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THE CENTER OF GRAVITY OF THE HUMAN BODY AS RELATED TO THE EQUIPMENT OF THE GERMAN INFANTRY

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THE CENTER OF GRAVITY OF THE HUMAN BODY AS RELATED TO THE EQUIPMENT OF THE GERMAN INFANTRY

By

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The action of the earth's power of attraction can be imagined as a force from every center of gravity of the body, acting in the direction of a line connecting each center with the center of the earth, and equal to the weight of the mass gathered at the original point. Because of the great distance from these points to the center of the earth, all these lines run parallel to each other. A resultant therefore exists for these forces of inertia. This resultant corresponds to a fixed straight line for every position of the body. This straight line remains unchanged if the body is displaced in space over not too great a distance, while still remaining parallel to itself, but does change its position within the body as soon as the latter is rotated about it. The straight line in the new position intersects the old one at a definite point. However, all the other lines that represent the positions of the resultants of the forces of inertia due to new rotation of the body also pass through this point. This point of intersection can thus be considered the common point of attachment for all the resultants of the forces of inertia for every possible body position. This is the body's center of gravity. The size of the resultant is the body weight, since the resultant is also the total of all the individual forces of inertia. We can therefore consider the weights of all the separate parts of the body as being united in the body's center of gravity. We shall call the direction of this resultant the body's gravity line. The gravity line is thus the line that connects the body's center of gravity with the center of the earth, and therefore does not change its absolute position in space if the center of gravity remains unchanged, no matter how the body may otherwise be rotated about this center. On the other hand, every rotation of the body on the center of gravity changes the position of the gravity line in relation to the body itself. As long as the gravity line intersects the body's supporting surface, the body is upheld, but as soon as this line falls outside the supporting surface, the body falls down.

The supporting surface is not identical with the portion of the body surface that actually touches the ground. This is only true when the supporting surface is in one piece and its edges are everywhere convex, as is the case with the base of a circular cone. But if the edges have indentations, that is to say are outwardly concave at any point, the portion that lies in the concavity must also be considered a supporting surface, all the way to the double tangents that are determined by these concavities. Conditions remain the same even if the concavities go all the way through, that is, if the surface consists of two or more pieces. Then the entire intervening space between the two double tangents also belongs to the entire supporting surface.

Thus the supporting surface for the human body, when standing erect on both feet, consists of the two soles and the space between the double tangents that can be made to contact the tip of the foot and the back of the heel.

A man can reduce his supporting surface to a minimum, when standing upright, by standing on one leg and raising himself on his toes. It reaches a maximum when the two legs are spread as far apart as possible, a position that is instinctively assumed in fencing or in standing on a swaying deck, for the gravity line can then swing back and forth a great deal without falling outside the supporting surface.

In the case of a single supporting point, the body is supported when the center of gravity lies in the vertical at that point. A supported center of gravity can assume three different positions in relation to its supporting point.

1) The center of gravity can coincide with the supporting point. It will then retain its position for any rotation of the body around this point, and the body will therefore always be balanced in any position. This is called indifferent equilibrium.

2) The center of gravity can lie above the supporting point. In this case every rotation of the body moves the center of gravity away from the vertical and lowers it at the same time. An outer force is then required to bring the center of gravity back to its original position, since it is no longer able to do this of its own accord, but always has a tendency to fall still more. Since the force of inertia gains a moment of rotation when the center of gravity leaves the vertical, this center will keep on falling until it reaches its lowest point. This is called labile equilibrium.

3) The center of gravity can lie below the supporting point. In this case any rotation of the body raises it, and creates a moment of rotation that brings the center of gravity, that always has a tendency to fall, back to its original position, the lowest of all. The body is therefore bound to return of its own accord to its position of equilibrium. This is called stable equilibrium.

Thus, in labile equilibrium, the center of gravity always assumes the highest position in rotating on the supporting point, while in stable equilibrium it always assumes the lowest.

The abstract lever arm created by the force of inertia, with any movement out of labile equilibrium, is the shorter, the nearer the center of gravity lies to the supporting point, and it becomes equal to zero when the two coincide, this condition being the characteristic of indifferent equilibrium. This is why the unsteadiness of position in labile equilibrium becomes greater as the distance increases between the center of gravity and the supporting point. Daily experience teaches the same thing, for the closer the entire body's center of gravity is to the ground, the more securely one stands, because the moment of rotation of the force of inertia is smaller, the lower the center of gravity lies, for the same movement out of rest position. This is why short people stand relatively more securely than tall ones.

An exact knowledge of the location of the center of gravity of the entire body, and above all of the separate sections of the limbs, together with a knowledge of the combined weights in the centers of gravity, is the absolute requirement for formulating the statics and mechanics of the human body, since this knowledge is what makes it possible to distinguish what forces must be overcome by each separate muscle with each body movement.

The question arises as to whether there can be any center of gravity in the body at all, because it is not a fixed mass but consists of separate limbs that are always changing their position in relation to each other and cannot even be considered masses in the mechanical sense, and because fluids are always flowing through the body. In the absolute sense the answer is certainly no. Mosso has actually proved, with his balance, that the center of gravity of the body is always changing, due to respiration and circulation (Archives italiennes de biologie, volume V, Turin, Löscher, 1884, page 131, "The use of the balance in the study of the circulation of human blood"). However, this change of position is so insignificant that it can be disregarded in connection with most problems of human statics and mechanics. In the same way, changes of form due to the softness of the tissues do have an effect on the location of the center of gravity, but this effect too is so small that we can still speak, within certain limits, of a real center of gravity.

Things are very different as far as the changing positions of the limbs are concerned. The joints make them able to move so far, in relation to each other, that the common center of gravity is greatly displaced by a change in their position. In the average position, the center of gravity lies inside the body, but the latter can be bent over so far that the common center falls entirely outside of it, just as in a curved arch. Every determination of the center of gravity is therefore valid for only one definite body position.

It is also evident that such determinations are bound to be individual only, up to a certain point, because of the difference in the distribution of the mass inside the body for each human being, according to age, sex, etc.

Mathematicians and anatomists began a long time ago to investigate the body's center of gravity, but they did so by methods that were bound to be incomplete because they dealt with the soft body, or reduced it to simple stereometric forms. It therefore seems necessary to use a new method in attacking this problem, a method that does away with these disadvantages by making the body so stiff and unyielding, without adding to its mass, that it retains its form unaltered in any position.

The mathematician Joh. Alphonsus Borellus (De motu animalium, Lugduni Batavorum, 1679) determined the center of gravity of the human body by having a nude man stretched out fully on a level board, the center of which was supported by a three-sided, prism-shaped wedge (page 176, proposition 134). Equilibrium then occurred when the upward prolongation of the cutting edge of the wedge lay between the buttocks and the pubis.

The results obtained by the brothers Wilh. and Ed. Weber (Mechanik der menschlichen Gehwerkzeuge, Göttingen, 1836, p. 114 and foll.) were more exact than those of Borellus. They did seek to balance a body by laying it on a board supported by a horizontal wedge, as did Borellus, but they did not move the board back and forth for this purpose. Instead, they first balanced the board before laying the body on it, and then they moved the body alone until it was in equilibrium. The resulting measurement was thus entirely independent of the board. Furthermore, they did not try to find the position of balance for the body itself directly, as this is quite impossible with such a labile arrangement, because the body will always tend to lie more to one side or to the other. Instead, they limited themselves to noting the drop of the board to one side, and then repeated the measurement after turning the body around on the board. Since one measurement of the location of the center of gravity was as much too high on one side as it was too low on the other, the mean between the two gave the true location. By making a series of independent measurements of the body of a man 1669.2 mm. tall, they found that the center of gravity lay 721.5 mm. from the crown, 947.7 mm. from the heel, and 87.7 mm. from the rotational axis of the hip. They then measured the vertical distance from the promontory to the rotational axis of the hip on a skeleton of the same height as the man, and found it to be 79 mm. Since this distance can vary little in people of the same height, it was possible to deduce a fourth distance in relation to the location of the center of gravity, the vertical distance from it to the promontory, which was 8.7 mm.

The center of gravity was higher, about at the level of the navel, when one leg was removed from the cadaver, and it lay still higher, at the extension of the breastbone, when both legs were removed.

According to the Weber brothers (as above, page 119), "when standing perfectly rigid and motionless, with no help from the muscles, and with knee and hip joints drawn up as high as possible above the location of balance, the center of gravity of the torso alone lies in a vertical line directly above the ankles, while the hip joint is in front of and the knee joint behind the vertical drawn through the center of gravity. The pressure exerted by the weight of the torso on its supports will then have the same effect as if the knee and the hip joints were stretched still more, but this is prevented by the elasticity of their tendons. The entire body is then supported, right down to the ankles, by the bones and tendons of the legs, and needs to be balanced by the muscles, as a single unyielding mass, only at the ankle joints."

Herrmann von Meyer is the man who studied the center of gravity, and human statics and mechanics generally, most profoundly. His works are to be found partly in Müller's Archiv, 1853 and 1854, as follows: "The Upright Standing Position," 1853, page 9; "The Upright Walk," 1853, page 365; "The Mechanics of the Knee Joint," 1853, page 497; "The Individual Characteristics of the Upright Walk," 1853, page 548; "The Normal Curve of the Spine," by Fr. Horner, with a postscript by Meyer, 1854, page 478; and partly in the treatises: "The Changing Locations of the Center of Gravity in the Human Body, a contribution to plastic anatomy," Leipsic, Engelmann, 1863, and "Statics and Mechanics of the Human Body," Leipsic, Engelmann, 1873.

Borellus and the Weber brothers measured the body in a recumbent position and were satisfied with determining the vertical distance of the center of gravity from the crown and the foot, assuming a complete bilaterial body symmetry that would make this center lie in the median plane. Meyer also sought to ascertain the location of the center of gravity from back to front, and in so doing used the upright position that he described and called "military." His method consisted of giving the body a forward bend at the ankles, without any other change of position, until danger of falling became evident at the axis drawn through the centers of the metatarsal heads of both great toes. "The torso and legs were also bent backward, as a single unit, by stretching the ankle joint until danger of falling became evident at the rear edge of the heel. The center of gravity thus moved all the way across the supporting surface. Exact measurements at definite recognizable points on the legs made it possible to determine the angle which they made with the horizontal, both for the original and for the two end positions. This indicated at once the angle covered by the motion of the ankle joint, and this angle, together with the horizontal displacement indicated by the length of the foot, determined the position of the gravity line for the original body position."

Meyer found the location of the center of gravity, for the upright position that he called "military," to be in the second sacral vertebra, or directly above it in the sacral cavity, and he found the gravity line that comes from it to be 5 cm. behind the axis of the hips and 3 cm. in front of the axis of the ankles ("Statics and Mechanics of the Human Body," Leipsic, 1873, page 204). He later determined the center of gravity for the separate parts of the body by reducing the head, the torso and the legs to simple mathematical shapes (ellipsoid and sphere). According to him, the center of gravity of the head lies a little in front of the atlanto-occipital joint. He remarks in this connection (Meyer-Horner, p. 499), that labile equilibrium is used as little in the support of the head by the atlas as in the support of the body by the hip or ankle joint. Combining the centers of gravity for head and torso then gave the location of their common center of gravity in the torso. It lies, according to Meyer, a little in front of the center of the forward edge of the tenth thoracic vertebra. He then divided the body into further parts by the same method, dividing the torso in the narrower sense into two ellipsoids, one for the thorax and one for the pelvis. He also determined the center of gravity of the entire leg, and stated that it lay 266 mm. from the trochanter, in the leg axis, for a leg 720 mm. long. Meyer obtained the weights of the different parts from Krause's "Manual of Anatomy." He then constructed a large number of figures in different poses and positions, with this material as a basis, and showed to what an amazing extent the location of the center of gravity changes, according to the position of the torso and the grouping of the extremities.

The next scientist to investigate the center of gravity of the human body was Harless ("Static Moments of the Dimensions of the Human Limbs," page 71 and following, in "Treatises of the Mathematical-Physical Class of the Royal

Academy of Sciences of Bavaria," volume 8, 1860, First Section, Munich, 1857). He repeated the Weber experiments on a well-built young man of twenty-four. He was 1655 mm. tall, and the mean distance from the center of gravity to the soles of the feet was 970 mm. For a total height of 1000 mm. this corresponds to 586.102 mm. from the soles and 413.898 mm. from the crown. According to Weber, the distance of the center of gravity from the heel was 947.7 mm. for a body 1669.2 mm. tall, or 432.24 mm. from the crown and 567.76 mm. from the heel for a full height of 1000 mm. The difference between the Harless and the Weber measurements was thus 1.8% of the full height. Valentin (Harless as above) found 429.19 as the distance from the crown. According to Meyer the distance from the crown was 768 mm. for a body 1897 mm. tall. This amounts to 404.96 from the crown and 595.14 from the sole for a height of 1000. In this case the difference from the Weber measurements was 2.73% of the full height.

According to Harless, the mean distance from the crown, for four different individuals, was 420.07 for a height of 1000. He thus tried to obtain comparable values for the individual measurements by making the body lengths all equal to 1000, and then calculating the distances from the center of gravity to the crown and the soles per thousand. In doing this, however, he did not take into consideration that the limbs can have very different proportions in different individuals, that the height of the shoulders varies extremely, that two bodies of the same height can have legs and torsos of different lengths, and that this would naturally bring about a different location for the center of gravity, due to the different distribution of the mass. For instance, a blacksmith with very heavy and high-set arms would have a higher center of gravity than would a tailor of the same height with weak arms and low shoulders. Furthermore, it could also happen, with the Harless hypothesis, that two bodies of the same height could have the same distance for the center of gravity to the crown, and yet show a great difference in the location of this center from the anatomical-mechanical point of view. One individual might have very short legs, and thus have the center of gravity far above the promontory, while another man of the same height but with long legs would have it below, although at the same distance from the crown. Thus determining the center of gravity by the distance from the crown is of less value than the anatomical-mechanical determination, and is not improved by taking the arithmetical mean of many measurements.

The main importance of Harless is that he, like Meyer and Horner, undertook to find the centers of gravity of the different body sections, but much more thoroughly and exactly. He made his measurements by the balance method, and thus obtained their vertical positions. He divided the masses of the limbs at the ends of the lever arms, that is to say, at the axes of the joints, because of their static moments. However, since he was using soft tissues, he removed the entire joint in making the cuts at the hips and shoulders. After repeating the Weber measurements on a living body, he used the body of a man who had been beheaded (Graf) to measure the centers of gravity of the separate body sections, and obtained the following values:

Body height	172.685	cm. 5'8"
Height of foot	6.0	cm.
Length of lower leg	42.9	cm.
Length of upper leg	44.9	cm.
Trochanter line to summit of crest of ilium	14.0	cm.
Hip bone level to summit of acromium process	39.0	cm.
Length of head from chin to crown	21.2	cm.
Front neck length	4.7	cm.
Length of foot	25.369	cm.
Length of entire arm	86.6	cm.
Length of upper arm	36.4	cm.
Length of forearm	29.889	cm.
Length of hand	20.314	cm.

The weights of the separate parts were as follows (from the body of a man who had been executed and therefore drained of blood):

Weight before execution	63970	g.
Head	4555	g.
Torso	29608	g.
Both arms	7540	g.
Both legs	22270	g.
One hand (mean)	540	g.
Each forearm	1160	g.
Each upper arm	2070	g.
Each upper leg	7165	g.
Each lower leg	2800	g.
Each foot	1170	g.

In determining the center of gravity of the torso, he assumed that it consisted of two truncated covers. He calculated the weight of the torso in this way, with the specific body weight considered as 1.066, in order to prove that considering the torso as a system of cones did not result in too great an error. He obtained 28.5153 kg. as the weight of the entire torso, while weighing it directly gave 29.608 kg. The specific weight he used was very high, as given by Harkness and Baumgärtner, while Hermann indicates 0.9213 as the average specific weight for normal corpses (Vierordt, Tables, Jena 1888). Since Harless used a body drained of blood, the difference between the calculation and the direct weighing of the blood-filled torso should have been much greater. Considering the torso as composed of mathematical shapes and at the same time assuming an equal distribution of mass throughout, would thus result in serious error as far as the weight is concerned. But since the masses in the torso are very unevenly distributed because of the position of the heavy spine toward the rear, determining the center of gravity by the aid of this mathematical fiction becomes more unreliable than ever.

Direct measurements for the location of the center of gravity in the extremities gave the following results:

1) Upper arm 36.4 cm. long { 17.621 cm. from upper end; 18.779 cm. from lower end

2) Forearm 29.889 cm. long { 13.122 cm. from upper end; 16.767 cm. from lower end

3) Hand 20.314 cm. long	9.623 cm. from upper end
	10.691 cm. from lower end
4) Upper leg 44.9 cm. long	20.995 cm. from upper end
	23.905 cm. from lower end
5) Lower leg 42.9 cm. long	15.455 cm. from upper end
	27.445 cm. from lower end
6) Foot 25.369 cm. long	11.664 cm. from rear end
	13.705 cm. from front end
7) Head 21.2 cm. long	7.7 cm. from crown
	13.5 cm. from chin

8) The center of gravity of the upper section of the torso lay lay 23.465 cm. from the lower edge and 17.53 cm. from the upper edge.
The center of gravity of the lower section of the torso lay 5.8899 cm. from the upper edge and 7.6101 from the lower edge.

He divided the torso into an upper section, that reached down to the crest of the hip bone, and a lower section that included the pelvis. It is as incomprehensible as it is absurd that in making his calculations he extended them to the hundredth and the thousandth part of a millimeter.

He then calculated the location of the common center of gravity of the executed man from his measurements for the distances of the centers of gravity from the ends of the limbs, and again found, after reduction to a height of 1000, that it lay at 413.65 cm. from the crown.

Before this he had determined the center of gravity directly, on a living, well-built young man of twenty-four, according to the directions of the Weber brothers, and found that it lay at 413.898 from the crown. Although these two results agree to a striking extent, this agreement cannot be considered a proof of the exactness either of his direct method or his indirect one, such as considering the torso formed of a system of geometrical shapes, because these results were obtained from two entirely different individuals. We merely wish to point out that the distance of the center of gravity from the crown proved to be 432.24 for a height of 1000 in the Weber experiments.

In a second treatise (as above, page 257 and following) Harless gives the measurements and weights for a second corpse, a young man of twenty-nine (Kefer) who was executed, and therefore drained of blood.

Entire length 167.85 cm.
Head 20.2 cm.
Torso 57.5 cm.
Upper leg 42.3 cm., lower leg 38.15 cm., height of foot 9.7 cm.
Length of foot 25 cm., width of foot at heel 6.3 cm., width at balls of toes 9.9 cm.
Upper arm right 30.6 cm., left 30 cm.
Forearm right 26.4 cm., left 26.1 cm.
Hand right 18.5 cm., left 18.9 cm.
Distance between two summits of crest of ilium 27.1 cm.
Distance between rotational points of shoulders 29.9cm.
Distance between rotational points of hips 16.8 cm.
Weight of corpse 47087 g.
Head alone 3747 g.
Torso drained of blood, without head or extremities 19846.5 g.
Upper arm right 1484.5 g., left 1411.3 g.
Forearm right 821.1 g., left 770.1 g.
Hand right 393.2 g., left 374 g.
Upper leg right 5947 g., left 5827 g.
Lower leg right 2242.6 g., left 2252.4 g.
Foot right 982.2 g., left 988.2 g.

Information on the location of the center of gravity is given later, with our own.

Since Harless determined the height alone of the centers of gravity, the location of all the centers in the other two directions still remains to be found. Such a determination of the centers of gravity for the entire systems can be controlled by calculating the same center from the directly measured centers of the individual sections, and this also offers a control that works backward for the experimental determination of the centers of gravity in the separate sections. Neither Meyer nor Harless did this, nor did they determine the center of gravity for the entire body or for a system of body parts both experimentally and by calculation for one and the same individual, this being the only way in which any mutual control of the two kinds of investigation is possible. Harless did compare his calculations, although they dealt only with the height, with values that he obtained experimentally, but the actual measurements had been made on a different individual.

In our own investigation of the center of gravity, that will now be described in detail, we avoided the errors due to dealing with soft material by freezing the bodies to a rigid mass. We froze the cadavers so hard with an artificial freezing mixture that they remained absolutely solid both as a whole body and later in sections, to the very end of the experiments.

The method used to determine the exact center of gravity in a solidly frozen cadaver or a section of it, a method that we worked out as a practical one through repeated experiments, was the following.

It would, of course, be quite sufficient to hang up the body twice by a cord, from different spots, for it would swing back and forth each time until the center of gravity was directly under the point of attachment and therefore in the prolongation of the cord. This center would then have to lie in the point of intersection of the prolongation of the two cords. However, this method was not used because of the practical difficulty of following the direction of the cord through the body, so that the results were bound to be inexact.

The body was therefore not hung up at a point by a cord, but was hung on an axis formed of a pointed iron rod, as slender as possible, but still strong enough and hard enough to prevent it from being bent by the weight of the body. Stronger steel rods were thus used for the whole body than for the limbs, and stronger ones for these than for the lower leg or the foot alone, as these needed relatively thin rods only. Hanging up the body, or part of it, by three different axes thus made it possible to determine three planes in each case, practically at right angles to each other, in each of which the center of gravity must lie, so that the latter was found to be at the point of intersection of all the planes. The rods were very hard, and had long sharp points, so that it was easy to drive them through all the tissues of the frozen cadaver and the bones as well, after starting them in the right direction with a blacksmith's hammer. Since they were perfectly round, they formed an axis around which the body swung easily back and forth in every case, until the center of gravity was directly under the axis. This location was easily and exactly reached because we always placed the axis as far as possible from the probable location of the center of gravity. Each plane in which the center lay was determined by two plumb lines hung from the axis, and the line where the plane cut the body surface was marked on the latter. This cutting line was found very exactly even when the plumb lines could not be hung right at the point of insertion of the axis, because the cords were made so long that they could be lined up by sighting across them below the body section.

Considered by itself, it would make no difference how this system of planes was related to the body. However, it proved useful to make one plane practically perpendicular to the longitudinal direction of the body or section. This was done by first laying the body or the section lengthwise across a sharp edge until it was balanced, upon which we drove in the first axis right above this edge and parallel to it, as near the body surface as possible. After determining and marking the location of the first plane on the body surface by sighting across the plumb lines, the body was hung up a second time. The second axis was driven in as far as possible from the marked transverse plane, in a sagittal direction and as nearly as possible in the median plane, and then the second plane was projected on the body as before. Then the point of intersection of this plane mark with the first plane mark had to be fixed by scratching it in.

Then the body was hung up for the third time in the same way, with the third axis as far forward as possible, that is to say, perpendicular to the second axis, and also as far as possible from the transverse plane. The body was then marked once more at the point of intersection of this last plane with the mark of the transverse plane.

The body was then sawed through the transverse plane. After this, two fine threads were stretched on the smooth and uniformly hard surface of the cross section, connecting the points of intersection on the body surface with each other. The point of intersection of the transverse plane with the two others thus determined the location of the center of gravity, which was indicated by the intersection of the two threads. This way in which the second and third axes were driven in, perpendicular to each other, made these threads cross each other every time almost at right angles, and this made it very easy to find the point of intersection.

Altogether we had four fresh, normally built male cadavers (suicides) at our disposal. The first one served only to determine the common center of gravity, as we were not allowed to cut it. The center of gravity of the whole body and of the torso alone could not be determined very exactly on the second cadaver because the body began to break up during the measurement and thus made it change its position. Therefore, only the last two cadavers served for all the measurements.

CADAVER NO. I

The first cadaver was that of a young man of eighteen, very well built and with fine muscles, 169 cm. tall, who had shot himself. The body reached the Anatomy Department in fresh condition and in strong rigor mortis. The abdomen was not distended, but rather sunken. As the body could not be cut, we could only carry out part of the above method in this case. We had to be satisfied with hanging up the body three times, as described, marking the levels of the three planes on the surface, and then transferring the points of plane intersection to the skeleton of a man of the same size by measuring the bone protuberances, and stretching the threads there.

The common center of gravity which we found in this way lay at the level of the lower edge of the second sacral vertebra, almost in the plane of the entry to the pelvis, 0.5 cm. to the right of the median plane, 4.5 cm. below the promontory, in the plane determined by the promontory and the center points of the two hip joints.

On being frozen, the body lay on its back on a horizontal board. The head was not raised, and the limbs were symmetrically arranged, with the legs stretched out and slightly rotated outward, and the arms slightly bent at the cubital joint, with a semi-pronation of the forearm.

CADAVER NO. II

This body belonged to a very muscular man of forty-five, who had hanged himself. The abdomen was not distended, but rather shrunken, so that the lower edge of the thorax was plainly visible. The body was 170 cm. long and weighed 75100 g. It was frozen in exactly the same position as cadaver I.

The head and the limbs were sawed off, after all the marks were made on the body as described, and put back into the freezing mixture to keep them frozen solid. Then the torso alone was hung up three times, and all the necessary marks were made on it to fix the position of the center of gravity. This took quite a long time, as our technique was not yet well enough developed to allow us to work fast enough and yet get all the measurements exact. During this time the torso began to thaw out, as already mentioned. There was nothing more we could do, since it was no longer possible to saw through the transverse planes, than to drive long thin steel pins into the body at the corresponding points and thus determine the intersecting lines of the planes and the common center of gravity, and the center of gravity of the torso alone, inside the body, as well as we were able.

All that we could determine exactly about the common center of gravity was that it lay near the promontory, but below it.

We were able to determine the center of gravity of the torso alone a little better. It lay near the median plane at the lower edge of the first lumbar vertebra, at the attachment of the ligaments. The weight of the torso alone was 36020 g.

We were able to determine all the values but these, that are only approximate, very exactly, according to the method described, on this cadaver and on the two that remained.

The center of gravity of the head lay right in the median plane, at the clivus, at the connection of the sphenoid and the occipital, below the slope of the pituitary cavity. However, it should be pointed out that we removed the head with part of the neck. The saw actually passed 5 cm. above the lower part of the throat and backward at an angle through the neck. The center of gravity as we found it was thus a little farther forward and lower down than that of a head without a throat. The head with neck weighed 5350 g.

The arms were sawed off from the armpits upward and in a sagittal direction, so that the saw went almost through the center of the head of the humerus in each case. The arms were weighed as soon as they were removed. The right arm weighed 4950 g. and the left arm 4790 g. The center of gravity of the entire arm was then determined, but at first the planes were merely marked on the surface. Then the upper arm and the forearm were sawed apart. The cut should have gone as nearly as possible through the head of the cubital joint, but it later developed that the saw had not hit the axis exactly, but cut through below it, so that the upper arm was in both cases a little too long in proportion to the forearm. However, it just happened that the center of gravity of the entire arm lay right in the plane of this cut. We then found the center of gravity of the upper arm alone, and then of the entire limb, although before doing so we weighed the upper arm, the forearm, and the hand. After that we determined the center of gravity of the forearm with the hand, again merely marking the planes for the time being. Then we removed the hand in the transverse plane of the bending axis, through the head of the capitulum, so that it was now possible to find the weight and the center of gravity of the forearm alone and of the hand alone, as well as of the forearm with the hand.

The length of the right upper arm, measured from the center of the head of the humerus to the cut surface, that went, in this cadaver, nearly through the articulating surface of the joint and not through the cubital axis, was 31.7 cm., and the weight was 2580 g. The right forearm without the hand, up to the cut surface that passed through the head of the capitulum, was 29.5 cm. long. The right forearm with the hand weighed 2370 g., the right forearm alone 1700 g., and the right hand alone 670 g.

