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WATERTOWN ARSENAL LABORATORY

EXPERIMENTAL REPORT

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TENSILE STRESS-STRAIN CURVES

BY

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TENSILE STRESS-STRAIN CURVES

OBJECT

To determine the relation between stress and strain in the initial part of the plastic region of the stress-strain curves particularly of plain carbon steels.

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SUMMARY

1. It is shown that for specimens of a number of steels the logarithm of the stress is essentially a linear function of the logarithm of the plastic strain (for strains from .01 to about .4).

2. From the data of Bridgman, it has been found that, for copper, this linearity extends to very large strains (3.5).

3. For the plain carbon steels studied, the logarithm of the slope of the linear part of the stress-strain curve is essentially a linear function of the logarithm of the yield strength.

4. If the logarithmic stress-strain curve is linear at least to the strain at which necking begins, the magnitude of the slope of the curve is

exactly equal to the value of the strain at which the maximum load occurs.

5. Based upon the above results, a simple method of determining the stress-strain curve is suggested. This method involves the determination of only the following values: the maximum load, the breaking load, the strains to maximum load and to fracture.

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INTRODUCTION

In previous papers[1 to 5], it has been pointed out that when the tensile stress (load divided by actual area) is plotted against the tensile strain (logarithm of the ratio of the initial to the actual area), the stress-strain curve is linear from about the strain at maximum load to the strain at fracture. In the design of most engineering structures, however, only the initial part of the stress-strain curve is of importance, for an engineering structure cannot suffer large plastic strains without becoming unstable and perhaps being rendered unserviceable. At present, it is difficult to compare the initial plastic parts of the stress-strain curves because of the difficulties encountered in comparing lines of continually changing slopes.

It was suggested by Norris[6] that the logarithm of the stress was very nearly a linear function of the "true tensile elongation"*. No attention has been given, however, to the relation between the stress and the "natural" strain in the initial plastic part of

*The ratio of the initial area to the actual area minus one.

the stress-strain curve.

RESULTS AND DISCUSSION

While a plastically deformed material is under stress, the actual deformation consists of two parts: the elastic and the plastic. The elastic contribution to the deformation involves a volume increase, while the plastic involves no change in volume. The relation between strain and area is given by the following equation:

$$\varepsilon = \ln \frac{A_o}{A} \qquad (1)$$

where ε is the strain, Ao and A, the initial instantaneous areas, respectively. This equation may be derived[2] from the fundamental definition of strain, i.e.:

$$d\varepsilon = \frac{dl}{l} \qquad (2)$$

if it is assumed that the volume of the material remains constant during deformation. It follows that this relation between strain and area is exactly consistent with the definition of strain, only if, ε, refers to the plastic (permanent) strain. The area, A, therefore, is not the area that is involved when a stress, S, is being applied but the area measured after the stress has been removed and the elastic strain has been relieved*. Consequently, the area, A, will be taken hereafter as the area of the test bar after the load has been removed. The difference between the strain as determined from the area measured under load and that determined from the area measured under no load is very small (.002 at 100,000 p.s.i. stress) and of no significance except for very small strains.

The logarithm of the tensile stress** has been plotted as a function of the logarithm of this plastic strain for specimens of a number of steels. When plotted in this manner, the data, from very small, strains ($\epsilon <$ .01) to strains of about .4, fall on

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*If the actual area under load has been measured, the plastic strain is equal to $\ln\frac{A_o}{A} - 2u(S/E)$, where A is the area under load, u, Poisson's ratio, S, the stress, and E, Young's modulus.

**The stress is obtained by dividing the load by the area and is therefore an average stress, which after necking commences, is subject to the correction due to the variation in stress across the necked diameter.

straight lines*.

In Figures 1, 2, 3, and 4, the data for variously heat-treated specimens of four plain carbon steels are plotted in this fashion. The data of Figures 1 and 2 are taken from a previous report[1], the data of Figure 3 are new, and that of Figure 4 are taken from the results of Gensamer and co-workers[4]**.

It is to be noted that for all specimens the logarithm of the stress is essentially a linear function of the logarithm of the strain, from strains of about .01 to about .4***. For strains larger than about .4, the data in most cases diverge upward from the straight lines.

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*For steels which exhibit a drop in load at yielding and a lower yield-point elongation, this linearity of the logarithmic stress-strain curve commences when the stress begins to rise. (after the lower yield-point elongation)

**The data of Gensamer et al are somewhat uncertain, for the published results do not extend to small strains.

***It should be noted that plotting of data on log-log paper should be carried out with discretion, for the scales are such to obscure small variations from general relationships. The justification for such plotting rests on the usefulness of the results.

This upward divergence of the logarithmic stress-strain curve from linearity is not always the case, however. In Figure 5, are plotted the logarithmic shear stress-strain curves for a mild steel[8]. Curve A was derived[8] from a tensile test with the use of the Von Mises' viewpoint, and Curve B was taken directly from a torsion test performed on the same material. The data for large shear strains derived from the tensile tests diverge upward from the straight line, while the torsion data diverge downward. This fundamental difference between the behavior in tension and torsion has been previously discussed in detail[8] and ascribed to the difference in orientation effects (of carbides) associated with the two types of deformation. Recent data obtained in this laboratory* on copper specimens in tension indicate that the linearity extends to the fracture strain, and results obtained by Bridgman[9] indicate that for copper in compression, the linearity extends to strains of about 3.5.

In Figure 6, the logarithm of the slopes of

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*To be presented in a subsequent report.

the logarithmic stress-strain curves have been plotted against the logarithm of the stress at a strain of .01. The logarithm of the slope varies nearly linearly with the logarithm of the stress for each carbon content. The points fall on nearly parallel straight lines for each carbon content independent of the type of structure of the steel. Some of the specimens were tempered martensite, some pearlite, and some bainite. The commonly held belief that the ratio of yield strength to tensile strength depends upon structure may be reconciled with the above statement if it is realized that fully quenched and tempered steels reveal a drop in load at yielding more readily than do steels having lamellar structures. The lower yield stress of a metal having a drop in load at yielding is greater than would be the yield strength (at a small strain) for a steel having a smooth stress-strain curve and the same tensile strength. Fully quenched steels may therefore, in many cases, appear to have a larger yield-tensile ratio than steels having a lamellar structure. The .59% C. steel specimen tempered at 1300° F. illustrates this phenomenon. This specimen reveals a drop in load at yielding and a pronounced lower yield-point elongation. As indicated in Figure 3, the stress at a strain of .01 obtained by extrapolating the smooth stress-strain curve is about 14,000 p.s.i. smaller than is the lower yield stress*.

