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HEAVY MILITARY EQUIPMENT DEPARTMENT

TECHNICAL INFORMATION SERIES

Author	Subject Category	No. R74EMH12
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Title

THE MEAN OF THE FIELD FOR CYLINDRICALLY SPREADING WAVES IN A RANDOM MEDIUM

G.E

[redacted]

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Summary

The mean value of the field for cylindrically spreading waves in a random medium is obtained when forward scattering is dominant. In addition, the region of validity of the solution is obtained.

These results are identical to the corresponding results for plane waves, except for cylindrical spreading loss.

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SECTION I

INTRODUCTION

In a previous paper (Ref. 1) the author derived the coherence function for the circumstances of the title. In this paper, the mean value of the field is obtained when forward scattering dominates. The region of validity of the solution is obtained.

It is shown that, except for cylindrical spreading loss, the mean value of the field is the same as that of the corresponding plane wave case treated by Tatarskii (Ref. 2). The region of validity is shown to be the same as that of the plane wave case.

As in Reference 1 long range propagation is assumed.

SECTION II

THE MEAN OF THE FIELD

Following Reference 1, let the source be monochromatic and cylindrical with radius ϵ such that

$$k\epsilon >> 1 \tag{2-1}$$

Let the source radiate into a statistically isotropic and homogeneous medium.

The wave equation in cylindrical coordinates is given by

$$\frac{\partial^2 \phi(\rho, s, t)}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \phi(\rho, s, t)}{\partial \rho} + \Delta(\rho) \phi(\rho, s, t)$$

$$= \frac{(1 + \mu(\rho, s))^2}{c^2} \frac{\partial^2 \phi(\rho, s, t)}{\partial t^2} \ ; \quad \rho \geq \epsilon \tag{2-2}$$

where s represents the pair (ϕ, z) and

$$\Delta(\rho) \equiv \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \tag{2-3}$$

Utilizing the fact that by hypothesis

$$\rho k >> 1 \tag{2-4}$$

one has

$$\phi(\rho, s, t) = b(\rho, s) \frac{e^{ik\rho}}{\sqrt{\rho}} e^{-i\omega t} \tag{2-5}$$

Assuming that $\mu(\rho, s)$ is small, so that

$$(1 + \mu)^2 = 1 + 2\mu \tag{2-6}$$

and substituting Equation (2-5) into Equation (2-2) yields (Ref. 1)

$$2ik \frac{\partial b(\rho, s)}{\partial \rho} + \Delta(\rho)\, b(\rho, s) + 2k^2 \mu(\rho, s)\, b(\rho, s) = - \frac{\partial^2 b(\rho, s)}{\partial \rho^2} \qquad (2\text{-}7)$$

where $b(\rho, s)$ satisfies the boundary condition

$$b(\epsilon, s) = 1 \qquad (2\text{-}8)$$

The term $\partial^2 b/\partial \rho^2$ contributes backscattering to the solution of Equation (2-7). It will be disregarded here, and its effect will be estimated in the next section.

Following Reference 1, let the index of refraction covariance function be

$$E\left(\mu(\rho, s)\, \mu(\rho', s')\right) = \overline{\mu^2} \exp\left[- \frac{\left(\rho - \rho' \cos(\phi - \phi')\right)^2}{a^2} \right]$$

$$\cdot \exp\left[- \frac{\rho'^2 \sin^2(\phi - \phi')}{a^2} \right] \exp\left[- \frac{(z - z')^2}{a^2} \right] \qquad (2\text{-}9)$$

where a is correlation distance of the index of refraction.

Making the long range propagation approximation (Ref. 1) one has

$$E\left(\mu(\rho, s)\, \mu(\rho', s')\right) \simeq \sqrt{\pi}\, \overline{\mu^2}\, a\, \delta\left(\rho - \rho' \cos(\phi - \phi')\right)$$

$$\cdot \exp\left[- \frac{\rho'^2 \sin^2(\phi - \phi')}{a^2} \right] \cdot \exp\left[- \frac{(z - z')^2}{a^2} \right] \qquad (2\text{-}10)$$

Disregarding $\partial^2 b/\partial\rho^2$ in Equation (2-7) the resulting equation may be written as

$$2ik \exp\left(ik\, \Phi(\rho, s)\right) \frac{\partial}{\partial \rho} \left[\exp\left(-ik\, \Phi(\rho, s)\right) b(\rho, s) \right] = - \Delta(\rho)\, b(\rho, s) \qquad (2\text{-}11)$$

where

$$\Phi(\rho\ ,\ s) \equiv \int_{\epsilon}^{\rho} \mu\,(\rho',\ s)\, d\rho' \tag{2-12}$$