The left upper arm, that was also cut through the articulating surface of the joint, was 31.5 cm. long and weighed 2560 g. The left forearm was 29.5 cm. long. The left forearm with the hand weighed 2230 g., the left forearm alone 1600 g., and the left hand alone 620 g.

The center of gravity of the right upper arm alone was 14.5 cm. from the center of the head of the humerus, and 17.2 cm. above the articulating surface of the cubital joint, in the medullary cavity of the humerus.

The center of gravity for the entire right upper limb lay in the cut surface that went through the articulating surface of the joint, and therefore a little below the cubital axis, at the lower end of the brachialis internus, near the cubital fold, 1.5 cm. in front of the bone, right between the head of the flexor and the head of the extensor.

The center of gravity of the right forearm lay 12.5 cm. below the articulating surface of the cubital joint and 17 cm. above the head of the capitulum, right in the mass of the flexor, and 1 cm. in front of the center of the interosseal ligament.

The center of gravity of the right forearm with hand lay 19 cm. below the articulating surface of the cubital joint and 10.5 cm. above the head of the capitulum, inside the flexor, 0.5 cm. in front of the attachment of the interosseal ligament to the radius.

The center of gravity of the right hand, with fingers half flexed, lay 5.5 cm. below the center of the head of the capitulum, in the palm, 1 cm. higher than the center of the third metacarpus head, corresponding to the attachment of the third finger, and 1 cm. in front of the bone.

The center of gravity of the left upper arm alone lay 13.3 cm. below the center of the head of the humerus and 18.2 cm. above the cut through the articulating surface of the cubital joint, also in the medullary cavity of the humerus.

The center of gravity of the entire upper limb lay also in the cut surface, through the articulating surface of the cubital joint and a little below its axis, in the center of the cubital fold, at the lower end of the brachialis internus and 2 cm. in front of the anterior supertrochlear cavity.

The center of gravity of the left forearm alone lay 12.4 cm. below the articulating surface of the cubital joint and 17.1 cm. above the head of the capitulum, 1 cm. in front of the center of the interosseal ligament, inside the flexor.

The center of gravity of the left forearm with hand lay 19 cm. below the articulating surface of the cubital joint and 10.5 cm. above the capitulum head, 1 cm. in front of the attachment of the interosseal ligament to the radius, in the flexor.

The center of gravity of the left hand, with fingers half flexed, lay in exactly the same location as in the right hand.

The lower limbs were removed by sawing through the hip joint close to Poupart's ligament, and later on we found that the cuts passed perfectly symmetrically through the heads of the hip bones.

The center of gravity of the entire leg was first determined and the points of intersection marked temporarily on the surface. Then the upper leg was removed by a transverse cut that passed through the axis of the knee joint. It was now possible to determine the center of gravity of the upper leg alone and also of the whole leg, since the latter lay in the upper leg region. Then the center of gravity of the lower leg with the foot was determined, and also temporarily marked on the surface, and finally the foot was removed by a transverse cut through the axis of the tibio-talus joint. Then the center of gravity of the lower leg alone, and after that of the lower leg with foot, were determined, the latter center lying in the lower leg region. Finally, we determined the center of gravity of the foot alone, by the method described. Each of these pieces was also weighed.

The weight of the entire right leg was 12120 g., of the left leg 11850 g. The length of the right upper leg, from the center of the head of the femur to the axis of the knee joint, was 44 cm., and the weight was 7650 g. The length of the right lower leg without the foot, from the knee axis to the axis of the tibio-talus joint, was 41.4 cm. The weight of the lower leg with the foot was 4470 g., of the lower leg alone 3210 g., and of the foot along 1100 g. The right foot alone was 28.5 cm. long and 7.8 cm. high. The left upper leg was 43.3 cm. long and weighed 7300 g. The left lower leg was 41.8 cm. long. The left lower leg with foot weighed 4500 g., the lower leg alone 3320 g., and the foot alone 1160 g. The left foot was 28.3 cm. long and 7.5 cm. high.

The center of gravity of the right upper leg alone lay 19 cm. below the center of the head of the joint and 25 cm. above the bending axis of the knee, 1.5 cm. behind the linea aspera, and somewhat toward the median.

The center of gravity of the entire right lower extremity lay 39 cm. below the center of the head of the hip bone and 5 cm. above the bending axis of the knee, right in the center of the rear edge of the bone.

The center of gravity of the right lower leg alone lay 17.4 cm. below the axis of the knee and 24 cm. above the tibio-talus axis, 1 cm. behind the center of the interosseal ligament, exactly in the center between tibia and fibula.

The center of gravity of the right lower leg with the foot lay 24.6 cm. below the bending axis of the knee and 16.8 cm. above the bending axis of the tibio-talus joint, directly behind the attachment of the interosseal ligament to the tibia.

The center of gravity of the right foot lay 11.5 cm. from the rear edge of the foot and 17 cm. from the tip. It was 6.5 cm. in front of the center of the tibio-talus joint, directly behind the joint between the second and third sphenoid, at the forward surface of the navicular.

The center of gravity of the left upper leg alone lay 19.3 cm. below the center of the head of the thigh bone and 24 cm. above the axis of the knee.

The center of gravity of the entire left leg lay 38.5 cm. below the center of the head of the hip bone and 4.8 cm. above the bending axis of the knee.

The center of gravity of the left lower leg alone lay 17.4 cm. below the bending axis of the knee and 24.4 cm. above the axis of the tibio-talus joint.

The center of gravity of the left lower leg with the foot lay 25.5 cm. below the knee axis and 16.3 cm. above the axis of the tibio-talus joint.

The center of gravity of the left foot lay 12 cm. from the rear edge of the foot and 16.3 cm. from the tip.

In this case the centers of gravity of the inside of the extremity lay in the same location as in the right extremity.

Weights for Cadaver II
(The figures indicate grams)

Parts of the body	Right		Left
Entire body		75100	
Head		5350	
Torso without limbs		36020	
Whole arm	4950		4790
Upper arm	2580		2560
Forearm with hand	2370		2230
Forearm without hand	1700		1600
Hand	670		620
Entire leg	12120		11890
Upper leg	7650		7300
Lower leg with foot	4470		4500
Lower leg without foot	3210		3320
Foot	1100		1160

Since these weights were obtained directly, the total of the separate weights cannot equal the entire weight, because there is always something lost with each sawing.

Dimensions of Cadaver No. II

Length of entire body, 170 cm.
Length of head, from below the chin to the crown in vertical projection, 21 cm.
Length of upper leg, right 44 cm., left 43.3 cm.
Length of lower leg, right 41.4 cm., left 41.8 cm.
Height of foot, right 7.8 cm., left 7.5 cm.
Length of foot, right 28.5 cm., left 28.3 cm.
Distance between centers of hip joints, 17 cm.
Length of upper arm, right 31.7 cm., left 31.5 cm.
Length of lower arm without hand, right 29.5 cm., left 29.5 cm.

The measurements were always taken from the axis of a joint (center of a joint) to another, as far as the cut through the joint hit the axis. Only in the case of the cubital joint did the cut pass through the articulating surface of the joint in both arms, and thus below the axis.

CADAVER NO. III

This was a fresh, very muscular corpse of normal build, of a manual worker of about fifty, who had hanged himself. The cadaver was frozen while fresh, lying on its back like the others, and was used for determining the centers of gravity after being frozen solid in the same way. The corpse was 166 cm. long and weighed 60750 g.

The center of gravity of the entire body lay in a transverse plane that passed through the lower edge of the first sacral vertebra, and thus not at the promontory itself, but close to it and somewhat below. It lay 0.2 cm. to the right of the median plane and 4 cm. in front of the upper linea transversa of the sacrum.

The center of gravity of the torso alone, without head or arms, lay in the center of the first lumbar vertebra, 2 cm. from the front surface and 1.4 cm. from the rear surface of the vertebra, and 0.3 cm. to the right of the median.

The center of gravity of the head lay in the fossa Tarini behind the slope of the pituitary cavity, right in the median plane.

In this cadaver the cut surfaces divided the heads of the shoulder joints exactly in two, while the cuts through the cubital joints and the wrists struck the bending axes.

The entire right upper extremity weighed 3550 g. The upper arm was 30.6 cm. long and weighed 1990 g. The right forearm without the hand was 26.3 cm. long. The right forearm with the hand weighed 1550 g., without the hand 1050 g., and the hand alone 500 g.

The entire left upper extremity weighed 3480 g. The left upper arm was 30.2 cm. long and weighed 1880 g. The left forearm without the hand was 27.1 cm. long. The left forearm with the hand weighed 1600 g., without the hand 1120 g., and the hand along 470 g.

The center of gravity of the right upper arm alone lay 13.4 cm. from the center of the head of the humerus, and 17.2 cm. above the axis of the cubital joint, at the rear edge of the humerus.

The center of gravity of the entire right upper extremity lay 28 cm. below the center of the head of the humerus and 2.6 cm. ~~below~~ above the cubital axis, 1.8 cm. in front of the humerus, in the brachialis internus.

The center of gravity of the right forearm alone lay 10.9 cm. below the axis of the cubital joint, 15.4 cm. above the center of the head of the capitulum, and 1 cm. in front of the center of the interosseal ligament, in the flexor, at an equal distance from the radius and the ulna.

The center of gravity of the right forearm with the hand lay 17.8 cm. below the cubital axis and 8.5 cm. above the center of the head of the capitulum, 0.7 cm. in front of the attachment of the interosseal ligament to the radius.

The center of gravity of the right hand, with flexed fingers, lay 5.9 cm. below the center of the head of the capitulum, about 1 cm. above the second metacarpus head, at equal distances, about 2 cm., from the second and third metacarpus bones, in the fold between the hand and the thumb, quite close to the skin of the palm.

The center of gravity of the left upper arm alone lay 13.7 cm. below the center of the head of the humerus and 16.5 cm. above the axis of the cubital joint, directly behind the humerus.

The center of gravity of the entire left upper extremity lay 29.1 cm. below the center of the head of the humerus, 1.1 cm. above the cubital axis, 1.4 cm. in front of the center of the humerus, in the brachialis internus.

The center of gravity of the left forearm alone lay 11 cm. below the cubital axis and 16.1 cm. above the center of the head of the capitulum, about 1.1 cm. in front of the center of the interosseal ligament, equally distant from the radius and the ulna, in the flexor.

The center of gravity of the left forearm with the hand lay 17.6 cm. below the cubital axis, 9.5 cm. above the center of the head of the capitulum, and 1 cm. in front of the interosseal ligament, nearer the radius.

The center of gravity of the left hand, with fingers a little less flexed than on the right hand, lay 5.7 cm. from the center of the head of the capitulum, 0.8 cm. from the center of the capitulum of the third metacarpus, in the palm, 1.2 cm. in front of the palmar edge of the third metacarpus, and equally distant from the second and the fourth metacarpus. In this case the center of gravity did not lie at the surface of the hand, but in the tendon of the third superficial finger flexor.

The entire right lower extremity, from the center of the head of the upper thigh bone to the sole of the foot, was 92.7 cm. long, and the weight was 10650 g.

The entire left lower extremity was 91.6 cm. long and weighed 10250 g.

The right upper leg was 42 cm. long and weighed 6690 g. The right lower leg alone was 43 cm. long, and the foot 26.5 cm. long and 7.7 cm. high. The weight of the right lower leg with the foot was 3950 g., of the lower leg alone 2870 g., and of the foot alone 1060 g.

The left upper leg was 41 cm. long and weighed 6220 g. The left lower leg alone was 42.9 cm. long, the foot 26.9 cm. long and 7.7 cm. high. The weight of the left lower leg with the foot was 3980 g., of the lower leg alone 2880 g., and of the foot alone 1090 g.

In order to avoid any misunderstanding it must be stressed once more that the lengths indicated are not the absolute lengths of the bones, but the distances between the respective axes of the joints or centers of the joints. This is why the upper leg seems to be too short and the lower leg too long.

The center of gravity of the right upper leg alone lay 19.7 cm. below the center of the head of the femur, 22.3 cm. above the knee axis, and 2.2 cm. behind the rear edge of the thigh bone, toward the median.

The center of gravity of the entire right lower extremity lay 37.7 cm. below the center of the head of the femur, 4.3 cm. above the axis of the knee, and directly at the rear edge of the thigh bone.

The center of gravity of the right lower leg alone lay 18.7 cm. below the knee axis, 24.3 cm. above the tibio-talus axis, and 1 cm. behind the interosseal ligament, somewhat nearer the tibia.

The center of gravity of the right lower leg with the foot lay 26.9 cm. below the knee axis and 16.1 cm. above the tibio-talus axis, right at the attachment of the interosseal ligament to the tibia.

The center of gravity of the right foot lay 11.4 cm. from the rear edge of the foot and 15.1 cm. from the tip, at the angle of the lower and outer edges of the third sphenoid, near its attachment to the navicular.

The center of gravity of the left upper leg alone lay 19.5 cm. below the center of the head of the femur, 21.5 cm. above the knee axis, 1.5 cm. behind the linea aspera of the thigh bone, toward the median.

The center of gravity of the entire left lower extremity lay 38.5 cm. below the center of the head of the femur, 2.5 cm. above the axis of the knee joint, and 0.7 cm. in front of the rear edge of the femur, somewhat to the side, in the bone itself.

The center of gravity of the left lower leg alone lay 17.7 cm. below the knee axis, 25.2 cm. above the tibio-talus axis, and 1 cm. behind the interosseal ligament, nearer the tibia, in the flexor.

The center of gravity of the left lower leg with the foot lay 26 cm. below the knee axis, 19.9 cm. above the axis of the ankle, at the point of attachment of the interosseal ligament to the tibia.

The center of gravity of the left foot lay 11.8 cm. from the rear edge of the foot and 15.1 cm. from the tip, in the angle between the third sphenoid and the cuboid on the plantar side, close to the joint of the third sphenoid and the navicular.

Weights of Cadaver No. III
(Figures equal grams)

Parts of the body	Right	Left
Entire body	60750	
Head without neck	4040	
Torso without extremities	28850	
Entire arm	3550	3480
Upper arm	1990	1880

Weights of Cadaver No. III (cont.)

Forearm with hand	1550	1600
Forearm without hand	1050	1120
Hand	500	470
Entire leg	10650	10250
Upper leg	6690	6220
Lower leg with foot	3950	3980
Lower leg without foot	2870	2880
Foot	1060	1090

Here, too, the sum of the weights of the separate parts is bound to be less than the total weight, because the weights of the whole limbs as well as of each section were taken separately, and there was a certain amount of loss due to sawing.

Dimensions of Cadaver No. III

Length of the whole cadaver, 166 cm.
Length of head, from below chin to the crown in vertical projection, 20.2cm.
Length of upper leg, right 42.0 cm., left 41.0 cm.
Length of lower leg, right 43.0 cm., left 42.9 cm.
Height of foot, right 7.7 cm., left 7.7 cm.
Length of foot, right 26.5 cm., left 26.9 cm.
Distance between centers of hip joints, 17.5 cm.
Length of upper arm, right 30.6 cm., left 30.2 cm.
Length of forearm without hand, right 26.3 cm., left 27.1 cm.

The measurements were always taken from one joint center, or joint axis, to the other.

CADAVER NO. IV

This was the fresh, muscular body, with little fat, and still in rigor mortis, of a laborer who had hanged himself. The abdomen was not distended, but somewhat sunken, so that the edge of the thorax, the xiphoid process and the edges of the iliac crest were clearly raised. The cadaver was frozen flat on its back, with the forearms slightly bent and in average pronation.

Wire nails were hammered in at definite points, and lines were drawn on the body, before freezing, in order to have exact marks in advance forsawing through the joint axes, because the even hardness of the frozen body made it difficult to find the bone protuberances.

1) Wire nails hammered in from the front to the rear through the centers of the humerus heads

2) The cubital axis determined and marked by a cross line

3) A nail hammered through the center region of the capitulum head on both sides

4) The anterior superior crests of the ilium marked

5) The upper edge of the great trochanter marked on both sides, in order to determine the height of the center of the head of the femur

6) The axis of the knee joint determined and marked onboth sides by a cross line

7) The axis of the tibio-talus joint determined and marked on both sides by a cross line

Projecting these lines and points on the horizontal surface of the table also made it much easier to measure the body dimensions.

The corpse was 168.8 cm. long and weighed 56090 g.

The center of gravity of the entire body lay in the cavity of the false pelvis. It lay in the median body plane, 2.1 cm. vertically below the promontory and 4.5 cm. above the connecting line between the centers of the heads of the thigh bones, at the same level as the upper edge of the third sacral vertebra, 7 cm. in front of it, and 7 cm. higher than the upper edge of the symphysis.

The center of gravity of the torso alone, without head and arms, lay at the upper edge of the first lumbar vertebra, at the front surface, 0.5 cm. to the right of the median, and 25.8 cm. above the connecting line between the centers of the heads of the thigh bones.

The center of gravity of the torso with head and arms lay nearly in the median plane, only 0.2 cm. to the right of it, and at the front edge of the lower surface of the eleventh thoracic vertebra, 29 cm. above the connecting line between the centers of the heads of the thigh bones.

The head was sawed off by placing the saw right under the chin, sloping upward and to the rear, in order to remove the head from the neck as completely as possible. It was found later that only the three upper cervical vertebrae remained attached to the skull.

The center of gravity of the head lay exactly in the median plane, 0.7 cm. behind the slope of the pituitary cavity, in the fossa Tarini at the base of the brain, right in the angle formed by the upper edge of the bridge with the posterior lamina perforata (figs. 1 and 2).

The entire right upper extremity weighed 3520 g., while the right upper arm was 32 cm. long and weighed 1730 g. The right lower arm without the hand was 27 cm. long, the right lower arm with the hand weighed 1790 g., without the hand 1300 g., and the hand alone 490 g.

The entire left upper extremity weighed 3710 g., the length of the left upper arm was 32 cm., and it weighed 2020 g. The left forearm without the hand was 27 cm. long. With the hand it weighed 1690 g., without the hand 1240 g., and the hand alone weighed 450 g.

Since the lengths of the two extremities and their sections differed by only a few millimeters, we took the mathematical mean in this case in order to obtain entirely symmetrical ratios. The weights we left as they were. We also had the cut surfaces and their centers of gravity for this cadaver copied, and the figures set up in plates I and II, to give an exact picture of the location of the centers of gravity in the right upper and lower extremities.

The center of gravity of the right upper arm alone lay 16.3 cm. below the head of the humerus and 15.7 cm. above the cubital axis, in the bone of the humerus itself, near the rear edge and somewhat toward the median.

The center of gravity of the entire right upper extremity lay 0.5 cm. below the cubital axis and 0.3 cm. in front of the bone, in the brachialis internus.

The center of gravity of the right forearm alone lay 11.4 cm. below the cubital axis and 15.6 cm. above the center of the head of the capitulum, 1.5 cm. in front of the center of the interosseal ligament, and somewhat nearer the radius than the ulna.

The center of gravity of the right forearm with the hand lay 17.7 cm. below the cubital axis and 9.3 cm. above the center of the head of the capitulum, 0.5 cm. in front of the interosseal ligament, and nearer the radius than the ulna, in the region of the flexors.

The center of gravity of the right hand, with flexed fingers, lay 5.5 cm. below the center of the head of the capitulum, between the third metacarpus and the palm, 1 cm. from the latter.

The center of gravity of the left upper arm alone lay 15.3 cm. below the center of the humerus and 16.7 cm. above the cubital axis, exactly as on the other side.

The center of gravity of the entire left upper extremity lay 0.5 cm. above the cubital axis, and 0.5 cm. in front of the bone.

The center of gravity of the left forearm alone lay 11.9 cm. below the cubital axis and 15.1 cm. above the center of the head of the capitulum, also in front of the center of the interosseal ligament, in the region of the flexors.

The center of gravity of the left forearm with the hand lay 17.9 cm. below the cubital axis and 9.1 cm. above the center of the head of the capitulum, 0.5 cm. in front of the interosseal ligament, nearer the radius than the ulna.

The center of gravity of the left hand, with flexed fingers, lay 5 cm. below the head of the capitulum, exactly in the same location as in the right hand.

The entire right lower extremity, from the center of the head of the femur to the soles of the feet, was 88 cm. long and weighed 10070 g.

The entire left lower extremity was also 88 cm. long, and weighed 10630 g.

The right upper leg was 40 cm. long and weighed 6150 g. The right lower leg alone was 41.5 cm. long, the foot 26.5 cm. long and 6.5 cm. high. The weight of the right lower leg with the foot was 3960 g., of the lower leg alone 2970 g., and of the foot 990 g.

The left upper leg was 40 cm. long and weighed 6750 g. The left lower leg was 41.5 cm. long, the foot 26.5 cm. long and 6.5 cm. high. The left lower leg with the foot weighed 3900 g., the lower leg alone 2900 g., and the foot 1000 g.

In this case, too, we used the arithmetical means of the lengths of the two extremities.

The center of gravity of the right upper leg alone lay 17 cm. below the center of the head of the femur and 23 cm. above the knee axis, 1.5 cm. behind the linea aspera, somewhat toward the median.

The center of gravity of the entire right lower extremity lay 35.5 cm. below the center of the head of the thigh bone, 4.5 cm. above the knee axis, and 1 cm. behind the thigh bone itself.

The center of gravity of the right lower leg alone lay 17 cm. below the knee axis, 24.5 cm. above the axis of the tibio-talus joint, 0.7 cm. behind the interosseal ligament, and a little nearer the tibia than the fibula.

The center of gravity of the right lower leg with the foot lay 25 cm. below the knee axis, 16.5 cm. above the axis of the tibio-talus joint, exactly at the attachment of the interosseal ligament to the tibia.

The center of gravity of the right foot lay 12 cm. in front of the rear edge of the foot, 14.5 cm. behind the tip, 3 cm. above the sole, and under the third sphenoid, near its forward edge.

Since the relative positions of the centers of gravity as to width and depth, for the left lower extremity, were practically the same as for the right one (Plate II), only the heights are given here.

The center of gravity of the left upper leg alone lay 15.5 cm. below the center of the head of the femur and 24.5 cm. above the knee axis.

The center of gravity of the entire left extremity lay 33 cm. below the center of the head of the femur and 7 cm. above the knee axis.

The center of gravity of the left lower leg alone lay 17.5 cm. below the knee axis and 24 cm. above the axis of the tibio-talus joint.

The center of gravity of the left lower leg with the foot lay 25.5 cm. below the knee axis and 16 cm. above the axis of the tibio-talus joint.

The center of gravity of the left foot lay exactly as in the right foot.

Weights of Cadaver No. IV
(Figures indicate grams)

Parts of the body		
Entire body	55700	
Head without neck	3930	
Torso without extremities	23780	
Entire arm	3520	3710
Upper arm	1730	2020
Forearm with hand	1790	1690
Forearm without hand	1300	1240
Hand	490	450
Entire leg	10110	10650
Upper leg	6150	6750
Lower leg with foot	3960	3900
Lower leg without foot	2970	2900
Foot	990	1000

In this case the differences due to the losses resulting from sawing, between the entire weight and the sum of the weights of the parts, have already been balanced, in order to be able to use the weights directly for later calculations.

Dimensions of Cadaver No. IV

Entire length, 168.8 cm.
Length of head, from below chin to crown in vertical projection, 21.3 cm.
Lower edge of chin to line connecting centers of humerus heads 10.1 cm.
Line connecting centers of humerus heads to line connecting centers of hip joints, 40 cm.

Length of upper leg, right 40 cm., left 40 cm.
Length of lower leg, right 41.5 cm., left 41.5 cm.
Height of foot, right 6.5 cm., left 6.5 cm.
Length of foot, right 26.5 cm., left 26.5 cm.
Distance between centers of hip joints, 17 cm.
Length of upper arm, right 32 cm., left 32 cm.
Length of forearm, right 27 cm., left 27 cm.
Distance between centers of humerus heads
(with shoulders drawn forward), 36 cm.

The means of the lengths of the extremities were taken and indicated for both, for the sake of later calculations.

Determining the centers of gravity in the separate sections of the limbs revealed that there was a fairly close and noteworthy relationship between the centers of gravity and the axes of the joints. Both in the upper arm and in the upper leg the center of gravity lay almost exactly in the straight line that passes through the center of the head of the joint on one hand, and the center of the cubital and knee joints on the other. In the lower leg the center of gravity lay almost in the straight line connecting the center of the knee axis with the center of the tibio-talus joint. In the forearm, when in average pronation, the center of gravity lay in the straight line that passed through the center of the capitulum head and the center of the cubital axis.

In considering the torso, the curve of the spine was used as a basis, just as the curve lay in the frozen cadaver in the recumbent position. The curve determined by the Weber brothers was not used, because it did not correspond entirely to natural conditions, but was obtained from the disemboweled cadaver, for the organs alter the curve of the spine a great deal, as proved by Parow (Virchows Archiv, Vol. 31).

It was also evident from the torso that the center of gravity of the torso alone, without head or arms, and the position that it assumed in freezing, had a definite relation to the limbs, for it lay practically in the straight line bisecting the line that joined the two hip centers and connecting it with the center of the atlanto-occipital joint.

Although it was already possible to determine from the findings that the center of gravity lay, for all the sections of the extremities, in the straight line that connected the center of each neighboring joint with the other, a fact that would greatly simplify further calculations, more work still remained to be done, namely to investigate the systematic arrangement of the centers of gravity more carefully in regard to individual deviations.

The relationships observed made it possible for us to construct an upright body position in which all the centers of gravity except that of the foot lay in a single frontal plane. All that was necessary was to bring the centers of all the main joints into this plane.

After this we constructed a skeleton holding this position, and compared it with a normal, well-built living body, in order to find out whether it was possible for the body to maintain such an upright position, in which the centers of all the main joints and the sectional centers of gravity that lay between them would fall in a single vertical frontal plane.

To do this, we took the measurements of cadaver No. IV and laid them out in life size on squared paper, both in profile and in a front view. Then we drew the skeleton according to these measurements, laid out the contours of the soft portions around it, and indicated the centers of gravity that had been found by direct measurement. Since we had originally marked the axes of the joints on the cadaver, and knew the locations of the centers of gravity inside the individual sections in relation to the joints and bones, it was easy to draw the bones and joints and centers of gravity without adding anything arbitrary (Plate III). We were also fortunately able to judge the thickness as well as the length ratios of bones and soft parts from our study of the sections.

We prepared this drawing in life size on squared paper, so that the joints and the centers of gravity could be plotted on a rectangular system of coordinates. Both views offered two projections, on two planes perpendicular to each other, and thus formed all three coordinates required for the rectangular system to be based on them.

We made the frontal plane the YZ-plane, with the line of intersection of the median body plane with this frontal plane as the Z-axis, and the line of intersection of the frontal plane with the horizontal ground plane on which the body stood, as the Y-axis. The vertical to this plane at point of intersection 0 of the two axes was therefore called the X-axis. The directions forward, to the right, and upward, respectively, for the X,Y, and Z axes, measuring from point of origin 0 on the graph, were called positive. With these points determined, the profile view presented the projection on the XZ-plane, and the horizontal ground plane coincided with the XY-plane.

A spinal curve was laid out in the plane passing through the center point between the two hip joints on one hand and the atlanto-occipital joint on the other, on the assumption that the center of gravity of the torso lay exactly in this plane. It was easy to make this agree very well with the photographs that showed the median section of the frozen cadavers.

The position of the arms was changed a little. The arms in the drawings were moved forward until the centers of the humerus heads and the centers of gravity of the separate sections lay in the XY-plane. The displacement was not large, so that it could not cause any real change in the location of the common center of gravity. On being photographed, the two drawings were reduced to one-tenth their original size, so that one millimeter in the plates corresponds to one centimeter in the original drawing. The drawings are found in plate III.