The equation of the straight lines of Figure 6 may be written as:

$$\log m = K - .94 \log I \qquad (3)$$

where m is the slope of the logarithmic stress-strain curve, I, is the stress at a strain of .01, and K, is a constant which depends upon the carbon content as indicated in Figure 7. Thus, for the plain carbon steels investigated, the yield strength and carbon content together will determine, with the use of Figure 7 and Equation 3, the slope of the logarithmic stress-strain curve (in the range of strain from .01 to .4).

For the steel specimens studied in the range from very small strains ($\rightarrow$ .01) to strains of at least .4, the following relation is valid:

$$\frac{d \ln S}{d \ln \varepsilon} = m \qquad (4)$$

where m is the slope of the logarithmic stress-strain

*The value of the stress at a strain of .01 used in this report for this specimen is taken as the extrapolated value.

curve. It follows from Equation 4 that:

$$\left(\frac{dS/S}{d\varepsilon/\varepsilon}\right) = \left(\frac{dS/S}{d\varepsilon}\right)\varepsilon = m \qquad (5)$$

From Equation 1, $d\varepsilon$ may be obtained, and, upon substitution:

$$\left(\frac{dS/S}{dA/A}\right)\varepsilon = -m \qquad (6)$$

Necking begins when the load reaches a maximum and therefore at the maximum load:

$$d\,L = d(AS) = 0 \qquad (7)$$

since $L = AS$

where L is the load, and S the stress. Carrying out the differentiation indicated in Equation 7, the following is obtained:

$$AdS + SdA = 0$$

$$\text{or} \quad \left(\frac{dS}{S} \Big/ \frac{dA}{A} \right) = -1 \tag{8}$$

which is the condition for necking. Substituting Equation 8 in Equation 6, the following relation results:

$$\varepsilon_n = m \tag{9}$$

where ε_n is the strain at which necking begins and at which the maximum load occurs. Therefore, the slope of the logarithmic stress-strain curve is equal exactly to the strain at the maximum load*.

This result permits the determination (at least for strains in the range of .01 to about .4) of stress-strain curves by a very simple procedure. It is only necessary to measure the initial diameter of the specimen, the maximum load and the elongation or the diameter at which the maximum load is reached. From these measurements, the stress, and the strain at maximum load may be determined, and one point may be

*This result depends, of course, upon the validity of the assumption that the logarithmic stress-strain curve is linear, at least to the strain at which the maximum load occurs, which assumption is valid for the steels studied.

plotted on the logarithmic stress-strain curve. A straight line with a slope exactly equal to the strain at the maximum load (ϵ_n) may then be drawn through this point. This logarithmic stress-strain curve may then be replotted as a normal stress-strain curve up to a strain of 0.4, as illustrated in Figures 8a and 8b. If the metal exhibits a drop in load at yielding and a lower yield-point elongation, it is necessary to measure the upper and lower yield stresses as well as the maximum load and strain to maximum load. The lower yield stress is laid off on the logarithmic stress-strain curve, as illustrated in Figure 8c. The true stress-strain curve can then be replotted as illustrated in Figure 8d, placing on this curve the upper yield stress.

The diameter at which necking begins may be determined very easily if the diameter over the gauge length of the tensile bar before testing is uniform. Before the maximum load is reached, the entire specimen deforms uniformly over the gauge length. When the maximum load is reached, the specimen begins to neck and the metal at the necked region continues to deform and the material away from the necked region ceases deforming. After the bar is broken, the diameter of the uniformly deformed but unnecked section of the specimen may be measured and the strain to the maximum load determined.

For specimens which are not exactly uniform over the gauge length, measurements of the diameter of the specimen may be made (at marked intervals along the specimen) before the specimen is tested. After the specimen is broken, the diameter at the same marked positions in the unnecked section of the bar may be measured. From the initial and final areas at these positions, the strain may be determined. However, since the bar is not uniform, this strain will not be equal to the strain at which the specimen began to neck. A chart has been prepared (Figure 8) by which the strain at necking can be determined from this measured strain.

The relation between the measured strain and the strain to necking will depend upon the ratio of the initial area of the specimen at which the measurement was made, to the initial area of the specimen at the point at which necking occurs. The relation between the two strains has been calculated (Appendix A) for several values of C where:

$$C = \ln \frac{A_{o1}}{A_{on}}$$

and where A_{o1} is the initial area of the specimen at the uniformly deformed but unnecked portion of the specimen at which the final diameter was measured and A_{on}, is the initial area of the specimen at the position at which necking occurred.

If it is desired to obtain the entire stress-strain curve, the final diameter and the breaking load* may be measured and a straight line drawn (on linear paper) between that part of the stress-strain curve which is linear on log-log paper (strain of about .4) to the point at which fracture occurs. This procedure is illustrated in Figures 8b and 8d.

*In normal tensile tests, the breaking load is difficult to determine accurately, and, therefore, special care must be taken to obtain accurate results.

APPENDIX A

Determination of Strain at Necking

The strain at which necking begins in a tensile-test bar may be simply found by measuring, after the test, the cross-section area of an unnecked portion of the bar, provided the cross-sectional areas before the test in both the necked and unnecked portions are known. Since within the range of uniform deformation, the logarithm of the stress S varies linearly with the logarithm of the plastic strain ε, there is obtained:

$$\ln S = m \ln \varepsilon + b \qquad (A1)$$

where m and b are constants. Moreover, since it has been shown that the constant of proportionality, m, is equal to the strain at necking ε_n, Equation A1 may be written:

$$\ln S = \varepsilon_n \ln \varepsilon + b$$

Therefore $$\ln S_n - \ln S_1 = \varepsilon_n (\ln \varepsilon_n - \ln \varepsilon_1)$$

or $$\ln \frac{S_n}{S_1} = \varepsilon_n \ln \frac{\varepsilon_n}{\varepsilon_1} \qquad (A2)$$

where the S_n and ε_n refer to the stress and strain at necking and S_1 and ε_1 to the maximum stress and strain occurring at an unnecked portion having the resulting area A_1. Since S_n and S_1 occur simultaneously the load:

$$A_n S_n = A_1 S_1$$

or

$$\frac{S_n}{S_1} = \frac{A_1}{A_n} \qquad \text{(A3)}$$

where A_n is the area at necking. Substituting Equation A3 in A2:

$$\varepsilon_n \ln \frac{\varepsilon_n}{\varepsilon_1} = \ln \frac{A_1}{A_n} = \ln \left(\frac{A_1}{A_{o1}}\right)\left(\frac{A_{on}}{A_n}\right)\left(\frac{A_{o1}}{A_{on}}\right)$$

$$= \ln \frac{A_{on}}{A_n} - \ln \frac{A_{o1}}{A_1} + \ln \frac{A_{o1}}{A_{on}}$$

where A_{on} and A_{o1} designate to the areas before the test of those portions, referred to above, which afterwards have areas A_n and A_1, respectively.

