extent, i.e.g. an infinite slit, or an infinite regular array of finite apertures. The rotations and translations must of course be mutually adapted.

The symmetry relations to be derived below follow rather simple from (10) (or (5) to (8), and the following important symmetries of the $\Gamma^{(j)}(\underline{x}', \underline{x}'')$

$$\Gamma(\underline{x}', \underline{x}'') = \Gamma(\underline{x}'', \underline{x}') \quad (14)$$

$$\Gamma(\underline{x}' + \underline{t}, \underline{x}'' + \underline{t}) = \Gamma(\underline{x}', \underline{x}'') \quad (15)$$

$$\Gamma(R\underline{x}', R\underline{x}'') = R \Gamma(\underline{x}', \underline{x}'') R \quad (16)$$

(where $R(a \ b) R = (R a)(R b)$)

$$\nabla' \times \Gamma^{(j)}(\underline{x}' + \underline{t}, \underline{x}'' + \underline{t}) = \nabla' \times \Gamma^{(j)}(\underline{x}', \underline{x}'') , \quad z', z'' \geq 0 \quad (17)$$

$$\nabla' \times \Gamma^{(j)}(R\underline{x}', R\underline{x}'') = R \nabla' \times \Gamma^{(j)}(\underline{x}', \underline{x}'') R , \quad z', z'' \geq 0 \quad (18)$$

Applying these results to (10), one finds

$$\begin{aligned} (k_x h_0) e^{i k y_0 \cdot \underline{s}'} &= -2 i k k_x \int_A dA(\underline{s}) \Gamma(\underline{s}', \underline{s}) \cdot \underline{E}(\underline{s}; \underline{m}_0, h_0) \\ &= -2 i k k_x \int_A dA(\underline{s}) \Gamma(\underline{s}', \underline{s} + \underline{t}) \cdot \underline{E}(\underline{s} + \underline{t}; \underline{m}_0, h_0) \\ &= -2 i k k_x \int_A dA(\underline{s}) \Gamma(\underline{s}' - \underline{t}, \underline{s}) \cdot \underline{E}(\underline{s} + \underline{t}; \underline{m}_0, h_0) \end{aligned}$$

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

Hence

$$F(\underline{s} + \underline{t}; \underline{n}_0, \underline{h}_0) = e^{i\kappa \underline{n}_0 \cdot \underline{t}} F(\underline{s}; \underline{n}_0, \underline{h}_0) \quad (19)$$

where $\underline{t}$ is one of the permissible translations or lattice vectors. In particular if Σ is composed of an infinite array of identical cells Σ_{lm} each with an aperture A_{lm} the permissible translations have the form

$$\underline{t} = \underline{t}_{lm} = l\underline{a} + m\underline{b} \quad (20)$$

$\underline{a}, \underline{b}$ being the lattice vectors, and l, m integers. The above result then implies that

$$F(\underline{s}; \underline{n}_0, \underline{h}_0) = e^{i\kappa \underline{n}_0 \cdot \underline{s}} f(\underline{s}; \underline{n}_0, \underline{h}_0) \quad (21)$$

where

$$f(\underline{s}; \underline{n}_0, \underline{h}_0) = \sum_p \sum_q f_{pq}(\underline{n}_0, \underline{h}_0) e^{2\pi i(p\alpha + q\beta) \cdot \underline{s}} \quad (22)$$

is a doubly periodic function over the lattice, $\underline{\alpha}, \underline{\beta}$ being the reciprocal lattice vectors (i.e. $\underline{\alpha} \cdot \underline{a} = 1, \underline{\alpha} \cdot \underline{b} = 0, \underline{\beta} \cdot \underline{a} = 0, \underline{\beta} \cdot \underline{b} = 1$). Hence

$$f(\underline{s} + \underline{t}_{lm}; \underline{n}_0, \underline{h}_0) = f(\underline{s}; \underline{n}_0, \underline{h}_0) \quad (23)$$

$f_{pq}(\underline{n}_0, \underline{h}_0)$ is given by

$$f_{pq}(\underline{n}_0, \underline{h}_0) = \frac{1}{|(\underline{a} \times \underline{b})|} \int_{A_{00}} dA_{00}(\underline{s}) f(\underline{s}; \underline{n}_0, \underline{h}_0) e^{-i(\kappa \underline{n}_0 + 2\pi p \underline{\alpha} + 2\pi q \underline{\beta}) \cdot \underline{s}} \quad (24)$$

Applying the rotational invariance in a similar way, one has

$$(\underline{k} \times \underline{h}_0) e^{i\kappa \underline{n}_0 \cdot \underline{s}'} = -2i\kappa \underline{k} \times \int_A dA(\underline{s}) \Gamma(\underline{s}', R\underline{s}) \cdot \underline{F}(R\underline{s}; \underline{n}_0, \underline{h}_0)$$

$$= -2i\kappa \underline{k} \times \int_A dA(\underline{s}) R \Gamma(R^{-1}\underline{s}', \underline{s}) R \cdot \underline{F}(R\underline{s}; \underline{n}_0, \underline{h}_0)$$

$$= -2i\kappa R [\underline{k} \times \int_A dA(\underline{s}) \Gamma(R^{-1}\underline{s}', \underline{s}) \cdot R^{-1} \underline{F}(R\underline{s}; \underline{n}_0, \underline{h}_0)]$$

THE PRINCETON UNIVERSITY RADOME STUDY

i.e.

$$(k \times R^{-1} h_0) e^{ik R^{-1} \underline{r}_0 \cdot \underline{s}'} = -2ik k \times \int_A dA(\underline{s}) \Gamma(\underline{s}', \underline{s}) \cdot R^{-1} \underline{F}(R \underline{s}; \underline{r}_0, h_0)$$

Hence

$$\underline{F}(R \underline{s}; R \underline{r}_0, R h_0) = R \underline{F}(\underline{s}; \underline{r}_0, h_0) \quad (25)$$

which is really not terribly surprising.

One may also consider the case where the screen and aperture ($\underline{s}$) are invariant under combined rotations R and translations $\underline{t}$ i.e. under operations

$$\underline{s} \rightarrow R \underline{s} + \underline{t}$$

Then

$$\begin{aligned} (k \times h_0) e^{ik \underline{r}_0 \cdot \underline{s}'} &= -2ik k \times \int_A dA(\underline{s}) \Gamma(\underline{s}', R \underline{s} + \underline{t}) \cdot \underline{F}(R \underline{s} + \underline{t}; \underline{r}_0, h_0) \\ &= -2ik k \times \int_A dA(\underline{s}) \Gamma(\underline{s}' - \underline{t}, R \underline{s}) \cdot \underline{F}(R \underline{s} + \underline{t}; \underline{r}_0, h_0) \\ &= -2ik k \times \int_A dA(\underline{s}) R \Gamma(R^{-1}(\underline{s}' - \underline{t}), \underline{s}) R \cdot \underline{F}(R \underline{s} + \underline{t}; \underline{r}_0, h_0) \\ &= -2ik R \left[k \times \int_A dA(\underline{s}) \Gamma(R^{-1}(\underline{s}' - \underline{t}), \underline{s}) \cdot R^{-1} \underline{F}(R \underline{s} + \underline{t}; \underline{r}_0, h_0) \right] \end{aligned}$$

i.e.

$$(k \times R^{-1} h_0) e^{ik R^{-1} \underline{r}_0 \cdot (\underline{s}' + \underline{t})} = -2ik k \times \int_A dA(\underline{s}) \Gamma(\underline{s}', \underline{s}) \cdot R^{-1} \underline{F}(R \underline{s} + \underline{t}; \underline{r}_0, h_0)$$

Thus

$$\underline{R}^{-1} \underline{F}(R \underline{s} + \underline{t}; R \underline{r}_0, R h_0) = e^{ik \underline{r}_0 \cdot \underline{t}} \underline{F}(\underline{s}; \underline{r}_0, h_0) \quad (26)$$

All these symmetries are reflected in analogous symmetries of the total fields, as may be seen from (5) to (8) and (14) to (18). In

Secret

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

particular, the electric field for $z \gg 0$ has exactly the same symmetry properties as $E(\underline{s}; \underline{n}_0, h_0)$, viz

$$\underline{E}(\underline{r} + \underline{t}; \underline{n}_0, \underline{e}_0) = e^{ik\underline{n}_0 \cdot \underline{t}} \underline{E}(\underline{r}; \underline{n}_0, \underline{e}_0) \quad (27)$$

so that

$$\underline{E}(\underline{s} + k\underline{z}; \underline{n}_0, \underline{e}_0) = e^{ik\underline{n}_0 \cdot \underline{s}} \underline{\tilde{E}}(\underline{s}, z; \underline{n}_0, h_0) \quad (28)$$

where $\underline{\tilde{E}}(\underline{s}, z; \underline{n}_0, h_0)$ is a doubly periodic function over the lattice. One also has

$$\underline{E}(R\underline{r}; R\underline{n}_0, R\underline{e}_0) = R \underline{E}(\underline{r}; \underline{n}_0, \underline{e}_0) \quad (29)$$

and, in the case of combined symmetry

$$R' \underline{E}(R\underline{r} + \underline{t}; R\underline{n}_0, R\underline{e}_0) = e^{ik\underline{n}_0 \cdot \underline{t}} \underline{E}(\underline{r}; \underline{n}_0, \underline{e}_0) \quad (30)$$

VI. THE FAR FIELD Noting the asymptotic expansion of the Green's function

$$G(\underline{r}_p, \underline{r}_q) \simeq \frac{e^{ik(\underline{r}_p - \underline{n}_p, \underline{r}_q)}}{4\pi r_p}, \quad \underline{n}_p = \frac{\underline{r}_p}{r_p} \quad (1)$$

which is valid for $r_p \rightarrow \infty$ and $\underline{r}_q$ lying in a fixed finite region; one finds after a little manipulation (cf. Schwinger⁽³⁾) that the far field for $z \gg 0$ is given by

$$\underline{E}(\underline{r}; \underline{n}_0, \underline{e}_0) \simeq \underline{n} \times \underline{Y}(\underline{n}; \underline{n}_0, h_0) \frac{e^{ikr}}{r}, \quad \underline{n} = \frac{\underline{r}}{r} \quad (2)$$

where

$$\underline{Y}(\underline{n}; \underline{n}_0, h_0) = \frac{ik}{2\pi} \int_A dA(\underline{s}) \underline{E}(\underline{s}; \underline{n}_0, h_0) e^{-ik\underline{n}_0 \cdot \underline{s}} \quad (3)$$

THE PRINCETON UNIVERSITY RADOME STUDY

Further calculation then shows that $\underline{Y}$ satisfies the following variational principle:

$$\begin{aligned} [h \cdot \underline{Y}(\underline{r}; \underline{r}_0, h_0)]^{-1} &= [h_0 \cdot \underline{Y}(-\underline{r}_0; -\underline{r}, h)]^{-1} \\ &= \frac{-4\pi \int_A dA(\underline{s}') \int_A dA(\underline{s}'') \underline{F}(\underline{s}'; \underline{r}_0, h_0) \cdot \underline{T}(\underline{s}', \underline{s}'') \cdot \underline{F}(\underline{s}''; -\underline{r}, h)}{(h \int_A dA(\underline{s}) \underline{F}(\underline{s}; \underline{r}_0, h_0) e^{-ik\underline{r}_0 \cdot \underline{s}})(h_0 \int_A dA(\underline{s}) \underline{F}(\underline{s}; -\underline{r}, h) e^{ik\underline{r}_0 \cdot \underline{s}})} \end{aligned} \quad (4)$$

is extremal for small variations of the aperture field $\underline{F}(\underline{s}; \underline{r}_0, h_0)$ about its correct value.

The translational symmetry does not yield any information about $\underline{Y}$, but from the the rotational symmetry one has

$$\begin{aligned} \underline{Y}(\underline{r}; \underline{r}_0, h_0) &= \frac{ik}{2\pi} \int_A dA(\underline{s}) \underline{F}(\underline{s}; \underline{r}_0, h_0) e^{-ik\underline{r}^{-1} \cdot \underline{r}_0 \cdot \underline{s}} \\ &= \frac{ik}{2\pi} \int_A dA(\underline{s}) R \underline{F}(\underline{s}; R^{-1}\underline{r}_0, R^{-1}h_0) e^{-ikR^{-1}\underline{r}_0 \cdot \underline{s}} \end{aligned}$$

so that

$$\underline{Y}(R\underline{r}; R\underline{r}_0, R h_0) = R \underline{Y}(\underline{r}; \underline{r}_0, h_0) \quad (5)$$

In the case of an infinite array of aperture these expressions must be modified, because the expansion (1) is only valid for $\underline{r}_0$ in a finite region. It is most convenient to expand separately about the corresponding origin in each cell. This yields

$$\underline{E}(\underline{r}; \underline{r}_0, \underline{e}_0) \approx \frac{ik}{2\pi} \sum_{\underline{l}} \sum_m \frac{e^{ik\underline{r}_{lm}}}{r_{lm}} \underline{r}_{lm} \times \int_{A_{lm}} dA(\underline{s}) \underline{F}(\underline{s}; \underline{r}_0, h_0) e^{-ik\underline{r}_{lm} \cdot \underline{s}} \quad (6)$$

where

$$\underline{r}_{lm} = \underline{r} - \underline{l}a - m\underline{b}$$

$$\underline{r}_{lm} = \underline{r}_{lm} / r_{lm}$$

Using the fact that

$$\underline{F}(\underline{s} + \underline{l}a + m\underline{b}; \underline{r}_0, h_0) = e^{ik\underline{r}_0 \cdot (\underline{l}a + m\underline{b})} \underline{F}(\underline{s}; \underline{r}_0, h_0)$$

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

and writing

$$Y_{oo}(\underline{r}; \underline{r}_o, h_o) = \frac{iK}{2\pi} \int_{A_{oo}} dA_{oo}(\underline{s}) F(\underline{s}; \underline{r}_o, h_o) e^{-iK \underline{r} \cdot \underline{s}} \quad (7)$$

one finds directly that

$$E(\underline{r}; \underline{r}_o, e_o) \approx \sum_l \sum_m (n_{lm} \times Y_{oo}(n_{lm}; \underline{r}_o, h_o)) \frac{e^{iK(r_{lm} + l n_o \cdot \underline{a} + m n_o \cdot \underline{b})}}{r_{lm}} \quad (8)$$

The far field problem is thus reduced to the determination of Y_{oo} and the evaluation of the sum (8).

This latter point can however be avoided if the unit cell Σ_{oo} has comparable dimensions to the wave length $\lambda = 2\pi/K$. To do this one applies the vector wave equation

$$\nabla^2 \underline{E} + K^2 \underline{E} = 0$$

directly to the $\underline{E}$ given by V.(28), viz

$$\underline{E}(\underline{s} + k \underline{z}; \underline{r}_o, e_o) = e^{iK n_o \cdot \underline{s}} \underline{E}(\underline{s}, \underline{z}; \underline{r}_o, h_o)$$

with

$$\underline{E}(\underline{s}, \underline{z}; \underline{r}_o, h_o) = \sum_p \sum_q \underline{E}_{pq}(\underline{z}; \underline{r}_o, h_o) e^{2\pi i(p\alpha + q\beta) \cdot \underline{s}} \quad (9)$$

since $\underline{E}(\underline{s}, \underline{z}; \underline{r}_o, h_o)$ is a doubly periodic function over the lattice. Since the functions $\exp(2\pi i \underline{s} \cdot (p\alpha + q\beta))$ form a complete set over the lattice, one obtains the following differential equations for the

$$\frac{d^2 \underline{E}_{pq}(\underline{z}; \underline{r}_o, h_o)}{d \underline{z}^2} + [K^2 - (2\pi p\alpha + 2\pi q\beta + K n_{ot})^2] \underline{E}_{pq}(\underline{z}; \underline{r}_o, h_o) = 0 \quad (10)$$

where

$$n_{ot} = n_o - (n_o \cdot \underline{k}) \underline{k}$$

is the transverse part of n_o

Thus $\underline{E}_{pq}(\underline{z}; \underline{r}_o, h_o)$ will be an exponentially decaying function of $\underline{z}$ for those p, q for which

$$K^2 < (2\pi p\alpha + 2\pi q\beta + K n_{ot})^2 \quad (11)$$

THE PRINCETON UNIVERSITY RADOME STUDY

For $p=q=0$, however, it is clear that

$$\xi_{00}(z; \underline{n}_0, \underline{h}_0) \sim e^{iK n_{0z} z} \quad (12)$$

In order that all other $\xi_{pq}(z; \underline{n}_0, \underline{h}_0)$ be exponentially decaying for $p, q \neq 0$ independently of $\underline{n}_0, \underline{\alpha}, \underline{\beta}$ must satisfy the inequalities

$$\alpha, \beta > \frac{K}{\pi} = \frac{2}{\lambda} \quad (13)$$

$\underline{\alpha}, \underline{\beta}$ are supposed "minimal" lattice vectors, so that

$$|l\underline{\alpha} + m\underline{\beta}| \geq \underline{\alpha} \text{ or } \underline{\beta}, \text{ for } l, m \geq 1$$

The lattice spacings a, b must therefore satisfy

$$a, b < \frac{\lambda}{2 \sin \Theta} = \frac{\pi}{K \sin \Theta} \quad (14)$$

where Θ is the angle between the two basic lattice vectors,

$$\cos \Theta = \frac{(\underline{a} \cdot \underline{b})}{ab}$$

Suppose condition (14) satisfied, and let $a > b$; then the exponential decay of all $\xi_{pq}(z; \underline{n}_0, \underline{h}_0)$ is stronger than that of

$$e^{-\frac{2\pi}{a \sin \Theta} \sqrt{1 - \frac{K a \sin \Theta}{\pi}} \cdot z} \quad (15)$$

Only the term in $\xi_{00}(z; \underline{n}_0, \underline{h}_0)$ used thus be considered in the far field. Thus the apparent periodicity of $\xi(\underline{r}; \underline{n}_0, \underline{e}_0)$ shown by V(28) has no significance for the far field one need therefore consider only the "00" term in the aperture field, i.e.

$$\begin{aligned} \xi(\underline{r}; \underline{n}_0, \underline{e}_0) &= \xi(\underline{r} + \underline{k}z; \underline{n}_0, \underline{e}_0) \\ &\approx e^{iK(\underline{n}_0 \cdot \underline{r} + n_{0z} z)} \xi_{00}(0; \underline{n}_0, \underline{h}_0) \\ &= e^{iK \underline{n}_0 \cdot \underline{r}} \xi_{00}(0; \underline{n}_0, \underline{h}_0) \end{aligned} \quad (16)$$

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

where

$$\underline{k} \times \underline{E}_{oo}(0; \underline{n}_o, h_o) = \underline{f}_{oo}(\underline{n}_o, h_o) \quad (17)$$

is given by equation V (24).

$$\begin{aligned} \underline{f}_{oo}(\underline{n}_o, h_o) &= \frac{1}{|(\underline{a} \times \underline{b})|} \int_{A_{oo}} dA_{oo}(\underline{s}) \underline{E}(\underline{s}; \underline{n}_o, h_o) e^{i \underline{k} \cdot \underline{n}_o \cdot \underline{s}} \\ &= \frac{2\pi}{i \kappa |(\underline{a} \times \underline{b})|} \underline{Y}_{oo}(\underline{n}_o; \underline{n}_o, h_o) \end{aligned} \quad (18)$$

Because of the plane wave character of this field,

$$\underline{n}_o \cdot \underline{E}_{oo}(0; \underline{n}_o, h_o) = 0 \quad (19)$$

Thus

$$\underline{E}_{oo}(0; \underline{n}_o, h_o) = \frac{-(\underline{n}_o \times \underline{f}_{oo}(\underline{n}_o, h_o))}{(\underline{k} \cdot \underline{n}_o)} \quad (20)$$

and therefore

$$\underline{E}(\underline{r}; \underline{n}_o, \underline{e}_o) = \frac{-2\pi i}{\kappa |(\underline{a} \times \underline{b})| (\underline{k} \cdot \underline{n}_o)} (\underline{n}_o \times \underline{Y}_{oo}(\underline{n}_o; \underline{n}_o, h_o)) e^{i \underline{k} \cdot \underline{n}_o \cdot \underline{r}} \quad (21)$$

Since

$$\underline{H}(\underline{r}; \underline{n}_o, h_o) = \underline{n}_o \times \underline{E}(\underline{r}; \underline{n}_o, \underline{e}_o) \quad (22)$$

the complex Poynting vector is

$$\begin{aligned} \underline{P}(\underline{n}_o, h_o) &= \frac{1}{2} (\underline{E}(\underline{r}; \underline{n}_o, \underline{e}_o) \times \underline{H}^*(\underline{r}; \underline{n}_o, h_o)) \\ &= \frac{2\pi}{\kappa^2} \cdot \frac{|\underline{n}_o \times \underline{Y}_{oo}(\underline{n}_o; \underline{n}_o, h_o)|^2}{|(\underline{a} \times \underline{b})|^2 (\underline{k} \cdot \underline{n}_o)^2} \end{aligned} \quad (23)$$

THE PRINCETON UNIVERSITY RADOME STUDY

Before concluding this section, which completes the most general considerations, it is important to note that because of the symmetry of $E(\underline{s}; \eta_0, h_0)$ (V (19)) and of $\Gamma(\underline{s}', \underline{s})$ (V (15)), $\Upsilon_{\infty}(\eta; \eta_0, h_0)$ also satisfies a variational principle like (4). (In fact $\Upsilon_{\infty}(\eta; \eta_0, h_0)$ may be regarded simply as the normalized $\Upsilon(\eta; \eta_0, h_0)$). Thus

$$\begin{aligned} [\eta \cdot \Upsilon_{\infty}(\eta; \eta_0, h_0)]^{-1} &= [\eta_0 \cdot \Upsilon_{\infty}(-\eta_0; -\eta, h)]^{-1} \\ &= \frac{-4\pi \int_{A_{\infty}} dA_{\infty}(\underline{s}') \int_{A_{\infty}} dA_{\infty}(\underline{s}'') E(\underline{s}'; \eta_0, h_0) \cdot \Gamma(\underline{s}', \underline{s}'') \cdot E(\underline{s}''; -\eta, h)}{(\eta \cdot \int_{A_{\infty}} dA_{\infty}(\underline{s}) E(\underline{s}; \eta_0, h_0) e^{i\kappa \eta_0 \cdot \underline{s}}) (\eta_0 \cdot \int_{A_{\infty}} dA_{\infty}(\underline{s}) E(\underline{s}; -\eta, h) e^{i\kappa \eta_0 \cdot \underline{s}})} \end{aligned} \quad (24)$$

VII. REFORMULATION OF THE VARIATIONAL PRINCIPLE The formulas of the previous sections give the general solution of the plane diffraction problem (with a perfectly conducting screen) in terms of the aperture field, which latter, is in its turn, determined by the integral equation V (10) or by the variational principles VI (4) or VI (24). It is clear that by interchanging the roles of $\underline{E}$ and $-\underline{H}$, and of $\Gamma^{(1)}$, $\Gamma^{(2)}$ one may obtain an entirely analogous formulation in terms of the screen currents.

