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COMPUTER PROGRAMS FOR CALCULATING SMALL DISTURBANCE TRANSONIC FLOWS ABOUT OSCILLATING AIRFOILS

SCIENCE APPLICATIONS, INCORPORATED

TECHNICAL REPORT AFFDL—TR—74—135

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AIR FORCE FLIGHT DYNAMICS LABORATORY
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Computer programs are described which implement a small disturbance potential flow theory for the two-dimensional unsteady transonic flow about thin airfoils undergoing low reduced frequency harmonic oscillations. The theory is based upon the treatment of the unsteady flow as a small perturbation to the steady transonic flow. Separating the perturbation potential into a steady and unsteady component results in a pair of coupled boundary (over)

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) value problems for the components. The governing equation for the steady perturbation potential is the usual nonlinear transonic potential equation and it is solved in computer program STRANS using the mixed differencing relaxation procedure of Murman and Cole. The governing equation for the unsteady perturbation potential is linear and, for the harmonic boundary disturbance considered, of mixed elliptic hyperbolic type depending on the local nature of the steady potential. Using a steady solution previously generated by STRANS, computer program UTRANS solves the unsteady potential equation by the same relaxation procedure. The solution procedures are found to be quite efficient, permitting the calculation of unsteady aerodynamic forces to engineering accuracy in a few minutes on a CDC 6600 computer.

FOREWORD

This computer program User's Manual was prepared by the Los Angeles Division of Science Applications, Incorporated, Los Angeles, California for the Vehicle Dynamics Division of the Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio. The computer programs were developed under Project 1370, "Dynamic Problems in Flight Vehicles," Task 137004, "Design Analysis," Contract F33615-74-C-3094. James J. Olsen and later Lt. William L. Holman of AFFDL/FYS were the Air Force task engineers.

R. M. Traci was the principal investigator for the study and J. L. Farr, Jr. developed the computer programs described in this report. Consultant E. D. Albano contributed to the development and implementation of the numerical method.

The authors submitted this report in October 1974 for publication as an AFFDL technical report to cover research performed from June through September 1974.

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont'd)

LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

a	Sound speed, coefficients in asymptotic expansion
c	Airfoil chord
C_p, $\overline{C}_p$	Unscaled and scaled pressure coefficient
ΔC_p	Jump in pressure coefficient over airfoil
C_ℓ, $\overline{C}_\ell$	Unscaled and scaled lift coefficient
EPSCØL,EPSGRD	Variables to control column and grid convergence
f,t	Airfoil shape function, thickness distribution
$H^{(2)}$	Hankel function of second kind
i	Designates imaginary part
i,j	Mesh indices
I_1, I_2	Integrals in farfield expression
k	Reduced frequency based on chord
$\hat{k}$	Ratio of scaled frequency to transonic similarity parameter
K	Transonic similarity paramter
M_∞	Freestream Mach number
M,NGFF,PGFF	Parameters in grid and circulation iteration schemes
R, R_1, S	Transformed coordinates
t, $\tilde{t}$	Physical and scaled time
U	Freestream velocity

V	Coefficient in transonic potential equation
$x, \tilde{x}$	Physical and scaled streamwise coordinate
$y, \tilde{y}$	Physical and scaled normal coordinate
α	Angle of attack
δ	Airfoil thickness parameter
Δ	Mesh spacing or jump in quantity
ε	Unsteady perturbation parameter
γ	Ratio of specific heats
γ_0, γ_1	Steady and unsteady circulation
ω, Ω	Physical and scaled frequency of oscillation
ω_j	Relaxation parameter
σ	Jump in potential across wake
$\phi, \tilde{\phi}$	Unscaled and scaled perturbation velocity potential
ψ	Total velocity potential

<u>Subscripts</u>

∞	Refers to freestream
i,j	Refers to finite difference grid points
ff	Refers to farfield
te	Refers to trailing edge of airfoil
u, ℓ	Refers to upper and lower airfoil surface
x, y, t	Refers to partial derivatives with respect to coordinates and time

Superscripts

^{0}, ^{1}	Refers to zero order steady and first order unsteady perturbations
E	Refers to elliptic, centered difference form
H	Refers to hyperbolic, backward difference form
ν	Refers to iteration number
+, -	Refers to above or below airfoil or wake

1.0 INTRODUCTION

Computer programs STRANS and UTRANS implement a small disturbance potential flow theory for the two-dimensional unsteady transonic flow about thin airfoils undergoing low reduced frequency harmonic oscillations. The theory is based on Landahl's suggestion[1,2] that a linear system can be obtained by considering the unsteady flow as a small perturbation to the mean nonuniform flow. The perturbation expansion approach has recently been developed with different emphasis in independent studies by Traci, et al.[3], and Ehlers[4]. The computer programs, documented in this users manual, were developed during the former study and detailed descriptions of the theory and numerical solution technique with calculated results are presented in the final report of that study[3].

In the perturbation expansion approach used, the perturbation potential function is expanded in a series of increasing powers of a small parameter which is a measure of the amplitude of an unsteady disturbance to the boundary. The resulting expansion of the unsteady potential equation results in a sequence of partial differential equations for the perturbation potentials. The zeroth order equation is the usual nonlinear steady transonic potential equation of mixed elliptic/hyperbolic type and is solved in STRANS using the mixed differencing, relaxation procedure of Murman and Cole[4]. The first order unsteady potential equation is linear and for harmonic boundary disturbances is also of the mixed elliptic/hyperbolic type, depending upon the steady solution. It is solved in UTRANS using the same numerical technique as used in STRANS.

The background, approximations and practical accuracy of the theory and numerical solution procedure are discussed in some detail in the companion report[3] to this users manual. As noted in that report, the method implemented in the present versions of the programs has not been completely verified as to its accuracy due to the dearth of reliable experimental data or accurate alternate solutions for comparison. This lack of empirical or analytical techniques for predicting unsteady aerodynamic forces in the transonic speed range is in fact the justification for publishing the preliminary versions of the computer programs described here. The approach embodied in the programs shows considerable promise of providing a practical method for filling this void. There are many ways theoretically, numerically and operationally that the present versions of STRANS and UTRANS may be improved. Work is continuing in this direction, so that it is anticipated that the efficiency and capability of the programs will continue to increase in the future.

The theory and practice of the computer program operation are discussed in the following sections. The small perturbation theory and numerical solution procedure are summarized in Sections 2.0 and 3.0, respectively. A description of the program's logical operation and a brief subroutine description are given in Section 4.0. Section 5.0 presents a complete description of the program input, with suggested values for various control variables, and the program output. Section 6.0 describes the program usage and includes suggestions for making effective use of the programs. Past experience with convergency, accuracy and efficiency is discussed as well as a summary of a recommended computational procedure. Sample cases which exercise all program options are presented in Section 7.0 with a complete specification of all input and sample output. Finally, complete FORTRAN listings of STRANS and UTRANS are presented in the appendices.

2.0 SMALL PERTURBATION THEORY FOR UNSTEADY TRANSONIC FLOW

Small disturbance theory is the principal analytical tool for all speed ranges and the transonic speed range is no exception. The transonic flow regime (defined roughly as $|1 - M_\infty| \sim \delta$ where M_∞ is the freestream Mach number and δ is the airfoil thickness ratio) is inherently nonlinear however, so that linear theories which apply to fully subsonic or supersonic flow are not valid. Special account must therefore be taken of the nonlinearity which, of course, severely complicates the formulation so that finite difference solution procedures are indicated. The theory summarized here is based upon the treatment of the unsteady flow as a small perturbation of the steady transonic flow. This effectively linearizes the oscillating component of the flow about the nonlinear steady solution. The resulting boundary value problems for the steady (solved in STRANS) and first order unsteady (solved in UTRANS) perturbation potentials are presented in this section.

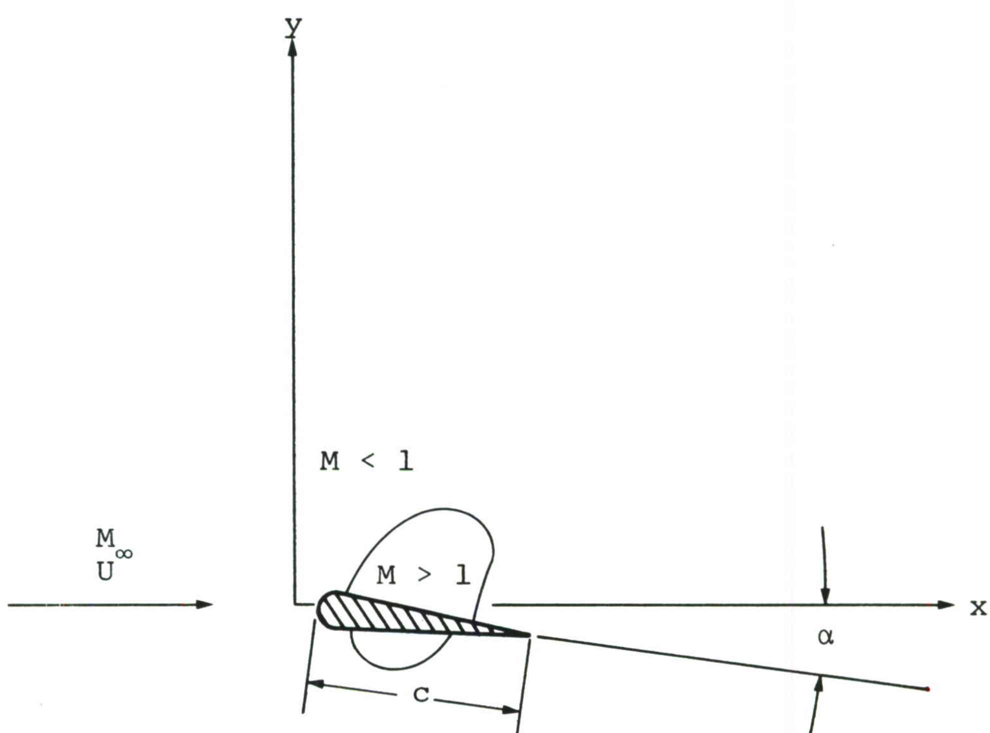


FIGURE 1. SCHEMATIC OF AIRFOIL GEOMETRY AND TRANSONIC FLOWFIELD

The problem of interest is the two-dimensional flow about an airfoil oscillating in shape, angle of attack or flap angle in the transonic flow speed range. Of particular interest are the steady and unsteady aerodynamic forces on the airfoil for use in flutter studies. The airfoil geometry, flowfield schematic and coordinate definition are given in Figure 1 above. Rectangular coordinates (x,y) are fixed to the airfoil leading edge and U, M_∞, a_∞ are the freestream velocity, Mach number and sound speed respectively. The airfoil has a thickness ratio δ which is the airfoil maximum thickness divided by its chord c and an angle of attack α. The assumption is made that $\delta << 1$ and α is of the same order of magnitude as δ. Also the oscillatory motion of the airfoil is assumed to be described by a small non-dimensional displacement $\varepsilon << \delta$ and a reduced frequency $k = \omega c/U$ based on airfoil chord where ω is the frequency of oscillation. The reduced frequency range treated by the present analysis in $k << 1$ or more explicitly k of order $\delta^{2/3}$.

Assuming inviscid, isentropic flow, the problem can be reduced to the solution of a single equation for a velocity potential plus the tangency boundary condition on the airfoil surface. As is well known the derivation of a small disturbance theory for transonic flows requires a singular perturbation approach. For the low frequency unsteady flow regime of interest, the following scaling is introduced:

$$\tilde{x} = \frac{x}{c} \ , \ \tilde{y} = \delta^{1/3}[(1+\gamma)M_\infty^2]^{1/3}\ \frac{y}{c}$$

$$\tilde{t} = \delta^{2/3}\ \frac{[(1+\gamma)M_\infty^2]^{2/3}}{M_\infty^2}\ \frac{U}{c}\ t \tag{1}$$

and the total potential is expanded about the uniform flow thusly:

$$\psi = Uc\tilde{x} + \frac{\delta^{2/3}Uc}{[(1+\gamma)M_\infty^2]^{1/3}}\ \tilde{\phi}(\tilde{x},\tilde{y},\tilde{t}) + \ldots \tag{2}$$

Retaining all terms to leading order in the total potential equation and boundary condition results in the following form for the unsteady small perturbation system:

$$(K-\tilde{\phi}_{\tilde{x}})\tilde{\phi}_{\tilde{x}\tilde{x}} + \tilde{\phi}_{\tilde{y}\tilde{y}} = 2\tilde{\phi}_{\tilde{x}\tilde{t}} \tag{3}$$

where the transonic similarity parameter is:

$$K = \frac{(1-M_\infty^2)}{[(\gamma+1)M_\infty^2]^{2/3}\delta^{2/3}}$$

with boundary conditions:

$$\tilde{\phi}_{\tilde{y}} = \frac{\partial}{\partial \tilde{x}} f_{u,\ell}(\tilde{x},\tilde{t}), \text{ on } \tilde{y} = \pm 0, \ 0 \leq x \leq 1 \tag{4}$$

$$[\tilde{\phi}_{\tilde{x}}] = 0 \text{ on } \tilde{y} = 0, \ \tilde{x} > 1 \tag{5}$$

$$\tilde{\phi}_{\tilde{x}}^2 + \tilde{\phi}_{\tilde{y}}^2 \rightarrow 0 \text{ as } \tilde{x}^2 + \tilde{y}^2 \rightarrow \infty \tag{6}$$

where $f_{u,\ell}$ is the unsteady airfoil shape function (Equation 7 below) on the upper and lower surfaces respectively, and where [] denotes a jump between 0^- and 0^+. It is noted that the airfoil tangency boundary condition (Equation 4) and Kutta condition (Equation 5) are applied in the small disturbance manner on $y = 0$ and in the low frequency, quasi-steady sense ($k \rightarrow 0$).

The system of Equations 3 through 6 provide a formulation of the unsteady airfoil problem in the nonlinear domain, which includes flowfields with shocks. The formulation is self consistent for similarity parameter K of order 1 and for reduced frequency k of order $\delta^{2/3}$.

The approach to solving the nonlinear system given above (Equations 3 through 6) is to expand the perturbation potential function in terms of the unsteady boundary disturbance $\varepsilon << 1$. From this point on all tildas ($\sim$) will be dropped with the understanding that all variables are scaled variables. Harmonic boundary disturbances are explicitly treated:

$$f_{u,\ell}(x,t) = f^0_{u,\ell}(x) + \varepsilon \, f^1_{u,\ell}(x)e^{i\Omega t} \tag{7}$$

where the scaled frequency is:

$$\Omega = \frac{M_\infty^2}{\delta^{2/3}[(1+\gamma)M_\infty^2]^{2/3}} k \qquad (8)$$

The perturbation potential is expanded as follows:

$$\phi(x,y,t) = \phi^0(x,y) + \varepsilon\, \phi^1(x,y)e^{i\Omega t} + \ldots \qquad (9)$$

Substituting this into the perturbation potential equation plus boundary conditions and combining terms results in the following pair of boundary value problems for ϕ^0 and ϕ^1 respectively.

$$\left.\begin{aligned} &(K-\phi^0_x)\phi^0_{xx} + \phi^0_{yy} = 0 \\ &\phi^0_y = f^{0\prime}_{u,\ell}(x), \text{ on } y = \pm 0,\ 0 \le x \le 1 \\ &[\phi^0_x] = 0, \text{ on } y = 0,\ x > 1 \\ &(\phi^0_x)^2 + (\phi^0_y)^2 \to 0, \text{ as } x^2 + y^2 \to 0 \end{aligned}\right\} \qquad (10)$$

and

$$\left.\begin{aligned} &(K-\phi^0_x)\phi^1_{xx} + \phi^1_{yy} - (\phi^0_{xx} + 2i\Omega)\phi^1_x = 0 \\ &\phi^1_y = f^{1\prime}_{u,\ell}(x), \text{ on } y = \pm 0,\ 0 \le x \le 1 \\ &[\phi^0_x] = 0, \text{ on } y = 0,\ x > 1 \\ &(\phi^1_x)^2 + (\phi^1_y)^2 \to 0, \text{ as } x^2 + y^2 \to \infty \end{aligned}\right\} \qquad (11)$$

Equation 10 is recognized as the usual formulation for steady transonic flow and Equation 11 is the formulation for the low frequency unsteady perturbation thereof. Note that the governing equation for ϕ^1 is linear but of the same mixed elliptic/hyperbolic type as the steady potential equation depending upon the steady solution. It is also noted that ϕ^1 is in general complex thereby permitting phase shifts between the field quantities and the boundary disturbance. A more general formulation including higher order unsteady modes and description of the nature of the equations is given in Reference 3.

The main physical quantities of interest are the pressure coefficient and airfoil force coefficients. The pressure coefficient, defined in the usual manner, is given by:

$$C_p = \frac{\delta^{2/3}}{[(1+\gamma)M_\infty^2]^{1/3}} (\overline{C}_p^0 + \varepsilon\, \overline{C}_p^1\, e^{i\Omega t}) \qquad (12)$$

where the steady and unsteady scaled pressure coefficients are given to leading order in the small disturbance and low frequency approximation by:

$$\overline{C}_p^0 = -\,2\phi_x^0,\ \overline{C}_p^1 = -\,2\phi_x^1 \qquad (13)$$

Also the lift coefficient is given by:

$$C_\ell = \frac{2\delta^{2/3}}{[(1+\gamma)M_\infty^2]^{1/3}} (\gamma_0 + \varepsilon\, \gamma_1 e^{i\Omega t}) \qquad (14)$$

where $\gamma_0 = [\phi^0]_{x=1}$ and $\gamma_1 = [\phi^1]_{x=1}$ are the steady and unsteady circulations about the airfoil. The finite difference forms used to calculate $\overline{C}_p^0$ and $\overline{C}_p^1$ are defined in the next section and the detailed definition of the calculated generalized force coefficients is presented in Section 4.0.

3.0 SUMMARY OF NUMERICAL SOLUTION PROCEDURE

The numerical solution procedures for the boundary value problems for the steady perturbation potential (Equation 10) and the unsteady perturbation potential (Equation 11), as implemented in computer programs STRANS and UTRANS respectively, are now summarized. As has been pointed out, both equations are of the same mixed elliptic/hyperbolic type and are solved using the mixed differencing relaxation procedure of Murman and Cole[5]. Summary of the procedure used in STRANS for calculating the steady solution is given in Section 3.1. Variations on the basic procedure, used in UTRANS for the unsteady perturbation are discussed in Section 3.2. It is noted that the finite difference equations which follow are written in terms of the actual FORTRAN variables used in the programs to as great a degree as possible. In addition, following usual programming practice, the FORTRAN variables are descriptive of the physical variables.

3.1 Numerical Solution Procedure for the Steady Potential

The difference scheme and relaxation solution procedure summarized in this section are based on the work of Murman[5,6,8], Cole[5] and Krupp[6,7]. They pointed out the essential ingredient for the success of relaxation procedures for the steady transonic potential equation. The key to their approach is to account for the local nature of the flow (elliptic in subsonic regions, hyperbolic in supersonic regions) in the finite difference approximation to the governing equations. The version of this technique, as implemented in STRANS, is summarized in this section. A more detailed description can be found in Reference 3.

Consider the steady, transonic potential equation, with boundary conditions, in the numerical solution domain indicated schematically below. Finite difference grid lines are defined in the region around the airfoil which is fixed on the slit $0 \leq x \leq 1$, $y = 0$. Values of the potential are defined at the intersection of the grid lines (i,j). The finite difference approximations to the governing equation at a general grid point account for the varying type of the equation as is summarized in Section 3.1.1. An iterative line relaxation solution procedure, also described in Section 3.1.1, uses the finite difference equations to successively update each point in the grid starting from some initial guess. The special numerical treatments required for the airfoil boundary, Kutta condition and far-field are described in Sections 3.1.2 to 3.1.4, respectively.

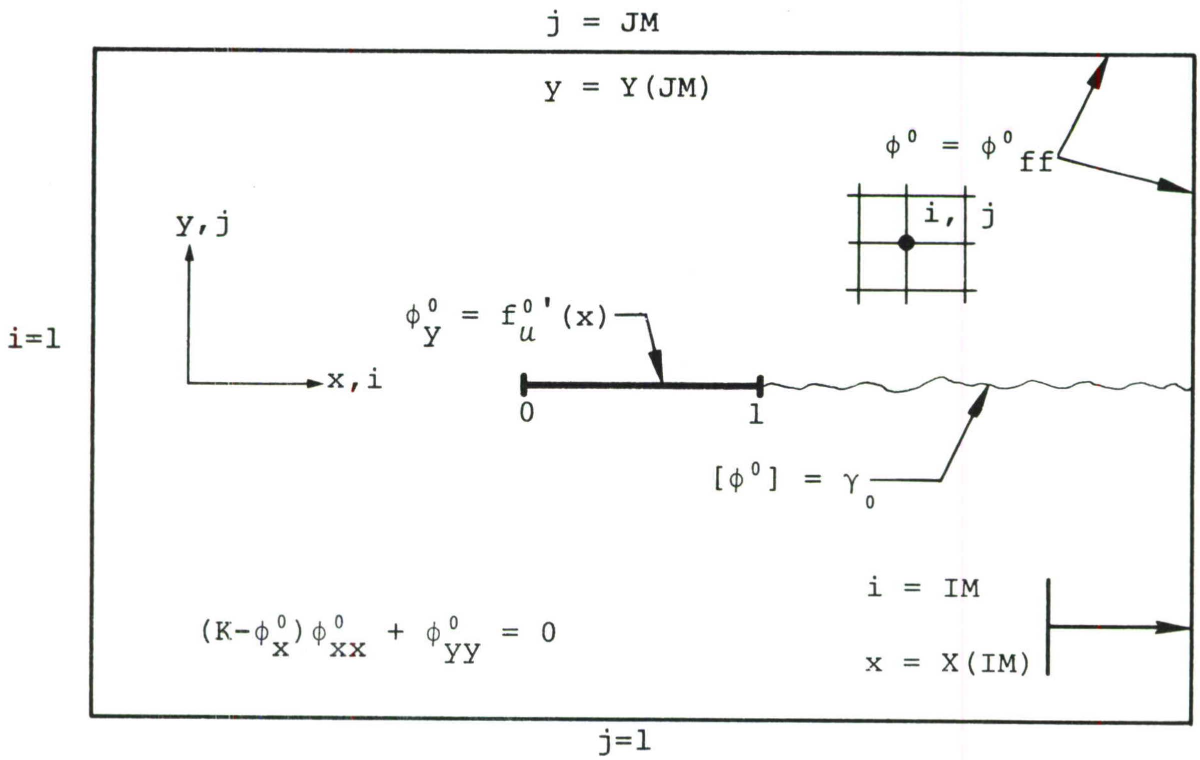


FIGURE 2. SCHEMATIC OF NUMERICAL SOLUTION DOMAIN

3.1.1 Finite Difference Forms and Iterative Solution Procedure

In the numerical scheme, a rectangular mesh with general grid line spacing, as indicated in the sketch (Figure 3), is used. Uneven grid spacing makes it possible to concentrate grid points near the airfoil and in regions where rapid changes in the potential or its derivative (wing leading edge, shocks, etc.) are expected. Expansion of the grid spacing away from the airfoil out to the boundaries of the mesh (i = 1 or IM, j = 1 or JM) permits economic use of grid points while maximizing resolution in the region of interest.

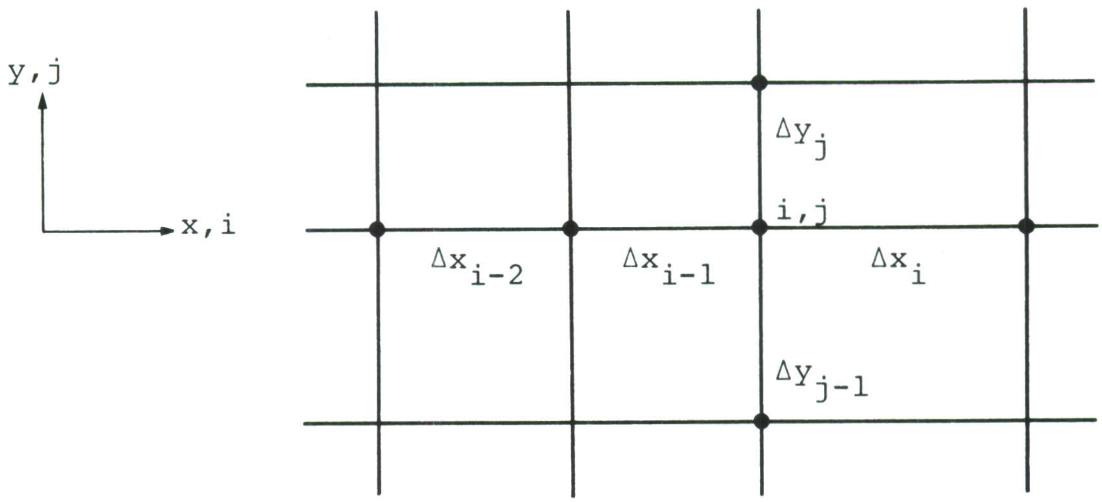


FIGURE 3. SKETCH OF LOCAL MESH ABOUT A COMPUTATIONAL POINT

At a general point (i,j) in the finite difference mesh, the solution begins by center differencing the coefficient of ϕ^0_{xx} in the following manner:

$$V_{i,j} = (K-\phi^0_x)_{i,j} = K - AX1_i \,(\phi_{i+1,j}-\phi_{i,j}) - AX2_i \,(\phi_{i,j}-\phi_{i-1,j}) \qquad (15)$$

with

$$AX1_i = \frac{\Delta x_{i-1}}{\Delta x_i (\Delta x_{i-1}+\Delta x_i)}$$

$$AX2_i = \frac{\Delta x_i}{\Delta x_{i-1} (\Delta x_{i-1}+\Delta x_i)}$$

Tests are then made on this coefficient to determine the nature of the potential equation at the point (i,j). If $V_{i,j} > 0$ the equation is elliptic so that ϕ^0_{xx} is center differenced and if $V_{i,j} < 0$ the equation is hyperbolic and backward differencing is used for both $V_{i,j}$ and ϕ^0_{xx}. An additional test is made to determine if the point is near the sonic line. If $V_{i,j} < 0$ but $V_{i-1,j} > 0$, the point is identified as a "parabolic point" and the center differenced form of $V_{i,j}$ is used but backward differenced form for ϕ^0_{xx} is used. The ϕ^0_{yy} term is always center differenced regardless of the above considerations. The finite difference form of the transonic potential equation can thus be summarized as follows:

$V_{i,j} > 0$, elliptic (subsonic)

$$V_{i,j}(\phi_{xx})^E_{i,j} + (\phi_{yy})_{i,j} = 0 \qquad (16)$$

$V_{i,j} < 0$, hyperbolic (supersonic)

$$V^H_{i,j}(\phi_{xx})^H_{i,j} + (\phi_{yy})_{i,j} = 0 \qquad (17)$$

$V_{i,j} < 0$, $V_{i-1,j} > 0$, parabolic (sonic)

$$V_{i,j}(\phi_{xx})^H_{i,j} = (\phi_{yy})_{i,j} = 0 \qquad (18)$$

where:

$$V^H_{i,j} = K - CX_{i-1}(\phi_{i,j}-\phi_{i-1,j}) - CX_{i-2}(\phi_{i-1,j}-\phi_{i-2,j}) \qquad (19)$$

$$(\phi_{xx})^E_{i,j} = BX1_i(\phi_{i+1,j}-\phi_{i,j}) - BX2_i(\phi_{i,j}-\phi_{i-1,j}) \qquad (20)$$

$$(\phi_{xx})^H_{i,j} = BX1_{i-1}(\phi_{i,j}-\phi_{i-1,j}) - BX2_{i-1}(\phi_{i-1,j}-\phi_{i-2,j}) \qquad (21)$$

$$(\phi_{yy})_{i,j} = AY1_j(\phi_{i,j+1}-\phi_{i,j}) - AY2_j(\phi_{i,j}-\phi_{i,j-1}) \qquad (22)$$

and where

$$BX1_i = 2\ AX1_i/\Delta x_{i-1},\ BX2_i = 2\ AX2_i/\Delta x_i$$

$$CX_i = 1/(2\Delta x_i) \qquad (23)$$

$$AY1_j = \frac{2}{\Delta y_j(\Delta y_j+\Delta y_{j-1})},\ AY2_j = \frac{2}{\Delta y_{j-1}(\Delta y_j+\Delta y_{j-1})}$$

Using the above forms, the finite difference equations are set up for each column (x = constant) in turn, taking into account airfoil, wake and farfield boundary conditions. This results in a sequence of nonlinear algebraic equations for each column of ϕ's, which can be solved by iteration or Newton's method. The method used in STRANS is to linearize the system of equations by using the previous iterate for the coefficient $V_{i,j} = K - \phi_x^0$. The resulting linear system is tridiagonal and is easily solved by Gaussian elimination. The iteration process is terminated when the difference between successive iterates, for all points in the column, is less than some specified small amount (EPSCØL). The process works quite well, requiring but three or four iterations to converge to good accuracy ($\Delta\phi < 10^{-5}$).

After each column is solved, it is relaxed with a variable relaxation factor which depends on the local nature of the solution.

$$\phi_{i,j}^{\nu+1} = \phi_{i,j}^{\nu} + \omega_j(\tilde{\phi}_{i,j}^{\nu+1}-\phi_{i,j}^{\nu}) \qquad (24)$$

$$j = 1, 2, \ldots JM$$

where ω is the relaxation factor and $\tilde{\phi}$ is the solution of of the system of equations for a column. Relaxation factors which have worked well are $\omega = 1.5 \rightarrow 1.7$ for elliptic points and $\omega = .75$ for hyperbolic points.

The column solution process described above is performed for each column in turn, starting from an initial guess. The initial guess typically is a previous calculation for the same or a similar airfoil at conditions of Mach number and angle of attack close to the problem of interest. If such a solution doesn't exist, the farfield expression is used as an initial guess with an arbitrary truncation of the singularity at $x = y = 0$. The grid is swept in the above manner until the change in ϕ for all grid points during one grid sweep is less than some arbitrary small amount (EPSGRD). Typically convergence is quite rapid initially but becomes slower as the iteration process proceeds.

A technique which considerably aids convergence is to begin the calculation on a relatively coarse grid and to periodically refine the grid as the solution proceeds. The rationale behind the technique is that the coarse grid speeds transmission of information (circulation, boundary conditions, etc.) throughout the grid. Successive refinement of the grid then increases solution accuracy with the interpolated values of ϕ from the coarse grid providing a good starting point. Currently the technique is implemented by a simple halving of the grid spacing.

3.1.2 Airfoil Boundary Condition

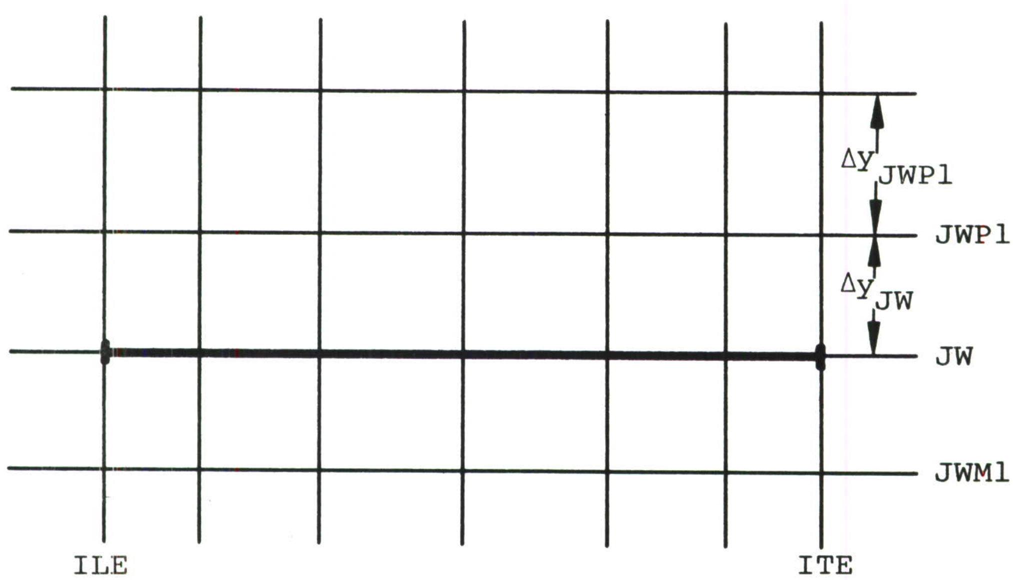


FIGURE 4. SCHEMATIC OF MESH NEAR AIRFOIL

As derived above, the airfoil tangency boundary condition is applied on the line $y = 0$, $0 \le x \le 1$, in the usual small perturbation sense, rather than on the physical boundary of the airfoil. This is a significant simplification relative to the finite difference approximation since it precludes the possibility of accounting for the details of the boundary geometry in the finite difference mesh. Thus the rectangular grid used throughout the solution field is also appropriate near the airfoil. The airfoil boundary conditions fit naturally into the finite difference equations for a column of grid points as required by the line relaxation procedure. The way that this is accomplished is now described.

The schematic given above defines the mesh structure near the slit on which the airfoil boundary conditions are applied; ILE and ITE define the columns at the leading edge and trailing edge respectively and JW defines the row location of the slit. For any column such that ILE $\le$ i $\le$ ITE the airfoil boundary condition effectively closes the system of equations for ϕ at grid points above and below the slit. The mathematical statement of the boundary condition is recalled:

$$\phi_y^0 = f_{u,\ell}^{0\,\prime}(x) \text{ on } y = \pm 0,\ 0 \le x \le 1 \tag{25}$$

where $f_{u,\ell}^{0\,\prime}$ are defined by the user in the functions FUP and FLP respectively. The essence of the method is to use Equation 25 in the finite difference approximation to the ϕ_{yy}^0 term of the potential equation at points just above (j = JW+1) and just below (j = JW-1) the airfoil. The form taken for this term is as follows:

$$(\phi_{yy})_{i,j} = \frac{2}{(\Delta y_j + 2\Delta y_{j-1})}\left\{\frac{(\phi_{i,j+1} - \phi_{i,j})}{\Delta y_j} - f_u^{0\,\prime}(x_i)\right\} \tag{26}$$

$$j = \text{JW+1},\ \text{ILE} \le i \le \text{ITE}$$

$$(\phi_{yy})_{i,j} = \frac{2}{(\Delta y_{j-1} + 2\Delta y_j)}\left\{f_\ell^{0\,\prime}(x_i) - \frac{(\phi_{i,j} - \phi_{i,j-1})}{\Delta y_{j-1}}\right\} \tag{27}$$

$$j = \text{JW-1},\ \text{ILE} \le i \le \text{ITE}$$

where subscript u,ℓ refer to upper or lower airfoil surface respectively. These finite difference forms effectively fix the boundary condition on the slit; $y = 0^+$ for the upper surface and $y = 0^-$ for the lower surface. They also are consistent with the second order accuracy of the differencing at general points in the mesh. The other terms in the potential equation are treated in the same manner as at general field points.

