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Notes on the Deflection of Jets by Insertion of Curved Surfaces, and on the Design of Bends in Wind Tunnels

By

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Summary.—It is a fact of common experience that small jets of water can be deflected by partial insertion of a finger or other curved surface. The finger experiences a force impelling it towards the centre of the jet but if this force is withstood by an equal and opposite force the fluid is consequentially drawn round the surface and comes off at an angle. Similar phenomena are observed (Squire[1]; Appendix on the Coanda Effect) with jets of air of high total head surrounded by air of small total head: the former air behaves of course almost like another (immiscible) fluid.

The phenomenon can be satisfactorily explained on classical potential theory. The pressure must be constant on all water-air boundaries, but lower on the surface (so that the force shall be towards the centre). In the sequel various *plane* flows are built up (in the Helmholtz-Kirchhoff manner) by assuming patterns in the hodograph plane, and these show precisely the characteristics observed in practice : and doubtless what is true of plane "jets," is true qualitatively of jets of more normal cross-section.

The principal application is presumably to the control of jet-propelled aircraft. But the *mathematics* has another application. If wind-tunnel bends are designed according to this pattern (with walls replacing the sides of the jet) the velocity will be constant along the walls except in a small limited region where deleterious effects of adverse velocity gradient can be checked (i) by moving walls (ii) by suction slots. Both alternatives are investigated in §§2 and 3; the latter is perhaps more satisfactory. A 180-deg. bend with all adverse gradient avoided by use of one suction slot is designed, and drawn out in Fig. 6. Its use would eliminate the need for cascades in wind tunnels and would reduce the number of bends from four to two.

1. *Examples of the Effect, Derived Mathematically.*—If $w(z) = \phi + i\psi$ is the complex potential of an infinitely long two-dimensional jet of water, then the region of flow is transformed by $w = w(z)$ into an infinite strip, which we take for convenience as $0 \leqslant \psi \leqslant \pi$ (if the velocity is 1 at infinity this makes the jet of width π there). A, B are the points $\phi = -\infty, \phi = +\infty$ (Fig. 1a). Put $\Omega = \log dw/dz = \tau - i\chi$. At all water-air boundaries $\tau = 0$, but on the deflecting surface $\tau > 0$, by the argument in the summary. A little thought shows that the boundary-shape in the Ω-plane must be as in Fig. 1b, where CXD is any smooth curve and β_1, β_2 any numbers greater than α. Since $\chi = \alpha$ at A and $-\alpha$ at B (points at infinity in the physical plane) the deflection is 2α.

An example easy to treat mathematically is that in which $\beta_1 = \beta_2 = \beta$ and CXD is a semi-circle (Fig. 1c). There is of course a unique transformation from Fig. 1a to Fig. 1c which makes A, B and the origins correspond : it is in fact

$$\left.\begin{aligned} &\zeta = -i\,[\Omega - \beta^2/\Omega]\,;\ \text{so that}\ \Omega = \tfrac{1}{2}i\zeta + \sqrt{(\beta^2 - \tfrac{1}{4}\zeta^2)}\,, \\ \text{and}\quad &\zeta = (\alpha + \beta^2/\alpha)\coth(w/2)\,. \end{aligned}\right\} \qquad (1)$$

(The subsidiary ζ-plane is shown in Fig. 1d.) Now $dz/dw = e^{-\Omega}$. This with (1) makes the boundary-shape deducible by numerical integration.

In fact, for $w = \phi$ (real), $|dz/dw| = 1$ and $dz/dw = e^{i\chi}$,

where
$$\chi = -\tfrac{1}{2}\left(\alpha + \frac{\beta^2}{\alpha}\right)\coth\frac{\phi}{2} - \sqrt{\left[\tfrac{1}{4}\left(\alpha + \frac{\beta^2}{\alpha}\right)^2\coth^2\frac{\phi}{2} - \beta^2\right]}. \qquad (2)$$