The direct solution of the integral equation V(10) seems to present almost insuperable difficulties, though exact solutions have been obtained in certain special cases (infinite half plane, circular aperture and for normal incidence) by what can only be regarded as mathematical tour de force. It is therefore necessary, in general, to proceed approximately, using the variational principles.

In order to illustrate a method that seems most suited to this, it is best to start with the case of Υ_{∞} since some of the previous results about $E(\underline{s}; \eta_0, h_0)$ are particularly adapted to the following formulation.

For reasons of symmetry one rewrites the variational principle in

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

the form:

extreme value of

$$\begin{aligned} T(\underline{n}'', \underline{h}''; \underline{n}', \underline{h}') &= \underline{h}'' \cdot \underline{Y}_{00}(\underline{n}''; \underline{n}', \underline{h}') = \underline{h}' \cdot \underline{Y}_{00}(-\underline{n}'; -\underline{n}'', \underline{h}'') \\ &= - \frac{(\int_{A_{00}} dA_{00}(\underline{s}) \underline{h}'' \cdot \underline{F}(\underline{s}; \underline{n}', \underline{h}') e^{-i\kappa \underline{n}'' \cdot \underline{s}}) (\int_{A_{00}} dA_{00}(\underline{s}) \underline{h}' \cdot \underline{F}(\underline{s}; -\underline{n}'', \underline{h}'') e^{i\kappa \underline{n}' \cdot \underline{s}})}{4\pi \int_{A_{00}} dA_{00}(\underline{s}') \int_{A_{00}} dA_{00}(\underline{s}'') \underline{F}(\underline{s}'; \underline{n}', \underline{h}') \cdot \underline{T}(\underline{s}', \underline{s}'') \cdot \underline{F}(\underline{s}''; -\underline{n}'', \underline{h}'')} \end{aligned} \quad (1)$$

Since $\underline{F}$ is zero on the screen, the integrals may be extended over the unit cells Σ_{00} instead of only the aperture portion of the unit cell.

It is more convenient to deal with the above in component form, and hence to introduce what might be called a "polarization" index i.e.

$$\underline{F} = (F_1, F_2) = \{F_\sigma\}$$

(σ thus refers to x, y components or s, φ components, or any components in the xy plane). From V(21) and (22) one has

$$F_\sigma(\underline{s}; \underline{n}', \underline{h}') = \sum_p \sum_q f_{\sigma pq}(\underline{n}', \underline{h}') e^{i(\kappa \underline{n}' + 2\pi p \underline{x} + 2\pi q \underline{y}) \cdot \underline{s}} \quad (2)$$

so that if one writes

$$F_\sigma(\underline{s}; \underline{n}', \underline{h}') = F'_\sigma(\underline{s}) \quad f_{\sigma pq}(\underline{n}', \underline{h}') = f'_{\sigma pq}$$

$$F_\sigma(\underline{s}; -\underline{n}'', \underline{h}'') = F''_\sigma(\underline{s}) \quad f_{\sigma pq}(-\underline{n}'', \underline{h}'') = f'_{\sigma pq}$$

one has

$$\begin{aligned} \int_{A_{00}} dA_{00}(\underline{s}) \underline{h}'' \cdot \underline{F}(\underline{s}; \underline{n}', \underline{h}') e^{-i\kappa \underline{n}'' \cdot \underline{s}} &= \int_{\Sigma_{00}} d\Sigma_{00}(\underline{s}) \left(\sum_\sigma \underline{h}''_\sigma F'_\sigma(\underline{s}) e^{-i\kappa \underline{n}'' \cdot \underline{s}} \right) \\ &= \int_{\Sigma_{00}} d\Sigma_{00}(\underline{s}) \left(\sum_\sigma \sum_p \sum_q \underline{h}''_\sigma f'_{\sigma pq} e^{i(2\pi p \underline{x} + 2\pi q \underline{y} + \kappa(\underline{n}' - \underline{n}'') \cdot \underline{s})} \right) \\ &= \sum_\sigma \sum_p \sum_q \underline{h}''_\sigma f'_{\sigma pq} D_{pq}(\kappa(\underline{n}' - \underline{n}'')) \end{aligned} \quad (3)$$

THE PRINCETON UNIVERSITY RADOME STUDY

where

$$\begin{aligned} D_{pq}(\kappa \underline{n}) &= \int_{\Sigma_{00}} d\underline{\Sigma}_{00}(\underline{s}) e^{i(\kappa \underline{n} + 2\pi p \underline{x} + 2\pi q \underline{y}) \cdot \underline{s}} \\ &= |(\underline{a} \times \underline{b})| \int_{-\frac{1}{2}}^{+\frac{1}{2}} d\underline{s} \int_{-\frac{1}{2}}^{+\frac{1}{2}} d\underline{\eta} e^{i(2\pi p \underline{s} + 2\pi q \underline{\eta} + \kappa \underline{s} \cdot \underline{n} + \kappa \underline{\eta} \cdot \underline{n})} \\ &= |(\underline{a} \times \underline{b})| \frac{\sin(\pi p + \frac{1}{2} \kappa \underline{n} \cdot \underline{a})}{\pi p + \frac{1}{2} \kappa \underline{n} \cdot \underline{a}} \cdot \frac{\sin(\pi q + \frac{1}{2} \kappa \underline{n} \cdot \underline{b})}{\pi q + \frac{1}{2} \kappa \underline{n} \cdot \underline{b}} \end{aligned}$$

(4)

Similarly

$$\int_{A_{00}} dA_{00}(\underline{s}) \underline{h}' \cdot \underline{\tilde{F}}(\underline{s}; -\underline{n}'', \underline{h}'') e^{i \kappa \underline{n}' \cdot \underline{s}} = \sum_{\sigma} \sum_p \sum_q h'_{\sigma} f'_{\sigma pq} D_{pq}(\kappa(\underline{n}' - \underline{n}''))$$

Writing

$$\begin{aligned} \int_{\Sigma_{00}} d\underline{\Sigma}_{00}(\underline{s}') \int_{\Sigma_{00}} d\underline{\Sigma}_{00}(\underline{s}'') e^{i(2\pi p' \underline{x} + 2\pi q' \underline{y} + \kappa \underline{n}') \cdot \underline{s}'} \Gamma_{\sigma' \sigma''}(\underline{s}', \underline{s}'') e^{i(2\pi p'' \underline{x} + 2\pi q'' \underline{y} + \kappa \underline{n}'') \cdot \underline{s}''} \\ = C_{\sigma' p' q', \sigma'' p'' q''}(\kappa \underline{n}', \kappa \underline{n}'') \end{aligned}$$

(5)

One has $\int_{A_{00}} dA_{00}(\underline{s}') \int_{A_{00}} dA_{00}(\underline{s}'') \underline{\tilde{F}}(\underline{s}'; \underline{n}', \underline{h}') \cdot \underline{\Gamma}(\underline{s}', \underline{s}'') \cdot \underline{\tilde{F}}(\underline{s}''; -\underline{n}'', \underline{h}'')$

$$= \sum_{\sigma'} \sum_{p'} \sum_{q'} \sum_{\sigma''} \sum_{p''} \sum_{q''} f'_{\sigma' p' q'} C_{\sigma' p' q', \sigma'' p'' q''}(\kappa \underline{n}', -\kappa \underline{n}'') f_{\sigma'' p'' q''} \quad (6)$$

METAL-RESONANT IRIS CONSTRUCTION OF RADOMES

$$\begin{aligned}
 & \text{Thus } \Upsilon(\underline{n}', \underline{h}'; \underline{n}, \underline{h}) \\
 &= \frac{- \sum_{\sigma} \sum_p \sum_q h_{\sigma}'' f_{\sigma p q}' D_{pq}(\kappa(\underline{n}' - \underline{n})) \sum_{\sigma'} \sum_{p'} \sum_{q'} h_{\sigma'}' f_{\sigma' p' q'}'' D_{p' q'}(\kappa(\underline{n}' - \underline{n}'))}{4\pi \sum_{\sigma'} \sum_{p'} \sum_{q'} \sum_{\sigma''} \sum_{p''} \sum_{q''} f_{\sigma' p' q'}' C_{\sigma' p' q', \sigma'' p'' q''}(\kappa \underline{n}', -\kappa \underline{n}'') f_{\sigma'' p'' q''}''} \quad (7)
 \end{aligned}$$

Actually the integral in the denominator of (1) will generally not converge, unless one integrates by parts. This usually leads to a result of the same form, but the matrix C in the denominator of (7) must be found indirectly (see below). In view of the formal identity one can however deal with the above expression as far as general remarks are concerned. Taking

$$\underline{n}' = -\underline{n}'' = \underline{n}, \quad \underline{h}' = \underline{h}'' = \underline{h}$$

and writing

$$h_{\sigma} D_{pq}(2\kappa \underline{n}) = M_u$$

$$C_{\sigma' p' q', \sigma'' p'' q''}(\kappa \underline{n}, \kappa \underline{n}) = C_{u', u''}$$

where u denotes the triad (σ, p, q) , (7) reduces to

$$\begin{aligned}
 \Upsilon(-\underline{n}, \underline{h}; \underline{n}, \underline{h}) &= \Upsilon \\
 &= -(\sum_u f_u M_u)^2 / 4\pi \sum_{u'} \sum_{u''} C_{u', u''} f_{u'} f_{u''} \quad (8)
 \end{aligned}$$

This has an extreme value for the correct choice of $\underline{E}$ (i.e. of the f_u)

$$\begin{aligned}
 \therefore 4\pi \Upsilon \sum_{u'} \sum_{u''} C_{u', u''} f_{u'} f_{u''} &= -(\sum_u f_u M_u)^2 \\
 \& 4\pi \Upsilon \sum_{u'} C_{u, u'} f_{u'} &= -(\sum_{u'} f_{u'} M_{u'}) M_u \quad (9)
 \end{aligned}$$

Hence

$$(\sum_{u'} C_{u, u'} f_{u'}) (\sum_{u''} f_{u''} M_{u''}) = M_u \sum_{u'} \sum_{u''} C_{u, u''} f_{u'} f_{u''}$$

THE PRINCETON UNIVERSITY RADOME STUDY

Whence

$$\sum_{u'} C_{u,u'} f_{u'} = \delta M_u \quad (10)$$

where δ is a constant introduced so that the normalization be correct. Purely formally one thus has the solution

$$f_{\sigma p q}(\underline{u}, \underline{h}) = \delta \sum_{\sigma'} \sum_{p'} \sum_{q'} [C^{-1}(\underline{u}, \underline{h})]_{\sigma p q, \sigma' p' q'} h_{\sigma'} D_{p' q'}(2\underline{u}) \quad (11)$$

Returning to the question of the integral in the denominator, one notes that

$$\Gamma(\underline{u}', \underline{u}'') = (I - \frac{1}{k^2} \nabla' \nabla'') G(\underline{u}', \underline{u}'')$$

Then

$$\underline{E}' \cdot \Gamma \cdot \underline{E}'' = (\underline{E}' \cdot \underline{E}'') G - \frac{1}{k^2} \underline{E}' \cdot \nabla' \nabla'' G \cdot \underline{E}''$$

Now

$$\begin{aligned} \nabla' \cdot \nabla'' (\underline{E}' \underline{E}'' G) &= \nabla' \cdot (\underline{E}' [(\nabla' \cdot \underline{E}'') G + \underline{E}'' \cdot \nabla' G]) \\ &= -(\nabla' \cdot \underline{E}') (\nabla' \cdot \underline{E}'') G + \nabla' \cdot \underline{E}' \nabla'' \cdot (\underline{E}'' G) + \nabla' \cdot \underline{E}'' \nabla' \cdot (\underline{E}' G) + \underline{E}' \cdot \nabla' \nabla'' G \cdot \underline{E}'' \end{aligned} \quad (12)$$

Since $\underline{E}$ is tangential to the edge of the aperture, $\underline{E}$ being normal to it, one finds (using Gauss' theorem) that

$$\int_A dA(\underline{s}') \int_A dA(\underline{s}'') \underline{E}' \cdot \Gamma \cdot \underline{E}'' = \int_A dA(\underline{s}') \int_A dA(\underline{s}'') \left\{ (\underline{E}' \cdot \underline{E}'') - \frac{1}{k^2} (\nabla' \cdot \underline{E}') (\nabla'' \cdot \underline{E}'') \right\} G$$

one then finds that C is given by

$$C_{\sigma' p' q', \sigma'' p'' q''}(\underline{u}', \underline{u}'') = C_{p' q', p'' q''}(\underline{u}', \underline{u}'') \left[\delta_{\sigma' \sigma''} + \frac{1}{k^2} (k \eta_{p'} + 2\pi p' \alpha_{p'} + 2\pi q' \beta_{p'}) (k \eta_{p''} + 2\pi p'' \alpha_{p''} + 2\pi q'' \beta_{p''}) \right]$$

with

$$C_{p' q', p'' q''}(\underline{u}', \underline{u}'') = \int_{A_{oo}} dA(\underline{s}') \int_{A_{oo}} dA(\underline{s}'') e^{i(2\pi p' \alpha + 2\pi q' \beta + k \eta) \cdot \underline{s}'} e^{i(2\pi p'' \alpha + 2\pi q'' \beta + k \eta) \cdot \underline{s}''}$$

METAL-RESONANT IRIS CONSTRUCTION OF RADOME

The above discussion also contains implicitly the case of a single arbitrary aperture A , and is, in fact merely a specialization taking the symmetry into account. The variational principle again has the form (1), with A_{∞} replaced by A . Now suppose the functions $\{\psi_{pq}(\underline{s})\}$ form a complete orthonormal set on A . One may then write

$$F_{\sigma}(\underline{s}; \underline{n}', h') = F'_{\sigma}(\underline{s}) = \sum_p \sum_q F'_{\sigma pq} \psi_{pq}(\underline{s})$$

$$F_{\sigma}(\underline{s}; \underline{n}'', h'') = F''_{\sigma}(\underline{s}) = \sum_p \sum_q F''_{\sigma pq} \psi_{pq}(\underline{s})$$

Placing

$$D_{pq}(K\underline{n}) = \int_A dA(\underline{s}) \psi_{pq}(\underline{s}) e^{iK\underline{n} \cdot \underline{s}}$$

$$C_{\sigma'p'q', \sigma''p''q''} = \int_A dA(\underline{s}') \int_A dA(\underline{s}'') \psi_{p'q'}(\underline{s}') \Gamma_{\sigma'\sigma''}(\underline{s}', \underline{s}'') \psi_{p''q''}(\underline{s}'')$$

(with the same reservation as before regarding the convergence of the integral), writing

$$M_u = h_{\sigma} D_{pq}(2K\underline{n})$$

$$C_{u'u''} = C_{\sigma'p'q', \sigma''p''q''}$$

and proceeding as formerly, one ends up with the equation

$$\sum_{u'} C_{u,u'} F_{u'} = \frac{i}{2K} M_u$$

so that formally

$$F_{\sigma pq}(\underline{n}, h) = \frac{i}{2K} \sum_{\sigma'} \sum_{p'} \sum_{q'} [C^{-1}(K\underline{n}, K\underline{n})]_{\sigma pq, \sigma'p'q'} h_{\sigma'} D_{p'q'}(2K\underline{n})$$

It is clear however, that if the partial integration is carried out $\sum$ will get much more messy than is necessary, if the $\psi_{pq}(\underline{s})$ are not chosen properly. It is by far the simplest, from this point of view, to do a simple Fourier analysis of F_{σ} , even if this is not well adapted to the shape of A ; this is, of course, what was done in the earlier part of this section.

THE PRINCETON UNIVERSITY RADOME STUDY

The purpose of this last development is to render approximations possible by reducing the infinite "Fourier" series for $F(\underline{x})$ in terms of the complete set $\psi_{pq}(\underline{x})$, to finite series with N terms which may be treated exactly. It may then be presumed that in the limit as $N \rightarrow \infty$ an exact solution of the original problem is obtained.

VIII. CONCLUDING REMARKS (December 14, 1951) The above report describes the general structure of the theory of electromagnetic diffraction by plane perfectly conducting screens.

All the known general results which seem relevant have been collected, and various symmetry properties of the solutions have been derived.

Analytical difficulties have so far prevented the deduction of any more explicit properties of the solution, on the basis of the formulae of section VI.

IX. REFERENCES Only the most recent useful references are cited below; though many of the ideas they contain are much older, a succinct and clear expression does not seem to have been given them till rather recently.

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APPENDIX F

AERODYNAMICS OF RADOMES

A study of the aerodynamic requirements placed on radomes has been made with a special view toward estimating the relative penalties involved in making changes from the optimum shape. Even though the electrical requirements are not yet fully formulated a realistic picture of possible modifications seems desirable.

A drag of a supersonic missile arises from the combined effects of normal pressure forces and viscosity. For a missile of good aerodynamic shape flying at 40,000 feet the drag will be about half pressure and half viscous. At sea level head pressure drag accounts for a major fraction of the total. Although a hemispherical radome is perfectly acceptable for subsonic missiles the high pressure encountered behind a detached shock in supersonic flight produces a very large drag. The variation with Mach number in head pressure drag of a hemisphere and 30° cone are shown in Fig. F1. Theory and experiment agree in showing that this can be steadily reduced as the fineness ratio, the ratio of nose length to body diameter, is made larger. Other considerations require a compromise so that most of the radomes presently contemplated are cones and ogives having fineness ratios of about 2 or 3.

I. DRAG PENALTY FROM NOSE ROUNDING The possibility of rounding off the nose of a radome immediately suggests itself, the thought being that electrical requirements might be met more easily without increasing the drag very much. By assuming that the drag of a rounded cone is equal to the weighted sum of cone and sphere values the drag of such a combination has been computed and is shown for 30° and 40° cones at $M = 2$ in Fig. F2. Rounding 20 % has very little effect and a 30° cone with 50 % rounding has no greater drag than an unmodified 40° cone. Experimental data for a family of successively blunter shapes tested in free flight has been obtained by the Langley Pilotless Aircraft Research Division* which show an initial decrease in drag with small rounding. This apparently results from a change in location of transition from laminar to turbulent boundary layer which in turn alters skin friction and the base pressure drag.

During the course of this study a report arrived on the "NACA Conference on Aerodynamic Design Problems of Supersonic Guided Missiles" at the Ames Aeronautical Laboratory on October 2, 3, 1951, which describes new results on the possibility of rounding off the nose of the radome. The following section is quoted from the paper of A. Seiff and C. A. Sandahl:

"If a cone of fineness ratio 3 is opened at the tip and a spherical tip inserted while the fineness ratio is held constant, there will be an increase in drag at the tip, but a decrease over the sides as a result of the decreased inclination of the sides to the stream. The increase at the tip occurs within a small frontal area whereas the decrease on the sides occurs over a

* NACA RM L51 E25.