Values of potential on the airfoil surface are not explicitly calculated in the scheme used here. These values are obtained only after the equations for a column of grid points have been solved and the values of $\phi^0_{i,j}$ relaxed. ϕ on the airfoil surface is then obtained by linear extrapolation from above, for the upper surface, and from below for the lower surface. The pressure coefficient on the airfoils is calculated, using the extrapolated values of potential, by center differencing using the following equation:

$$\overline{C}^0_p = -2\phi^0_x = -\ 2AX1_i(\phi_{i+1,j}-\phi_{i,j}) - 2AX2_i(\phi_{i,j}-\phi_{i-1,j}) \quad (28)$$

The airfoil circulation γ_{te} is another quantity that is of interest and which is needed for the treatment of the Kutta condition discussed in the next section. This is calculated in a straightforward manner, using the values of potential at the trailing edge as follows:

$$(\gamma_0)_{te} = [\phi^0]_{x=1} = \phi^0_u|_{x=1} - \phi^0_\ell|_{x=1} \quad (29)$$

where, as before, subscripts u and ℓ refer to the extrapolated values on the upper and lower airfoil surfaces respectively.

The generalized forces on the airfoil are calculated in STRANS using a trapezoidal rule integration over the airfoil of the difference in upper and lower surface pressure coefficients, which are calculated using Equation 28. The assumed forms of the generalized forces are described in Section 5.0 on program output.

3.1.3 Kutta Condition

The numerical treatment of the Kutta condition is an essential part of the calculation for lifting airfoils and is worth discussing in some detail. Consideration of airfoils with camber or angle of attack requires the calculation of the circulation (γ_0) around the airfoil. This is not known apriori and must be a part of the overall solution process. Application of the Kutta condition requires that the circulation equal the jump in potential across a "wake" extending from the airfoil to downstream infinity. Incorporation of the wake in the finite difference scheme involves differencing the ϕ_{yy} term for points near the wake in such a manner as to take into account the jump in ϕ. The method for accomplishing this, and the iteration method for calculating γ_0 are now discussed.

The potentials just above ($y = 0^+$) and below ($y = 0^-$) the wake are defined as ϕ^+ and ϕ^- respectively, and an average potential at the wake is defined in the obvious manner:

$$\bar{\phi} = \frac{\phi^+ + \phi^-}{2} \tag{30}$$

The potentials ϕ^+ and ϕ^- satisfy the nonlinear transonic potential equation and it can easily be shown that $\bar{\phi}$ likewise satisfies the same differential equation. Also, using continuity of pressure and the potential equation it can be shown that:

$$\phi^+_{yy} = \phi^-_{yy} = \bar{\phi}_{yy} \tag{31}$$

This plus the fact that $\phi^+ = \bar{\phi} + \sigma$, where σ is the jump in potential across the wake, provides a means for accounting for the Kutta condition in the finite difference approximation to the potential equation. For grid points on the wake ($j = JW$, $i > ITE$) the average potential is calculated and used in the differencing of ϕ_{yy} for points just above and just below the wake. The difference forms for ϕ_{yy} at these points are given by:

$\underline{j = JW + 1}$

$$\phi_{yy}\big|_{i,j} = AY1_j\phi_{i,j+1} - (AY1_j + AY2_j)\phi_{i,j} + AY2_j\left(\bar{\phi}_i + \frac{\sigma_i}{2}\right) \tag{32}$$

$\underline{j = JW \text{ (wake)}}$

$$\bar{\phi}_{yy}\Big|_{i,j} = AY1_j \phi_{i,j+1} - AY1_j \left(\bar{\phi}_i + \frac{\sigma_i}{2}\right) - AY2_j \left(\bar{\phi}_i - \frac{\sigma_i}{2}\right) + AY2_j \phi_{i,j-1} \tag{33}$$

$\underline{j = JW - 1}$

$$\phi_{yy}\Big|_{i,j} = AY1_j \left(\bar{\phi}_i - \frac{\sigma_i}{2}\right) - (AY1_j + AY2_j)\phi_{i,j} + AY2_j \phi_{i,j-1} \tag{34}$$

where AY1 and AY2 were defined earlier; and where

$$\sigma_i = \frac{(x_i - 1)}{(X(IM)-1)} (\gamma_{ff} - \gamma_{te}) + \gamma_{te} \tag{35}$$

where σ_i is the jump in potential across the wake at any x station and γ_{ff} and γ_{te} are the circulation in the farfield and at the trailing edge respectively. At any stage of the iteration process, σ_i is taken as the linearly interpolated value between the current value of γ_{te} evaluated as the jump in potential at the trailing edge ($x = 1$), and $\gamma_{ff}(x = X(IM))$. In the converged solution $\gamma_{ff} = \gamma_{te}$ so that $\Delta\phi(x>1) = \gamma$ is a constant along the wake thereby automatically satisfying the condition of pressure continuity $[\phi_x] = 0$ along the wake.

The differencing scheme given above is incorporated into the process for setting up the system of algebraic equations for a column of grid points with $x_i > 1$. The scheme is symmetric with respect to points above and below the wake, and retains the second order accuracy of the difference approximation for ϕ_{yy}. $\bar{\phi}$ on the wake is calculated as any other grid point since, as noted earlier, it satisfies the transonic potential equation.

It remains only to define a scheme for updating γ_{ff} based on the calculated values of γ_{te}. This is done by over-relaxing on previous values of γ_{te}.

$$\gamma_{ff} = \gamma_{te}^{(1)} + PGFF\left(\gamma_{te}^{(NGFF)} - \gamma_{te}^{(1)}\right) \tag{36}$$

where NGFF is the number of grid sweeps before γ_{ff} is updated and PGFF is a weighting factor. It is recommended that NGFF be selected such that NGFF > 1.5M where M is the number of grid points between the airfoil and the farfield. This guarantees that information from the previous update of the farfield has had time to reach the airfoil.

3.1.4 Farfield Boundary Condition

The completion of the finite difference analog to the steady transonic flow problem requires that boundary conditions be fixed on the far boundary in some manner. This is accomplished in STRANS in two different ways depending upon the freestream Mach number, M_∞. For $M_\infty < 1$, the potential on all boundary points (I = 1, IM and J = 1, JM) is specified as a Dirichlet boundary condition and for $M_\infty > 1$ the small perturbation form of the characteristic relations is used to relate ϕ_y to ϕ_x.

For a subsonic freestream, analytical expressions are available which are valid far from the airfoil. The steady subsonic farfield has been studied most extensively by Klunker[9] and his result is used in STRANS. The farfield representation describes thickness and lift effects to leading order and is given by:

$$\phi^0_{ff} = \frac{1}{\pi\sqrt{K}} \frac{x}{x^2+Ky^2} \left\{ \int_0^1 t(\xi)d\xi + \int\int_{-\infty}^{\infty} (\phi^0_x)^2 d\xi d\eta \right\} + \gamma_{ff}\left[\arctan\left(\frac{x}{Ky}\right) + \frac{\pi}{2}\,\mathrm{sgn}\; y\right] \tag{37}$$

$$M_\infty < 1,\; x^2 + Ky^2 \to \infty$$

where $t(\xi)$ is the airfoil thickness distribution. The doublet strength due to the nonuniform flow is given by the integral of $(\phi^0_x)^2$ over the solution field. This is evaluated in STRANS periodically (every NGFF iterations) as the iterative solution proceeds using a trapezoidal rule integration scheme. Similarly, in calculations for airfoils with lift, the circulation γ_{ff} is updated (also every NGFF iterations). Thus as the doublet strength and airfoil circulation are refined, Equation 37 is used to update ϕ^0_{ff} on the boundaries of the finite difference grid. Suggestions for locating and updating the farfield are given in Section 6.0 below.

STRANS can also treat supersonic freestream Mach numbers by setting a characteristic boundary condition on the farfield boundaries. The upstream boundary is a completely uniform flow so that $\phi^0 \equiv 0$ is set on i = 1, j = 1, JM. Because of the backward differencing used for supersonic grid points, no prescription is required on the downstream boundary as long as all points are supersonic, as they should be in a converged solution. If during the iteration procedure a grid point on the boundary becomes subsonic, then a zero gradient ($\phi_x^0 = 0$) boundary condition is used. On the top and bottom of the grid, it is assumed that the disturbance from the body is weak so that the characteristic relation on the incoming characteristic applies. The first order approximation to this relation is:

$$\phi_y^0 = \pm\sqrt{-K}\,\phi_x^0 \text{ on } y = \begin{cases} Y(JM) \\ Y(1) \end{cases} \tag{38}$$

This is incorporated in the finite difference procedure in the same manner as the airfoil boundary condition. That is, on the top and bottom of the grid $(\phi_{yy})_{i,j}$ is differenced as a one-sided difference using Equation 38 in the finite difference forms.

3.2 Numerical Solution Method for First Order Unsteady Perturbation Potential

As noted before, the boundary value problem for the first order unsteady perturbation potential ϕ^1 is of the same mixed elliptic/hyperbolic type as the boundary value problem for the steady perturbation potential. The governing equation for ϕ^1 is linear which would suggest that many of the powerful techniques of classical analysis could be applied. The equation is seriously complicated, however, by the fact that various coefficients are functions of the nonlinear steady potential, for which the only general solution methods are numerical. In view of this complication and the success of the mixed differencing relaxation procedure for the steady potential equation of similar type, it was decided to use this same method in UTRANS to develop unsteady solutions. The application of the method to the unsteady potential equation involves but slight variations on the procedure described in detail in Section 3.1. These variations are emphasized in the summary of the solution method given in this section.

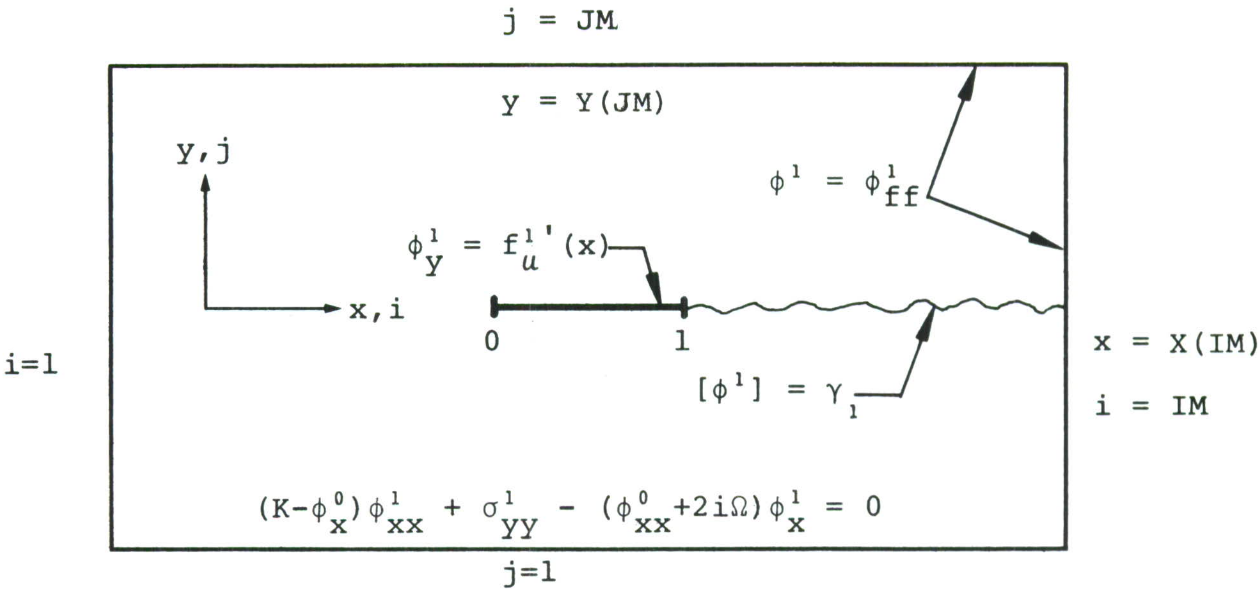


FIGURE 5. SCHEMATIC OF NUMERICAL SOLUTION DOMAIN

Consider the boundary value problem for the unsteady transonic potential, with numerical solution domain indicated schematically in Figure 5. It is recalled that the governing equation is linear and, for $\Omega \neq 0$, complex. Also the equation is of mixed elliptic/hyperbolic type with the local nature determined by the steady perturbation potential ϕ^0. Because of the strong similarity and dependence of ϕ^1 on ϕ^0, a possible solution approach would be to solve for both simultaneously. However, because of possible different stability and convergence requirements, it was decided to use a sequential approach. That is, since ϕ^0 does not depend on ϕ^1 it is solved independently by computer program STRANS, and the resulting solution stored on magnetic tape. The converged solution so obtained, is then used in the solution process for the corresponding ϕ^1. This approach has an added benefit in that ϕ^0 need not be regenerated for each unsteady boundary disturbance of interest.

As mentioned, the numerical solution proceeds in a similar manner to that for ϕ^0. The local nature of the equation at each grid point is determined by the corresponding value of $(K-\phi^0_x)_{i,j} = V_{i,j}$ at the same grid point. Then if $K - \phi^0_x > 0$ (elliptic) the x derivatives of ϕ^1 are center differenced and if $K - \phi^0_x < 0$ (hyperbolic) the x derivatives are backward differenced. Near the sonic line at so-called "parabolic points," (defined in Section 3.1) the centered difference form for $V_{i,j}$ and backward difference form for the x derivatives of ϕ^1 are used. The finite difference approximation to the unsteady potential equation are thus summarized as follows:

$\underline{V_{i,j} > 0, \text{ elliptic}}$

$$V_{i,j}(\phi^1_{xx})^E_{i,j} + (\phi^1_{yy})_{i,j} - [2i\Omega+(\phi^0_{xx})^E_{i,j}](\phi^1_x)^E_{i,j} = 0 \quad (39)$$

$\underline{V_{i,j} < 0, \text{ hyperbolic}}$

$$V^H_{i,j}(\phi^1_{xx})^H_{i,j} + (\phi^1_{yy})_{i,j} - [2i\Omega+(\phi^0_{xx})^H_{i,j}](\phi^1_x)^H_{i,j} = 0 \quad (40)$$

$\underline{V_{i,j} < 0, \; V_{i-1,j} > 0, \text{ parabolic}}$

$$V_{i,j}(\phi^1_{xx})^H_{i,j} + (\phi^1_{yy})_{i,j} - [2i\Omega+(\phi^0_{xx})^H_{i,j}](\phi^1_x)^H_{i,j} = 0 \quad (41)$$

where $V_{i,j}$ and $V^H_{i,j}$ and the elliptic (superscript E) and hyperbolic (superscript H) difference forms are as defined in Section 3.1 above.

Using the above forms, the finite difference equations are set up for each column (x = constant) in the grid and the resulting sequence of linear algebraic equations is solved by Gaussian elimination. The equations in this case are linear so that no iterative solution is required. Since the ϕ's are in general complex the above process requires complex operations which are straightforward on modern machines. After each column is solved, it is relaxed with a variable relaxation factor, depending on the local nature of the solution, as before. This process is repeated for each column in turn, sweeping the grid from left to right until the change in ϕ^1 for all grid points during one grid sweep is less than some arbitrary small amount (EPSGRD). A grid halving routine has been implemented in UTRANS in the same manner as described for STRANS, with similar improvements in the efficiency of the method.

The numerical treatment of the body boundary condition and Kutta condition for the unsteady perturbation potential are the same as described earlier for ϕ^0. The only difference is that all quantities such as the circulation and jump in ϕ across the wake are in general complex. The iteration procedure for $(\gamma_1)_{ff}$ is the same as that given in Section 3.1.3 above.

The present version of UTRANS accounts on option for two rigid body modes of oscillation; pitch and control surface oscillation. The specification of a non-zero hinge point indicates the desired mode is control surface oscillation. The respective airfoil boundary conditions are specified in subroutine INITAL and are simply:

$$\phi^1_{y|u,\ell} = -1 \text{ on } y = \pm 0,\ x_h < x < 1 \tag{42}$$

where x_h is the hinge point. Unsteady components of the generalized forces are also calculated by a trapezoidal rule integration of the unsteady pressure perturbation. Complete definitions of the forces are given in Section 5.0.

The farfield boundary conditions for ϕ^1 are defined from an asymptotic expression for subsonic freestream flow and a form of the unsteady characteristic relation for supersonic freestream flows. For $M_\infty < 1$ an asymptotic expression is used to fix values of ϕ^1 in the Dirichlet sense on all four boundaries of the grid, thereby closing the sequence of equations for ϕ^1 at a column of grid points. The expression was derived in Reference 3 and is given by:

$$\frac{4\sqrt{K}\ \phi^1_{ff}(x,y)}{\Omega\gamma_1 y} \simeq -\left(i\hat{k}\int_0^1 e^{-i\hat{k}\xi}\Delta\phi(\xi)\,d\xi + e^{-ik}\right)e^{i\hat{k}x}\,\frac{H_1^{(2)}(R_1)}{R_1} + I_1 + I_2 \quad , \qquad \text{for } R_1 \gg 1 \tag{43}$$

where I_1 is given by:

$$I_1 \sim \frac{2}{\sqrt{\pi}}\, e^{i\left(\frac{\pi}{4} - S(R_1)\right)} \sum_{n=0}^{\infty} \frac{a_n}{(2i)^n} \left\{ \sum_{\nu=0}^{\infty} (-i)^\nu \left.\frac{d^\nu g}{ds^\nu}\right|_{S(R_1)} \right\} \tag{44}$$

where

$$S(R) = \sqrt{R^2 - K\hat{k}^2 y^2} + R$$

$$g(S) = \frac{S^{n-1/2}(S^2 - K\hat{k}^2 y^2)}{(S^2 + K\hat{k}^2 y^2)^{n+3/2}}$$

$$a_n = \frac{(16-1)(16-3^2)\ldots[16-(2n-1)^2]}{n!2^{2n}}$$

and I_2 is:

$$I_2 = \begin{cases} 0 & \text{for } x \leq 1 \\ -2i \displaystyle\int_{\hat{k}\sqrt{K}\,y}^{R_1} \sin\sqrt{R^2 - K\hat{k}^2 y^2}\, \frac{H_2^{(2)}(R)}{R}\, dR & \text{for } x > 1 \end{cases} \quad (45)$$

where $\hat{k} = \Omega/K$, $R = \hat{k}\sqrt{(x-\xi)^2 + ky^2}$ and $R_1 = \hat{k}\sqrt{(x-1)^2 + Ky^2}$ and $H_i^{(2)}$ is the Hankel function of the second kind of ith order. The integral I_2 cannot be evaluated in closed form but, since it is over a finite range, can be integrated numerically with little difficulty. It is noted that the far-field approximation depends on the solution through the unsteady component of the airfoil circulation (γ_1) and also through the "airfoil integral." The airfoil contribution to the farfield involves an integral over the airfoil ($0 \leq x \leq 1$) of the jump in potential between the upper and lower surface. Thus as the airfoil circulation and potential distribution is updated, the farfield is periodically updated during the solution process, using Equation 43.

Supersonic freestream flow can also be treated by UTRANS in much the same manner as described above for STRANS. The appropriate small perturbation approximation to the characteristic relation for the unsteady perturbation potential is:

$$\phi_y^1 = \pm\left(\sqrt{-K} + \frac{\phi_x^0}{\sqrt{-K}}\right)\phi_x^1 \pm i\,\frac{\Omega}{\sqrt{-K}}\,\phi^1 \tag{46}$$

$$\text{on } y = \begin{cases} Y(JM) \\ Y(1) \end{cases}$$

This is incorporated into the finite difference approximation to ϕ_{yy} at the top and bottom of the grid using a one sided difference approximation as before. Upstream and downstream sides of the grid are treated as described above.

4.0 PROGRAM DESCRIPTIONS

Computer programs STRANS and UTRANS, used in conjunction, implement the theory and numerical solution procedure for unsteady transonic flow described in the previous two sections. As described above, the boundary value problems for the steady perturbation potential (Equation 10) and the unsteady perturbation potential (Equation 11) are solved in STRANS and UTRANS respectively using a finite difference relaxation procedure. The use and manipulation of magnetic tapes forms an integral part of the operation of each program as well as serving as the necessary "data link" between the two programs. As a result the user is assumed to have some familiarity with the use of tapes and their manipulation with control cards. The reading and writing of data files on magnetic tape is described in the next section and motivated in Section 6.0. In this section, the logical flow of the STRANS and UTRANS programs is described and a brief summary of each subroutine is presented. Both programs are quite similar in logical approach and operation, so that they are described together. Differences between the programs are highlighted with appropriate comments as needed.

The logical flow of the STRANS program is shown schematically in Figure 6 below. The general logic of the UTRANS program is almost identical with minor exceptions noted in the description below. The calculation is begun by reading card input and, if a restart is being performed, a tape dump. In UTRANS the tape dump of the steady solution being perturbed is also read. All finite difference coefficients and airfoil boundary conditions are initialized in a call to INITAL and subsonic farfield quantities are initialized in a call to FARFLD. If a restart is not being performed, initial values for ϕ at all grid points are determined by the linearized subsonic or supersonic solution. The computational cycle is executed by setting up the tridiagonal equations for a column of grid points using the mixed differencing finite difference equations. The equations are solved iteratively in STRANS and in one pass in UTRANS, by Gaussian ellimination in a call to TRI. Each column is solved and relaxed in turn proceding through the grid from left to right. The grid is swept iteratively in this manner until the change in ϕ for all grid points is less than EPSGRD(1). A call to PRINT prints out the airfoil pressure coefficients every NPRINT iterations, and the farfield is updated every NGFF iterations, in FARFLD.

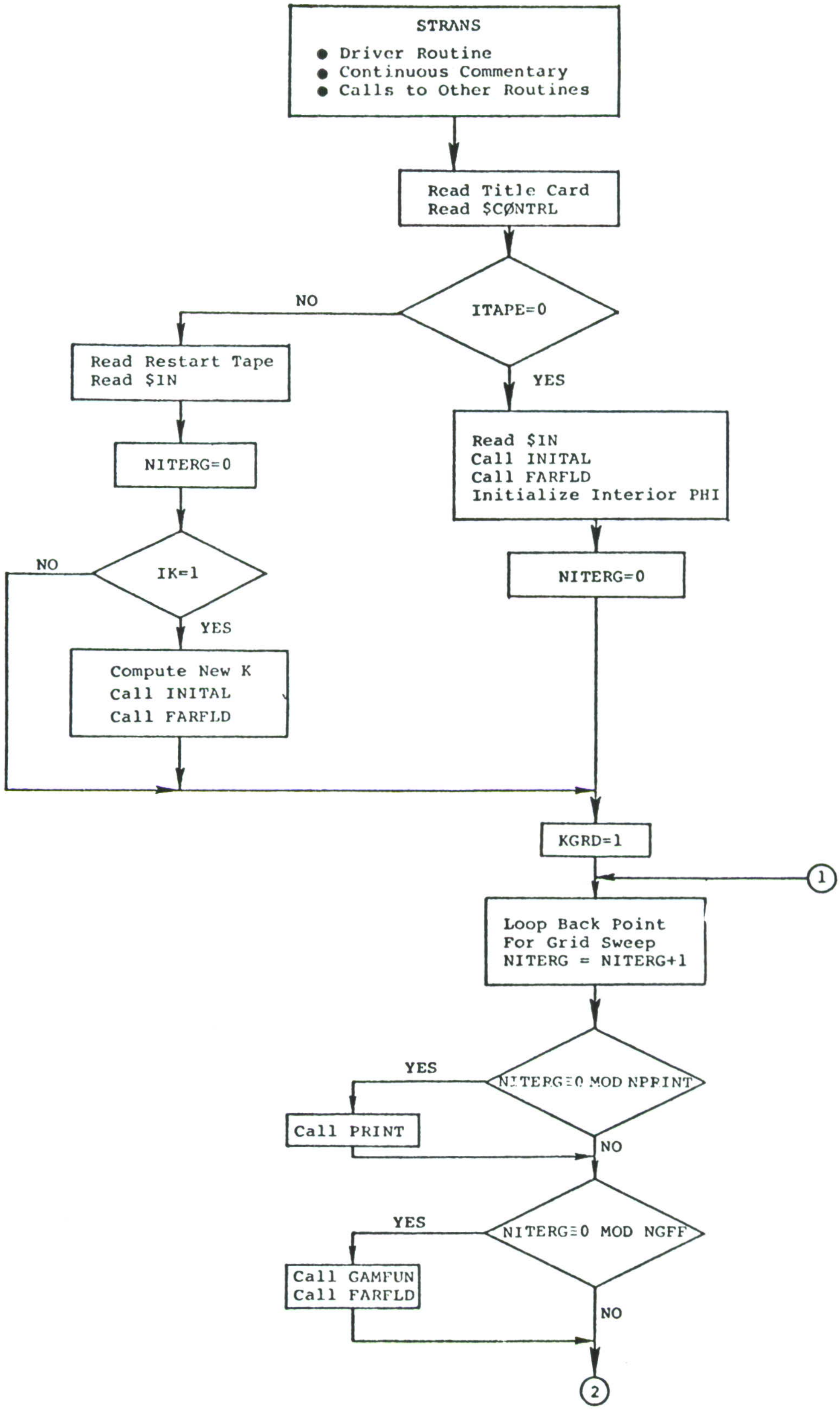


FIGURE 6. LOGICAL FLOW OF STRANS PROGRAM

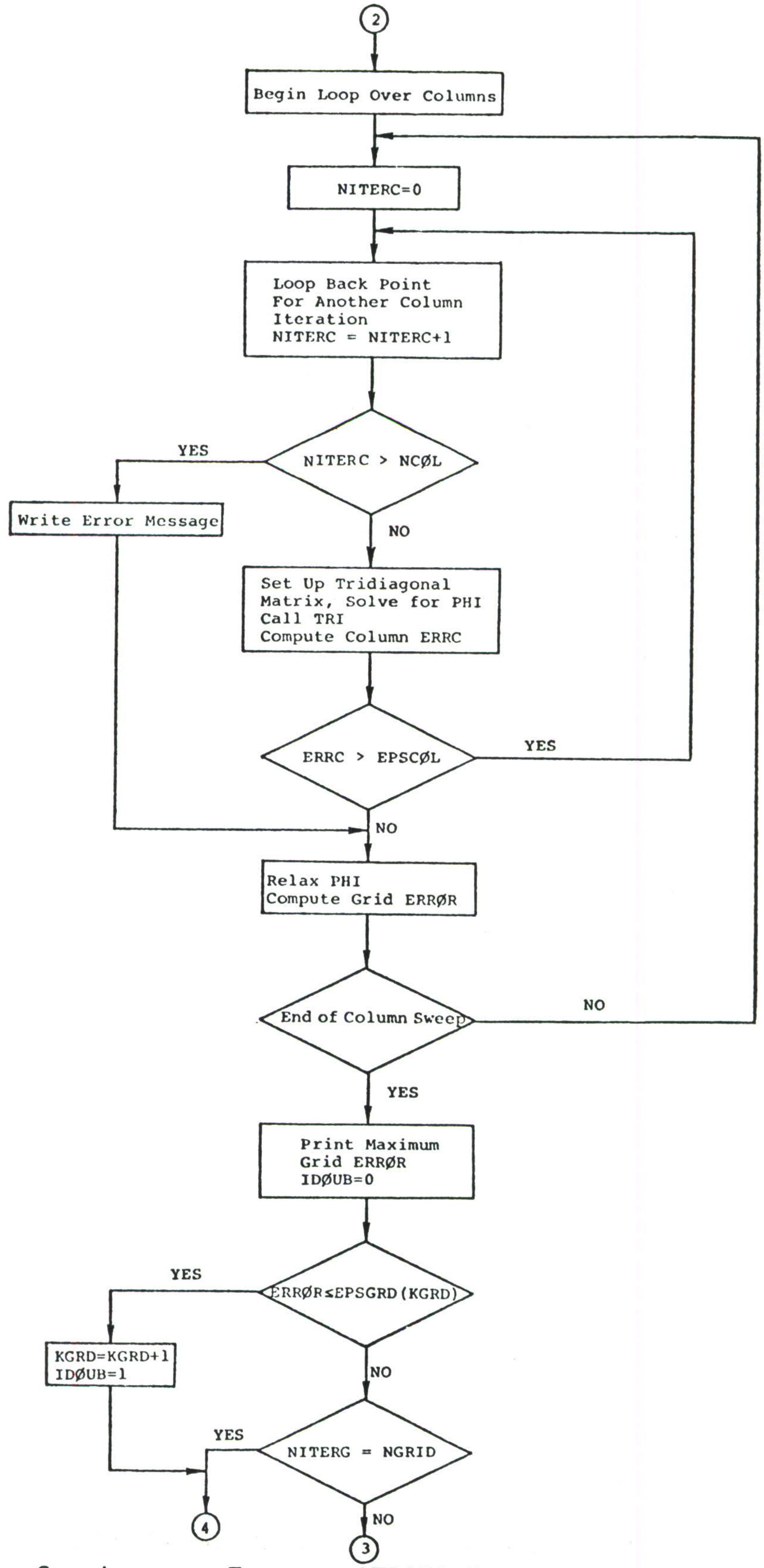


FIGURE 6. LOGICAL FLOW OF STRANS PROGRAM (CONTINUED)

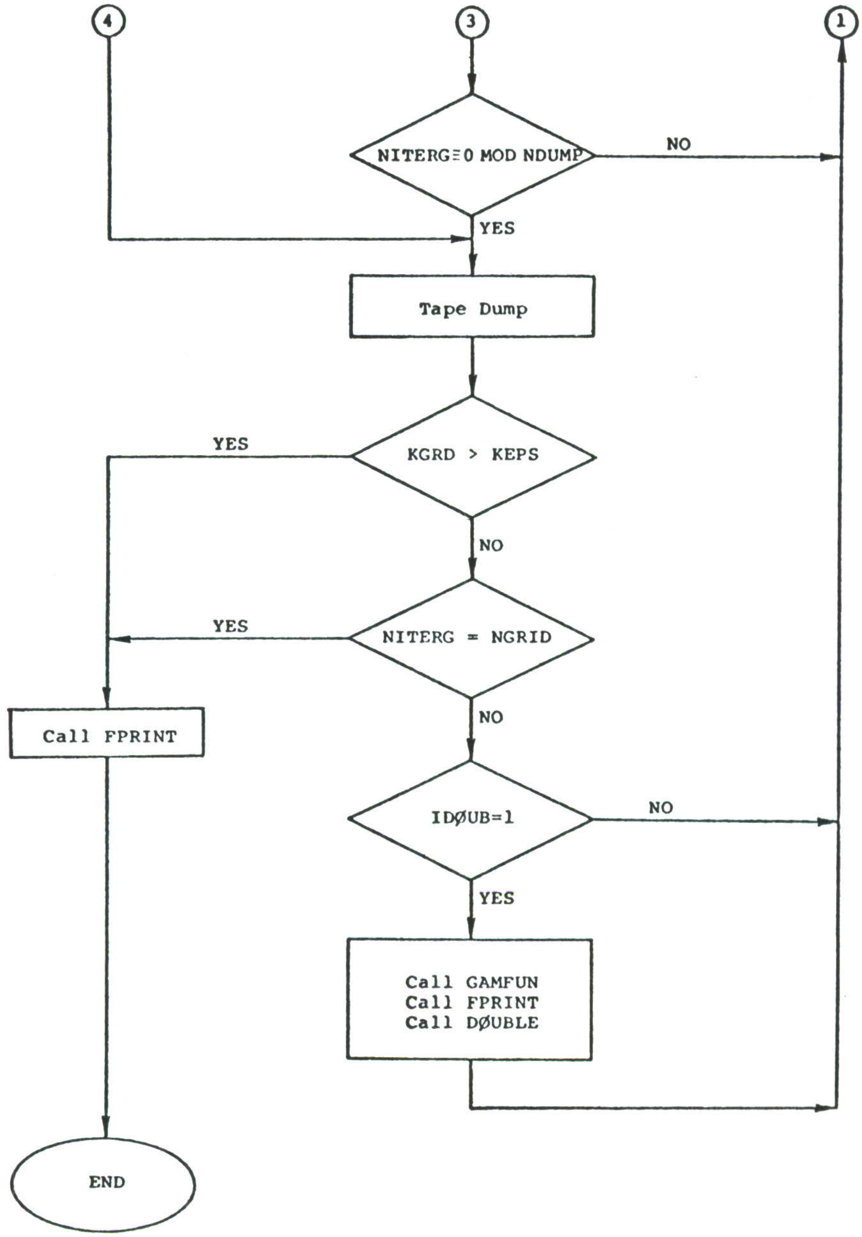


FIGURE 6. LOGICAL FLOW OF STRANS PROGRAM (CONTINUED)

When the converged solution is obtained, a tape dump of all relevant input and calculated quantities is performed and a call to FPRINT calculates and prints out the airfoil pressure and force coefficients. Various diagnostic prints are also performed in STRANS and UTRANS after every grid iteration, when the farfield is updated, when the grid is refined and when a tape dump is performed. Finally, if desired, the grid is refined by doubling all grid lines in a call to subroutine DØUBLE, and the computational cycle is begun anew until the new solution has again converged or until the maximum number of iterations (NGRID) has been exceeded. In either case, a final tape dump and final print are executed.

A summary of each subroutine is now presented.

STRANS/UTRANS

These are the driver routines for the respective programs. The logical flow of the mixed differencing relaxation procedure as just described is controlled by these routines and all operations including input, initialization, finite difference solution and output are performed either internally or by calls to the various subroutines described below.