We naturally had to investigate whether the position thus constructed occurred in real life, and to what extent the recumbent position could be transferred to the upright one.

It was, of course, not to be expected that the curve of the spine would be exactly the same in both body positions. It will, however, suffice if they are fairly similar, for it is impossible to measure all the smallest curves of the spine directly on the living body. Even Parow's coordinate gauge (Virchows Archiv, Vol. 31, p. 105 and foll.), that he used to construct the spinal curves from the location of the spinal processes, gave, as we know, only approximate results. We hoped, however, that the following method would be satisfactory.

A well-built, muscular young man, a soldier, was photographed in profile, after having the center points of the joints marked on his body just as they were in the constructed drawing, and having the center of gravity of the head projected to the side of the head and marked on it. Two long plumb lines determined the YZ-plane. The model had to change his position until he fell of his own accord into a pose in which someone standing at a distance would sight all the marks for the joints as lying in the vertical plane of the plumb lines (the YZ-plane). He was then photographed in this position, as shown in plate IV. The left arm had to be bent, in order not to hide the center point of the hip joint. Even if this did displace the center of gravity somewhat, the displacement was so small, as compared with the advantage in making the joint visible, that it was not worth considering. Later calculations will show that the center of gravity was displaced only 7 mm. upward, 4 mm. forward, and 1 mm. to the right.

Comparison of plates III and IV shows that there is a good correspondence between the real position and the constructed one, and that our constructed position can by no means be called forced, but very natural. The thorax bulges forward a little more in the live figure, and the lower part of the back is drawn in a little more, but these are differences that can very well be considered individual and variable. The nature of the spinal curve and the position of the head are exactly the same as in the constructed figure in plate III, even if the curves of the separate sections of the spine may seem to be a little more pronounced.

The most important points, the construction of the lower extremities, the position of the pelvis, and the position of the head, are exactly like those we constructed.

The question may arise as to whether this position is very comfortable. It was not our purpose to determine this, but we shall come back to this question later on. We had just one goal from the first, to standardize a natural position that would serve as a suitable starting position for our measurements and calculations. We shall call this position the normal one.

The centers of gravity for the systems of limbs and for the whole body were now caculated for the normal body position, from the centers of gravity of the separate sections, in order to have a control for our measurements of the centers of gravity both of the separate limbs and of the whole body.

If the centers of the joints in the normal position, as well as the measured centers of the different limbs, are plotted on the above system of coordinates, the following coordinate values will be obtained, and can be read off directly from plate III.

Coordinates for the Centers of the Joints, the Crown, the Tip of the Foot and the Lower Edge of the Bent Hand (with flexed fingers)

Coordinates for		x	y	z
1) Crown		0	0	168.8
2) Atlanto-occipital joint		0	0	154.0
3) Hip	r	0	+ 8.5	88.0
	l	0	- 8.5	88.0
4) Knee	r	0	+ 8.5	48.0
	l	0	- 8.5	48.0
5) Tibio-talus joint	r	0	+ 8.5	6.5
	l	0	- 8.5	6.5

6) Tip of foot	r	+ 20.5	+ 13.0	0
	ℓ	+ 20.5	- 13.0	0
7) Shoulder	r	0	+ 18.0	137.0
	ℓ	0	- 18.0	137.0
8) Cubital joint	r	0	+ 18.0	105.0
	ℓ	0	- 18.0	105.0
9) Wrist	r	0	+ 18.0	78.0
	ℓ	0	- 18.0	78.0
10) Lower edge of bent hand (with flexed fingers)	r	0	+ 18.0	67.5
	ℓ	0	- 18.0	67.5

NOTE: The figures that indicate millimeters in the drawing in plate III actually indicate centimeters, as the drawings are reproduced with a ten-time reduction. The same is true for the following coordinates for the centers of gravity.

Coordinates for the Centers of Gravity

Coordinates for		x	y	z
(1,2) Head		0	0	157.8
(2,3) Torso		0	+ 0.5	113.8
(3,4) Upper leg	r	0	+ 8.5	71.0
	ℓ	0	- 8.5	72.5
(4,5) Lower leg	r	0	+ 8.5	31.0
	ℓ	0	- 8.5	30.5
(5,6) Foot	r	+ 6.5	+10.3	3.0
	ℓ	+ 6.5	-10.3	3.0
(7,8) Upper arm	r	0	+18.0	120.7
	ℓ	0	-18.0	121.7
(8,9) Forearm	r	0	+18.0	93.6
	ℓ	0	-18.0	93.1
(9,10) Hand	r	0	+18.0	72.5
	ℓ	0	-18.0	73.0
(1,6,7,10) Whole body		0	0	92.5
(1,3,7,10) Torso + head + arms		0	0	117.0
(3,6) Whole leg	r	0	+ 8.5	52.5
	ℓ	0	- 8.5	55.0
(4,6) Lower leg + foot	r	0	+ 8.5	23.0
	ℓ	0	- 8.5	22.5
(7,10) Whole arm	r	0	+18.0	104.5
	ℓ	0	-18.0	105.5
(8,10) Forearm + hand	r	0	+18.0	87.3
	ℓ	0	-18.0	87.1

The numbers in parentheses in front of the words "head," "torso," etc., indicate the numbering of the joints, "tips of feet," etc., as given in the preceding table of coordinates. They offer a convenient means of identifying a body section. For instance, the upper leg is marked (3,4) because it lies between joints 3 and 4. This makes it possible to recognize the coordinates of the center points of the joints, as distinguished from those of the centers of gravity, without a great many words. For instance, the coordinates for the centers of the hip joints can be indicated as x_3, y_3, and z_3, and those for the center of gravity of the upper leg as $x_{3,4}$, $y_{3,4}$, $z_{3,4}$. If the right or left side is also to be indicated, this can be done by marking the coordinates for the center of the left hip joint as x'_3, y'_3, and z'_3, while the coordinates for the center of gravity of the left upper leg would be marked $x'_{3,4}$, $y'_{3,4}$, and $z'_{3,4}$, the prime being omitted for the right side. We shall use this means of identification later on. The coordinates for the whole body will simply be marked x_0, y_0, z_0.

Since the weights of the separate sections are necessary to our calculations, they are set up once more in the accompanying table, in the same way as the coordinates.

In the calculations, the only points of interest in regard to the weights are their ratios, and so they have been reduced to figures per ten thousand, as this is an advantage in the calculation of the common center of gravity. These figures are tabulated in the column next to the real weight.

Parts of the Body		Weight in grams for Cadaver No. IV	Weight ratios per 10,000 for whole body
(1,2) Head		3930	705.5
(2,3) Torso		23780	4270
(3,4) Upper leg	r	6150	1104
	ℓ	6750	1212
(4,5) Lower leg	r	2970	533
	ℓ	2900	520.5
(5,6) Foot	r	990	178
	ℓ	1000	179.5
(7,8) Upper arm	r	1730	310.5
	ℓ	2020	362.5
(8,9) Forearm	r	1300	233
	ℓ	1240	222.5
(9,10) Hand	r	490	88
	ℓ	450	81
(1,6,7,10) Whole body		55700	10000

The weight p, for the right upper leg, for instance, can be designated by $p_{3,4}$, and the weight for the upper left leg by $p'_{3,4}$, to correspond with the markings used for the center of gravity coordinates, etc.

Calculation of the Common Center of Gravity for the Entire Body and for Whole Body Sections, from the Centers of Gravity and the Weights of Separate Limbs

Since forces of inertia are parallel forces, we must first indicate how a resultant is obtained from two or more parallel forces. Let us assume that A and B are points of attachment of two parallel forces in Figure 3, with strengths p_1 and p_2, while AC and BD are lines that represent graphically, by their length and direction, the size and direction of these two forces. Now, if two equal but opposed forces ofany desired size are placed in the direction of the line connecting points A and B, so that one force is attached at A and the other at B, this will not alter the action of the original system of forces. These two new forces are indicated in the figure by lines AE and BF, and their strength by q. If the two forces acting at A are now combined to form a resultant AG, and those at B to form a resultant BH, the total of these two resultants is still equivalent to the two original forces p_1 and p_2. The same thing remains true if the two resultants are moved to a common point of intersection J of their directions, so that J becomes the common point of attachment for them both. These two forces, JQ and JR, can still be combined into a further resultant JM, that will therefore also be the resultant of the original parallel forces.

The size of this resultant is equal to the sum of the two parallel forces, or $p_1 + p_2$, for it is easy to note from the figure that

$$\triangle GCA \cong \triangle QKJ \cong \triangle RLM, \text{ and that therefore}$$

$$\triangle HDB \cong \triangle RLJ \cong \triangle QKM. \text{ Therefore,}$$

$$JK = AC = p_1, \text{ and } KM = BD = p_2;$$

$$\text{therefore } JM = p_1 + p_2.$$

It also follows, from the congruence of the triangles, that JM is parallel to the two original forces. This resultant JM of the parallel forces intersects, in point N, the line that connects the two points of attachment A and B. The distances of this point N from A and B are related to each other as the reverse of the relation of the parallel forces acting upon A and B, for, from the similarity of triangles ANJ and QKJ, there follows the proportion,

$$AN : JN = q : p_1,$$

and from the similarity of triangles BNJ and RLJ there follows the proportion,

$$BN : JN = q : p_2.$$

Therefore $JN \cdot q = AN \cdot p_1 = BN \cdot p_2$, and therefore

$$AN : BN = p_2 : p_1$$

If the two parallel forces were exerted in another direction but remained parallel to each other, and also remained fixed to points A and B, the same reasoning would bring us to exactly the same conclusion, that the direction of the new resultants of the parallel forces intersects line AB, connecting the two forces, in a point whose distances from A and B are in reverse ratio to forces p_1 and p_2. Therefore, this point of intersection is again point N. We thus come to the conclusion that the direction of all resultants, obtained by any kind of rotation of forces p_1 and p_2 around their points of attachment, all pass through one and the same point N, if the forces continue to be parallel after rotation. If forces p_1 and p_2 are forces of inertia, point N is thus, according to our previous definition, their center of gravity.

Now if x_1, y_1, and z_1 are coordinates of the point of attachment A of force p_1, and x_2, y_2, and z_2, of point of attachment B of force p_2, and if the coordinates of center of gravity N are indicated by x_0, y_0, and z_0, then ratios

$$(x_0 - x_1) : (x_2 - x_0) = p_2 : p_1$$
$$(y_0 - y_1) : (y_2 - y_0) = p_2 : p_1$$
$$(z_0 - z_1) : (z_2 - z_0) = p_2 : p_1$$

are true, as a result of the location of point N on the connecting line AB. From this we find that

$$x_0 = \frac{p_1x_1 + p_2x_2}{p_1 + p_2}$$

$$y_0 = \frac{p_1y_1 + p_2y_2}{p_1 + p_2}$$

$$z_0 = \frac{p_1z_1 + p_2z_2}{p_1 + p_2}$$

for coordinates x_0, y_0, and z_0 of the center of gravity.

Now let us assume a system of three parallel forces p_1, p_2, and p_3, whose points of attachment have respectively the coordinates x_1, x_2, and x_3; y_1, y_2, and y_3; z_1, z_2, and z_3. Then these forces can also be combined into one resultant, which is reached by first obtaining the resultant of p_1 and p_2, and then combining this resultant with the third force p_3, to obtain the final resultant. Therefore, coordinates x'_0, y'_0, and z'_0, of the center of gravity of the two forces $(p_1 + p_2)$ and p_3, whose points of attachment have the coordinates x_0, y_0, and z_0, and x_3, y_3, and z_3, have finally to be calculated according to the method described. Then

$$x'_0 = \frac{(p_1 + p_2)x_0 + p_3x_3}{(p_1 + p_2) + p_3} = \frac{p_1x_1 + p_2x_2 + p_3x_3}{p_1 + p_2 + p_3}$$

$$y'_0 = \frac{(p_1 + p_2)y_0 + p_3y_3}{(p_1 + p_2) + p_3} = \frac{p_1y_1 + p_2y_2 + p_3y_3}{p_1 + p_2 + p_3}$$

$$z'_0 = \frac{(p_1 + p_2)z_0 + p_3z_3}{(p_1 + p_2) + p_3} = \frac{p_1z_1 + p_2z_2 + p_3z_3}{p_1 + p_2 + p_3}$$

These three equations reveal the law according to which n parallel forces p1, p2, p3 .... pn can be combined into one resultant, and according to which coordinates x_0, y_0, and z_0 indicate the common center of gravity for all n forces of inertia. When we universalize this:

$$x_0 = \frac{p_1x_1 + p_2x_2 + p_3x_3 + \cdots p_nx_n}{p_1 + p_2 + p_3 + \cdots p_n} = \frac{\Sigma p_ix_i}{\Sigma p_i}$$

$$y_0 = \frac{p_1y_1 + p_2y_2 + p_3y_3 + \cdots p_ny_n}{p_1 + p_2 + p_3 + \cdots p_n} = \frac{\Sigma p_iy_i}{\Sigma p_i}$$

$$z_0 = \frac{p_1z_1 + p_2z_2 + p_3z_3 + \cdots p_nz_n}{p_1 + p_2 + p_3 + \cdots p_n} = \frac{\Sigma p_iz_i}{\Sigma p_i}$$

As seen from these three equations, the values of x_0, y_0, and z_0 do not change if we substitute for the real weights p_i the ratios between the separate weights, as for instance the ratios that result from representing the body weight as the figure 10000. The advantage of this is that the calculation of the coordinates of the common center of gravity in the body, for any position whatever, can be done with one convenient divisor in all three equations, namely 10000. Therefore, these ratios, as they are given on page 41 , must be understood in the following calculations under the symbol p_i.

The following values are obtained for cadaver IV, for products p_iz_i*, for the purpose of calculating coordinate z_0 for the center of gravity of the whole body in normal position, from the coordinate values and ratios given on page (40 and 41).

Head	111327.9
Torso	485926
Right upper leg	78384
Left upper leg	87870
Right lower leg	16523
Left lower leg	15875.25
Right foot	534
Left foot	538.5
Right upper arm	37477.35
Left upper arm	44116.25

NOTE. - These products are used because they recur in the following calculations.

Right forearm	21808.8
Left forearm	20714.75
Right hand	6380
Left hand	5913
Total	933388.8

By dividing by $\sum p_i = 10000$ we obtain, to one decimal point, a value of 93.3 cm. for z_0, while the directly measured value of the Z-coordinate was 92.5 cm.

Absolute exactness can naturally not be attained, first because crumbs fall away in sawing the separate sections, which decreases the weight, and second because the position the body assumed on being measured could not correspond absolutely to our normal position. If we disregard the center of gravity for the feet and consider only that of the mass balanced on the tibio-talus joint, we find that

$$\sum p_i z_i = 933388.8 - (534 + 538.5) = 932316.3$$

This must now be divided by 10000 - (178 + 179.5) = 9642.5, and so the Z-coordinate for the center of gravity of the body, without feet, becomes, if we distinguish the coordinates in question as x'_0, y'_0, and z'_0

$$z'_0 = \frac{932316.3}{9642.5} = 96.7 \text{ cm.}$$

We have the following values for products $p_i y_i$, to use in calculating the y_0 coordinates of the common center of gravity,

Head	0		
Torso	+ 2135		
Right upper leg	+ 9384	Left upper leg	- 10302
Right lower leg	+ 4530.5	Left lower leg	- 4424.25
Right foot	+ 1833.4	Left foot	- 1848.85
Right upper arm	+ 5589	Left upper arm	- 6526
Right forearm	+ 4194	Left forearm	- 4005
Right hand	+ 1584	Left hand	- 1458
Total	29249.9	Total	- 28563.1

therefore $\sum p_i y_i = 29249.9 - 28563.1 = 686.8$ and

$$y_0 = \frac{686.8}{10000} = + 0.07 \text{ cm.}$$

This calculation thus gives a mere deflection of 0.07 cm. to the right of the median plane, while our direct measurements placed the center of gravity right in it.

For the Y-coordinate of the center of gravity of the body, without feet, we find that

$$y'_0 = \frac{696.8 - (1833.4 - 1848.85)}{9642.5} = \frac{702.25}{9642.5} = + 0.07 \text{ cm.}$$

thus the same value exactly.

In calculating the x_0 coordinate for the common center of gravity, only products $p_i x_i$, that belong to the center of gravity for the two feet, have a value other than zero in our normal position, so that

$$x_0 = \frac{178 \cdot 6.5 + 179.5 \cdot 6.5}{10000} = \frac{2323.75}{10000} = 0.2 \text{ cm}$$

that is to say, the common center of gravity lies, for our position of the feet, 2mm. in front of the frontal plane in which lie all the centers of gravity except those of the feet.

The X-coordinates for the common center of gravity without the feet is on the contrary zero, because all the x_i values equal zero.

We also calculated the coordinates for the centers of gravity of the limbs and their combinations, that we had previously obtained by direct measurement, in addition to the coordinates for the common center of gravity. We found them to be,

For the Entire Leg

Right	$p_i z_i$
Upper leg	78384
Lower leg	16523
Foot	534
Total	95441

Left	$p_i z_i$
Upper leg	87870
Lower leg	15875.25
Foot	538.5
Total	104283.75

Weights

Upper leg	1104
Lower leg	533
Foot	178
Total	1815, therefore

$$z_{3,6} = \frac{95441}{1815} = 52.6 \text{ cm.}$$

$$y_{3,6} = + 8.7 \text{ cm.}$$

$$x_{3,6} = + 0.6 \text{ cm.}$$

Weights

Upper leg	1212
Lower leg	520.5
Foot	179.5
Total	1912, therefore,

$$z'_{3,6} = \frac{104283.75}{1912} = 54.5 \text{ cm}$$

$$y'_{3,6} = - 8.7 \text{ cm.}$$

$$x'_{3,6} = - 0.6 \text{ cm.}$$

The coordinate values obtained by calculation and those given on page 40 as having been measured directly should be compared with these as well as with all the following calculations.

Lower Leg + Foot

Right

$p_i z_i$

Lower leg	16523
Foot	534
Total	17057

Weights

Lower leg	533
Foot	178
Total	711, therefore

$$z_{4,6} = \frac{17057}{711} = 24.0 \text{ cm.}$$

$$y_{4,6} = + 9.0 \text{ cm.}$$

$$x_{4,6} = + 1.6 \text{ cm.}$$

Left

$p_i z_i$

Lower leg	15875.52
Foot	538.5
Total	16413.75

Weights

Lower leg	520.5
Foot	179.5
Total	700, therefore

$$z'_{4,6} = \frac{16413.75}{700} = 23.5 \text{ cm.}$$

$$y'_{4,6} = - 9.0 \text{ cm.}$$

$$x'_{4,6} = - 1.7 \text{ cm.}$$

Entire Arm

Right

$p_i z_i$

Upper arm	37477.35
Forearm	21808.8
Hand	6380
Total	65666.15

Left

$p_i z_i$

Upper arm	44116.25
Forearm	20714.75
Hand	5913
Total	70744

Weights

Upper arm	310.5
Forearm	233
Hand	88
Total	631.5, therefore,

$$z_{7,10} = \frac{65666.15}{631.5} = 104.0 \text{ cm}$$

$y_{7,10} = + 18$ cm.

$x_{7,10} = 0$ cm.

Weights

Upper arm	362.5
Forearm	222.5
Hand	81
Total	666 therefore,

$$z'_{7,10} = \frac{70744}{666} = 106.2 \text{ cm.}$$

$y'_{7,10} = - 18$ cm.

$x'_{7,10} = 0$ cm.

Forearm + Hand

Right
$P_i z_i$

Forearm	21808.8
Hand	6380
Total	28188.8

Weights

Forearm	233
Hand	88
Total	321, therefore,

$$z'_{8,10} = \frac{28188.8}{321} = 87.8 \text{ cm.}$$

$y'_{8,10} = + 18$ cm.

$x'_{8,10} = 0$ cm.

Left
$P_i z_i$

Forearm	20714.75
Hand	5913
Total	26627.75

Weights

Forearm	222.5
Hand	81
Total	303.5, therefore

$$z'_{8,10} = \frac{26627.75}{303.5} = 87.7 \text{ cm.}$$

$y'_{8,10} = - 18$ cm.

$x'_{8,10} = 0$ cm.

Torso + Head
$P_i z_i$

Torso	485925
Head	111327.9
Total	597253.9

Weights

Torso	4270
Head	705.5
Total	4975.5, therefore

$$z_{1,3} = \frac{597253.9}{4975.5} = 120.0 \text{ cm}$$

$$y_{1,3} = \frac{2135}{4975.5} = 0.4 \text{ cm}$$

$$x_{1,3} = 0 \text{ cm.}$$

Torso + Head + Arms

$p_i z_i$

Torso		485926
Head		111327.9
Entire arm	r.	65666.15
	l.	70744
Total		733664.05

Weights

Torso		4270
Head		705.5
Entire arm	r.	631.5
	l.	666
Total		6273,

therefore

$$z_{1,3,7,10} = \frac{733664.05}{6273} = 117.0 \text{ cm.}$$

$p_i y_i$

Torso	2135	Head	0
Upper arm r.	5589	Upper arm l.	- 6525
Forearm r.	4194	Forearm l.	- 4005
Hand r.	1584	Hand l.	- 1458
Total	+ 13502	Total	-11988,

therefore

$$y_{1,3,7,10} = \frac{13502 - 11988}{6273} = \frac{1514}{6273} = + 0.2 \text{ cm.}$$

$$x_{1,3,7,10} = 0$$

If we assume a uniform and symmetrical distribution of the body masses, it will be possible to take the arithmetical mean of the center of gravity coordinates on page 40 and of the weight ratios on page 41 . Then, if we also place the center of gravity of the torso right in the median plane, the mean coordinate values of the centers of gravity will be:

Parts of the body	x	y	z
Head	0	0	157.8
Torso	0	0	113.8
Both upper legs	0	± 8.5	71.75
Both lower legs	0	± 8.5	30.75
Both feet	+ 6.5	±10.3	3
Both upper arms	0	±18	121.2
Both forearms	0	±18	93.35
Both hands	0	±18	72.75

The positive values of y refer to the right side of the body, and the negative ones to the left.

If we assume a total weight of 10000, it follows that the means for the weights and the weight ratios are:

Mean weights		Mean weight ratios	(in whole numbers)
Whole body	55700 g	10000	10000
Head	3930 g	705.5	706
Torso	23780 g	4270	4270
Upper leg	6450 g	1158	1158
Lower leg	2935 g	526.75	527
Foot	995 g	178.75	179
Upper arm	1875 g	336.5	336
Forearm	1270 g	227.75	228
Hand	470 g	84.5	84

Calculating the center of gravity coordinates, assuming a uniform and symmetrical distribution of the masses, gives the following results:

Entire Body

	$p_1 z_1$
Head	111327.9
Torso	485926
Both upper legs	166173

Both lower legs	32395.125
Both feet	1072.5
Both upper arms	81567.6
Both forearms	42520.925
Both hands	12294.75
Total	933277.8

$$z_0 = \frac{933277.8}{10000} = 93.3 \text{ cm}$$

$$y_0 = 0 \text{ cm.}$$

$$x_0 = \frac{2 \cdot 6.5 \cdot 178.75}{10000} = 0.2 \text{ cm.}$$

If the feet are not counted, it follows, for $\Sigma p_i z_i$ that

Whole body	933277.8
Both feet	-1072.5
Total	932205.3

Weights

Whole body	10000
Both feet	-357.5
Total	9642.5

and therefore,

$$z'_0 = \frac{932205.3}{9642.5} = 96.7 \text{ cm.}$$

$$y'_0 = 0 \text{ cm.}$$

$$x'_0 = 0 \text{ cm.}$$

Whole Leg
(right or left)
$p_i z_i$

Upper leg	83086.5
Lower leg	16197.5625
Foot	536.25
Total	99820.3125

Weights

Upper leg	1158
Lower leg	526.75
Foot	178.75
Total	1863.5

therefore

$$^*z_{3,6} = \frac{99820.3125}{1863.5} = 53.6 \text{ cm.}$$

	$P_i y_i$
Upper leg	9843
Lower leg	4477.375
Foot	1841.125
Total	16161.5

therefore,

$$y_{3,6} = \frac{16161.5}{1863.5} = 8.7 \text{ cm.}$$

$(y'_{3,6} = 8.7$ cm. for the left leg)

$$x_{3,6} = +\ 0.6 \text{ cm.}$$

Lower Leg + Foot
(right or left)

	$P_i z_i$
Lower leg	16197.5625
Foot	536.25
Total	16733.8125

Weights

Lower leg	526.75
Foot	178.75
Total	705.5

therefore

$$z_{4,6} = \frac{16733.8125}{705.5} = 23.7 \text{ cm.}$$

$$y_{4,6} = \pm\ 9.0 \text{ cm.}$$

$$x_{4,6} = +\ 1.6 \text{ cm.}$$

Entire Arm
(right or left)

	$P_i z_i$
Upper arm	40783.8
Forearm	21260.4625
Hand	6147.375
Total	68191.6375

NOTE - See page 41 for explanation of figures below x,y and z.

Weights

Upper arm	336.5
Forearm	227.75
Hand	84.5
Total	648.75

therefore,

$$z_{7,10} = \frac{68191.6375}{648.75} = 105.1 \text{ cm.}$$

$$y_{7,10} = \pm 18 \text{ cm.}$$

$$x_{7,10} = 0 \text{ cm.}$$

Forearm + Hand
(right or left)

Forearm	21260.4625
Hand	6147.375
Total	27407.8375

Weights

Forearm	227.75
Hand	84.5
Total	312.25

therefore,

$$z_{8,10} = \frac{27407.8375}{312.25} = 87.8 \text{ cm.}$$

$$y_{8,10} = \pm 18 \text{ cm.}$$

$$x_{8,10} = 0 \text{ cm.}$$

Head + Torso + Both arms
$p_1 z_1$

Head	111327.9
Torso	485926
Both upper arms	81567.6
Both forearms	42520.925
Both hands	12294.75
Total	733637.175

12541 0.15
1383.?75

Weights

Head	705.5
Torso	4270
Both upper arms	673
Both forearms	455.5
Both hands	169
Total	6273,

therefore,

$$z_{1,3,7,10} = \frac{733637.175}{6273} = 117.0 \text{ cm.}$$

$$y_{1,3,7,10} = 0 \text{ cm.}$$

$$x_{1,3,7,10} = 0 \text{ cm.}$$

Head + Torso

$p_i z_i$ for

Head	111327.9
Torso	485926
Total	597253.9

Weights

Head	705.5
Torso	4270
Total	4975.5,

therefore

$$z_{1,3} = \frac{597253.9}{4975.5} = 120.0 \text{ cm.}$$

$$y_{1,3} = 0 \text{ cm.}$$

$$x_{1,3} = 0 \text{ cm.}$$

One result, among others, of the great agreement between the calculated centers of gravity and those obtained by direct measurement, as indicated on page 40, of the same cadaver, is that our construction of the upright body position does not differ to any important degree from the relationship of the body sections to each other in the horizontal recumbent position.

The findings obtained have been set up in the following table to give a general view of the locations of the centers of gravity, and at the same time offer a comparison of the data obtained from the different cadavers.

The first table gives the results in detail, while the second one gives the ratios for the distances between the centers of gravity of the separate limbs and the center points of the joints, the length of each limb being assumed as 1, and the arithmetical means of these ratios.

(First table on page 57, other tables on pages 62 and 63.)