THE PRINCETON UNIVERSITY RADOME STUDY

large frontal area. The net effect on drag depends on the balancing of these opposite tendencies. This effect can be calculated by using experimental data for the foredrag of hemispheres and assuming that the side pressures will be the same as on a pointed cone of the same slope. The results of such a calculation are shown in (Fig. F3) where increment of wave drag of the blunt shapes over that of the cone is plotted as a function of the ratio of tip radius to maximum radius for Mach numbers 1.5, 3, and 6. A small initial reduction in drag coefficient is indicated at all Mach numbers. The effect on these curves of the assumption that the side pressures remain conical is uncertain in the absence of experimental data. Therefore, the drag coefficients of the family of bodies shown at the left in (Fig. F4) were measured in the Ames supersonic free-flight wind tunnel and two of the experimental curves are shown. The bluntest shape in this family had a tip radius of 0.5 but the curves are extended toward an end point at a tip radius of 1 based on the best available values of the foredrag of a hemisphere. As can be seen by comparing, at a Mach number of 6, the calculated curve with experiment, the measured initial drag reduction is greater in both magnitude and extent than predicted so that favorable effect of the spherical tip on the side pressures is indicated. The comparison at a Mach number of 6 is typical of those obtained. The calculated curve is least quantitative in the region of the optimum bluntness. It is exact at either extreme of bluntness, zero or maximum, by definition. It is quantitatively useful for predicting the variation of drag with bluntness for tip radii greater than 0.5.

The abscissa scale of (Fig. F4) extends out to a fully blunt hemispherical tip. The data in the right half of the figure apply, therefore, to noses suitable for radar. The incremental drag penalties at high bluntness are very severe, in the order of several tenths in drag coefficient, and the severity increases with Mach number. The maximum bluntness which can be used with zero penalty ranges from about 0.4 at a Mach number of 1.2 to about 0.2 at a Mach number of 6.

So far, the effect of fineness ratio on drag has not been discussed. For severely blunt shapes, with tip radii equal to or greater than 0.5, the effect of fineness ratio is small since the drag of the spherical tip dominates the nose drag. However for basically slender shapes, the fineness ratio is an important variable as is shown in (Fig. F5). In this figure, the increment in foredrag, pressure plus friction, over that of the same shape at fineness ratio 4 is plotted as a function of the fineness ratio. These results were obtained from theory for cones and ogives. A single curve can be faired reasonably well through data for both shapes at three Mach numbers: 2, 5, and 8.

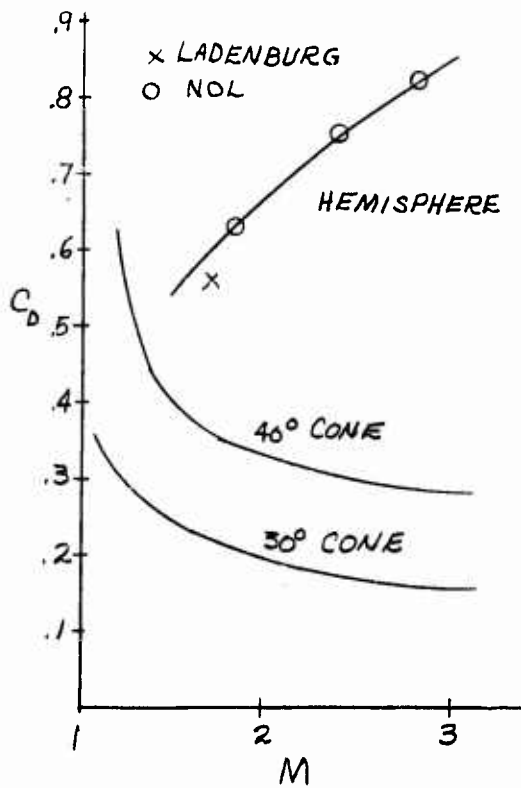


FIG. F1. COMPARISON OF PRESSURE DRAG ON SPHERES & CONES.

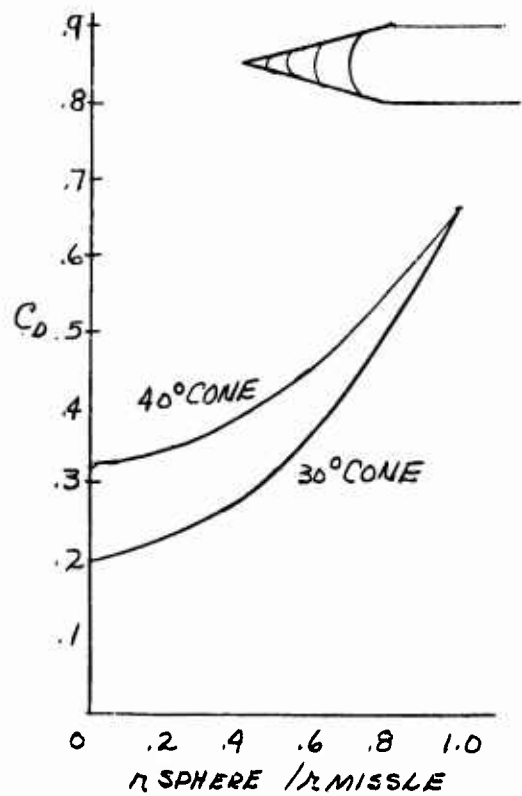


FIG. F2. EFFECT ON HEAD PRESSURE DRAG OF ROUNDING A CONICAL NOSE

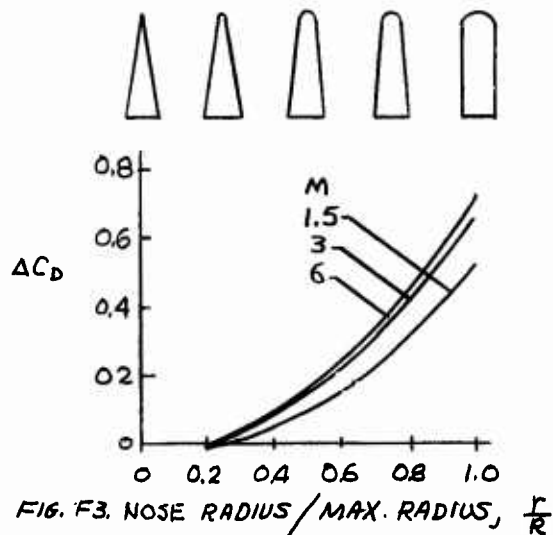


FIG. F3. NOSE RADIUS/MAX. RADIUS, r/R

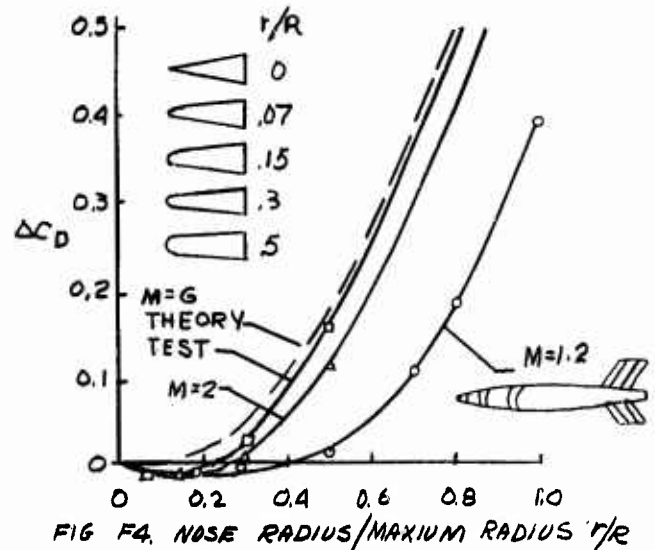
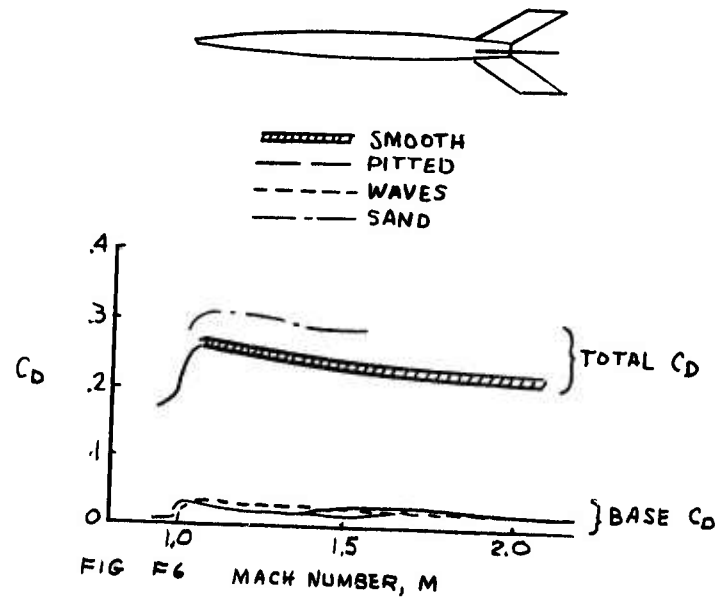
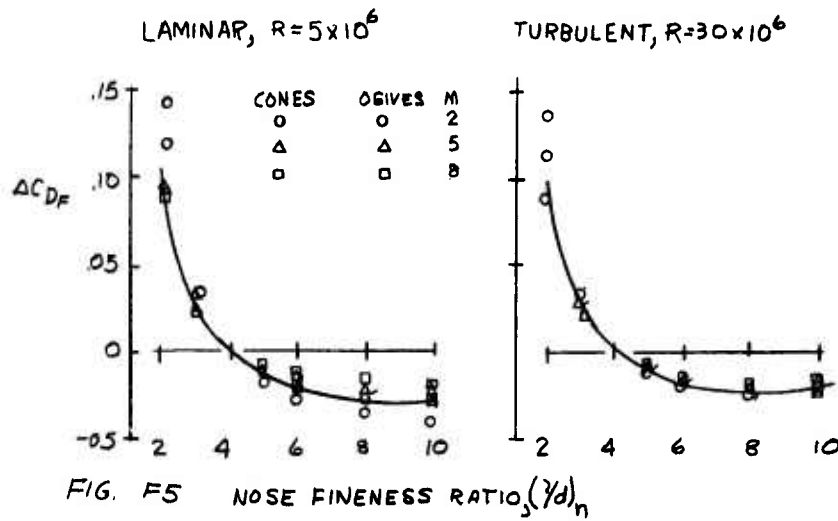


FIG. F4. NOSE RADIUS/MAXIMUM RADIUS r/R



Two boundary-layer conditions are postulated, laminar flow at a Reynolds number of 5×10^6 representing a condition of low skin friction, and turbulent flow at a Reynolds number of 30×10^6 representing a typical high-friction condition. The two curves are nearly identical except that the high-friction curve shows a definite tendency to turn up at the extremes of fineness ratio plotted, whereas the low-friction curve continues down. These curves show relatively large drag changes with small changes in fineness ratio up to a fineness ratio of about 6. Beyond a fineness ratio of 6, the drag changes are smaller and other considerations might outweigh the drag improvement."

II. SURFACE ROUGHNESS. "The final subject to be considered in this paper is the effect of body surface condition on the drag. This subject is one of great interest in connection with the mass production of missiles where perfect surfaces cannot be expected. Some data of this type are now available from flight tests of the RM-10 research vehicle at Mach numbers up to 2. The Reynolds numbers of the investigation were of the order of 50 million so that the boundary layers were predominantly turbulent. Several models with highly polished surfaces were tested to establish the drag coefficient for the smooth condition. The effect of roughness protruding from the surface was investigated with a body thickly coated with 0.01-inch-diameter sand particles. This dimension can be compared with the maximum body radius which was 3 inches. A second model was made to simulate a partly ground aluminum casting. Its surface was about 70 percent smooth, the remainder being pitted below the level at which grinding was stopped. Although the pits were only about 0.002 inch deep, the model looked and felt rough. The effect of longitudinal waves, such as might occur in the manufacture of metal bodies by the spinning technique, was investigated with a third model having waves about 0.02 inch deep and about 0.5 inch long extending over the entire length. The results of these tests are shown in (Fig. F6.) It will be noted that neither the wavy surface nor the pitted surface of the simulated casting caused any measurable increase in drag. The sand-coated surface, however, increased the drag substantially. It was established that this drag change was not a change in base drag. It appears from these exploratory tests that, at very high Reynolds numbers at which the boundary layers are naturally turbulent, some degree of surface roughness due to pitting or waviness can be tolerated but that roughness projecting from the surface can cause substantial drag increase."

Secret

THE PRINCETON UNIVERSITY RADOME STUDY

III. CONICAL WINDSHIELD. One quite different approach to reducing head drag is the proposal to mount a small conical windshield on a sting ahead of the real nose. Experiments indicate, however, that the cone must be fairly large to give an appreciable reduction in pressure on the main body.* This would doubtless distort the radiation pattern and introduce serious structural problems in the radome as well. Such a scheme doesn't seem to be very fruitful for solving this problem.

IV. TEMPERATURES AND PRESSURES. In addition to consideration of the changes in drag resulting from modifications of shape some attention should be paid to temperatures and maximum pressures occurring in the radome.

As the thermal conductivity of most proposed radome materials is low the skin temperature will attain the aerodynamic recovery temperature soon after the missile is launched. At the nose of a blunt radome the temperature is equal to the stagnation value which is a function of the ambient temperature and flight Mach number. Elsewhere the surface temperature varies according to the relation.

$$T_{\text{surface}} = T_{\text{stream}} + \Theta (T_{\text{stagnation}} - T_{\text{stream}})$$

For turbulent boundary layers $\Theta \approx .91$ and for laminar boundary layers $\Theta \approx .85$. Table FI gives some typical values computed assuming an ambient temperature of 100° F.

TABLE FI

<u>Mach Number</u>	<u>Stagnation Temp.</u>	<u>Surface Temp. (turbulent boundary layer on 30° cone)</u>
0.6	140 °F	130 °F
1.5	352	334
2.0	548	515
2.5	800	746

Generally a laminar boundary layer exists over the fore part of a missile but undergoes transition to the turbulent type some distance from the nose, depending on nose shape, heat transfer rate, surface smoothness, and flight Mach number. The temperatures given in the table for a 30° cone are therefore upper limits, slightly lower values being appropriate in the region of laminar flow. At present little information is available on transition points.

For flight $M = 2$ the temperature on the inner surface of a foam material radome may be estimated in the following way. Assume an initial temperature of 100° F throughout at time zero.

$$\frac{\text{Radome thickness}}{\text{}} = 0.8 \text{ cm}$$

* NACA RM L8107a

Secret

AERODYNAMICS OF RADOMES

Thermal diffusivity = .0016 cm²/sec (10 lbs/ft³ ground cork assumed to have similar thermal properties)

Original temperature = 100° F

Outer surface temperature = 500° F for t > 0

Then simple transient heat flow theory gives variations in inner surface temperature as follows:

Time	32 sec	40	80	160	320
Inner surface temp.	110° F	121	192	310	430

One may conclude that quite small changes of temperature will occur in the inner portion of the radome during flights presently contemplated.

In Table FII values of stagnation pressure and surface pressure on a 40° conical radome are given for sea level flight at various Mach numbers. At 20,000 feet these values should be divided by 2; at 30,000 feet by 3.

<u>TABLE FII</u>		
<u>Mach number</u>	<u>Stagnation pressure</u>	<u>Surface pressure on 40° cone</u>
1.2	2.4 atm.	1.6 atm.
1.5	3.4	1.6
2.0	5.6	1.9
2.5	8.5	2.3

The forces near the nose of a blunt radome may be quite high compared to those over a pointed body. Since the area to which these apply is small for small amounts of rounding sufficiently strong materials may well be available.

It seems reasonable to conclude that no great difficulties either in loss of missile range or in material properties required are introduced by small amounts of nose rounding if this seems desirable from the electrical point of view.

APPENDIX G

MATERIALS FOR RADOMES

INTRODUCTION. This section is not intended to be a survey of materials which have been used or which could be used to build radomes. It is rather a discussion of how we can best outline a materials programme which will be of most assistance in developing radomes of satisfactory electrical performance. Then, and only then, will we be in a position to specify the properties we need and to look for the best materials giving these properties.

GENERAL DISCUSSION. A considerable amount of work has gone into the development of materials which might be useful for the construction of radomes for guided missiles. The unfortunate part of the whole thing is that since we do not know yet how to design such radomes of a good aerodynamic shape to give errors as small as the systems designers call for we are not in any position even to begin to specify the properties of the materials out of which the radomes should be made.

It would be very convenient if this report could be written by opening a number of materials handbooks and picking out those materials which satisfy a certain set of specifications and then recommending one or more of these for use or at least as the basis of an advanced development programme.

Since we are so far away from this happy state of things, any materials programme should be so directed as to give us the properly shaped pieces we need for an intelligent investigation of ways and means of reducing radome errors to the level where they can be tolerated. This means that the electrical design should take precedence over the mechanical design while we keep, if at all possible, to those shapes which are aerodynamically to be recommended.

The main job of the materials people is then to produce, out of materials of the widest possible range of electrical properties, those shapes and designs which will allow the rapid testing of those ideas which intelligent thinking on electrical design may show to have good possibilities. They must be prepared to modify and retest any shapes and designs as a result of the analysis of previous tests. Where possible, therefore, those methods of fabrication which allow the maximum flexibility of construction should be used rather than methods requiring costly molds or mandrels which cannot be easily adjusted as to shape. In some cases, of course, the precision with which each test piece has to be made calls for expensive forms and here we do not have much choice.

In looking for a material of certain electrical properties consideration should be given to any material which can easily be machined even if it should not have the mechanical strength and absence of porosity which would ultimately be needed. If it can tell us what kind of electrical properties we need, then we can start the search for a material giving

Secret

THE PRINCETON UNIVERSITY RADOME STUDY

these properties and having also the mechanical properties necessary for the application.

Also, in trying out some idea we may have to take extraordinary measures to hold certain thickness, shape or other tolerances within limits which would seem prohibitively narrow for any production process. This should not discourage the testing of such materials since our first important objective is not to go into production on radomes but rather to acquire the basic knowledge necessary for the eventual proper design and construction of radomes.

MATERIALS AVAILABLE. For the testing of many of the current radome ideas particularly where the dielectric constant is low some of the very many types of plastics or fibre glass reinforced plastics are the obvious materials to use. Methods of fabrication have been worked out to build radomes in the form either of half wave plates of low dielectric constant or either single or double type A sandwiches.

Where higher dielectric constant is required one may of course try to develop a plastic material by additions of titania or even electrically conducting material but it would seem simpler to use some ceramic material. Most dense fired ceramic bodies will have dielectric constants of 6 or greater and can be made without great difficulty with values approaching 100 for the frequencies to be used. For instance, mixtures of steatite and titania give values varying from about 6 to over 90 which vary almost linearly with the composition. Actually, because of thickness tolerances we probably do not want to go beyond about 10 in which case any one of a number of materials or mixtures of such materials as alumina, zircon, steatite, cordierite, etc. could be used to give the desired properties. Careful slip casting using only crushed vitrified raw materials could give the desired homogeneity and precision of forming with extremely small firing shrinkage or warping. The possibility of machining the green ware gives a considerable flexibility as far as thickness and possibly shape are concerned. These materials offer good possibilities for achieving the high transmission at a wide range of angles for all polarizations which is theoretically possible with approximately half wave thicknesses of high dielectric constant material.

A type of material which might be useful in a testing programme would be one of the very light inorganic insulating materials. An example would be the hydrated calcium silicates produced under the name of Kaylo by the Owens Illinois Glass Company. Here we have a body with a density of about 10 lbs. per cubic foot which while somewhat weak in structure could easily be machined to various desired shapes to test out rapidly any theories on low dielectric constant material. Such material when fired is converted to wollastonite still in a very light form and machinable. This might be useful where the extremely low dielectric loss of wollastonite might be desired.

MATERIALS FOR RADOMES

If ceramic materials could be developed with a magnetic permeability appreciably greater than unity at the frequencies contemplated then we might have the possibility of balancing the ratio of permeability to dielectric constant so as to give zero reflection at some angle of incidence. Such an additional adjustable factor in design might be very useful. Some investigation of the ferromagnetic spinels or some of the rare earth compounds with this in mind might yield useful results.

As an ultimate material of construction glass might offer some possibilities because of the precision which can be achieved in the production of large pieces and because of its great homogeneity and moderately high dielectric constant. However, as a part of the programme of exploring types of materials it lacks flexibility in shape and thickness design.

For type B sandwiches it will probably be the case that for most of the possible choices of materials which will give a skin of sufficient strength the dielectric constant of the skin will be in the region where it can most readily be made of plastic while the core will have a dielectric constant which could be achieved most easily with a ceramic material. Such a composite might have very desirable mechanical properties. The lower elastic constants of the plastic might act as a cushion to protect the somewhat greater brittleness of the ceramic core. Such a development would of course, depend on the development of the ability to make and fire ceramic shapes with the necessary electrical properties.

This discussion might be summed up best in the form of actual recommendations as to the directions a materials programme should take so as best to fit into the general plan for the development of satisfactory radomes.

RECOMMENDATIONS.

1. That there should be developed the necessary techniques for making radome shapes out of several electrical different types of materials regardless of mechanical properties to aid in a measurement program to understand the causes of beam deflection.
2. That, until we are in better position to specify the electrical properties needed for low beam deflection, considerable stress be laid on any methods of fabrication which allows easy adjustment of radome design (thickness, taper and other factors) without change in very expensively made molds or mandrels.
3. That some effort should be applied to the development of materials with magnetic permeabilities appreciably greater than unity at the frequencies to be used.
4. That the three phases of radome development
 - (a) Material handling and fabrication
 - (b) Shape and electrical design
 - (c) Testing

be carried on with the maximum cooperation on the part of the participating personnel.