But by the definition of strain:

$$\varepsilon_n = \ln \frac{A_{on}}{A_n}$$

$$\varepsilon_1 = \ln \frac{A_{o1}}{A_1}$$

and we define $C = \ln \frac{\lambda_{o1}}{\lambda_{on}}$. Then:

$$\varepsilon_n \ln \frac{\varepsilon_n}{\varepsilon_1} = \varepsilon_n - \varepsilon_1 + C \qquad (A4)$$

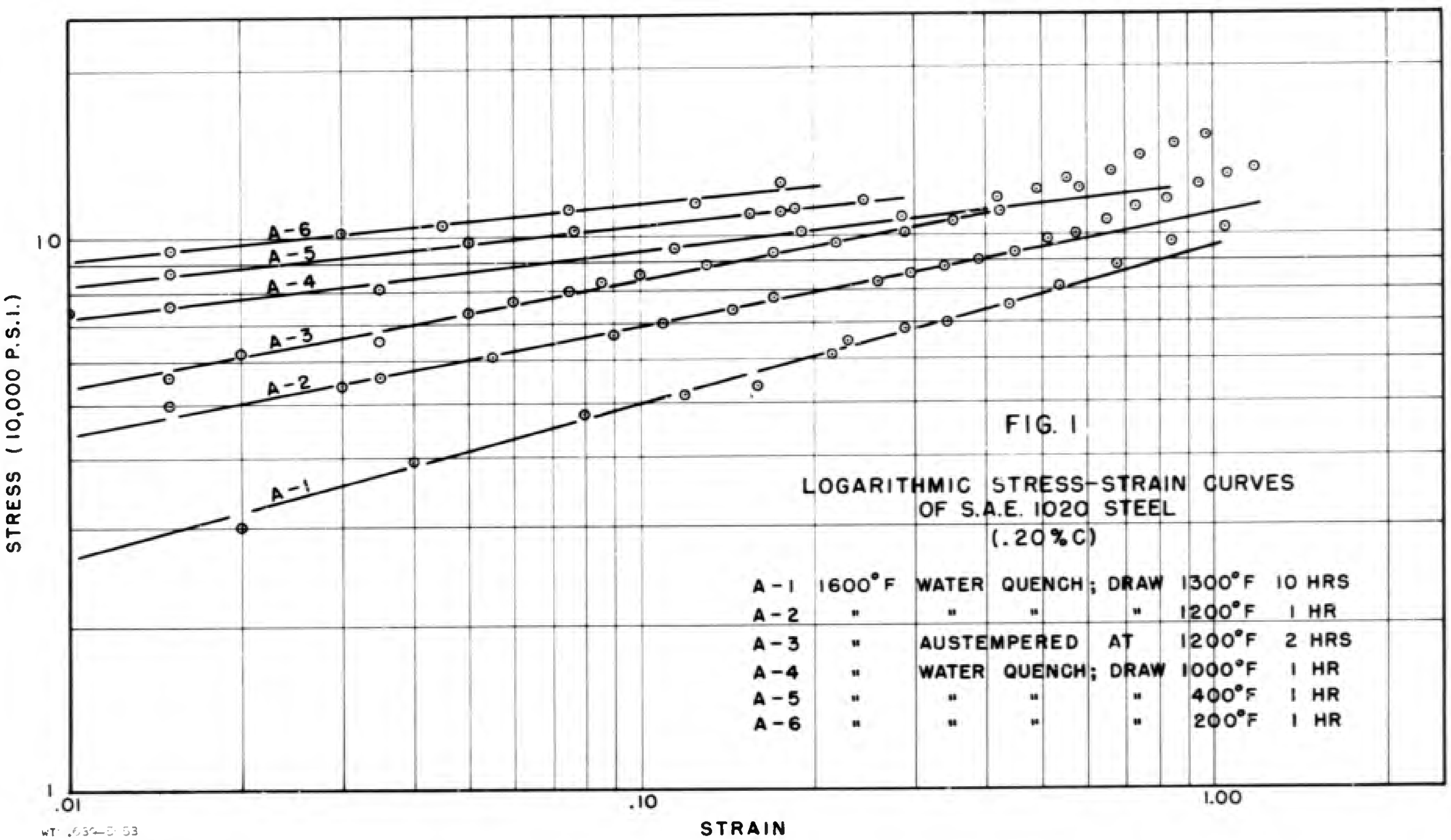


FIG. 1

LOGARITHMIC STRESS-STRAIN CURVES
OF S.A.E. 1020 STEEL
(.20%C)

A-1	1600°F	WATER QUENCH;	DRAW	1300°F	10 HRS
A-2	"	"	"	1200°F	1 HR
A-3	"	AUSTEMPERED	AT	1200°F	2 HRS
A-4	"	WATER QUENCH;	DRAW	1000°F	1 HR
A-5	"	"	"	400°F	1 HR
A-6	"	"	"	200°F	1 HR