For $w = \phi + i\pi$, $\coth w/2 = \tanh \phi/2$ and $|dz/dw| = 1$ only for $\tanh\frac{\phi}{2} > \frac{2\beta\alpha}{\alpha^2+\beta^2}$. For $\tanh\frac{\phi}{2} < \frac{2\beta\alpha}{\alpha^2+\beta^2}$, we have $\frac{dz}{dw} = \frac{ds}{d\phi}e^{i\chi}$ where

$$\left.\begin{aligned} \frac{ds}{d\phi} &= e^{-\tau} = \exp\left[-\sqrt{\left\{\beta^2 - \frac{1}{4}\left(\alpha + \frac{\beta^2}{\alpha}\right)^2 \tanh^2\frac{\phi}{2}\right\}}\right] \\ \text{and} \qquad \chi &= -\tfrac{1}{2}\left(\alpha + \frac{\beta^2}{\alpha}\right)\tanh\frac{\phi}{2}. \end{aligned}\right\} \quad \ldots \quad (3)$$

We have to calculate

$$x = \int \frac{ds}{d\phi}\cos\chi\, d\phi \text{ and } y = \int \frac{ds}{d\phi}\sin\chi\, d\phi \quad \ldots \quad (4)$$

If $t = \tanh\frac{1}{2}\phi$ is taken as the independent variable this can be done in quite a short time. (A separate integration from $w = 0$ to $w = i\pi$ is advisable to give the width of the jet at its narrowest place and so the relative position of its two boundaries.) Fig. 2 shows the shape for $\alpha = 30$ deg., $\beta = 60$ deg. and Fig. 3 that for $\alpha = 45$ deg., $\beta = 90$ deg.

An example was attempted with $\alpha = 90$ deg. (deflection 180 deg.), but the jet partially crossed itself, making the calculation physically meaningless.

Deflection by a convex surface will always be qualitatively as in Figs. 2 and 3. The jet will be constricted before reaching the surface and will open out again after leaving it. The points where it reaches and leaves the surface will be points of inflection of the boundary streamline.

In Fig. 2, the maximum velocity is attained at P and there is an adverse velocity gradient on PQ. But it seems unlikely that this could cause separation at a point R before Q, because the pressure at and near R is less than the pressure of the air outside and this would suck the jet back again on to the surface as far as Q.

2. *Wind-tunnel Bends.* (i) *Moving Walls.*—One possible application of §1 to wind-tunnel design is to replace the water-air boundaries by fixed walls and the deflecting surface by a rotating cylinder. This would counteract the effect of adverse velocity gradient in the only region where it occurs.

The difficulty (for a 90 deg. bend, the important case) is that in Fig. 3 the deflecting surface is not nearly cylindrical and to make it so would greatly alter the flow in the rest of the channel. This can be partially overcome as follows.

First choose a contour in the hodograph plane more likely to give a cylindrical deflecting surface. This is Fig. 1e, a right-angled isosceles triangle. The transformation of Fig. 1e into Fig. 1a is via Fig. 1f (u-plane) and Fig. 1g (ζ-plane) :—

$$u = -\frac{K\Omega}{\beta}(1+i),\ sn(u) = \frac{\sqrt{2}}{\zeta - \sqrt{(1-\zeta^2)}},\ \zeta = \zeta_0 \coth\frac{w}{2}, \quad \ldots \quad (5)$$

where $sn(u)$ and K are formed with modulus $k = 2^{-1/2}$.

$\Omega = i\alpha$ becomes $u = \frac{\alpha}{\beta}K(1-i)$ and we find

$$\frac{\sqrt{2}}{sn(u)} = \frac{1 + i cn^2\left(\frac{\alpha}{\beta}K\right)}{\sqrt{2}\, sn\left(\frac{\alpha}{\beta}K\right) dn\left(\frac{\alpha}{\beta}K\right)},$$

so that

$$\zeta = -\frac{1}{\sqrt{2}\, sn\left(\frac{\alpha}{\beta}K\right) dn\left(\frac{\alpha}{\beta}K\right)}.$$

If $\alpha = \frac{1}{2}\beta$, this gives $\zeta = \sqrt{[\frac{1}{2}(1+\sqrt{2})]} = 1{\cdot}0987 = \zeta_0$. ($A$, B correspond to $\pm\,\zeta_0$ in the ζ-plane.)