APPENDIX H

POLARIZATION EFFECTS

The merits of different types of polarization depend to a large extent on the amount of depolarization by the target, the amount of double reflection, and the design difficulties of different scan systems. A better understanding of the antenna, radome, target interaction is required before an accurate evaluation can be made. The discussion below ignores such difficulties and assumes negligible depolarization and double reflection. (Such reflection would reverse the rotation sense of a received circularly polarized signal). Using these simplifications it is apparent that with axially symmetric radomes radome errors are reduced if the direction of the polarization vector is symmetric with respect to the radome axis as the antenna direction is moved around the radome axis. (See Fig. H1) If Z is the direction of the radome axis, Θ the angle of the scanner axis with respect to Z , and Φ the angle of the scanner axis in the XY plane, it can be seen that if radome errors are removed at one value of Φ they will be removed at all values of Φ .

When conical scan is being used, the polarization vector should have a mirror symmetry with respect to the line between the axis of the radome and the axis of rotation of the scanner. Such symmetry will mean that if the beam transmission changes when the antenna is pointing on one side of the line it will change by exactly the same amount at a symmetrical point on the other side of the line. If the beam deviates an amount $+\Delta\Phi$ when on one side, it will deviate $-\Delta\Phi$ when on the other. The cross over point of the lobes will have no error in Φ . Radome errors should be primarily in Θ .

With active systems such symmetry can be achieved by the use of rotating linear polarization. A simple type of nutating scan would not suffice. This is illustrated in Fig. H1 where small scan cycles are shown in relation to an axially symmetric radome. Circular polarization will also give the same symmetry.

In semi-active systems axial symmetry of the receiving antenna with respect to the radome can be obtained as above by either rotating linear polarization or circular polarization. However, to obtain symmetry of the received signal with respect to the radome and antenna a circularly polarized transmitter is required.

In semi-active systems cross-talk due to polarization should also be reduced by the use of a symmetric system. The cause of such cross-talk is discussed in some detail in a subsequent part of this appendix. Hughes Aircraft Company Falcon Progress Report 40-23-P page 47 also points out the advantages of circular polarization with regard to cross-talk and bore-sight errors.

In an active system, cross-polarization errors with linear polarization should be much smaller than with a semi-active system. To make the errors appreciable the angle of the E vector must be rotated on reflection. Such a reflection can also be expected to

THE PRINCETON UNIVERSITY RADOME STUDY

to convert circular polarization into elliptical causing errors of the same order. Cross-talk error may be introduced by beam intensity variations due to different transmission coefficients for $E_{\perp}$ and $E_{\parallel}$ in passage through the radome. Consider a vertically polarized plane wave moving along the axis of a radome. If the transmission $T_{\parallel}$ of $E_{\parallel}$ is greater than $T_{\perp}$ of $E_{\perp}$ more power will be transmitted from the top and bottom of the radome than from the sides. This will cause a distortion of the lobes. In a nutating scan if the axis of scan is lined up with the axis of the radome, the lobe strength will vary. This causes no zero error when the scanner is lined up with the radome axis but it will cause the gain of the system to be greater for an up-down error than for a left-right error. For example consider a case where the beam transmission varies sinusoidally by about 10% being larger in the up-down than in the left-right direction. If the signal strength without the radome would vary sinusoidally throughout a 100% modulation conical scan cycle, the radome would cause the apparent angular position of the target on the circle of scan to shift 10° . However with the target located on the radome axis no boresight error would occur.

The following analysis is by Dr. Samuel Silver of the University of California, Berkeley.

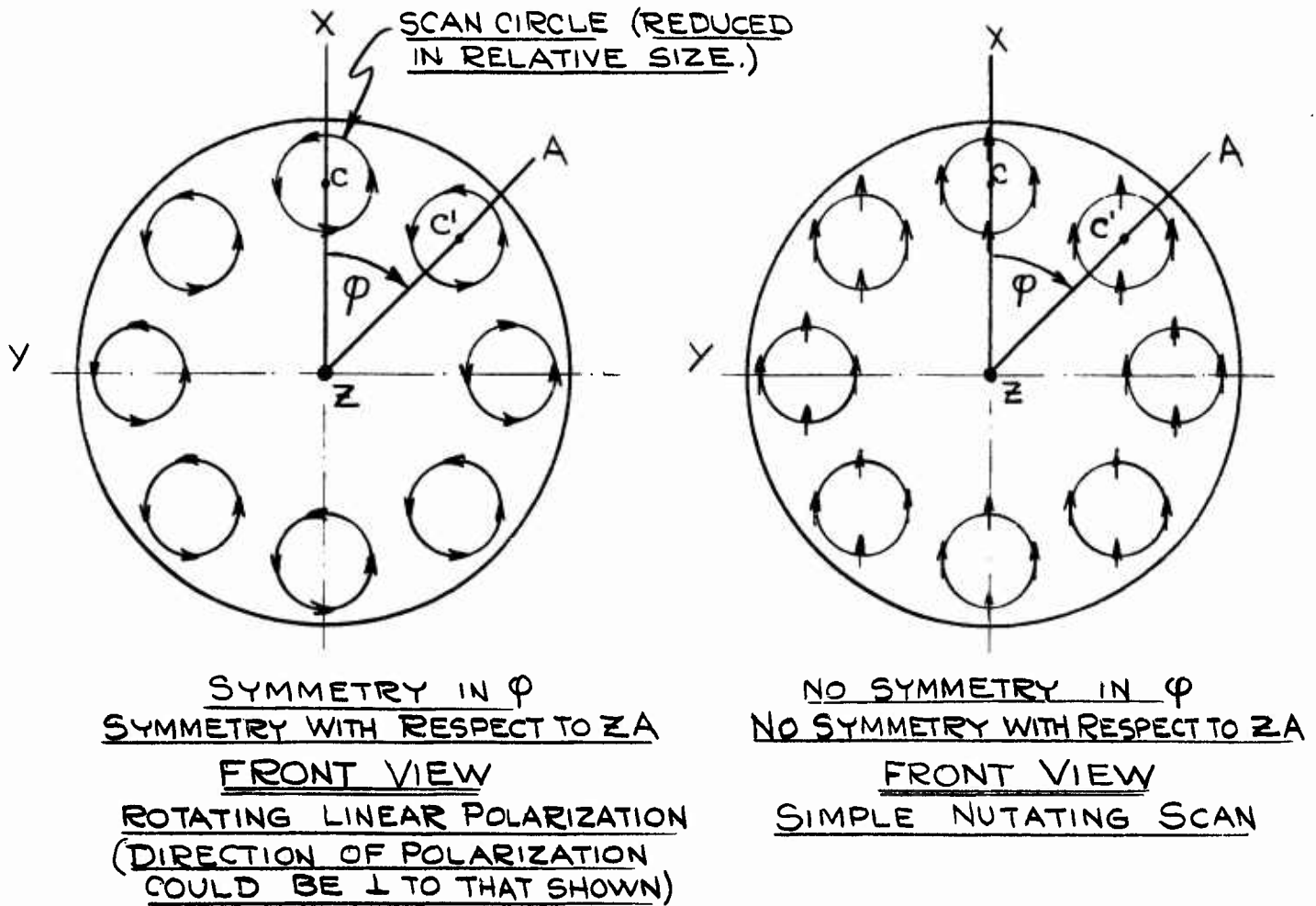
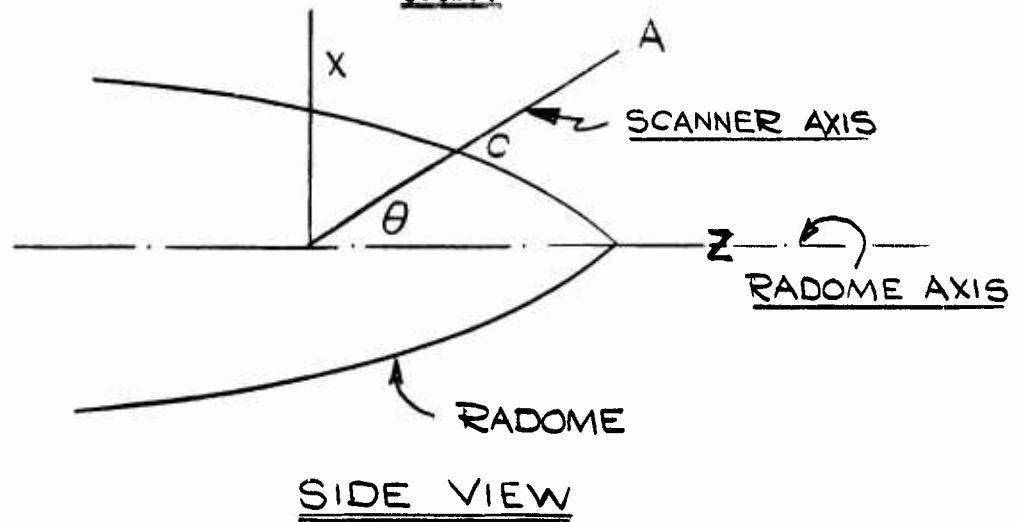
POLARIZATION ERRORS IN SEMI-ACTIVE SYSTEMS

There is a diversity of opinion as to which polarization, linear or circular, is more effective in reducing the radome error. The question cannot really be resolved until the nature of the interaction between the tracking antenna and its radome is understood so thoroughly that the relative importance of a number of factors that contribute to the error can be evaluated. The choice of polarization is also dependent on target characteristics. Even less seems to be known about that subject than about the radome-antenna problem.

Various aspects of the radome-antenna interaction can be analyzed on the basis of simple optical concepts. In the following sections these aspects of the interaction will be examined in relation to the choice of polarization. The discussion will be concerned with the case where the missile uses a conical lobing receiving antenna to track a target that is being illuminated by a radar on the parent aircraft. The conical lobing antenna will be assumed to consist of a paraboloidal or spherical reflector with a simple feed at its focus. We will examine three possible polarization techniques:

I. The target is illuminated with linear polarization; the tracking antenna is linearly polarized and the conical scan is executed in such a way as to maintain the polarization of the rotating beam fixed in direction--nutating linear polarization scan.

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FIG. H 1

SYMMETRY OF DIRECTION OF POLARIZATION OF
E VECTOR WITH RESPECT TO RADOME AXIS.

POLARIZATION EFFECTS

II. The target is illuminated with circular polarization; the tracking antenna is circularly polarized in the sense appropriate for the reception of the target signal (assuming a single reflection from the target surface is involved) and the scan is executed so that the sense of the circular polarization is maintained fixed over the scan cycle---nutating circular polarization scan.

III. The target is illuminated with circular polarization; the tracking antenna is linearly polarized and the scan is executed in such a manner that the direction of polarization rotates with the scan---rotating linear polarization.

The difference between the nutating and rotating polarization is shown in Fig. H2. The dotted circle represents the path of the nose of the main beam of the tracking antenna.

THE FREE ANTENNA PROBLEM. It will be instructive to examine the possibility of getting tracking errors with the antenna alone---no radome. It is known that such errors exist; the following discussion may throw some light on how they arise. Let us first review the polarization characteristics of an idealized antenna consisting of a simple linearly polarized feed at the focus of a paraboloidal reflector (Fig. H3). Because of the curvature of the reflector, the primary feed electric field distribution is transformed into an aperture field distribution having both x- and y- components. The x- component of the aperture distribution which is parallel to the axis of polarization of the feed is known as the principal polarization component. It has the symmetry property

$$(1) \quad E_x(x,y) = E_x(x,-y) = E_x(-x,-y) = E_x(-x,y).$$

This component gives rise to the main beam of the antenna, which because of the uniform phase characteristic of the aperture distribution and the symmetry property stated above, has its peak intensity along the axis of the reflector---the z- axis.

The y- component of the aperture distribution, known as the cross-polarization component, has the symmetry property

$$(2) \quad E_y(x,y) = E_y(-x,-y) = -E_y(x,-y) = -E_y(-x,y).$$

There is an effective change in phase of 180° in the cross-polarization component from any one quadrant of the aperture to the adjacent one. Because of this symmetry characteristic, the main part of the cross-polarization in the far-zone pattern appears in the form of four lobes whose peaks lie in the planes making 45° with the xz and yz planes. The latter are null-planes of the cross-polarization patterns.

THE PRINCETON UNIVERSITY RADOME STUDY

Suppose now that the feed is swung downward in the xz- plane (or the feed is kept fixed and the reflector is tipped). The first order effect is that the radiation pattern as a whole swings upward by roughly the same amount.* In the space-fixed xyz axes, the yz- plane is no longer a null-plane of cross-polarization but instead there is a distribution of the form sketched in Fig. H4. If the feed is swung upward in the xz- plane through the same amount, the radiation pattern swings downward and again there is cross-polarization the yz- plane having a distribution of the form shown in Fig. H4. The amplitude of the distribution is the same, but the sense of the polarization is reversed so that it is effectively 180° different in phase from the previous case. Let us designate the two cases as (A) and (B), respectively.

Consider now a plane wave to be incident on the antenna in the yz- plane, that is, the propagation vector of the plane wave lies in the yz- plane and makes an angle say Θ_0 with the z- axis. If the electric vector of the plane wave is strictly in the x-direction, the cross-polarization of the antenna pattern is inoperative in the reception process and the signals V_A and V_B that are received for the respective feed positions A and B are equal. If the difference between the A and B position-signals is taken as an error voltage to indicate vertical deviation of an oncoming wave from the yz- plane, the tracking system would receive no vertical error information under the assumed state of the plane wave polarization. On the other hand, if the electric vector in the plane wave is not strictly in the x- direction, but has a component in the yz- plane, the cross-polarization component is operative with the result that

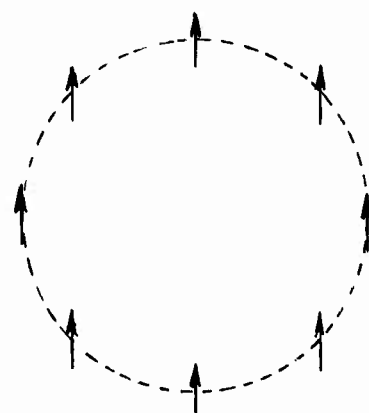
$$(3) \quad V_A = V_{||} + V_{\perp}$$

$$V_B = V_{||} - V_{\perp}$$

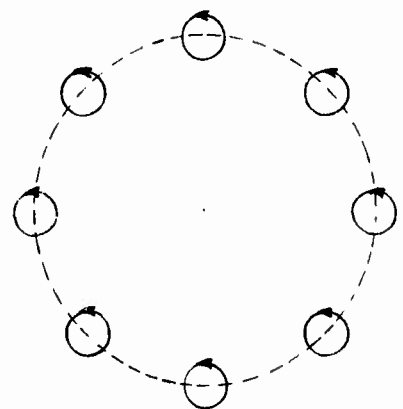
where $V_{||}$ and $V_{\perp}$ are the voltage amplitudes generated in the antenna feed line by the principal and cross-polarization components, respectively, in position A. A difference signal is then communicated by the antenna to the tracking system which is interpreted by the latter as representing vertical deviation of the oncoming plane wave. We thus have a cross-talk antenna error; that is, a left-right deviation in the yz- plane produces an up-down error signal.

One arrives at the same conclusion as to the possibility of cross-talk error even if one uses a more sophisticated formulation for the error signal. In the scanning process, one more generally measures the received voltage continuously over the scan cycle. If ψ is the scan angle, measured from the xz- plane, and $V(\psi)$ is the feed-line voltage (the scan envelope part) the error information communicated to the tracking system consists of the weighted averages

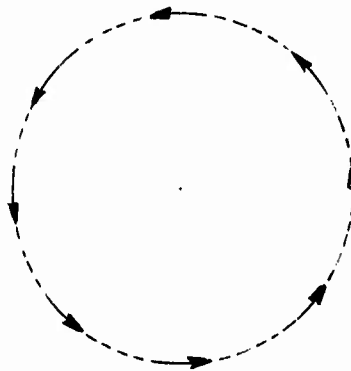
* The main beam swings through an angle equal to about 0.8 the feed swing angle.



(a) NUTATING LINEAR POLARIZATION



(b) NUTATING CIRCULAR POLARIZATION



(c) ROTATING LINEAR POLARIZATION

FIG. H2

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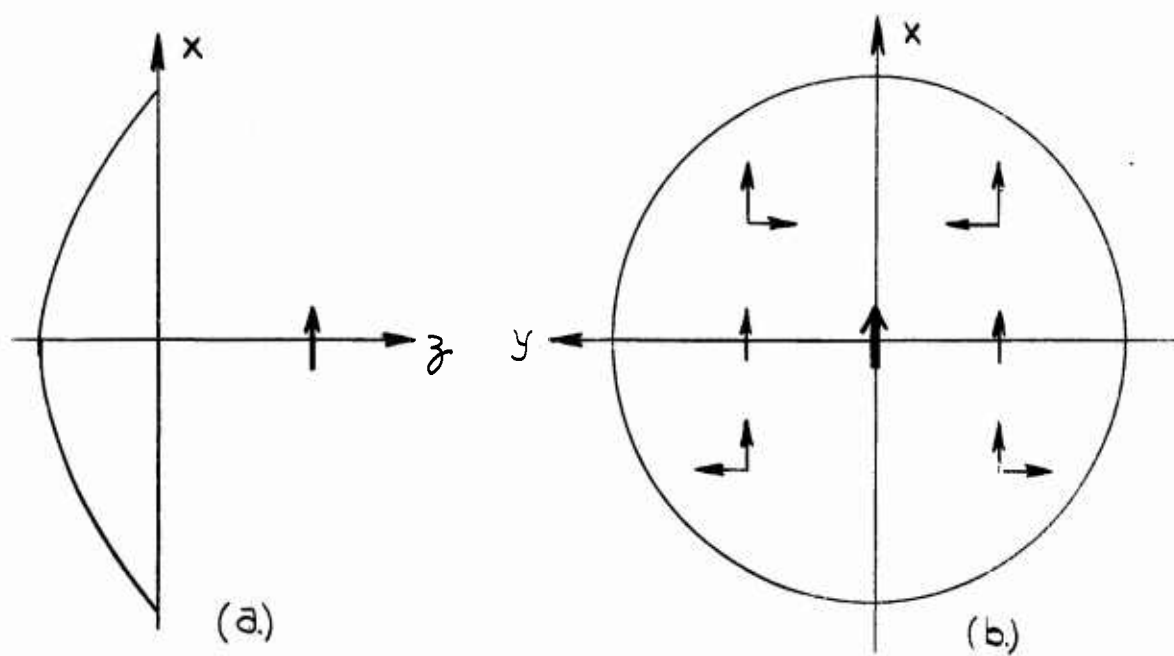


FIG. H 3

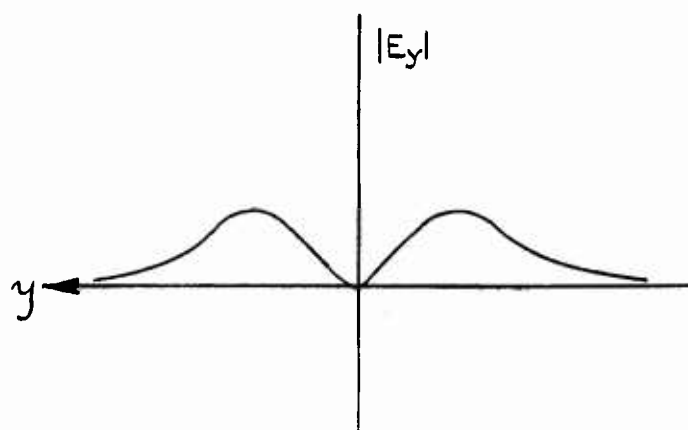


FIG. H 4

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POLARIZATION EFFECTS

$$(4a) \quad \int_0^{2\pi} |V(\psi)| \cos \psi \, d\psi \quad \text{--- xz- plane deviation}$$

$$(4b) \quad \int_0^{2\pi} |V(\psi)| \sin \psi \, d\psi \quad \text{--- yz- plane deviation}$$

The two parts $V_{||}(\psi)$ and $V_{\perp}(\psi)$ contributed by the principal and cross-polarization components will have the form shown in Fig. H5 for the case of the incident plane wave discussed above. It is readily seen that the superposition of these to form $V(\psi)$ will result in non-vanishing values for both (4a) and (4b).

In practice with guided missiles the polarization of the incident wave will generally not be parallel to the feed polarization axis. This is especially true in the operational problem with which we are concerned since the tracking antenna receives its signal from the target by a skew reflection or by diffraction. The extent to which cross-polarization is significant is essentially a statistical factor that is related to the probability of any given configuration of illuminating radar, target, and tracking antenna. The cross-talk error may be expected to be negligible in the case of large diameter-to-wavelength (D/λ) ratios because (a) the cross-polarization intensity is considerably lower than the main beam intensity and (b) the beam tilt required to obtain suitable cross-over in the conical scan is so small that the xz and yz planes remain virtually null-planes of cross-polarization. More data are needed on the cross-polarization intensity as a function of diameter-to-wavelength ratio before the analysis can be made quantitative.