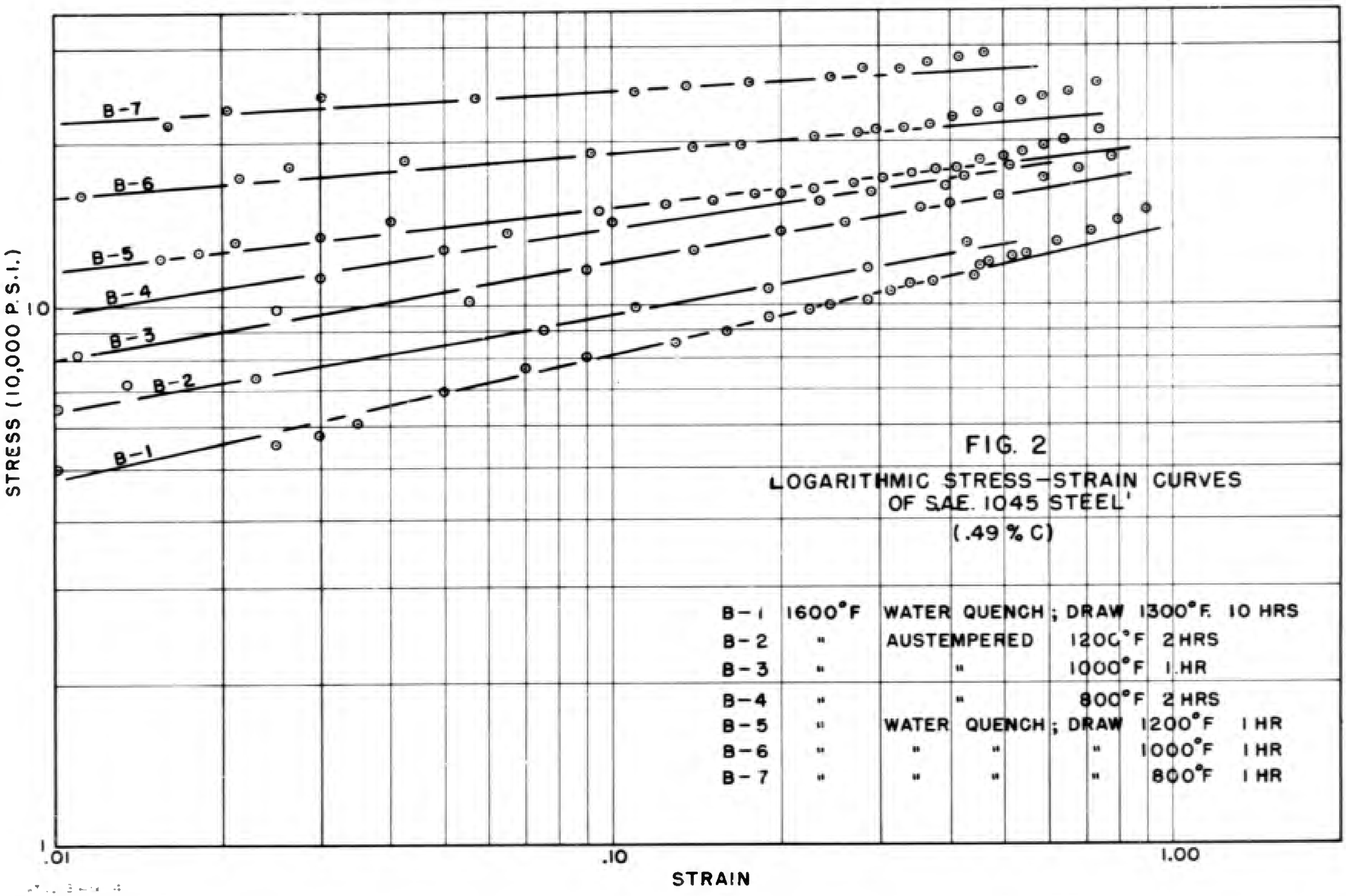
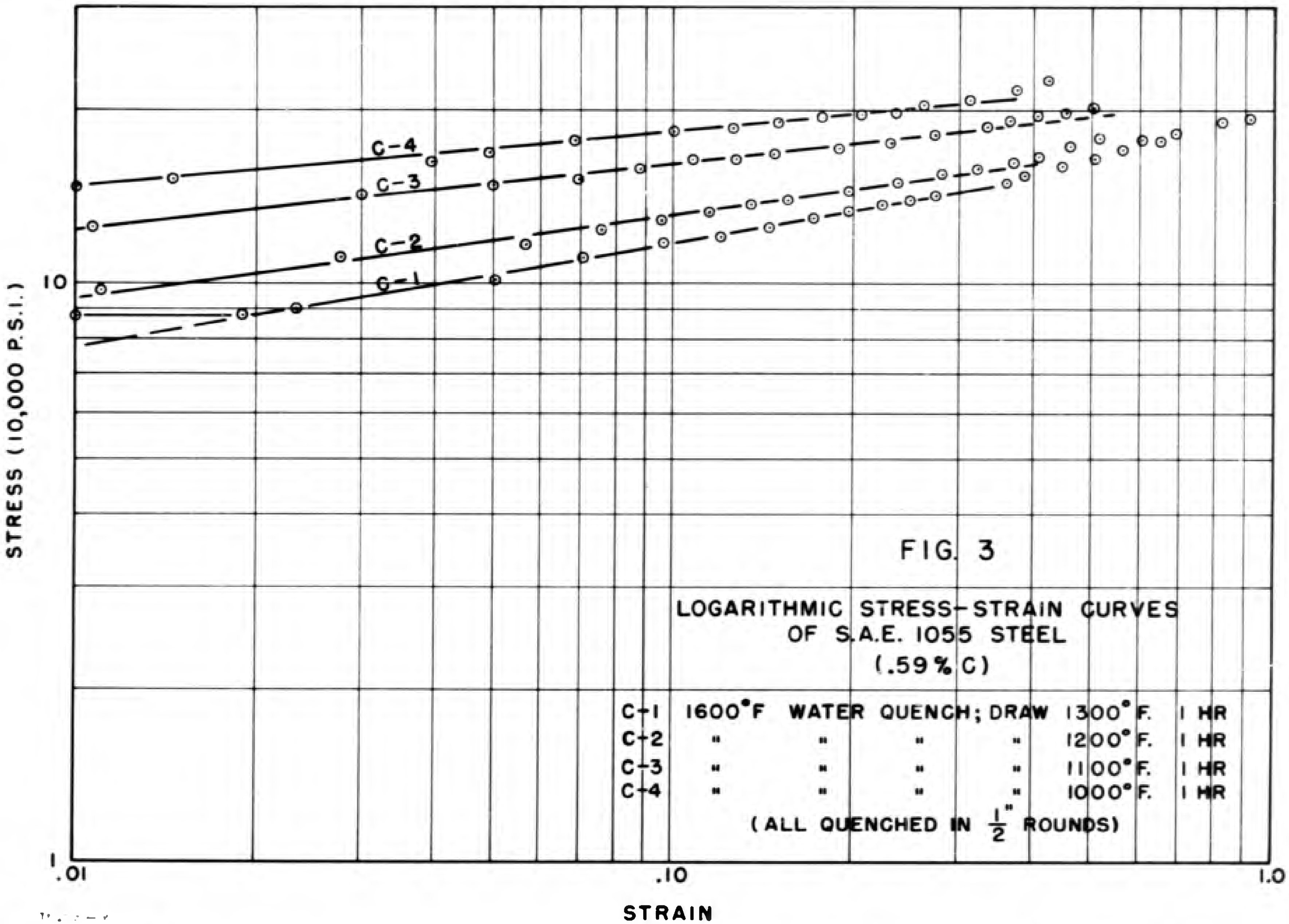