DØUBLE

This routine refines the grid spacing by doubling the number of grid lines. New grid points are added in both the X and Y direction by placing grid lines midway between the existing ones. Initial values for ϕ at the new grid points are determined by a linear interpolation. Calls are made to INITAL and FARFLD, to reevaluate finite difference coefficients and regenerate farfield values for ϕ respectively.

FARFLD

The subsonic farfield is calculated and updated in this routine using the asymptotic solutions for the steady (Equation 37) or unsteady (Equation 43) perturbation potentials. In the STRANS version, the doublet strength (DØUBLT) is also updated here by performing a trapezoidal rule integration over the finite difference grid of the expression:

$$DØUBLT = DØUB + \iint (\phi_x^0)^2 dxdy$$

FL (in STRANS only)

This is a function statement which contains the airfoil lower surface shape function. This function appears in the supersonic small disturbance solution which is used in STRANS as an initial guess for ϕ^0 below the airfoil for supersonic freestream flows.

FLP (in STRANS only)

This is a function statement which contains the airfoil lower surface slope distribution used in the linearized tangency boundary condition. This function is called from subroutine INITAL and its value at each grid point on the lower surface of the airfoil is stored in the FPL array.

FPRINT

This routine produces the final print and is called when the solution has converged to the desired accuracy or when the problem is terminated for reaching the maximum number of grid iterations allowed (NGRID). The unscaled pressure coefficients above and below the airfoil and the airfoil force coefficients are also calculated and printed out in this routine.

FU (in STRANS only)

This is a function statement which contains the airfoil upper surface shape function. This function appears in the supersonic small disturbance solution which is used in STRANS as an initial guess for ϕ^0 above the airfoil for supersonic freestream flows.

FUP (in STRANS only)

This is a function statement which contains the airfoil upper surface slope distribution used in the linearized tangency boundary condition. This function is called from subroutine INITAL and its value at each grid point on the upper surface of the airfoil is stored in the FPU array.

The doublet strength due to airfoil thickness (DØUB) must also be given in this subroutine. This quantity is defined by an integral of the airfoil thickness distribution function (normalized to airfoil thickness):

$$DØUB = \int_0^1 t(\xi)d\xi$$

GAMFUN

This routine performs the extrapolation to update farfield circulation (GAMFF).

HANKEL (in UTRANS only)

This subroutine calculates the first and second order Hankel functions of the second kind for real arguments using curvefits developed in Reference 10. These functions are used in the calculation of the farfield for the unsteady perturbation potential.

INITAL

The finite difference coefficients AX1, AX2, BX1, BX2, CX, AY1 and AY2 and ΔX(DX) and ΔY(DY) are computed in this subroutine. The airfoil boundary conditions FPU and FPL are also set here, using functions FUP and FLP respectively.

PRINT

This routine computes and prints the scaled pressure coefficients above and below the airfoil every NPRINT grid iterations.

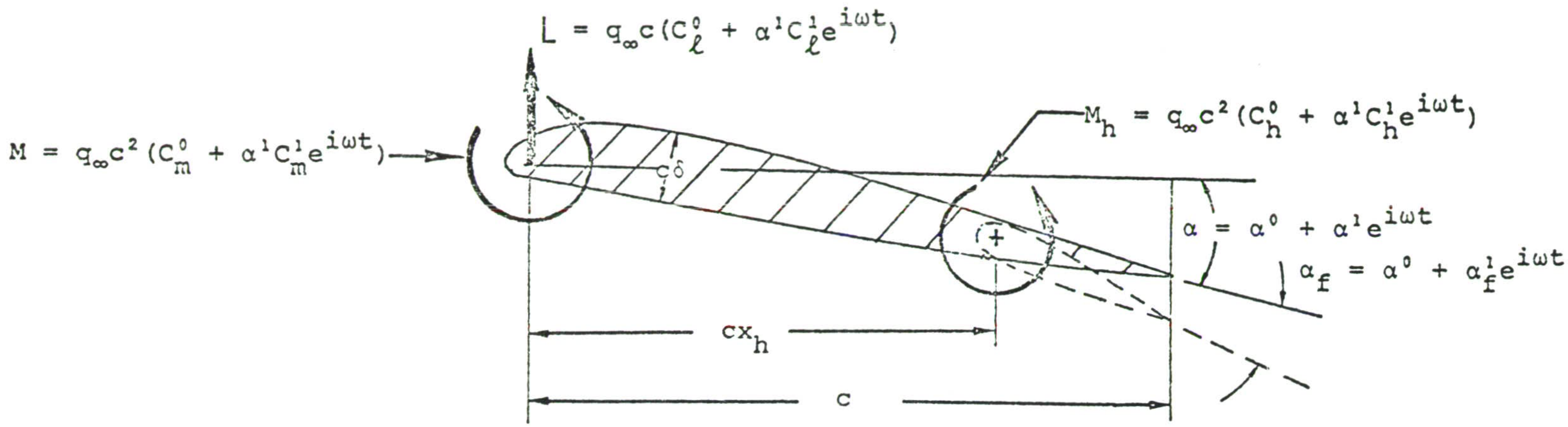
TRI

This routine solves a system of tridiagonal equations using Gaussian elimination.

5.0 INPUT AND OUTPUT

A description of the input required to run STRANS and UTRANS, and the output associated with each program are presented in this section. All card input is entered using the standard CDC NAMELIST package with the exception of a title card.

Figure 7 below is presented to summarize the definitions of various important parameters used in STRANS and UTRANS. It is reiterated that STRANS calculates the steady transonic flow about a general thin airfoil with mean angle of attack α^0 and with a control surface with hinge point x_h and mean flap angle α_f^0. UTRANS calculates the unsteady perturbation to this steady flow consisting of an harmonic oscillation in angle of attack about the nose or of a control surface oscillation. The output parameters of greatest interest are the airfoil pressure and force coefficients which are printed out once the final converged solution is obtained. The definitions of the scaled and unscaled forms of the pressure coefficients are shown in Figure 7. Also defined in the figure are the relations used in the programs for the steady and unsteady components of the generalized lift, moment about $x = 0$ and hinge moment coefficients. It is noted that counterclockwise moments are defined positive and that the unsteady perturbations to all coefficients are given per unit angle of oscillation. The unsteady perturbations to pressure and force coefficients are in general imaginary so that the real and imaginary components are printed out in the final print of UTRANS.



$$C_p = \frac{C_p}{\overline{C}_p}\left(\overline{C}_p^0 + \frac{\alpha^1}{\delta}\,\overline{C}_p^1\, e^{i\omega t}\right) \quad \text{(Unscaled)}$$

$$\overline{C}_p^0 = -2\phi_x^0\,,\quad \overline{C}_p^1 = -2\phi_x^1 \quad \text{(Scaled)}$$

} Pressure Coefficient

STRANS		UTRANS
$C_p = \frac{C_p}{\overline{C}_p}\,\overline{C}_p^0$		$C_p = \frac{1}{\delta}\frac{C_p}{\overline{C}_p}\,\overline{C}_p^1$
$C_\ell^0 = \int_0^1 (C_{p\ell}^0 - C_{pu}^0)\,dx$	(Lift)	$C_\ell^1 = \int_0^1 (C_{p\ell}^1 - C_{pu}^1)\,dx$
$C_m^0 = \int_0^1 (C_{p\ell}^0 - C_{pu}^0)\,x\,dx$	(Moment)	$C_m^1 = \int_0^1 (C_{p\ell}^1 - C_{pu}^1)\,x\,dx$
$C_h^0 = \int_{x_h}^1 (C_{p\ell}^0 - C_{pu}^0)(x-x_h)\,dx$	(Hinge Moment)	$C_h^1 = \int_{x_h}^1 (C_{p\ell}^1 - C_{pu}^1)(x-x_h)\,dx$

FIGURE 7. DEFINITION OF PRESSURE AND FORCE COEFFICIENTS

5.1 STRANS Input

The input for STRANS is now considered in three sets. Recommended and/or typical values for some of the input variables which control the numerical scheme, appear in parentheses. Also presented at the end of this section is a description of the restart capability which requires input from a magnetic tape dump of a previous calculation.

First Set

BCD title card containing any information in columns 1 through 80 (Format 8A10). This can be used to define the case being run and is printed out on the last page of output which presents the final converged results.

Second Set

The second set of data is read in under NAMELIST name $CØNTRL. The single variable read defines the use of the restart option. Some comments concerning the mechanics of the use of this option are given at the end of the section.

NAME	DESCRIPTION
ITAPE	This is a flag for using a restart tape. ITAPE = 0 means the problem is being started from scratch (iteration 0) using an initial guess defined in STRANS. ITAPE = 1 means the problem is being restarted from a previous run which is to be read from a dump tape.

Third Set

The third set of data is read in under NAMELIST name $IN, and includes all of the variables required to define a problem, and control the numerical iteration procedure.

NAME	DESCRIPTION
X	An array containing the streamwise grid coordinates; IM of them
Y	An array containing the normal grid coordinates; JM of them
IM	Number of grid points in the streamwise direction (maximum of 100)
JM	Number of grid points in the normal direction (maximum of 100)
ILE	I location of airfoil leading edge (X(ILE))
ITE	I location of airfoil trailing edge (X(ITE))
JW	J location of airfoil (Y(JW))
M8	Freestream Mach number
GAM	γ, ratio of specific heats
DEL	Airfoil thickness ratio in percent
ALPHA	Airfoil angle of attack in radians
ALPHAF	Flap angle in radians
XH	X location of the hinge point on the airfoil ($0 \leq XH \leq 1$)
GAMFF	Initial guess for the airfoil circulation; to be used in the initialization of the farfield
NGFF	Every NGFF grid iterations the farfield circulation and doublet strength are updated. This also causes the farfield to be updated. (∿10)
PGFF	Relaxation parameter used in the iteration for the airfoil circulation in the farfield (∿1.5)
ØMEGAH	Relaxation parameter for hyperbolic grid points (∿.75)

NAME	DESCRIPTION
ØMEGAE	Relaxation parameter for elliptic grid points (∿1.7)
ØMEGAP	Relaxation parameter for parabolic grid points (∿.75)
EPSCØL	Convergence criteria for column solution. The change in $\vert\phi^0\vert$ during a column iteration at every point in the column must be less than EPSCØL for convergence to occur (∿5 x 10^{-5})
NCØL	Maximum number of column iterations allowed. Note that if NCØL iterations is reached without convergence, a printout of the degree of convergence is given and the calculations proceed as if convergence had occurred (∿10)
EPSGRD	An array containing criteria to control grid convergence. The change in $\vert\phi^0\vert$ at every grid point during one grid sweep must be less than EPSGRD(1) for convergence to occur. The number of grid lines is then doubled if KEPS > 1 and EPSGRD(2) is used to determine grid convergence, and so on. KEPS values of EPSGRD are input.
KEPS	The number of EPSGRD's input (maximum of 3). The grid is refined KEPS-1 times.
NGRID	Maximum number of grid iterations allowed. When the number of grid iterations equals NGRID the calculation is terminated.
NDUMP	Binary tape dump frequency. Every NDUMP grid iterations current values of all variables will be dumped on tape. Note that a tape dump occurs automatically whenever the grid converges or the number of grid iterations equals NGRID (set equal to large number if a dump of only the final iteration is desired).
NPRINT	Every NPRINT grid iterations the scaled pressure coefficient above and below the airfoil is printed.

NAME	DESCRIPTION
IK	Setting IK = 1, flags the use of a previous solution as an initial guess for the current problem where the Mach number, airfoil thickness or shape and/or angle of attack may be different.

The input data listed above are necessary to initiate a calculation for which no initial guess is available. Most calculations, however, are performed as restarts using data which has been stored as binary files on the restart tape according to the format described in Section 5.2. This use of the restart capability is an inherent aspect of the recommended computational procedure as is discussed in some detail in Section 6.0. Some brief comments describing the initiation of a calculation using the restart capability are pertinent at this juncture.

It is noted that the restart or dump tape (TAPE7) may be manipulated in any way desired using the appropriate control cards. In general the tape will contain data from many runs, stored as individual binary files. For restarting the STRANS program, the desired file from the restart tape (TAPE7) is copied to a disc file (TAPE8). The user is reminded to rewind TAPE8. TAPE7 is then positioned at the end of the last file on the tape so that new dumps can be written by the program without losing any of the old data. The first two sets of data are then input with ITAPE=1. In the third set of data the following control variables are needed as input:

ØMEGAH, ØMEGAE, ØMEGAP, EPSCØL, EPSGRD, NDUMP, NCØL, NGRID, NGFF, PGFF, KEPS and NPRINT.

The remaining input variables are stored on the restart tape and need not be input unless the restart option is being used to run a new case. If a new case is being run, IK must be set to 1 which allows the Mach number, airfoil thickness, airfoil angle of attack, flap angle and/or the hinge position (M8, DEL, ALPHA, ALPHAF and XH) to be changed. If the airfoil angle of attack and/or the flap angle are changed a new guess for the farfield circulation (GAMFF) can and should be made.

5.2 STRANS Output

The output from STRANS consists of three parts: (i) a continuous commentary which describes the progress of the iterative solution procedure, (ii) a final print summarizing results of interest from the final converged solution, and (iii) a binary tape dump of all pertinent input and calculated parameters.

The continuous commentary consists of various print statements executed in the main program STRANS or the subroutine PRINT which describe the current state of the solution as well as the occurrence of various "milestones" in the iteration process. The only print that occurs every iteration is the value of the maximum change in ϕ^0 throughout the grid during one grid iteration. When a column iteration fails to converge, a print occurs which defines the degree of column convergence and the j location of the most poorly converged point. Every NGFF iterations, the subsonic farfield is updated and the new values of farfield circulation (GAMFF), airfoil circulation (GAMTE) and nonlinear doublet strength (DØUBLT) are printed. The user can examine the effect of degree of convergence on the solution by specifying a print of the scaled pressure coefficients on the upper and lower airfoil surfaces every NPRINT iterations. Finally, a descriptive print occurs at certain milestone points such as the doubling of grid lines, the occurrence of a binary tape dump and solution convergence.

The final print is executed in subroutine FPRINT when the solution has converged to the desired accuracy or when the number of grid iterations equals NGRID. The print is self-explanatory and includes the input parameters which define the problem and various calculated quantities of interest. The calculated quantities are of course based on the final converged solution. The steady force coefficients, as defined in Figure 7, are printed out as well as the upper and lower surface pressure coefficients with corresponding airfoil coordinate.

The most important form of STRANS output is the binary tape dump of all input parameters defining the problem and of the most recent values of ϕ^0 at all grid points. A tape dump occurs automatically if the solution has converged to the desired accuracy or if the number of grid iterations equals NGRID. The user may also specify that such a dump occur every NDUMP grid iterations. The tape so generated, not only forms a permanent record of the results of a calculation for possible future editing and examination but also forms a necessary part of the computational procedure. Most important is its use as required input for a UTRANS calculation. However, it may also be used to restart the calculation to refine accuracy or convergence or be used as the initial guess for ϕ^0 throughout the grid for a similar calculation, as described in Section 5.1.

The format used for writing and reading the binary tape is given in the following FORTRAN statements:

```
      WRITE (7)      NITERG, IM, IM1, JM, JM1, JW, JWP1,
                     JWM1, ITE, ILE, K, DEL, ALPHA,
                     GAMTE1, GAMFF, NDB, XH, M8, GAM,
                     DYBU1, DYBU2, DYBL1, DYBL2, DØUB,
                     DØUBLT, ALPHAF

      WRITE (7)      (X(I), I = 1, IM)

      WRITE (7)      (Y(I), I = 1, JM)

      WRITE (7)      (FPU(I), FPL(I), PHIUB(I), I = ILE,
                     ITE)

      WRITE (7)      (AX1(I), AX2(I), BX1(I), BX2(I),
                     CX(I), DX(I), I = 1, IM1)

      WRITE (7)      (AY1(I), AY2(I), DY(I), I = 1,JM1)

      WRITE (7)      (PHI(I), I = 1,L)

      END FILE 7

        where IM1 = IM-1, JM1 = JM-1, L = IM*JM
```

Any information may be retrieved from the tape by using the appropriate READ statements as is done in the restart option described above.

5.3 UTRANS Input

The input for UTRANS consists of normal card input plus input from a binary file which contains the steady solution generated by an STRANS run. UTRANS also has a restart capability which is implemented in exactly the same manner as previously described in Section 5.1 for STRANS and elaborated upon at the end of this section. The required input is now described and some comments are presented at the end of this section pertaining to the tape read of the steady solution. As before, the card input is described in three sets.

First Set

BCD title card containing any information in columns 1 through 80 (Format 8A10).

Second Set

The second set of data is read in under NAMELIST name $CØNTRL.

NAME	DESCRIPTION
ITAPE	This is a flag for using a restart tape. ITAPE=0 means the problem is being started from scratch (iteration 0), ITAPE=1 means the problem is being restarted from a previous run using the restart tape. Note that a tape is also used for the input of steady results independent of the value of ITAPE.

Third Set

The third set of data is read in under NAMELIST name $IN.

NAME	DESCRIPTION
X	An array containing the streamwise grid coordinates; IM of them.
Y	An array containing the normal grid coordinates; JM of them.
IM	Number of grid points in the streamwise direction (maximum of 100).

NAME	DESCRIPTION
JM	Number of grid points in the normal direction (maximum of 100).
ILE	I location of airfoil leading edge (X(ILE)).
ITE	I location of airfoil trailing edge (X(ITE)).
JW	J location of airfoil (Y(JW)).
SMALLK	Reduced frequency based on chord = $\omega c/U$.
XH	X location of the hinge point on the airfoil. Set XH=0 for oscillation in pitch ($0 \leq XH \leq 1$).
GAMFF	Initial guess for the airfoil circulation used in the initialization of the farfield. Note that GAMFF is a complex number.
NGFF	Every NGFF grid iterations the airfoil circulation in the farfield is updated. This also causes the farfield to be updated ($\sim$10).
PGFF	Relaxation parameter used in the iteration for the airfoil circulation in the farfield ($\sim$1.5).
ØMEGAH	Relaxation parameter for hyperbolic grid points ($\sim$.75).
ØMEGAE	Relaxation parameter for elliptic grid points ($\sim$1.7).
ØMEGAP	Relaxation parameter for parabolic grid points ($\sim$.75).
EPSGRD	An array containing criteria to control grid convergence. The change in $\|\phi^1\|$ at every grid point during one grid sweep must be less than EPSGRD(1) for convergence to occur. The number of grid lines is then doubled if KEPS > 1 and EPSGRD(2) is used to determine grid convergence and so on. KEPS values of EPSGRD are input.
KEPS	The number of EPSGRD's input (maximum of 3).

NAME	DESCRIPTION
NGRID	Maximum number of grid iterations allowed. When the number of grid iterations equals NGRID the calculation is terminated.
NDUMP	Binary tape dump frequency. Every NDUMP grid iterations current values of all variables will be dumped on tape. Note that a tape dump occurs automatically whenever the grid converges or the number of grid iterations equals NGRID. (Set equal to large number if a dump of only the final iteration is desired.)
NPRINT	Every NPRINT grid iterations the scaled upper and lower surface pressure coefficient per unit angle of oscillation is printed.
IK	Setting IK=1 allows the user to use a previous solution as an initial guess for the current problem where the reduced frequency and/or mode of oscillation is different.

It is recalled, that the solution for the unsteady perturbation potential, implemented in UTRANS, requires the solution of the steady potential, generated by STRANS. This is accomplished by reading the appropriate file on a dump tape generated by STRANS, in much the same way as is done in the restart option. It is instructive to briefly describe the UTRANS restart including the tape read of the steady solution.

Restarting the UTRANS program is only slightly more complicated than STRANS. In this case, two tape dumps or files are required. First the file containing the desired steady tape dump is copied from TAPE7 to a disc file, TAPE8. Next the file containing the desired unsteady tape dump is copied from TAPE7 to a disc file, TAPE9, and TAPE8 and TAPE9 are rewound for reading by the program. TAPE7 is then positioned at the end of the last file on the tape in preparation for accepting a new tape dump. The first two sets of data are input as before (be sure to set ITAPE=1). In the third set of data the following variables are necessary:

ØMEGAH, ØMEGAE, ØMEGAP, EPSGRD, NDUMP, NGRID, NGFF, PGFF, KEPS and NPRINT.

Again there is an option (IK=1) which allows the user to change the reduced frequency and/or the mode of oscillation (SMALLK and XH). In either case a new guess for the farfield circulation (GAMFF) should be made.

5.4 UTRANS Output

The output from UTRANS is very similar to that of STRANS and includes a continuous commentary, final print and binary tape dump.

The printed output is of the form described above for STRANS. The only difference is that the field variables in UTRANS are complex so that the real and imaginary parts are printed out in that order. The descriptive prints are all the same with the deletion of the unneeded comment on column convergence. The final print is executed in subroutine FPRINT when the solution has converged or has reached the maximum number of iterations desired by the user (NGRID). The print includes all important input variables which define both the steady solution being perturbed and the unsteady solution being generated. Also, various calculated quantities, based on the final converged solution, are printed. These include the real and imaginary parts of the unsteady contribution (per unit angle of oscillation) to the aerodynamic force coefficients as defined in Figure 7 above. Also unsteady contributions to the upper and lower surface pressure coefficients (per unit angle of oscillation) are printed for every computational point on the airfoil. It is again noted that these are complex so that the real and imaginary parts are printed out in order.

The other form of UTRANS output is the binary tape dump of all input parameters and the most recent values of ($Re\phi^1$, $Im\phi^1$) at all grid points. As before the tape dump occurs automatically at normal program termination or at the users discretion every NDUMP iterations. The format used for writing and reading the binary tape is given in the following FORTRAN statements:

```
WRITE (7)      NITERG, IM, IM1, JM, JM1, JW,
               JWP1, JWM1, ILE, ITE, GAMTE1,
               GAMFF, ØMEG, SMALLK, DYBU1, DYBU2,
               DYBL1, DYBL2, NDØUB, XH

WRITE (7)      (X(I), I = 1, IM)

WRITE (7)      (Y(I), I = 1, JM)

WRITE (7)      (FPU(I), FPL(I), PHIUB(I), I =
               ILE, ITE)

WRITE (7)      (AX1(I), AX2(I), BX1(I), BX2(I),
               CX(I), DX(I), I = 1, IM1)

WRITE (7)      (AY1(I), AY2(I), DY(I), I = 1, JM1)

WRITE (7)      (PHI(I), I = 1, L)

WRITE (7)      (CØNAFF(I), PHIAFF(I), I = 1, NPT)

END FILE 7

  where IM1 = IM-1, JM1 = JM-1, L = IM*JM, NPT =
               2*(IM-3) +3*JM
```

It is again noted that the above formats can be used to retrieve data from the dump tape using the appropriate READ statements.

6.0 PROGRAM USAGE

A summary of the basic information needed for using the STRANS and UTRANS programs and some hints on setting up and executing a calculation are presented in this section. The comments given here are incorporated in the sample cases, presented in the next section.

By way of introduction, it should be noted that the programs are not now and may never be of a degree of sophistication for which they may be treated as "black boxes" for the calculation of transonic aerodynamic coefficients. Finite difference numerical schemes for nonlinear problems such as transonic flow are not yet sufficiently advanced to permit the calculation of general cases without the care, attention and often agonizing of the user. Considerable judgment is required of the user in exercising the programs, and this can only come through experience with similar calculations. Section 6.1 is included with the hope of providing some guidance concerning various important computational details. In addition, Section 6.2 provides a summary of recommended procedures for exercising the programs to provide the first time user with some baseline experience level based on calculations performed to date with the programs.

6.1 Computational Details

In this section the general structure of the computer programs is first discussed followed by comments on a variety of details for setting up and running a calculation.

In their present configuration, both STRANS and UTRANS allow a maximum of 10000 computational grid points and the number of grid lines in the streamwise and normal directions must each be less than or equal to 100. In this configuration, STRANS requires 63_8K words to load and 51_8K words to execute while UTRANS requires 144_8K words to load and 132_8K words to execute. Greater storage is required by UTRANS because the unsteady potential is complex so that $\mathrm{Re}\phi^1$ and $\mathrm{Im}\phi^1$ must be stored in addition to storage required for the ϕ^0 solution being perturbed. The programs have been exercised to date on CDC 6600 and 7600 computers. Based on calculations performed to date on the CDC 7600, using the extended FORTRAN compiler (FTN) with full optimization (ØPT=2), typical execution times are 4.8×10^{-5} CPU sec/grid point/iteration for STRANS and 6.6×10^{-5} CPU sec/grid point/iteration for UTRANS. Execution times on the CDC 6600 using the same compiler are generally a factor of 4 greater. The execution times given here are useful for estimating computer time requirements for a calculation.

Specification of Airfoil Shape: All quantities required to perform an STRANS calculation are input to the program via cards or magnetic tape (restart) with the exception of the airfoil shape. As indicated in Section 4.0, the airfoil upper and lower surface shape functions (normalized to the airfoil thickness) are given in function subprograms FU and FL respectively and the upper and lower surface slopes ($f_u^{0\prime}(x)$ and $f_\ell^{0\prime}(x)$) are given in FUP and FLP respectively. FU and FL are needed only for initial calculations with a supersonic freestream and are used in the initial guess for values of ϕ^0 in the grid. $f_u^{0\prime}$ and $f_\ell^{0\prime}$ and the doublet strength due to thickness, DØUB, need to be specified for both subsonic and supersonic freestream cases as they are used in the airfoil boundary condition. The functions presently programmed in STRANS are for a symmetric circular arc airfoil. Thus to run calculations for another airfoil, the respective statements need to be changed in the function subprograms. The specification of airfoil slopes presents some problem since most airfoil shapes are not defined by simple analytic functions but by tabulated points. The recommended procedure for calculating the needed slopes is to perform a cubic spline fit to the data. However, good results have been obtained with STRANS using simpler methods such as linear interpolation to calculate slopes. The effect of angle of attack (ALPHA) and flap angle (ALPHAF) on the airfoil shape and slope functions are treated automatically in STRANS using the specified input quantities.

UTRANS automatically treats two rigid body modes of oscillation: pitch and control surface rotation. No program modifications are required as the different airfoil boundary conditions are determined internally using the specified input. The flag for one mode or the other is the hinge point (XH); if XH=0 the pitch mode is treated and if XH≠0 ($0 \leq XH \leq 1$) control surface oscillations are considered. The method is not restricted to these modes but would require modifications to FPU(I) and FPL(I) in subroutine INITAL to treat various flexible modes of oscillation.

Grid Design: The variable mesh size technique, used in STRANS and UTRANS, allows considerable flexibility in the choice of a computational grid. The basic reason for this capability is that grid points may be concentrated near the airfoil, where a high degree of resolution is desired. The grid spacing may then be expanded away from the airfoil out to the boundaries of the mesh, thereby providing for an economic use of grid points. Economy of use is particularly important for UTRANS calculations since three values (ϕ^0, Reϕ^1, Imϕ^1) must be stored at each nodal point.

The basic consideration in grid selection is that grid spacing vary smoothly so that sudden jumps in Δx or Δy should be avoided. As with most numerical techniques, considerations of error in the finite difference approximation indicate that the ratio of the spacing between adjacent grid points lie between 1/2 and 2. Some additional guidelines for grid design are that grid points be concentrated near the leading edge to resolve the blunt stagnation point singularity and the pressure singularity in a calculation with lift. If shocks are expected in the calculation, grid points can be concentrated near their expected location. Choice of a grid for unsteady calculations follows similar guidelines with regard to concentration of grid points. Unsteady calculations in general involve lift perturbations to the steady flow so that the leading edge or hinge point singularity must be resolved. Also, if the steady flow has shocks or rapid compressions, wave energy will be concentrated there so that a concentration of grid points is required to resolve the resulting unsteady pressure peak. It should be noted that UTRANS requires values of ϕ^0, generated by STRANS, at each point in the grid. No provision is made for interpolating, so that the UTRANS calculation must have the same basic grid as the STRANS calculation being perturbed. A provision is made which allows the ϕ^0 data to be from a more refined grid which was obtained using the grid halving technique on the equivalent UTRANS grid. The utility of grid halving is described below.

A basic grid which has provided accurate results in calculations performed to date consists of about 50 grid lines in both normal and streamwise directions. Approximately half of the grid columns (X = constant) are concentrated on the airfoil and the grid rows (Y = constant) are divided equally above and below the airfoil. Slight variations on this grid are given in the test cases below. Numerical studies which further refined the basic grid described here, have shown little change in the results for pressure coefficient on the airfoils, for example, with the singular exception of increased resolution of a shock wave.

It is judged that the basic grid is sufficiently accurate for most engineering applications.

Farfield Location and Update: The only other question in grid design is how far to expand the grid out to the farfield. The fact that the perturbation flowfield extends to large lateral distances for transonic flows, makes this an especially important question. The lateral extent of the nonlinear flowfield, varies considerably depending upon the transonic similarity parameter, so that no specific answer to the question is possible. In general though, the farfield must be far enough away from the airfoil that: (i) for subsonic freestream flows, the asymptotic expressions used for setting ϕ_{ff}^{0} (Equation 37) or ϕ_{ff}^{1} (Equation 43) be an accurate representation of the solution, and (ii) for supersonic freestreams, the perturbation is weak and the flow is entirely supersonic on the boundary. For subsonic flow, considerations of accuracy of the analytical farfield expressions indicate that Y(JM), Y(1) = ±6 and X(IM), X(1) = ±3 are sufficient at least for $M_\infty \lesssim .95$, $\delta \leq 0.1$, $\alpha \lesssim 2^o$ and Ω and $K \sim 1$. It is noted in passing, that numerical studies have shown that the farfield approximation has a weak effect on steady airfoil pressures and a slightly stronger effect on the unsteady pressures on the airfoil.

It is recalled that both the steady and unsteady subsonic farfields depend upon the solution through the nonlinear doublet strength (DØUBLT) and the airfoil circulation (GAMFF). These quantities are thus part of the solution process (as described in Section 3.0) and couple back into the solution through their effect on the farfield and wake. The user can control the iteration process for DØUBLT and GAMFF by specifying NGFF, the number of iterations between farfield updates and PGFF, the relaxation parameter for updating GAMFF. It is suggested that NGFF be somewhat greater than the number of grid points between the airfoil and the farfield to allow the updated farfield enough iterations to influence the entire grid before updating again. A suggested value for PGFF is 1.5 which has the effect of an overrelaxation on past values of airfoil circulation. Such values for both parameters have worked well in the past although no detailed attempt at determining optimum values has yet been made.

Relaxation Factors: The matter of the choice of optimum relaxation parameters (ω) is a complex one and has only been studied in detail for simple cases. In the case of the steady transonic potential equation, values varying from $\omega_e = 1.4 \rightarrow 1.9$ for elliptic points and $\omega_h = 0.75 \rightarrow 0.9$

for hyperbolic points have been used. The "best" value can vary significantly with respect to the transonic similarity parameter K, however limited numerical studies[3] indicate a value for $\omega_e \sim 1.7$ is close to optimum for slightly supercritical flows. Similar studies[3] have indicated that the relaxation procedure for ϕ^1 (UTRANS) may become unstable for values of ω_h much greater than 0.75 and the best values for ω_e and ω_h seem to be strongly related to the unsteady similarity parameters K and Ω.

Accuracy and Convergence: Questions of accuracy and convergence of the numerical schemes implemented in STRANS and UTRANS are of considerable importance relative to the utility of the programs as engineering tools. The accuracy of the small perturbation approximation for steady flows has been discussed in many studies[4,5,6] and STRANS results are consistent with these findings[3]. The question of the accuracy of the linearized unsteady perturbation approximation used in UTRANS is an open one but results to date[3] have been quite promising. Of particular interest to the user of the programs is the degree of convergence and grid definition required for practical accuracy. Clearly the trade-off, which plagues any numerical technique, is the cost of computer time. Some comments concerning this question are therefore appropriate.

The use of successive grid refinements to speed convergence is implemented in both STRANS and UTRANS in subroutine DØUBLE by a simple halving of the grid spacing. The technique has been found to be quite beneficial to the overall efficiency of a calculation. It is implemented by starting with a coarse grid and specifying KEPS > 1 and EPSGRD(I), I = 1, KEPS. The iteration procedure is continued on the coarse grid until the change in ϕ^0 or $|\phi^1|$ over one iteration is everywhere less than EPSGRD(1), a suggested value for which is 10^{-3}. A greater degree of convergence on an interim coarse grid seems to be wasteful. The grid lines are then doubled and the iteration procedure continued until the change in ϕ is less than EPSGRD(2). If this is the final grid refinement desired, a suggested value for EPSGRD(2) is 10^{-4}. Convergence to this accuracy for UTRANS runs requires about three or four minutes on a CDC 6600 including time expended on both coarse and refined grids. STRANS runs may be of comparable or in general shorter duration depending upon similarity parameters. It is believed that convergence to higher accuracy is not profitable since changes in airfoil pressure distribution are undetectable (less than one percent) for convergence between 10^{-4} and 10^{-5} for example.

An important aspect of the numerical solution procedure is the iteration process for determining the airfoil circulation. This procedure and the corresponding treatment of the Kutta condition and updating of the farfield have been discussed above. Updating the circulation and jump in the potential along the wake has a highly nonlinear effect on the convergence rate. Typically, $(\Delta\phi)_{max}$ increases by an order of magnitude when the circulation is updated. Convergence is then quite rapid until the next update of circulation and farfield values. Thus an additional consideration in terminating the iteration process, is the variation in circulation from one update to the next. The change in circulation (GAMFF) over NGFF iterations as well as the change in the nonlinear doublet strength (DØUBLT) in STRANS calculations are good indicators of the global convergence of the solution. In general, convergence to EPSGRD of 10^{-4} is equivalent to a change in GAMFF or DØUBLT over NGFF iterations of less than 10^{-3}. If this is not true, it is suggested that the calculation be continued to a greater degree of grid convergence. This will insure convergence of integral quantities such as airfoil force coefficients to acceptable engineering accuracy.

It is reiterated that the comments given in this section are intended only to summarize past experience with the programs and the suggested values for various parameters are not to be taken as hard and fast rules. Experimentation with such parameters are encouraged and in fact essential to the efficient operation of the programs.