Let us now consider the case of a circularly polarized antenna and a circularly polarized wave incident in the same manner as in the preceding case. For purposes of analysis, we will conceive of the feed as being formed by the addition of a linearly polarized element along the y-axis with a 90° phase lag behind the element along the x-axis. It is then obvious that the aperture distribution contributed by the added feed element is obtained by physical rotation of Fig. H3(b) through 90° . The principal polarization components of the two feed elements combine in the same sense as the feed elements while the cross-polarization components combine in the opposite sense. This is illustrated by Figs. H 6(a) and H 6(b), respectively. It is seen that the cross-polarization lobes in this case are circularly polarized in the opposite sense to that of the principal polarization. Now if the oncoming wave is circularly polarized for reception by the principal lobe, it cannot excite the system through the cross-polarization lobes. When the beam is tilted for conical scanning, the effect of cross-polarization in the principal planes is negligible. The cross-talk is not strictly equal to zero because the beam tilt is accompanied by distortion which produces a small ellipticity in the polarization. This, however, will be of smaller order than the cross-talk obtained in the linearly polarized case. If the signal is elliptically polarized, cross-talk will develop, just as in the linear case when the electric vector is not parallel to the feed polarization. The elliptically polarized wave can be resolved into circularly polarized waves of opposite sense. If the ratio of the amplitudes of the two

THE PRINCETON UNIVERSITY RADOME STUDY

circular polarization components is the same as the ratio of the components of linearly polarized wave, the cross-talk error obtained from the circularly polarized antenna will be of the same order of magnitude as the error obtained in the linearly polarized case. From an operational stand-point, the question again reduces to target behavior--to what extent does the target depolarize a circularly polarized wave. The target characteristics are a function of aspect and the probability of depolarizing configurations becomes the basic question in evaluating the circular polarization technique.

The third polarization technique that receives circular polarization with a rotating linearly polarized feed system leads to about the same result as the circular-to-circular technique. Taking again the case of the plane wave incident in the yz plane, we find that the feed line voltage including primary and cross-polarization contributions has the property

$$(5) \quad V(\psi) = V(\pi - \psi)$$

When this symmetry property is imposed on the integrals (4a) and (4b) it is found that (4a) vanishes. Thus there is no cross-talk error. If the incident wave is elliptically polarized, the symmetry property (5) no longer holds and an error results.

With an ideal antenna, the cross-talk error which has been shown to be possible would in any case be quite small. If the reflector is properly illuminated and the symmetry of the aperture distribution is maintained, the cross-polarization lobes are relatively small, and for the cross-over angles used they do not intrude into the tracking cone sufficiently to produce much effect. However, in practice an antenna is mounted on structures which are not designed to maintain any special electrical symmetry. These structures are energized by the spill-over radiation and the field that is diffracted around the edge of the reflector and contribute to the total radiation pattern. The effect is two-fold: the main beam is distorted and cross-polarization is enhanced. Both effects will lead to cross-talk errors.

RADOME EFFECTS. The effect of cross-polarization becomes more significant when the antenna is enclosed by a radome. The antenna and radome together constitute the radiating system and the pattern is determined by the field distribution over the outer surface of the radome. The pattern of the radome-antenna system is a function of the orientation of the antenna and consequently varies over the scan cycle. In the course of the scan, the pattern changes both its shape and polarization characteristics; the effect of this, together with the fact that the scan reference axis is established in terms of the free space antenna pattern, is to give rise to boresight and cross-talk errors.

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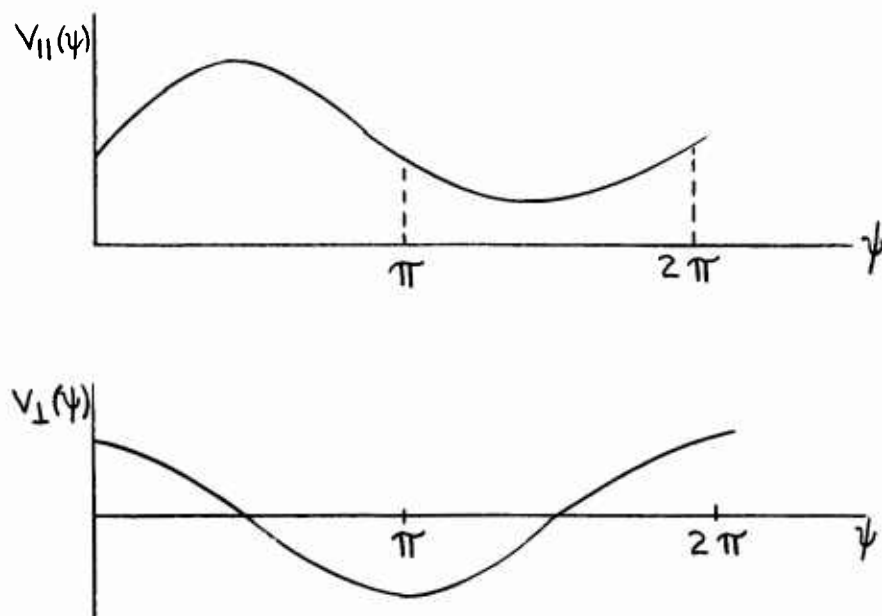
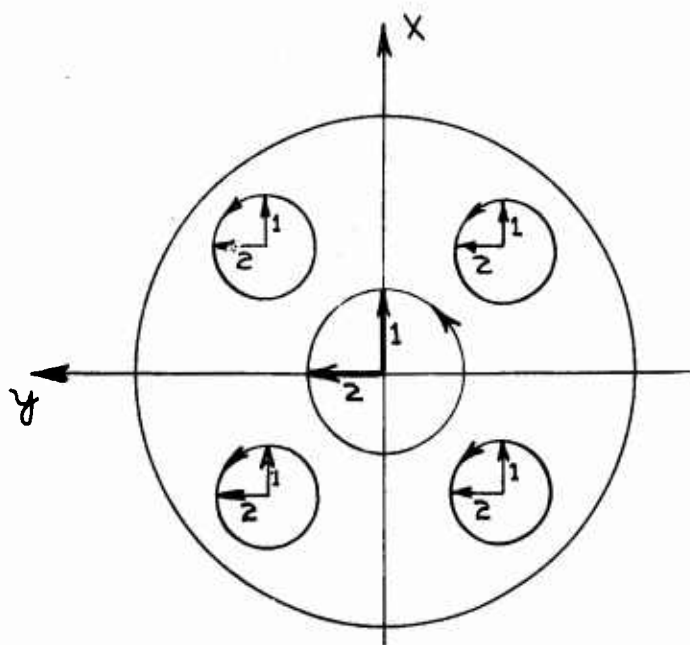
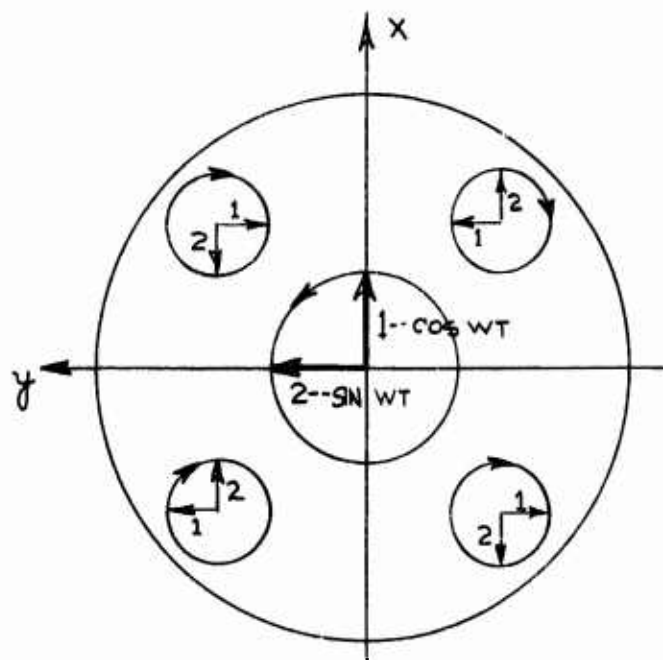


FIG. H 5



(a) PRIMARY POLARIZATION HAS SAME SENSE AS FEED TO OBSERVER ON AXIS OF THE SYSTEM.



(b) CROSS-POLARIZATION LOBES ARE R.H. CIRCLES WHEN FEED IS L.H. CIRCLES.

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FIG. H 6

POLARIZATION EFFECTS

The dimensions of the radome are necessarily such that all of it is in the immediate near zone of the antenna. The radome is supposed to be a streamlined figure of revolution which for our purposes can be taken to be simply a cone. As is shown in Fig. H7 the diameter of the base of the figure is not much larger than the diameter of the reflector. The interaction between the radome and antenna is, therefore, quite complex and such factors as multiple scattering and induction field contributions play an important part in determining the characteristics of the system. In this connection one aspect of the scanning technique should be noted: when the antenna is operating without the radome there is no significant difference between the conical scanning procedure wherein the feed is offset from the paraboloid axis and rotated and that in which the feed-to-vertex axis is kept fixed while the reflector is tipped and rotated. Inside the radome, however, the two procedures present quite different variations of the internal geometry and because of multiple scattering the two techniques give rise to different variations of the pattern over the scan cycle. It would be worthwhile to explore this point experimentally.

The following discussion is based on what may be termed a zero order approximate treatment of the effect of the radome. Only the distortion of an otherwise parallel beam of rays in a single transmission process through the curved dielectric wall is considered. It is assumed that at each point where the incident beam strikes the wall the transmission is essentially the same as that of a plane wave incident on a plane dielectric slab.

The distortion by the radome arises from the fact that the transmission characteristics are functions of the angle of incidence and the orientation of the electric vector with respect to the plane of incidence. For the wall thickness such as are likely to be used for a guided missile radome, the angular dispersion of the rays caused by the curvature is quite small as is illustrated by Fig. H8. This factor is, therefore, of secondary importance. The major factor is the amplitude and phase distortion resulting from varying angles of incidence and orientations of the electric vector. This dependence is illustrated by the transmission coefficients for a plane wave incident on a plane homogeneous dielectric slab; Let θ_i be the angle of incidence, ϵ_0 and ϵ_1 be the permittivities of free space and the slab, respectively, d be the thickness of the slab; the ratios of transmitted electric field amplitude to the incident electric field amplitude for the respective cases (a) when the electric vector is perpendicular to the plane of incidence and (b) when the electric vector is parallel to the plane of incidence are

$$(6a) \quad \left(\frac{E_t}{E_i} \right)_\perp = \frac{2 e^{j(k_0 d \cos \theta_i)}}{2 \cos(k_1 d \cos \theta_r) + j \left[\frac{Z_{10}}{A_{10}} + \frac{A_{10}}{Z_{10}} \right] \sin(k_1 d \cos \theta_r)}$$

$$(6b) \quad \left(\frac{E_t}{E_i} \right)_\parallel = \frac{2 e^{j(k_0 d \cos \theta_i)}}{2 \cos(k_1 d \cos \theta_r) + j \left[Z_{10} A_{10} + \frac{1}{Z_{10} A_{10}} \right] \sin(k_1 d \cos \theta_r)}$$

THE PRINCETON UNIVERSITY RADOME STUDY

where

$$K_0 = \omega (\mu \epsilon_0)^{1/2}, \quad K = \omega (\mu \epsilon_2)^{1/2}, \quad Z_{10} = (\epsilon_0/\epsilon_1)^{1/2}$$

$$\cos \theta_r = \sqrt{1 - Z_{10}^2 \sin^2 \theta_i}$$

$$A_{10} = \cos \theta_r / \cos \theta_i$$

The transmission coefficients are complex quantities and differ both in magnitude and phase. The range of variation with angle of incidence is quite sensitive to the electrical thickness of the slab (thickness measured in units of wavelength in dielectric). A complete discussion with numerical values is given in Vol. 26 of the M.I.T. Radiation Laboratory Series. The magnitude of the transmission coefficient for the perpendicular electric vector case decreases monotonically with increasing angle of incidence. In the case of the parallel component, the transmission coefficient first increases slowly until the Brewster's angle,

$$\tan \theta_{iB} = \frac{1}{Z_{10}}$$

is reached and then decreases rapidly with increasing angle of incidence. By the use of so-called sandwich-type walls made up of several layers of dielectrics of suitably chosen thickness and dielectric constants, the difference between the two transmission coefficients and their dependence on the angle of incidence can be adjusted some extent. A new wall construction with improved transmission characteristics was reported recently by the radome group of Hughes Aircraft Company¹.

The effect of the radome is most marked when the antenna is looking through the apex region. Let us consider the special case of zero offset angle when the axis of the reflector coincides with the axis of the cone. Leaving the question of scanning aside for the moment we shall take the feed also to be located on the axis and consider the case of linear polarization. Fig. H9 shows the projected aperture as seen looking down the axis. The small cross-polarization component

1. L. L. Bailin, B. Golvin, and A. H. McEuen, Some Considerations of Radome Errors, presented at the R.D.B. Symposium on Antennas and Radomes held August 1, 1951 in San Francisco. In this paper the depolarization effect was discussed also but from the receiving standpoint.

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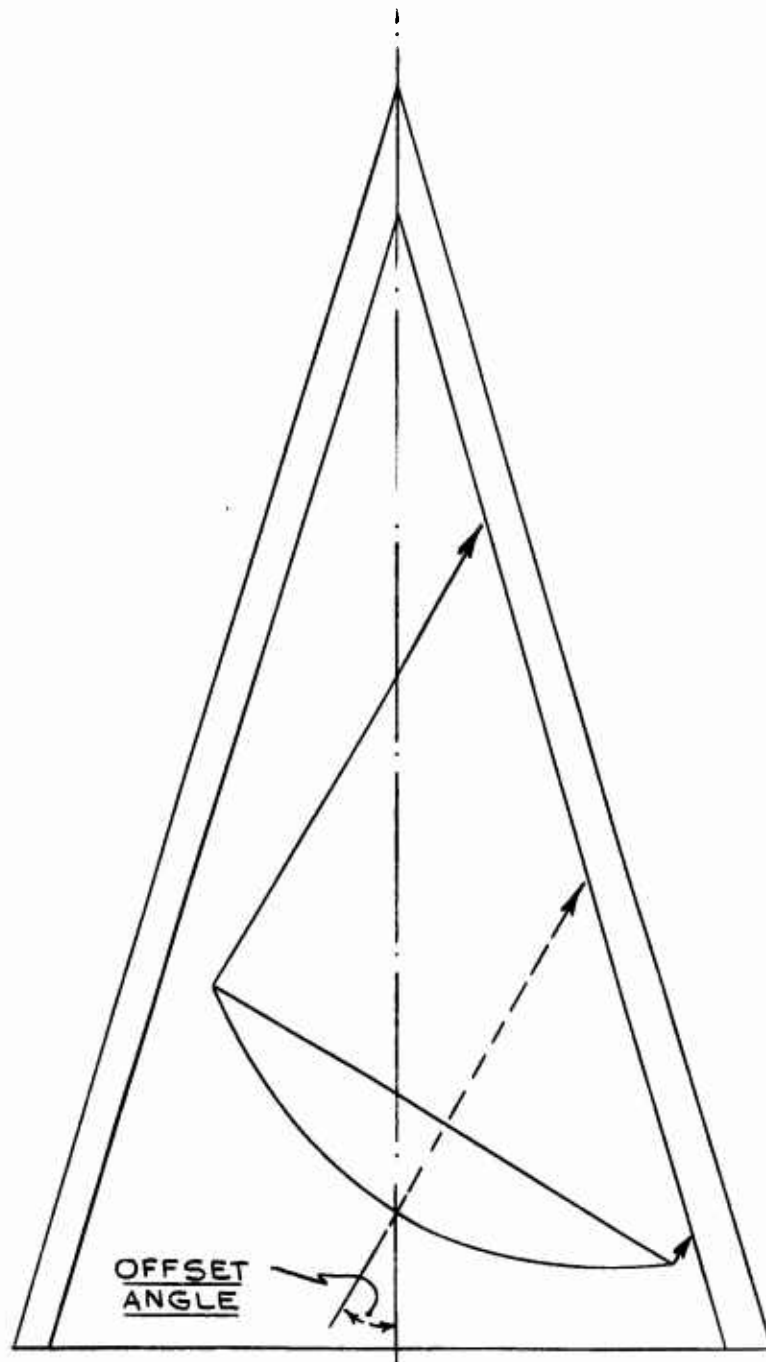


FIG. H 7

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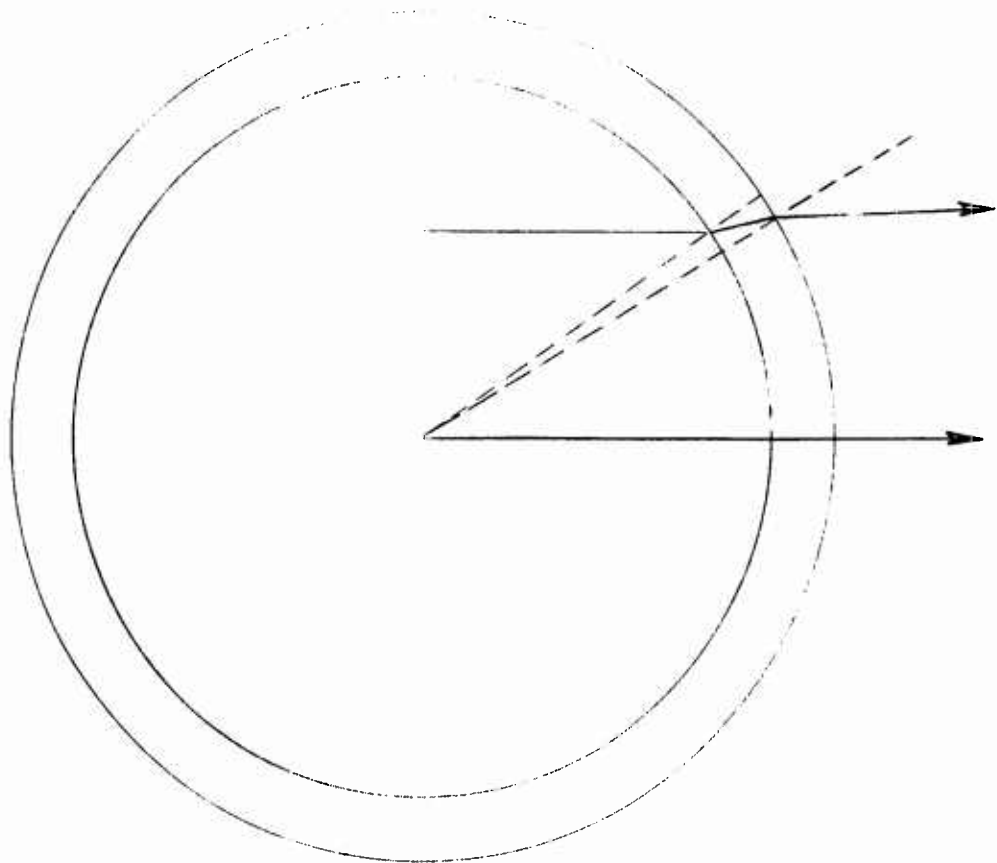


FIG. H 8

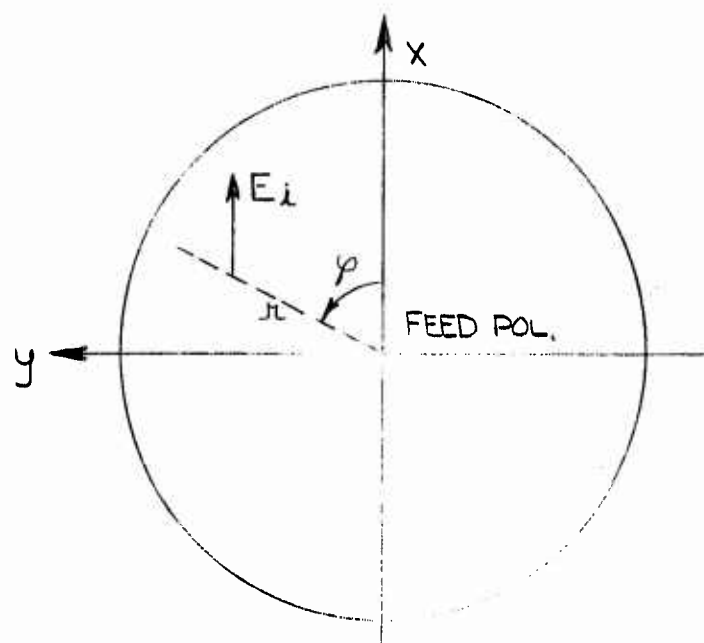


FIG. H 9

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POLARIZATION EFFECTS

in the aperture distribution of the reflector itself is being neglected. It is quite evident that for this particular configuration the angle between the incident electric vector E_i and the plane of incidence varies over the entire range of 0° to 90° . On resolving E_i into components parallel to and perpendicular to the plane of incidence and recombining after transmission, we find that at any point (μ, φ) on the surface the transmitted field has the components

$$(7a) \quad E_x = E_i (\tau_{||} \cos^2 \varphi + \tau_{\perp} \sin^2 \varphi)$$

$$(7b) \quad E_y = \frac{E_i}{2} (\tau_{||} - \tau_{\perp}) \sin 2\varphi$$

where $\tau_{||}$, $\tau_{\perp}$ are the transmission coefficients. The radome has the two-fold effect of generating cross-polarization and of distorting the amplitude and phase distribution of the principal polarization component. As a result of the latter, the main beam is broadened and takes on different beam widths in the two principal planes. In the special case illustrated in Fig. H9, the angle of incidence is the same for all rays in the system and $\tau_{||}$ and $\tau_{\perp}$ do not vary with either μ or φ . This is a special property of the cone. In any other streamlined figure the angle of incidence is a function of μ and correspondingly $\tau_{||}$ and $\tau_{\perp}$ are functions of μ . In any case the cross-polarization has the same symmetry as the cross-polarization component in the free antenna system. However, when the beam is displaced for scanning the symmetry property of $\tau_{||}$ and $\tau_{\perp}$ in relation to the beam axis is destroyed with the result that the main lobe and cross-polarization lobes are distorted. When a nutating scan is used, the distortion causes a boresight error unless the polarization of the signal from the target is precisely parallel to the polarization of the feed.* We have already seen how cross-polarization gives rise to cross-talk error in the free antenna system. The rotating polarization scan, on the other hand, maintains fixed geometrical relationships between the polarization and the plane of incidence at every point in the beam over the scan cycle. The surfaces of constant intensity generated in the scanning process are, therefore, right circular cones whose axes coincide with the scan axis. If the signal from the target is circularly polarized, this type of scan produces neither boresight nor cross-talk error for the zero offset position.