FIG. 2
LOGARITHMIC STRESS-STRAIN CURVES OF S.A.E. 1045 STEEL (.49 % C)



C-4
C-3
C-2
C-1
10
1
STRESS (10,000 P.S.I.)
.01
.10
1.0
STRAIN
FIG. 3
LOGARITHMIC STRESS-STRAIN CURVES
OF S.A.E. 1055 STEEL
(.59% C)
C-1 1600°F WATER QUENCH; DRAW 1300° F. 1 HR
C-2 " " " " 1200° F. 1 HR
C-3 " " " " 1100° F. 1 HR
C-4 " " " " 1000° F. 1 HR
(ALL QUENCHED IN 1/2" ROUNDS)

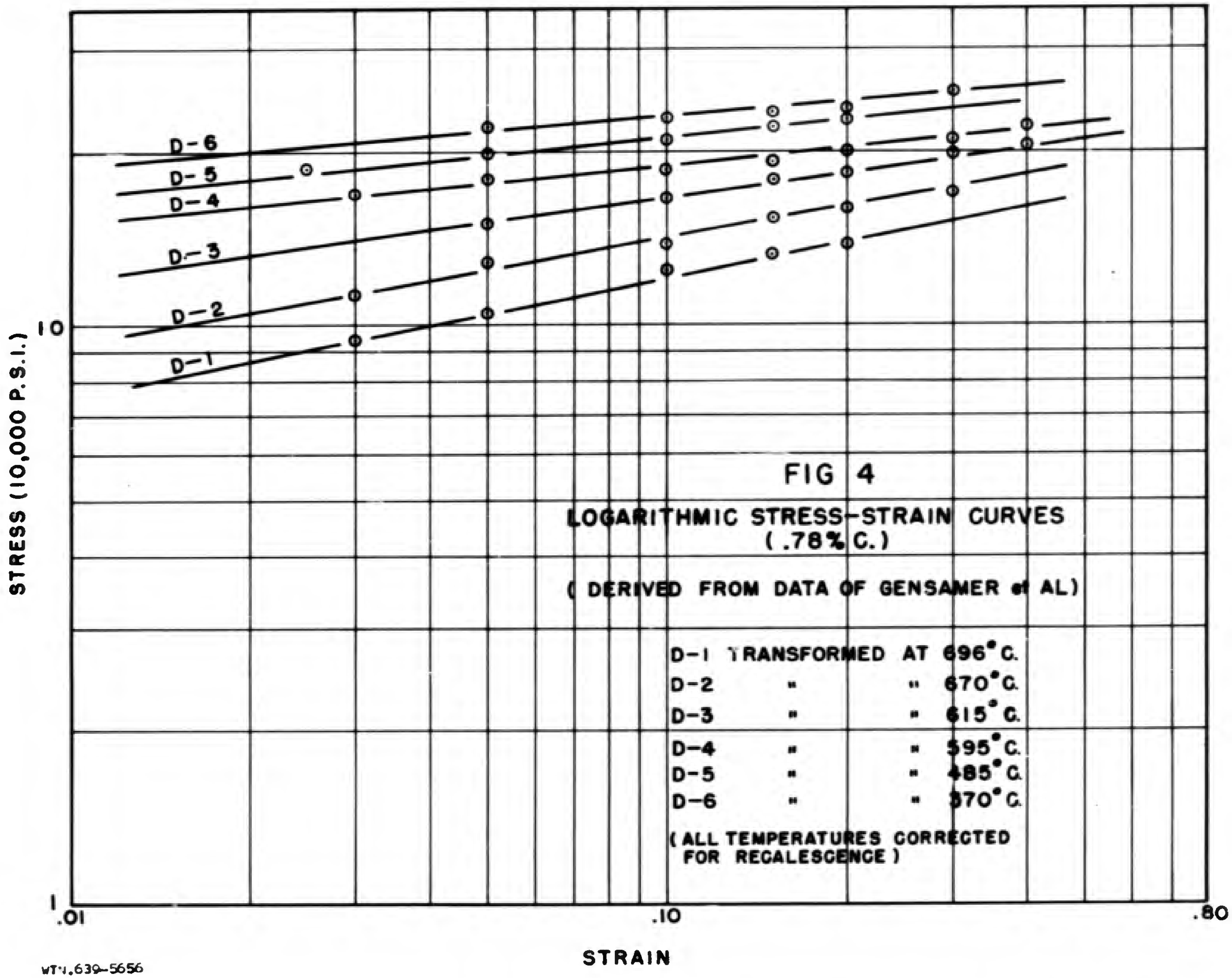