Multiplying both sides of Equation (2-11) by exp $(-ik\,\Phi)$, integrating over ρ , and multiplying the resulting equation by exp $(i\Phi)$ yields

$$2ik\, b(\rho\ ,\ s) = 2ik\, \exp\left(ik \int_{\epsilon}^{\rho} \mu\,(\rho',\ s)\, d\rho' \right)$$

$$- \int_{\epsilon}^{\rho} \exp\left(ik \int_{\rho'}^{\rho} \mu\,(\rho'',\ s)\, d\rho'' \right) \Delta(\rho')\, b(\rho',\ s)\, d\rho' \tag{2-13}$$

where the boundary condition

$$b(\epsilon,\ s) = 1 \tag{2-14}$$

was used in obtaining Equation (2-13).

Let it be assumed that $\int_{\epsilon}^{\rho} \mu\,(\rho',\ s)\, d\rho'$ is Gaussian either because $\mu\,(\rho\ ,\ s)$ satisfies and appropriate central limit theorem or because $\mu\,(\rho\ ,\ s)$ is a Gaussian field. In Reference 1 it is shown that $b(\rho\ ,s)$ is a functional of $\mu\,(\rho',\ s)$ with $\epsilon \leq \rho' \leq \rho$. Then, due to the sharply peaked nature of the correlation function of $\mu\,(\rho\ ,\ s)$, $\int_{\rho'}^{\rho} \mu(\rho'',\ s)\, d\rho''$ is statistically independent of $b(\rho',\ s)$. Therefore, taking the expectation value of both sides of Equation (2-13) yields

$$2ik\, E(b\,(\rho\ ,\ s)\,) = 2ik\, E\left(\exp\left(ik \int_{\epsilon}^{\rho} \mu\,(\rho',\ s)\, d\rho' \right) \right)$$

$$- \int_{\epsilon}^{\rho} E\left(\exp\left(ik \int_{\rho'}^{\rho} \mu(\rho'',\ s)\, d\rho'' \right) \right)$$

$$\cdot\ \Delta\,(\rho')\, E\left(b\,(\rho',\ s) \right) d\rho' \tag{2-15}$$

Using the fact that $\mu(\rho, s)$ is statistically homogeneous and isotropic it can be shown (Ref. 1) that $b(\rho, s)$ is statistically homogeneous and isotropic in s. Therefore $E(b(\rho, s))$ is a function of ρ only, so that

$$\Delta(\rho') E\left(b(\rho', s)\right) = 0 \tag{2-16}$$

and

$$E\left(b(\rho, s)\right) = E\left(\exp\left(ik \int_{\epsilon}^{\rho} \mu(\rho', s)\, d\rho'\right)\right)$$

$$\equiv \exp\left(- \frac{k^2}{2} E\left(\Phi^2(\rho, s)\right)\right) \tag{2-17}$$

Using Equation (2-10) one obtains

$$E\left(\Phi^2(\rho, s)\right) \simeq \sqrt{\pi}\, \overline{\mu^2}\, a \int_{\epsilon}^{\rho} \int_{\epsilon}^{\rho} \delta(\rho_1 - \rho_2)\, d\rho_1\, d\rho_2$$

$$= \sqrt{\pi}\, \overline{\mu^2}\, a\rho \qquad \rho >> \epsilon, \rho >> a \tag{2-18}$$

Finally, one has

$$E\left(b(\rho, s)\right) = \exp\left(- \frac{\sqrt{\pi}}{2} \overline{\mu^2} k^2 a\rho\right) \tag{2-19}$$

This result is identical to the corresponding result for the plane wave case, obtained by Tatarskii (Ref. 2).

In the next section the region of validity of the forward scattering equation is obtained.

SECTION III

THE REGION OF VALIDITY OF THE FORWARD SCATTERING EQUATION

Let $B(\rho, s)$ be the solution of Equation (2-7), so that

$$2ik \frac{\partial B(\rho, s)}{\partial \rho} + \Delta(\rho) B(\rho, s) + 2k^2 \mu(\rho, s) B(\rho, s) = -\frac{\partial^2 B(\rho, s)}{\partial \rho^2} \quad (3\text{-}1)$$