* The error is a function of the angle between the axis of the cone and the direction of propagation of the incident wave from the target.

THE PRINCETON UNIVERSITY RADOME STUDY

Returning to the non-scanning condition at the zero offset position, we see that if the feed polarization should be along the y-axis in Fig. H9, the transmitted field would have components

$$(8a) \quad E_x = \frac{E'_i}{2} (\tau_{||} - \tau_{\perp}) \sin 2\varphi$$

$$(8b) \quad E_y = \frac{E'_i}{2} (\tau_{||} \sin^2 \varphi + \tau_{\perp} \cos^2 \varphi)$$

where E'_i is the amplitude of the incident field. If the feed is circularly polarized, then even should the magnitude of E'_i and E_i be equal, the principal polarization of the transmitted field is elliptically polarized and the cross component is circularly polarized in the opposite sense. In that case, if the signal from the target is circularly polarized appropriately for the sense of the feed polarization, the cross-polarization has a very much reduced effect. The effect does not vanish completely because of the distortion that sets in when the beam is tipped off from the cone axis for conical scanning.

As the offset angle is increased, the superiority of the rotating polarization scan over the nutating circular polarization scan decreases and in fact when the apex area is not involved, the nutating circular polarization scan becomes superior. At the larger offset angles the variation of beam shape with scan is less with the nutating c.p. scan than with the rotating polarization scan. This is so because when the apex is no longer involved the variation of the direction of the polarization with respect to the plane of incidence for the case of fixed polarization is quite small over the main portion of the parallel ray system. Rotating the polarization, on the other hand, results in a continuous variation of the transmitted field intensity with corresponding variations in beam shape. The main lobe obtained from a circularly polarized feed is elliptically polarized at the larger offset angle. This is readily obvious from a consideration of the orientation of the two component feed polarizations with respect to the plane of incidence at every point on the surface. This represents an effective loss in efficiency for receiving a circularly polarized signal. However, the ellipticity will vary only slightly over the scan cycle. Since cross-polarization of the system is elliptically polarized in the opposite sense the cross-talk error is reduced and overall performance is superior to that of a linearly polarized feed system.

It should be emphasized that the conclusions as to the superiority of circular polarization and rotating polarization over the nutating linear polarization are based on the assumption that the target has a negligible depolarizing effect on circular polarization, and that consequently regardless of the changing orientation of the target with respect to missile the polarization of the signal with respect to the radome-antenna beam is a fixed factor. This is in contrast with the case of

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POLARIZATION EFFECTS

linear polarization for which changing configurations of missile with respect to target and vice versa would be accompanied by boresight and cross-talk error. However, if the target does have a depolarizing effect on circular polarization, and one which is a function of the scattering angle, it may be that very little is gained by the circular polarization techniques. This question can only be resolved by further study of target characteristics, especially a study of the full space scattering pattern.

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APPENDIX I

TEST EQUIPMENT

The need for fundamental knowledge concerning the sources of radome error cannot be overemphasized. So little data are available that it is impossible to determine accurately the most important criteria for a good radome. As a result in considering tests to be made it is desirable to plan on overmeasuring until criteria are better established. The following recommendations of tests and equipment are merely intended as a basis for discussion among those involved in the problem. Although the procedures recommended and systems described are for the most part quite standard, it was considered that a listing would be useful. It is desirable that all groups concerned have the same kind of test equipment in order to be able to interpret each other's data more easily.

Equipment is required to obtain the following classes of information:

- I. Fundamental knowledge about various types of radome materials. This testing can be done most easily using flat panels of the materials.
- II. Knowledge of the sources of amplitude and phase distortions caused by radomes.
- III. Knowledge of the sizes of errors actually obtained in systems using various radomes.
- IV. Production test data.

I. FLAT PANEL TESTING. Electrical properties which should be measured are the amplitude and phase of the transmitted wave and the amplitude of the reflected wave. This should be done for both polarizations and for angles of incidence between 0 and approximately 80 degrees. (In the case of reflection any opalescence in general can be ignored and angles of incidence and reflection made equal.) These measurements should be done so as to give data for one plane wave and for an infinite panel. Theoretical work can be done most easily using these approximations.

It is apparent that to obtain measurements approximating those for a single plane wave, one of the two antennas involved in such a system must be able to receive only one wave, i.e. it must have a narrow beam or be a long distance from the panel being tested. To obtain meaningful infinite panel information from a small sample it is necessary to ensure that the transmitted radiation does not hit the edges of the panel or, alternatively, that the receiver is unable to receive radiation which has hit the edges of the panel. This means that one of the antennas should be so close to the panel or have such a narrow beam and small side lobes that the edges of the panel are not irradiated. It appears that the conditions requiring the smallest panel for test purposes are obtained with

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one antenna close to the panel and with a fairly broad beam angle while the other antenna is large enough to have a narrow beam angle and at a distance great enough that it doesn't enter the beam of the first antenna.

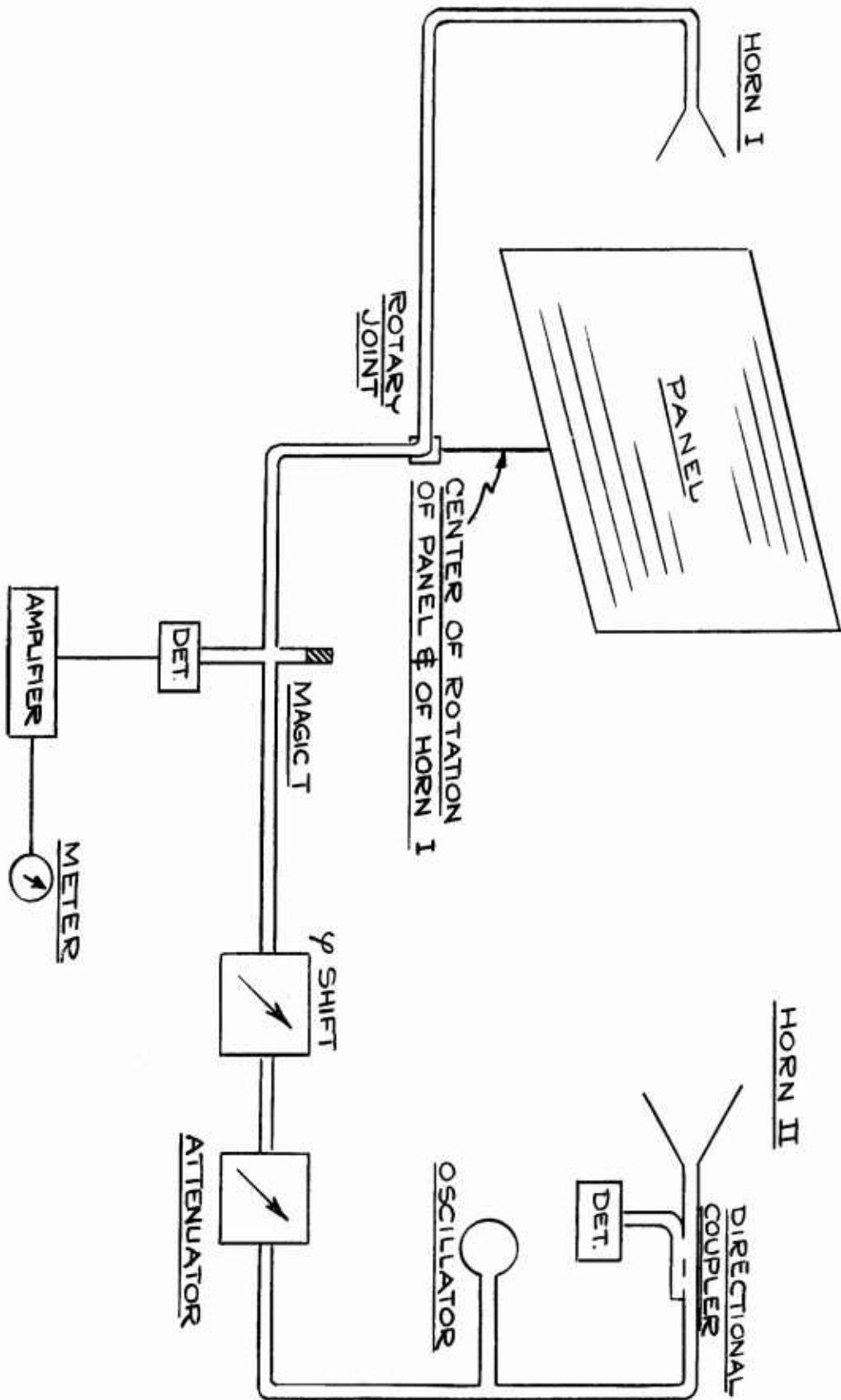
Obviously the maximum half angle of the beam from the antenna which does not "see" the edges of the panel must be smaller than 90° minus the maximum angle of incidence. At an 80° angle of incidence this requires a half angle width of less than 10° . Order-of-magnitude calculations indicate that a panel width of over $2\frac{1}{2}$ meters is required if 80° incidence is to be measured and $1\frac{1}{4}$ meter if 75° is to be measured. It is apparent that for many types of preliminary tests a panel of this size would be difficult to obtain. If small panels of a few square cm. area are to be tested, the same equipment and methods could be used, but in this case the edges of the panel would be irradiated. The edge effects could be allowed for by making measurements on panels of different sizes; or possibly the measurements of smaller panels could be confined to small angles.

In Fig. II is a block diagram of a system for making the reflection measurements described above. The amplitude (and phase) of a known amount of radiation from the transmitter can be compared with that received at the receiving antenna before and after reflection or transmission. In measuring reflections at angles close to zero degrees antenna I would obviously block the radiation to antenna II. But in this region measurements could of course be made by receiving and transmitting from the same horn.

II. FUNDAMENTAL RADOME TESTS. It is apparent that among the fundamental radome tests should be measurements on phase fronts (See Appendix B). The amplitude along the phase fronts or constant amplitude fronts should also be determined. If this is done at several distances from the radome and at all angles at which the radome would be used, the data should help isolate the actual causes of radome error. The use of conical scan would only complicate the analysis of the data. Both polarizations should be studied. Since a plane wave approximation is helpful in much theoretical work, antennas should be chosen which produce a beam giving the most easily analyzed pattern rather than the antennas actually used in specific systems. Different sizes of antennas should be used with different radomes but it would be advantageous for all groups to use a standard antenna for testing radomes of the same approximate size since this would make the interchange of information more productive. Although it would be advantageous to make phase front measurements on an actual system, this problem should be treated independently. Some work has been done on this type of measurement, but most of the equipment on which reports are available require several operations per measurement. So much data are necessary that equipment would be desirable giving phase information in a continuous manner. The operation could be manual or automatic.

Some types of equipment suitable for making these measurements are described below. Phase measurements in these systems are made with the R.F. directly. This phase information could be obtained in most cases

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FLAT PANEL TEST EQUIPMENT

FIG. I-1

at an I.F. frequency if desired. (That this and other variations should be considered is apparent from the successful operation of the phase measuring equipment of the Air Force Cambridge Research Center ⁽¹⁾).

In the systems described, the antenna inside the radome has been used as transmitter and a probe as receiver, but this relationship could of course be reversed if desirable. It should be possible to move the radome in both azimuth and elevation angles with respect to the antenna. A point of constant phase could be tracked with the probe and the location of the probe and the amplitude of the wave at this point recorded. Alternatively a constant amplitude front could be tracked. These measurements should be made with the radome in different positions and with the probe at various distances from the radome. Fig. 12 is a block diagram of a system for doing this. Two detectors and D.C. amplifiers are required. Errors could occur due to unequal responses of the detectors and amplifier drifts.

If the signal were modulated before hitting the detector a single detector and A.C. amplification could be used. The voltage out of the amplifier would be proportional to $AC \cos \Phi \cos 2\pi f t$ where A is the unmodulated reference signal amplitude, C the amplitude of a modulation side band, Φ the phase difference between the received and reference signals and f the modulation frequency. A phase sensitive detector would be required to detect the sign of the signal and if $\Phi = 90^\circ$, the phase tracked in such a system, the detected signal would be zero. If Φ is less than 90° the signal is positive, if more than 90° it is negative.

The received signal could be modulated in several ways. The signal from the transmitter could be modulated at the transmitter but the reference signal then should be unmodulated. This requires a separate oscillator and modulated amplifier at the transmitter. Alternatively if the signal from the oscillator were fed into a crystal and the crystal bias voltage changed at an audio frequency, the reflected signal from this crystal would be modulated. It could be separated from the direct signal with a directional coupler and this low power signal then radiated from the antenna.

The above discussion has been concerned only with obtaining phase information. One method of obtaining information concerning the amplitude along a constant phase front is as follows: On the phase front plotted by the equipment above there would be a $\frac{\lambda}{4}$ phase difference between the signal received by the probe and the reference signal. If an additional $\frac{\lambda}{4}$ phase delay were artificially introduced, the phase delay Φ would be $\frac{\lambda}{2}$, $\cos \Phi = -1$ and the output signal would become proportional to $-AC \cos 2\pi f t$. The value of AC could then be obtained directly from the phase detector. A value proportional to A^2 could be obtained separately if desired.

One arrangement of this type of system (see Fig. 13) could have the transmission and detection at the scanner. In such a system the detecting crystal at the receiving probe would be the modulating

(1) Automatic Antenna Wavefront Plotter, Barrett and Barnes, Electronics. January 1952 p. 120.

THE PRINCETON UNIVERSITY RADOME STUDY

element. The modulated signal would be reradiated and received back at the scanner. Signals reflected from places other than the probe would be unmodulated. The received signal would be separated from the transmitted and compared with a reference signal in a magic T circuit. The additional unmodulated component present due to reflections from other objects would produce additional cross terms in detection. As long as this reflected amplitude is small compared to the reference signal it can be ignored. The general operation of the system would be the same as that previously described. The comparison of amplitudes would be made by inserting a $\frac{\lambda}{8}$ delay in the transmission line.

In such a system the signal passes through the radome twice. This produces no error in any phase front measured but the rate of change of phase with distance is twice as large. In the usual direct transmission system the detector compares amplitudes but in the case of a reflected wave system the detector would compare amplitudes squared. In such a system the probe could be very simple and small, thus making measurements inside of a radome easier.

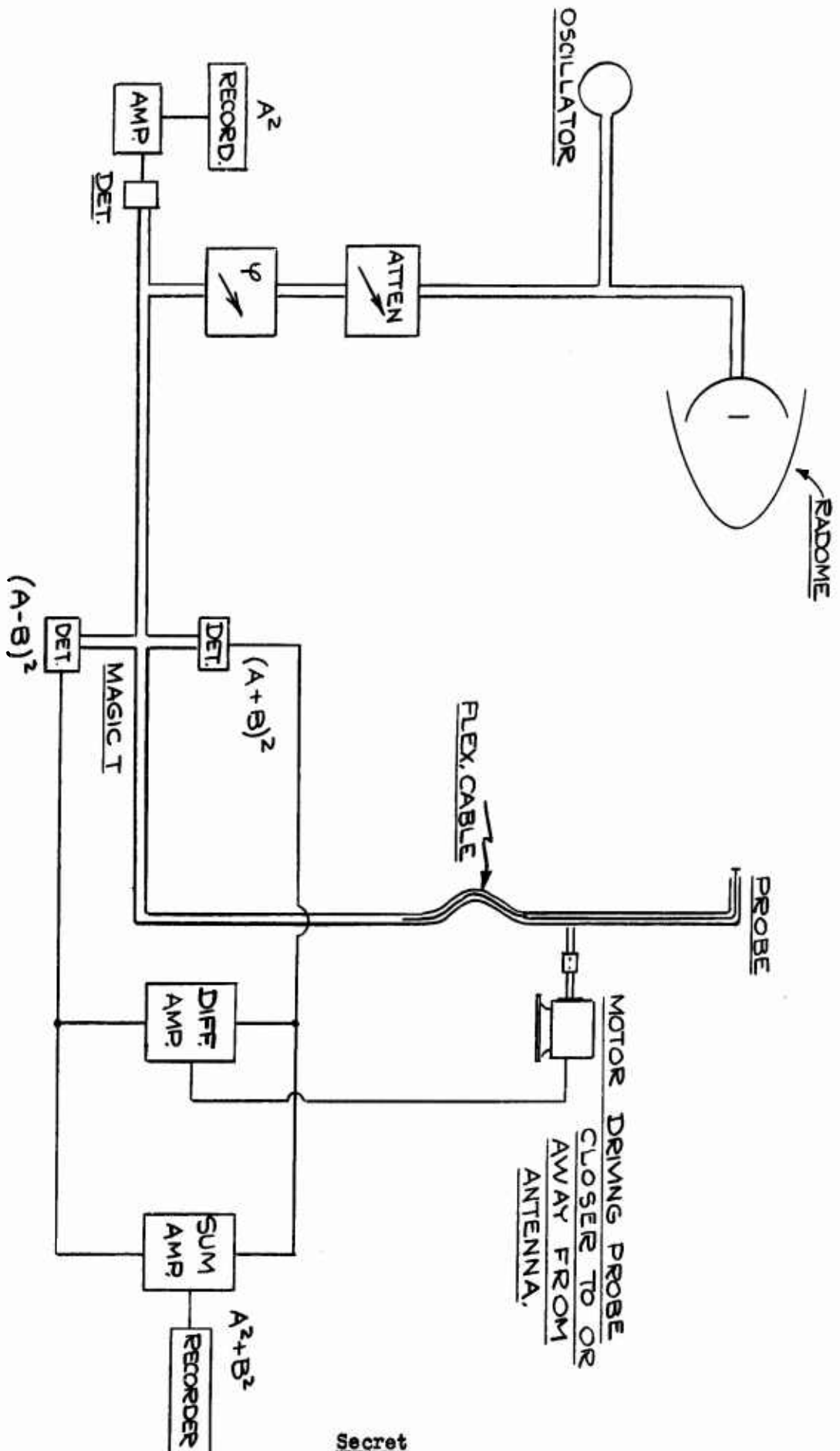
In the above discussion perfect square law detectors and modulation with equal side bands have been assumed. This is of course not completely true but expected deviations should not cause large errors.

III. EQUIPMENT FOR SYSTEM DESIGN WORK. The purpose of this equipment is to compare radomes in a manner approximating the actual system use, and to find the optimum conditions for the use of a given radome. Such equipment should do the following things:

1. Measure radome error in terms of degrees per degree.
2. Allow the testing of different radomes.
3. Allow variation of the position of the antenna along the radome axis.
4. Allow the use of different feeds including circularly polarized feed.
5. Allow measurement at all useful angles.
6. Allow use of different sizes of antennas and the corresponding nod angles.

It would be very advantageous to have the system semiautomatic. Part or all of the equipment might be adaptable to testing of specific systems. The following is a discussion of equipment which satisfies the above criteria for a conical scan system. For a single lobe system considerable variation might be necessary.