With $t = \tanh \frac{1}{2}\phi$ as before, we have for $w = \phi$ (real), $\zeta = \zeta_0 t^{-1}$ and

$$\left.\begin{aligned} \chi &= \frac{\beta}{K} sn^{-1} \left\{ \sqrt{\left[\frac{1}{2}\left(1 + \frac{t}{\zeta_0}\right)\right]} - \sqrt{\left[\frac{1}{2}\left(1 - \frac{t}{\zeta_0}\right)\right]} \right\}, \\ \frac{ds}{dt} &= \frac{2}{1 - t^2}. \end{aligned}\right\} \qquad (6)$$

For $w = \phi + i\tau$, $\zeta = \zeta_0 t$, and, for $|t| > \zeta_0^{-1}$, (6) holds with t^{-1} for t in the expression for χ. But for $|t| < \zeta_0^{-1}$ we have

$$\left.\begin{aligned} \tau &= \frac{\beta}{2K} cn^{-1} \left\{ \frac{\zeta_0 t - \sqrt{(1 - \zeta_0^2 t^2)}}{\zeta_0 t + \sqrt{(1 - \zeta_0^2 t^2)}} \right\}, \\ \chi &= \beta - \tau, \\ \frac{ds}{dt} &= \frac{2}{1 - t^2} e^{-\tau}, \end{aligned}\right\} \qquad (7)$$

In Fig. 4 the boundary is drawn out for $\alpha = \frac{1}{4}\pi$, $\beta = \frac{1}{2}\pi$. The dotted circular cylinder could replace the firm line and the resulting shape be taken as a wind-tunnel bend in the hope that flow changes resulting from the substitution would be small.

An alternative procedure (not gone through here) would be to take the expression

$$\tau = \frac{\beta}{2K} cn^{-1} \frac{\zeta - \sqrt{(1 - \zeta^2)}}{\zeta + \sqrt{(1 - \zeta^2)}} \qquad (8)$$

as a first approximation to the velocity for $|\zeta| < 1$ and then to construct χ according to the condition $d\chi/ds =$ const. Thus

$$\frac{d\chi}{d\zeta} = \frac{d\chi}{d\phi} \frac{\zeta_0}{\zeta_0^2 - \zeta^2} = \frac{d\chi}{ds} e^{-\tau} \frac{\zeta_0}{\zeta_0^2 - \zeta^2} \qquad (9)$$

gives χ as a function of ζ for $|\zeta| < 1$. But we know that $\tau = 0$ for $|\zeta| > 1$. From these two pieces of data we can obtain the value of τ for $|\zeta| < 1$ and of χ for $|\zeta| > 1$ by Poisson's integral for the upper half-plane, applied to the function $\frac{\Omega}{\sqrt{(1 - \zeta^2)}}$. Hence a shape with the critical portion much more nearly cylindrical would be found.

Finally it may be noted that if a *rectangle* is taken as the contour in the Ω-plane, the region of adverse gradient would be confined to a *straight* portion of wall, where it might be overcome by a moving *belt*, though this would be mechanically more difficult.

3. *Wind-tunnel Bends.* (ii) *Suction Slots.*—A more satisfactory scheme seems to be the use of a suction slot to overcome a designed discontinuous drop in velocity along the surface. An Ω-plane as in Fig. 1h would achieve this, transforming into Fig. 1a by

$$e^{i\Omega\pi/\sigma} = \frac{e^{w + \alpha\pi/\sigma} + 1}{e^{w} + e^{\alpha\pi/\sigma}} \qquad (10)$$

This gives on the boundary

$$\chi = \frac{\sigma}{\pi} \log \left| \frac{e^{w + \alpha\pi/\sigma} + 1}{e^{w} + e^{\alpha\pi/\sigma}} \right|. \qquad (11)$$

Fig. 5 shows the shape derived from $\alpha = \pi/4$, $\sigma = \pi^2/12$.

More interesting is the possibility of obtaining a 180 deg. bend in this way without the channel crossing itself. If the y-axis is taken as the line of symmetry of the bend, this means that the increase in x between the points $y = 0$ and $y = \infty$ on the outer wall should exceed π, the breadth of the channel—or in mathematical terms, that

$$\int_0^\infty \cos\left(\frac{\sigma}{\pi}\log\left[\frac{e^{\phi+\pi^2/2\sigma}+1}{e^{\phi}+e^{\pi^2/2\sigma}}\right]\right) d\phi \quad \ldots \quad (12)$$

should exceed π.