6.2 Summary of Recommended Computational Procedures

Certain standard practices used in any large scale numerical computations should be followed in exercising the STRANS and UTRANS programs. For instance, when initiating a series of calculations for a new airfoil it is advisable to make a short run of perhaps 25 cycles. In this way all input quantities can be verified and possible adjustments to the grid may be desired based on the progress of the calculation. Such a calculation would also be useful in estimating convergence and therefore computer time requirements for later calculations. The peculiarities of transonic flow computations make certain additional procedures advisable. One such general procedure which has worked well is described in this section. It makes good use of the restart option as well as the successive grid refinement technique.

It is noted that the relaxation procedure for, in this case, the transonic potential equation is a systematic method for getting from one solution to another or from an initial guess to the desired solution. The procedure summarized here thus makes use of this inherent characteristic to generate a matrix of approximate solutions for various Mach numbers and angles of attack for a given airfoil, which can be used as relatively good initial guesses for more refined or highly convergent solutions. For steady solutions the procedure involves initiating a calculation for a given airfoil on a relatively coarse grid (10 x 10 to 25 x 25) for a subcritical Mach number for solutions with $M_\infty < 1$ or a completely supersonic Mach number if solutions with $M_\infty > 1$ are desired. In either case, STRANS uses the linearized subsonic or supersonic solutions respectively to generate an initial guess. This initial case is relaxed to a relatively loose degree of convergence ($\sim 10^{-3}$) and the restart option is used in subsequent runs to bootstrap up or down in Mach number if subsonic or supersonic solutions respectively are desired. These interim solutions can then be used to bootstrap up in angle of attack. Increments in Mach number of about .05 and in angle of attack of about 1° have generally been used to date in generating these initial solutions. Any one of these calculations can later be further refined using the grid halving technique and ultimately continued to the recommended 10^{-4} convergence level.

The procedure for unsteady calculations with UTRANS is quite similar. In this case, the procedure is initiated on a coarse grid for the quasi-steady or $k = 0$ case, using any fully refined and converged steady solution. The reduced frequency is incrementally increased in steps of about 0.1 in subsequent calculations, until the desired value is reached. As before the solution can then be refined and continued to the recommended convergence level.

Clearly many variations on the above procedure are possible and the procedure is by no means optimum for any given calculation. It is for this reason that the systematic procedure outlined above has not been programmed into STRANS or UTRANS.

7.0 SAMPLE CASES

Detailed input and sample output for sequences of STRANS and UTRANS runs are presented in this section. The sample cases attempt to implement, at least in part, the suggestions presented in the previous section.

7.1 STRANS Test Cases

A sequence of computer runs are described in this section, which calculate the steady transonic flow over a 10 percent thick, symmetric circular arc airfoil for the following conditions:

$$M_\infty = 0.7 \begin{cases} \alpha = 0^\circ \\ \alpha = 2^\circ \end{cases}$$

$$M_\infty = 0.8 \begin{cases} \alpha = 0^\circ \\ \alpha = 2^\circ \end{cases}$$

The individual runs required to complete these cases (i.e., achieve convergence $\sim 10^{-4}$ on a refined grid) are described in the run log given in Table 1. The table lists the restart tape read by each run, the grid(s) used, total grid iterations, convergence achieved on each grid and finally the tape dump generated. The "coarse" 29×29 grid on which the calculations were initiated is given in the input to Run 1S below. The "refined" 54×55" grid is generated by halving the grid spacing as described earlier. Runs 2S, 5S, 6S, 7S and 10S, marked by * in the table, are believed to be of engineering accuracy with respect to grid refinement and convergence.

The runs described here implement two approaches to the calculation of lifting cases, using interim solutions, and it is worth commenting briefly on the relative efficiency of the approaches. Runs 1, 3, 4 and 5, for $M_\infty = 0.7$, generate a solution with $\alpha = 2^\circ$ by increasing α in increments using the interim coarse grid solutions. That is, Run 3 uses file 1S.a as its initial guess for $\alpha = 1^\circ$ and Run 4 uses file 3S as its initial guess for $\alpha = 2^\circ$. After achieving a certain degree of convergence on the coarse grid Run 4 refines the grid and the solution is completed in Run 5. The sequence of $M_\infty = 0.8$ runs (Runs 7, 8, 9, 10) on the other hand use, in each case, the refined grid solution as an initial guess for the successively larger angles of attack. Accounting for the fact that iterations on the refined grid require about a factor of 4 more computing time, it would seem that the sequence of runs for the $M_\infty = 0.7$, $\alpha = 2^\circ$ solution required almost a factor of 4 less computing time than the $M_\infty = 0.8$, $\alpha = 2^\circ$ solution. The reason for the increased efficiency is that the convergence of lifting cases is generally controlled by the iteration process for determining the circulation. Thus it would seem to be most efficient to achieve relatively good convergence for the circulation before proceeding to the refined grid. This of course neglects the fact that supercritical cases such as $M_\infty = 0.8$ generally require greater computing effort than subcritical ($M_\infty = 0.7$) cases. However the increased effort for supercritical cases is not a factor of 4 by any means so that this comparison would seem to justify the conclusion given. The comparison of "bootstrapping" techniques given here, also reinforces the point made earlier that the user can effect considerable savings by exercising some judgment in the way calculations are performed.

TABLE 1. SEQUENCE OF RUNS FOR STRANS SAMPLE CASES (* CONVERGED SOLUTION)

Run	M_∞	α	Restart Tape Used	Grid Type	Grid Iterations	Convergence Achieved	Tape Dump Generated
1S	.7	0°	--	29×29 54×55	58	10^{-3} 10^{-3}	1S.a 1S.b
* 2S	.7	0°	1S.b	54×55	85	10^{-4}	2S
3S	.7	1°	1S.a	29×29	44	10^{-3}	3S
4S	.7	2°	3S	29×29 54×55	92	10^{-3} 10^{-3}	4S.a 4S.b
* 5S	.7	2°	4S.b	54×55	100	1.2×10^{-4}	5S
* 6S	.75	0°	2S	54×55	68	10^{-4}	6S
* 7S	.8	0°	6S	54×55	75	10^{-4}	7S
8S	.8	1°	7S	54×55	100	10^{-3}	8S
9S	.8	2°	8S	54×55	100	1.6×10^{-3}	9S
*10S	.8	2°	9S	54×55	200	4.7×10^{-4}	10S

7.1.1 Input for STRANS Test Cases

The card input for each of the STRANS runs described above is given in this section:

- Run 1S: no tape read, generate files 1S.a, 1S.b.

*** CIRCULAR ARC ***

```
$CØNTRL
ITAPE=0,
$END
$IN
X(1)=-4.,-3.2,-2.4,-1.6,-1.,-.6,-.35,-.2,-.075,0.,.05,.11,
.18,.26,.36,.5,.64,.74,.82,.88.,94,1.,1.1,1.25,1.45,1.75,
2.25,3.,3.75,
Y(1)=-7.,-5.4,-3.8,-2.7,-1.9,-1.4,-1.1,-.85,-.65,-.5,-.36,
-.23,-.1366,-.05,0.,.05,.1366,.23,.36,.5,.65,.85,1.1,1.4,
1.9,2.7,3.8,5.4,7.,
IM=29,
JM=29,
ILE=10,
ITE=22,
JW=15,
M8=.7,
GAM=1.4,
DEL=.1,
ALPHA=0.,
GAMFF=0.,
ØMEGAH=.75,
ØMEGAE=1.5,
ØMEGAP=.75,
EPSCØL=5.E-5,
EPSGRD(1)=1.E-3,1.E-3,
XH=0.,
NDUMP=2000,
NCØL=10,
NGRID=150,
NGFF=10,
PGFF=1.5,
KEPS-2,
NPRINT=10,
ALPHAF=0.,
$END
```

- Run 2S: read file 1S.b, generate file 2S.

```
*** CIRCULAR ARC ***

$CØNTRL
ITAPE=1,
$END
$IN
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSCØL=5.E-5,
EPSGRD(1)=1.E-4,
NDUMP=2000,
NCØL=10,
NGRID=100,
NGFF=10,
PGFF=1.5,
KEPS=1,
NPRINT=10,
$END
```

- Run 3S: read file 1S.a, generate file 3S.

```
*** CIRCULAR ARC ***

$CØNTRL
ITAPE=1,
$END
$IN
ALPHA=.0174532925,
GAMFF=1.,
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSCØL=5.E-5,
EPSGRD(1)=1.E-3,
NDUMP=2000,
NCØL=10,
NGRID=100,
NGFF=10,
PGFF=1.5,
KEPS=1,
NPRINT=10,
IK=1,
$END
```

- Run 4S: read file 3S, generate files 4S.a and 4S.b

```
            ***  CIRCULAR ARC  ***

$CØNTRL
ITAPE=1,
$END
$IN
ALPHA=.034906585,
GAMFF=1.,
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSCØL=5.E-5,
EPSGRD(1)=1.E-3,1.E-3,
NDUMP=2000,
NCØL=10,
NGRID=150,
NGFF=10,
PGFF=1.5,
KEPS=2,
NPRINT=10,
IK=1,
$END
```

- Run 5S: read file 4S.b, generate file 5S.

```
            ***  CIRCULAR ARC ***

$CØNTRL
ITAPE=1,
$END
$IN
ØMEGAH=.75,
ØMEGAE=1.7,
ØEMGAP=.75,
EPSCØL=5.E-5,
EPSGRD(1)=1.E-4,
NDUMP=2000,
NCØL=10,
NGRID=100,
NGFF=10,
PGFF=1.5,
KEPS=1,
NPRINT=10,
$END
```

- Run 6S: read file 2S, generate file 6S.

*** CIRCULAR ARC ***

```
$CØNTRL
ITAPE=1,
$END
$IN
M8=.75,
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSCØL=5.E-5,
EPSGRD(1)=1.E-4,
NDUMP=2000,
NCØL=10,
NGRID=100,
NGFF=10,
PGFF=1.5,
KEPS=1,
NPRINT=10,
IK=1,
$END
```

- Run 7S: read file 6S, generate file 7S.

*** CIRCULAR ARC ***

```
$CØNTRL
ITAPE=1,
$END
$IN
M8=.8,
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSCØL=5.E-5,
EPSGRD(1)=1.E-4,
NDUMP=2000,
NCØL=10,
NGRID=100,
NGFF=10,
PGFF=1.5,
KEPS=1,
NPRINT=10,
IK=1,
$END
```

- Run 8S: read file 7S, generate file 8S.

*** CIRCULAR ARC ***

```
$CØNTRL
ITAPE=1,
$END
$IN
ALPHA=.0174532925,
GAMFF=1.,
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSCØL=5.E-5,
EPSGRD(1)=1.E-4,
NDUMP=2000,
NCØL=10,
NGRID=100,
NGFF=10,
PGFF=1.5,
KEPS=1,
NPRINT=10,
IK=1,
$END
```

- Run 9S: read file 8S, generate file 9S.

*** CIRCULAR ARC ***

```
$CØNTRL
ITAPE=1,
$END
$IN
ALPHA=.034906585,
GAMFF=1.,
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSCØL=5.E-5,
EPSGRD(1)=1.E-4,
NDUMP=2000,
NCØL=10,
NGRID=100,
NGFF=10,
PGFF=1.5,
KEPS=1,
NPRINT=10,
IK=1,
$END
```

- Run 10S: read file 9S, generate file 10S.

*** CIRCULAR ARC ***

```
$CØNTRL
ITAPE=1,
$END
$IN
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSCØL=5.E-5,
EPSGRD(1)=1.E-4,
NDUMP=2000,
NCØL=10,
NGRID=2000,
NGFF=10,
PGFF=1.5,
KEPS=1,
NPRINT=10,
$END
```

7.1.2 Sample Output for STRANS Test Cases

The following pages contain a sample of the continuous commentary output for the first 9 cycles of run 1S in addition to the final printed output for all runs.

• Sample Output from Run 1S

```
SIMILARITY PARAMETER (K) =    .21247E+01
SCALING FACTOR (CP/CPBAR) =    .20411E+00

AT ITERATION     1 COLUMN    9 FAILED TO CONVERGE ERR =    .40315E-03 J= 11

AT ITERATION     1 COLUMN   10 FAILED TO CONVERGE ERR =    .52978E-03 J= 10

AT ITERATION     1 COLUMN   11 FAILED TO CONVERGE ERR =    .33828E-03 J= 11

AT ITERATION     1 COLUMN   12 FAILED TO CONVERGE ERR =   -.72338E-04 J=  9

AT ITERATION     1 THE MAXIMUM ERROR =  -.11291E+01 AND OCCURRED AT NODE  304

AT ITERATION     2 COLUMN   12 FAILED TO CONVERGE ERR =    .80174E-03 J= 10

AT ITERATION     2 COLUMN   13 FAILED TO CONVERGE ERR =   -.21155E-03 J=  9

AT ITERATION     2 COLUMN   14 FAILED TO CONVERGE ERR =   -.14111E-03 J= 11

AT ITERATION     2 COLUMN   15 FAILED TO CONVERGE ERR =    .13400E-03 J=  8

AT ITERATION     2 COLUMN   16 FAILED TO CONVERGE ERR =   -.28489E-03 J= 10

AT ITERATION     2 THE MAXIMUM ERROR =  -.19514E+01 AND OCCURRED AT NODE  362

AT ITERATION     3 COLUMN   14 FAILED TO CONVERGE ERR =   -.61267E-04 J= 12

AT ITERATION     3 COLUMN   15 FAILED TO CONVERGE ERR =   -.25586E-03 J= 10

AT ITERATION     3 THE MAXIMUM ERROR =   .84634E+00 AND OCCURRED AT NODE  393

AT ITERATION     4 COLUMN   14 FAILED TO CONVERGE ERR =    .10505E-03 J= 12

AT ITERATION     4 COLUMN   15 FAILED TO CONVERGE ERR =    .10476E-03 J= 10

AT ITERATION     4 THE MAXIMUM ERROR =  -.40767E+00 AND OCCURRED AT NODE  275

AT ITERATION     5 COLUMN   15 FAILED TO CONVERGE ERR =    .10977E-03 J= 12

AT ITERATION     5 THE MAXIMUM ERROR =  -.50022E+00 AND OCCURRED AT NODE  247

AT ITERATION     6 THE MAXIMUM ERROR =  -.34276E+00 AND OCCURRED AT NODE  218

AT ITERATION     7 THE MAXIMUM ERROR =  -.26471E+00 AND OCCURRED AT NODE  189

AT ITERATION     8 THE MAXIMUM ERROR =  -.17040E+00 AND OCCURRED AT NODE  160

AT ITERATION     9 THE MAXIMUM ERROR =  -.99641E-01 AND OCCURRED AT NODE  131

AT ITERATION     9 SCALED PRESSURE COEFFICIENT, UPPER (ILE TO ITE) =
 -.57853E+00  -.15848E+01  -.14662E+01  -.21306E+01  -.25423E+01  -.26149E+01  -.26104E+01  -.22773E+01  -.18205E+01  -.13008E+01
 -.79889E+00  -.17902E+00   .68001E+00

AT ITERATION     9 SCALED PRESSURE COEFFICIENT, LOWER (ILE TO ITE) =
 -.57853E+00  -.15848E+01  -.14662E+01  -.21306E+01  -.25423E+01  -.26149E+01  -.26104E+01  -.22773E+01  -.18205E+01  -.13008E+01
 -.79889E+00  -.17902E+00   .68001E+00

UPDATE GAMFF AND FARFIELD AT ITERATION   10 GAMFF =  0.        GAMTE =  0.        DOUBLET =   .77020E+00
```

RUN 1S

*** CIRCULAR ARC ***

```
MACH NUMBER =    .70000E+00
SIMILARITY PARAMETER (K) =    .21247E+01
THICKNESS RATIO =    .10000E+00
AIRFOIL ANGLE OF ATTACK (RADIANS) =  0.
HINGE POINT =  0.
FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =    .20411E+00
SCALED AIRFOIL CIRCULATION =  0.
LIFT COEFFICIENT BASED ON CIRCULATION (2*CP/CPBAR*CIRCULATION) =  0.

AIRFOIL FORCE COEFFICIENTS

  LIFT =  -.30527E-12
  MOMENT ABOUT (X=0) =  -.64593E-11
  HINGE MOMENT =  -.64593E-11

PRESSURE COEFFICIENTS ON THE AIRFOIL (UNSCALED)

  AIRFOIL COORDINATE =
    0.          .25000E-01   .50000E-01   .80000E-01   .11000E+00   .14500E+00   .18000E+00   .22000E+00   .26000E+00   .31000E+00
    .36000E+00  .43000E+00   .50000E+00   .57000E+00   .64000E+00   .69000E+00   .74000E+00   .78000E+00   .82000E+00   .85000E+00
    .88000E+00  .91000E+00   .94000E+00   .97000E+00   .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
    .32144E+00  .16996E+00   .73050E-01  -.18271E-01  -.90736E-01  -.15948E+00  -.21618E+00  -.26990E+00  -.31420E+00  -.35805E+00
   -.39098E+00 -.41995E+00  -.43030E+00  -.42254E+00  -.39456E+00  -.36002E+00  -.31671E+00  -.27179E+00  -.21632E+00  -.16741E+00
   -.11056E+00 -.43872E-01   .37581E-01   .14069E+00   .29417E+00

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
    .32144E+00  .16996E+00   .73050E-01  -.18271E-01  -.90736E-01  -.15948E+00  -.21618E+00  -.26990E+00  -.31420E+00  -.35805E+00
   -.39098E+00 -.41995E+00  -.43030E+00  -.42254E+00  -.39456E+00  -.36002E+00  -.31671E+00  -.27179E+00  -.21632E+00  -.16741E+00
   -.11056E+00 -.43872E-01   .37581E-01   .14069E+00   .29417E+00
```

RUN 2S

```
***   CIRCULAR ARC   ***

MACH NUMBER =    .70000E+00
SIMILARITY PARAMETER (K) =    .21247E+01
THICKNESS RATIO =    .10000E+00
AIRFOIL ANGLE OF ATTACK (RADIANS) =  0.
HINGE POINT =  0.
FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =    .20411E+00
SCALED AIRFOIL CIRCULATION =  0.
LIFT COEFFICIENT BASED ON CIRCULATION (2*CP/CPBAR*CIRCULATION) =  0.


AIRFOIL FORCE COEFFICIENTS

  LIFT =  -.25171E-13
  MOMENT ABOUT (X=0) =  -.14231E-11
  HINGE MOMENT =  -.14231E-11


PRESSURE COEFFICIENTS ON THE AIRFOIL (UNSCALED)

  AIRFOIL COORDINATE =
    0.           .25000E-01   .50000E-01   .80000E-01   .11000E+00   .14500E+00   .18000E+00   .22000E+00   .26000E+00   .31000E+00
    .36000E+00   .43000E+00   .50000E+00   .57000E+00   .64000E+00   .69000E+00   .74000E+00   .78000E+00   .82000E+00   .85000E+00
    .88000E+00   .91000E+00   .94000E+00   .97000E+00   .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
    .32429E+00   .17649E+00   .81831E-01  -.67862E-02  -.77197E-01  -.14403E+00  -.19910E+00  -.25133E+00  -.29435E+00  -.33725E+00
   -.36936E+00  -.39762E+00  -.40724E+00  -.39838E+00  -.37095E+00  -.33957E+00  -.29759E+00  -.25552E+00  -.20460E+00  -.15935E+00
   -.10621E+00  -.42625E-01   .35753E-01   .13640E+00   .28880E+00

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
    .32429E+00   .17649E+00   .81831E-01  -.67862E-02  -.77197E-01  -.14403E+00  -.19910E+00  -.25133E+00  -.29435E+00  -.33725E+00
   -.36936E+00  -.39762E+00  -.40724E+00  -.39838E+00  -.37095E+00  -.33957E+00  -.29759E+00  -.25552E+00  -.20460E+00  -.15935E+00
   -.10621E+00  -.42625E-01   .35753E-01   .13640E+00   .28880E+00
```

RUN 3S

```
*** CIRCULAR ARC ***

MACH NUMBER =    .70000E+00
SIMILARITY PARAMETER (K) =    .21247E+01
THICKNESS RATIO =    .10000E+00
AIRFOIL ANGLE OF ATTACK (RADIANS) =    .17453E-01
HINGE POINT =  0.
FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =    .20411E+00
CRITICAL PRESSURE COEFFICIENT (SONIC) =  -.86735E+00
SCALED AIRFOIL CIRCULATION =    .38690E+00
LIFT COEFFICIENT BASED ON CIRCULATION (2*CP/CPBAR*CIRCULATION) =    .15794E+00

AIRFOIL FORCE COEFFICIENTS

  LIFT =    .12256E+00
  MOMENT ABOUT (X=0) =    .38857E-01
  HINGE MOMENT =    .38857E-01

PRESSURE COEFFICIENTS ON THE AIRFOIL (UNSCALED)

  AIRFOIL COORDINATE =
    0.            .50000E-01    .11000E+00    .18000E+00    .26000E+00    .36000E+00    .50000E+00    .64000E+00    .74000E+00    .82000E+00
    .88000E+00    .94000E+00    .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
    .99576E-02   -.93743E-01   -.20453E+00   -.30310E+00   -.38432E+00   -.44650E+00   -.46754E+00   -.41531E+00   -.33176E+00   -.23331E+00
   -.13657E+00   -.14147E-01    .15985E+00

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
    .37371E+00    .16060E+00    .19648E-01   -.10636E+00   -.21221E+00   -.30021E+00   -.35508E+00   -.33519E+00   -.27310E+00   -.19024E+00
   -.10464E+00    .67024E-02    .16934E+00
```

RUN 4S

```
*** CIRCULAR ARC ***

MACH NUMBER =    .70000E+00
SIMILARITY PARAMETER (K) =    .21247E+01
THICKNESS RATIO =    .10000E+00
AIRFOIL ANGLE OF ATTACK (RADIANS) =    .34907E-01
HINGE POINT =  0.
FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =    .20411E+00
CRITICAL PRESSURE COEFFICIENT (SONIC) =  -.86735E+00
SCALED AIRFOIL CIRCULATION =    .81265E+00
LIFT COEFFICIENT BASED ON CIRCULATION (2*CP/CPBAR*CIRCULATION) =    .33174E+00


AIRFOIL FORCE COEFFICIENTS

  LIFT =    .27899E+00
  MOMENT ABOUT (X=0) =    .84621E-01
  HINGE MOMENT =    .84621E-01


PRESSURE COEFFICIENTS ON THE AIRFOIL (UNSCALED)

  AIRFOIL COORDINATE =
   0.           .25000E-01   .50000E-01   .80000E-01   .11000E+00   .14500E+00   .18000E+00   .22000E+00   .26000E+00   .31000E+00
   .36000E+00   .43000E+00   .50000E+00   .57000E+00   .64000E+00   .69000E+00   .74000E+00   .78000E+00   .82000E+00   .85000E+00
   .88000E+00   .91000E+00   .94000E+00   .97000E+00   .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
  -.26426E+00  -.24284E+00  -.28125E+00  -.32452E+00  -.36378E+00  -.40450E+00  -.43990E+00  -.47430E+00  -.50265E+00  -.52789E+00
  -.54212E+00  -.55193E+00  -.53961E+00  -.51084E+00  -.46300E+00  -.41765E+00  -.36258E+00  -.30998E+00  -.24826E+00  -.19449E+00
  -.13398E+00  -.63048E-01   .22357E-01   .13090E+00   .29058E+00

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
   .83600E+00   .52923E+00   .39136E+00   .26419E+00   .16570E+00   .74213E-01   .26464E-04  -.69236E-01  -.12648E+00  -.18339E+00
  -.22631E+00  -.27297E+00  -.29772E+00  -.30454E+00  -.29158E+00  -.26988E+00  -.23746E+00  -.20216E+00  -.15719E+00  -.11575E+00
  -.67154E-01  -.84498E-02   .64310E-01   .15912E+00   .30379E+00
```

```
*** CIRCULAR ARC ***

MACH NUMBER =    .70000E+00
SIMILARITY PARAMETER (K) =    .21247E+01
THICKNESS RATIO =    .10000E+00
AIRFOIL ANGLE OF ATTACK (RADIANS) =    .34907E-01
HINGE POINT =  0.
FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =    .20411E+00
CRITICAL PRESSURE COEFFICIENT (SONIC) =  -.86735E+00
SCALED AIRFOIL CIRCULATION =    .82024E+00
LIFT COEFFICIENT BASED ON CIRCULATION (2*CP/CPBAR*CIRCULATION) =    .33484E+00


AIRFOIL FORCE COEFFICIENTS

  LIFT =    .28190E+00
  MOMENT ABOUT (X=0) =    .85654E-01
  HINGE MOMENT =    .85654E-01


PRESSURE COEFFICIENTS ON THE AIRFOIL (UNSCALED)

  AIRFOIL COORDINATE =
    0.          .25000E-01   .50000E-01   .80000E-01   .11000E+00   .14500E+00   .18000E+00   .22000E+00   .26000E+00   .31000E+00
    .36000E+00  .43000E+00   .50000E+00   .57000E+00   .64000E+00   .69000E+00   .74000E+00   .78000E+00   .82000E+00   .85000E+00
    .88000E+00  .91000E+00   .94000E+00   .97000E+00   .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
   -.26120E+00 -.23757E+00  -.27332E+00  -.31760E+00  -.35598E+00  -.39578E+00  -.43043E+00  -.46405E+00  -.49151E+00  -.51739E+00
   -.53392E+00 -.54137E+00  -.53064E+00  -.50201E+00  -.45573E+00  -.41169E+00  -.35776E+00  -.30664E+00  -.24702E+00  -.19543E+00
   -.13605E+00 -.66206E-01   .18601E-01   .12608E+00   .28586E+00

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
    .84398E+00  .53905E+00   .40236E+00   .27638E+00   .17855E+00   .87805E-01   .14471E-01  -.54354E-01  -.11110E+00  -.16878E+00
   -.21411E+00 -.25968E+00  -.28592E+00  -.29350E+00  -.28217E+00  -.26201E+00  -.23091E+00  -.19729E+00  -.15451E+00  -.11524E+00
   -.68146E-01 -.10728E-01   .61207E-01   .15487E+00   .29937E+00
```

RUN 5S

RUN 6S

```
*** CIRCULAR ARC ***

MACH NUMBER =   .75000E+00
SIMILARITY PARAMETER (K) =   .16625E+01
THICKNESS RATIO =   .10000E+00
AIRFOIL ANGLE OF ATTACK (RADIANS) =  0.
HINGE POINT =  0.
FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =   .19493E+00
SCALED AIRFOIL CIRCULATION =  0.
LIFT COEFFICIENT BASED ON CIRCULATION (2*CP/CPBAR*CIRCULATION) =  0.


AIRFOIL FORCE COEFFICIENTS

  LIFT =  -.29814E-14
  MOMENT ABOUT (X=0) =  -.57375E-12
  HINGE MOMENT =  -.57375E-12


PRESSURE COEFFICIENTS ON THE AIRFOIL (UNSCALED)

  AIRFOIL COORDINATE =
   0.           .25000E-01   .50000E-01   .80000E-01   .11000E+00   .14500E+00   .18000E+00   .22000E+00   .26000E+00   .31000E+00
   .36000E+00   .43000E+00   .50000E+00   .57000E+00   .64000E+00   .69000E+00   .74000E+00   .78000E+00   .82000E+00   .85000E+00
   .88000E+00   .91000E+00   .94000E+00   .97000E+00   .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
   .35184E+00   .19767E+00   .97493E-01   .31042E-02  -.72789E-01  -.14608E+00  -.20782E+00  -.26791E+00  -.31893E+00  -.37167E+00
  -.41280E+00  -.45051E+00  -.46376E+00  -.45161E+00  -.41505E+00  -.37481E+00  -.32313E+00  -.27314E+00  -.21443E+00  -.16350E+00
  -.10480E+00  -.35762E-01   .48164E-01   .15491E+00   .31444E+00

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
   .35184E+00   .19767E+00   .97493E-01   .31042E-02  -.72789E-01  -.14608E+00  -.20782E+00  -.26791E+00  -.31893E+00  -.37167E+00
  -.41280E+00  -.45051E+00  -.46376E+00  -.45161E+00  -.41505E+00  -.37481E+00  -.32313E+00  -.27314E+00  -.21443E+00  -.16350E+00
  -.10480E+00  -.35762E-01   .48164E-01   .15491E+00   .31444E+00
```

RUN 7S

*** CIRCULAR ARC ***

```
MACH NUMBER =    .80000E+00
SIMILARITY PARAMETER (K) =    .12552E+01
THICKNESS RATIO =    .10000E+00
AIRFOIL ANGLE OF ATTACK (RADIANS) =  0.
HINGE POINT =  0.
FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =    .18673E+00
SCALED AIRFOIL CIRCULATION =  0.
LIFT COEFFICIENT BASED ON CIRCULATION (2*CP/CPBAR*CIRCULATION) =  0.


AIRFOIL FORCE COEFFICIENTS

  LIFT =    .66414E-14
  MOMENT ABOUT (X=0) =  -.23138E-12
  HINGE MOMENT =  -.23138E-12


PRESSURE COEFFICIENTS ON THE AIRFOIL (UNSCALED)

  AIRFOIL COORDINATE =
   0.          .25000E-01   .50000E-01   .80000E-01   .11000E+00   .14500E+00   .18000E+00   .22000E+00   .26000E+00   .31000E+00
    .36000E+00  .43000E+00   .50000E+00   .57000E+00   .64000E+00   .69000E+00   .74000E+00   .78000E+00   .82000E+00   .85000E+00
    .88000E+00  .91000E+00   .94000E+00   .97000E+00   .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
    .38919E+00  .23013E+00   .12548E+00   .26033E-01  -.55096E-01  -.13515E+00  -.20456E+00  -.27482E+00  -.33779E+00  -.40859E+00
   -.47077E+00 -.55728E+00  -.61714E+00  -.59487E+00  -.49735E+00  -.41907E+00  -.34730E+00  -.28432E+00  -.21469E+00  -.15667E+00
   -.91665E-01 -.16930E-01   .72305E-01   .18436E+00   .34958E+00

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
    .38919E+00  .23013E+00   .12548E+00   .26033E-01  -.55096E-01  -.13515E+00  -.20456E+00  -.27482E+00  -.33779E+00  -.40859E+00
   -.47077E+00 -.55728E+00  -.61714E+00  -.59487E+00  -.49735E+00  -.41907E+00  -.34730E+00  -.28432E+00  -.21469E+00  -.15667E+00
   -.91665E-01 -.16930E-01   .72305E-01   .18436E+00   .34958E+00
```

```
*** CIRCULAR ARC ***

MACH NUMBER =   .80000E+00
SIMILARITY PARAMETER (K) =   .12552E+01
THICKNESS RATIO =   .10000E+00
AIRFOIL ANGLE OF ATTACK (RADIANS) =   .17453E-01
HINGE POINT =  0.
FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =   .18673E+00
SCALED AIRFOIL CIRCULATION =   .40315E+00
LIFT COEFFICIENT BASED ON CIRCULATION (2*CP/CPBAR*CIRCULATION) =   .15056E+00


AIRFOIL FORCE COEFFICIENTS

  LIFT =   .13090E+00
  MOMENT ABOUT (X=0) =   .44130E-01
  HINGE MOMENT =   .44130E-01


PRESSURE COEFFICIENTS ON THE AIRFOIL (UNSCALED)

  AIRFOIL COORDINATE =
    0.          .25000E-01   .50000E-01   .80000E-01   .11000E+00   .14500E+00   .18000E+00   .22000E+00   .26000E+00   .31000E+00
    .36000E+00   .43000E+00   .50000E+00   .57000E+00   .64000E+00   .69000E+00   .74000E+00   .78000E+00   .82000E+00   .85000E+00
    .88000E+00   .91000E+00   .94000E+00   .97000E+00   .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
    .19939E+00   .90329E-01   .18702E-02  -.84270E-01  -.15646E+00  -.22940E+00  -.29400E+00  -.36052E+00  -.42115E+00  -.48785E+00
   -.55887E+00  -.65093E+00  -.71085E+00  -.72955E+00  -.56639E+00  -.42539E+00  -.35866E+00  -.29467E+00  -.22411E+00  -.16560E+00
   -.99700E-01  -.23968E-01   .66341E-01   .17972E+00   .34729E+00

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
    .56931E+00   .36152E+00   .24253E+00   .13095E+00   .41434E-01  -.45337E-01  -.11933E+00  -.19309E+00  -.25829E+00  -.33048E+00
   -.39393E+00  -.46965E+00  -.51948E+00  -.51289E+00  -.45323E+00  -.39578E+00  -.32955E+00  -.26966E+00  -.20185E+00  -.14462E+00
   -.80692E-01  -.70752E-02   .80960E-01   .19160E+00   .35420E+00
```

RUN 3S

```
***  CIRCULAR ARC  ***


MACH NUMBER =    .80000E+00
SIMILARITY PARAMETER (K) =    .12552E+01
THICKNESS RATIO =    .10000E+00
AIRFOIL ANGLE OF ATTACK (RADIANS) =    .34907E-01
HINGE POINT =  0.
FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =    .18673E+00
CRITICAL PRESSURE COEFFICIENT (SONIC) =  -.46875E+00
SCALED AIRFOIL CIRCULATION =    .97550E+00
LIFT COEFFICIENT BASED ON CIRCULATION (2*CP/CPBAR*CIRCULATION) =    .36430E+00


AIRFOIL FORCE COEFFICIENTS

  LIFT =    .32412E+00
  MOMENT ABOUT (X=0) =    .11572E+00
  HINGE MOMENT =    .11572E+00


PRESSURE COEFFICIENTS ON THE AIRFOIL (UNSCALED)

  AIRFOIL COORDINATE =
    0.            .25000E-01   .50000E-01   .80000E-01   .11000E+00   .14500E+00   .18000E+00   .22000E+00   .26000E+00   .31000E+00
     .36000E+00   .43000E+00   .50000E+00   .57000E+00   .64000E+00   .69000E+00   .74000E+00   .78000E+00   .82000E+00   .85000E+00
     .88000E+00   .91000E+00   .94000E+00   .97000E+00   .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
    -.68274E-01  -.10524E+00  -.16742E+00  -.23314E+00  -.29193E+00  -.35424E+00  -.41143E+00  -.47015E+00  -.53137E+00  -.60894E+00
    -.67616E+00  -.75811E+00  -.82708E+00  -.88358E+00  -.91442E+00  -.63863E+00  -.33422E+00  -.28604E+00  -.22257E+00  -.16644E+00
    -.10146E+00  -.25815E-01   .64962E-01   .17929E+00   .34854E+00

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
     .79911E+00   .52875E+00   .39331E+00   .26734E+00   .16771E+00   .72730E-01  -.68172E-02  -.84761E-01  -.15247E+00  -.22592E+00
    -.28870E+00  -.35909E+00  -.40436E+00  -.41556E+00  -.38780E+00  -.34698E+00  -.29221E+00  -.23915E+00  -.17673E+00  -.12285E+00
    -.62044E-01   .86373E-02   .93837E-01   .20155E+00   .36080E+00
```

RUN 9S

```
                                  *** CIRCULAR ARC ***

MACH NUMBER =    .80000E+00
SIMILARITY PARAMETER (K) =    .12552E+01
THICKNESS RATIO =    .10000E+00
AIRFOIL ANGLE OF ATTACK (RADIANS) =    .34907E-01
HINGE POINT =  0.
FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =    .18673E+00
CRITICAL PRESSURE COEFFICIENT (SONIC) =  -.46875E+00
SCALED AIRFOIL CIRCULATION =    .11621E+01
LIFT COEFFICIENT BASED ON CIRCULATION (2*CP/CPBAR*CIRCULATION) =    .43399E+00


AIRFOIL FORCE COEFFICIENTS

  LIFT =    .38667E+00
  MOMENT ABOUT (X=0) =    .14156E+00
  HINGE MOMENT =    .14156E+00


PRESSURE COEFFICIENTS ON THE AIRFOIL (UNSCALED)

  AIRFOIL COORDINATE =
   0.           .25000E-01   .50000E-01   .80000E-01   .11000E+00   .14500E+00   .18000E+00   .22000E+00   .26000E+00   .31000E+00
    .36000E+00  .43000E+00   .50000E+00   .57000E+00   .64000E+00   .69000E+00   .74000E+00   .78000E+00   .82000E+00   .85000E+00
    .88000E+00  .91000E+00   .94000E+00   .97000E+00   .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
   -.15102E+00 -.16475E+00  -.21795E+00  -.27711E+00  -.33174E+00  -.39073E+00  -.44480E+00  -.51159E+00  -.57537E+00  -.63999E+00
   -.70288E+00 -.78587E+00  -.85921E+00  -.92200E+00  -.97067E+00  -.72759E+00  -.41040E+00  -.32431E+00  -.24572E+00  -.18247E+00
   -.11260E+00 -.33191E-01   .60682E-01   .17760E+00   .34822E+00

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
    .86554E+00  .57670E+00   .43574E+00   .30675E+00   .20420E+00   .10680E+00   .25587E-01  -.53621E-01  -.12204E+00  -.19577E+00
   -.25824E+00 -.32741E+00  -.37190E+00  -.38502E+00  -.36286E+00  -.32690E+00  -.27663E+00  -.22678E+00  -.16765E+00  -.11628E+00
   -.57492E-01  .11374E-01   .94879E-01   .20106E+00   .35958E+00
```

RUN 10S

7.2 UTRANS Test Cases

A sequence of UTRANS runs are described in this section, which calculate the unsteady flow perturbation for the circular arc airfoil oscillating in angle of attack at the following conditions:

$$M_\infty = 0.7 \;,\; \alpha = 0^\circ \begin{cases} k = 0.0 \\ k = 0.2 \end{cases}$$

$$M_\infty = 0.8 \;,\; \alpha = 0^\circ \begin{cases} k = 0.0 \\ k = 0.2 \end{cases}$$

The individual runs required to complete these cases (i.e., achieve convergence of $\sim 10^{-4}$ on a refined grid) are described in the run log in Table 2. The table presents the mean flow conditions (M_∞, α^0) and reduced frequency (k), the tape dump of the steady solution and restart tape read for each run, the grid(s) used, total grid iterations, convergence achieved on each grid and finally the tape dump generated. All runs are on the same coarse and refined grids described above. Runs 2U, 5U, 7U and 10U, marked by * in the table, are the final converged results for the cases listed above. The runs were performed in the order shown and implement the "bootstrap" technique for getting from one unsteady solution to the other as described in the previous section. The required input for each run and sample output are now presented.