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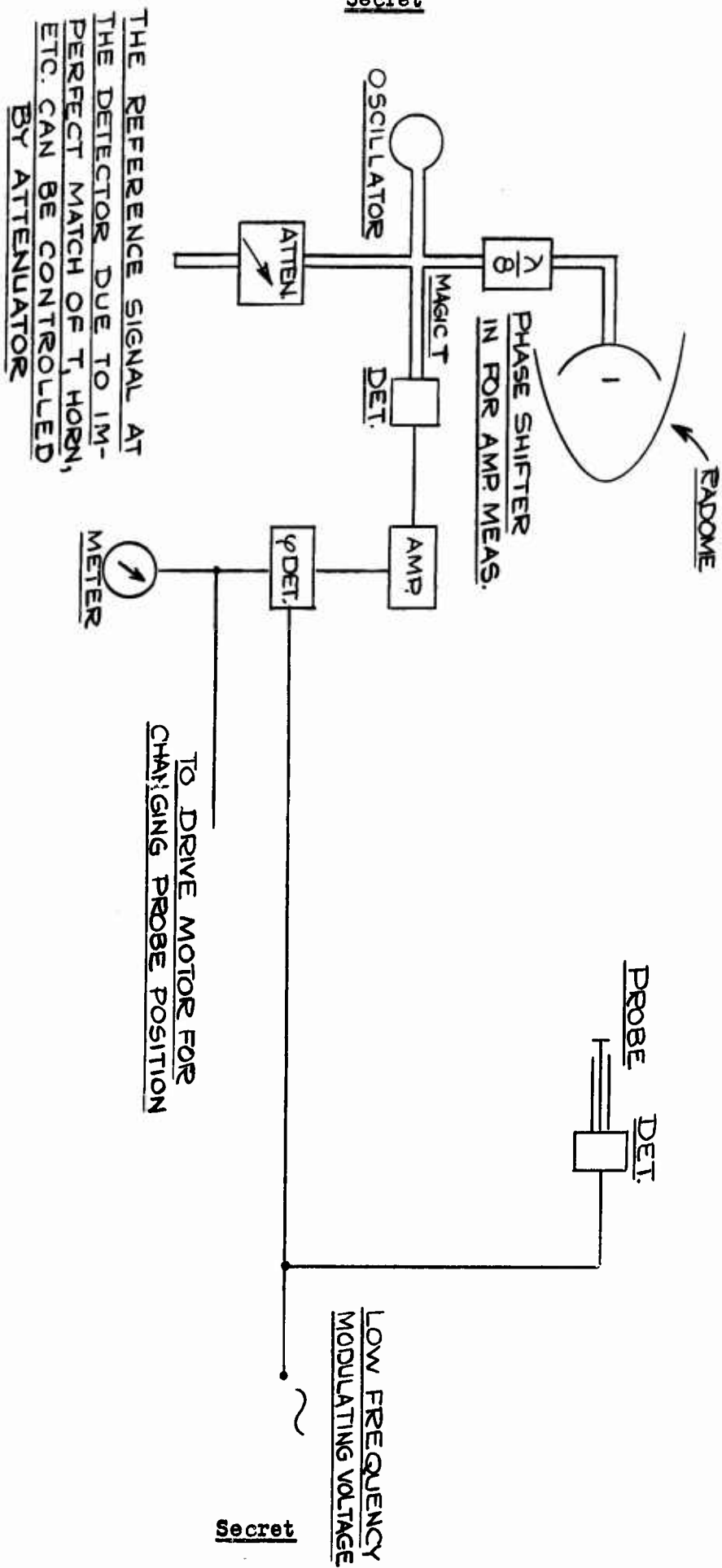
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EQUIPMENT FOR TRACKING CONSTANT PHASE FRONT

(D.C. AMPLIFICATION)

FIG. I-2

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EQUIPMENT FOR TRACKING CONSTANT PHASE FRONT.
(A.C. AMPLIFICATION)

FIG. I-3

TEST EQUIPMENT

Three obvious alternative procedures are as follows:

- A. The conically scanning antenna would be lined up on the receiving probe and the radome placed over the antenna thus shifting the zero. The probe then would be moved until it is at the new zero. In taking data the radome would be rotated about the scanner, the probe moved to the new zero and both the radome position and zero position recorded. See Fig. 14.
- B. Operation would be the same as in System A except that instead of moving the probe the antenna would be rotated to reach a new zero.
- C. The conically scanning antenna would be lined up on the receiving probe and the radome placed over the antenna thus shifting the zero. The amplitude of the modulated signal received because of the zero shift would be measured and the phase of the modulation compared with that of the scanner. The error signal obtained in this way could be compared with an error signal obtained from the system by a movement of the probe with respect to the scanner system or movement of scanner system with respect to the probe. In operation the radome would be moved and the amplitudes of the elevation and azimuth error signals recorded, possibly in a continuous manner. See Fig. 15.

To provide for semiautomatic operation, the first two systems would require an automatic follow up system to cause the probe or scanner to seek a new null position as the radome is shifted. System C would not require a servo system to give semiautomatic operation. However, it would not provide error information directly in degrees per degree. From the size and phase of the error signal the error in degrees per degree could be calculated approximately but not exactly. The information given by system C might be as useful as far as system performance is concerned as that given by systems A and B. The simplicity of system C would make it more easily adaptable to specific system testing using the actual scanner of the radar system. Systems A and B give slightly different information. In system A the radome position would be measured directly with respect to the scanner and in system B it would appear with respect to the original line of sight. In practice the differences between measurements from the two systems should be small.

In comparing systems A and B the advantages depend primarily on the relative difficulty of designing the antenna and the probe. A possible advantage of leaving the complexities in the probe (as in system B) is that the system would be more readily adaptable to the study of any specific radar and radome system, since the probe would be the same in these cases. Some equipment similar to types described above is already being used. A detailed report is available on the system in use at the U.S. Naval Air Development Center, Johnsville, Pennsylvania⁽²⁾.

(2)

Report No. NADC-EL-5101 Aeronautical Electronics and Electrical Laboratory, U.S. Naval Air Development Center, Johnsville, Pa.

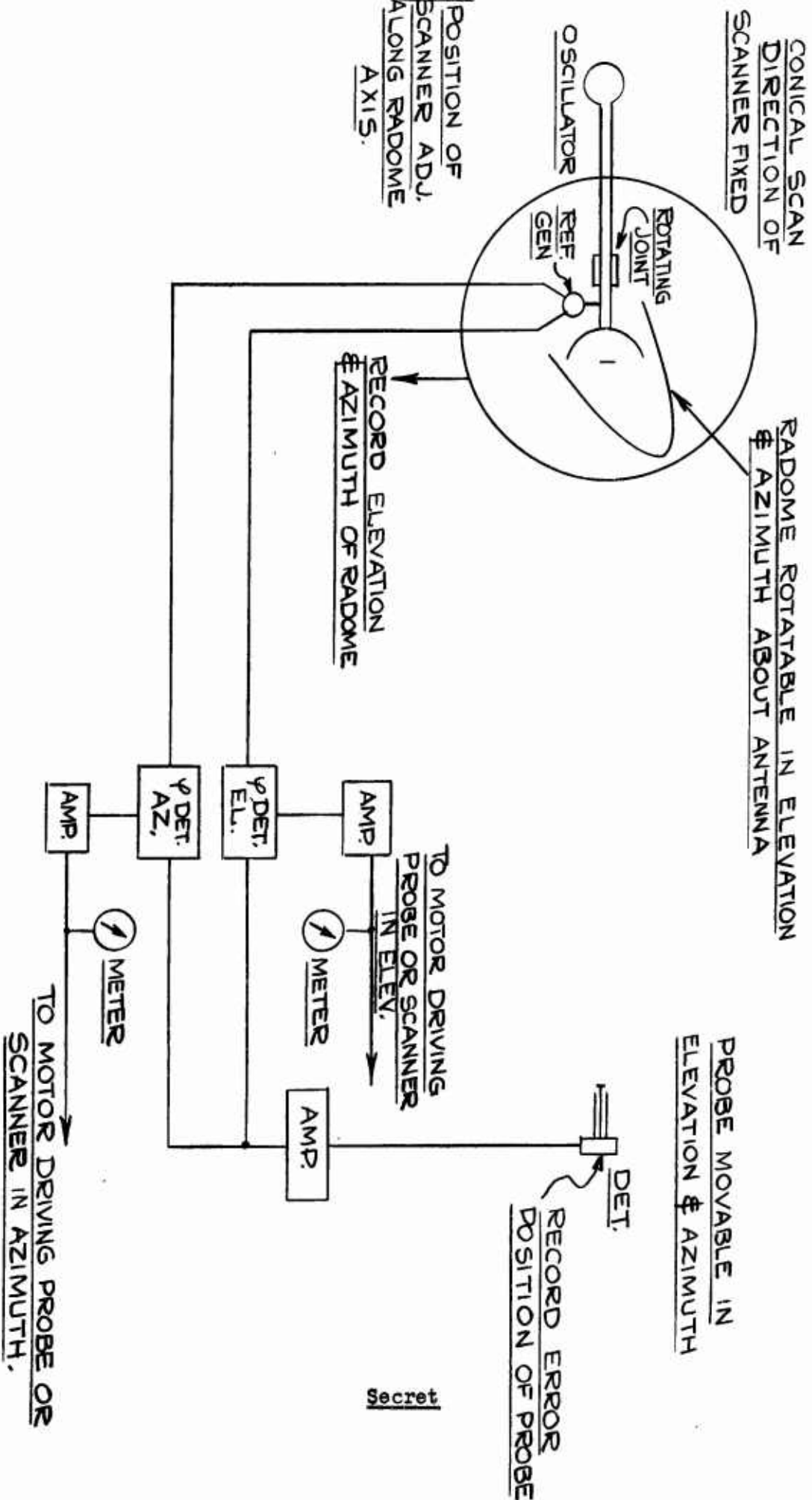
THE PRINCETON UNIVERSITY RADOME STUDY

IV. PRODUCTION TEST EQUIPMENT. It is impossible to determine the minimum criteria for such testing with the data available at present. It seems that error tests similar to those outlined for design testing might be advisable. Equipment similar to that for design testing in which the actual scanner and radome of the system are used would probably serve the purpose.

Although all groups working with radomes should have equipment applicable to their special problems, it is important that wherever possible the equipment used by different groups be standardized. This would reduce the cost of the equipment and make the interchange of data much more productive.

To summarize it is the conclusion of this study group that it is extremely important to organize an extensive radome measurements program. Furthermore, particular emphasis should be placed upon measurements that give an indication as to why a radome is not working rather than a mere indication that it is not working. A number of suggestions regarding useful measuring equipments have been made.

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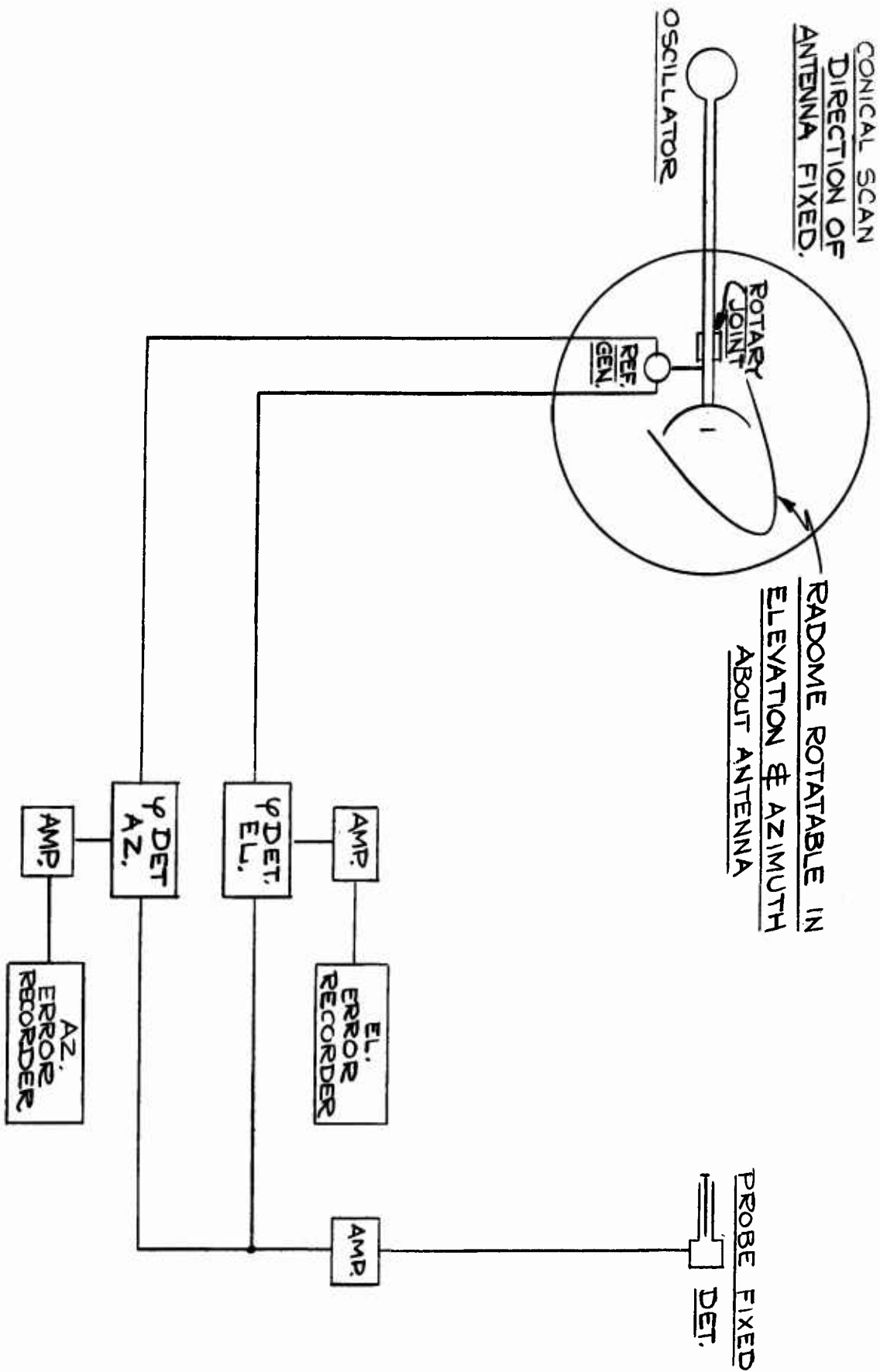


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SYSTEM TEST EQUIPMENT TYPE "A"

FIG. I - 4

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SYSTEM TEST EQUIPMENT - TYPE C

FIG. 1-5

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APPENDIX J

ALTERNATIVES TO DISH TYPE ANTENNAS

Because of the radome problem associated with the dish type homing system and the problem of multiple target resolution associated with the interferometer type, it seems desirable to consider other antenna configurations to see if one exists which may combine the advantages of both without attendant disadvantages. Although it is apparent that there are a number of alternatives to the two types used, only a few possibilities will be discussed below.

I. LINEAR ARRAYS A. An antenna system in the form of two crossed linear arrays with one of the arrays used for transmitting and the other for receiving could be used. The drag of two such arrays mounted at the nose of the missile would probably be small. (See Fig. J1)

The general features of the system are as follows: Each linear array would have a radiation pattern broad in one direction and narrow in the other, one pattern in transmission and the other in reception. The pattern of each array either in transmission or reception would be a section of a cone with the angle of the cone determined by the phase variation along the array. Signals could be received only from targets in the line of intersection of the transmission and reception patterns. (See Fig. J2). By changing the phase delays to different parts of the array, the angle of a cone with respect to its array could be varied. The phase delay could be obtained by mechanically adjusting the width of the transmission line feeding the array using a system similar to that of the AN/APQ-7(Eagle) antenna. If the angles of the cones were modulated a small amount sinusoidally 90° out of phase by a rotating shaft along the length of the wave guide, an effect similar to conical scan would be produced.

Below is a comparison of the characteristics of such a system with the standard dish type. Although each array would be covered with a radome, radome errors would probably be negligible because of their cylindrical structure. Moreover, error due to phase variations becomes smaller as the length of the array is increased. The beam could be made more narrow than that of a dish type system. The moving parts of a gimbled antenna would be replaced by those of a variable width wave guide. To obtain a $\pm 30^\circ$ beam angle variation the Eagle antenna required an R.F. switch to reverse the feed. Since this angle limitation was due to the phase errors produced when the guide was made narrower to achieve a greater phase delay, it should be possible to obtain greater angles of scan in the short array required for a missile. The movement of the beam would not be directly in azimuth and elevation as in the dish type system. Modulation of the two beam angles 90° out of phase would produce a conical scan or circular variation of beam direction when the beam is in the forward direction, but as the beam angle is increased the beam would sweep out a more elliptical path unless corrections were introduced. The aerodynamic properties of such an antenna would have to be studied in detail. With antennas

THE PRINCETON UNIVERSITY RADOME STUDY

extending out to the missile diameter a narrower beam width should be obtained than with a dish type antenna in the same size missile. The drag of such a structure should be small compared to the total drag. However, the location at the nose would contribute somewhat to missile instability, possibly a serious objection.

In comparison with an interferometer system, the beam would be of equivalent sharpness, multiple target discrimination possible and the antenna gain greater, comparable to the gain of a dish type system. The complicated scheme for obtaining angle rates required in an interferometer system would not be needed.

II. LINEAR ARRAYS B An alternative type of system using crossed linear arrays combines lobe switching of the transmitting antenna with simultaneous lobing of the receiving antenna. The arrangement of the crossed arrays differs from that described above in that the transmitting array would consist of one section sloping back on the side of the nose. This shape would probably produce less instability than a straight array of the same length at the nose. The conical lobe from this array could be moved in angle in the same manner as that described above. On the other hand, the lobe of the receiving antenna would be straight ahead and not moved in angle. It would be kept on target by rolling the missile. Since the fixed beam of the receiving antenna would eliminate trouble due to reflection from the body of the missile, it could be located in structures back nearer the center of the missile.

In Fig. J3 are shown two views of the head of the missile indicating the transmitting and receiving antennas. The transmitting antenna would radiate its energy with a conical radiation pattern having the line

$\odot A$ as the axis of the cone. The radiation would be thrown into a sector of the surface of the cone as shown in Fig. J4. The cone angle $\Theta + \infty$ could be varied by adjusting the phase velocity along the array, as in the system described above. This of course is the means for adjusting the off-set angle Θ . In addition to this adjustment the beam must be wobbled perhaps 1° . To summarize, the transmitting antenna pattern would consist of a curved surface (the surface of a cone) perhaps 2° in thickness and 30° of the total circumference of the cone.

The receiving antenna consisting of two linear arrays would receive fixed overlapping lobes with the median plane as the cross over. In-phase and out-of-phase signals would be received from these antennas using a bridge circuit in a simultaneous lobing system. This antenna pattern would be fixed with respect to the missile and the missile roll stabilized to keep it in the main lobe. As is discussed later, roll stabilization need be only good enough to keep the target in the main lobe. Small departures from perfect stabilization should produce negligible target angle rate errors as the angle data is obtained by taking the difference between the angle error signal from the radar and from a gyro.

It should be noted that a strong signal would be received only when the transmitting antenna pattern maximum intersects that of the

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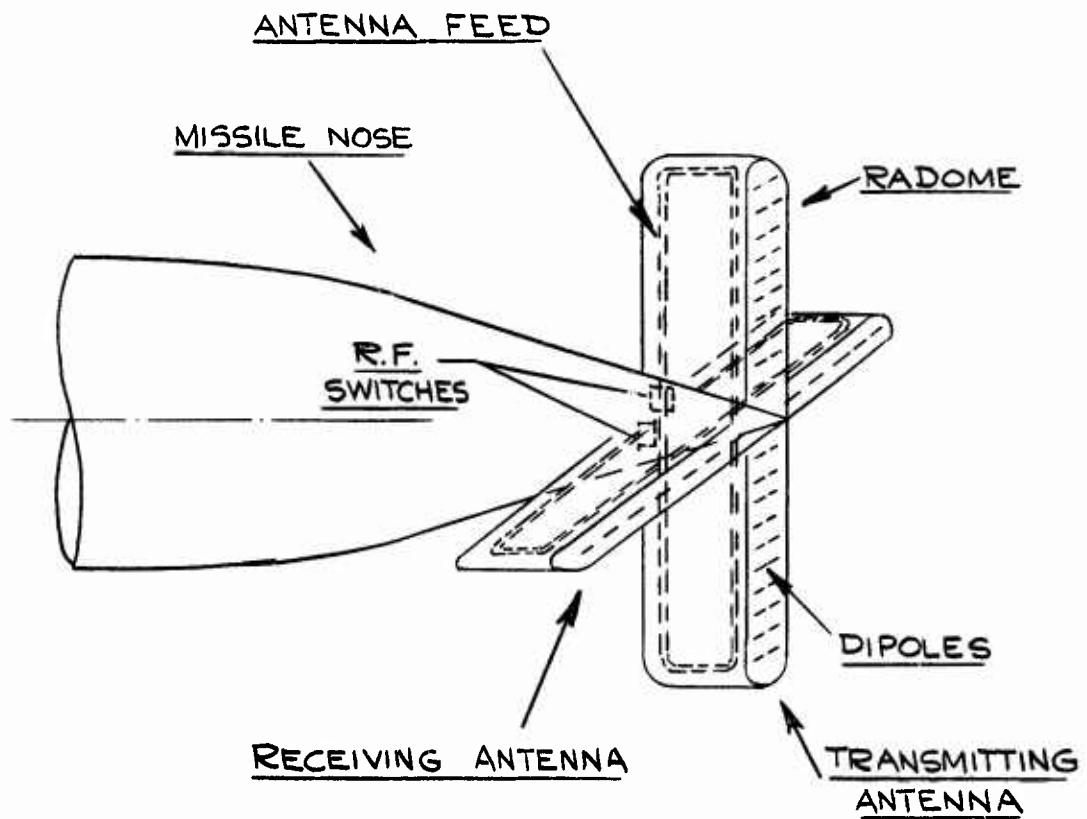