LOCATIONS OF THE CENTERS OF GRAVITY OF THE ENTIRE BODY
AND OF THE SEPARATE LIMBS FOR THE THREE CADAVERS

	CADAVER II	CADAVER III	CADAVER IV
Total length	170 cm.	166 cm.	168.8 cm.
Total weight	75100 g.	60750 g.	56090 g. (see page 34)
Center of gravity of whole body	Near promontory but below it	4 cm. in front of lower edge of first sacral vertebra, 0.2 cm. to right	2.1 cm. vertically below promontory, 7 cm. in front of upper edge of third sacral vertebra
Center of gravity of head	With neck. At clivus, exactly in median plane, at connection of sphenoid and occipital	Without neck. Fossa Tarini, behind slope of pituitary cavity, in median plane	Without neck. Exactly as in cadaver III, 0.7 cm. behind slope of pituitary cavity, in fossa Tarini, median plane
Center of gravity of torso alone, without head or arms	At front surface of lower edge of first lumbar vertebra, near median plane	Center of first lumbar vertebra, 2 cm. from front surface, 1.4 cm. from rear surface of vertebra, 0.3 cm. to right of median plane	At upper edge of first lumbar vertebra and at front surface of vertebra, 0.5 cm. to right
Entire lower extremity, right	Center of gravity in upper leg, 39 cm. from above, 5 cm. from below directly on bone, center	Center of gravity in upper leg, 37.7 cm. from above, 4.3 cm. from below, directly behind bone, center	Center of gravity in upper leg, 35.5 cm. from above, 4.5 from below, 1 cm. behind bone

Entire lower extremity, left	38.5 cm. from above, 4.8 cm. from below, same as right	38.5 cm. from above, 2.5 cm. from below, in bone itself, 0.7 cm. in front of rear edge and somewhat outward, length 50.7 cm.	In upper leg, 33 cm. from above, 7 cm. from below, otherwise same as right
Upper leg alone, right	Length 44 cm., weight 7650 g., center of gravity 19 cm. from above and 25 cm. from below, 1.5 cm. behind linea aspera, toward right	Length 42 cm., weight 6690 g. center of gravity 19.7 cm. from above and 22.3 cm. from below, 2.2 cm. behind linea aspera	Length 40 cm. (mean), weight 6150 g., center of gravity 17 cm. from above,* 1.5 cm. behind linea aspera *and 23 cm. from below
Upper leg alone, left	Length 43.3 cm., weight 7300 g., center of gravity 19.3 cm. from above and 24 cm. from below	Length 41 cm., weight 6220 g., center of gravity 19.5 cm. from above and 21.5 cm. from below, 1.5 cm. behind linea aspera	Length 40 cm. (mean), weight 6750 g., center of gravity 15.5 cm. from above and 24.5 cm. from below, same as right
Lower leg with foot, right	Center of gravity in lower leg, 24.6 cm. from above and 16.8 cm. from below, directly behind attachment of interosseal ligament to tibia (length 49.2 cm., lower leg plus height of foot)	Center of gravity in lower leg, 26.9 cm. from above and 16.1 cm. from below, exactly at attachment of interosseal ligament to tibia (length 50.7 cm., lower leg plus height of foot)	Center of gravity in lower leg, 25 cm. from above and 16.5 cm. from below, exactly at attachment of interosseal ligament to tibia (length 48 cm., lower leg plus height of foot)
Lower leg with foot, left	25.5 cm. from above and 16.3 cm. from below, otherwise same as right (length 49.3 cm.)	26 cm. from above and 19.9 cm. from below, otherwise exactly the same as right (length 50.6 cm.)	25.5 from above and 16 cm. from below, otherwise same as right (length 48 cm.)
Foot alone, right	Length 28.5 cm., height 7.8 cm., weight 1100 g., center of gravity 11.5 cm. from rear, 17 cm. from	Length 26.5 cm., height 7.7 cm., weight 1060 g., 11.4 cm. from rear and 15.1 cm. from front, at	Length 26.5 cm., height 6.5 cm., weight 990 g., center of gravity 12 cm. from rear and 14.5 cm. from

	front, 6.5 cm. in front of center of tibio-talus joint, in front of surface of navicular, between second and third sphenoid	angle of lower and outer edge of third cuneiform, near attachment to navicular	front, under third cuneiform, near its forward edge, 3 cm. above sole
Foot alone, left	Length 28.3 cm., height 7.5 cm., weight 1160 g., center of gravity 12 cm. from rear, 16.3 cm. from front, otherwise same as right	Length 26.9 cm., height 7.7 cm., weight 1090 g., center of gravity 11.8 cm. from rear and 15.1 cm. from front, at front surface of navicular between third cuneiform and cuboid	Length 26.5 cm., height 6.5 cm., weight 1000 g., center of gravity 12 cm. from rear and 14.5 cm. from front, otherwise same as right
Lower leg alone, right	Length 41.4 cm., weight 3210 g., center of gravity 17.4 cm. from above and 24 cm. from below, 1 cm. behind center of interosseal ligament	Length 43 cm., weight 2870 g., center of gravity 18.7 cm. from above and 24.3 cm. from below, somewhat behind interosseal ligament	Length 41.5 cm. (mean), weight 2970 g., center of gravity 17 cm. from above and 24.5 cm. from below, behind interosseal ligament
Lower leg alone, left	Length 41.8 cm., weight 3320 g., center of gravity 17.4 cm. from above and 24.4 cm. from below, and same as right	Length 42.9 cm., weight 2880 g., center of gravity 17.7 cm. from above and 25.2 cm. from below, 1 cm. behind interosseal ligament	Length 41.5 cm. (mean), weight 2900 g., center of gravity 17.5 cm. from above and 24 cm. from below, and same as right
Entire upper extremity, right	Center of gravity in upper arm, somewhat below cubital axis, 1.5 cm. in front of bone	Center of gravity in upper arm, 28 cm. from above and 2.6 cm. from below, 1.8 cm. in front of humerus	Center of gravity in upper arm, 0.5 cm. below cubital axis, 0.3 cm. in front of bone

Entire upper extremity, left	Same as on right	29.1 cm. from above and 1.1 cm. from below, 1.4 cm. in front of center of humerus	0.5 cm. above cubital axis and 0.5 cm. in front of bone
Upper arm alone, right	Length, to articulation surface of cubital joint, 31.7 cm., weight 2580 g., center of gravity 14.5 cm. from above (center of head of humerus), 17.2 cm. above cubital axis, in medullary cavity of humerus, near rear edge	Length 30.6 cm., weight 1990 g., center of gravity 13.4 cm. from above and 17.2 cm. from below, at rear surface of humerus	Length 32.0 cm., weight 1730 g., center of gravity 16.3 cm. from above and 15.7 cm. from below, in humerus itself, near rear edge
Upper arm alone, left	Length 31.5 cm. (to articulating surface of cubital joint), weight 2560 g., center of gravity 13.3 cm. from above and 18.2 cm. from below, in medullary cavity	Length 30.2 cm., weight 1880 g., center of gravity 13.7 cm. from above and 16.5 cm. from below, at rear surface of humerus	Length 32 cm., weight 2020 g., center of gravity 15.3 cm. from above and 16.7 cm. from below, otherwise same as right
Forearm with hand, right	Center of gravity in forearm, 19 cm. from cubital axis, 10.5 cm. from head of capitulum, 0.5 cm. in front of attachment of interosseal ligament to radius, length, 41.5 cm. from cubital axis to lower edge of flexed fingers	Center of gravity in forearm, 17.8 cm. from above and 8.5 cm. from below, 0.7 cm. in front of attachment of interosseal ligament to radius, length 37.5 cm. from cubital axis to lower edge of flexed fingers	Center of gravity in forearm, 17.7 cm. from above and 9.3 cm. from below, 0.5 cm. from interosseal ligament, nearer radius, length 37.5 cm. from cubital axis to lower edge of flexed fingers
Forearm with hand, left	Center of gravity 19 cm. from above and 10.5 cm. from below, 1 cm. in front of attachnent of interosseal ligament to radius, length 41.5 cm.	17.6 cm. from above and 9.5 cm. from below, 1 cm. in front of interosseal ligament, nearer radius, length 38.0 cm.	17.9 cm. from above and 9.1 cm. from below, 0.5 cm. in front of interosseal ligament, nearer radius, length 37.5 cm.

Forearm alone, right	Length 29.5 cm., weight 1700 g., center of gravity 12.5 cm. from above and 17 cm. from below, 1 cm. in front of interosseal ligament	Length 26.3 cm., weight 1050 g., center of gravity 10.9 cm. from above and 15.4 cm. from below, 1 cm. in front of center of interosseal ligament	Length 27 cm. (mean), weight 1300 g., center of gravity 11.4 cm. from above and 15.6 cm. from below, 1.5 cm. in front of center of interosseal ligament
Forearm alone, left	Length 29.5 cm., weight 1600 g., center of gravity 12.4 cm. from above and 17.1 cm. from below, 1 cm. in front of center of interosseal ligament	Length 27.1 cm., weight 1120 g., center of gravity 11.0 cm. from above and 16.1 cm. from below, 1.1 cm. in front of center of interosseal ligament	Length 27 cm. (mean), weight 1240 g., center of gravity 11.9 cm. from above and 15.1 cm. from below, otherwise same as right
Hand alone, right, with slightly bent fingers	Weight 670 g., center of gravity 5.5 cm. from above (center of head of capitulum), in palm, 1 cm. in front of center of head of third metacarpus	Weight 500 g., center of gravity 5.9 cm. from above, 1 cm. in front of capitulum of second metacarpus, near skin of palm	Weight 490 g., center of gravity 5.5 cm. from above, between third metacarpus and palmar surface, 1 cm. from palmar surface
Hand alone, left, with slightly bent fingers	Weight 620 g., and same as right	Weight 470 g., center of gravity 5.7 cm. from above, 0.8 cm. from center of third metacarpus head in palm	Weight 450 g., center of gravity 5 cm. from above, otherwise same as right

RATIOS OF DISTANCES OF THE CENTERS OF GRAVITY FROM THE JOINT AXES OR THE CENTERS OF THE RESPECTIVE JOINTS

(distances between joint axes or axial centers of a limb reduced to 1)

PARTS OF BODY		CADAVER II		CADAVER III		CADAVER IV	
		from above	from below	from above	from below	from above	from below
Upper arm*	r.	-	-	0.438	0.562	0.509	0.491
	l.	-	-	0.454	0.546	0.478	0.522
Forearm	r.	-	-	0.414	0.586	0.422	0.578
	l.	-	-	0.406	0.594	0.441	0.559
Upper leg	r.	0.432	0.568	0.469	0.531	0.425	0.575
	l.	0.446	0.554	0.476	0.524	0.3875	0.6125
Lower leg	r.	0.420	0.580	0.435	0.565	0.410	0.590
	l.	0.416	0.584	0.413	0.587	0.422	0.578
Foot (length from front to rear reduced to 1)	r.	0.404	0.596	0.430	0.570	0.453	0.547
	l.	0.424	0.576	0.439	0.561	0.453	0.547
Forearm + hand (length from cubital axis to lower edge of flexed fingers reduced to 1)	r.	-	-	0.475	0.525	0.472	0.528
	l.	-	-	0.463	0.537	0.477	0.523
Lower leg + foot (distance from knee axis to sole reduced to 1)	r.	0.500	0.500	0.531	0.469	0.521	0.479
	l.	0.517	0.483	0.514	0.486	0.531	0.469

*) NOTE: The ratios for the upper extremities of cadaver II are not included because in this case the cuts passed through the articulating surface of the cubital joint and not through the axis. These figures, therefore, cannot be compared with the corresponding ones for cadavers III and IV.

Arithmetical Means, for the Three Cadavers, of the Distance Between the Centers of Gravity and the Joint Axes or Joint Centers for each Limb

Parts of the Body		Arithmetical Mean of Ratios			
		One side	Both sides	One side	Both sides
Upper arm	r.	0.4735	0.470	0.5265	0.530
	ℓ.	0.466		0.534	
Forearm	r.	0.418	0.421	0.582	0.579
	ℓ.	0.4235		0.5765	
Upper leg	r.	0.442	0.439	0.558	0.561
	ℓ.	0.4365		0.5635	
Lower leg	r.	0.422	0.4195	0.578	0.5805
	ℓ.	0.417		0.583	
Foot	r.	0.429	0.434	0.571	0.566
	ℓ	0.439		0.561	
Forearm + hand	r.	0.4735	0.472	0.5265	0.528
	ℓ.	0.470		0.530	
Lower leg + foot	r.	0.517	0.519	0.483	0.481
	ℓ	0.521		0.479	

We obtain a distance ratio of 0.609 from above and 0.391 from below, in our position of the body, for the center of gravity of the torso alone, when the distance between the atlanto-occipital joint and the line connecting the centers of the hip joints is made equal to 1. By limiting the distance ratios for the sections of the extremities to two decimal points, we obtain the following table:

Parts of the Body	From above	From below
Upper arm	0.47	0.53
Forearm	0.42	0.58
Upper leg	0.44	0.56
Lower leg	0.42	0.58
Foot	0.43	0.57

These figures do not differ very much from the fractions 4/9 and 5/9, so that we can say, with a certain amount of approximation, that the center of gravity of the separate extremities lies above the center of the latter, and divides the distance between the two joint axes or center points by a ratio of 4 to 5.

The thing to do, if the approximate location of the center of gravity of an extremity must be determined in a hurry, is to measure the distance between its axes, divide this by 9, place the center of gravity at 4/9 of the distance below the proximal joint, and in a straight line with the center of the next joint. The center of gravity lies almost at the edge of the bone in the upper leg and the upper arm, and about 1 cm. from the center of the interosseal ligament, toward the curved side, in the lower leg and forearm.

The direct measurements, made by Harless, of the distances between the centers of gravity of the separate sections of the limbs and the ends of the joints, agree very well with our own, if we take the means of the individual findings. Complete agreement can of course not be expected, first because there are individual differences, and second because the soft material was bound to introduce errors for Harless. We too have observed individual differences in the three cadavers, but they were generally smaller than those Harless showed in his two series of measurements, so that, as could be foreseen, the latter cannot be assumed to be due to individual differences alone.

We reduced the Harless findings, as they are given on pages 93 and 271 to a basis of 1, in order to make possible a direct comparison with our own, and calculated the mean for the two sides of the body, when this had not already been done.

Ratios for Distance of Centers of
Gravity for the Two Harless Cadavers (Graf and Kefer)

	Graf		Kefer		Arithmetical Mean of both	
	from above	from below	from above	from below	from above	from below
Upper arm	0.485	0.515	0.429	0.571	0.457	0.543
Forearm	0.439	0.561	0.410	0.590	0.4245	0.5755
Upper leg	0.468	0.532	0.424	0.576	0.446	0.554
Lower leg	0.360	0.640	0.469	0.531	0.4145	0.5855
Foot	0.460	0.540	0.436	0.564	0.448	0.552

These figures should be compared with our findings on page 63 .

The Harless ratios are also very close, on the average, to the fractions 4/9 and 5/9.

If we now compare our findings with those obtained by Meyer through direct measurement, the main difference proves to be the location of the common center of gravity in the body. According to Meyer, it lies, for the upright position, quite far behind the hip joint, in the second sacral vertebra or above it in the sacral cavity, something that we never found in our own measurements. However, we must clearly point out that we determined the center of gravity with the cadaver in the recumbent position, and only constructed later on an upright position that proved to be a possible one by comparison with a living body, while Meyer and Horner together observed the living body directly. It is perfectly true that the body can be held in such a position that the center of gravity falls behind the hip joint, in the spinal cavity and even farther back. All that is necessary is to bend the body backward, with the shoulders pulled back hard, so that the body forms a convex curve toward the front. The center of gravity will then lie behind the hip joint, and the greater the curve the farther back it will lie. But this Meyer position is in any case no starting position for measurements, as it cannot be standardized. It is also no "military" position, as Meyer calls it, and it still remains to be proved whether it is specially stable or not. We shall return to this position later on.

Furthermore, when Meyer says (Müllers Archiv, 1853, page 25) that the flection planes in the tibio-talus joint form an obtuse angle toward the front that is increased still more by turning the toes outward, and that this causes a locking of the ankle joint that prevents the leg from bending forward against the foot when there is a simultaneous locking of the knee and hip joints, this is true in the case of the first ankle joint only, but not of the foot as such. If the leg is firmly twisted, it is quite possible to bend the spine fairly far back, as seen in Meyer's well-known figure, for in that case the torso is supported by the Bertini ligament. However, the leg is not locked at the ankle joint, in the upright position, even with a very wide spread of the feet, for movement is still possible in the second tarsal joint. This makes a good deal of bending possible even if the knee and hip joints are locked while the feet are turned far out. This is easy to prove on one's own body. If this were not the case, it would be quite impossible to walk on a very steep inclined plane.

DETERMINING THE LOCATION OF THE CENTER OF GRAVITY ON THE LIVING BODY, IN DIFFERENT POSITIONS AND WITH DIFFERENT LOADS

The measuring and weighing that we have done have made it possible to determine the center of gravity on the living body at any one time. This requires that the body be plotted on a graph of space coordinates. Two projections are enough, on one plane each, the most convenient being a sagittal and a frontal plane. The best way to obtain them is to photograph the front view and the profile. The horizontal ground surface should always be used as the XY-plane, and the vertical that bisects the line connecting the centers of the hip joints as the Z-axis. If the X-axis is permitted to run in a sagittal direction, the Y-axis will have to have a frontal direction. This in itself determines a system of right-angle coordinates, when it is also decided that the positive direction of the X-axis shall be forward, of the Y-axis to the right, and of the Z-axis upward.

Two plumb lines were used to help in placing the body exactly, in photographing it, to indicate the proper direction of the axis in all the photographs. Actually, two plumb lines were used, so that the YZ-plane could also be obtained for the normal position. Then we marked the centers of the joints on the photographs right on the spots where they belonged, as indicated by measurements made on the living body in the same position, by projecting them on the body surface. Trying to mark them on the skin before taking the photographs is impractical, because the displacement of the skin when the body moves changes their position. This method could therefore only be used when standing, this time, in the normal position, because the locations of the center points would have to be marked over again on the body surface by projection for every other position of the body. The centers of the joints, such as the elbow and knee, were also found, because we discovered that the center of gravity of a limb always lies very nearly in the line that connected the centers of neighboring joints. After determining the ratio of the distance between a limb's center of gravity and the center of the neighboring joint (see page 62), we were able to calculate, from the coordinates of the centers of the joints, the coordinates of the center of gravity lying between them.

If x_i, y_i, and z_i, and also x_k, y_k, and z_k, are the coordinates of the centers of the joints of a limb, and ε_i and ε_k are the ratios, as given on page 62 , of the distances of a center of gravity from the neighboring joint centers, we then have the following proportions for the center of gravity in calculating the coordinates $x_{i,k}$, $y_{i,k}$, and $z_{i,k}$,

$$(x_i - x_{i,k}) : (x_{i,k} - x_k) = \varepsilon_i : \varepsilon_k ,$$
$$(y_i - y_{i,k}) : (y_{i,k} - y_k) = \varepsilon_i : \varepsilon_k ,$$
$$(z_i - z_{i,k}) : (z_{i,k} - z_k) = \varepsilon_i : \varepsilon_k ,$$

from which it follows that

$$(\varepsilon_i + \varepsilon_k)\, x_{i,k} = {}_k x_i + \varepsilon_i x_k,$$
$$(\varepsilon_i + \varepsilon_k)\, y_{i,k} = {}_k y_i + \varepsilon_i y_k,$$
$$(\varepsilon_i + \varepsilon_k)\, z_{i,k} = {}_k z_i + \varepsilon_i z_k,$$

Now since ratios ε_i and ε_k are calculated for a total length of 1 for a limb,

$$\varepsilon_i + \varepsilon_k = 1,$$

and therefore

$$x_{i,k} = {}_k x_i + {}_i x_k ,$$
$$y_{i,k} = {}_k y_i + {}_i y_k ,$$
$$z_{i,k} = {}_k z_i + {}_i z_k ,$$

The quantities to be used as the values of ratios ε_i and ε_k are the arithmetical means of their values taken from the three center of gravity determinations, calculated to two decimal points, as shown on page 63 . We thus have the table,

Body part	ε_i	ε_k
Torso	0.61	0.39
Upper leg	0.44	0.56
Lower leg	0.42	0.58
Foot	0.43	0.57
Upper arm	0.47	0.53
Forearm	0.42	0.58

in which ε_1 always belongs to the proximal joint. The ratios for the torso are not included in the above table, because the torso does not form as unchangeable a mass as do the limbs. However, the extent to which it does change its form in the body position we are now investigating is not very important, so that in this case we can use the ratios as already obtained for our normal position without any error worth mentioning, as later calculations will show. We found that when the distance of the atlanto-occipital joint from the line connecting the centers of the hip joints was 66 cm., the distance of the center of gravity from the atlanto-occipital joint was 40.2 cm., and from the center of the common axis of the hip joints, 25.8 cm. It is from this that we calculated the above values of ε_1 and ε_k. In order to simplify the calculation we have further assumed that the center of gravity of the torso falls, for our body position, exactly in the median plane.

We have not indicated any ε_1 and ε_k ratios for the hand, as the position of the hands and fingers varies too much. However, it is usually easy to recognize the coordinates of the hand's center of gravity from the pictures fairly well, and measure it directly.

We do not recommend taking the averages between our own and the Harless values in order to obtain the weights of the different parts of the living body, because Harless weighed a body that had been drained of blood. Even the averages from our own three weighings would not be much more exact, for the individuals weighed were far too few. It is better to use the weights of cadavers whose length ratios and whose weights appear to be as nearly as possible those of the living figure. The model we used had about the same length ratios and weighed about as much as cadaver IV. The difference in weight was only 3 kg., so that we had to adjust the different weights of cadaver IV, weighing 55700, to the weight of our living model, which was 58700.

This meant multiplying the weights of cadaver IV by the ratio

$$\frac{58700}{55700} = 1.05386$$

The weights we obtained in this way for our living model, that were used as a basis for all further calculations, were, when corrected to within 10 g.,

Total weight	58700 g.
Head	4140 g.
Torso	25060 g.
One upper leg	6800 g.
One lower leg	3090 g.
One foot	1050 g.
One upper arm	1980 g.
One forearm	1340 g.
One hand	490 g.

Our model was photographed in the following positions, the same man in both, in one-tenth life size, so that millimeters in the photograph correspond to centimeters in real life.

1 - Plate IV - normal position, as described on page 38 ,

2 - Plate V - easy position, as he placed himself without any direction,

3 - Plate VI - military position.

A number of photographs were then taken of positions in which the model carried military equipment, all of them of the nude figure without the pack. They were,

4 - Plates VII and VIII - presenting arms,

5 - Plates IX and X - firing,

6 - Plates XI and XII - aiming rifle with one arm outstretched,

and positions of the nude soldier in complete military equipment with loaded cartridges,

7 - Plates XIII and XIV - standing and shouldering arms,

8 - Plates XV and XVI - firing.

Profile views were sufficient for the first three positions, because of the symmetrical arrangement of the body, but in the others we had to take both a front and a profile view. The left forearm also had to be bent somewhat at the elbow in the first two positions, to make the mark on the hip joint visible, while a slight forward slope of the arm was sufficient in the military position. It is very easy to make a slight correction in the calculations that will bring the figures back to the symmetrical values.

The coordinates of the centers of the joints were determined from the photographs as follows:

First, we determined the centers of the joints on the same live model, standing in the same position, and projected them to the body surface, as may be seen in the plates. Then we laid transparent paper ruled in millimeter squares over the photograph, so that the Z-axis ran in the direction of the plumb line and through the center of the line connecting the two hip centers, and so that the point of origin of the coordinates lay at the horizontal ground level. This made it possible to obtain all three coordinates for positions 4 to 8 from both views, allowing for the foreshortening and lengthening due to perspective. It is true that we only obtained two coordinates, X and Z, for positions 1 to 3, from just one photograph, but since the distances for the symmetrically placed limbs had been measured on the living figure, we also had the Y-coordinates, on assuming complete bilateral symmetry.

POSITION OF THE BODY WITHOUT A LOAD

I. NORMAL POSITION - PLATE IV

Coordinates for the Centers of the Joints

Joints		x	y	z
Atlanto-occipital		0	0	152
Hip	r	0	+ 8.5	87
	l	0	- 8.5	87
Knee	r	0	+ 8.5	47
	l	0	- 8.5	47
Tibio-talus	r	0	+ 8.5	6
	l	0	- 8.5	6
Rear edge of foot	r	- 4	+ 6	4
	l	- 4	- 6	4
Tip of foot	r	+20	+16	1.5
	l	+20	-16	1.5
Humeroscapular	r	0	+18	134
	l	0	-18	134
Cubital	r	0	+18	103
	l	0	-18	103
Wrist	r	0	+18	76
	l	+19	-11	108

From these we calculated the following center of gravity coordinates, with the exception of those for the head and hands, that were obtained direct from the figure. We used the ε_i and ε_k values on page 68, with the aid of formulas

$$x_{i,k} = \varepsilon_k x_i + \varepsilon_i x_k$$

$$y_{i,k} = \varepsilon_k y_i + \varepsilon_i y_k$$

$$z_{i,k} = \varepsilon_k z_i + \varepsilon_i z_k$$

Center of Gravity Coordinates for the Normal Position

(Plate IV)

		x	y	z	
Head		0	0	156	
Torso		0	0	112.4	← 112.8 ?
Upper leg	r	0	+ 8.5	69.4	
	l	0	- 8.5	69.4	
Lower leg	r	0	+ 8.5	29.8	
	l	0	- 8.5	29.8	
Foot	r	+ 6.3	+10.3	3	
	l	+ 6.3	-10.3	3	
Upper arm	r	0	+18	119.4	
	l	0	-18	119.4	
Forearm	r	0	+18	91.7	
	l	+ 8	-15.1	105.1	
Hand	r	0	+18	71	
	l	+23	- 8	108.5	

Using these coordinates and the weights on page 70, we have the following calculation,

Calculation of the Center of Gravity for the x_0, y_0, and z_0 Coordinates of the Common Center of Gravity

Parts of the Body		$p_i x_i$	$p_i y_i$	$p_i z_i$
Head		0	0	645840
Torso		0	0	2816744
Upper leg	r	0	+57800	471920
	l	0	-57800	471920
Lower leg	r	0	+26265	93083
	l	0	-26265	93083
Foot	r	+ 6615	+10815	3150
	l	+ 6615	-10815	3150
Upper arm	r	0	+35640	236412
	l	0	-35640	236412
Forearm	r	0	+24120	122878
	l	+10720	-20234	140934
Hand	r	0	+ 8820	32790
	l	+11270	- 3920	53165
Total		+35220	+ 8786	5421379

therefore

$$x_0 = + \frac{35220}{58700}, \quad y_0 = + \frac{8786}{58700}, \quad z_0 = \frac{5421379}{58700},$$

or,

$$x_0 = + 0.6, \quad y_0 = + 0.1, \quad z_0 = 92.4$$

If the bent forearm is dropped and placed symmetrically with the right arm, the three coordinates for its center of gravity are 0, -18, 91.7, and for the left hand, 0, -18, and 71.

This makes the three coordinates for the common center of gravity for normal position,

$$x_0 = + \frac{13230}{58700} = + 0.2$$

$$y_0 = 0$$

$$z_0 = \frac{5385048}{58700} = 91.7$$

Therefore, as already indicated, raising the left forearm only displaces the common center of gravity 0.7 cm. higher, 0.4 cm. farther forward, and 0.1 cm. farther to the right.