Put the contents of the round brackets equal to $\frac{1}{2}\pi - \theta$. The integral becomes

$$\frac{\pi}{\sigma}\int_0^{\pi/2}\sin\theta\left(\frac{1}{e^{\theta\pi/\sigma}-1}+\frac{1}{1-e^{\pi(\theta-\pi)/\sigma}}\right)d\theta$$

$$=\frac{\pi}{\sigma}\int_0^{\pi/2}\sin\theta\left(\sum_1^\infty e^{-n\theta\pi/\sigma}+\sum_0^\infty e^{n\pi(\theta-\pi)/\sigma}\right)d\theta$$

$$=\frac{\pi}{\sigma}\left[\sum_1^\infty\left(\frac{1-\frac{n\pi}{\sigma}e^{-n\pi^2/2\sigma}}{1+\left(\frac{n\pi}{\sigma}\right)^2}\right)+\sum_0^\infty\left(e^{-n\pi^2/\sigma}\,\frac{1+\frac{n\pi}{\sigma}e^{n\pi^2/2\sigma}}{1+\left(\frac{n\pi}{\sigma}\right)^2}\right)\right]$$

$$=\frac{\pi}{\sigma}\left(1+\sum_1^\infty\frac{1+e^{-n\pi^2/\sigma}}{1+\left(\frac{n\pi}{\sigma}\right)^2}\right)$$

$$=\frac{\pi}{2}\left(\frac{1}{\sigma}+\coth\sigma\right)+\frac{\pi}{\sigma}\sum_1^\infty\frac{e^{-n\pi^2/\sigma}}{1+\left(\frac{n\pi}{\sigma}\right)^2}, \quad \ldots \quad (13)$$

the series term being quite negligible for σ not large. The condition that the channel shall not cross itself is then

$$\frac{1}{\sigma}+\coth\sigma>2\,;\ \textit{i.e.},\ \sigma<1\cdot23 \quad \ldots \quad (14)$$

In practice rather more must be the case and considerable space must be left between the two interior tunnel walls. For $\sigma = 0\cdot904$, $\sigma^{-1} + \coth\sigma = 2\frac{1}{2}$, there is half a tunnel's breadth between the tunnel walls. The result is shown in Fig. 6.

REFERENCE

No.	*Author*	*Title*
1	H. B. Squire, D. J. Harper, J. Bekassy and W. Chester.	Wind-tunnel Tests of Oblique Jet Units R.A.E. Report Aero. 2007. A.R.C. 8449. January, 1945. (Unpublished).

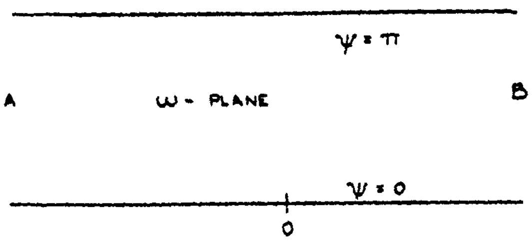

FIG. 1a.

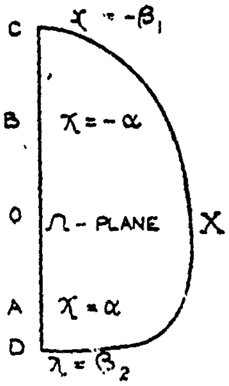

FIG. 1b.

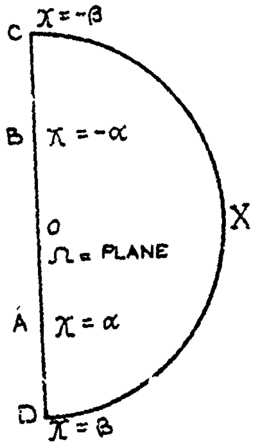

FIG. 1c.

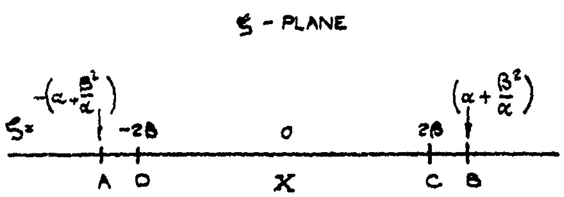

FIG. 1d.

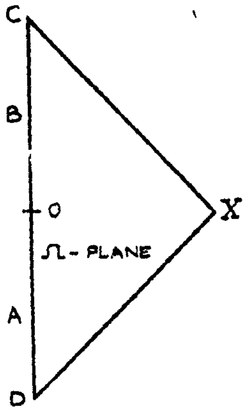

FIG. 1e.