FIG 4

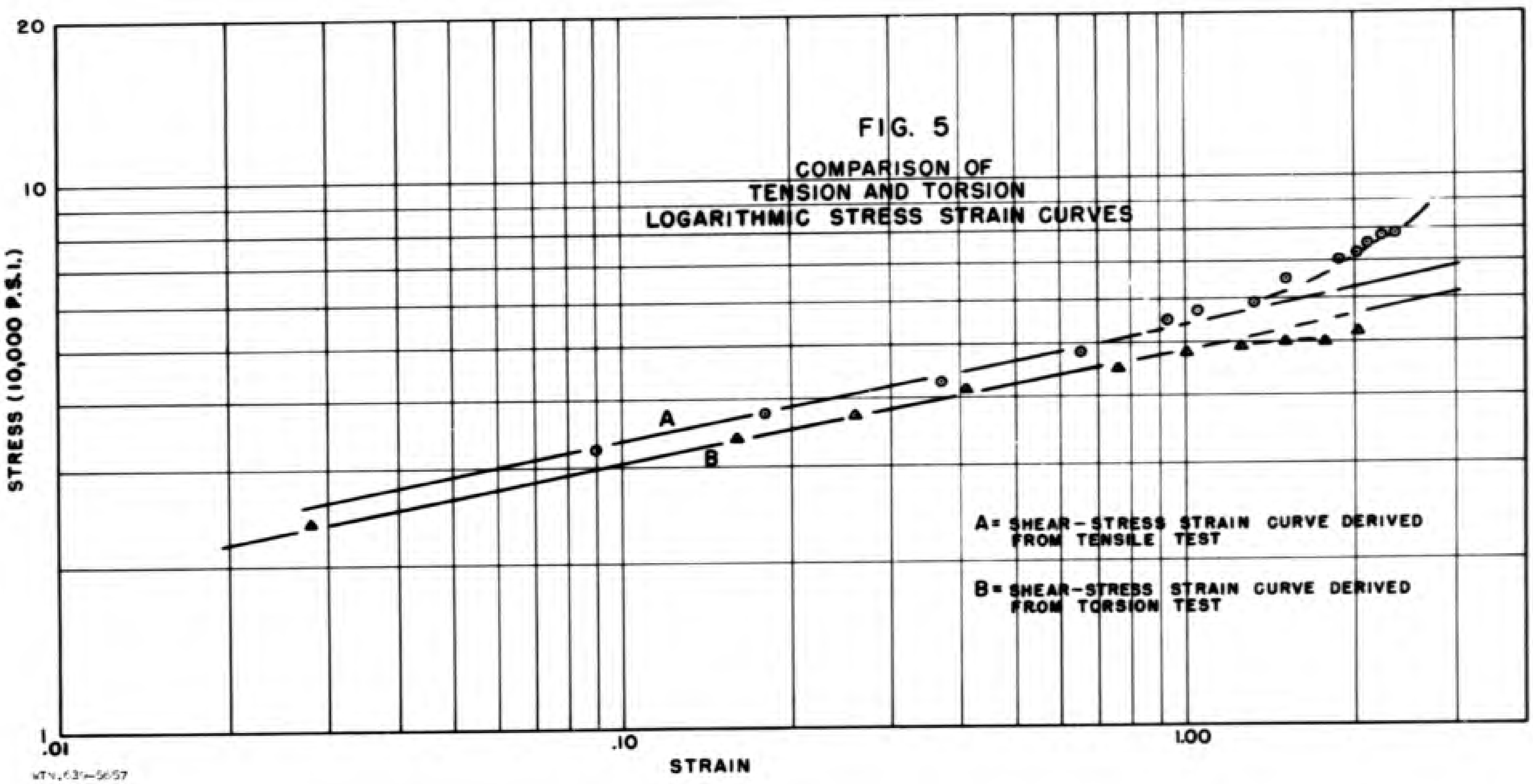
LOGARITHMIC STRESS-STRAIN CURVES (.78% C.)

(DERIVED FROM DATA OF GENSAMER et AL)

WT.1.639-5656

FIG. 5

COMPARISON OF
TENSION AND TORSION
LOGARITHMIC STRESS STRAIN CURVES



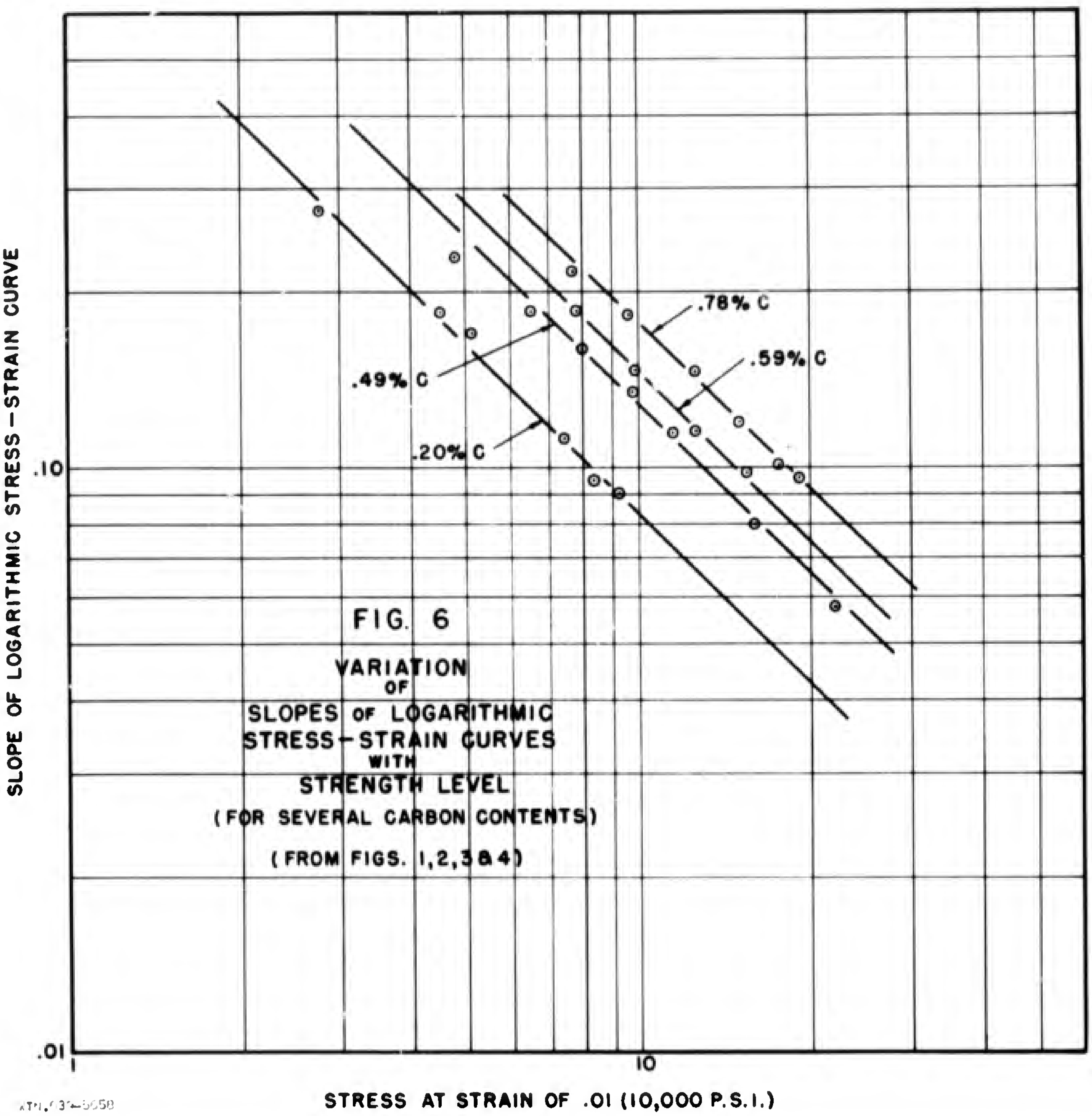


FIG. 6

VARIATION OF SLOPES OF LOGARITHMIC STRESS–STRAIN CURVES WITH STRENGTH LEVEL (FOR SEVERAL CARBON CONTENTS)