The common center of gravity thus lies, for the absolutely symmetrical and normal position of the body, 4.7 cm. above the line that connects the centers of the hip joints, while our direct measurements showed that it lay 4.5 cm. above this line.

The supporting surface of an upright human body consists, as we have shown, of the area limited by the outer contours of the feet and the outer double tangents that pass through the rear edges and the tips of the feet. Also, the stability of the position will generally be greatest when the gravity line passes through the center of the supporting surface. The point of intersection of the gravity line with the supporting surface is shown in figure 4, which consists of the projection of the feet on the XY-plane. This point of intersection lies much nearer the rear than the front of the supporting surface, and so the stability of the body in this position is greater in resisting a push from the rear than from the front, because the body leans a little more backward than forward, and can thus fall backward more easily. This shows that our normal position is not the most stable one possible.

On the other hand, this position offers great advantages for making measurements and calculations, because it presents a definite point of origin, due to the fact that it makes the centers of gravity of the separate sections of the body all lie one above the other in one vertical plane, and makes them easy to determine on the living body, as shown in plate IV.

Schmidt has shown, in a very remarkable work on the horizontal level of the human skull (Archiv für Anthropologie, Vol. IX, 1876, pp. 25 and foll.), based on the careful measurements of skulls of live adult Germans, that the plane that passes through the outer openings of the ears and the lower orbital rim is not the physiological horizon. This plane rises above the latter, as determined by a large series of observations, at an average angle of 5 1/2° to 5 3/4°. Schmidt considers the physiological horizon (as above, page 35) as "the horizontal plane when holding the head straight; the head is straight when it is held upright and rests on the spine with the least possible muscular effort, and with the gaze horizontal." He also found that soldiers and ex-soldiers were especially likely to hold the head in this way.

He summed up his results by saying that "the plane that connects the attachment of the zygomatic arch above the openings of the ears with the lower rim of the orbital cavity, the plane of the Göttingen Anthropological Congress, is the best horizontal to be found. It approaches best the true physiological horizon and is the most stable of all the suggested normal planes."

However, it is impossible to determine exactly the level position of the head and the horizontal plane of the skull that is based on it, without knowing the head's center of gravity. Human beings have no ligament in the nape of the neck, and the neck muscles cannot be considered as head supports, since nowhere in the body do the muscles act as permanent carriers. If we assume, in correspondence with this, that the head is held in labile equilibrium during life, even if not continuously, then the figure we constructed as shown in plate III holds the head in the straight normal position. If a line is drawn on this figure, connecting the center of the ear opening with the lower orbital rim, it is found to diverge by about 8° from the horizontal, and therefore the present angle approaches Schmidt's angle very closely. However, a series of measurements made on macerated skulls show fairly large individual deviations on this point, and it is better to accept wide limits for this measurement, rather than go into degrees and fractions.

We shall have to leave further study of this ratio to the anthropologists, and be satisfied with presenting this mere indication, since only a great number of very exact measurements can lead to any useful results.

Furthermore, the normal position offers a great advantage when it is necessary to find the center of gravity directly on the living form, without any pictures.

All we need to know, in that case, is the direction of the individual limbs and the degree to which they diverge from the normal in their two joint centers, that is to say, away from the vertical. As we have seen, we have first to calculate the coordinates of the centers of the joints, and then calculate the coordinates of the centers of gravity from them as described. Therefore, what must be shown here is merely how to calculate the coordinates for the centers of the joints after bending the limb, from its coordinates before bending (or in the normal position).

Let us assume, in figure 5, that M_1 and M_k are the center points of the joints of a limb, with M_1 as the proximal joint. If this limb M_1M_k is in the normal position it must be parallel to the Z-axis. Now, if we rotate the limb on the joint center M_k about an angle α, out of the normal position, in a plane that forms angle φ with the XZ-plane, joint center M_1 must assume the position M'_1. Angle α is therefore the angle, which the new position of line $M_kM'_1$ forms with the positive direction of the Z-axis. The coordinates of point M'_1 can therefore be calculated from the coordinates of the point M_k, that remains fixed, as follows.

If the length of the limb, that is to say, the distance between the two neighboring joint centers, is called a, then $M_kAM'_1$ forms a right-angle triangle with the right angle at A, the acute angle α, and the length of the hypothenuse a. Then M_kA = a sine α. However, this distance M_kA is also the hypothenuse of a right-angle triangle ABM_k, having its right angle at B and the acute angle φ. In this right-angle triangle the two perpendiculars M_kB and AB are exactly the distances by which coordinate x'_1 is greater than x_k, and coordinate y'_1 is greater than y_k. Coordinate z'_1 is greater than z_k by distance AM'_1. If we also consider that AM'_1 is a perpendicular in the first right-angle triangle, then

$$x'_1 = x_k + a \text{ sine } \alpha \cos \varphi$$

$$y'_1 = y_k + a \text{ sine } \alpha \text{ sine } \varphi$$

$$z'_1 = z_k + a \cos \alpha$$

II. THE EASY NATURAL POSITION, PLATE V

Coordinates for the Centers of the Joints

Joints		x	y	z
Atlanto-occipital		1.5	0	150.5
Ankle	r	0	+ 8.5	87
	l	0	- 8.5	87
Knee	r	- 1	+ 7	47
	l	- 1	- 7	47
Tibio-talus	r	- 5	+ 4.5	6
	l	- 5	- 4.5	6
Rear edge of foot	r	- 9	+ 2	4
	l	- 9	- 2	4

Tip of foot	r	+ 15	+ 12	1.5
	ℓ	+ 15	- 12	1.5
Humeroscapular	r	- 1	+ 18	133
	ℓ	- 1	- 18	133
Cubital	r	- 3	+ 18	102
	ℓ	- 3	- 18	102
Wrist	r	+ 1	+ 18	75.5
	ℓ	+ 17	- 13	102.5

From this we calculate

Coordinates for the Centers of Gravity of the Easy Position, Plate V

Centers for		x	y	z
Head		- 1	0	154.5
Torso		- 0.6	0	111.8
Upper leg	r	- 0.4	+ 7.8	69.4
	ℓ	- 0.4	- 7.8	69.4
Lower leg	r	- 2.7	+ 6	29.8
	ℓ	- 2.7	- 6	29.8
Foot	r	+ 1.3	+ 6.3	3
	ℓ	+ 1.3	- 6.3	3
Upper arm	r	- 1.9	+18	118.4
	ℓ	- 1.9	-18	118.4
Forearm	r	- 1.3	+18	90.9
	ℓ	+ 5.4	-15.9	102.2
Hand	r	0	+18	70.5
	ℓ	+20.5	-10	102

These coordinates make it possible for us to calculate Coordinates x_0, y_0, z_0 for the Common Center of Gravity

		p_1x_1	p_1y_1	p_1z_1
Head		- 4140	0	639630
Torso		-15036	0	2801708
Upper leg	r	- 2720	+53040	471920
	ℓ	- 2720	-53040	471920
Lower leg	r	- 8343	+18540	92082
	ℓ	- 8343	-18540	92082
Foot	r	+ 1365	+ 6615	3150
	ℓ	+ 1365	- 6615	3150
Upper arm	r	- 3762	+35640	234432
	ℓ	- 3762	-35640	234432
Forearm	r	- 1742	+24120	121806
	ℓ	+ 7236	-21306	136948
Hand	r	0	+ 8820	34545
	ℓ	+10045	- 4900	49980
Total		-30557	+ 6734	5387785

therefore,

$$x_0 = -\frac{30557}{58700}, \quad y_0 = +\frac{6734}{58700}, \quad z_0 = \frac{5387785}{58700}$$

or

$$x_0 = -0.5 \quad y_0 = +0.1 \quad z_0 = 91.8$$

If the bent left forearm is stretched out again, so that it is symmetrical with the right one, the three coordinates for its center of gravity will be,

For the left forearm:	-1.3	-18	90.9
For the left hand:	0	-18	70.5

As a result, the center of gravity coordinates for the entire body in the easy, natural position, are

$$x_0 = -\frac{49580}{58700} = -0.8$$

$$y_0 = 0$$

$$z_0 = \frac{5357208}{58700} = 91.3$$

In this case, raising the left forearm drew the center of gravity 0.3 cm. farther forward, 0.1 cm. to the right, and 0.5 cm. higher up.

The common center of gravity for the easy, natural position therefore lies, when both sides are perfectly symmetrical, 7.3 cm. above the center of the hip joint and 0.8 cm. toward the rear, therefore always above the hip socket.

The projection of the feet on the XY-plane and the point of intersection of the gravity line with the supporting surface are shown in figure 6. In this case the gravity line falls 4 cm. farther forward than in the normal position, and thus makes the present position more stable.

III. THE MILITARY POSITION, PLATE VI

Coordinates of the Joint Centers, obtained from the Photographs

for		x	y	z
Atlanto-occipital		+ 5	0	152
Hip	r	0	+ 8.5	87
	l	0	- 8.5	87

Knee	r	- 4.5	+ 6	47
	ℓ	- 4.5	- 6	47
Tibio-talus	r	- 7	+ 4.5	6
	ℓ	- 7	- 4.5	6
Rear edge of foot	r	-11	+ 2	4
	ℓ	-11	- 2	4
Tip of foot	r	+13	+12	1.5
	ℓ	+13	-12	1.5
Humeroscapular	r	+ 4.5	+18	133
	ℓ	+ 4.5	-18	133
Cubital	r	+ 0.5	+18	102.5
	ℓ	+ 0.5	-18	102.5
Wrist	r	0	+18	76
	ℓ	+ 7.5	-18	76.5

These make it possible to calculate the following:

Center of Gravity Coordinates for the Military Position, Plate VI

		x	y	z
Head		+ 6	0	156
Torso		+ 2	0	112.4
Upper leg	r	- 2	+ 7.4	69.4
	ℓ	- 2	- 7.4	69.4
Lower leg	r	- 5.6	+ 5.4	29.8
	ℓ	- 5.6	- 5.4	29.8
Foot	r	- 0.7	+ 6.3	3
	ℓ	- 0.7	- 6.3	3
Upper arm	r	+ 2.6	+18	118.7
	ℓ	+ 2.6	-18	118.7
Forearm	r	+ 0.3	+18	91.4
	ℓ	+ 3.4	-18	91.6
Hand	r	0	+18	70
	ℓ	+ 7.5	-18	70.5

With the help of these coordinates we are able to obtain

Calculations of the x_0, y_0, and z_0 Coordinates for the Common Center of Gravity

		$p_i x_i$	$p_i y_i$	$p_i z_i$
Head		+ 24840	0	645840
Torso		+ 50120	0	2816744
Upper leg	r	- 13600	+ 50320	471920
	ℓ	- 13600	- 50320	471920
Lower leg	r	- 17304	+ 16686	92082
	ℓ	- 17304	- 16686	92082
Foot	r	- 735	+ 6615	3150
	ℓ	- 735	- 6615	3150
Upper arm	r	+ 5148	+ 35640	235026
	ℓ	+ 5148	- 35640	235026

Forearm	r	+ 402	+ 24120	122476
	l	+ 4556	- 24120	122744
Hand	r	0	+ 8820	34300
	l	+ 3675	- 8820	34545
	Total	+30611	0	5381005

therefore,

$$x_0 = + \frac{30611}{58700}, \quad y_0 = \frac{0}{58700}, \quad z_0 = \frac{5381005}{58700}$$

or

$$x_0 = + 0.5, \quad y_0 = 0, \quad z_0 = 91.7$$

In this case the left forearm was not raised, but drawn a little farther forward together with the upper arm than the military position required, in order to make the points visible. However, the two arms must be kept absolutely symmetrical, and in that case the coordinates are:

For the left forearm	+ 0.3	- 18[illegible]	91.4
For the hand	0	- 18[illegible]	70.0

As a result, the coordinates for the common center of gravity in the military position are,

$$x_0 = + \frac{22782}{58700} = + 0.4,$$

$$y_0 = 0$$

$$z_0 = \frac{5380492}{58700} = 91.7$$

This incorrect position of the left arm therefore hardly altered the common center of gravity at all, for the latter only moved 1 mm. forward, remaining at the same height as before.

The common center of gravity for the military position is thus at the same level as for the normal position. It lies, in this case too, 4.7 cm. above the center of the hip joints, and it is drawn forward only 0.4 cm. within the body, so that it still lies vertically above the hip joints. On the other hand, the model's sloping posture draws the gravity line much farther forward through the supporting surface than is the case in the normal position.

Figure 7 indicates the position of the gravity line in relation to the supporting surface.

The gravity line intersects the supporting surface at a point that lies 7.2 cm. in front of the tibio-talus joint, so that it lies nearer the balls of the feet than the heels. In the normal position the body was more sensitive to a push from the front, but in this case it is more sensitive to a push from the rear, because the body is more inclined to fall forward than backward. The easy position, plate V, lies between these two, as far as stability is concerned, being almost equally stable against a push from the back or the front.

This unsatisfactory placement of the gravity line in the military position causes severe muscular strain, that leadsto great fatigue on being held for any length of time. It is, therefore, a very undesirable position, if anyone must hold it for a long time, and can only be considered useful if it is intended to make it easier to step out at the beginning of a march.

When Meyer states, as has previously been quoted, that the center of gravity for the upright and stable position, which he calls "military," lies in the second sacral vertebra or immediately below it in the sacral cavity, in such a way that the gravity line that emerges from it passes down 5 cm. behind the axis of the hips, the fact is that this does not correspond either to the military position or to the comfortable and stable position, as has just been proved. The Meyer position should therefore be called extremely forced, with far too great a rear bend of the thorax. This makes it highly uncomfortable and makes it require far too much exertion to hold.

POSITIONS OF THE BODY WITH A LOAD

The good will of the military authorities made it possible for us to have a complete military outfit of infantry equipment placed at our disposal. We wish to thank Major General Walde and Major Meissner, of the Prinz

Johann Georg, 8th Infantry Regiment No. 107, for their kind support of our experiment, by both word and deed, for we would otherwise have been unable to carry it out.

The soldier's clothing had, of course, to be completely discarded, so that we could show the body forms and the marks of the joint centers clearly. That is why we have omitted any calculation of the center of gravity of the helmet. Of course, this means an omission of part of the soldier's load, but since the clothing fits all around the body, its omission should not cause any definite displacement of the center of gravity in one direction or another. The heavy boots are partly balanced by the helmet.

The belt and its lock are also disregarded, as being part of the clothing.

Infantry Equipment No. M/87

It would be quite sufficient for our purpose to list the weight of the rifle, bayonet with spade, filled cartridge cases, filled knapsack, bread sack with content and water bottle. However, the entire contents of the knapsack are listed below, so that anyone can figure out in detail what objects increase the weight of the knapsack and what objects can be omitted to decrease it, and thus change the center of gravity.

Weight of the New Infantry Equipment M/87

Articles	kg.	g.
1 knapsack		
1 bag for knapsack		
1 knapsack harness		
1 rear cartridge case		
40 loaded cartridges		
1 shirt		
1 pr. drawers		
1 pr. white trousers		
2 pr. clouts		
2 pr. socks		
3 handkerchiefs		
1 pr. laced boots		
1 pay book		

1 song book 1 clothes brush 1 shoe brush 1 shoe polish brush 1 spreading brush 1 sewing kit 3 cans meat 4 cans gun grease, leather polish and blacking 3 cans vegetables 3 portions hard tack 3 bags (salt, coffee, rice) 1 field cap 1 coat with two straps 1 cooking kit with two straps	12	250
1 body belt with lock and pocket 2 front cartridge cases 60 loaded cartridges 1 bayonet 1 spade and sheath 1 bread sack and content 1 water bottle	7	300
1 rifle 1 rifle strap 1 rear sight cover	4	700

Total weight of equipment 24 kg. 250 g.

The actual weight of the equipment at our disposal was 24 kg. 100 g. The difference of 150 g. was due to the fact that we figured too low a weight for the contents of the filled bread sack. This was because this weight is very variable, and we wanted to make it equal to that of the bayonet and the space, which made calculation easier.

Weights and Centers of Gravity of the Separate Articles of Equipment We Used

1 - Knapsack with rear cartridge case and 40 loaded cartridges		12250 g.
2.- Two front cartridge cases, with 30 loaded cartridges each		3180 g.
3 - Bayonet and spade		1570 g.
4 - Filled bread sack and water bottle		1570 g.
5 - Rifle		4700 g.
Total		23270 g.
6 - Helmet	525 g.	
7 - Belt and lock	305 g.	
Total	830 g.	+ 830 g.
Total		24100 g.

Centers of Gravity

These were found by hanging up the articles three different times.

1. The center of gravity of the packed knapsack (figs. 8 and 9), with the rear cartridge case filled, lay 155 mm. above the lower edge of the knapsack (not the cartridge case), and 185 mm. below the upper edge of the knapsack (not the coat), right in the median plane that divides the knapsack into right and left halves, 70 mm. behind the plane at the edge of the front surface as such, and 100 mm. behind the center of the bulge, where it bulges forward because of the packing.

2. The center of gravity of one front cartridge case (figs. 10 and 11) lay 50 mm. from above and 34 mm. from below, 22 mm. from the center of the front surface.

3. The center of gravity of the bayonet with spade and sheath (fig. 12) lay 265 mm. below the upper edge of the sheath, somewhat forward.

4. The common center of gravity of the filled bread sack and water bottle, that changed according to the amounts contained, was assumed to be symmetrical with the common center of gravity of the spade and bayonet.

5. The center of gravity of the M/71.84 rifle, without muzzle or sighting covers, lay in a transverse plane of the rifle that passed through the octagonal piece, 20 mm. from the rear edge of the base of the sight, 15 mm. below the upper surface and 26 mm. above the lower surface of the stock (see fig. 13).

MILITARY POSITION, PRESENTING ARMS, PLATES VII AND VIII

Coordinates for the Centers of the Joints

Joints		x	y	z
Atlanto-occipital		+ 3	+ 1.5	151
Hip	r	0	+ 8.5	87
	l	0	- 8.5	87
Knee	r	- 5.5	+ 6	47
	l	- 5.5	- 6	47
Tibio-talus	r	-10	+ 4.5	6
	l	-10	- 4.5	6
Rear edge of foot	r	-14	+ 2	4
	l	-14	- 2	4
Tip of foot	r	+10	+12	1.5
	l	+10	-12	1.5
Humeroscapular	r	+ 5.5	+18	133
	l	+ 5.5	-18	131
Cubital	r	+15	+13.5	103
	l	+ 4	-20	101
Wrist	r	+23	+ 7.5	81
	l	+25.5	- 7.5	102

From this we were able to calculate

Coordinates for the Centers of Gravity for the Military Position, by Presenting Arms, Plates VII and VIII

		x	y	z
Head		+ 4	+ 1.5	155
Torso		+ 1.2	+ 0.6	122
Upper leg	r	- 2.4	+ 7.4	69.4
	l	- 2.4	- 7.4	69.4
Lower leg	r	- 7.4	+ 5.4	29.8
	l	- 7.4	- 5.4	29.8
Foot	r	- 3.7	+ 6.3	3
	l	- 3.7	- 6.3	3
Upper arm	r	+10	+15.9	118.9
	l	+ 4.8	-18.9	116.9
Forearm	r	+18.4	+11	93.8
	l	+13	-14.8	101.4
Hand	r	+27	+ 4.5	77
	l	+29.5	- 3.5	103
Rifle		+27	- 0.5	104.5

In this case as in all later ones we determined the center of gravity coordinates for the rifle, the head and the hands directly from the photographs.

Calculation of the x_0, y_0, z_0 Coordinates for the Common Center of Gravity

		p_1x_1	p_1y_1	p_1z_1
Head		+ 16560	+ 6210	645840
Torso		+ 30072	+15036	2806720
Upper leg	r	- 16320	+50320	471920
	l	- 16320	-50320	471920
Lower leg	r	- 22866	+16686	92082
	l	- 22866	-16686	92082
Foot	r	- 3885	+ 6615	3150
	l	- 3885	- 6615	3150
Upper arm	r	+ 19800	+31482	235422
	l	+ 9504	-37422	231462
Forearm	r	+ 24656	+14740	125692
	l	+ 17420	-19832	135876
Hand	r	+ 13230	+ 2205	37730
	l	+ 14455	- 1715	50470
Rifle		+126900	- 2350	491150
	Total	+186455	+ 8354	5894666

therefore

$$x_0 = + \frac{186455}{63409}, \quad y_0 = + \frac{8354}{63400}, \quad z_0 = \frac{5894666}{63400}$$

or

$$x_0 = + 2.9 \qquad y_0 = + 0.1 \qquad z_0 = 93.0$$

Figure 14 shows the supporting surface and the point of intersection S for the gravity line, for the "present arms" position in plates VII and VIII.

In this case the gravity line lies much nearer the tip than the back of the foot, so that this position is very unstable and makes it easy to fall forward. The center of gravity lies 12.9 cm. in front of the tibio-talus joint, 2.9 cm. in front of and 6 cm. above the hip joint. Because of the rifle and the resulting position of the left forearm, the center of gravity is raised 1.3 cm. higher than in the military position in plate VI. In order to decide to what extent the rifle alone has drawn the center of gravity forward and upward in this case, we have to calculate the center of gravity coordinates for the same position of the arm without rifle. Then we obtained

$$x_0 = + \frac{[illegible]}{58700} = + 1.0$$

$$y_0 = + \frac{10704}{58700} = + 0.2$$

$$z_0 = \frac{5403516}{58700} \quad 92.1$$

Thus the rifle alone displaces the center of gravity upward by 0.9 cm., and forward by 1.9 cm., which can be explained by the fact that the center of gravity is already displaced upward and forward by the bent arms, in contrast to the purely military position in plate VI. Furthermore, it must be noted that the body itself is drawn farther forward in the " resent arms" position, because of the weight of the extended rifle, than is the case in the military position without rifle, and that this alone would cause a forward displacement of the center of gravity.

V. POSITION OF SOLDIER FIRING, WITHOUT PACK, PLATES IX AND X

Coordinates for Centers of Joints in Firing Position, without Pack

		x	y	z
Atlanto-occipital		+ 3.5	+ 3.5	147
Hip	r	- 5	+ 7.5	86
	l	+ 5	- 7.5	86
Knee	r	- 9.5	+11	46.5
	l	+ 8	-13	46.5
Tibio-talus	r	-11	+16	6
	l	+ 7	-19	6
Rear edge of foot	r	-14.5	+12.5	4
	l	+ 2	-18.5	4
Tip of foot	r	+ 5	+29.5	1.5
	l	+28	-18.5	1.5
Humeroscapular	r	-16	+15	137
	l	+ 9.5	-14	132.5
Cubital	r	- 2.5	+37	133
	l	+38.5	- 3.5	120
Wrist	r	+16.5	+15.5	141.5
	l	+54	+ 8	141

From this we obtain

Coordinates for the Centers of Gravity for the Firing Position, without Pack, Plates IX and X

		x	y	z
Head		+ 4.5	+ 5	151
Torso		+ 1.4	+ 1.4	109.8
Upper leg	r	- 7	+ 9	68.6
	ℓ	+ 6.3	- 9.9	68.6
Lower leg	r	-10.1	+13.1	29.5
	ℓ	+ 7.6	-15.5	29.5
Foot	r	- 6.1	+19.8	3
	ℓ	+13.2	-18.5	3
Upper arm	r	- 9.7	+25.3	135.1
	ℓ	+23.1	- 9.1	126.6
Forearm	r	+ 5.5	+28	136.6
	ℓ	+45	+ 1.3	128.8
Hand	r	+20	+12.5	142.5
	ℓ	+57.5	+ 8.5	142.5
Rifle		+48	+ 9.5	146

These coordinates then give us

Coordinates x_0, y_0, z_0 for the Common Center of Gravity

		p_1x_1	p_1y_1	p_1z_1
Head		+ 18630	+20700	625140
Torso		+ 35084	+35084	2751588
Upper leg	r	- 47600	+61200	466480
	ℓ	+ 42840	-67320	466480
Lower leg	r	- 31209	+40479	91155
	ℓ	+ 23484	-47895	91155
Foot	r	- 6405	+20790	3150
	ℓ	+ 13860	-19425	3150
Upper arm	r	- 19206	+50094	267498
	ℓ	+ 45738	-18018	250668
Forearm	r	+ 7370	+37520	183044
	ℓ	+ 60300	+ 1742	172592
Hand	r	+ 9800	+ 6125	69825
	ℓ	+ 28175	+ 4165	69825
Rifle		+225600	+44650	686200
	Total	+406461	+169891	6197950

therefore,

$$x_0 = +\frac{406461}{63400}, \quad y_0 = +\frac{169891}{63400}, \quad z_0 = \frac{6197950}{63400}$$

or

$$x_0 = +6.4, \quad y_0 = +2.7, \quad z_0 = 97.8$$

In this case the center of gravity lies 11.8 cm. above the line connecting the hip joint centers, and 7.1 cm. higher than in our normal position. It lies over the promontory, about at the level of the cartilage between the fourth and fifth lumbar vertebrae, about 5.5 cm. forward, about 1 cm. to the left of the median, and 7 cm. in front of the center of the line connecting the hip centers.

Figure 15 shows the relation of the gravity line to the supporting surface.

The figure has much greater supporting surface with this position of the feet, as the figure shows, because the gravity line lies nearer the front edge than the rear edge of the supporting surface and thus makes the position more secure against the recoil of the rifle. The position of the right foot farther back and farther outward is a further advantage, visible in the drawing, for it increases the distance between the gravity line and the rear edge of the supporting surface still more, in the negative direction of the X-axis, which is the direction of the rifle's recoil.

VI. POSITION OF THE NUDE FIGURE WITH RIFLE EXTENDED AND OUTSTRETCHED ARM, PLATES XI AND XII

Coordinates of the Centers of the Joints

		x	y	z
Atlanto-occipital		0	+ 3	150
Hip	r	0	+ 8.5	87
	l	0	- 8.5	87
Knee	r	- 2	+ 9	47
	l	- 2	- 9.5	47
Tibio-talus	r	- 4	+11	6
	l	- 4	-12.5	6
Rear edge of foot	r	- 8.5	+ 9	4
	l	- 8.5	-10.5	4
Tip of foot	r	+16.5	+16	1.5
	l	+16.5	-17.5	1.5
Humeroscapular	r	+ 2.5	+17.5	137
	l	- 5	-16.5	131
Cubital	r	+29.5	+15.5	131
	l	- 5	-18	100
Wrist	r	+53.5	+12.5	140.5
	l	+ 3.5	-21	74

From these we obtain

Coordinates for the Centers of Gravity of the Nude Figure with Rifle Extended, Plates XI and XII

		x	y	z
Head		+ 1	+ 4	154
Torso		0	+ 1.2	111.6
Upper leg	r	- 1.1	+ 8.7	69.4
	ℓ	- 1.1	- 8.9	69.4
Lower leg	r	- 2.8	+ 9.8	29.8
	ℓ	- 2.8	- 10.8	29.8
Foot	r	+ 2.3	+ 12	3
	ℓ	+ 2.3	- 13.5	3
Upper arm	r	+ 15.2	+ 16.6	134.2
	ℓ	- 5	- 17.2	116.4
Forearm	r	+ 39.6	+ 14.2	135
	ℓ	- 1.4	- 19.3	89.1
Hand	r	+ 57	+ 11	140.5
	ℓ	+ 6	- 22	69
Rifle		+ 86.5	+ 7	143.5

These give us

Calculations for the x_0, y_0, and z_0 Coordinates for the Common Center of Gravity

		$p_i x_i$	$p_i y_i$	$p_i z_i$
Head		+ 4140	+ 16560	637560
Trunk		0	+ 30072	2796696
Upper leg	r	- 7480	+ 59160	471920
	ℓ	- 7480	- 60520	471920
Lower leg	r	- 8652	+ 30282	92082
	ℓ	- 8652	- 33372	92082
Foot	r	+ 2415	+ 12600	3150
	ℓ	+ 2415	- 14175	3150
Upper arm	r	+ 30096	+ 32868	265716
	ℓ	- 9900	- 34056	230472
Forearm	r	+ 53064	+ 19028	180900
	ℓ	- 1876	- 25862	119394
Hand	r	+ 27930	+ 5390	68845
	ℓ	+ 2940	- 10780	33810
Rifle		+ 406550	+ 32900	674450
	Total	+ 485510	+ 60095	6152147

therefore

$$x_0 = + \frac{485510}{63400}, \quad y_0 = + \frac{60095}{63400}, \quad z_0 = \frac{6152147}{63400}$$

or

$$x_0 = + 7.7, \quad y_0 = + 0.9, \quad z_0 = 97.0$$

In this case the center of gravity lies 10 cm. above and 7.7 cm. in front of the center of the hip joint, about at the level of the center of the fifth lumbar vertebra. It rises entirely out of the pelvis, for it lies farther forward than the front edge of the pelvis. It is also displaced 0.9 cm. to the right of the median plane.