FIG. J1
LINEAR ARRAYS A

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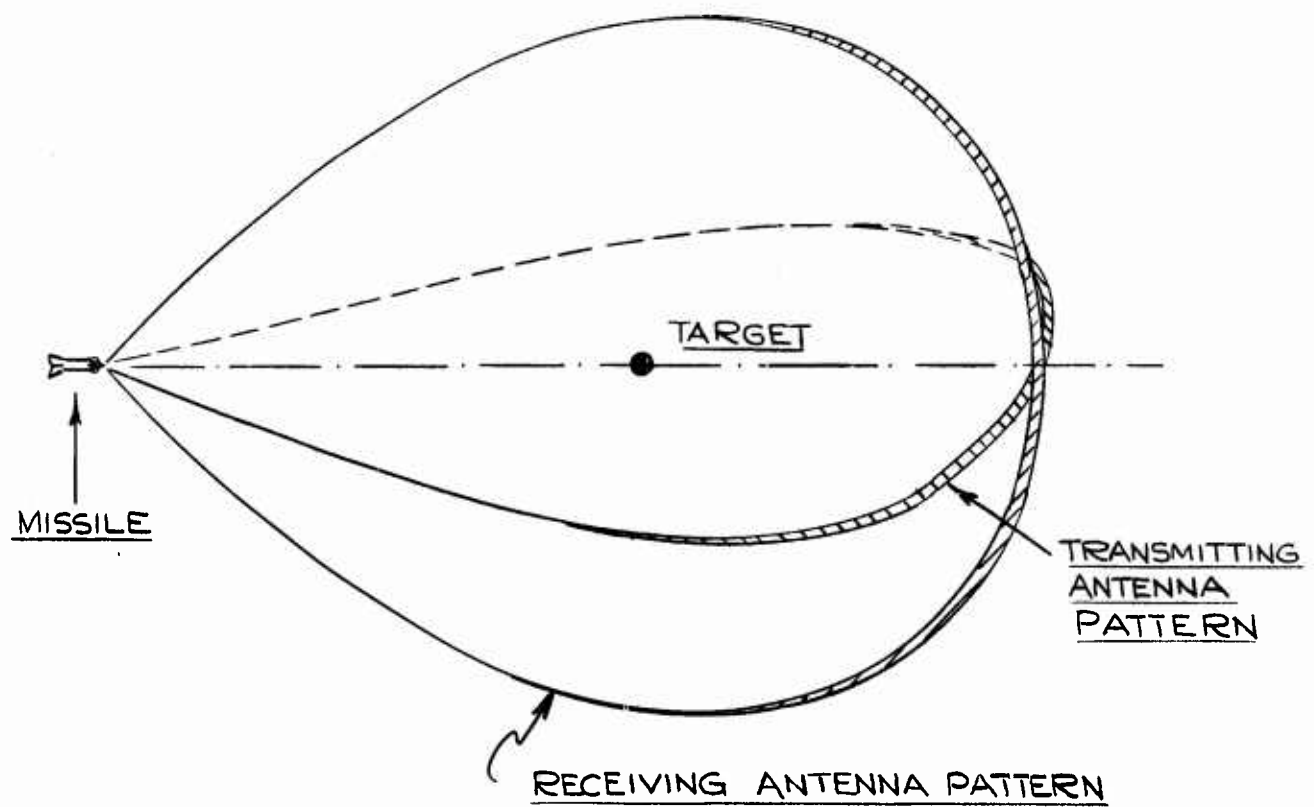
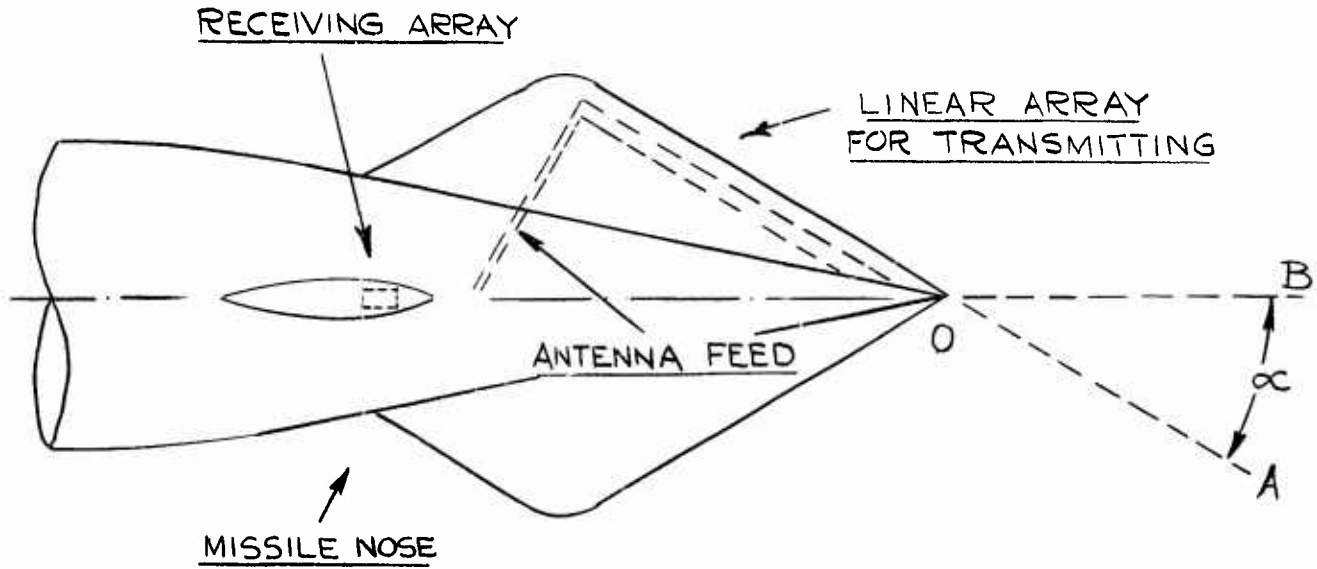


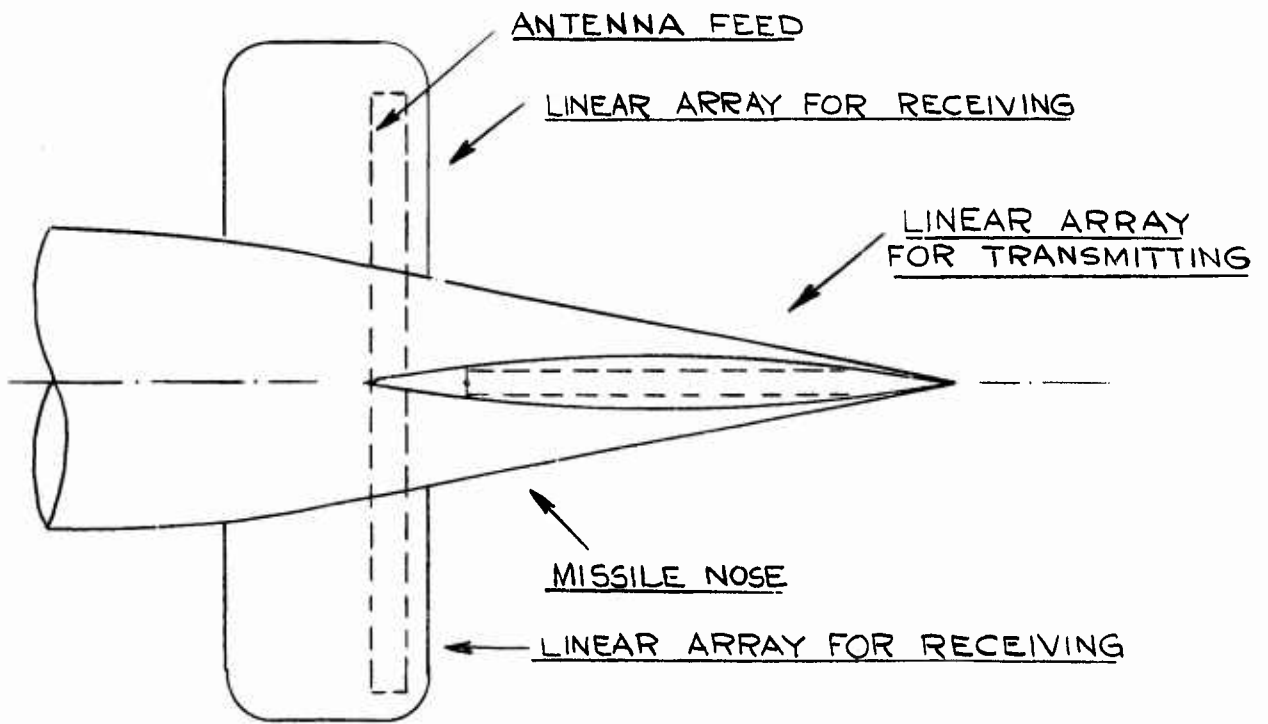
FIG. J 2
BEAM SHAPE OF LINEAR ARRAYS A

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SIDE VIEW



PLAN VIEW

FIG. J3
LINEAR ARRAYS B

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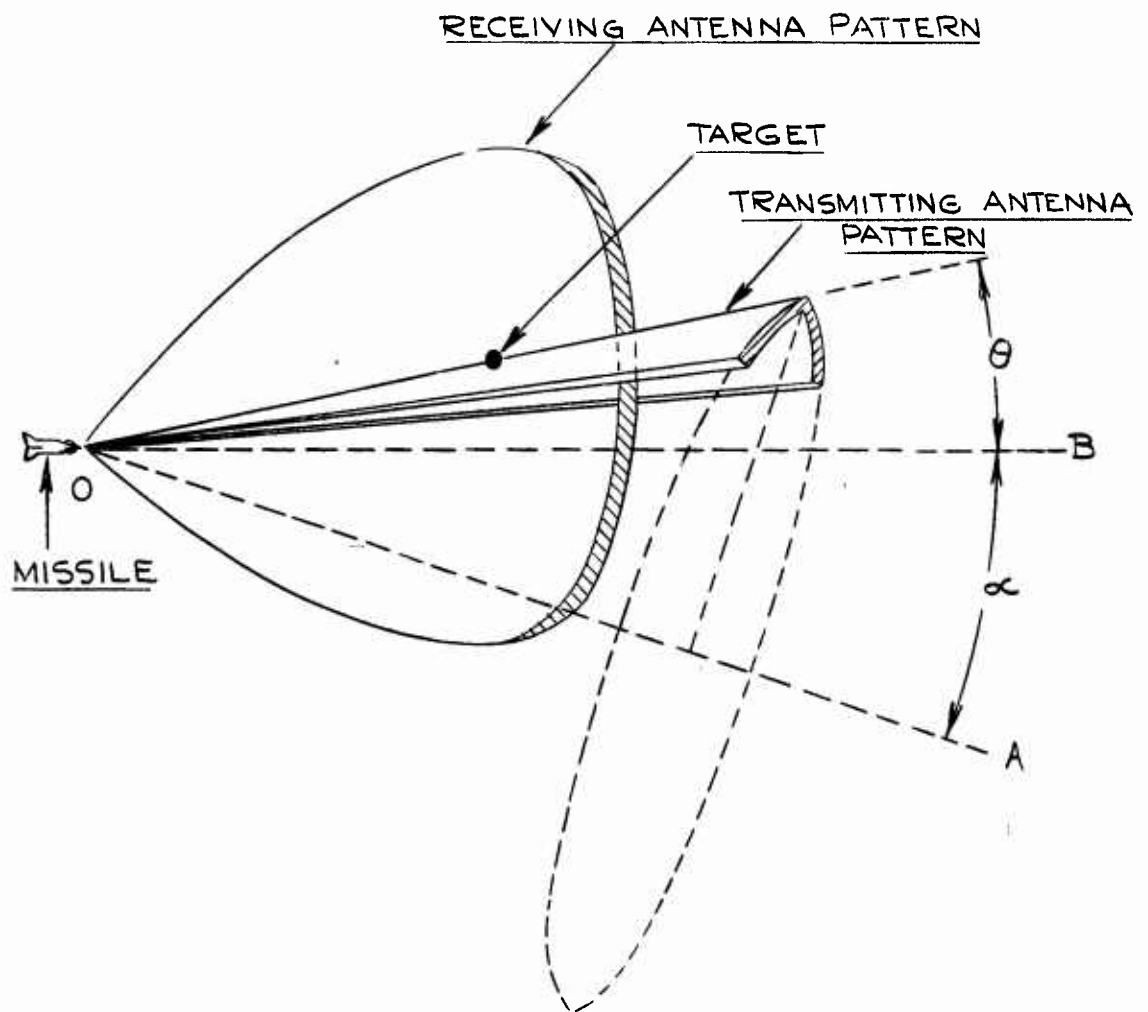


FIG.-J 4
BEAM SHAPE OF LINEAR ARRAYS B

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ALTERNATIVES TO DISH TYPE ANTENNAS

receiving pattern. Effectively this intersection would define a line in space (actually a rough cone of perhaps 2° half angle.) Furthermore the axis of this cone would be defined by the following properties of the signal: large in-phase signal, no out-of-phase signal, and absence of the switching frequency in the modulation of the in-phase signal. Thus these properties would define a line in space. The error signals would appear as an out-of-phase antenna pattern signal and as modulation of the in-phase pattern signal. The phases of these error signals would determine the sign of the angle errors.

The error signal appearing at the switching frequency would be used to servo the cone angle $\Theta + \alpha$ and its rate would be compared with a gyro rate signal to give one component of the target angle. The simultaneous lobing error signal would control the roll of the missile to keep the receiving antenna pattern on the target and its rate would be compared with a gyro rate signal to measure the other component of the target angle. Then these two target angle rates would call for lateral accelerations of the missile. As the roll position of the missile would be indeterminant for the target on the axis of the missile, the angle Θ should probably be greater than 5° . In case of a tail chase this might cause trouble, meaning that probably it would be desirable to obtain the lateral acceleration by means of wing controls rather than body angle of attack. The roll stabilization required by this system would present very serious problems because of the necessity for control of the roll of the missile by the launching system before launching. In addition excessively large roll rates might be required when the missile is close to the target.

III. COMBINATION OF DISH AND INTERFEROMETER ANTENNAS Another approach to the antenna problem is to combine a dish system with a type of interferometer. This can also be considered similar in some ways to a single lobing system with receiving antennas separated from the dish and mounted outside of the main radome.

A dish type antenna inside of a radome would be used for transmission and controlled in a crude manner to direct its radiation at the target. The target would not have to be in the center of the beam. Two pairs of small (approx. 1λ) receiving antennas would be located outside of the main radome, one pair for azimuth and one for elevation. Each pair would have a multiple conical lobed receiving pattern. By changing the relative phases of the antennas the cone angles of the lobes could be moved about the line between the two antennas. Multiple target discrimination would be obtained since these antennas receive radiation only from targets illuminated by the transmitting antenna. See Figs. J5 and J6. The small receiving antennas could be located away from the body of the missile so as not to be disturbed by reflection from the missile. Any interference with the transmitted beam which would not prevent power from reaching the target could be tolerated. Since the location of the transmitted beam is not critical, no error could occur due to the main radome and each of the receiving antennas would have a small spherical radome contributing only small errors.

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THE PRINCETON UNIVERSITY RADOME STUDY

The antenna gain of such a system would be smaller than that of the dish type system but much larger than that of an interferometer type. The circuitry probably would be as involved as that of the interferometer. There would be no multiple target interference. The mechanical problem of phasing the antennas would be simpler than that of the linear array and the angle of coverage greater. However, there would be the additional problem of moving the transmitting antenna.

The above discussion is primarily to suggest possible approaches to the radome problem. Many difficulties inherent in these systems may have been overlooked and many other combinations may exist.

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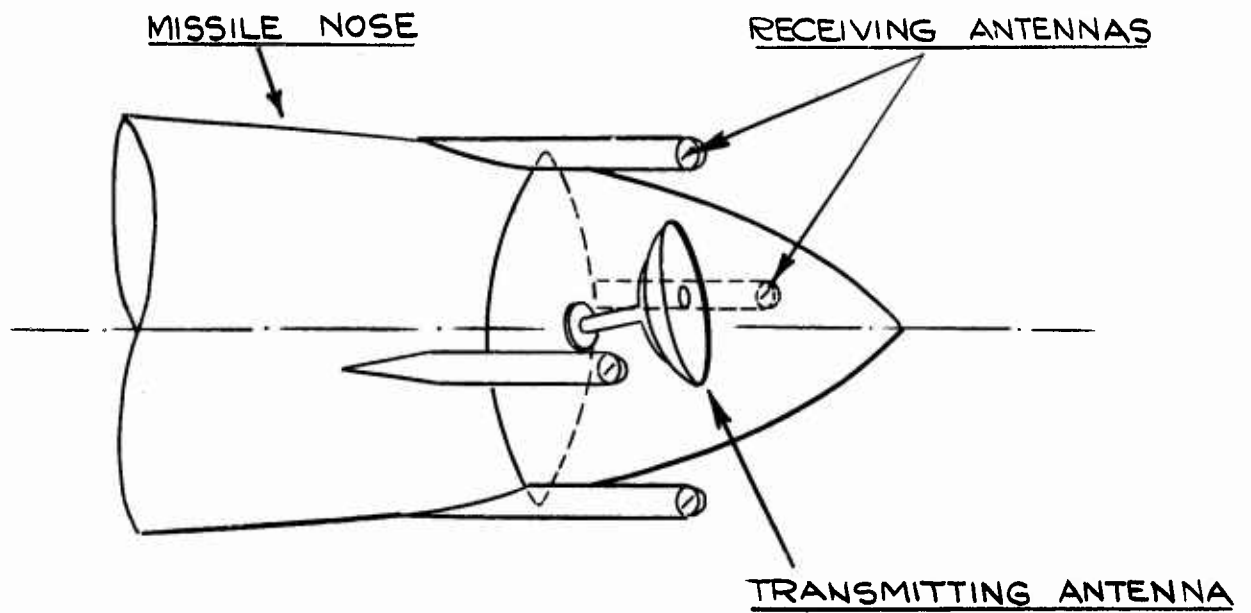


FIG. J5

COMBINATION OF DISH AND
INTERFEROMETER ANTENNAS

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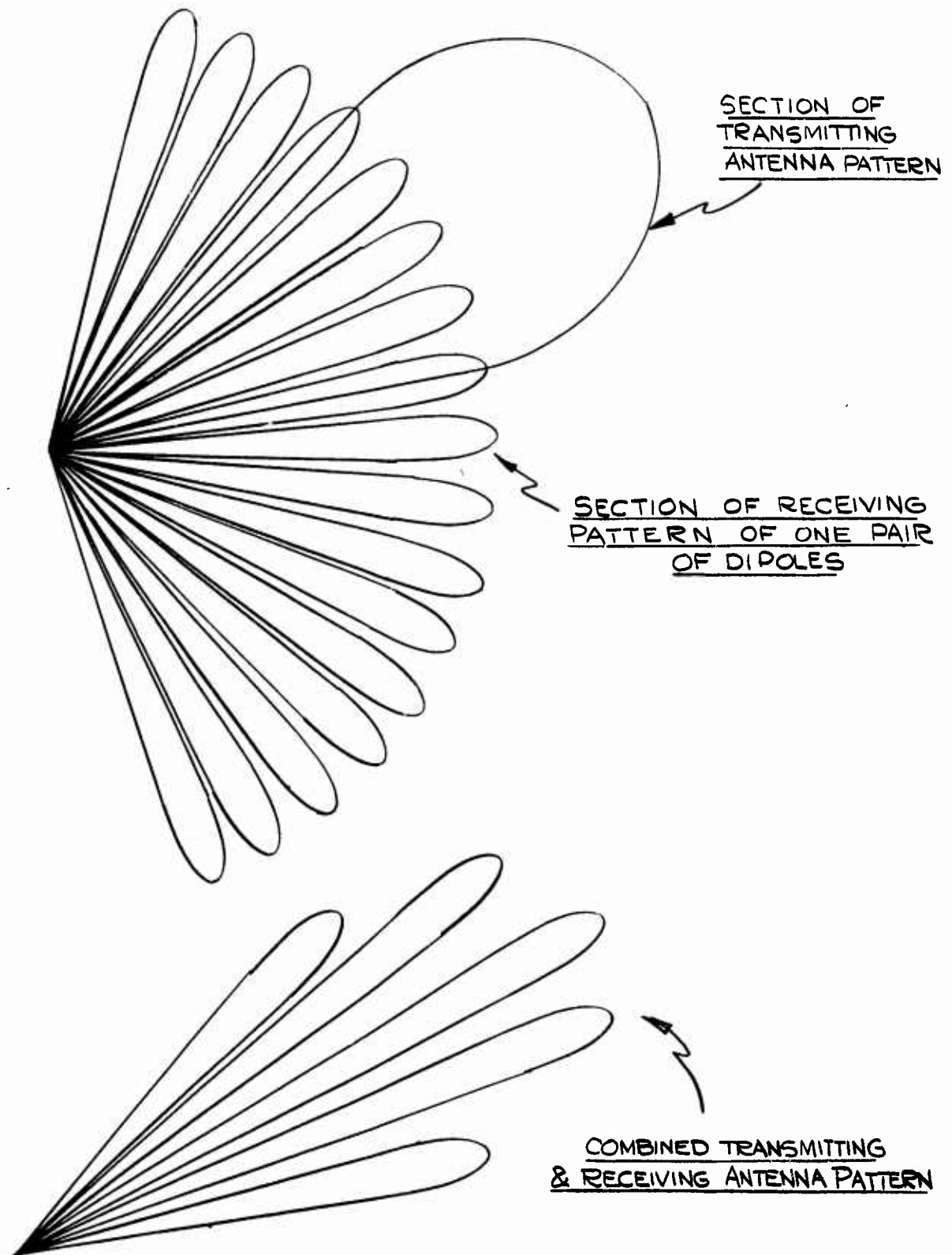


FIG. J 6

BEAM SHAPE OF COMBINATION
DISH AND INTERFEROMETER ANTENNAS

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APPENDIX K

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