(FROM FIGS. 1,2,3 & 4)

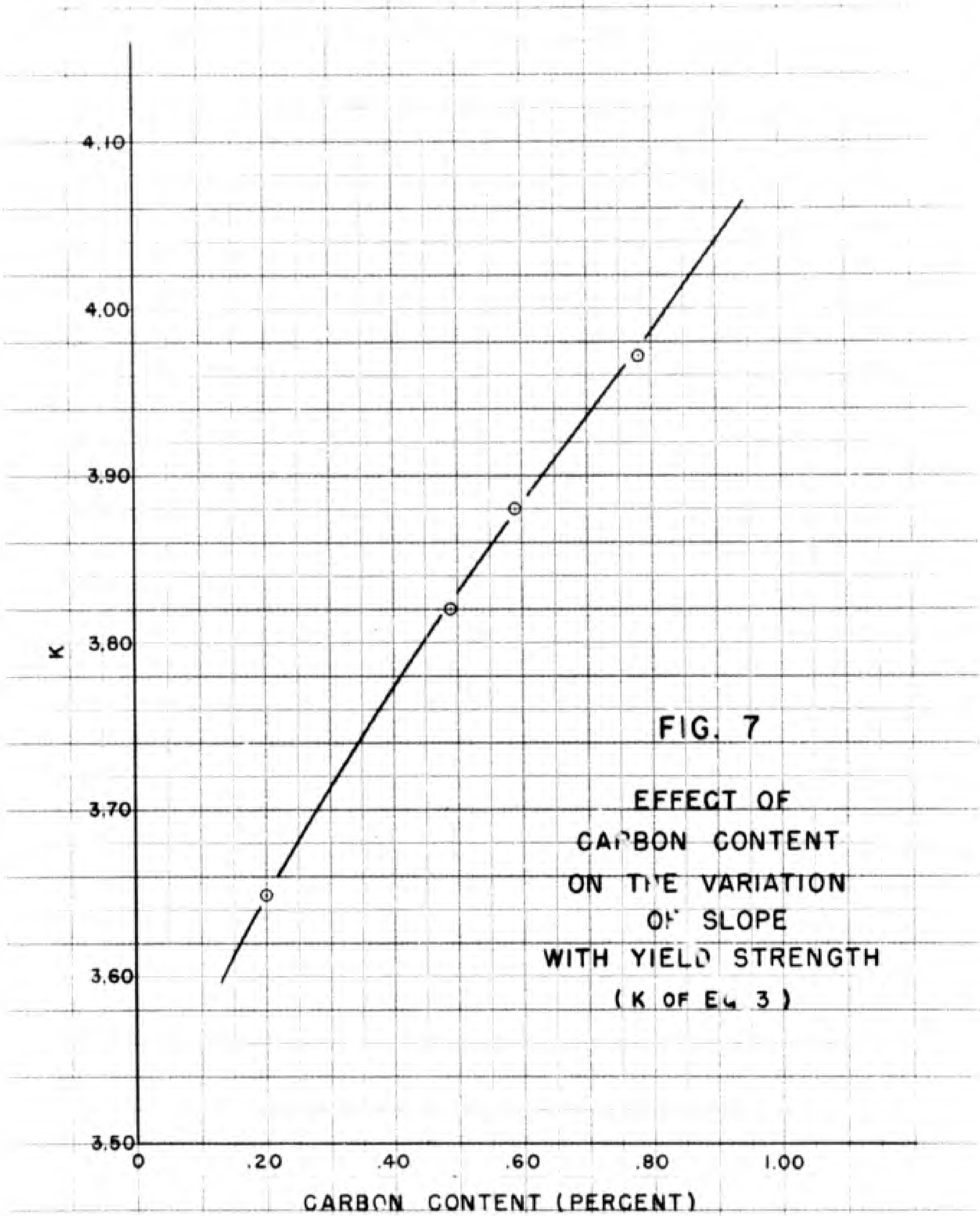


FIG. 7

EFFECT OF
CARBON CONTENT
ON THE VARIATION
OF SLOPE
WITH YIELD STRENGTH
(K OF EQ. 3)