Figure 16 shows the relationship of the gravity line to the supporting surface.

With the body in this position, the gravity line lies fairly far forward in relation to the supporting surface, but not so far forward as in presenting arms, as might have been expected. This is because the model has drawn his body back for some distance, while in presenting arms he was forced to lean forward. On the other hand, the center of gravity in this case is drawn about 5 cm. farther forward inside the body than in presenting arms, so that it lies almost outside the body, due to the rear bend. It is impossible to hold the military position when loaded with the rifle in this way, and it is just as impossible to assume a military position when heavily loaded from the front, as with the heavy drum.

In order to determine the effect that the advanced rifle has on the center of gravity in this position, it is necessary to find the x_0, y_0, and z_0 coordinates for the center of gravity of this position of the body without the rifle. Thus we find that

$$x_0 = + \frac{78960}{58700} = + 1.3,$$

$$y_0 = + \frac{27195}{58700} = + 0.4,$$

$$z_0 = \frac{5477697}{58700} = 93.3$$

This means that the advanced rifle displaces the center of gravity 6.4 cm. farther forward, 0.5 cm. farther to the right, and 3.7 cm. higher up, for the same body position.

VII. MILITARY POSITION WITH FULL PACK, SHOULDERING ARMS
PLATES XIII AND XIV

Coordinates for the Joint Centers

		x	y	z
Atlanto-occipital		+ 3	+ 1	151
Hip	r	0	+ 8.5	87
	ℓ	0	- 8.5	87
Knee	r	- 5.5	+ 6	47
	ℓ	- 5.5	- 6	47
Tibio-talus	r	- 11	+ 4.5	6
	ℓ	- 11	- 4.5	6
Rear edge of foot	r	- 15	+ 2	4
	ℓ	- 15	- 2	4
Tip of foot	r	+ 9	+ 12	1.5
	ℓ	+ 9	- 12	1.5
Humeroscapular	r	+ 3	+ 19	132
	ℓ	+ 6.5	- 18	131
Cubital	r	- 1	+ 21	101
	ℓ	+ 4	- 27.5	102
Wrist	r	+ 0.5	+ 24	74
	ℓ	+ 26	- 18	91

As a result, we have

Coordinates for the Centers of Gravity in the Military Position with Full Pack, Shouldering Arms, Plates XIII and XIV

		x	y	z
Head		+ 4	+ 1	155
Torso		+ 1.2	+ 0.4	112
Upper leg	r	- 2.4	+ 7.4	69.4
	ℓ	- 2.4	- 7.4	69.4
Lower leg	r	- 7.8	+ 5.4	29.8
	ℓ	- 7.8	- 5.4	29.8
Foot	r	- 4.7	+ 6.3	3
	ℓ	- 4.7	- 6.3	3
Upper arm	r	+ 1.1	+ 19.9	117.4
	ℓ	+ 5.3	- 22.5	117.4
Forearm	r	- 0.4	+ 22.3	89.7
	ℓ	+ 13.2	- 23.5	97.4
Hand	r	+ 0.5	+ 23.5	69
	ℓ	+ 29.5	- 15.5	90

Rifle		+ 6	- 11.5	143
Knapsack with rear cartridge case		- 15.5	+ 0.4	114.5
Front cartridge cases	r	+ 16	+ 12.5	95.5
	ℓ	+ 16	- 12.5	95.5
Bayonet and spade		- 7.5	- 18	75
Bread sack and water bottle		- 7.5	+ 18	75

Using these coordinates and the weights of the articles of equipment, we obtain

Calculation of the x_0, y_0, and z_0 Coordinates for the Common Center of Gravity

		p_1x_1	p_1y_1	p_1z_1
Head		+ 16560	+ 4140	641700
Torso		+ 30072	+ 10024	2806720
Upper leg	r	- 16320	+ 50320	471920
	ℓ	- 16320	- 50320	471920
Lower leg	r	- 24102	+ 16686	92082
	ℓ	- 24102	- 16686	92082
Foot	r	- 4935	+ 6615	3150
	ℓ	- 4935	- 6615	3150
Upper arm	r	+ 2178	+ 39402	232452
	ℓ	+ 10494	- 44550	232452
Forearm	r	- 536	+ 29882	120198
	ℓ	+ 17688	- 31490	130516
Hand	r	+ 245	+ 11515	33810
	ℓ	+ 14455	- 7595	44100
Rifle		+ 28200	- 54050	672100
Knapsack with rear cartridge case		-189875	+ 4900	1402625
Front cartridge cases	r	+ 25440	+ 19875	151845
	ℓ	+ 25440	- 19875	151845
Bayonet and spade		- 11775	- 28260	117750
Bread sack and water bottle		- 11775	+ 28260	117750
	Total	-133903	- 37822	7990167

Therefore,

$$x_0 = -\frac{133903}{81970}, \quad y_0 = -\frac{37822}{81970}, \quad z_0 = \frac{7990167}{81970},$$

or

$$x_0 = -1.6, \quad y_0 = -0.5, \quad z_0 = 97.5$$

In this case the center of gravity lies 10.5 cm. above the center of the hip joint and 1.6 cm. farther back, and thus somewhat above the center of the fifth lumbar vertebra, in the vertebra itself. It does not lie right in the median plane, but 0.5 cm. to the left. This is due to the position of the rifle, resting as it does on the left shoulder.

Fig. 17 shows the relationship of the gravity line to the supporting surface.

In order to determine the effect of the rifle alone and of the knapsack on the center of gravity in this case, we have only to calculate the coordinates for the center of gravity, first omitting those for the rifle alone, and then omitting those for the other articles of equipment.

The coordinates for the common center of gravity without the rifle are

$$x_0 = + \frac{162103}{77270} = - 2.1$$

$$y_0 = + \frac{16228}{77270} = + 0.2$$

$$z_0 = \frac{7318067}{77270} = 94.7$$

In this position the model again has the left arm bent at the elbow, and the hand held forward and somewhat toward the center, as this is necessary for holding the rifle. The symmetrical position of the other hand must be assumed, in order to determine to what extent this displaces the center of gravity forward, to the right, and upward. The coordinates for the common center of gravity, with full equipment, but without the rifle, and with the arms in a symmetrical position, are then,

$$x_0 = - \frac{202853}{77270} = - 2.6$$

$$y_0 = + \frac{19064}{77270} = + 0.2$$

$$z_0 = \frac{7297459}{77270} = 94.4$$

This shows that the position of the left arm has drawn the center of gravity 0.5 cm. farther forward and 0.3 cm. higher up, while the displacement from the median remains the same, or 0.2 cm. The center of gravity did not fall in the median plane this time, as would be expected, because the model leaned the upper part of the body somewhat to the right, as may be seen from plate XIII.

Omitting the centers of gravity for all the equipment, including the rifle, gives the coordinates for the common center of gravity, in the position which the body assumed because of the load, as

$$x_0 = + \frac{442}{58700} = 0$$

$$y_0 = + \frac{11328}{58700} = + 0.2$$

$$z_0 = \frac{5376252}{58700} = 91.6$$

If the left arm is placed symmetrically to the right one, with the body in this position, without pack or rifle, the coordinates for the common center of gravity become

$$x_0 = - \frac{40308}{58700} = - 0.7$$

$$y_0 = + \frac{14164}{58700} = + 0.2$$

$$z_0 = \frac{5355644}{58700} = 91.2$$

The center of gravity coordinates for the pack (without the rifle) must now be omitted in order to prove the effect of the rifle alone in this position of the body with rifle sloping. We then find that

$$x_0 = + \frac{28642}{63400} = + 0.5$$

$$y_0 = - \frac{42722}{63400} = - 0.7$$

$$z_0 = \frac{6048352}{63400} = 95.4$$

The results obtained for the coordinates of the common center of gravity in these individual cases are tabulated below, to make it easy to compare the results for the military position.

	Position in Plate			Symmetrical Position		
	x_0	y_0	z_0	x_0	y_0	z_0
Body with full pack and rifle	- 1.6	- 0.5	97.5			
Body with full pack without rifle	- 2.1	+ 0.2	94.7	- 2.6	+ 0.2	94.4
Body without pack with rifle	+ 0.5	- 0.7	95.4			
Body alone	0	+ 0.2	91.6	- 0.7	+ 0.2	91.2

Thus, as this table shows, the center of gravity is displaced very little by the pack alone, for it lies only 3.2 cm. higher and 1.9 cm. farther back, as seen by comparing the figures for the symmetrical position of the body. Since the radius of the hip joint is generally 2.5 cm., adding the new pack does not even push the gravity line outside the joint. This is what makes the new style pack so excellent.

These calculations can be controlled by using direct measurements, as was done with cadaver no. III. When the body was frozen solid in the recumbent position, the pack (without rifle) was hung on it as well as possible, and the center of gravity was found in the regular way. It proved to be displaced by 2.5 cm. to the rear and 3.5 cm. upward. The upward displacement agrees to within 2 mm. with our calculation, while the rear displacement differs by 6 mm. The difference was caused by the fact that the frozen cadaver could not take on the same position as the live model, whose back bent slightly under the weight of the knapsack.

The table on page 96 also shows that the rifle over the left shoulder causes a displacement, for this body position, of 0.5 cm. forward, 0.9 cm. to the left, and 3.8 cm. upward.

VIII. SOLDIER WITH FULL PACK, IN FIRING POSITION, PLATES XV AND XVI

Coordinates for the Centers of the Joints

		x	y	z
Atlanto-occipital		+ 1	+ 7	147
Hip	r	- 5	+ 7	86
	l	+ 5	- 7	86
Knee	r	- 12	+ 9	46.5
	l	+ 5	- 14	46.5
Tibio-talus	r	- 16	+ 15	6
	l	+ 2.5	- 21	6
Rear edge of foot	r	- 19.5	+ 11	4
	l	- 2.5	- 20.5	4
Tip of foot	r	0	+ 28	1.5
	l	+ 23.5	- 20.5	1.5
Humeroscapular	r	- 19	+ 14.5	135
	l	+ 7.5	- 12.5	133
Cubital	r	- 5	+ 36.5	128
	l	+ 36.5	+ 1.5	120
Wrist	r	+ 16.5	+ 18.5	141
	l	+ 49.5	+ 11.5	141

From this we obtain

Coordinates for the Centers of Gravity with the Soldier with Full Pack, in Firing Position, Plates XV and XVI

		x	y	z
Head		+ 1.5	+ 8.5	151
Torso		+ 0.4	+ 2.7	109.8
Upper leg	r	- 8.1	+ 7.9	68.6
	l	+ 5	- 10.1	68.6
Lower leg	r	- 13.7	+ 11.5	29.5
	l	+ 4	- 16.9	29.5
Foot	r	- 11.1	+ 18.3	3
	l	+ 8.7	- 20.5	3
Upper arm	r	- 12.4	+ 24.8	131.7
	l	+ 21.1	- 7.3	126.9
Forearm	r	+ 4	+ 28.9	133.5
	l	+ 42	+ 4	128.8
Hand	r	+ 19.5	+ 16	142.5
	l	+ 53	+ 12	142.5
Rifle		+ 47.5	+ 13	146
Knapsack with rear cartridge case		- 18	- 8	114
Front cartridge cases	r	- 2.5	+ 15	98
	l	+ 16	- 3.5	98
Bayonet and spade		+ 2	- 21	74
Bread sack and water bottle		- 12	+ 12	74

By using these coordinate values, we obtain

Calculation of the x_0, y_0, and z_0 Coordinates for the Common Center of Gravity

		p_ix_i	p_iy_i	p_iz_i
Head		+ 6210	+ 35190	625140
Torso		+ 10024	+ 67662	2751588
Upper leg	r	- 55080	+ 53720	466480
	l	+ 34000	- 68680	466480
Lower leg	r	- 42333	+ 35535	91155
	l	+ 12360	- 52221	91155
Foot	r	- 11655	+ 19215	3150
	l	+ 9135	- 21525	3150
Upper arm	r	- 24552	+ 49104	260766
	l	+ 41778	- 14454	251262
Forearm	r	+ 5360	+ 38726	178890
	l	+ 56280	+ 5360	172592
Hand	r	+ 9555	+ 7840	69825
	l	+ 25970	+ 5880	69825
Rifle		+223250	+ 61100	686200
Knapsack with rear cartridge case		-220500	- 98000	1396500
Front cartridge cases	r	- 3975	+ 23850	155820
	l	+ 25440	- 5565	155820
Bayonet and spade		+ 3140	- 32970	116180
Bread sack and water bottle (full)		- 18840	+ 18840	116180
	Total	+ 85567	+128607	8128158

therefore,

$$x_0 = +\frac{85567}{81970}, \quad y_0 = +\frac{128607}{81970}, \quad z_0 = \frac{8128158}{81970}$$

or

$$x_0 = +1.0 \quad y_0 = +1.6 \quad z_0 = 99.2$$

In this case the center of gravity lies 13.2 cm. above the center of the hip joint, about at the level of the lower third of the fourth lumbar vertebra, at the forward edge of the vertebra itself. It also lies a little more than 1 cm. to the right of the median plane, which is not easy to determine because of the rotation of the spine.

Figure 18 shows the relationship of the gravity line to the supporting surface.

In order to determine the effect of the pack on the center of gravity in the firing position, coordinates x_0, y_0, and z_0 have been calculated, omitting all the articles belonging to the pack except the rifle. We then find that

$$x_0 = + \frac{300302}{63400} = + 4.7$$

$$y_0 = + \frac{222452}{63400} = + 3.5$$

$$z_0 = \frac{6187658}{63400} = 97.6$$

This shows that the center of gravity for the soldier in the firing position, without a pack, but with the torso in the same position as the one he assumed in firing with the pack, lies at the level of the next to the last cartiledge, 3 cm. in front of it, and about 1 cm. to the right of the median. It thus lies about 5 cm. farther forward than the line connecting the hip joint centers. The center of gravity is therefore drawn backward about 3.5 cm. and upward 1.6 cm. by the presence of the pack.

In the firing position that the model without pack assumed, the center of gravity lay almost at the same level, but about 7 cm. in front of the line connecting the hip joint centers, and thus 2 cm. farther forward than in the position he assumed in the experiment with the pack, after discarding the latter. This shows that the pack made the back curve a little, and this displaced the center of gravity of the torso slightly backward.

Since the center of gravity was displaced considerably to the rear by the pack when in the firing position, the gravity line would be displaced just as far for the same position, and this would cause an undesirable placement of the latter in relation to the supporting surface, as far as stability against a push from the rear was concerned. We have seen, from the firing position without pack, that placing the gravity line nearer the front than the rear edge of the supporting surface increases the stability of the firing position. The model will therefore have to lean forward a little more in firing with a pack to reach the same stability as he did without it. However, comparing figures 18 and 15 shows that the gravity line intersects the supporting surface at the same spot in both cases, and this proves that the model has automatically leaned forward a little more.

The Importance of the Relation of the Location of the Gravity Line to the Supporting Surface

The location of the gravity line in relation to the supporting surface, as shown by the foot projections, indicates only to what extent the stability of the particular position of the loaded or unloaded body that we have investigated is affected. It leads to no conclusions as to the desirable or undesirable location of the center of gravity within the body. All the centers of gravity for the individual positions are therefore included in plate XVII, in order to give a clear picture of the different locations of this center in the body itself. However, the Z-coordinates must be increased by 1 or 2 cm. respectively to make them usable with our drawing, because the centers of the hip joints in the living model were, for most positions, 1 cm. lower, and in the firing position 2 cm. lower, than in our symmetrical figure. Furthermore, we have to consider that the line connecting the centers of the hip joints, in the firing position, lay outside the frontal plane, and that the X and Y coordinates had to be corrected accordingly. The projection of the feet indicates the degree of rotation of the line connecting the hip joint centers. Taking all this into consideration, we obtain the following values for the x'_0, y'_0, and z'_0 centers of gravity coordinates as plotted on our normal figure in plate III.

	x'_0	y'_0	z'_0
1 - Normal position, plate IV	+0.2	0	92.7
2 - Easy position, plate V	-0.8	0	92.3
3 - Military position without pack, plate VI	+0.4	0	92.7

4 - Military position without pack, presenting arms, plates VII and VIII	+2.9	+0.1	94
5 - Firing position, without pack, plates IX and X	+7	-1	99.8
6 - Rifle advanced with right arm extended, plates XI and XII	+7.7	+0.9	98
7 - Military position with full pack, shouldering arms, plates XIII and XIV	-1.6	-0.5	98.5
8 - Firing position with full pack, plates XV and XVI	+1.5	+1	101.2

These locations for the centers of gravity indicate at once in what position the model must stand to place the gravity line as nearly as possible in the center of the supporting surface, what locations of the center of gravity strain the muscles the most because of the bend that must be made, and also what location is closest to our normal one.

The most undesirable location for the center of gravity seems to be the one with the freely extended rifle, and the one in firing position without the pack seems to be almost as bad, while the last one, the firing position with the pack, seems to be the best of all. The reason is that in firing without the pack the center of gravity moves out of the pelvis, while in firing with the pack it lies vertically above the heads of the hip joints. However, two points must be noted in this connection. One is that in firing without a pack the gravity line falls automatically almost in the center of the supporting surface, and the other is that in firing with a pack the torso has to lean forward to make the same thing possible.

As far as the height of the center of gravity is concerned, both firing positions are equally undesirable from every point of view.

Further practical consequences of this can be seen at once from the pictures in plate XVII.

Effect of the Pliability of the Torso on the Location of the Common Center of Gravity

Since the torso is movable because of the pliability of the spine, and therefore cannot be treated as a stiff mass, the location of its center of gravity cannot be expected to be in the same spot for every position of the torso, as it was in the frozen cadaver, that is to say, in the plane determined by the center of the atlanto-occipital joint and the centers of the two hip joints. On bending the torso back strongly, the center of gravity will remain at about the same level, but drawn a little farther back, while on bending still more it moves forward, not backward. All of this must be taken into consideration. Plates IV and VI of the live model in the normal and military positions show, from a study of the figures alone, that the center of gravity of the torso lies in front of the above plane. Furthermore, comparing these plates with the shape of the torso in plate III shows that the center of gravity due to the strong rear bend of the torso, in plates IV and VI, cannot lie as far forward as in plate III. If we assume the center of gravity of the torso, in these two positions, to be in the above plane, as we did in our calculations, it is placed a little to the rear of the true location. If, on the other hand, we assume that the center of gravity in these positions is the one we found in our direct measurements, we are placing it a little in front of the true location. The center of gravity as shown in plates IV and VI lies farther back from the plane through the atlanto-occipital joint and the hips than it does in any other plate. The spot in which we located the center of gravity in the cadaver was in this case 2.5 cm. behind this plane. These indications show that there is a play of 2.5 cm. for the center of gravity of the torso. Displacing it by 1 cm. forward or backward would, however, also mean a simultaneous displacement of $\frac{25060}{58700} = 0.427$ cm. forward or backward for the common center of gravity, for the whole body without a load, but only a displacement of 0.395 cm. or 0.306 cm. for the body loaded with the rifle alone, or with the rifle and the pack. The largest amount of play, or 2.5 cm., for the torso in plates IV and VI, thus amounts to only a play of 1.07 cm. for the common center of gravity itself. For instance, if we assume that it lies in the center of this margin of error, this removes our X-coordinates for the center of gravity only 0.5 cm. from their real value, that is to say, the center of gravity actually falls only 0.5 cm. farther forward than we had calculated. Our assumption that the center of gravity of the torso always lies in the plane between the atlanto-occipital joint and the hip centers therefore leads to no error worth mentioning, in our body position.

The head of the hip joint in the male adult has an average diameter of 5 cm., and so the center of gravity of the torso can move much farther back or forth, up to about 2 cm., without making the frontal plane, that lies between the atlanto-occipital joint and the center of gravity of the torso, move out of the heads of the hip joints.

Effect of the Ground Slope on the Position of the Body

We took a wood board about fifty centimeters wide and one meter long and raised and lowered it slowly with a windlass into different sloping positions, whose angles could be read off easily on a protractor by the use of a plumb line, in order to determine to what extent the ground slope affects the body position, and how the soldier's pack affects it also.

With the board at an angle of 32°, and thus a steep ground surface, the soldier loaded with the regulation pack could still stand up, but he could not step forward any more on the rising plane, and kept his body bent far forward. On the other hand, he could still walk up this sloping plane without the pack, and even if he placed his knapsack on his head and the belt with the front cartridge belt around his neck.

In spite of chalking both boards and socks well (he had taken off his shoes), his feet finally began to slip, which complicated the experiment. So we made a new series of experiments, in which sliding was completely eliminated, and only the slope counted, by nailing a small board across the large one, with the heels of the boots resting against it. Three young soldiers took part in this experiment.

Soldier A could still stand, with his body bent far forward, and with the pack, on a plane at an angle of 41 1/2°, without pack up to 47°, and when he carried the knapsack on his chest up to 50°.

Soldier B could stand with pack up to 41°, without pack up to 47-49°, and with the pack on his chest up to 52°.

Soldier C could stand with pack up to 42°, without pack up to 48°, and with the pack on his chest up to 48°.

We do not intend to carry this investigation any farther. It will rather be the task of the military authorities to study, on the basis of broad empiric investigations, to what extent the new pack hinders the soldier in his movements on uneven ground.

The load pressing on the body when standing on uneven ground is bound to alter the position of the lower part of the back, and make the body bend forward in order not to fall backward, just as the displacement of the center of gravity toward the rear by the pack causes a forward bend of the body that draws the center of gravity above the supporting surface and thus makes the upright position more secure. The steeper the slope and the farther back the center of gravity, the greater the bend of the body will be. However, this bend can only take place at the ankles, the hips, and the spine. People with stiff joints, that is to say, older people as a whole, will therefore find it harder to climb mountains than young people with supple joints. It is evident that shifting the pack to the upper portion of the back will make it much easier to bend forward and correct this displacement of the center of gravity, because the number of vertebrae at this spot increase the pliability.

It is thus not mere chance, or a local fashion, that makes the north Germans carry their packs in the center of the back, while the south Germans and those down in the Alps and into Italy carry every load on the head, as high up on the body as is possible. Even at the Main river we see how market women carry their baskets on their heads, while in the Berner Oberland cheeses weighing a hundred pounds are raised as high as possible on the back, and in Capri the young women carry heavy trunks and building stones on their heads up the steep road to Anacapri.

Walking on steep ground with a heavy load is in any case difficult, and requires much more careful balancing than when marching on level ground. It is especially easy to fall backward if the sacrum is loaded, or in striking a steep slope when jumping over a ditch, or when moving over very uneven ground.

Our purpose in these last remarks has been to point out a further practical consequence of our investigation on the location of the center of gravity, without wishing to venture into a field in which we are not competent.

Translated by -
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October 16, 1944

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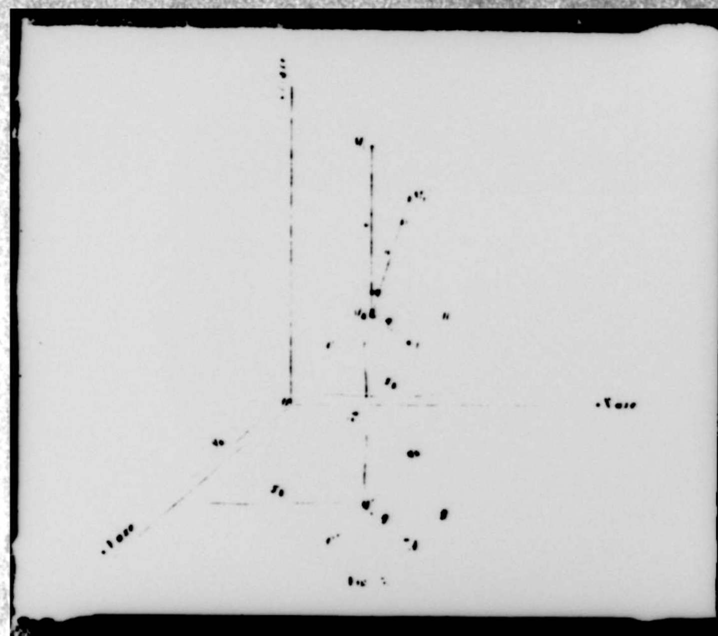

Figure 5

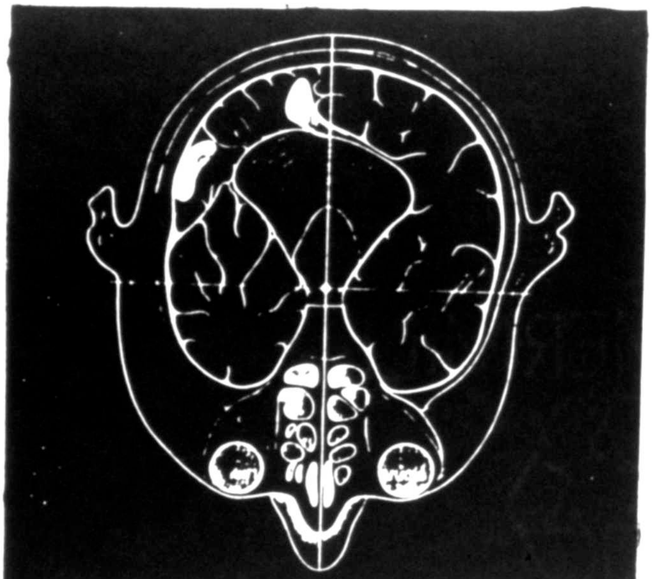

Fig. 1.

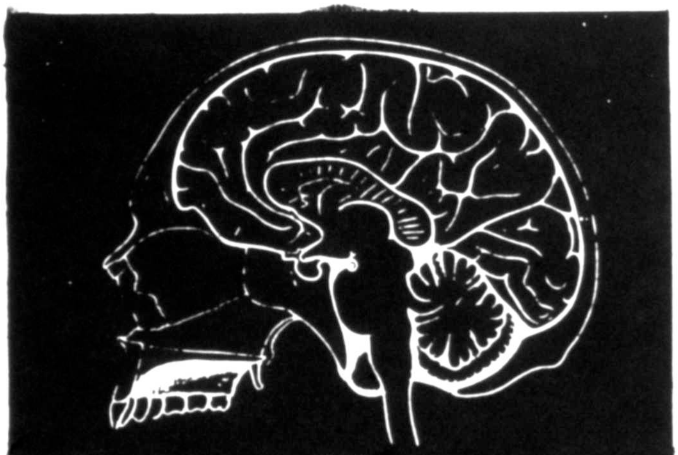

Fig. 2

Schwerpunktslage im Kopfe. ½ der naturlichen Grosse. ⊙ Schwerpunkt.