Table 2. Sequence of Runs for UTRANS Sample Cases (* Converged Solution)

Run	M_∞	α	k	STRANS Tape	Restart Tape	Grid Type	Grid Iterations	Convergence Achieved	Tape Dump Generated
1U	.7	0^o	0.0	2S	--	29×29 54×55	45	3×10^{-3} 10^{-3}	1U.a 1U.b
* 2U	.7	0^o	0.0	2S	1U.b	54×55	200	1.85×10^{-4}	2U
3U	.7	0^o	0.1	2S	1U.a	29×29	127	10^{-3}	3U
4U	.7	0^o	0.2	2S	3U	29×29 54×55	136	10^{-3} 10^{-3}	4U.a 4U.b
* 5U	.7	0^o	0.2	2S	4U.b	54×55	200	10^{-4}	5U
6U	.8	0^o	0.0	7S	1U.a	29×29 54×55	189	10^{-3} 10^{-3}	6U.a 6U.b
* 7U	.8	0^o	0.0	7S	6U.b	54×55	199	10^{-4}	7U
8U	.8	0^o	0.1	7S	6U.a	29×29	187	10^{-3}	8U
9U	.8	0^o	0.2	7S	8U	29×29 54×55	175	10^{-3} 10^{-3}	9U.a 9U.b
*10U	.8	0^o	0.2	7S	9U.b	54×55	200	10^{-4}	10U

7.2.1 Input for UTRANS Test Cases

The card input for each of the UTRANS runs described above is given in this section.

- Run 1U: read file 2S, no restart tape read; generate files 1U.a and 1U.b.

```
              ***  CIRCULAR ARC  ***

$CØNTRL
ITAPE=0,
$END
$IN
X(1)=-4.,-3.2,-2.4,-1.6,-1.,-.6,-.35,-.2,-.075,0.,.05,.11,
.18,.26,.36,.5,.64,.74,.82,.88,.94,1.,1.1,1.25,1.45,1.75,
2.25,3.,3.75,
Y(1)=-7.,-5.4,-3.8,-2.7,-1.9,-1.4,-1.1,-.85,-.65,-.5,-.36,
-.23,-.1366,-.05,0.,.05,.1366,.23,.36,.5,.65,.85,1.1,1.4,
1.9,2.7,3.8,5.4,7.,
GAMFF=(2.,0.),
IM=29,
JM=29,
ILE=10,
ITE=22,
JW=15,
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSGRD(1)=3.E-3,1.E-3,
NDUMP=2000,
NGRID=200,
NGFF=10,
PGFF=1.5,
KEPS=2,
NPRINT=10,
SMALLK=0.,
XH=0.,
$END
```

- Run 2U: read file 2S, restart file 1U.b; generate file 2U.

*** CIRCULAR ARC ***

```
$CØNTRL
ITAPE=1,
$END
$IN
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSGRD(1)=1.E-4,
NDUMP=2000,
NGRID=200,
NGFF=10,
PGFF=1.5,
KEPS=1,
NPRINT=10,
$END
```

- Run 3U: read file 2S, restart file 1U.a; generate file 3U.

*** CIRCULAR ARC ***

```
$CØNTRL
ITAPE=1,
$END
$IN
GAMFF=(2.3,-1.),
SMALLK=.1,
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSGRD(1)=1.E-3,
NDUMP=2000,
NGRID=200,
NGFF=10,
PGFF=1.5,
KEPS=1,
NPRINT=10,
IK=1,
$END
```

- Run 4U: read file 2S, restart file 3U, generate file 4U.

```
                *** CIRCULAR ARC ***

$CØNTRL
ITAPE=1,
$END
$IN
GAMFF=(2.,-.5),
SMALLK=.2,
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSGRD(1)=1.E-3,1.E-3,
NDUMP=2000,
NGRID=200,
NGFF=10,
PGFF=1.5,
KEPS=2,
NPRINT=10,
IK=1,
$END
```

- Run 5U: read file 2S, restart file 4U; generate file 5U.

```
                *** CIRCULAR ARC ***

$CØNTRL
ITAPE=1,
$END
$IN
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSGRD(1)=1.E-4,
NDUMP=2000,
NGRID=200,
NGFF=10,
PGFF=1.5,
KEPS=1,
NPRINT=10,
$END
```

- Run 6U: read file 7S, restart file 1U.a; generate files 6U.a, 6U.b.

```
        ***   CIRCULAR ARC   ***

$CØNTRL
ITAPE=1,
$END
$IN
GAMFF=(2.6,0.),
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSGRD(1)=1.E-3,1.E-3,
NDUMP=2000,
NGRID=200,
NGFF=10,
PGFF=1.5,
KEPS=2,
NPRINT=10,
IK=1,
$END
```

- Run 7U: read file 7S, restart file 6U.b; generate file 7U.

```
        ***   CIRCULAR ARC   ***

$CØNTRL
ITAPE=1,
$END
$IN
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSGRD(1)=1.E-4,
NDUMP=2000,
NGRID=200,
NGFF=10,
PGFF=1.5,
KEPS=1,
NPRINT=10,
$END
```

- Run 8U: read file 7S, restart file 6U.a; generate file 8U.

*** CIRCULAR ARC ***

```
$CØNTRL
ITAPE=1,
$END
$IN
GAMFF=(3.8,-1.),
SMALLK=.1,
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSGRD(1)=1.E-3,
NDUMP=2000,
NGRID=100,
NGFF=10,
PGFF=1.5,
KEPS=1,
NPRINT=10,
IK=1,
$END
```

- Run 9U: read file 7S, restart file 8U; generate files 9U.a and 9U.b.

*** CIRCULAR ARC ***

```
$CØNTRL
ITAPE=1,
$END
$IN
GAMFF=(3.5,-1.5),
SMALLK=.2,
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSGRD(1)=1.E-3,1.E-3,
NDUMP=2000,
NGRID=200,
NGFF=10,
PGFF=1.5,
KEPS=2,
NPRINT=10,
IK=1,
$END
```

- Run 10U: read file 7S, restart file 9U.b; generate file 10U.

*** CIRCULAR ARC ***

```
$CØNTRL
ITAPE=1,
$END
$IN
ØMEGAH=.75,
ØMEGAE=1.7,
ØMEGAP=.75,
EPSGRD(1)=1.E-4,
NDUMP=2000,
NGRID=200,
NGFF=10,
PGFF=1.5,
KEPS=1,
NPRINT=10,
$END
```

7.2.2 Sample Output for UTRANS Test Cases

The following pages contain a sample of the continuous commentary output for the first 19 cycles of run 1U in addition to the final printed output for all runs.

● Sample Output for Run 1U

```
SIMILARITY PARAMETER (K) =    .21247E+01
SCALING FACTOR (CP/CPHAR) =    .20411E+00

AT ITERATION    1 THE MAXIMUM ERROR =  -.15922E+00 -0.        AND OCCURRED AT NODE  393

AT ITERATION    2 THE MAXIMUM ERROR =  -.11673E+00 -0.        AND OCCURRED AT NODE  335

AT ITERATION    3 THE MAXIMUM ERROR =  -.86551E-01 -0.        AND OCCURRED AT NODE  277

AT ITERATION    4 THE MAXIMUM ERROR =   .59566E-01  0.        AND OCCURRED AT NODE  303

AT ITERATION    5 THE MAXIMUM ERROR =   .44078E-01  0.        AND OCCURRED AT NODE  302

AT ITERATION    6 THE MAXIMUM ERROR =   .34331E-01  0.        AND OCCURRED AT NODE  273

AT ITERATION    7 THE MAXIMUM ERROR =   .24071E-01  0.        AND OCCURRED AT NODE  272

AT ITERATION    8 THE MAXIMUM ERROR =   .17737E-01  0.        AND OCCURRED AT NODE  243

AT ITERATION    9 THE MAXIMUM ERROR =   .12534E-01  0.        AND OCCURRED AT NODE  242

AT ITERATION    9 SCALED PRESSURE COEFFICIENT, UPPER (ILE TO ITE) =
 -.52380E+01 -0.        -.35303E+01 -0.        -.31102E+01 -0.        -.27441E+01 -0.        -.23983E+01 -0.
 -.20266E+01 -0.        -.15247E+01 -0.        -.10564E+01 -0.        -.74481E+00 -0.        -.52239E+00 -0.
 -.36767E+00 -0.        -.20894E+00 -0.        -.45323E-01 -0.

AT ITERATION    9 SCALED PRESSURE COEFFICIENT, LOWER (ILE TO ITE) =
  .52380E+01 -0.         .35303E+01 -0.         .31102E+01 -0.         .27441E+01 -0.         .23983E+01 -0.
  .20268E+01 -0.         .15247E+01 -0.         .10564E+01 -0.         .74481E+00 -0.         .52239E+00 -0.
  .36767E+00 -0.         .20894E+00 -0.         .45323E-01 -0.

UPDATE GAMFF AND FARFIELD AT ITERATION   10 GAMFF =   .22529E+01  0.        GAMTE =   .21923E+01  0.

AT ITERATION   10 THE MAXIMUM ERROR =  -.20743E+00  0.        AND OCCURRED AT NODE  797

AT ITERATION   11 THE MAXIMUM ERROR =   .14285E+00  0.        AND OCCURRED AT NODE  797

AT ITERATION   12 THE MAXIMUM ERROR =  -.97979E-01  0.        AND OCCURRED AT NODE  797

AT ITERATION   13 THE MAXIMUM ERROR =   .67245E-01  0.        AND OCCURRED AT NODE  797

AT ITERATION   14 THE MAXIMUM ERROR =  -.46500E-01  0.        AND OCCURRED AT NODE  797

AT ITERATION   15 THE MAXIMUM ERROR =  -.32033E-01 -0.        AND OCCURRED AT NODE  799

AT ITERATION   16 THE MAXIMUM ERROR =   .22230E-01  0.        AND OCCURRED AT NODE  799

AT ITERATION   17 THE MAXIMUM ERROR =  -.15157E-01 -0.        AND OCCURRED AT NODE  799

AT ITERATION   18 THE MAXIMUM ERROR =   .10557E-01  0.        AND OCCURRED AT NODE  799

AT ITERATION   19 THE MAXIMUM ERROR =   .82213E-02  0.        AND OCCURRED AT NODE  544

AT ITERATION   19 SCALED PRESSURE COEFFICIENT, UPPER (ILE TO ITE) =
 -.52857E+01 -0.        -.35983E+01 -0.        -.31636E+01 -0.        -.27880E+01 -0.        -.24466E+01 -0.
 -.20770E+01 -0.        -.15937E+01 -0.        -.11313E+01 -0.        -.83185E+00 -0.        -.61358E+00 -0.
 -.45350E+00 -0.        -.29855E+00 -0.        -.13773E+00 -0.
```

RUN 1U

*** CIRCULAR ARC ***

MACH NUMBER = .70000E+00
SIMILARITY PARAMETER (K) = .21247E+01
THICKNESS RATIO = .10000E+00
MEAN AIRFOIL ANGLE OF ATTACK (RADIANS) = 0.
REDUCED FREQUENCY (BASED ON CHORD) = 0.
SCALED FREQUENCY (OMEGA) = 0.
HINGE POINT = 0.
MEAN FLAP ANGLE (RADIANS) = 0.
CP SCALING FACTOR (CP/CPBAR) = .20411E+00
SCALED AIRFOIL CIRCULATION = .23186E+01 0.
LIFT COEFFICIENT BASED ON CIRCULATION (PER UNIT PITCH ANGLE IN RADIANS) - .94648E+01 0.

UNSTEADY FORCE COEFFICIENTS (PER UNIT PITCH ANGLE IN RADIANS)

LIFT = .79730E+01 0.
MOMENT ABOUT (X=0) = .24268E+01 0.
HINGE MOMENT = .24268E+01 0.

PRESSURE COEFFICIENTS ON THE AIRFOIL, UNSCALED (PER UNIT PITCH ANGLE IN RADIANS)

AIRFOIL COORDINATE =

0.	.25000E-01	.50000E-01	.80000E-01	.11000E+00
.14500E+00	.18000E+00	.22000E+00	.26000E+00	.31000E+00
.36000E+00	.43000E+00	.50000E+00	.57000E+00	.64000E+00
.69000E+00	.74000E+00	.78000E+00	.82000E+00	.85000E+00
.88000E+00	.91000E+00	.94000E+00	.97000E+00	.10000E+01

AIRFOIL PRESSURE COEFFICIENTS, UPPER =

-.15560E+02 -0.	-.10959E+02 -0.	-.95889E+01 -0.	-.84105E+01 -0.	-.75633E+01 -0.
-.68357E+01 -0.	-.62863E+01 -0.	-.57843E+01 -0.	-.53666E+01 -0.	-.49211E+01 -0.
-.45159E+01 -0.	-.39836E+01 -0.	-.34657E+01 -0.	-.29582E+01 -0.	-.24649E+01 -0.
-.21270E+01 -0.	-.18022E+01 -0.	-.15537E+01 -0.	-.13158E+01 -0.	-.11432E+01 -0.
-.97022E+00 -0.	-.79366E+00 -0.	-.61114E+00 -0.	-.41487E+00 -0.	-.19552E+00 -0.

AIRFOIL PRESSURE COEFFICIENTS, LOWER =

.15576E+02 -0.	.10955E+02 -0.	.95841E+01 -0.	.84041E+01 -0.	.75580E+01 -0.
.68309E+01 -0.	.62789E+01 -0.	.57788E+01 -0.	.53624E+01 -0.	.49154E+01 -0.
.45102E+01 -0.	.39798E+01 -0.	.34625E+01 -0.	.29551E+01 -0.	.24621E+01 -0.
.21245E+01 -0.	.18000E+01 -0.	.15517E+01 -0.	.13141E+01 -0.	.11417E+01 -0.
.96892E+00 -0.	.79252E+00 -0.	.61017E+00 -0.	.41404E+00 -0.	.19483E+00 -0.

RUN 20

```
*** CIRCULAR ARC ***

MACH NUMBER =    .70000E+00
SIMILARITY PARAMETER (K) =    .21747E+01
THICKNESS RATIO =    .10000E+00
MEAN AIRFOIL ANGLE OF ATTACK (RADIANS) =  0.
REDUCED FREQUENCY (BASED ON CHORD) =  0.
SCALED FREQUENCY (OMEGA) =  0.
HINGE POINT =  0.
MEAN FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =    .20411E+00
SCALED AIRFOIL CIRCULATION =    .23729E+01  0.
LIFT COEFFICIENT BASED ON CIRCULATION (PER UNIT PITCH ANGLE IN RADIANS) =    .96867E+01  0.


UNSTEADY FORCE COEFFICIENTS (PER UNIT PITCH ANGLE IN RADIANS)

  LIFT =    .81682E+01  0.
  MOMENT ABOUT (X=0) =    .24909E+01  0.
  HINGE MOMENT =    .24909E+01  0.


PRESSURE COEFFICIENTS ON THE AIRFOIL, UNSCALED (PER UNIT PITCH ANGLE IN RADIANS)

  AIRFOIL COORDINATE =
    0.                  .25000E-01          .50000E-01          .80000E-01          .11000E+00
     .14500E+00         .18000E+00          .22000E+00          .26000E+00          .31000E+00
     .36000E+00         .43000E+00          .50000E+00          .57000E+00          .64000E+00
     .69000E+00         .74000E+00          .78000E+00          .82000E+00          .85000E+00
     .88000E+00         .91000E+00          .94000E+00          .97000E+00          .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
    -.15852E+02 -0.     -.11169E+02 -0.     -.97817E+01 -0.     -.85864E+01 -0.     -.77293E+01 -0.
    -.69927E+01 -0.     -.64335E+01 -0.     -.59252E+01 -0.     -.55034E+01 -0.     -.50465E+01 -0.
    -.46343E+01 -0.     -.40901E+01 -0.     -.35618E+01 -0.     -.30415E+01 -0.     -.25357E+01 -0.
    -.21880E+01 -0.     -.18545E+01 -0.     -.15984E+01 -0.     -.13513E+01 -0.     -.11705E+01 -0.
    -.99030E+00 -0.     -.80829E+00 -0.     -.61988E+00 -0.     -.41786E+00 -0.     -.19554E+00 -0.

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
     .15852E+02 -0.      .11169E+02 -0.      .97817E+01 -0.      .85864E+01 -0.      .77293E+01 -0.
     .69927E+01 -0.      .64335E+01 -0.      .59252E+01 -0.      .55034E+01 -0.      .50465E+01 -0.
     .46343E+01 -0.      .40901E+01 -0.      .35618E+01 -0.      .30414E+01 -0.      .25357E+01 -0.
     .21879E+01 -0.      .18545E+01 -0.      .15983E+01 -0.      .13513E+01 -0.      .11704E+01 -0.
     .99024E+00 -0.      .80822E+00 -0.      .61981E+00 -0.      .41778E+00 -0.      .19547E+00 -0.
```

```
*** CIRCULAR ARC ***

MACH NUMBER =    .70000E+00
SIMILARITY PARAMETER (K) =    .21247E+01
THICKNESS RATIO =    .10000E+00
MEAN AIRFOIL ANGLE OF ATTACK (RADIANS) =  0.
REDUCED FREQUENCY (BASED ON CHORD) =    .10000E+00
SCALED FREQUENCY (OMEGA) =    .20414E+00
HINGE POINT =  0.
MEAN FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =    .20411E+00
SCALED AIRFOIL CIRCULATION =    .21954E+01  -.45432E+00
LIFT COEFFICIENT BASED ON CIRCULATION (PER UNIT PITCH ANGLE IN RADIANS) =    .89619E+01  -.18546E+01


UNSTEADY FORCE COEFFICIENTS (PER UNIT PITCH ANGLE IN RADIANS)

  LIFT =    .69145E+01  -.13763E+01
  MOMENT ABOUT (X=0) =    .21963E+01  -.38830E+00
  HINGE MOMENT =    .21963E+01  -.38830E+00


PRESSURE COEFFICIENTS ON THE AIRFOIL, UNSCALED (PER UNIT PITCH ANGLE IN RADIANS)

  AIRFOIL COORDINATE =
   0.                      .50000E-01              .11000E+00              .18000E+00              .26000E+00
    .36000E+00             .50000E+00              .64000E+00              .74000E+00              .82000E+00
    .88000E+00             .94000E+00              .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
   -.10350E+02   .24169E+01  -.70632E+01   .16066E+01  -.62370E+01   .13943E+01  -.55210E+01   .12030E+01  -.48630E+01   .10198E+01
   -.41568E+01   .81925E+00  -.32252E+01   .56957E+00  -.23053E+01   .35860E+00  -.16755E+01   .23951E+00  -.12011E+01   .16473E+00
   -.86237E+00   .11754E+00  -.53674E+00   .75074E-01  -.23638E+00   .33744E-01

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
    .10350E+02  -.24169E+01   .70632E+01  -.16066E+01   .62370E+01  -.13943E+01   .55210E+01  -.12030E+01   .48630E+01  -.10198E+01
    .41568E+01  -.81925E+00   .32252E+01  -.56957E+00   .23053E+01  -.35860E+00   .16755E+01  -.23951E+00   .12011E+01  -.16473E+00
    .86237E+00  -.11754E+00   .53674E+00  -.75074E-01   .23638E+00  -.33744E-01
```

RUN

RUN 4U

*** CIRCULAR ARC ***

```
MACH NUMBER =    .70000E+00
SIMILARITY PARAMETER (K) =    .21247E+01
THICKNESS RATIO =    .10000E+00
MEAN AIRFOIL ANGLE OF ATTACK (RADIANS) =  0.
REDUCED FREQUENCY (BASED ON CHORD) =    .20000E+00
SCALED FREQUENCY (OMEGA) =    .40828E+00
HINGE POINT =  0.
MEAN FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =    .20411E+00
SCALED AIRFOIL CIRCULATION =    .19162E+01  -.69863E+00
LIFT COEFFICIENT BASED ON CIRCULATION (PER UNIT PITCH ANGLE IN RADIANS) =    .78225E+01  -.28519E+01


UNSTEADY FORCE COEFFICIENTS (PER UNIT PITCH ANGLE IN RADIANS)

  LIFT =    .66261E+01  -.23466E+01
  MOMENT ABOUT (X=0) =    .20655E+01  -.62770E+00
  HINGE MOMENT =    .20655E+01  -.62770E+00


PRESSURE COEFFICIENTS ON THE AIRFOIL, UNSCALED (PER UNIT PITCH ANGLE IN RADIANS)

  AIRFOIL COORDINATE =
    0.              .25000E-01      .50000E-01      .80000E-01      .11000E+00
    .14500E+00      .18000E+00      .22000E+00      .26000E+00      .31000E+00
    .36000E+00      .43000E+00      .50000E+00      .57000E+00      .64000E+00
    .69000E+00      .74000E+00      .78000E+00      .82000E+00      .85000E+00
    .88000E+00      .91000E+00      .94000E+00      .97000E+00      .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
   -.12482E+02  .52751E+01 -.88056E+01  .36808E+01 -.77198E+01  .32049E+01 -.67874E+01  .27912E+01 -.61220E+01  .24902E+01
   -.55529E+01  .22260E+01 -.51228E+01  .20194E+01 -.47378E+01  .18276E+01 -.44199E+01  .16638E+01 -.40793E+01  .14834E+01
   -.37722E+01  .13190E+01 -.33659E+01  .11063E+01 -.29629E+01  .90850E+00 -.25580E+01  .72735E+00 -.21530E+01  .56552E+00
   -.18695E+01  .46356E+00 -.15929E+01  .37302E+00 -.13773E+01  .30831E+00 -.11635E+01  .24915E+00 -.10041E+01  .20832E+00
   -.84670E+00  .17041E+00 -.68712E+00  .13411E+00 -.52253E+00  .99087E-01 -.34653E+00  .63957E-01 -.15977E+00  .28969E-01

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
    .12482E+02 -.52751E+01  .88056E+01 -.36808E+01  .77198E+01 -.32049E+01  .67874E+01 -.27912E+01  .61220E+01 -.24902E+01
    .55529E+01 -.22260E+01  .51228E+01 -.20194E+01  .47378E+01 -.18276E+01  .44199E+01 -.16638E+01  .40793E+01 -.14834E+01
    .37722E+01 -.13190E+01  .33659E+01 -.11063E+01  .29629E+01 -.90850E+00  .25580E+01 -.72735E+00  .21530E+01 -.56552E+00
    .18695E+01 -.46356E+00  .15929E+01 -.37302E+00  .13773E+01 -.30831E+00  .11635E+01 -.24915E+00  .10041E+01 -.20832E+00
    .84670E+00 -.17041E+00  .68712E+00 -.13411E+00  .52253E+00 -.99087E-01  .34653E+00 -.63957E-01  .15977E+00 -.28969E-01
```

RUN 5U

```
*** CIRCULAR ARC ***

MACH NUMBER =    .70000E+00
SIMILARITY PARAMETER (K) =    .21247E+01
THICKNESS RATIO =    .10000E+00
MEAN AIRFOIL ANGLE OF ATTACK (RADIANS) =  0.
REDUCED FREQUENCY (BASED ON CHORD) =    .20000E+00
SCALED FREQUENCY (OMEGA) =    .40828E+00
HINGE POINT =  0.
MEAN FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =    .20411E+00
SCALED AIRFOIL CIRCULATION =    .18655E+01  -.67502E+00
LIFT COEFFICIENT BASED ON CIRCULATION (PER UNIT PITCH ANGLE IN RADIANS) =    .76155E+01  -.27556E+01


UNSTEADY FORCE COEFFICIENTS (PER UNIT PITCH ANGLE IN RADIANS)

  LIFT =    .64565E+01  -.22713E+01
  MOMENT ABOUT (X=0) =    .20152E+01  -.61039E+00
  HINGE MOMENT =    .20152E+01  -.61039E+00


PRESSURE COEFFICIENTS ON THE AIRFOIL, UNSCALED (PER UNIT PITCH ANGLE IN RADIANS)

  AIRFOIL COORDINATE =
   0.                 .25000E-01        .50000E-01        .80000E-01        .11000E+00
    .14500E+00        .18000E+00        .22000E+00        .26000E+00        .31000E+00
    .36000E+00        .43000E+00        .50000E+00        .57000E+00        .64000E+00
    .69000E+00        .74000E+00        .78000E+00        .82000E+00        .85000E+00
    .88000E+00        .91000E+00        .94000E+00        .97000E+00        .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
   -.12098E+02  .50577E+01 -.85458E+01  .35349E+01 -.74989E+01  .30819E+01 -.65999E+01  .26881E+01 -.59582E+01  .24014E+01
   -.54100E+01  .21498E+01 -.49967E+01  .19536E+01 -.46238E+01  .17700E+01 -.43164E+01  .16133E+01 -.39846E+01  .14397E+01
   -.36849E+01  .12815E+01 -.32855E+01  .10756E+01 -.28901E+01  .88408E+00 -.24921E+01  .70837E+00 -.20959E+01  .55147E+00
   -.18183E+01  .45232E+00 -.15483E+01  .36417E+00 -.13384E+01  .30122E+00 -.11332E+01  .24396E+00 -.98131E+00  .20429E+00
   -.82999E+00  .16729E+00 -.67675E+00  .13215E+00 -.51775E+00  .97990E-01 -.34705E+00  .63630E-01 -.16098E+00  .28734E-01

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
    .12098E+02 -.50577E+01  .85458E+01 -.35349E+01  .74989E+01 -.30819E+01  .65999E+01 -.26881E+01  .59582E+01 -.24014E+01
    .54100E+01 -.21498E+01  .49967E+01 -.19536E+01  .46238E+01 -.17700E+01  .43164E+01 -.16133E+01  .39846E+01 -.14397E+01
    .36849E+01 -.12815E+01  .32855E+01 -.10756E+01  .28901E+01 -.88408E+00  .24921E+01 -.70837E+00  .20959E+01 -.55147E+00
    .18183E+01 -.45232E+00  .15483E+01 -.36417E+00  .13384E+01 -.30122E+00  .11332E+01 -.24396E+00  .98131E+00 -.20429E+00
    .82999E+00 -.16729E+00  .67675E+00 -.13215E+00  .51775E+00 -.97990E-01  .34705E+00 -.63630E-01  .16098E+00 -.28734E-01
```

RUN 6U

```
*** CIRCULAR ARC ***

MACH NUMBER =   .80000E+00
SIMILARITY PARAMETER (K) =   .12552E+01
THICKNESS RATIO =   .10000E+00
MEAN AIRFOIL ANGLE OF ATTACK (RADIANS) =  0.
REDUCED FREQUENCY (BASED ON CHORD) =  0.
SCALED FREQUENCY (OMEGA) =  0.
HINGE POINT =  0.
MEAN FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =   .18673E+00
SCALED AIRFOIL CIRCULATION =   .37887E+01  0.
LIFT COEFFICIENT BASED ON CIRCULATION (PER UNIT PITCH ANGLE IN RADIANS) =   .14149E+02  0.

UNSTEADY FORCE COEFFICIENTS (PER UNIT PITCH ANGLE IN RADIANS)

  LIFT =   .12461E+02  0.
  MOMENT ABOUT (X=0) =   .42040E+01  0.
  HINGE MOMENT =   .42040E+01  0.

PRESSURE COEFFICIENTS ON THE AIRFOIL, UNSCALED (PER UNIT PITCH ANGLE IN RADIANS)

  AIRFOIL COORDINATE =
    0.                  .25000E-01          .50000E-01          .80000E-01          .11000E+00
     .14500E+00         .18000E+00          .22000E+00          .26000E+00          .31000E+00
     .36000E+00         .43000E+00          .50000E+00          .57000E+00          .64000E+00
     .69000E+00         .74000E+00          .78000E+00          .82000E+00          .85000E+00
     .88000E+00         .91000E+00          .94000E+00          .97000E+00          .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
    -.16725E+02 -0.     -.12260E+02 -0.     -.10894E+02 -0.     -.97583E+01 -0.     -.89909E+01 -0.
    -.83897E+01 -0.     -.79976E+01 -0.     -.77255E+01 -0.     -.76046E+01 -0.     -.76860E+01 -0.
    -.86676E+01 -0.     -.10490E+02 -0.     -.11087E+02 -0.     -.84556E+01 -0.     -.36506E+01 -0.
    -.19577E+01 -0.     -.16400E+01 -0.     -.13624E+01 -0.     -.11139E+01 -0.     -.94359E+00 -0.
    -.78266E+00 -0.     -.62771E+00 -0.     -.47406E+00 -0.     -.31535E+00 -0.     -.14599E+00 -0.

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
     .16720E+02 -0.      .12257E+02 -0.      .10892E+02 -0.      .97552E+01 -0.      .89872E+01 -0.
     .83865E+01 -0.      .79935E+01 -0.      .77218E+01 -0.      .76010E+01 -0.      .76816E+01 -0.
     .86629E+01 -0.      .10485E+02 -0.      .11080E+02 -0.      .84496E+01 -0.      .36474E+01 -0.
     .19558E+01 -0.      .16385E+01 -0.      .13612E+01 -0.      .11129E+01 -0.      .94278E+00 -0.
     .78200E+00 -0.      .62718E+00 -0.      .47364E+00 -0.      .31503E+00 -0.      .14574E+00 -0.
```