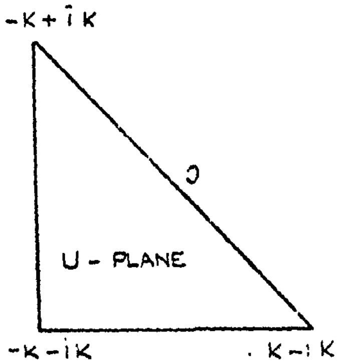

FIG. 1f.

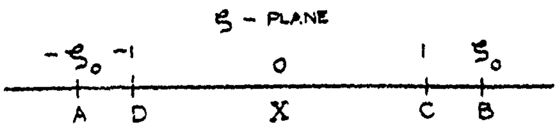

FIG. 1g.

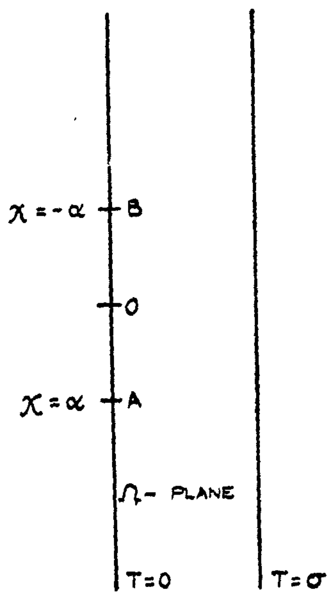

FIG. 1h

FIG. 1.—Conformal Transformations Employed.

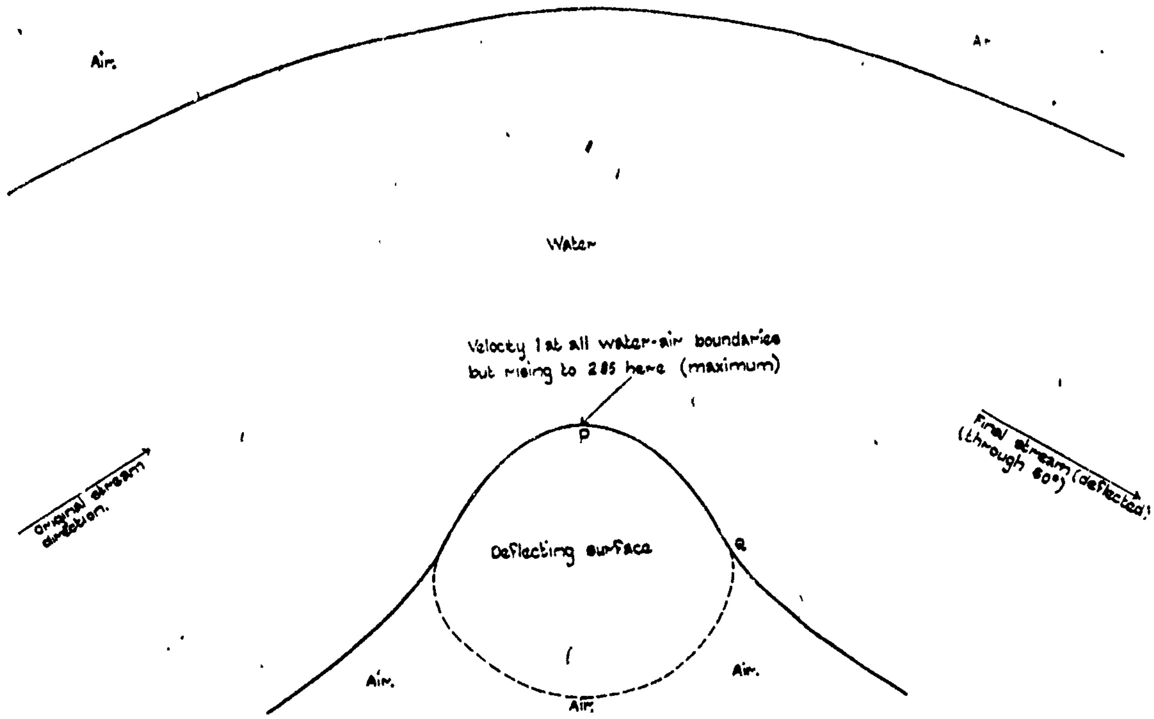
Air.
Water
Velocity 1 at all water-air boundaries
but rising to 2.85 here (maximum)
P
Q
Original stream direction.
Final stream (deflected) (through 80°)
Deflecting surface
Air.
Air.
Air.