Figs. 1 and 2 - Center of gravity of the head - 1/2 life size

° Center of gravity

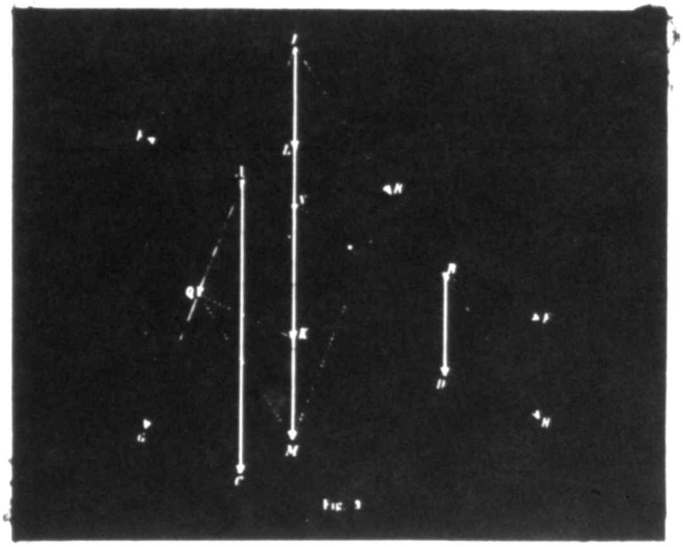

Figure 3

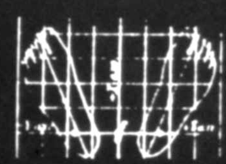

Fig. 4. [illegible] der naturlichen Grosse

Die Unterstutzungsflache und der Schnittpunkt S der Schwerlinie mit derselben fur die Normalstellung auf Taf. IV. Der Anfangspunkt O der Coordinaten ist die Projection des Centrums der Verbindungslinie der Huftgelenkmittelpunkte. ⊙ Mittelpunkt des Tibio-Talusgelenkes. + Schwerpunkt des Fusses.

Fig: 4 - Supporting surface and point of intersection S of the gravity line with it, for the normal position in Plate IV.

Point of origin O on the graph is the projection of the center of the line connecting the hip joint centers:

⊙ - Center of tibio-talus joint + - Center of gravity of the foot

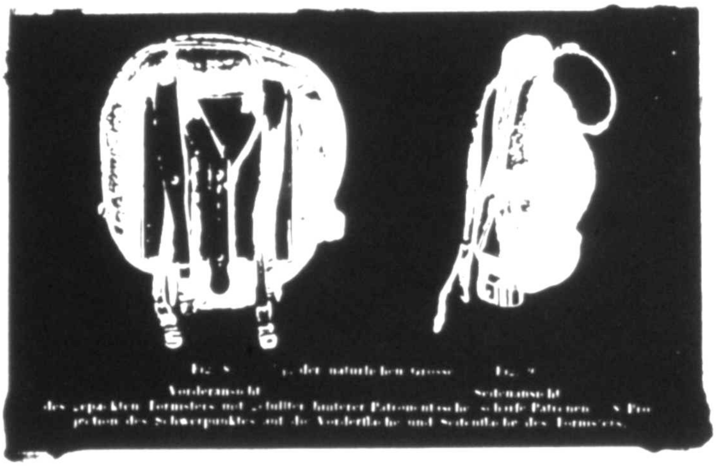

Fig. 8 - Front view, and Fig. 9 - Side view of packed knapsack with filled rear cartridge case and loaded cartridges. S - projection of the center of gravity on the front surface and on the side surface of the knapsack

Fig. 10 - Rear view, and Fig. 11 - Side view of the front cartridge case filled with loaded cartridges; S - projection of the center of gravity on the rear and side surfaces of the case

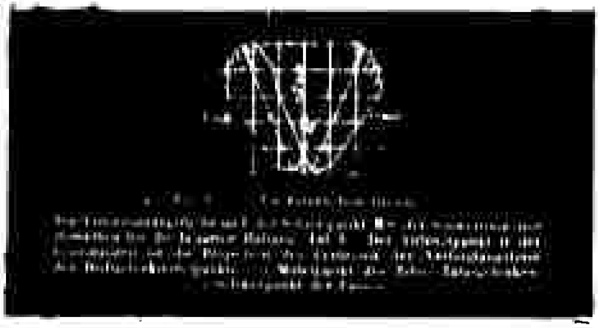

Fig. 6 - Supporting surface and point of intersection S of the gravity line with it, for the easy position, Plate V.

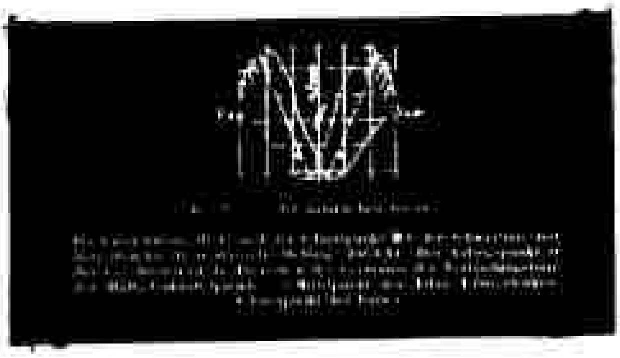

Fig. 7 - Supporting surface and point of intersection S of the gravity line with it, for the military position, Plate VI.

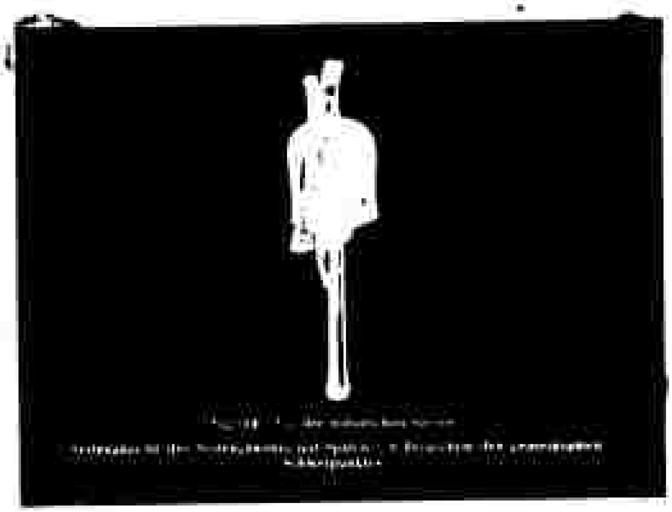

Fig. 12 - Side view of bayonet and spade.
S - Projection of the common center of gravity

Fig. 13 - Side view of rifle M/71.84. S - projection of the center of gravity on the upper surface of the stock

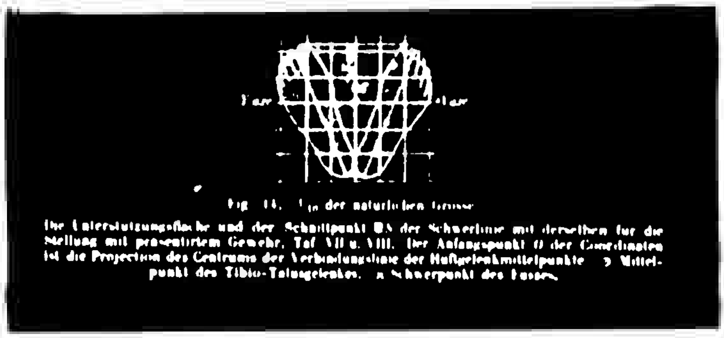

Fig. 14 - For soldier presenting arms, Plates VII and VIII.

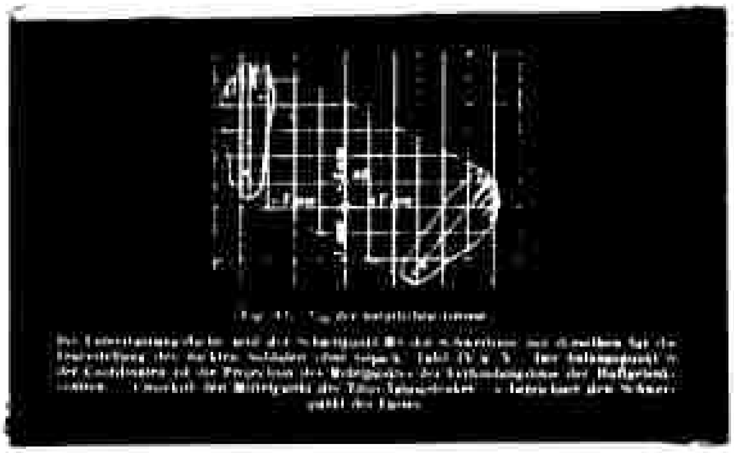

Fig. 15 - For soldier without pack, firing position, Plates IX and X

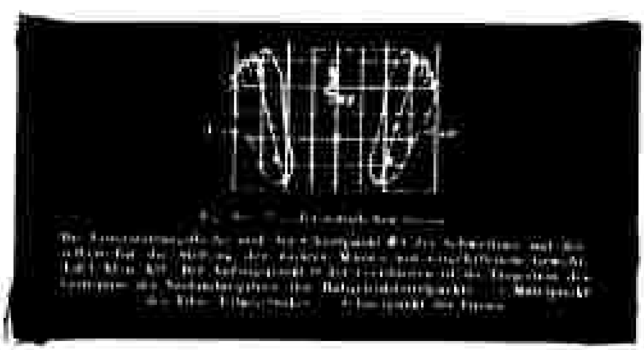

Fig. 16 - For soldier with extended rifle - Plates XI and XII

SUPPORTING SURFACE AND POINT OF INTERSECTION OF THE GRAVITY LINE WITH IT

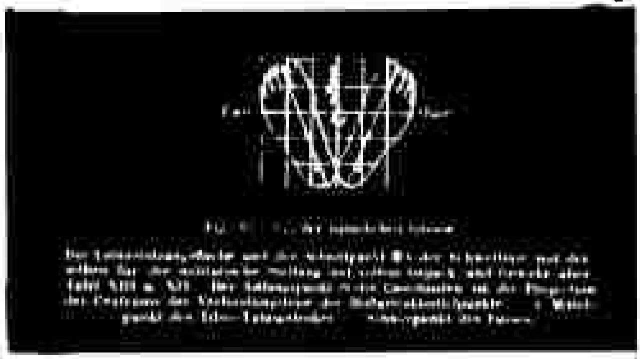

Fig. 17 - For the military position with full pack, shouldering arms, Plates XIII and XIV

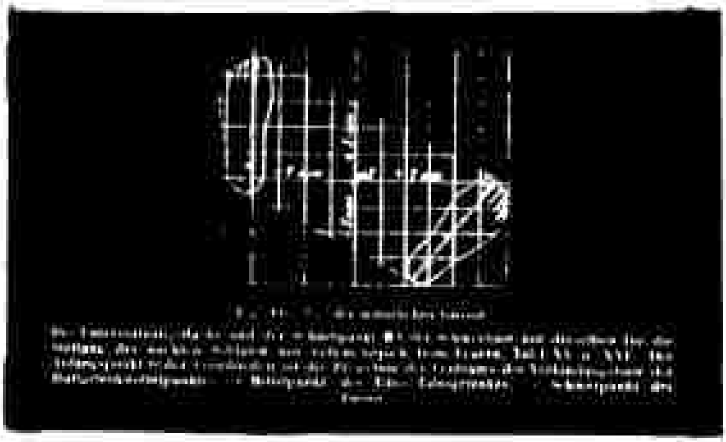

Fig. 17 - For the soldier with full pack, in firing position, Plates XV and XVI

SUPPORTING SURFACE AND POINT OF INTERSECTION OF THE GRAVITY LINE WITH IT

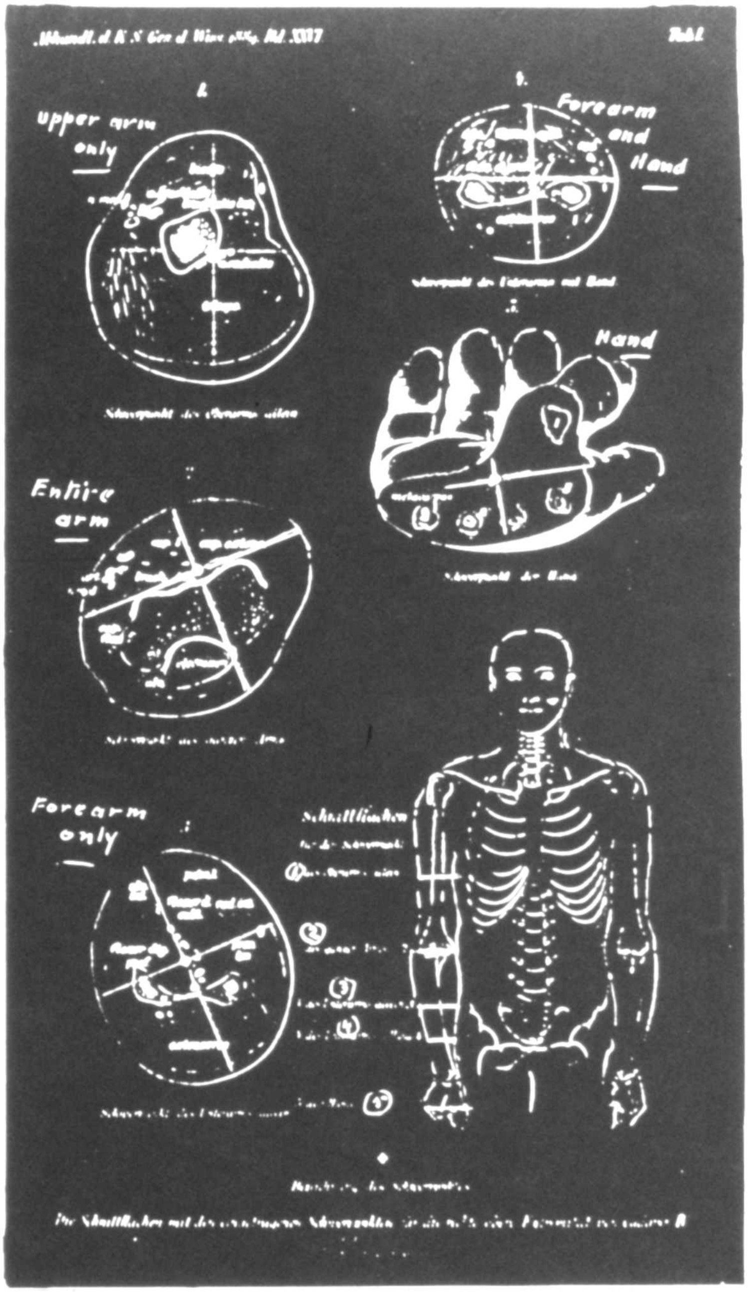

PLATE I

Cutting surfaces and centers of gravity drawn in, for the right upper extremity of cadaver IV

Cutting surfaces for 1) upper arm only; 2) entire arm; 3) forearm only; 4) forearm with hand; 5) hand

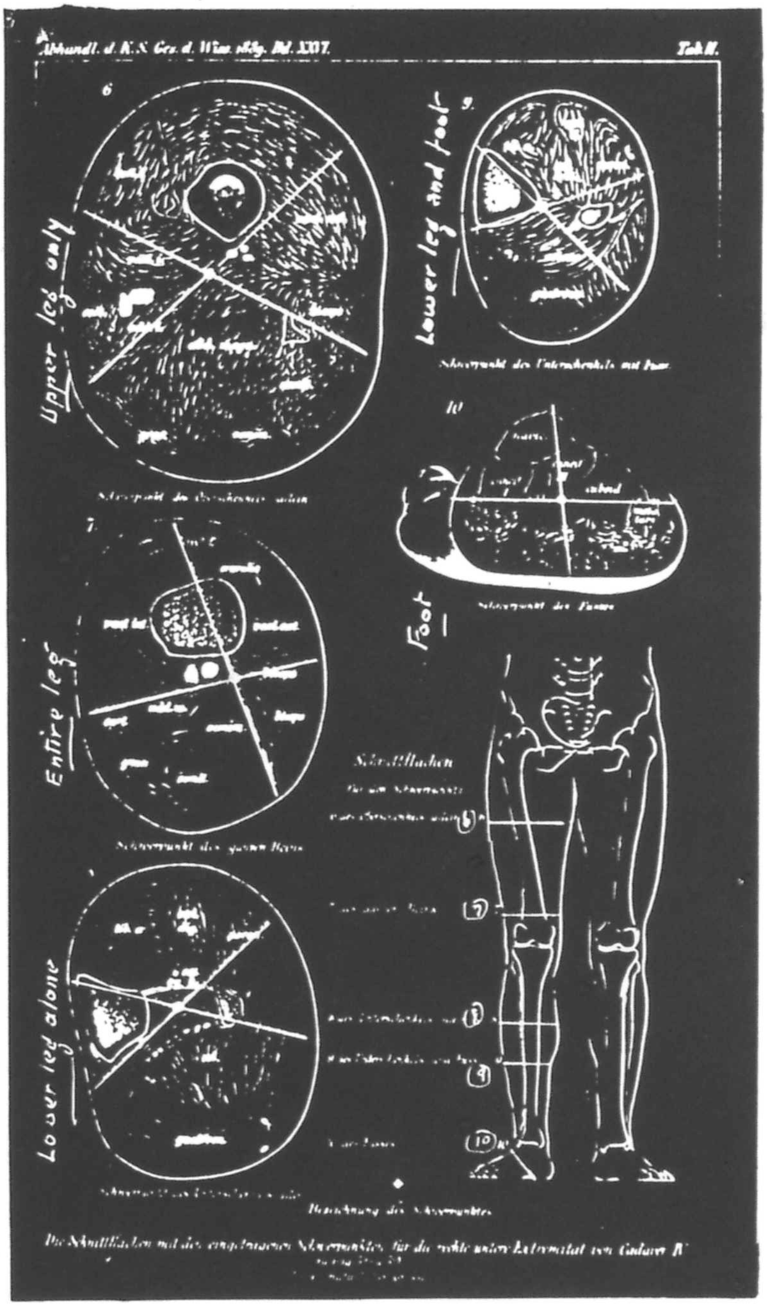

PLATE II

Cutting surfaces and centers of gravity drawn in, for the right lower extremity of cadaver IV

Cutting surfaces for 6) upper leg alone; 7) entire leg; 8) lower leg only; 9) lower leg with foot; 10) foot only

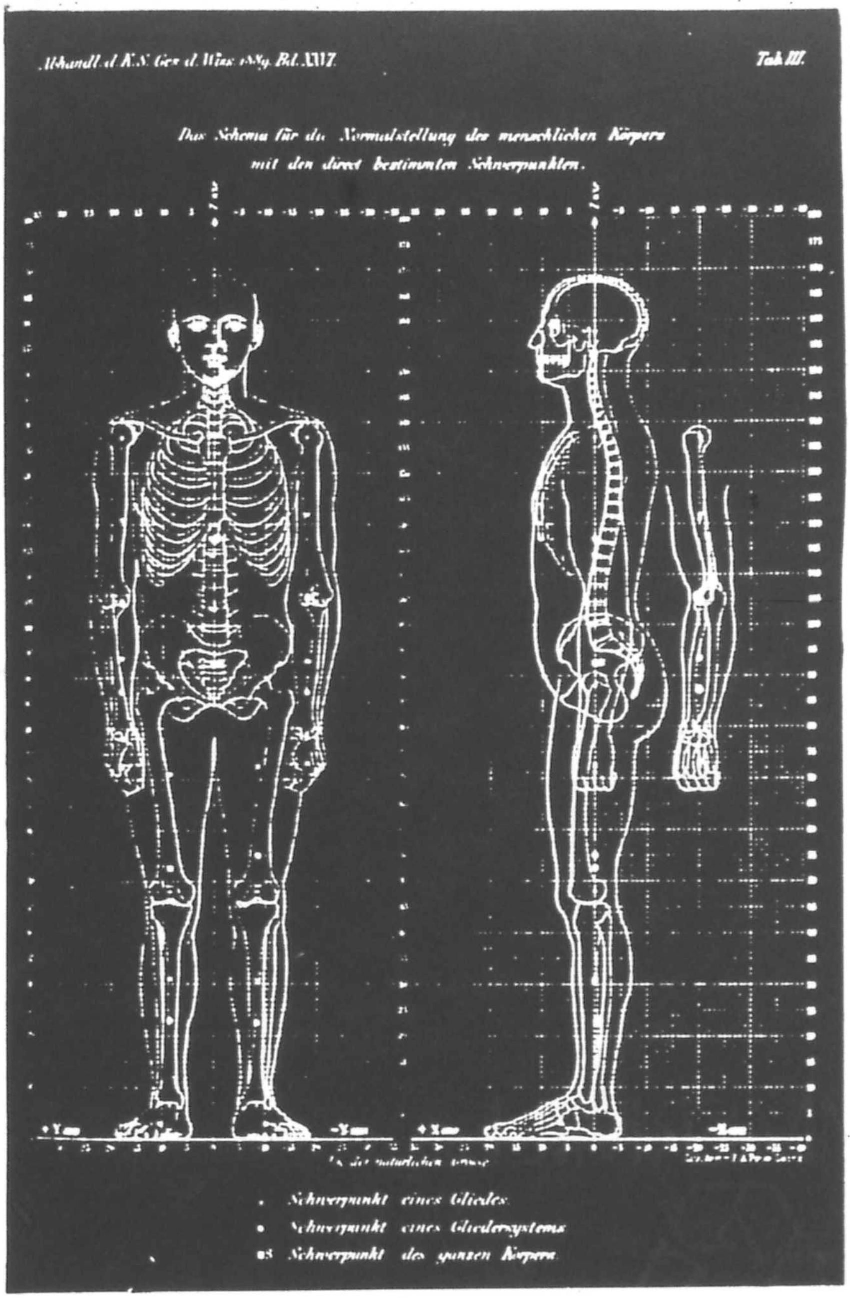

PLATE III

Diagram of the normal position, with centers of gravity measured directly

- • - Center of gravity of a limb
- ▫ - Center of gravity of a group of limbs
- ⊡S - Center of gravity of entire body

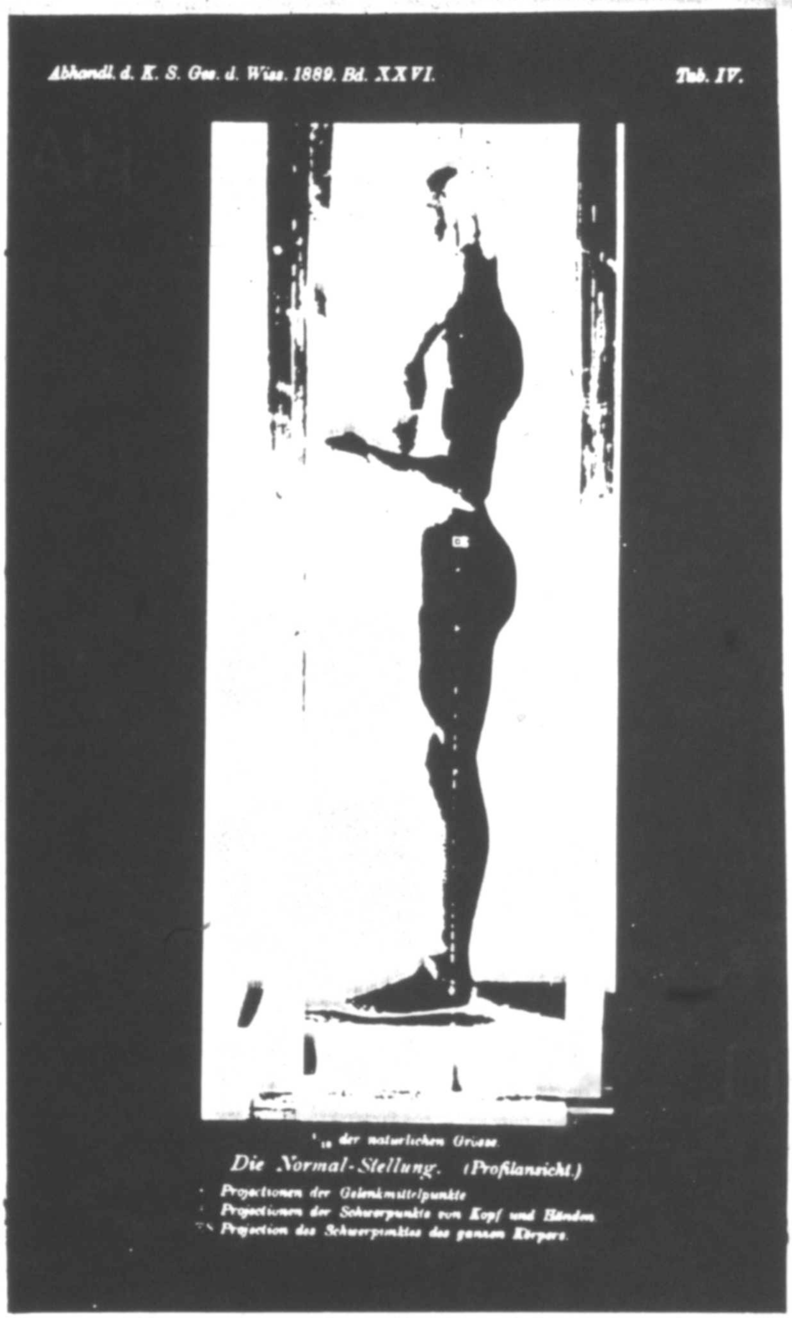

PLATE IV

Normal position - profile

Projection of centers of the joints
Projections of centers of gravity of head and hands
Projection of center of gravity of entire body