ILLUSTRATIONS OF THE METHOD OF OBTAINING
THE TWO TYPES OF STRESS—STRAIN CURVES

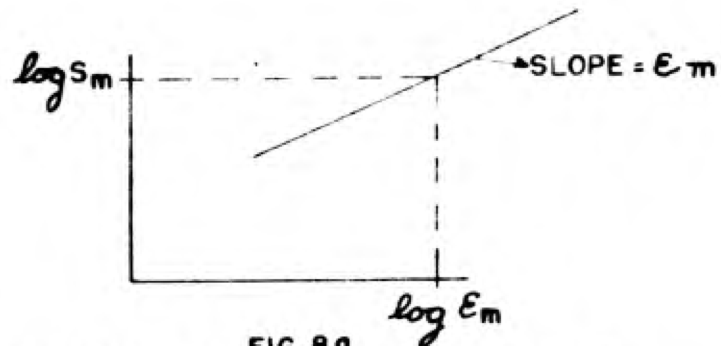


FIG. 8a
LOGARITHMIC STRESS—STRAIN CURVE

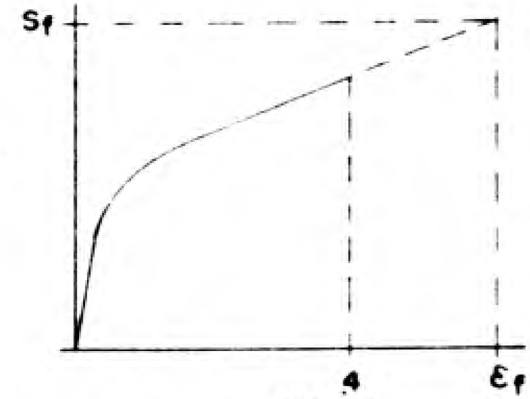


FIG. 8b
TRUE STRESS—STRAIN CURVE

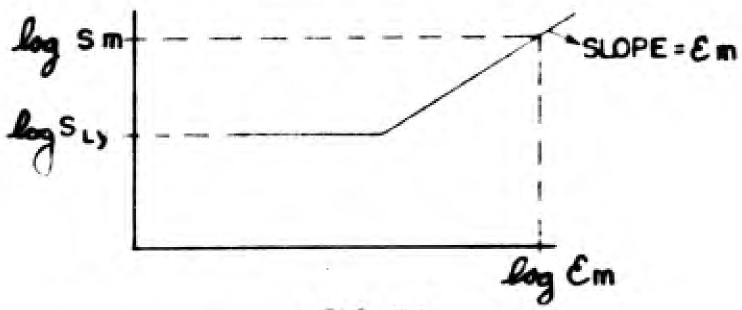


FIG. 8c
LOGARITHMIC STRESS—STRAIN CURVE

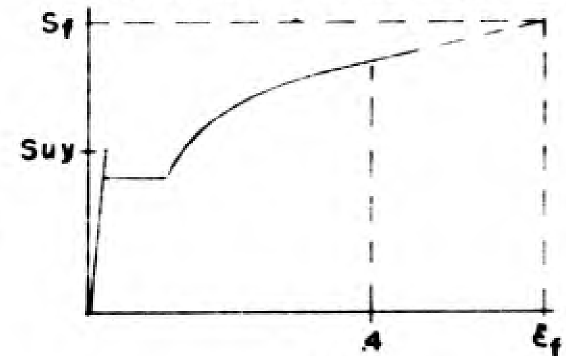


FIG. 8d
TRUE STRESS—STRAIN CURVE

LEGEND

S_m—STRESS AT MAXIMUM LOAD
S_f—STRESS AT FRACTURE
S_{Ly}—LOWER YIELD STRESS

S_{uy}—UPPER YIELD STRESS
e_m—STRAIN AT MAXIMUM LOAD
$\mathcal{E}_f$—STRAIN AT FRACTURE

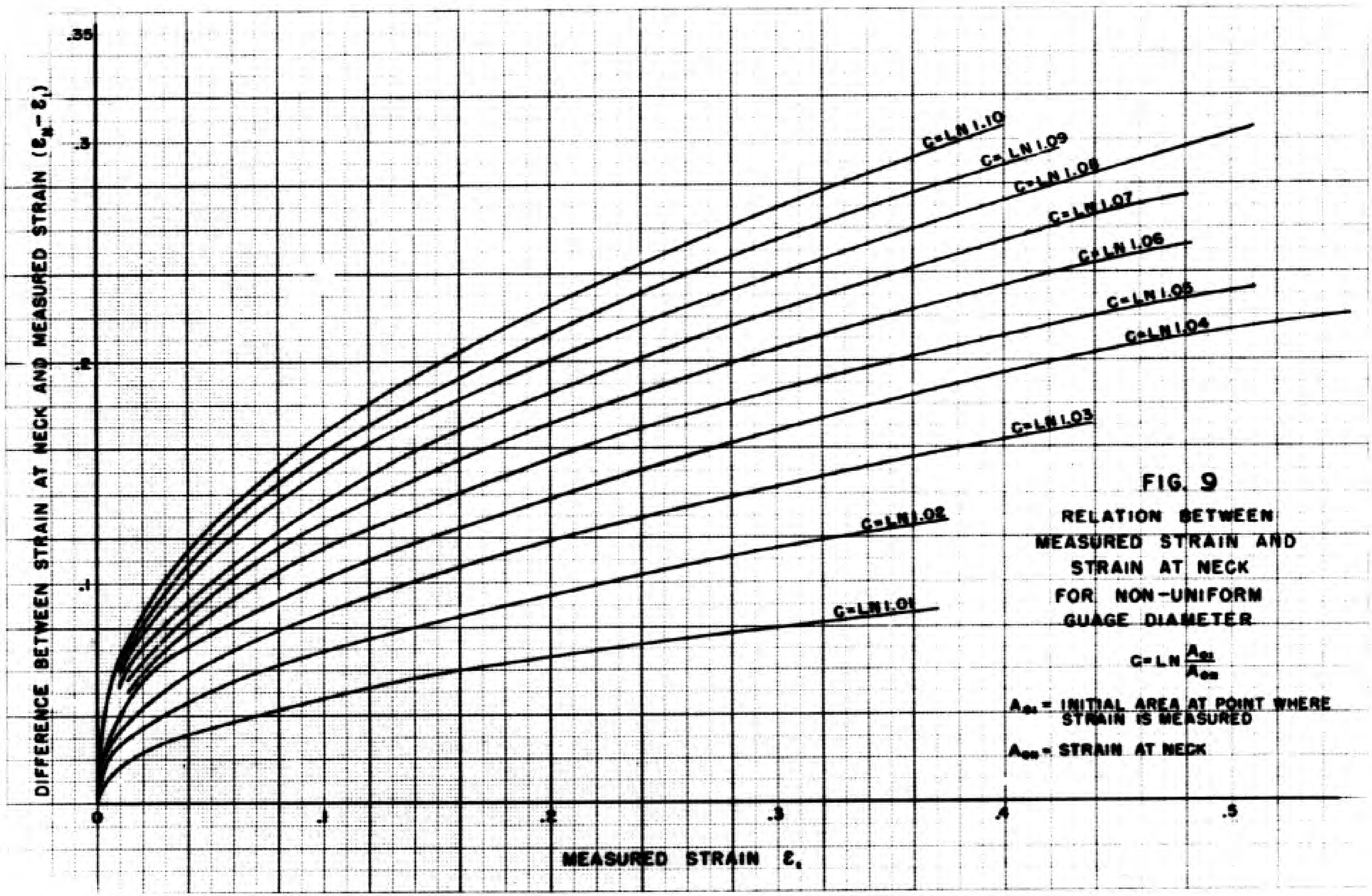


FIG. 9

RELATION BETWEEN MEASURED STRAIN AND STRAIN AT NECK FOR NON-UNIFORM GUAGE DIAMETER

$$C = LN \frac{A_{01}}{A_{0N}}$$

A_{01} = INITIAL AREA AT POINT WHERE STRAIN IS MEASURED

A_{0N} = STRAIN AT NECK

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