FIG. 2.

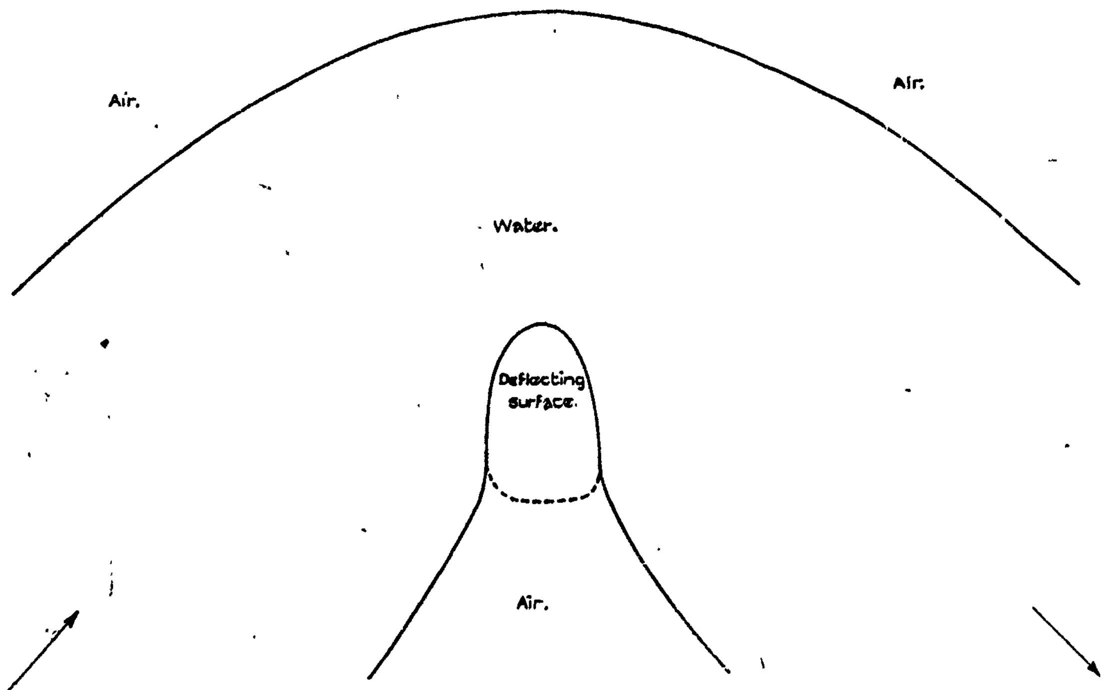
Air.
Air.
Water.
Deflecting surface.
Air.

FIG. 3.

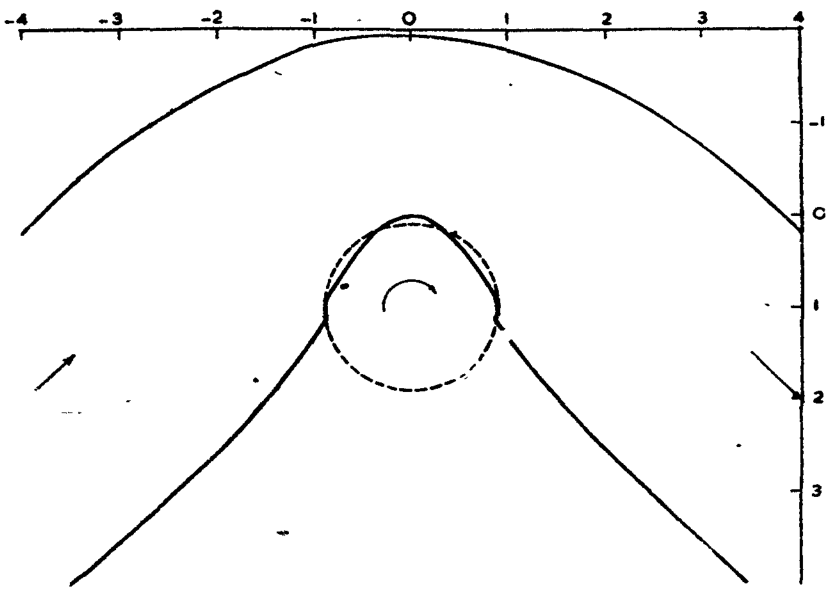

FIG. 4.—Bend with Region of Adverse Gradient replaced by Rotating Cylinder.