RUN 7U

```
***   CIRCULAR ARC   ***

MACH NUMBER =    .60000E+00
SIMILARITY PARAMETER (K) =    .12552E+01
THICKNESS RATIO =    .10000E+00
MEAN AIRFOIL ANGLE OF ATTACK (RADIANS) =  0.
REDUCED FREQUENCY (BASED ON CHORD) =  0.
SCALED FREQUENCY (OMEGA) =  0.
HINGE POINT =  0.
MEAN FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =    .18673E+00
SCALED AIRFOIL CIRCULATION =    .38117E+01  0.
LIFT COEFFICIENT BASED ON CIRCULATION (PER UNIT PITCH ANGLE IN RADIANS) =    .14235E+02  0.

UNSTEADY FORCE COEFFICIENTS (PER UNIT PITCH ANGLE IN RADIANS)

  LIFT =    .12544E+02  0.
  MOMENT ABOUT (X=0) =    .42374E+01  0.
  HINGE MOMENT =    .42374E+01  0.

PRESSURE COEFFICIENTS ON THE AIRFOIL, UNSCALED (PER UNIT PITCH ANGLE IN RADIANS)

 AIRFOIL COORDINATE =
   0.                .25000E-01        .50000E-01        .80000E-01        .11000E+00
   .14500E+00        .18000E+00        .22000E+00        .26000E+00        .31000E+00
   .36000E+00        .43000E+00        .50000E+00        .57000E+00        .64000E+00
   .69000E+00        .74000E+00        .78000E+00        .82000E+00        .85000E+00
   .88000E+00        .91000E+00        .94000E+00        .97000E+00        .10000E+01

 AIRFOIL PRESSURE COEFFICIENTS, UPPER =
  -.16753E+02 -0.   -.12288E+02 -0.   -.10926E+02 -0.   -.97933E+01 -0.   -.90284E+01 -0.
  -.84303E+01 -0    -.80410E+01 -0.   -.77720E+01 -0.   -.76541E+01 -0.   -.77398E+01 -0.
  -.87296E+01 -0.   -.10568E+02 -0.   -.11177E+02 -0.   -.85303E+01 -0.   -.36855E+01 -0.
  -.19745E+01 -0.   -.16521E+01 -0.   -.13725E+01 -0.   -.11222E+01 -0.   -.95098E+00 -0.
  -.78911E+00 -0.   -.63324E+00 -0.   -.47856E+00 -0.   -.31887E+00 -0.   -.14785E+00 -0.

 AIRFOIL PRESSURE COEFFICIENTS, LOWER =
   .16753E+02 -0.    .12288E+02 -0.    .10926E+02 -0.    .97933E+01 -0.    .90283E+01 -0.
   .84303E+01 -0.    .80410E+01 -0.    .77719E+01 -0.    .76541E+01 -0.    .77398E+01 -0.
   .87296E+01 -0.    .10558E+02 -0.    .11177E+02 -0.    .85302E+01 -0.    .36854E+01 -0.
   .19745E+01 -0.    .16520E+01 -0.    .13724E+01 -0.    .11221E+01 -0.    .95093E+00 -0.
   .78905E+00 -0.    .63317E+00 -0.    .47850E+00 -0.    .31880E+00 -0.    .14779E+00 -0.
```

RUN 8U

```
*** CIRCULAR ARC ***

MACH NUMBER =   .80000E+00
SIMILARITY PARAMETER (K) =   .12552E+01
THICKNESS RATIO =   .10000E+00
MEAN AIRFOIL ANGLE OF ATTACK (RADIANS) =  0.
REDUCED FREQUENCY (BASED ON CHORD) =   .10000E+00
SCALED FREQUENCY (OMEGA) =   .22314E+00
HINGE POINT =  0.
MEAN FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =   .18673E+00
SCALED AIRFOIL CIRCULATION =   .28805E+01  -.11692E+01
LIFT COEFFICIENT BASED ON CIRCULATION (PER UNIT PITCH ANGLE IN RADIANS) =   .10757E+02  -.43662E+01

UNSTEADY FORCE COEFFICIENTS (PER UNIT PITCH ANGLE IN RADIANS)

  LIFT =   .86944E+01  -.34476E+01
  MOMENT ABOUT (X=0) =   .30917E+01  -.10579E+01
  HINGE MOMENT =   .30917E+01  -.10579E+01

PRESSURE COEFFICIENTS ON THE AIRFOIL, UNSCALED (PER UNIT PITCH ANGLE IN RADIANS)

  AIRFOIL COORDINATE =
    0.                    .50000E-01          .11000E+00          .18000E+00          .26000E+00
     .36000E+00           .50000E+00          .64000E+00          .74000E+00          .82000E+00
     .88000E+00           .94000E+00          .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
    -.87434E+01  .41591E+01 -.62767E+01  .29666E+01 -.57660E+01  .27373E+01 -.54345E+01  .25909E+01 -.53007E+01  .25371E+01
    -.62078E+01  .27552E+01 -.77793E+01  .27090E+01 -.40044E+01  .10075E+01 -.10266E+01  .92282E-01 -.81437E+00  .11692E+00
    -.57668E+00  .84766E-01 -.34838E+00  .50727E-01 -.14995E+00  .21973E-01

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
     .87434E+01 -.41591E+01  .62767E+01 -.29666E+01  .57660E+01 -.27373E+01  .54345E+01 -.25909E+01  .53007E+01 -.25371E+01
     .62078E+01 -.27552E+01  .77793E+01 -.27090E+01  .40044E+01 -.10075E+01  .10266E+01 -.92282E-01  .81437E+00 -.11692E+00
     .57668E+00 -.84766E-01  .34838E+00 -.50727E-01  .14995E+00 -.21973E-01
```

RUN 9U

```
***  CIRCULAR ARC  ***

MACH NUMBER =    .80000E+00
SIMILARITY PARAMETER (K) =    .12552E+01
THICKNESS RATIO =    .10000E+00
MEAN AIRFOIL ANGLE OF ATTACK (RADIANS) =  0.
REDUCED FREQUENCY (BASED ON CHORD) =    .20000E+00
SCALED FREQUENCY (OMEGA) =    .44629E+00
HINGE POINT =  0.
MEAN FLAP ANGLE (RADIANS) =  0.
CP SCALING FACTOR (CP/CPBAR) =    .18673E+00
SCALED AIRFOIL CIRCULATION =    .21245E+01  -.13303E+01
LIFT COEFFICIENT BASED ON CIRCULATION (PER UNIT PITCH ANGLE IN RADIANS) =    .79341E+01  -.49680E+01


UNSTEADY FORCE COEFFICIENTS (PER UNIT PITCH ANGLE IN RADIANS)

  LIFT =    .70755E+01  -.43429E+01
  MOMENT ABOUT (X=0) =    .26016E+01  -.12927E+01
  HINGE MOMENT =    .26016E+01  -.12927E+01


PRESSURE COEFFICIENTS ON THE AIRFOIL, UNSCALED (PER UNIT PITCH ANGLE IN RADIANS)

  AIRFOIL COORDINATE =
   0.                      .25000E-01             .50000E-01             .80000E-01             .11000E+00
    .14500E+00             .18000E+00             .22000E+00             .26000E+00             .31000E+00
    .36000E+00             .43000E+00             .50000E+00             .57000E+00             .64000E+00
    .69000E+00             .74000E+00             .78000E+00             .82000E+00             .85000E+00
    .88000E+00             .91000E+00             .94000E+00             .97000E+00             .10000E+01

  AIRFOIL PRESSURE COEFFICIENTS, UPPER =
   -.82102E+01   .63720E+01  -.60290E+01   .46916E+01  -.53606E+01   .41984E+01  -.48086E+01   .37948E+01  -.44380E+01   .35266E+01
   -.41538E+01   .33237E+01  -.39769E+01   .31992E+01  -.38660E+01   .31225E+01  -.38370E+01   .31037E+01  -.39338E+01   .31765E+01
   -.46472E+01   .34583E+01  -.60816E+01   .38381E+01  -.68851E+01   .37322E+01  -.57771E+01   .23118E+01  -.28027E+01   .54385E+00
   -.15017E+01   .19272E+00  -.12219E+01   .19820E+00  -.10132E+01   .16330E+00  -.82829E+00   .12901E+00  -.70093E+00   .10383E+00
   -.57979E+00   .81078E-01  -.46393E+00   .60712E-01  -.34913E+00   .41392E-01  -.23066E+00   .22714E-01  -.10624E+00   .87445E-02

  AIRFOIL PRESSURE COEFFICIENTS, LOWER =
    .82102E+01  -.63720E+01   .60290E+01  -.46916E+01   .53606E+01  -.41984E+01   .48086E+01  -.37948E+01   .44380E+01  -.35266E+01
    .41538E+01  -.33237E+01   .39769E+01  -.31992E+01   .38660E+01  -.31225E+01   .38370E+01  -.31037E+01   .39338E+01  -.31765E+01
    .46472E+01  -.34583E+01   .60816E+01  -.38381E+01   .68851E+01  -.37322E+01   .57771E+01  -.23118E+01   .28027E+01  -.54385E+00
    .15017E+01  -.19272E+00   .12219E+01  -.19820E+00   .10132E+01  -.16330E+00   .82829E+00  -.12901E+00   .70093E+00  -.10383E+00
    .57979E+00  -.81078E-01   .46393E+00  -.60712E-01   .34913E+00  -.41392E-01   .23066E+00  -.22714E-01   .10624E+00  -.87445E-02
```

*** CIRCULAR ARC ***

MACH NUMBER = .80000E+00
SIMILARITY PARAMETER (K) = .12552E+01
THICKNESS RATIO = .10000E+00
MEAN AIRFOIL ANGLE OF ATTACK (RADIANS) = 0.
REDUCED FREQUENCY (BASED ON CHORD) = .20000E+00
SCALED FREQUENCY (OMEGA) = .44629E+00
HINGE POINT = 0.
MEAN FLAP ANGLE (RADIANS) = 0.
CP SCALING FACTOR (CP/CPBAR) = .18673E+00
SCALED AIRFOIL CIRCULATION = .21320E+01 -.12718E+01
LIFT COEFFICIENT BASED ON CIRCULATION (PER UNIT PITCH ANGLE IN RADIANS) = .79619E+01 -.47494E+01

UNSTEADY FORCE COEFFICIENTS (PER UNIT PITCH ANGLE IN RADIANS)

LIFT = .70984E+01 -.41516E+01
MOMENT ABOUT (X=0) = .25988E+01 -.12261E+01
HINGE MOMENT = .25988E+01 -.12261E+01

PRESSURE COEFFICIENTS ON THE AIRFOIL, UNSCALED (PER UNIT PITCH ANGLE IN RADIANS)

AIRFOIL COORDINATE =

0.	.25000E-01	.50000E-01	.80000E-01	.11000E+00
.14500E+00	.18000E+00	.22000E+00	.26000E+00	.31000E+00
.36000E+00	.43000E+00	.50000E+00	.57000E+00	.64000E+00
.69000E+00	.74000E+00	.78000E+00	.82000E+00	.85000E+00
.88000E+00	.91000E+00	.94000E+00	.97000E+00	.10000E+01

AIRFOIL PRESSURE COEFFICIENTS, UPPER =

-.62744E+01	.61103E+01	-.60801E+01	.45046E+01	-.54092E+01	.40358E+01	-.48541E+01	.36517E+01	-.44828E+01	.33979E+01
-.41980E+01	.32061E+01	-.40198E+01	.30885E+01	-.39083E+01	.30170E+01	-.38792E+01	.30006E+01	-.39763E+01	.30712E+01
-.46864E+01	.33333E+01	-.61072E+01	.36741E+01	-.68911E+01	.35477E+01	-.57472E+01	.21556E+01	-.27663E+01	.47423E+00
-.14779E+01	.15860E+00	-.12009E+01	.16735E+00	-.99538E+00	.13824E+00	-.81372E+00	.11014E+00	-.68994E+00	.90752E-01
-.57286E+00	.73264E-01	-.46006E+00	.57023E-01	-.34803E+00	.41497E-01	-.23212E+00	.26149E-01	-.10798E+00	.11288E-01

AIRFOIL PRESSURE COEFFICIENTS, LOWER =

.62744E+01	-.61103E+01	.60801E+01	-.45046E+01	.54092E+01	-.40358E+01	.48541E+01	-.36517E+01	.44828E+01	-.33979E+01
.41980E+01	-.32061E+01	.40198E+01	-.30885E+01	.39083E+01	-.30170E+01	.38792E+01	-.30006E+01	.39763E+01	-.30712E+01
.46864E+01	-.33333E+01	.61072E+01	-.36741E+01	.68911E+01	-.35477E+01	.57472E+01	-.21556E+01	.27663E+01	-.47423E+00
.14779E+01	-.15860E+00	.12009E+01	-.16735E+00	.99538E+00	-.13824E+00	.81372E+00	-.11014E+00	.68994E+00	-.90752E-01
.57286E+00	-.73264E-01	.46006E+00	-.57023E-01	.34803E+00	-.41497E-01	.23212E+00	-.26149E-01	.10798E+00	-.11288E-01

RUN 10U

8.0 REFERENCES

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2. Landahl, M. T., "Linearized Theory for Unsteady Transonic Flow," in Symposium Transsonicum (ed. by K. Oswatitsch), Springer Verlag/Berlin, pp. 414-439.

3. Traci, R. M., Albano, E. D., Farr, Jr., J. L., Cheng, M. K., "Small Disturbance Transonic Flows About Oscillating Airfoils," AFFDL-TR-74-37, April 1974.

4. Ehlers, F. E., "A Finite Difference Method for the Solution of the Transonic Flow Around Harmonically Oscillating Wings," NASA CR-2257, January 1974.

5. Murman, E. M., and Cole, J. D., "Calculation of Plane Steady Transonic Flows," AIAA Paper 70-188, June 1970.

6. Murman, E. M., and Krupp, J. A., "The Numerical Calculations of Steady Transonic Flows Past Thin Lifting Airfoils and Slender Bodies," AIAA Paper no. 71-566, June 1971.

7. Krupp, J. A., "The Numerical Calculation of Plane Steady Transonic Flows Past Thin Lifting Airfoils," Boeing Scientific Research Laboratories, D180-12958-1, June 1971.

8. Murman, E. M., "Analysis of Embedded Shock Waves Calculated by Relaxation Methods," Proceedings AIAA Computational Fluid Dynamics Conference, Palm Springs, Cal., July 1973.

9. Klunker, E. B., "Contribution to Methods for Calculating the Flow about Thin Lifting Wings at Transonic Speeds," NASA TN D-6530, Nov. 1971.