Suppose Equation (3-1) were to be solved by successive approximation. Then, $B(\rho, s)$ is expanded in a perturbation series

$$B(\rho, s) = b(\rho, s) + b_1(\rho, s) + \ldots \quad (3\text{-}2)$$

where

$$2ik \frac{\partial b(\rho, s)}{\partial \rho} + \Delta(\rho) b(\rho, s) + 2k^2 \mu(\rho, s) b(\rho, s) = 0 \quad (3\text{-}3a)$$

and

$$2ik \frac{\partial b_1(\rho, s)}{\partial \rho} + \Delta(\rho) b_1(\rho, s) + 2k^2 \mu(\rho, s) b_1(\rho, s) = -\frac{\partial^2 b(\rho, s)}{\partial \rho^2} \quad (3\text{-}3b)$$

Using the same type of manipulations which led to Equation (2-13), and the boundary condition

$$b_1(\epsilon, s) = 0 \quad (3\text{-}4)$$

Equation (3-3b) becomes

$$2ik\, b_1(\rho, s) = -\int_{\epsilon}^{\rho} \exp\left(ik \int_{\rho'}^{\rho} \mu(\rho'', s)\, d\rho'' \right) \cdot \left[\Delta(\rho') b_1(\rho', s) + \frac{\partial^2 b(\rho', s)}{\rho'^2} \right] d\rho' \quad (3\text{-}5)$$

From the properties of $b(\rho, s)$ and $\mu(\rho, s)$ one finds that $b_1(\rho, s)$ is statistically isotropic and homogeneous in s, and that

$\int_{\rho'}^{\rho} \mu(\rho'', s)\, d\rho''$ is statistically independent of $b_1(\rho', s)$. Therefore taking the expectation value of both sides of Equation (3-5) yields

$$2ik\, E(b_1, (\rho, s)) = - \int_{\epsilon}^{\rho} E\left(\exp\left(ik \int_{\rho'}^{\rho} \mu(\rho'', s)\, d\rho''\right)\right)$$

$$\cdot \frac{\partial^2 E\left(b(\rho', s)\right)}{\partial \rho^2} \cdot d\rho' \tag{3-6}$$

Equations (2-9) and (2-19) one has

$$E\left(\exp\left(ik \int_{\rho'}^{\rho} \mu(\rho'', s)\, d\rho''\right)\right)$$

$$= \exp\left(-\frac{k^2}{2} \cdot \sqrt{\pi}\, \overline{\mu^2}\, a \int_{\rho'}^{\rho} \int_{\rho'}^{\rho} \delta(\rho_1 - \rho_2)\, d\rho_1\, d\rho_1\right)$$

$$= \exp\left(-\frac{\sqrt{\pi}}{2}\, \overline{\mu^2}\, k^2 a\, (\rho - \rho')\right) \tag{3-7}$$

and

$$\frac{\partial^2 E\left(b(\rho', s)\right)}{\partial \rho'^2} = \left(\frac{\sqrt{\pi}}{2}\, \overline{\mu^2}\, k^2 a\right)^2 \exp\left(-\frac{\sqrt{\pi}}{2}\, \overline{\mu^2}\, k^2\, a \rho'\right) \tag{3-8}$$

Substituting Equations (3-7) and (3-8) into Equation (3-6) yields

$$E\left(b_1(\rho, s)\right) = i\frac{\pi}{8}^{\frac{3}{2}}\left(\overline{\mu^2}\,ka\right)^2 (k\rho)\exp\left(-\frac{\sqrt{\pi}}{2}\,\overline{\mu^2}\,k^2 a\rho\right)$$

$$\equiv i\frac{\pi}{8}^{\frac{3}{2}}\left(\overline{\mu^2}\,ka\right)^2 k\rho\, E\left(b(\rho, s)\right) \qquad \rho >> \epsilon, \rho >> a \tag{3-9}$$

Then

$$\frac{\left| E\left(b_1(\rho, s)\right)\right|}{E\left(b(\rho, s)\right)} = \frac{\pi}{8}^{\frac{3}{2}}\left(\overline{\mu^2}\,ka\right)^2 (k\rho) \tag{3-10}$$

Thus, if

$$\frac{\pi}{8}^{\frac{3}{2}}\left(\overline{\mu^2}\,ka\right)^2 (k\rho) \ll 1 \tag{3-11}$$

the forward scattering equation is valid. These results are identical to the corresponding results for the plane wave case obtained by Tatarskii (Ref. 2).

SECTION IV

REFERENCES

1. M. M. Fitelson, "The Coherence for Cylindrically Spreading Waves in a Random Medium", to be published in JASA.
2. V. I. Tatarskii, "The Effects of the Turbulent Atmosphere on Wave Propagation", pp 377-381, 401-404. Israel Program for Scientific Translation, 1971. (Available in U. S. through NTIS).