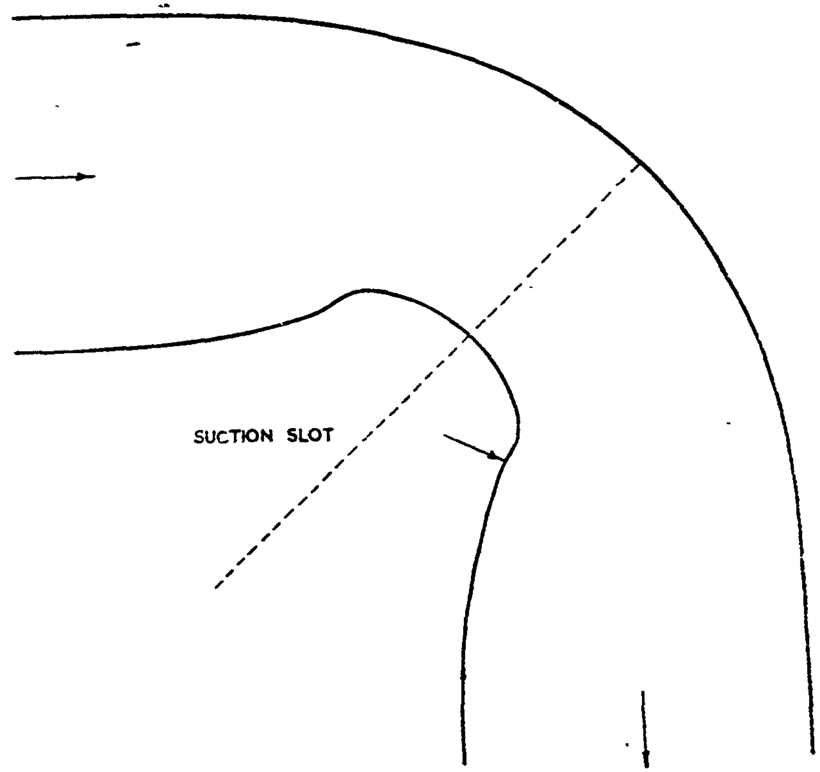

FIG. 5.—90° Bend with Suction Slot.

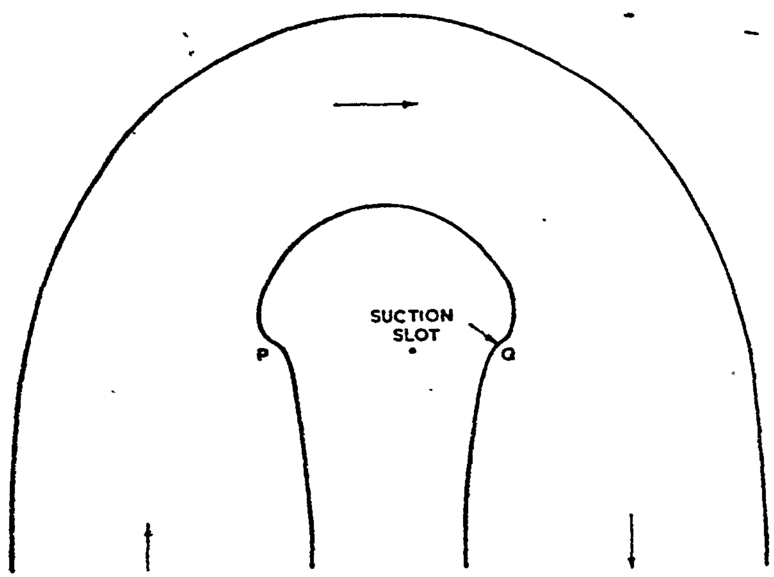

FIG. 6.—180° Bend; Velocity 2·47 on PQ, and Unity on all other Parts of the Boundary

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ABSTRACT:

Phenomenon of deflection of jets by curved surfaces is applied to wind-tunnel bends so that the velocity will be constant along the walls except in a small limited region where deleterious effects of adverse-velocity gradient can be checked by moving walls or by suction slots. A 180-degree bend with all adverse gradient avoided by use of one suction slot is designed and drawn. Its use would eliminate the need for cascades in wind tunnels and would reduce the number of bends from four to two.

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