10. Abramowitz and Stegun, Handbook of Mathematical Functions, Dover Publications, New York, 1965.

APPENDIX A

FORTRAN LISTING OF STRANS

A FORTRAN listing of the source deck for the STRANS program is presented in the following pages. The program, as configured here, requires 63_8K words to load and 51_8K words to execute.

```
      PROGRAM STRANS (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7,
     1  TAPE8)
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ITE,ILE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  ALPHA,DEL,M8,GAM,K,NDB,XH,TITLE(8),DOUB,DOUBLT,ALPHAF
      COMMON /COEFF/ A(99),B(99),C(99),D(99),PHI(10000)
      COMMON /GAMMA/ GAMTE1,GAMTE,PGFF,GAMFF
      DIMENSION PHIOC(99),PHIOG(99),OMEGA(99),V(99),EPSGRD(3)
      NAMELIST /IN/ X,Y,M8,GAM,DEL,ALPHA,GAMFF,IM,JM,ILE,ITE,JW,
     1  OMEGAH,OMEGAE,OMEGAP,EPSCOL,EPSGRD,XH,NDUMP,NCOL,NGRID,
     2  NGFF,PGFF,KEPS,IK,NPRINT,ALPHAF
      NAMELIST /CONTRL/ ITAPE
C
C  TO RESTART PROGRAM, COPY THE DATA FOR RESTART FROM TAPE7 TO A DISC
C  FILE TAPE8, POSITION TAPE7 AT THE END OF THE LAST FILE ON THE TAPE
C  SO NEW DATA MAY BE WRITTEN ON THE TAPE WITHOUT LOSING ANY OF THE
C  OLD DATA
C
      READ (5,912) (TITLE(I),I=1,8)
      READ (5,CONTRL)
      IF (ITAPE.EQ.0) GO TO 10
C  READ DATA FROM RESTART TAPE
      READ (8) NITERG,IM,IM1,JM,JM1,JW,JWP1,JWM1,ITE,ILE,K,DEL,
     1  ALPHA,GAMTE1,GAMFF,NDB,XH,M8,GAM,DYBU1,DYBU2,DYBL1,DYBL2,
     2  DOUB,DOUBLT,ALPHAF
      READ (8) (X(I),I=1,IM)
      READ (8) (Y(I),I=1,JM)
      READ (8) (FPU(I),FPL(I),PHIUB(I),I=ILE,ITE)
      READ (8) (AX1(I),AX2(I),BX1(I),BX2(I),CX(I),DX(I),I=1,IM1)
      READ (8) (AY1(I),AY2(I),DY(I),I=1,JM1)
      L=IM*JM
      READ (8) (PHI(I),I=1,L)
      IK=0
      READ (5,IN)
C  THE IK OPTION IS USED TO BOOT STRAP TO DIFFERENT MACH NUMBERS,
C  AIRFOIL THICKNESSES AND/OR ANGLES OF ATTACK
C  MAKE SURE YOU HAVE INPUT THE NEW M8, DEL AND/OR ALPHA
      IF (IK.EQ.0) GO TO 4
      K=(1.-M8**2)/((1.+GAM)*DEL*M8**2)**.6666666667
      CALL INITAL
      CALL FARFLD
    4 CONTINUE
      SK=SQRT(ABS(K))
      GAMTE=GAMTE1
      WRITE (6,900)
      WRITE (6,901) NITERG
      NITERG=0
      GO TO 15
C  START PROBLEM FROM SCRATCH
   10 CONTINUE
      READ (5,IN)
      K=(1.-M8**2)/((1.+GAM)*DEL*M8**2)**.6666666667
      SK=SQRT(ABS(K))
```

```
      GAMTE=GAMFF
      GAMTE1=GAMFF
      NITERG=0
      NDB=0
      IM1=IM-1
      JM1=JM-1
      JWP1=JW+1
      JWM1=JW-1
      IMJM=IM*JM
      DO 5 L=1,IMJM
      PHI(L)=0.
    5 CONTINUE
C   INITIALIZE FINITE DIFFERENCE COEFFICIENTS AND FARFIELD
      CALL INITAL
      CALL FARFLD
      IF (K.GT.0.) GO TO 45
C   INITIAL GUESS FOR SUPERSONIC CASE (INTERIOR ONLY)
      DO 40 I=3,IM1
      M=(I-1)*JM
      DO 41 J=2,JM1
      L=M+J
      PHI(L)=0.
      SKY=SK*ABS(Y(J))
      IF (X(I).LT.SKY.OR.X(I).GT.SKY+1.) GO TO 41
      IF (Y(J).GE.0.) PHI(L)=-FU(X(I)-SKY)/SK
      IF (Y(J).LT.0.) PHI(L)=-FL(X(I)-SKY)/SK
   41 CONTINUE
   40 CONTINUE
      GO TO 46
   45 CONTINUE
C   INITIAL GUESS FOR SUBSONIC CASE (INTERIOR ONLY)
      CON=DOUB/(3.14159265*SK)
      DO 6 I=3,IM1
      M=(I-1)*JM
      DO 7 J=2,JM1
      L=M+J
      IF (X(I).EQ.0..AND.Y(J).EQ.0.) GO TO 8
      PHI(L)=CON*X(I)/(X(I)**2+K*Y(J)**2)
      IF (ABS(PHI(L)).GT.1.) PHI(L)=SIGN(1.,X(I))
      GO TO 7
    8 CONTINUE
      PHI(L)=PHI(L-JM)
    7 CONTINUE
    6 CONTINUE
   46 CONTINUE
      L=(ILE-2)*JM+JW
      PHIUB(ILE)=PHI(L)
   15 CONTINUE
      WRITE (6,IN)
      WRITE (6,900)
      CPCPB=DEL**.6666666667/((1.+GAM)*M8**2)**.3333333333
      WRITE (6,913) K,CPCPB
      KGRD=1
   50 CONTINUE
      ERROR=0.
```

```
      NIT=NITERG
      NITERG=NITERG+1
      IF (MOD(NITERG,NPRINT).EQ.0) CALL PRINT(NIT)
      IF (K.LT.0.) GO TO 51
      IF (MOD(NITERG,NGFF).NE.0) GO TO 51
      CALL GAMFUN
      CALL FARFLD
      WRITE (6,910) NITERG,GAMFF,GAMTE,DOUBLT
   51 CONTINUE
      DO 1 I=2,JM1
      V(I)=K
    1 CONTINUE
C   BEGIN LOOP ON GRID
      DO 100 I=3,IM1
C   CHECK FOR AIR FOIL
      IFLAG=0
      IF (ILE.LE.I.AND.I.LE.ITE) IFLAG=1
      M=(I-1)*JM
C   SAVE THIS COLUMN OF PHI
      DO 2 J=2,JM1
      L=M+J
      PHIOG(J)=PHI(L)
    2 CONTINUE
C   LOOP BACK POINT FOR COLUMN ITERATION
      NITERC=0
  150 CONTINUE
      NITERC=NITERC+1
      IF (NITERC.GT.NCOL) GO TO 294
C   SAVE PREVIOUS PHI FOR COLUMN ITERATION
      DO 3 J=2,JM1
      L=M+J
      PHIOC(J)=PHI(L)
    3 CONTINUE
C   BEGIN LOOP ON COLUMN
      DO 160 J=2,JM1
C   CALCULATE CELL INDICES
      L=M+J
      LR=L+JM
      LL=L-JM
      LLL=LL-JM
      LB=L-1
      LT=L+1
      TPHIR=PHI(LR)
      TPHIL=PHI(LL)
      IF (I.EQ.ILE-1.AND.J.EQ.JW) PHI(LR)=.5*(PHI(L+JM)+PHIUB(ILE))
      IF (I.EQ.ITE+1.AND.J.EQ.JW) PHI(LL)=.5*(PHIUB(ITE)+PHI(L-JM))
C   SET UP TRIDIAGONAL MATRIX TO SOLVE FOR PHI(I,J)
C   A * PHI(I,J+1) + B * PHI(I,J) + C * PHI(I,J-1) = D
      IF (IFLAG.EQ.1.AND.J.EQ.JWP1) GO TO 250
      IF (IFLAG.EQ.1.AND.J.EQ.JWM1) GO TO 270
      IF (IFLAG.EQ.1.AND.J.EQ.JW) GO TO 290
      PART=0.
      IF (I.LE.ITE) GO TO 201
C   KUTTA CONDITION
      SIGI=(X(I)-1.)*(GAMFF-GAMTE)/(X(IM1)-1.)+GAMTE
```

```
      IF (J.EQ.JWM1) PART=.5*AY1(J)*SIGI
      IF (J.EQ.JW) PART=.5*(AY1(J)-AY2(J))*SIGI
      IF (J.EQ.JWP1) PART=-.5*AY2(J)*SIGI
  201 CONTINUE
      VV=K-AX1(I)*(PHI(LR)-PHI(L))-AX2(I)*(PHI(L)-PHI(LL))
      IF (VV.LT.0.) GO TO 220
C   ELLIPTIC
      OMEGA(J)=OMEGAE
      B(J)=-(VV*(BX1(I)+BX2(I))+AY1(J)+AY2(J))
      D(J)=-VV*(BX1(I)*PHI(LR)+BX2(I)*PHI(LL))+PART
      IF (J.EQ.2) GO TO 202
      IF (J.EQ.JM1) GO TO 203
      A(J)=AY1(J)
      C(J)=AY2(J)
      GO TO 200
C   BOTTOM BOUNDARY
  202 CONTINUE
      A(J)=AY1(J)
      D(J)=D(J)-AY2(J)*PHI(LB)
      GO TO 200
C   TOP BOUNDARY
  203 CONTINUE
      C(J)=AY2(J)
      D(J)=D(J)-AY1(J)*PHI(LT)
      GO TO 200
  220 CONTINUE
      IF (V(J).GT.0.) GO TO 240
C   HYPERBOLIC
      OMEGA(J)=OMEGAH
      VV=K-CX(I-1)*(PHI(L)-PHI(LL))-CX(I-2)*(PHI(LL)-PHI(LLL))
      B(J)=VV*BX1(I-1)-AY1(J)-AY2(J)
      D(J)=VV*(BX1(I-1)*PHI(LL)+BX2(I-1)*(PHI(LL)-PHI(LLL)))+PART
      IF (J.EQ.2) GO TO 222
      IF (J.EQ.JM1) GO TO 223
      A(J)=AY1(J)
      C(J)=AY2(J)
      GO TO 200
C   BOTTOM BOUNDARY
  222 CONTINUE
      A(J)=AY1(J)
      IF (K.LT.0.) GO TO 224
      D(J)=D(J)-AY2(J)*PHI(LB)
      GO TO 200
  224 CONTINUE
      B(J)=B(J)-2.*SK*CX(I-1)/DY(2)
      D(J)=D(J)-2.*SK*(CX(I-1)*PHI(LL)-CX(I-2)*(PHI(LL)-PHI(LLL)))/DY(2)
      GO TO 200
C   TOP BOUNDARY
  223 CONTINUE
      C(J)=AY2(J)
      IF (K.LT.0.) GO TO 225
      D(J)=D(J)-AY1(J)*PHI(LT)
      GO TO 200
  225 CONTINUE
      B(J)=B(J)-2.*SK*CX(I-1)/DY(J-1)
```

```
      D(J)=D(J)-2.*SK*(CX(I-1)*PHI(LL)-CX(I-2)*(PHI(LL)-PHI(LLL)))/
     1  DY(J-1)
      GO TO 200
C  PARABOLIC
  240 CONTINUE
      OMEGA(J)=OMEGAP
      B(J)=VV*BX1(I-1)-AY1(J)-AY2(J)
      D(J)=VV*(BX1(I-1)+BX2(I-1))*PHI(LL)-VV*BX2(I-1)*PHI(LLL)+PART
      IF (J.EQ.2) GO TO 242
      IF (J.EQ.JM1) GO TO 243
      A(J)=AY1(J)
      C(J)=AY2(J)
      GO TO 200
C  BOTTOM BOUNDARY
  242 CONTINUE
      A(J)=AY1(J)
      D(J)=D(J)-AY2(J)*PHI(LB)
      GO TO 200
C  TOP BOUNDARY
  243 CONTINUE
      C(J)=AY2(J)
      D(J)=D(J)-AY1(J)*PHI(LT)
      GO TO 200
C  BODY BOUNDARY ABOVE
  250 CONTINUE
      VV=K-AX1(I)*(PHI(LR)-PHI(L))-AX2(I)*(PHI(L)-PHI(LL))
      IF (VV.LT.0.) GO TO 260
C  ELLIPTIC
      OMEGA(J)=OMEGAE
      A(J)=DYBU1
      B(J)=-(DYBU1+VV*(BX1(I)+BX2(I)))
      C(J)=0.
      D(J)=DYBU2*FPU(I)-VV*(BX1(I)*PHI(LR)+BX2(I)*PHI(LL))
      GO TO 200
  260 CONTINUE
      IF (V(J).GT.0.) GO TO 265
C  HYPERBOLIC
      OMEGA(J)=OMEGAH
      VV=K-CX(I-1)*(PHI(L)-PHI(LL))-CX(I-2)*(PHI(LL)-PHI(LLL))
      A(J)=DYBU1
      B(J)=VV*BX1(I-1)-DYBU1
      C(J)=0.
      D(J)=DYBU2*FPU(I)+VV*(BX1(I-1)*PHI(LL)+BX2(I-1)*
     1  (PHI(LL)-PHI(LLL)))
      GO TO 200
C  PARABOLIC
  265 CONTINUE
      OMEGA(J)=OMEGAP
      A(J)=DYBU1
      B(J)=VV*BX1(I-1)-DYBU1
      C(J)=0.
      D(J)=DYBU2*FPU(I)+VV*(BX1(I-1)*PHI(LL)+BX2(I-1)*
     1  (PHI(LL)-PHI(LLL)))
      GO TO 200
C  BODY BOUNDARY BELOW
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```
  270 CONTINUE
      VV=K-AX1(I)*(PHI(LR)-PHI(L))-AX2(I)*(PHI(L)-PHI(LL))
      IF (VV.LT.0.) GO TO 280
C  ELLIPTIC
      OMEGA(J)=OMEGAE
      A(J)=0.
      B(J)=-(DYBL1+VV*(BX1(I)+BX2(I)))
      C(J)=DYBL1
      D(J)=-DYBL2*FPL(I)-VV*(BX1(I)*PHI(LR)+BX2(I)*PHI(LL))
      GO TO 200
  280 CONTINUE
      IF (V(J).GT.0.) GO TO 285
C  HYPERBOLIC
      OMEGA(J)=OMEGAH
      VV=K-CX(I-1)*(PHI(L)-PHI(LL))-CX(I-2)*(PHI(LL)-PHI(LLL))
      A(J)=0.
      B(J)=VV*BX1(I-1)-DYBL1
      C(J)=DYBL1
      D(J)=-DYBL2*FPL(I)+VV*(BX1(I-1)*PHI(LL)+BX2(I-1)*(PHI(LL)-
     1  PHI(LLL)))
      GO TO 200
C  PARABOLIC
  285 CONTINUE
      OMEGA(J)=OMEGAP
      A(J)=0.
      B(J)=VV*BX1(I-1)-DYBL1
      C(J)=DYBL1
      D(J)=-DYBL2*FPL(I)+VV*(BX1(I-1)*PHI(LL)+BX2(I-1)*
     1  (PHI(LL)-PHI(LLL)))
      GO TO 200
  290 CONTINUE
C  BODY BOUNDARY J=JW
      A(J)=0.
      B(J)=1.
      C(J)=0.
      D(J)=PHI(L)
  200 CONTINUE
      PHI(LR)=TPHIR
      PHI(LL)=TPHIL
  160 CONTINUE
C  TRIDIAGONAL MATRIX IS SET NOW SOLVE FOR COLUMN OF PHI
      CALL TRI (I)
C  CHECK FOR COLUMN CONVERGENCE OF PHI
      DO 295 J=2,JM1
      L=M+J
      JERROR=J
      ERRC=PHIOC(J)-PHI(L)
      IF (ABS(ERRC).GT.EPSCOL) GO TO 150
  295 CONTINUE
  294 CONTINUE
      IF (NITERC.GT.NCOL) WRITE (6,904) NITERG,I,ERRC,JERROR
C  CONVERGED, RELAX PHI, FIND ERROR, CAL. V, AND MOVE TO NEXT COLUMN
      DO 296 J=2,JM1
      L=M+J
      ERR=OMEGA(J)*(PHI(L)-PHIOG(J))
```

```
      PHI(L)=PHIOG(J)+ERR
      V(J)=K-AX1(I)*(PHI(L+JM)-PHI(L))-AX2(I)*(PHI(L)-PHI(L-JM))
      IF (ABS(ERR).LT.ABS(ERROR)) GO TO 296
      ERROR=ERR
      LERROR=L
  296 CONTINUE
      IF (IFLAG.NE.1) GO TO 100
      L=M+JW
      PHI(L)=PHI(L-1)+DY(JWM1)*(PHI(L-1)-PHI(L-2))/DY(JW-2)
      PHIUB(I)=PHI(L+1)-DY(JW)*(PHI(L+2)-PHI(L+1))/DY(JWP1)
      V(JW)=K-AX1(I)*(PHI(L+JM)-PHI(L))-AX2(I)*(PHI(L)-PHI(L-JM))
      IF (I.EQ.ITE) GAMTE=PHIUB(I)-PHI(L)
      IF (ABS(GAMFF).LE.1.E-8) GAMTE=0.
      IF (K.LT.0.) GAMFF=GAMTE
      IF (I.EQ.ITE.AND.NITERG.EQ.1) GAMTE1=GAMTE
  100 CONTINUE
C  PRINT OUT ERROR AFTER GRID SWEEP
      WRITE (6,905) NITERG,ERROR,LERROR
      IDOUB=0
      IF (ABS(ERROR).LE.EPSGRD(KGRD)) GO TO 300
      IF (NITERG.EQ.NGRID) GO TO 310
      IF (MOD(NITERG,NDUMP).EQ.0) GO TO 310
      GO TO 50
  300 CONTINUE
      KGRD=KGRD+1
      IDOUB=1
      GO TO 310
  301 CONTINUE
      IF (K.LT.0.) GO TO 302
      CALL GAMFUN
      WRITE (6,910) NITERG,GAMFF,GAMTE,DOUBLT
  302 CONTINUE
      CALL FPRINT
      WRITE (6,900)
      WRITE (6,906) NITERG
      CALL DOUBLE
      WRITE (6,914) IM,JM,JW,ILE,ITE
      WRITE (6,902)
      WRITE (6,903) (X(I),I=1,IM)
      WRITE (6,911)
      WRITE (6,903) (Y(I),I=1,JM)
      GO TO 50
  310 CONTINUE
C  TAPE DUMP
      WRITE (7) NITERG,IM,IM1,JM,JM1,JW,JWP1,JWM1,ITE,ILE,K,DEL,
     1  ALPHA,GAMTE1,GAMFF,NDB,XH,M8,GAM,DYBU1,DYBU2,DYBL1,DYBL2,
     2  DOUB,DOUBLT,ALPHAF
      WRITE (7) (X(I),I=1,IM)
      WRITE (7) (Y(I),I=1,JM)
      WRITE (7) (FPU(I),FPL(I),PHIUB(I),I=ILE,ITE)
      WRITE (7) (AX1(I),AX2(I),BX1(I),BX2(I),CX(I),DX(I),I=1,IM1)
      WRITE (7) (AY1(I),AY2(I),DY(I),I=1,JM1)
      L=IM*JM
      WRITE (7) (PHI(I),I=1,L)
      END FILE 7
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      WRITE (6,907) NITERG
      CALL PRINT(NITERG)
      IF (KGRD.GT.KEPS) GO TO 320
      IF (NITERG.EQ.NGRID) GO TO 330
      IF (IDOUB.EQ.1) GO TO 301
      GO TO 50
  320 CONTINUE
      WRITE (6,908) NITERG
      GO TO 350
  330 CONTINUE
      WRITE (6,909) NITERG
  900 FORMAT (1H1)
  901 FORMAT (1H ,/,* CASE IS BEING RESTARTED AT ITERATION*I5)
  902 FORMAT (1H ,/,* X(I),I=1,IM*)
  903 FORMAT (10E13.5)
  904 FORMAT (1H ,/,* AT ITERATION*I5* COLUMN*I4* FAILED TO CONVERGE*
     1  * ERR =*E13.5* J=*I3)
  905 FORMAT (1H ,/,* AT ITERATION*I5* THE MAXIMUM ERROR =*E13.5
     1  * AND OCCURRED AT NODE*I5)
  906 FORMAT (1H ,/,* THE NUMBER OF NODES IS BEING DOUBLED AT ITERATION*
     1  I5)
  907 FORMAT (1H ,/,* TAPE HAS BEEN DUMPED AT ITERATION*I5)
  908 FORMAT (1H ,/,* SOLUTION HAS CONVERGED TO DESIRED ACCURACY AT ITER
     1ATION*I5)
  909 FORMAT (1H ,/,* MAXIMUM NUMBER OF ITERATIONS HAS BEEN REACHED, CAS
     1E IS BEING TERMINATED AT ITERATION*I5)
  910 FORMAT (1H ,/,* UPDATE GAMFF AND FARFIELD AT ITERATION*I5
     1  * GAMFF =*E13.5* GAMTE =*E13.5* DOUBLET =*E13.5)
  911 FORMAT (1H ,/,* Y(I),I=1,JM*)
  912 FORMAT (8A10)
  913 FORMAT (1H ,/,* SIMILARITY PARAMETER (K) =*E13.5,/,* SCALING FACTO
     1R (CP/CPBAR) =*E13.5)
  914 FORMAT (1H ,/,* IM =*I4* JM =*I4* JW =*I4* ILE =*I4* ITE =*I4)
  350 CONTINUE
      CALL FPRINT
      END
      SUBROUTINE DOUBLE
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ITE,ILE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  ALPHA,DEL,M8,GAM,K,NDB,XH,TITLE(8),DOUB,DOUBLT,ALPHAF
      COMMON /COEFF/ A(99),B(99),C(99),D(99),PHI(10000)
      COMMON /GAMMA/ GAMTE1,GAMTE,PGFF,GAMFF
      NDB=NDB+1
C   CHANGE GRID INDICES
      IMAX=2*IM-4
      IMAX1=IMAX-1
      JMAX=2*JM-3
      JMAX1=JMAX-1
      JWN=2*JW-2
      JWNP1=JWN+1
      JWNM1=JWN-1
      ITEN=2*ITE-3
      DIST=1./(1.+DY(JWP1)/DY(JW))
```

```
      IK=ITE
C   CHANGE X
      L=IM
      DO 30 I=5,IMAX1,2
      II=IMAX1-I+5
      L=L-1
      X(II)=X(L)
      X(II-1)=.5*(X(L)+X(L-1))
   30 CONTINUE
      X(IMAX)=2.*X(IMAX1)-X(IMAX1-1)
      DELT=X(4)-X(3)
      X(2)=X(3)-DELT
      X(1)=X(2)-DELT
C   FIND NEW LEADING EDGE
      DO 31 I=3,IMAX1
      IF (X(I).GE.0.) GO TO 32
   31 CONTINUE
   32 ILEN=I
C   CHANGE Y
      L=JM
      DO 40 I=2,JMAX1,2
      II=JMAX1-I+2
      L=L-1
      Y(II)=Y(L)
      Y(II-1)=.5*(Y(L)+Y(L-1))
      IF (II-1.EQ.JWNP1) Y(II-1)=Y(L-1)+DIST*(Y(L)-Y(L-1))
      IF (II-1.EQ.JWNM1) Y(II-1)=Y(L)+DIST*(Y(L-1)-Y(L))
   40 CONTINUE
      Y(JMAX)=2.*Y(JMAX1)-Y(JMAX1-1)
      Y(1)=2.*Y(2)-Y(3)
C   MOVE THE PHI(S) (INTERIOR ONLY) AND INTERPOLATE FOR PHI (I=ODD)
      L=IM1*JM
      DO 10 I=3,IMAX1,2
      II=IMAX1-I+3
      LL=(II-1)*JMAX
      DO 15 J=2,JMAX1,2
      JJ=JMAX1-J+2
      LLL=LL+JJ
      L=L-1
      PHI(LLL)=PHI(L)
      PHI(LLL-1)=.5*(PHI(L)+PHI(L-1))
      IF (JJ.NE.JWN) GO TO 15
      PHI(LLL-1)=PHI(L)+DIST*(PHI(L-1)-PHI(L))
      IF (II.GT.ITEN) GO TO 16
      IF (II.GE.ILEN.AND.II.LE.ITEN) GO TO 17
      PHI(LLL+1)=PHI(L)+DIST*(PHI(L+1)-PHI(L))
      GO TO 15
   16 CONTINUE
      SIGI=(X(II)-1.)*(GAMFF-GAMTE)/(X(IMAX1)-1.)+GAMTE
      PDUM=PHI(L)+.5*SIGI
      PHI(LLL+1)=PDUM+DIST*(PHI(L+1)-PDUM)
      PDUM=PHI(L)-.5*SIGI
      PHI(LLL-1)=PDUM+DIST*(PHI(L-1)-PDUM)
      GO TO 15
   17 CONTINUE
```

```
      PHI(LLL+1)=PHIUB(IK)+DIST*(PHI(L+1)-PHIUB(IK))
      IK=IK-1
   15 CONTINUE
      L=L-2
   10 CONTINUE
C   FILL IN OTHER PHI(S)  (I=EVEN)
      IMAX2=IMAX1-1
      DO 20 I=4,IMAX2,2
      II=IMAX2-I+4
      L=(II-1)*JMAX
      DO 25 J=2,JMAX1
      JJ=JMAX1-J+2
      LL=L+JJ
      PHI(LL)=.5*(PHI(LL+JMAX)+PHI(LL-JMAX))
   25 CONTINUE
   20 CONTINUE
      IM=IMAX
      IM1=IMAX1
      JM=JMAX
      JM1=JMAX1
      JW=JWN
      JWP1=JWNP1
      JWM1=JWNM1
      ITE=ITEN
      ILE=ILEN
      KTE=(ITE-1)*JM+JW
      PHI(KTE+JM)=.5*PHI(KTE)+.25*GAMTE+.5*PHI(KTE+2*JM)
C   INITIALIZE FINITE DIFFERENCE COEFFICIENTS AND FARFIELD
      CALL INITAL
      CALL FARFLD
      L=(ILE-1)*JM+JW
      PHIUB(ILEN)=PHI(L+1)-DY(JW)*(PHI(L+2)-PHI(L+1))/DY(JWP1)
      RETURN
      END
      SUBROUTINE FARFLD
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ITE,ILE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  ALPHA,DEL,M8,GAM,K,NDB,XH,TITLE(8),DOUB,DOUBLT,ALPHAF
      COMMON /COEFF/ A(99),B(99),C(99),D(99),PHI(10000)
      COMMON /GAMMA/ GAMTE1,GAMTE,PGFF,GAMFF
      IF (K.LT.0.) GO TO 25
C   SUBSONIC FARFIELD
      SUM=0.
      DO 1 I=4,IM1
      SUM1=0.
      M=(I-1)*JM
      PO=PHI(M+2)-PHI(M-JM+2)
      IFLAG=0
      IF (I.GE.ILE+1.AND.I.LE.ITE) IFLAG=1
      DO 2 J=3,JM1
      L=M+J
      IF (IFLAG.EQ.1.AND.J.EQ.JWP1) PO=PHIUB(I)-PHIUB(I-1)
      PN=PHI(L)-PHI(L-JM)
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```
      IF (J.EQ.JW.AND.I.EQ.ITE+1) PN=PN-.5*GAMTE1
      SUM1=SUM1+(PN+PO)**2*DY(J-1)
      PO=PN
    2 CONTINUE
      SUM=SUM+.25*SUM1/DX(I-1)
    1 CONTINUE
      SUM=.25*SUM
      DOUBLT=DOUB+SUM
      SK=SQRT(K)
      CON=DOUBLT/(3.14159265*SK)
      CON1=1./6.2831853
      CON2=1.570796325
      DO 10 I=1,IM
      M=(I-1)*JM
      J3=JM1
      IF (I.LE.2.OR.I.EQ.IM) J3=1
      DO 15 J=1,JM,J3
      L=M+J
      IF (Y(J).EQ.0.) GO TO 20
      PHI(L)=CON*X(I)/(X(I)**2+K*Y(J)**2)+GAMFF*CON1*(ATAN(X(I)
     1  /(SK*Y(J)))+SIGN(CON2,Y(J)))
      GO TO 15
   20 CONTINUE
      PHI(L)=CON/X(I)
   15 CONTINUE
   10 CONTINUE
      RETURN
   25 CONTINUE
C  SUPERSONIC FARFIELD
      DO 30 I=1,IM
      M=(I-1)*JM
      J3=JM1
      IF (I.LE.2.OR.I.EQ.IM) J3=1
      DO 35 J=1,JM,J3
      L=M+J
      PHI(L)=0.
   35 CONTINUE
   30 CONTINUE
      RETURN
      END
      FUNCTION FL (CAC)
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ITE,ILE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  ALPHA,DEL,M8,GAM,K,NDB,XH,TITLE(8),DOUB,DOUBLT,ALPHAF
C  AIRFOIL LOWER SURFACE SHAPE FUNCTION
      FL=-.5+2.*(CAC-.5)**2-ALPHA*CAC/DEL
      IF (CAC.GT.XH) FL=FL-ALPHAF*(CAC-XH)/DEL
      RETURN
      END
      FUNCTION FLP (CAC)
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
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```
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ITE,ILE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  ALPHA,DEL,M8,GAM,K,NDB,XH,TITLE(8),DOUB,DOUBLT,ALPHAF
C   AIRFOIL LOWER SURFACE SLOPE DISTRIBUTION
      FLP=4.*(CAC-.5)-ALPHA/DEL
      IF (CAC.GT.XH) FLP=FLP-ALPHAF/DEL
      RETURN
      END
      SUBROUTINE FPRINT
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ITE,ILE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  ALPHA,DEL,M8,GAM,K,NDB,XH,TITLE(8),DOUB,DOUBLT,ALPHAF
      COMMON /COEFF/ A(99),B(99),C(99),D(99),PHI(10000)
      COMMON /GAMMA/ GAMTE1,GAMTE,PGFF,GAMFF
      CPCPB=DEL**.6666666667/((1.+GAM)*M8**2)**.3333333333
      PART=.5*((X(ITE+1)-1.)*(GAMFF-GAMTE)/(X(IM1)-1.)+GAMTE)
      IJW=ITE*JM+JW
      PHI(IJW)=PHI(IJW)-PART
      L=(ILE-2)*JM+JW
      PHIUB(ILE-1)=PHI(L)
      PHIUB(ITE+1)=PHI(IJW)+2.*PART
C   COMPUTE CP LOWER (A) AND CP UPPER (B)
      DO 10 I=ILE,ITE
      M=(I-1)*JM
      L=M+JW
      A(I)=-2.*(AX1(I)*(PHI(L+JM)-PHI(L))+AX2(I)*(PHI(L)-PHI(L-JM)))
     1  *CPCPB
      B(I)=-2.*(AX1(I)*(PHIUB(I+1)-PHIUB(I))+AX2(I)*(PHIUB(I)-
     1  PHIUB(I-1)))*CPCPB
   10 CONTINUE
      PHI(IJW)=PHI(IJW)+PART
C   COMPUTE AIRFOIL FORCE COEFFICIENTS
      C10=A(ILE)-B(ILE)
      CLIFT=C10*X(ILE)
      C20=CLIFT
      CMOM=.5*CLIFT*X(ILE)
      C30=0.
      CHINGE=C30
      IF (XH.GE.X(ILE)) GO TO 25
      C30=C10*(X(ILE)-XH)
      CHINGE=.5*C30*(X(ILE)-XH)
   25 CONTINUE
      ILE1=ILE+1
      DO 30 I=ILE1,ITE
      C1=A(I)-B(I)
      C2=C1*X(I)
      IF (X(I).GT.XH) C3=C1*(X(I)-XH)
      CLIFT=CLIFT+.5*(C1+C10)*DX(I-1)
      CMOM=CMOM+.5*(C2+C20)*DX(I-1)
      DXX=DX(I-1)
      IF (X(I).GT.XH.AND.X(I-1).LE.XH) DXX=X(I)-XH
      CHINGE=CHINGE+.5*(C3+C30)*DXX
      C10=C1
      C20=C2
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      C30=C3
   30 CONTINUE
      CIRLIF=2.*GAMTE*CPCPB
      CPCRIT=-2.*CPCPB*K
      WRITE (6,900)
      WRITE (6,901) (TITLE(I),I=1,8)
      WRITE (6,902) M8
      WRITE (6,903) K
      WRITE (6,904) DEL
      WRITE (6,905) ALPHA
      WRITE (6,916) XH
      WRITE (6,917) ALPHAF
      WRITE (6,906) CPCPB
      WRITE (6,918) CPCRIT
      WRITE (6,907) GAMTE
      WRITE (6,908) CIRLIF
      WRITE (6,909)
      WRITE (6,910) CLIFT,CMOM,CHINGE
      WRITE (6,911)
      WRITE (6,912)
      WRITE (6,915) (X(I),I=ILE,ITE)
      WRITE (6,913)
      WRITE (6,915) (B(I),I=ILE,ITE)
      WRITE (6,914)
      WRITE (6,915) (A(I),I=ILE,ITE)
  900 FORMAT (1H1)
  901 FORMAT (30X,8A10)
  902 FORMAT (1H ,/,1H ,/,1H ,/,* MACH NUMBER =*E13.5)
  903 FORMAT (* SIMILARITY PARAMETER (K) =*E13.5)
  904 FORMAT (* THICKNESS RATIO =*E13.5)
  905 FORMAT (* AIRFOIL ANGLE OF ATTACK (RADIANS) =*E13.5)
  906 FORMAT (* CP SCALING FACTOR (CP/CPBAR) =*E13.5)
  907 FORMAT (* SCALED AIRFOIL CIRCULATION =*E13.5)
  908 FORMAT (* LIFT COEFFICIENT BASED ON CIRCULATION*27H (2*CP/CPBAR*CI
     1RCULATION) =E13.5)
  909 FORMAT (1H ,/,1H ,/,* AIRFOIL FORCE COEFFICIENTS*)
  910 FORMAT (1H ,/,3X*LIFT =*E13.5,/,3X*MOMENT ABOUT (X=0) =*E13.5,/,
     1  3X*HINGE MOMENT =*E13.5)
  911 FORMAT (1H ,/,1H ,/,* PRESSURE COEFFICIENTS ON THE AIRFOIL (UNSCAL
     1ED)*)
  912 FORMAT (1H ,/,3X*AIRFOIL COORDINATE =*)
  913 FORMAT (1H ,/,3X*AIRFOIL PRESSURE COEFFICIENTS, UPPER =*)
  914 FORMAT (1H ,/,3X*AIRFOIL PRESSURE COEFFICIENTS, LOWER =*)
  915 FORMAT (3X10E13.5)
  916 FORMAT (* HINGE POINT =*E13.5)
  917 FORMAT (* FLAP ANGLE (RADIANS) =*E13.5)
  918 FORMAT (* CRITICAL PRESSURE COEFFICIENT (SONIC) =*E13.5)
      RETURN
      END
      FUNCTION FU (CAC)
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ITE,ILE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  ALPHA,DEL,M8,GAM,K,NDB,XH,TITLE(8),DOUB,DOUBLT,ALPHAF
```

```
C   AIRFOIL UPPER SURFACE SHAPE FUNCTION
      FU=.5-2.*(CAC-.5)**2-ALPHA*CAC/DEL
      IF (CAC.GT.XH) FU=FU-ALPHAF*(CAC-XH)/DEL
      RETURN
      END
      FUNCTION FUP (CAC)
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ITE,ILE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  ALPHA,DEL,M8,GAM,K,NDB,XH,TITLE(8),DOUB,DOUBLT,ALPHAF
C   AIRFOIL UPPER SURFACE SLOPE DISTRIBUTION
      FUP=-4.*(CAC-.5)-ALPHA/DEL
      IF (CAC.GT.XH) FUP=FUP-ALPHAF/DEL
C   DOUB IS DOUBLET STRENGTH DUE TO THICKNESS
      DOUB=.6666666667-2.*DEL**2
      DOUBLT=DOUB
      RETURN
      END
      SUBROUTINE GAMFUN
      COMMON /GAMMA/ GAMTE1,GAMTE,PGFF,GAMFF
      GAMFF=GAMTE1+PGFF*(GAMTE-GAMTE1)
      GAMTE1=GAMTE
      RETURN
      END
      SUBROUTINE INITAL
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ITE,ILE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  ALPHA,DEL,M8,GAM,K,NDB,XH,TITLE(8),DOUB,DOUBLT,ALPHAF
C   CALCULATE DX
      DO 15 I=1,IM1
      DX(I)=X(I+1)-X(I)
   15 CONTINUE
C   CALCULATE DY
      DO 25 I=1,JM1
      DY(I)=Y(I+1)-Y(I)
   25 CONTINUE
      DO 30 I=2,IM1
      AX1(I)=DX(I-1)/(DX(I)*(DX(I-1)+DX(I)))
      AX2(I)=DX(I)/(DX(I-1)*(DX(I-1)+DX(I)))
      BX1(I)=2.*AX1(I)/DX(I-1)
      BX2(I)=2.*AX2(I)/DX(I)
      CX(I)=.5/DX(I)
   30 CONTINUE
      CX(1)=.5/DX(1)
      DO 40 I=2,JM1
      AY1(I)=2./(DY(I)*(DY(I)+DY(I-1)))
      AY2(I)=2./(DY(I-1)*(DY(I)+DY(I-1)))
   40 CONTINUE
      IF (K.GT.0.) GO TO 50
C   TOP AND BOTTOM AND RHS BOUNDARY CONDITIONS
      AX1(IM1)=0.
      AX2(IM1)=0.
```

```
      BX1(IM1)=0.
      BX2(IM1)=2./DX(IM1-1)**2
      AY1(2)=2./DY(2)**2
      AY2(2)=0.
      AY1(JM1)=0.
      AY2(JM1)=2./DY(JM1-1)**2
   50 CONTINUE
      DYBU1=2./((DY(JWP1)+2.*DY(JW))*DY(JWP1))
      DYBU2=DY(JWP1)*DYBU1
      DYBL1=2./((DY(JW-2)+2.*DY(JWM1))*DY(JW-2))
      DYBL2=DY(JW-2)*DYBL1
C   SET AIRFOIL BOUNDARY CONDITION
      DO 45 I=ILE,ITE
      FPU(I)=FUP(X(I))
      FPL(I)=FLP(X(I))
   45 CONTINUE
      RETURN
      END
      SUBROUTINE PRINT (NITERG)
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ITE,ILE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  ALPHA,DEL,M8,GAM,K,NDB,XH,TITLE(8),DOUB,DOUBLT,ALPHAF
      COMMON /COEFF/ A(99),B(99),C(99),D(99),PHI(10000)
      COMMON /GAMMA/ GAMTE1,GAMTE,PGFF,GAMFF
      PART=.5*((X(ITE+1)-1.)*(GAMFF-GAMTE)/(X(IM1)-1.)+GAMTE)
      IJW=ITE*JM+JW
      PHI(IJW)=PHI(IJW)-PART
      L=(ILE-2)*JM+JW
      PHIUB(ILE-1)=PHI(L)
      PHIUB(ITE+1)=PHI(IJW)+2.*PART
C   COMPUTE CP LOWER (A) AND CP UPPER (B)
      DO 297 I=ILE,ITE
      M=(I-1)*JM
      L=M+JW
      A(I)=-2.*(AX1(I)*(PHI(L+JM)-PHI(L))+AX2(I)*(PHI(L)-PHI(L-JM)))
      B(I)=-2.*(AX1(I)*(PHIUB(I+1)-PHIUB(I))+AX2(I)*(PHIUB(I)-
     1  PHIUB(I-1)))
  297 CONTINUE
      PHI(IJW)=PHI(IJW)+PART
      WRITE (6,911) NITERG
      WRITE (6,903) (B(I),I=ILE,ITE)
      WRITE (6,912) NITERG
      WRITE (6,903) (A(I),I=ILE,ITE)
  903 FORMAT (10E13.5)
  911 FORMAT (1H ,/,* AT ITERATION*I5* SCALED PRESSURE COEFFICIENT, UPPE
     1R (ILE TO ITE) =*)
  912 FORMAT (1H ,/,* AT ITERATION*I5* SCALED PRESSURE COEFFICIENT, LOWE
     1R (ILE TO ITE) =*)
      RETURN
      END
      SUBROUTINE TRI (I)
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
```

```
     1 CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2 IM,IM1,JM,JM1,JW,JWP1,JWM1,ITE,ILE,DYBU1,DYBU2,DYBL1,DYBL2,
     3 ALPHA,DEL,M8,GAM,K,NDB,XH,TITLE(8),DOUB,DOUBLT,ALPHAF
      COMMON /COEFF/ A(99),B(99),C(99),D(99),PHI(10000)
      DO 10 KK=3,JM1
      J=JM1-KK+3
      P=A(J-1)/B(J)
      B(J-1)=B(J-1)-P*C(J)
      D(J-1)=D(J-1)-P*D(J)
   10 CONTINUE
      M=(I-1)*JM
      PHI(M+2)=D(2)/B(2)
      DO 20 J=3,JM1
      L=M+J
      PHI(L)=(D(J)-PHI(L-1)*C(J))/B(J)
   20 CONTINUE
      RETURN
      END
```