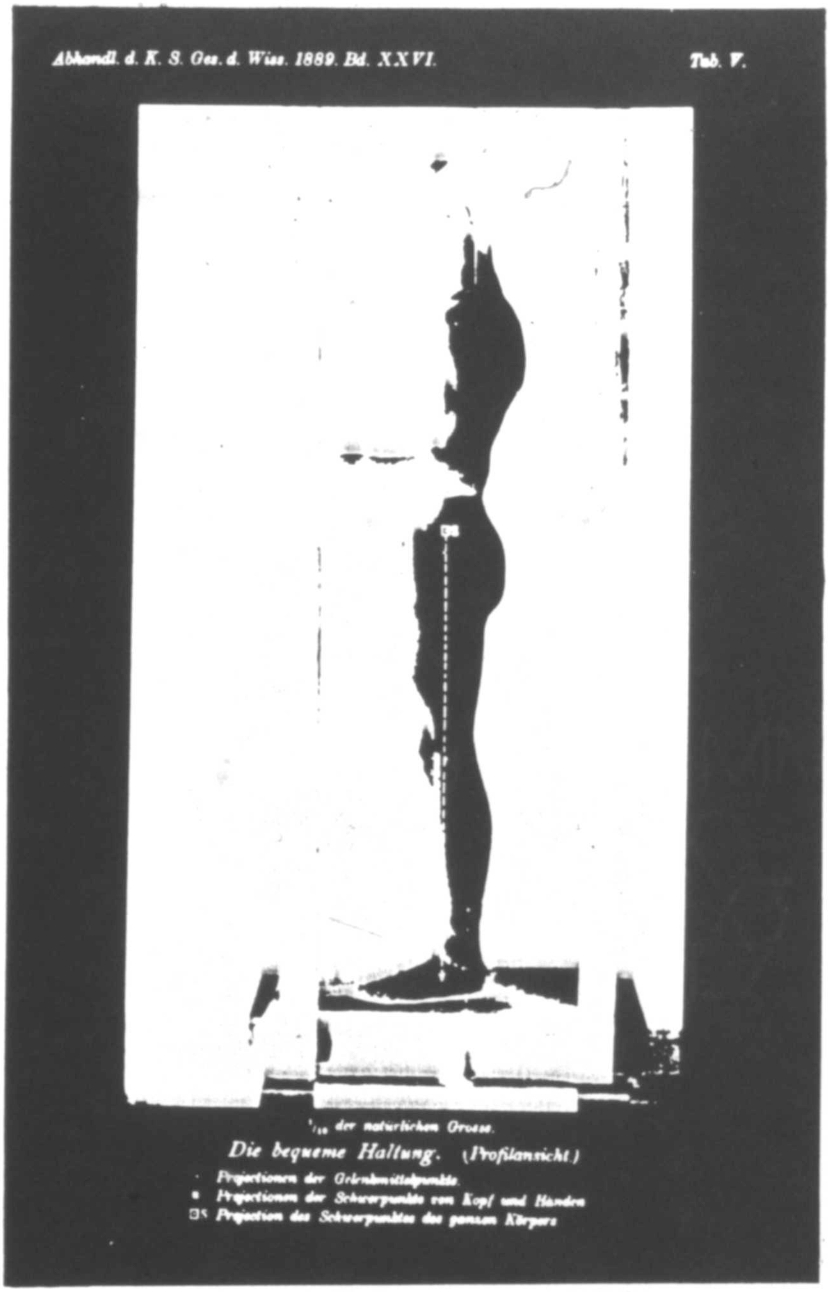

PLATE V

Easy position - profile

Projection of the centers of the joints
Projection of centers of gravity of head and hands
Projection of center of gravity of entire body

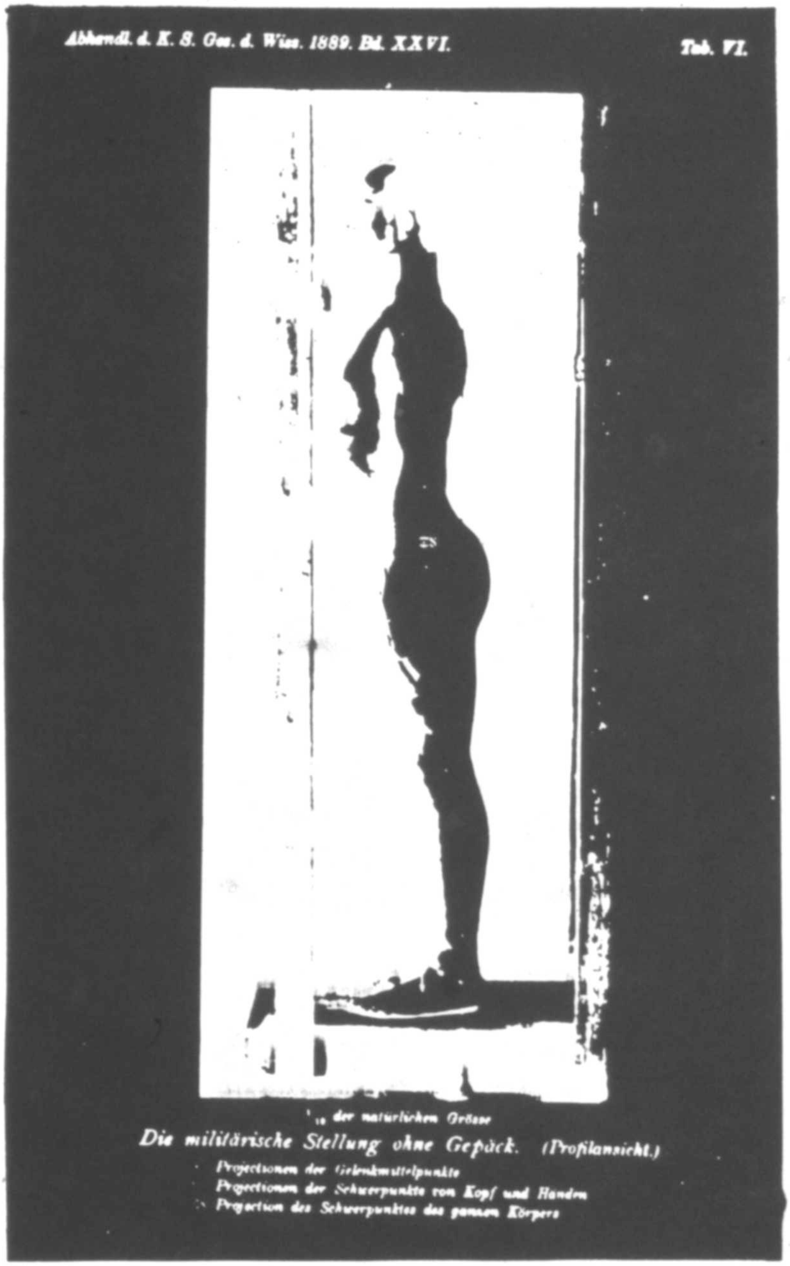

PLATE VI

Military position without pack

Projection of the centers of the joints
Projection of centers of gravity of head and hands
Projection of center of gravity of entire body

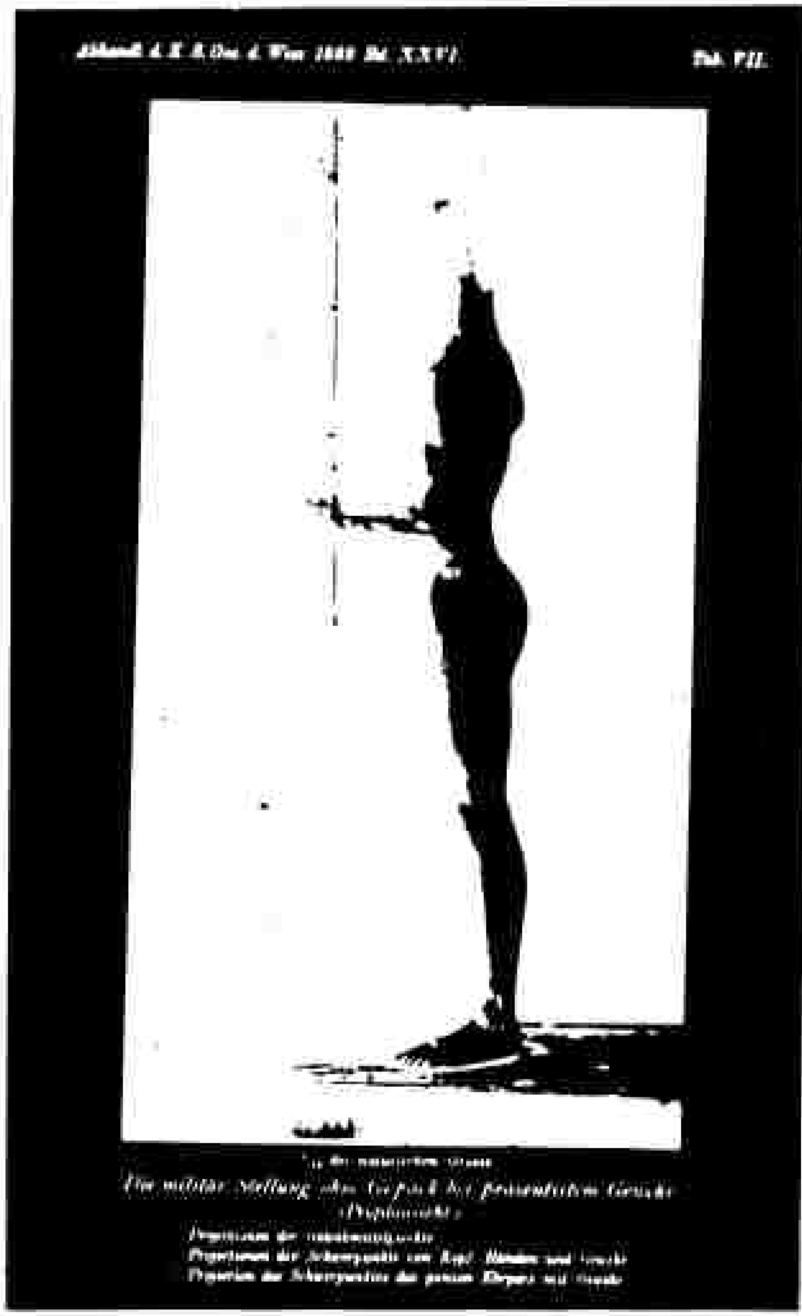

PLATE VII

Military position without pack, presenting arms (profile)

Projection of the centers of the joints
Projection of centers of gravity of head and hands
Projection of center of gravity of entire body

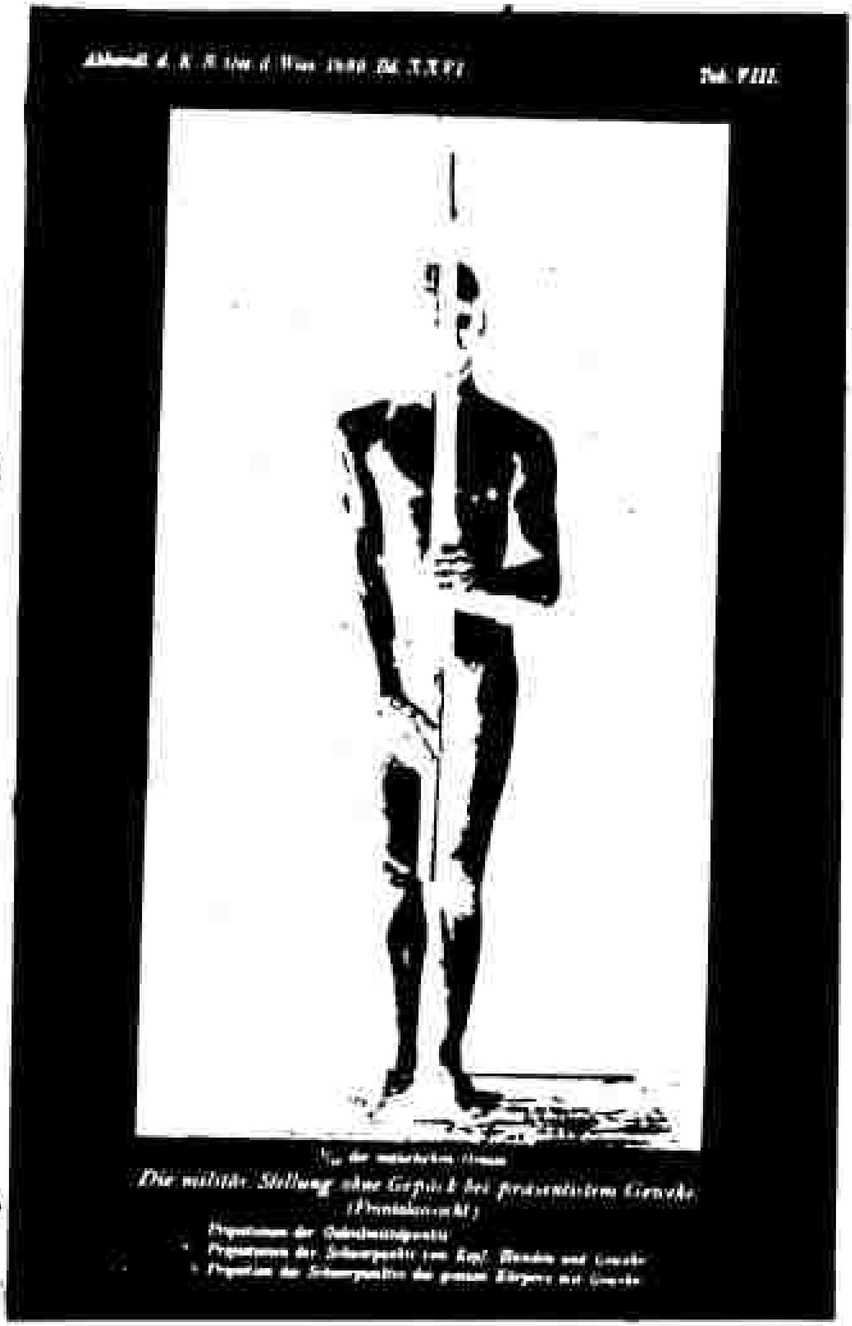

PLATE VIII

Military position without pack, presenting arms (front view)

Projection of centers of joints
Projection of centers of gravity of head and hands
Projection of center of gravity of entire body

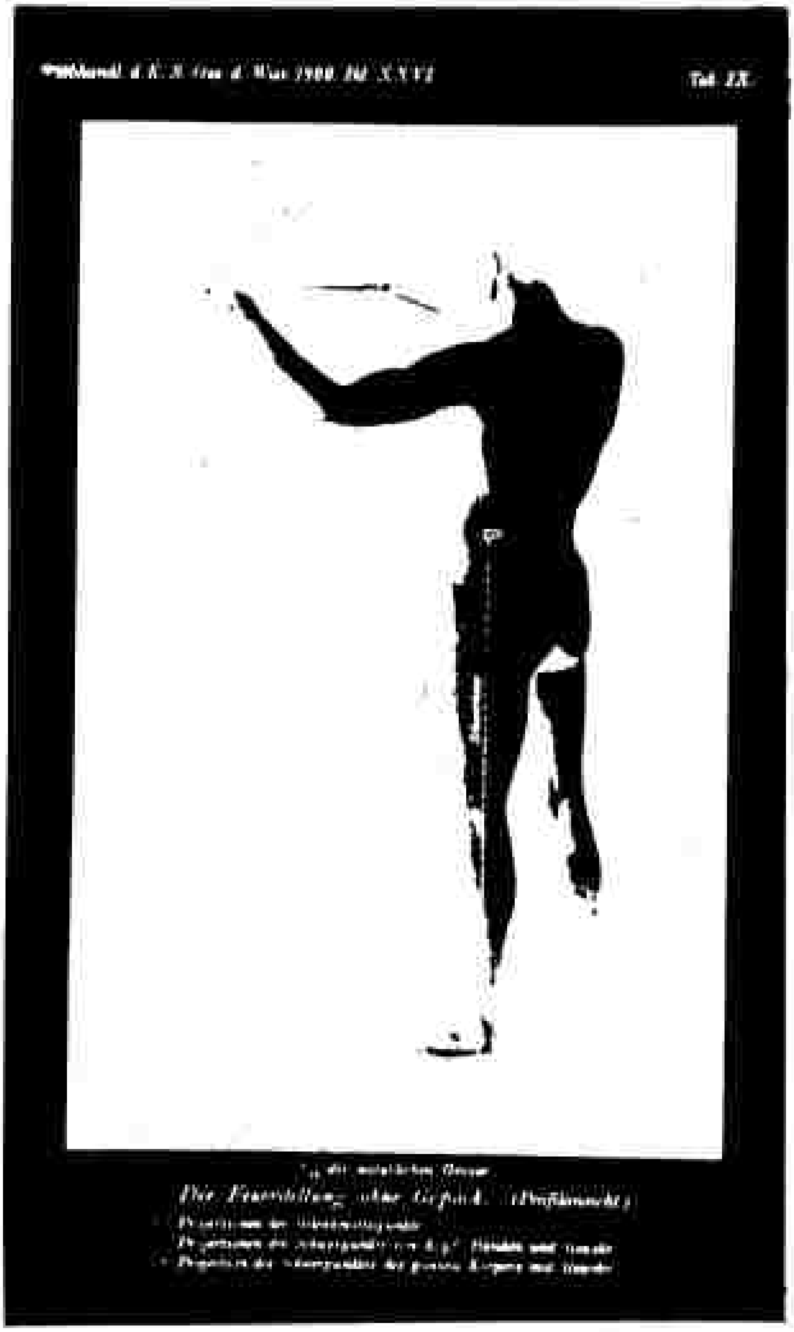

PLATE IX

Firing position without pack (profile)

Projection of centers of joints
Projection of centers of gravity of head and hands
Projection of centbr of gravity of entire body

PLATE X

Firing position without pack (front view)

Projection of centers of joints
Projection of centers of gravity of head and hands
Projection of center of gravity of entire body

PLATE XI

Position with extended rifle and outstretched right arm (profile)

Projection of centers of the joints
Projection of centers of gravity of head and hands
Projection of center of gravity of entire body

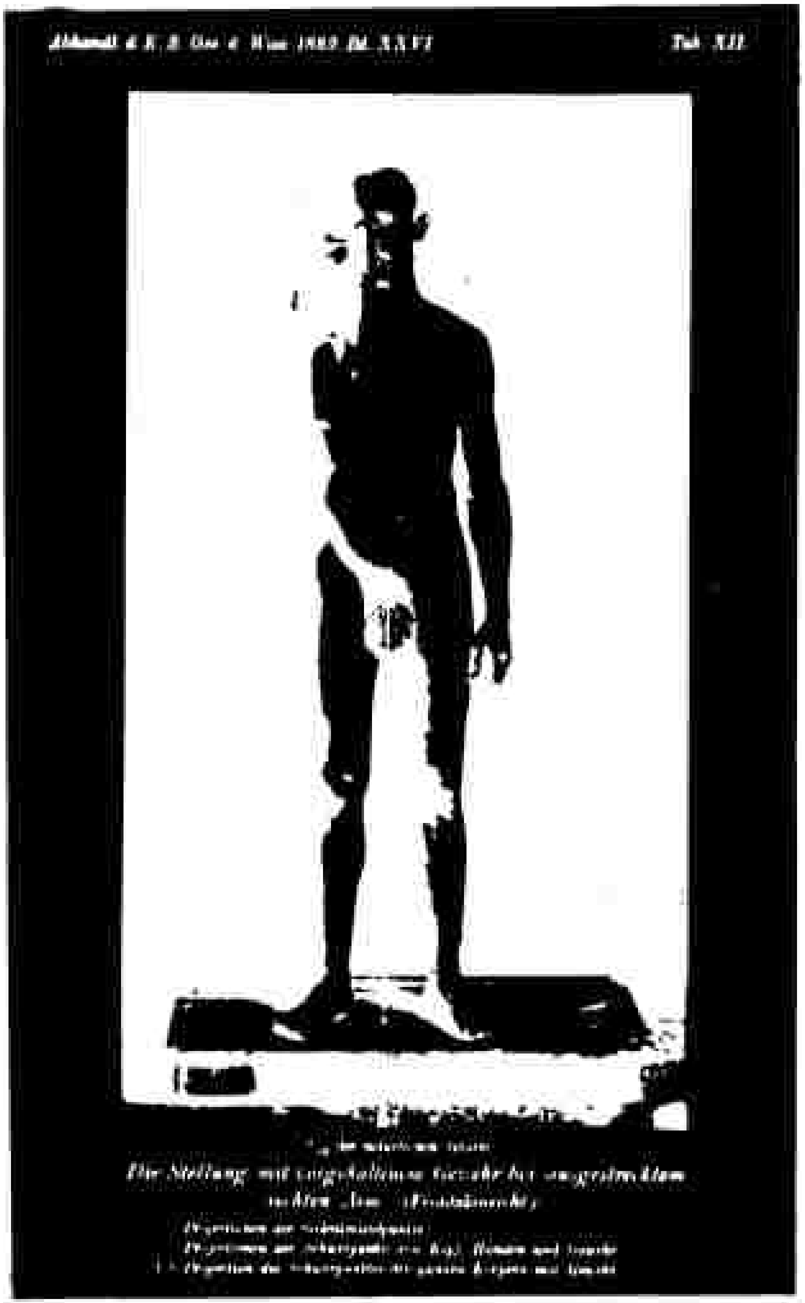

PLATE XII

Position with extended rifle and outstretched right arm (front view)

Projections of centers of the joints
Projection of centers of gravity of head and hands
Projection of center of gravity of entire body

PLATE XIII

Military position with full pack and shouldering arms (profile)

Projection of the centers of the joints
Projection of centers of gravity of head and hands
Projection of center of gravity of entire body

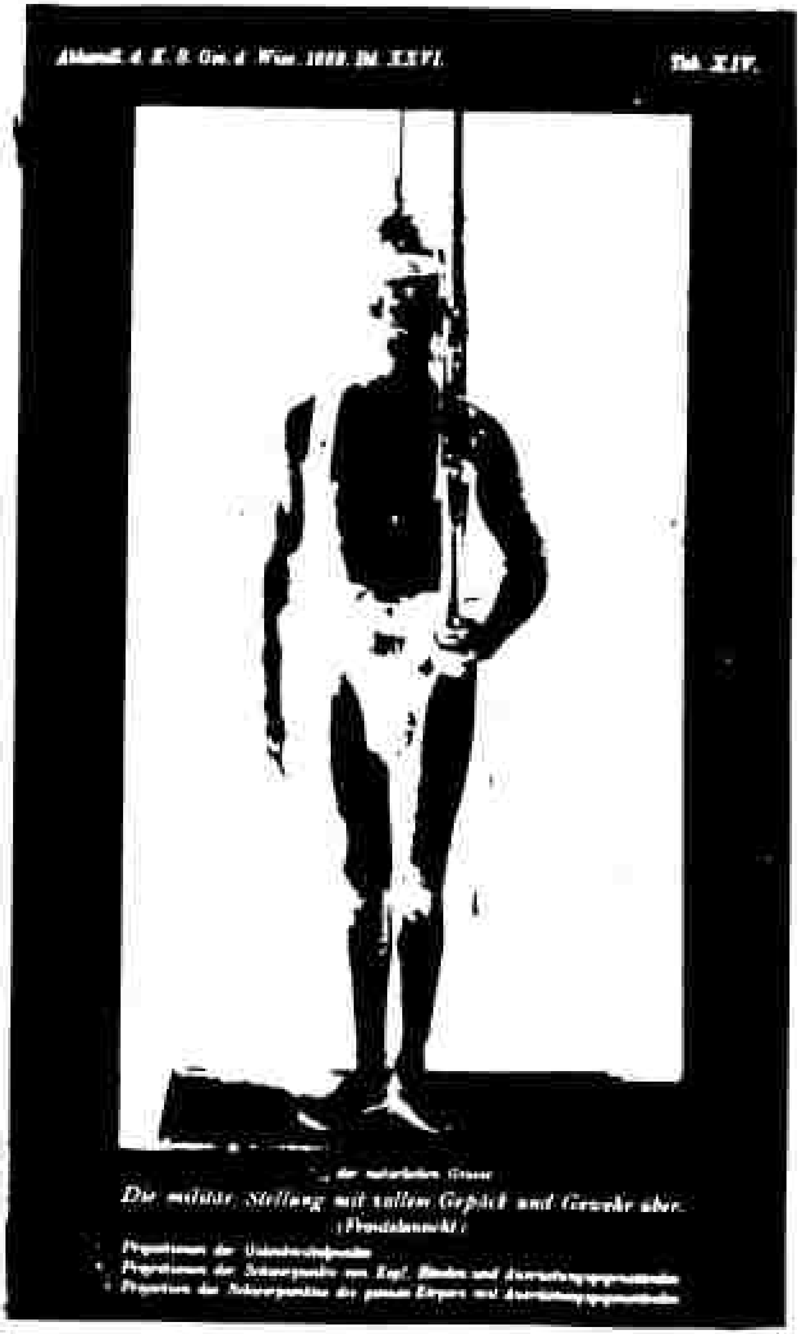

PLATE XIV

Military position with full pack and shouldering arms (front view)

Projection of centers of the joints
Projection of centers of gravity of head and hands
Projection of center of gravity of entire body

PLATE XVI

Firing position with full pack (front view)

Projection of centers of the joints
Projection of centers of gravity of head and hands
Projection of center of gravity of entire body

PLATE XV

Firing position with full pack (profile)

Projection of centers of the joints
Projection of centers of gravity of head and hands
Projection of center of gravity of entire body

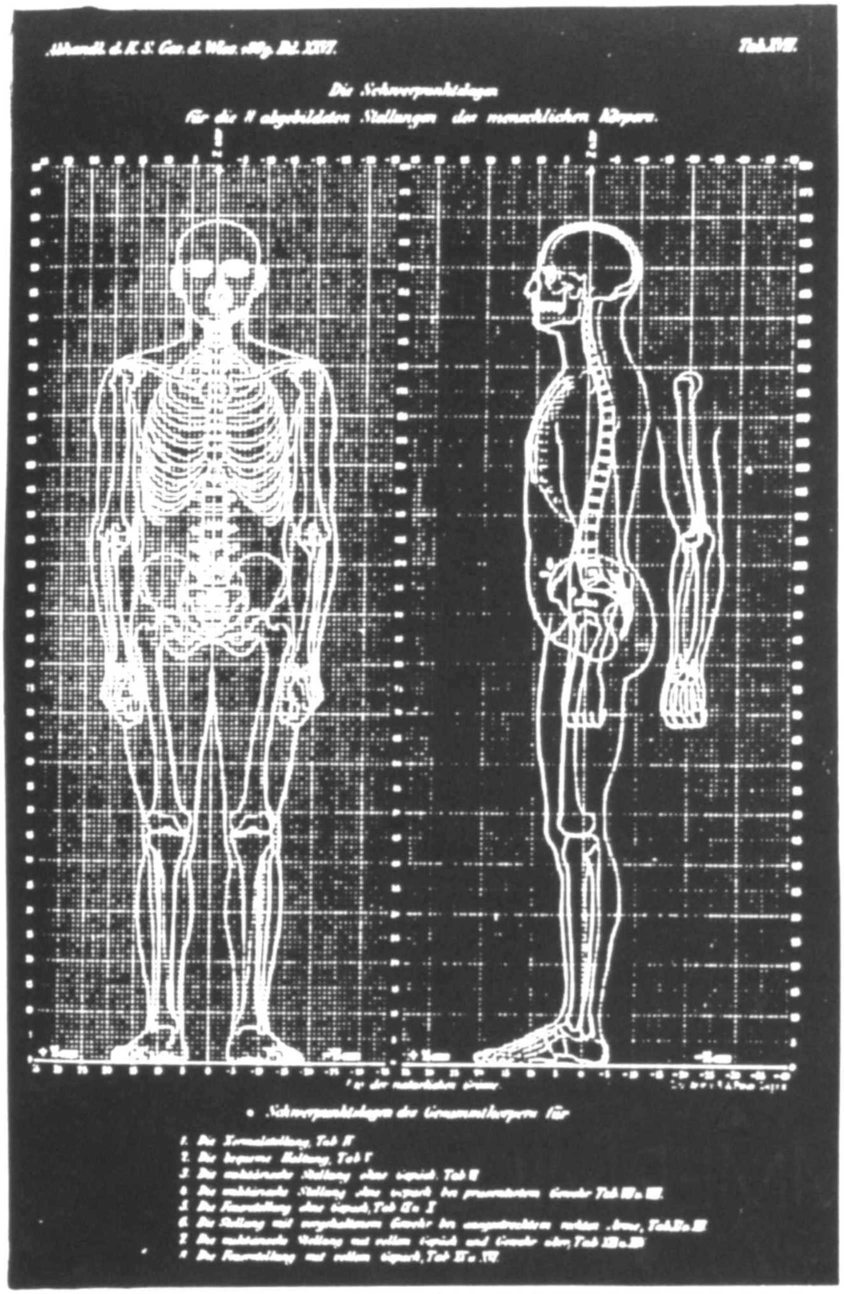

PLATE XVII

CENTERS OF GRAVITY FOR THE EIGHT POSITIONS AS

ALREADY ILLUSTRATED