APPENDIX B

FORTRAN LISTING OF UTRANS

A FORTRAN listing of the source deck for the UTRANS program is presented in the following pages. The program, as configured here, requires 144_8K words to load and 132_8K words to execute.

```
      PROGRAM UTRANS (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7,
     1  TAPE8,TAPE9)
      COMPLEX PHIUB,B,D,PHI,GAMTE1,GAMTE,GAMFF,PHIOG,PART,ERR,
     1  ERROR,OMEG2I,TPHIL,TPHIR,SIGI,GAMFFS,CONAFF,PHIAFF
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ILE,ITE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  K,SMALLK,OMEG,XH,NDOUB,CPCPB,TITLE(8),M8,DEL,ALPHA,ALPHAF
      COMMON /COEFF/ A(99),B(99),C(99),D(99),PHI(10000)
      COMMON /GAMMA/ GAMTE1,GAMTE,PGFF,GAMFF,GAMFFS
      COMMON /STEADY/ PHIS(10000),AX1S(99),AX2S(99),BX1S(99),
     1  BX2S(99),CXS(99),PHIUBS(99)
      COMMON /PHIAIR/ CONAFF(500),PHIAFF(500)
      DIMENSION PHIOG(99),OMEGA(99),V(99),EPSGRD(3)
      NAMELIST /IN/ X,Y,GAMFF,IM,JM,ILE,ITE,JW,OMEGAH,OMEGAE,OMEGAP,
     1  EPSGRD,NDUMP,NGRID,NGFF,PGFF,KEPS,NPRINT,SMALLK,IK,XH
      NAMELIST /CONTRL/ ITAPE
C
C   TO START PROGRAM, STEADY (PHIS) DATA IS TO BE READ FROM A DISC
C   FILE TAPE8. UNSTEADY DATA WILL NOW BE WRITTEN ON TAPE7.
C
C   TO RESTART PROGRAM, AGAIN STORE STEADY DATA AS ABOVE AND STORE
C   THE UNSTEADY DATA ON A DISC FILE TAPE9. NEW UNSTEADY DATA WILL NOW
C   BE WRITTEN ON TAPE7.
C
C   READ STEADY SOLUTION
      READ (8) DUM,IMS,IM1S,JMS,JM1S,JWS,DUM,DUM,ITES,ILES,K,DEL,ALPHA,
     1  DUM,DUM,NDB,DUM,M8,GAM,DUM,DUM,DUM,DUM,DUM,DUM,ALPHAF
      READ (8) DUM
      READ (8) DUM
      READ (8) (DUM,DUM,PHIUBS(I),I=ILES,ITES)
      READ (8) (AX1S(I),AX2S(I),BX1S(I),BX2S(I),CXS(I),DUM,I=1,IM1S)
      READ (8) DUM
      L=IMS*JMS
      READ (8) (PHIS(I),I=1,L)
C   MODIFY LEADING AND TRAILING EDGE PHI
      L=(ILES-1)*JMS+JWS
      PHIS(L)=.5*(PHIS(L)+PHIUBS(ILES))
      L=(ITES-1)*JMS+JWS
      PHIS(L)=.5*(PHIS(L)+PHIUBS(ITES))
      SK=SQRT(ABS(K))
      CPCPB=DEL**.6666666667/((1.+GAM)*M8**2)**.3333333333
      READ (5,911) (TITLE(I),I=1,8)
      READ (5,CONTRL)
      IF (ITAPE.EQ.0) GO TO 10
C   READ DATA FROM RESTART TAPE
      READ (9) NITERG,IM,IM1,JM,JM1,JW,JWP1,JWM1,ILE,ITE,GAMTE1,
     1  GAMFF,OMEG,SMALLK,DYBU1,DYBU2,DYBL1,DYBL2,NDOUB,XH
      READ (9) (X(I),I=1,IM)
      READ (9) (Y(I),I=1,JM)
      READ (9) (FPU(I),FPL(I),PHIUB(I),I=ILE,ITE)
      READ (9) (AX1(I),AX2(I),BX1(I),BX2(I),CX(I),DX(I),I=1,IM1)
      READ (9) (AY1(I),AY2(I),DY(I),I=1,JM1)
      L=IM*JM
```

```
      READ (9) (PHI(I),I=1,L)
      NPT=2*(IM-3)+3*JM
      READ(9) (CONAFF(I),PHIAFF(I),I=1,NPT)
      GAMTE=GAMTE1
      GAMFFS=GAMFF
      IK=0
      READ (5,IN)
      WRITE (6,900)
      WRITE (6,901) NITERG
      NITERG=0
      WRITE (6,913) K,CPCPB
C   THE IK OPTION IS USED TO BOOT STRAP TO DIFFERENT REDUCED FREQUENCIES
C   AND/OR MODES OF OSCILLATION
      IF (IK.EQ.0) GO TO 15
      OMEG=SMALLK*M8**2/((1.+GAM)*DEL*M8**2)**.6666666667
      IFAR=0
      IF (OMEG.GT.0.) IFAR=1
      CALL FARFLD (IFAR)
      GO TO 15
C   START PROBLEM FROM SCRATCH
   10 CONTINUE
      READ (5,IN)
      NITERG=0
      NDOUB=0
      OMEG=SMALLK*M8**2/((1.+GAM)*DEL*M8**2)**.6666666667
      GAMTE=GAMFF
      GAMTE1=GAMFF
      IM1=IM-1
      JM1=JM-1
      JWP1=JW+1
      JWM1=JW-1
      ERR=CMPLX(0.,0.)
      DO 3 I=ILE,ITE
      PHIUB(I)=ERR
    3 CONTINUE
      IF (K.LT.0.) GO TO 20
C   INITIAL GUESS FOR SUBSONIC CASE (INTERIOR ONLY)
      DO 6 I=3,IM1
      M=(I-1)*JM
      DO 7 J=2,JM1
      L=M+J
      IF (Y(J).EQ.0.) GO TO 8
      PHI(L)=.1591549431*GAMFF*(ATAN(X(I)/(SK*Y(J)))+SIGN(1.570796325,
     1  Y(J)))
      GO TO 7
    8 CONTINUE
      PHI(L)=CMPLX(0.,0.)
    7 CONTINUE
    6 CONTINUE
      GO TO 30
   20 CONTINUE
C   INITIAL GUESS FOR SUPERSONIC CASE (INTERIOR ONLY)
      DO 21 I=3,IM1
      M=(I-1)*JM
      DO 22 J=2,JM1
```

```
      L=M+J
      SKY=SK*ABS(Y(J))
      PHI(L)=CMPLX(0.,0.)
      IF (X(I).LT.SKY+XH.OR.X(I).GT.SKY+1.) GO TO 22
      PHI(L)=(X(I)-SKY-XH)*SIGN(1.,Y(J))/SK
   22 CONTINUE
   21 CONTINUE
   30 CONTINUE
C  INITIALIZE FINITE DIFFERENCE COEFFICIENTS AND FARFIELD
      CALL INITAL
      IFAR=1
      CALL FARFLD (IFAR)
   15 CONTINUE
      OMEG2I=CMPLX(0.,2.*OMEG)
      WRITE (6,IN)
      WRITE (6,900)
      IF (ITAPE.EQ.0) WRITE (6,913) K,CPCPB
      KGRD=1
C  RE-CYCLE POINT FOR GRID ITERATION
   50 CONTINUE
      ERROR=CMPLX(0.,0.)
      NIT=NITERG
      NITERG=NITERG+1
      IF (MOD(NITERG,NPRINT).EQ.0) CALL PRINT(NIT)
      IF (K.LT.0.) GO TO 51
      IF (MOD(NITERG,NGFF).NE.0) GO TO 51
      GAMFFS=GAMFF
      CALL GAMFUN
      IFAR=0
      CALL FARFLD (IFAR)
      WRITE (6,910) NITERG,GAMFF,GAMTE
   51 CONTINUE
      DO 1 I=2,JM1
      V(I)=K
    1 CONTINUE
C  BEGIN LOOP ON GRID
      INCR=2**(NDB-NDOUB)
      IS=3-INCR
      DO 100 I=3,IM1
      IS=IS+INCR
C  CHECK FOR AIR FOIL
      IFLAG=0
      IF (ILE.LE.I.AND.I.LE.ITE) IFLAG=1
      M=(I-1)*JM
      MS=(IS-1)*JMS
C  SAVE THIS COLUMN OF PHI
      DO 2 J=2,JM1
      L=M+J
      PHIOG(J)=PHI(L)
    2 CONTINUE
C  BEGIN LOOP ON COLUMN
      JS=2-INCR
      DO 150 J=2,JM1
      JS=JS+INCR
C  CALCULATE CELL INDICES
```

```
      L=M+J
      LR=L+JM
      LL=L-JM
      LLL=LL-JM
      LB=L-1
      LT=L+1
C   CALCULATE CELL INDICES FOR PHIO
      LS=MS+JS
      LSR=LS+JMS
      LSL=LS-JMS
      LSLL=LSL-JMS
C   CALCULATE V AND PHIXX FROM STEADY SOLUTION
      VV=K-AX1S(IS)*(PHIS(LSR)-PHIS(LS))-AX2S(IS)*(PHIS(LS)-PHIS(LSL))
      VVS=VV
      IF (VV.LT.0.) GO TO 60
C   ELLIPTIC
      PHIXX=BX1S(IS)*(PHIS(LSR)-PHIS(LS))-BX2S(IS)*(PHIS(LS)-PHIS(LSL))
      OMEGA(J)=OMEGAE
      GO TO 199
   60 CONTINUE
      OMEGA(J)=OMEGAP
      IF (V(J).GT.0.) GO TO 65
C   HYPERBOLIC
      OMEGA(J)=OMEGAH
      VV=K-CXS(IS-1)*(PHIS(LS)-PHIS(LSL))-CXS(IS-2)*
     1  (PHIS(LSL)-PHIS(LSLL))
   65 CONTINUE
C   PARABOLIC
      PHIXX=BX1S(IS-1)*(PHIS(LS)-PHIS(LSL))-BX2S(IS-1)*
     1  (PHIS(LSL)-PHIS(LSLL))
  199 CONTINUE
      V(J)=VVS
      TPHIR=PHI(LR)
      TPHIL=PHI(LL)
      IF (I.EQ.ILE-1.AND.J.EQ.JW) PHI(LR)=.5*(PHI(L+JM)+PHIUB(ILE))
      IF (I.EQ.ITE+1.AND.J.EQ.JW) PHI(LL)=.5*(PHIUB(ITE)+PHI(L-JM))
C   SET UP TRIDIAGONAL MATRIX TO SOLVE FOR PHI(I,J)
C   A=PHI(I,J+1)    B=PHI(I,J)    C=PHI(I,J-1)    D=CONSTANT
      IF (IFLAG.EQ.1.AND.J.EQ.JWP1) GO TO 250
      IF (IFLAG.EQ.1.AND.J.EQ.JWM1) GO TO 270
      IF (IFLAG.EQ.1.AND.J.EQ.JW) GO TO 290
      PART=CMPLX(0.,0.)
      IF (I.LE.ITE) GO TO 201
C   KUTTA CONDITION
      SIGI=(X(I)-1.)*(GAMFF-GAMTE)/(X(IM1)-1.)+GAMTE
      IF (J.EQ.JWM1) PART=.5*AY1(J)*SIGI
      IF (J.EQ.JW) PART=.5*(AY1(J)-AY2(J))*SIGI
      IF (J.EQ.JWP1) PART=-.5*AY2(J)*SIGI
  201 CONTINUE
      IF (VVS.LT.0.) GO TO 220
C   ELLIPTIC
      B(J)=-(VV*(BX1(I)+BX2(I))+AY1(J)+AY2(J)+(OMEG2I+PHIXX)*
     1  (AX2(I)-AX1(I)))
      D(J)=-VV*(BX1(I)*PHI(LR)+BX2(I)*PHI(LL))+(OMEG2I+PHIXX)*
     1  (AX1(I)*PHI(LR)-AX2(I)*PHI(LL))+PART
```

```
      IF (J.EQ.2) GO TO 202
      IF (J.EQ.JM1) GO TO 203
      A(J)=AY1(J)
      C(J)=AY2(J)
      GO TO 200
C  BOTTOM BOUNDARY
  202 CONTINUE
      A(J)=AY1(J)
      D(J)=D(J)-AY2(J)*PHI(LB)
      GO TO 200
C  TOP BOUNDARY
  203 CONTINUE
      C(J)=AY2(J)
      D(J)=D(J)-AY1(J)*PHI(LT)
      GO TO 200
C  HYPERBOLIC AND PARABOLIC
  220 CONTINUE
      B(J)=VV*BX1(I-1)-AY1(J)-AY2(J)-(OMEG2I+PHIXX)*CX(I-1)
      D(J)=VV*(BX1(I-1)*PHI(LL)+BX2(I-1)*(PHI(LL)-PHI(LLL)))-
     1  (OMEG2I+PHIXX)*(CX(I-1)*PHI(LL)+CX(I-2)*(PHI(LL)-PHI(LLL)))
     2  +PART
      IF (J.EQ.2) GO TO 222
      IF (J.EQ.JM1) GO TO 223
      A(J)=AY1(J)
      C(J)=AY2(J)
      GO TO 200
C  BOTTOM BOUNDARY
  222 CONTINUE
      A(J)=AY1(J)
      IF (K.LT.0.) GO TO 224
      D(J)=D(J)-AY2(J)*PHI(LB)
      GO TO 200
  224 CONTINUE
      BT=SK+.5*(K-VV)/SK
      B(J)=B(J)-2.*CMPLX(BT*CX(I-1),OMEG/SK)/DY(2)
      D(J)=D(J)-2.*BT*(CX(I-1)*PHI(LL)-CX(I-2)*(PHI(LL)-PHI(LLL)))/DY(2)
      GO TO 200
C  TOP BOUNDARY
  223 CONTINUE
      C(J)=AY2(J)
      IF (K.LT.0.) GO TO 225
      D(J)=D(J)-AY1(J)*PHI(LT)
      GO TO 200
  225 CONTINUE
      BT=SK+.5*(K-VV)/SK
      B(J)=B(J)-2.*CMPLX(BT*CX(I-1),OMEG/SK)/DY(J-1)
      D(J)=D(J)-2.*BT*(CX(I-1)*PHI(LL)-CX(I-2)*(PHI(LL)-PHI(LLL)))/
     1  DY(J-1)
      GO TO 200
C  BODY BOUNDARY ABOVE
  250 CONTINUE
      IF (VVS.LT.0.) GO TO 260
C  ELLIPTIC
      A(J)=DYBU1
      B(J)=-(VV*(BX1(I)+BX2(I))+DYBU1+(OMEG2I+PHIXX)*(AX2(I)-AX1(I)))
```

```
      C(J)=0.
      D(J)=DYBU2*FPU(I)-VV*(BX1(I)*PHI(LR)+BX2(I)*PHI(LL))+(OMEG2I+
     1  PHIXX)*(AX1(I)*PHI(LR)-AX2(I)*PHI(LL))
      GO TO 200
C   HYPERBOLIC AND PARABOLIC
  260 CONTINUE
      A(J)=DYBU1
      B(J)=VV*BX1(I-1)-DYBU1-(OMEG2I+PHIXX)*CX(I-1)
      C(J)=0.
      D(J)=DYBU2*FPU(I)+VV*(BX1(I-1)*PHI(LL)+BX2(I-1)*(PHI(LL)-
     1  PHI(LLL)))-(OMEG2I+PHIXX)*(CX(I-1)*PHI(LL)+CX(I-2)*(PHI(LL)-
     1  PHI(LLL)))
      GO TO 200
C   BODY BOUNDARY BELOW
  270 CONTINUE
      IF (VVS.LT.0.) GO TO 280
C   ELLIPTIC
      A(J)=0.
      B(J)=-(DYBL1+VV*(BX1(I)+BX2(I))+(OMEG2I+PHIXX)*(AX2(I)-AX1(I)))
      C(J)=DYBL1
      D(J)=-DYBL2*FPL(I)-VV*(BX1(I)*PHI(LR)+BX2(I)*PHI(LL))+(OMEG2I+
     1  PHIXX)*(AX1(I)*PHI(LR)-AX2(I)*PHI(LL))
      GO TO 200
C   HYPERBOLIC AND PARABOLIC
  280 CONTINUE
      A(J)=0.
      B(J)=VV*BX1(I-1)-DYBL1-(OMEG2I+PHIXX)*CX(I-1)
      C(J)=DYBL1
      D(J)=-DYBL2*FPL(I)+VV*(BX1(I-1)*PHI(LL)+BX2(I-1)*(PHI(LL)
     1  -PHI(LLL)))-(OMEG2I+PHIXX)*(CX(I-1)*PHI(LL)+CX(I-2)*(PHI(LL)
     2  -PHI(LLL)))
      GO TO 200
C   BODY BOUNDARY J=JW
  290 CONTINUE
      A(J)=0.
      B(J)=CMPLX(1.,0.)
      C(J)=0.
      D(J)=PHI(L)
  200 CONTINUE
      PHI(LR)=TPHIR
      PHI(LL)=TPHIL
  150 CONTINUE
C   TRIDIAGONAL MATRIX IS SET NOW SOLVE FOR COLUMN OF PHI
      CALL TRI (I)
C   RELAX PHI, FIND ERROR, AND MOVE TO NEXT COLUMN
      DO 295 J=2,JM1
      L=M+J
      ERR=OMEGA(J)*(PHI(L)-PHIOG(J))
      PHI(L)=PHIOG(J)+ERR
      IF (CABS(ERR).LT.CABS(ERROR)) GO TO 295
      ERROR=ERR
      LERROR=L
  295 CONTINUE
      IF (IFLAG.NE.1) GO TO 100
      L=M+JW
```

```
      PHI(L)=PHI(L-1)+DY(JWM1)*(PHI(L-1)-PHI(L-2))/DY(JW-2)
      PHIUB(I)=PHI(L+1)-DY(JW)*(PHI(L+2)-PHI(L+1))/DY(JWP1)
      IF (I.EQ.ITE) GAMTE=PHIUB(I)-PHI(L)
      IF (CABS(GAMFF).LE.1.E-8) GAMTE=CMPLX(0.,0.)
      IF (K.LT.0.) GAMFF=GAMTE
      IF (I.EQ.ITE.AND.NITERG.EQ.1) GAMTE1=GAMTE
  100 CONTINUE
C   PRINT OUT ERROR AFTER EACH GRID SWEEP
      WRITE (6,905) NITERG,ERROR,LERROR
      IDOUB=0
      IF (CABS(ERROR).LE.EPSGRD(KGRD)) GO TO 300
      IF (NITERG.EQ.NGRID) GO TO 310
      IF (MOD(NITERG,NDUMP).EQ.0) GO TO 310
      GO TO 50
  300 CONTINUE
      KGRD=KGRD+1
      IDOUB=1
      GO TO 310
  301 CONTINUE
      IF (K.LT.0.) GO TO 302
      GAMFFS=GAMFF
      CALL GAMFUN
      WRITE (6,910) NITERG,GAMFF,GAMTE
  302 CONTINUE
      CALL FPRINT
      WRITE (6,900)
      WRITE (6,906) NITERG
      CALL DOUBLE
      WRITE (6,914) IM,JM,JW,ILE,ITE
      WRITE (6,902)
      WRITE (6,903) (X(I),I=1,IM)
      WRITE (6,904)
      WRITE (6,903) (Y(I),I=1,JM)
      GO TO 50
  310 CONTINUE
C   TAPE DUMP
      WRITE (7) NITERG,IM,IM1,JM,JM1,JW,JWP1,JWM1,ILE,ITE,GAMTE1,
     1  GAMFF,OMEG,SMALLK,DYBU1,DYBU2,DYBL1,DYBL2,NDOUB,XH
      WRITE (7) (X(I),I=1,IM)
      WRITE (7) (Y(I),I=1,JM)
      WRITE (7) (FPU(I),FPL(I),PHIUB(I),I=ILE,ITE)
      WRITE (7) (AX1(I),AX2(I),BX1(I),BX2(I),CX(I),DX(I),I=1,IM1)
      WRITE (7) (AY1(I),AY2(I),DY(I),I=1,JM1)
      L=IM*JM
      WRITE (7) (PHI(I),I=1,L)
      NPT=2*(IM-3)+3*JM
      WRITE (7) (CONAFF(I),PHIAFF(I),I=1,NPT)
      END FILE 7
      WRITE (6,907) NITERG
      CALL PRINT(NITERG)
      IF (KGRD.GT.KEPS) GO TO 320
      IF (NITERG.EQ.NGRID) GO TO 330
      IF (IDOUB.EQ.1) GO TO 301
      GO TO 50
  320 CONTINUE
```

```
      WRITE (6,908) NITERG
      GO TO 350
  330 CONTINUE
      WRITE (6,909) NITERG
  900 FORMAT (1H1)
  901 FORMAT (1H ,/,* CASE IS BEING RESTARTED AT ITERATION*I5)
  902 FORMAT (1H ,/,* X(I),I=1,IM*)
  903 FORMAT (10E13.5)
  904 FORMAT (1H ,/,* Y(I),I=1,JM*)
  905 FORMAT (1H ,/,* AT ITERATION*I5* THE MAXIMUM ERROR =*2E13.5
     1  * AND OCCURRED AT NODE*I5)
  906 FORMAT (1H ,/,* THE NUMBER OF NODES IS BEING DOUBLED AT ITERATION*
     1  I5)
  907 FORMAT (1H ,/,* TAPE HAS BEEN DUMPED AT ITERATION*I5)
  908 FORMAT (1H ,/,* SOLUTION HAS CONVERGED TO DESIRED ACCURACY AT ITER
     1ATION*I5)
  909 FORMAT (1H ,/,* MAXIMUM NUMBER OF ITERATIONS HAS BEEN REACHED, CAS
     1E IS BEING TERMINATED AT ITERATION*I5)
  910 FORMAT (1H ,/,* UPDATE GAMFF AND FARFIELD AT ITERATION*I5
     1  * GAMFF =*2E13.5* GAMTE =*2E13.5)
  911 FORMAT (8A10)
  913 FORMAT (1H ,/,* SIMILARITY PARAMETER (K) =*E13.5,/,* SCALING FACTO
     1R (CP/CPBAR) =*E13.5)
  914 FORMAT (1H ,/,* IM =*I4* JM =*I4* JW =*I4* ILE =*I4* ITE =*I4)
  350 CONTINUE
      CALL FPRINT
      END
      SUBROUTINE DOUBLE
      COMPLEX PHIUB,B,D,PHI,GAMTE1,GAMTE,GAMFF,GAMFFS,SIGI,PDUM
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ILE,ITE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  K,SMALLK,OMEG,XH,NDOUB,CPCPB,TITLE(8),M8,DEL,ALPHA,ALPHAF
      COMMON /COEFF/ A(99),B(99),C(99),D(99),PHI(10000)
      COMMON /GAMMA/ GAMTE1,GAMTE,PGFF,GAMFF,GAMFFS
      NDOUB=NDOUB+1
C   CHANGE GRID INDICES
      IMAX=2*IM-4
      IMAX1=IMAX-1
      JMAX=2*JM-3
      JMAX1=JMAX-1
      JWN=2*JW-2
      JWNP1=JWN+1
      JWNM1=JWN-1
      ITEN=2*ITE-3
      DIST=1./(1.+DY(JWP1)/DY(JW))
      IK=ITE
C   CHANGE X
      L=IM
      DO 30 I=5,IMAX1,2
      II=IMAX1-I+5
      L=L-1
      X(II)=X(L)
      X(II-1)=.5*(X(L)+X(L-1))
```

```
   30 CONTINUE
      X(IMAX)=2.*X(IMAX1)-X(IMAX1-1)
      DELT=X(4)-X(3)
      X(2)=X(3)-DELT
      X(1)=X(2)-DELT
C   FIND NEW LEADING EDGE
      DO 31 I=3,IMAX1
      IF (X(I).GE.0.) GO TO 32
   31 CONTINUE
   32 ILEN=I
C   CHANGE Y
      L=JM
      DO 40 I=2,JMAX1,2
      II=JMAX1-I+2
      L=L-1
      Y(II)=Y(L)
      Y(II-1)=.5*(Y(L)+Y(L-1))
      IF (II-1.EQ.JWNP1) Y(II-1)=Y(L-1)+DIST*(Y(L)-Y(L-1))
      IF (II-1.EQ.JWNM1) Y(II-1)=Y(L)+DIST*(Y(L-1)-Y(L))
   40 CONTINUE
      Y(JMAX)=2.*Y(JMAX1)-Y(JMAX1-1)
      Y(1)=2.*Y(2)-Y(3)
C   MOVE THE PHI(S) (INTERIOR ONLY) AND INTERPOLATE FOR PHI (I=ODD)
      L=IM1*JM
      DO 10 I=3,IMAX1,2
      II=IMAX1-I+3
      LL=(II-1)*JMAX
      DO 15 J=2,JMAX1,2
      JJ=JMAX1-J+2
      LLL=LL+JJ
      L=L-1
      PHI(LLL)=PHI(L)
      PHI(LLL-1)=.5*(PHI(L)+PHI(L-1))
      IF (JJ.NE.JWN) GO TO 15
      PHI(LLL-1)=PHI(L)+DIST*(PHI(L-1)-PHI(L))
      IF (II.GT.ITEN) GO TO 16
      IF (II.GE.ILEN.AND.II.LE.ITEN) GO TO 17
      PHI(LLL+1)=PHI(L)+DIST*(PHI(L+1)-PHI(L))
      GO TO 15
   16 CONTINUE
      SIGI=(X(II)-1.)*(GAMFF-GAMTE)/(X(IMAX1)-1.)+GAMTE
      PDUM=PHI(L)+.5*SIGI
      PHI(LLL+1)=PDUM+DIST*(PHI(L+1)-PDUM)
      PDUM=PHI(L)-.5*SIGI
      PHI(LLL-1)=PDUM+DIST*(PHI(L-1)-PDUM)
      GO TO 15
   17 CONTINUE
      PHI(LLL+1)=PHIUB(IK)+DIST*(PHI(L+1)-PHIUB(IK))
      IK=IK-1
   15 CONTINUE
      L=L-2
   10 CONTINUE
C   FILL IN OTHER PHI(S) (I=EVEN)
      IMAX2=IMAX1-1
      DO 20 I=4,IMAX2,2
```

```
      II=IMAX2-I+4
      L=(II-1)*JMAX
      DO 25 J=2,JMAX1
      JJ=JMAX1-J+2
      LL=L+JJ
      PHI(LL)=.5*(PHI(LL+JMAX)+PHI(LL-JMAX))
   25 CONTINUE
   20 CONTINUE
      IM=IMAX
      IM1=IMAX1
      JM=JMAX
      JM1=JMAX1
      JW=JWN
      JWP1=JWNP1
      JWM1=JWNM1
      ITE=ITEN
      ILE=ILEN
      KTE=(ITE-1)*JM+JW
      PHI(KTE+JM)=.5*PHI(KTE)+.25*GAMTE+.5*PHI(KTE+2*JM)
C  INITIALIZE FINITE DIFFERENCE COEFFICIENTS AND FARFIELD
      CALL INITAL
      L=(ILE-1)*JM+JW
      DO 50 I=ILE,ITE
      PHIUB(I)=PHI(L+1)-DY(JW)*(PHI(L+2)-PHI(L+1))/DY(JWP1)
      L=L+JM
   50 CONTINUE
      L=(ILE-2)*JM+JW
      PHI(L)=.5*(PHI(L-JM)+.5*(PHIUB(ILE)+PHI(L+JM)))
      IFAR=1
      CALL FARFLD (IFAR)
      RETURN
      END
      SUBROUTINE FARFLD (IFAR)
      COMPLEX PHIUB,B,D,PHI,GAMTE1,GAMTE,GAMFF,GAMFFS,PART1,PART2,
     1  PART3,PART4,PART5,PART3N,H1,H2,CONST,CONAFF,PHIAFF
      REAL K,KH,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ILE,ITE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  K,SMALLK,OMEG,XH,NDOUB,CPCPB,TITLE(8),M8,DEL,ALPHA,ALPHAF
      COMMON /COEFF/ A(99),B(99),C(99),D(99),PHI(10000)
      COMMON /GAMMA/ GAMTE1,GAMTE,PGFF,GAMFF,GAMFFS
      COMMON /PHIAIR/ CONAFF(500),PHIAFF(500)
      DIMENSION EX(16),WT(16)
      DATA (EX(I),I=1,16)/.0483076657,.1444719616,.2392873623,
     1  .3318686023,.4213512761,.5068999089,.5877157572,.6630442669,
     2  .7321821187,.794483796,.8493676137,.8963211558,.9349060759,
     3  .9647622556,.9856115115,.9972638618/
      DATA (WT(I),I=1,16)/.0965400885,.0956387201,.0938443991,
     1  .0911738786,.087652093,.0833119242,.0781938958,.0723457941,
     2  .0658222228,.0586840935,.0509980593,.042835898,.0342738629,
     3  .0253920653,.0162743947,.00701861/
      DATA RSTAR /10./
      IF (K.LT.0.) GO TO 90
C  SUBSONIC FARFIELD
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      IF (IFAR.EQ.0) GO TO 60
      SK=SQRT(K)
      KH=OMEG/K
      IF (KH.GE..05) GO TO 30
      LFF=0
      DO 10 I=1,IM
      M=(I-1)*JM
      J3=JM1
      IF (I.LE.2.OR.I.EQ.IM) J3=1
      DO 15 J=1,JM,J3
      L=M+J
      LFF=LFF+1
      CONAFF(LFF)=CMPLX(0.,0.)
      PHIAFF(LFF)=CONAFF(LFF)
      IF (Y(J).EQ.0.) GO TO 20
      PHI(L)=.1591549431*GAMFF*(ATAN(X(I)/(SK*Y(J)))+SIGN(1.570796325,
     1  Y(J)))
      GO TO 15
   20 CONTINUE
      PHI(L)=CMPLX(0.,0.)
   15 CONTINUE
   10 CONTINUE
      RETURN
   30 CONTINUE
      CONST=.25*OMEG*GAMFF/SK
      LFF=0
      ZIP=0.
      PART2=CMPLX(0.,0.)
      PART4=PART2
      DO 35 I=ILE,ITE
      PART1=(PHIUB(I)-PHI((I-1)*JM+JW))*CEXP(CMPLX(0.,-KH*X(I)))
      PART4=PART4+.5*(PART1+PART2)*(X(I)-ZIP)
      ZIP=X(I)
      PART2=PART1
   35 CONTINUE
      DO 40 I=1,IM
      M=(I-1)*JM
      J3=JM1
      IF (I.LE.2.OR.I.EQ.IM) J3=1
      DO 45 J=1,JM,J3
      LFF=LFF+1
      L=M+J
      IF (Y(J).EQ.0.) GO TO 46
      PART3=CMPLX(0.,0.)
      R=KH*SQRT((X(I)-1.)**2+K*Y(J)**2)
      CALL HANKEL (R,H1,H2)
      CONAFF(LFF)=H1*CMPLX(0.,-.25*SK*KH**2*Y(J)/R)*CEXP(CMPLX(0.,
     1  KH*X(I)))
      PHIAFF(LFF)=PART4*CONAFF(LFF)
      PART1=-CEXP(CMPLX(0.,KH*(X(I)-1.)))*H1/R
      RI=RSTAR
      IF (R.GT.RSTAR) RI=R
      S=ABS(KH*(X(I)-1.))+RI
      SS=SQRT(S)
      G=S**2+K*(KH*Y(J))**2
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```
      GM=S**2-K*(KH*Y(J))**2
      AA=GM/(SS*G**1.5)
      BB=(1.5*SS+.5*K*(KH*Y(J))**2/(SS*S))/G**1.5-3.*SS*GM/G**2.5
      CC=3.75*SS*GM/G**2.5
      DD=(9.375*S*SS-1.875*K*(KH*Y(J))**2/SS)/G**2.5-18.75*S*SS*
     1  GM/G**3.5
      PART2=1.128379167*CEXP(CMPLX(0.,.7853981635-S))*CMPLX(BB+CC,AA-DD)
      PART5=CMPLX(0.,0.)
      IF (R.GE.RSTAR) GO TO 47
      RI=R
      RMID=.5*(RSTAR+RI)
      RAD=.5*(RSTAR-RI)
      SN=1.
C   GAUSSIAN QUADRATURE
      DO 41 IJ=1,32
      IA=IABS(IJ-16)
      IF (IJ.LE.16) IA=IA+1
      IF (IJ.GT.16) SN=-1.
      RI=RMID+RAD*EX(IA)*SN
      CALL HANKEL (RI,H1,H2)
      PART3N=CEXP(CMPLX(0.,-SQRT(RI**2-K*(KH*Y(J))**2)))*H2/RI
      PART5=PART5+WT(IA)*PART3N
   41 CONTINUE
      PART5=PART5*RAD
   47 CONTINUE
      IF (X(I).LE.1.) GO TO 44
      RI=KH*SK*ABS(Y(J))
      RMID=.5*(R+RI)
      RAD=.5*(R-RI)
      SN=1.
C   GAUSSIAN QUADRATURE
      DO 42 IJ=1,32
      IA=IABS(IJ-16)
      IF (IJ.LE.16) IA=IA+1
      IF (IJ.GT.16) SN=-1.
      RI=RMID+RAD*EX(IA)*SN
      CALL HANKEL (RI,H1,H2)
      PART3N=SIN(SQRT(RI**2-K*(KH*Y(J))**2))*H2/RI
      PART3=PART3+WT(IA)*PART3N
   42 CONTINUE
      PART3=PART3*CMPLX(0.,-2.*RAD)
   44 CONTINUE
      PHI(L)=(PART1+PART2+PART3+PART5)*CONST*Y(J)+PHIAFF(LFF)
      GO TO 45
   46 CONTINUE
      PHI(L)=CMPLX(0.,0.)
      CONAFF(LFF)=PHI(L)
      PHIAFF(LFF)=PHI(L)
   45 CONTINUE
   40 CONTINUE
      RETURN
   60 CONTINUE
      CONST=GAMFF/GAMFFS
      LFF=0
      ZIP=0.
```

```
      KH=OMEG/K
      PART2=CMPLX(0.,0.)
      PART3=PART2
      DO 80 I=ILE,ITE
      PART1=(PHIUB(I)-PHI((I-1)*JM+JW))*CEXP(CMPLX(0.,-KH*X(I)))
      PART3=PART3+.5*(PART1+PART2)*(X(I)-ZIP)
      ZIP=X(I)
      PART2=PART1
   80 CONTINUE
      DO 70 I=1,IM
      M=(I-1)*JM
      J3=JM1
      IF (I.LE.2.OR.I.EQ.IM) J3=1
      DO 75 J=1,JM,J3
      L=M+J
      LFF=LFF+1
      PHI(L)=PHI(L)-PHIAFF(LFF)
      PHIAFF(LFF)=PART3*CONAFF(LFF)
      PHI(L)=PHI(L)*CONST+PHIAFF(LFF)
   75 CONTINUE
   70 CONTINUE
      RETURN
C  SUPERSONIC FARFIELD
   90 CONTINUE
      DO 91 I=1,IM
      M=(I-1)*JM
      J3=JM1
      IF (I.LE.2.OR.I.EQ.IM) J3=1
      DO 92 J=1,JM,J3
      L=M+J
      PHI(L)=CMPLX(0.,0.)
   92 CONTINUE
   91 CONTINUE
      RETURN
      END
      SUBROUTINE FPRINT
      COMPLEX PHIUB,B,D,PHI,GAMTE1,GAMTE,GAMFF,GAMFFS,CLIFT,CMOM,
     1  CHINGE,C1,C2,C3,C10,C20,C30,CIRLIF,PART
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ILE,ITE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  K,SMALLK,OMEG,XH,NDOUB,CPCPB,TITLE(8),M8,DEL,ALPHA,ALPHAF
      COMMON /COEFF/ A(99),B(99),C(99),D(99),PHI(10000)
      COMMON /GAMMA/ GAMTE1,GAMTE,PGFF,GAMFF,GAMFFS
      CPDEL=CPCPB/DEL
      PART=.5*((X(ITE+1)-1.)*(GAMFF-GAMTE)/(X(IM1)-1.)+GAMTE)
      IJW=ITE*JM+JW
      PHI(IJW)=PHI(IJW)-PART
      L=(ILE-2)*JM+JW
      PHIUB(ILE-1)=PHI(L)
      PHIUB(ITE+1)=PHI(IJW)+2.*PART
C  COMPUTE CP LOWER (B) AND CP UPPER (D)
      DO 10 I=ILE,ITE
      M=(I-1)*JM
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```
      L=M+JW
      B(I)=-2.*(AX1(I)*(PHI(L+JM)-PHI(L))+AX2(I)*(PHI(L)-PHI(L-JM)))
     1  *CPDEL
      D(I)=-2.*(AX1(I)*(PHIUB(I+1)-PHIUB(I))+AX2(I)*(PHIUB(I)-
     1  PHIUB(I-1)))*CPDEL
   10 CONTINUE
      PHI(IJW)=PHI(IJW)+PART
C  COMPUTE UNSTEADY FORCE COEFFICIENTS
      C10=B(ILE)-D(ILE)
      CLIFT=C10*X(ILE)
      C20=CLIFT
      CMOM=.5*CLIFT*X(ILE)
      C30=CMPLX(0.,0.)
      CHINGE=C30
      IF (XH.GE.X(ILE)) GO TO 25
      C30=C10*(X(ILE)-XH)
      CHINGE=.5*C30*(X(ILE)-XH)
   25 CONTINUE
      ILE1=ILE+1
      DO 30 I=ILE1,ITE
      C1=B(I)-D(I)
      C2=C1*X(I)
      IF (X(I).GT.XH) C3=C1*(X(I)-XH)
      CLIFT=CLIFT+.5*(C1+C10)*DX(I-1)
      CMOM=CMOM+.5*(C2+C20)*DX(I-1)
      DXX=DX(I-1)
      IF (X(I).GT.XH.AND.X(I-1).LE.XH) DXX=X(I)-XH
      CHINGE=CHINGE+.5*(C3+C30)*DXX
      C10=C1
      C20=C2
      C30=C3
   30 CONTINUE
      CIRLIF=2.*GAMTE*CPDEL
      WRITE (6,900)
      WRITE (6,901) (TITLE(I),I=1,8)
      WRITE (6,902) M8
      WRITE (6,903) K
      WRITE (6,904) DEL
      WRITE (6,922) ALPHA
      WRITE (6,905) SMALLK
      WRITE (6,906) OMEG
      WRITE (6,907) XH
      WRITE (6,923) ALPHAF
      WRITE (6,908) CPCPB
      WRITE (6,909) GAMTE
      IF (XH.GT.0.) GO TO 40
      WRITE (6,910) CIRLIF
      WRITE (6,911)
      WRITE (6,913) CLIFT,CMOM,CHINGE
      WRITE (6,914)
      GO TO 45
   40 CONTINUE
      WRITE (6,921) CIRLIF
      WRITE (6,912)
      WRITE (6,913) CLIFT,CMOM,CHINGE
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      WRITE (6,915)
   45 CONTINUE
      WRITE (6,916)
      WRITE (6,917) (X(I),I=ILE,ITE)
      WRITE (6,918)
      WRITE (6,919) (D(I),I=ILE,ITE)
      WRITE (6,920)
      WRITE (6,919) (B(I),I=ILE,ITE)
  900 FORMAT (1H1)
  901 FORMAT (30X,8A10)
  902 FORMAT (1H ,/,1H ,/,1H ,/,* MACH NUMBER =*E13.5)
  903 FORMAT (* SIMILARITY PARAMETER (K) =*E13.5)
  904 FORMAT (* THICKNESS RATIO =*E13.5)
  905 FORMAT (* REDUCED FREQUENCY (BASED ON CHORD) =*E13.5)
  906 FORMAT (* SCALED FREQUENCY (OMEGA) =*E13.5)
  907 FORMAT (* HINGE POINT =*E13.5)
  908 FORMAT (* CP SCALING FACTOR (CP/CPBAR) =*E13.5)
  909 FORMAT (* SCALED AIRFOIL CIRCULATION =*2E13.5)
  910 FORMAT (* LIFT COEFFICIENT BASED ON CIRCULATION (PER UNIT PITCH AN
     1GLE IN RADIANS) =*2E13.5)
  911 FORMAT (1H ,/,1H ,/,* UNSTEADY FORCE COEFFICIENTS (PER UNIT PITCH
     1ANGLE IN RADIANS)*)
  912 FORMAT (1H ,/,1H ,/,* UNSTEADY FORCE COEFFICIENTS (PER UNIT FLAP A
     1NGLE IN RADIANS)*)
  913 FORMAT (1H ,/,3X*LIFT =*2E13.5,/,3X*MOMENT ABOUT (X=0) =*2E13.5,/,
     1  3X*HINGE MOMENT =*2E13.5)
  914 FORMAT (1H ,/,1H ,/,* PRESSURE COEFFICIENTS ON THE AIRFOIL, UNSCAL
     1ED (PER UNIT PITCH ANGLE IN RADIANS)*)
  915 FORMAT (1H ,/,1H ,/,* PRESSURE COEFFICIENTS ON THE AIRFOIL, UNSCAL
     1ED (PER UNIT FLAP ANGLE IN RADIANS)*)
  916 FORMAT (1H ,/,3X*AIRFOIL COORDINATE =*)
  917 FORMAT (3XE13.5,13XE13.5,13XE13.5,13XE13.5,13XE13.5)
  918 FORMAT (1H ,/,3X*AIRFOIL PRESSURE COEFFICIENTS, UPPER =*)
  919 FORMAT (3X10E13.5)
  920 FORMAT (1H ,/,3X*AIRFOIL PRESSURE COEFFICIENTS, LOWER =*)
  921 FORMAT (* LIFT COEFFICIENT BASED ON CIRCULATION (PER UNIT FLAP ANG
     1LE IN RADIANS) =*2E13.5)
  922 FORMAT (* MEAN AIRFOIL ANGLE OF ATTACK (RADIANS) =*E13.5)
  923 FORMAT (* MEAN FLAP ANGLE (RADIANS) =*E13.5)
      RETURN
      END
      SUBROUTINE GAMFUN
      COMPLEX GAMTE1,GAMTE,GAMFF,GAMFFS
      COMMON /GAMMA/ GAMTE1,GAMTE,PGFF,GAMFF,GAMFFS
      GAMFF=GAMTE1+PGFF*(GAMTE-GAMTE1)
      GAMTE1=GAMTE
      RETURN
      END
      SUBROUTINE HANKEL (R,H1,H2)
      COMPLEX H0,H1,H2
      IF (R.GT.3.) GO TO 10
      X2=(R/3.)**2
      X4=X2*X2
      X6=X4*X2
      X8=X6*X2
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      X10=X8*X2
      X12=X10*X2
      A0=1.-2.2499997*X2+1.2656208*X4-.3163866*X6+.0444479*X8-.0039444
     1  *X10+.00021*X12
      B0=2./3.14159265*ALOG(.5*R)*A0+.36746691+.60559366*X2-.74350384
     1  *X4+.25300117*X6-.04261214*X8+.00427916*X10-.00024846*X12
      A1=R*(.5-.56249985*X2+.21093573*X4-.03954289*X6+.00443319*X8
     1  -.00031761*X10+.00001109*X12)
      B1=(2./3.14159265*R*ALOG(.5*R)*A1-.6366198+.2212091*X2+2.1682709
     1  *X4-1.3164827*X6+.3123951*X8-.0400976*X10+.0027873*X12)/R
      GO TO 20
   10 CONTINUE
      XI1=3./R
      XI2=XI1*XI1
      XI3=XI2*XI1
      XI4=XI3*XI1
      XI5=XI4*XI1
      XI6=XI5*XI1
      F=.79788456-.00000077*XI1-.0055274*XI2-.00009512*XI3+.00137237
     1  *XI4-.00072805*XI5+.00014476*XI6
      TH=R-.78539816-.04166397*XI1-.00003954*XI2+.00262573*XI3
     1  -.00054125*XI4-.00029333*XI5+.00013558*XI6
      A0=F*COS(TH)/SQRT(R)
      B0=F*SIN(TH)/SQRT(R)
      F1=.79788456+.00000156*XI1+.01659667*XI2+.00017105*XI3-.00249511
     1  *XI4+.00113653*XI5-.00020033*XI6
      TH1=R-2.35619449+.12499612*XI1+.0000565*XI2-.00637879*XI3
     1  +.00074348*XI4+.00079824*XI5-.00029166*XI6
      A1=F1*COS(TH1)/SQRT(R)
      B1=F1*SIN(TH1)/SQRT(R)
   20 CONTINUE
      H0=CMPLX(A0,-B0)
      H1=CMPLX(A1,-B1)
      H2=2.*H1/R-H0
      RETURN
      END
      SUBROUTINE INITAL
      COMPLEX PHIUB
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ILE,ITE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  K,SMALLK,OMEG,XH,NDOUB,CPCPB,TITLE(8),M8,DEL,ALPHA,ALPHAF
C  CALCULATE DX
      DO 15 I=1,IM1
      DX(I)=X(I+1)-X(I)
   15 CONTINUE
C  CALCULATE DY
      DO 25 I=1,JM1
      DY(I)=Y(I+1)-Y(I)
   25 CONTINUE
      DO 30 I=2,IM1
      AX1(I)=DX(I-1)/(DX(I)*(DX(I-1)+DX(I)))
      AX2(I)=DX(I)/(DX(I-1)*(DX(I-1)+DX(I)))
      BX1(I)=2.*AX1(I)/DX(I-1)
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      BX2(I)=2.*AX2(I)/DX(I)
      CX(I)=.5/DX(I)
   30 CONTINUE
      CX(1)=.5/DX(1)
      DO 40 I=2,JM1
      AY1(I)=2./(DY(I)*(DY(I)+DY(I-1)))
      AY2(I)=2./(DY(I-1)*(DY(I)+DY(I-1)))
   40 CONTINUE
      IF (K.GT.0.) GO TO 50
C  TOP AND BOTTOM AND RHS BOUNDARY CONDITIONS
      AX1(IM1)=0.
      AX2(IM1)=0.
      BX1(IM1)=0.
      BX2(IM1)=2./DX(IM1-1)**2
      AY1(2)=2./DY(2)**2
      AY2(2)=0.
      AY1(JM1)=0.
      AY2(JM1)=2./DY(JM1-1)**2
   50 CONTINUE
      DYBU1=2./((DY(JWP1)+2.*DY(JW))*DY(JWP1))
      DYBU2=DY(JWP1)*DYBU1
      DYBL1=2./((DY(JW-2)+2.*DY(JWM1))*DY(JW-2))
      DYBL2=DY(JW-2)*DYBL1
C  SET AIRFOIL BOUNDARY CONDITION
      DO 45 I=ILE,ITE
      FPU(I)=0.
      IF (X(I).GE.XH) FPU(I)=-1.
      FPL(I)=FPU(I)
   45 CONTINUE
      RETURN
      END
      SUBROUTINE PRINT (NITERG)
      COMPLEX PHIUB,B,D,PHI,GAMTE1,GAMTE,GAMFF,GAMFFS,PART
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ILE,ITE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  K,SMALLK,OMEG,XH,NDOUB,CPCPB,TITLE(8),M8,DEL,ALPHA,ALPHAF
      COMMON /COEFF/ A(99),B(99),C(99),D(99),PHI(10000)
      COMMON /GAMMA/ GAMTE1,GAMTE,PGFF,GAMFF,GAMFFS
      PART=.5*((X(ITE+1)-1.)*(GAMFF-GAMTE)/(X(IM1)-1.)+GAMTE)
      IJW=ITE*JM+JW
      PHI(IJW)=PHI(IJW)-PART
      L=(ILE-2)*JM+JW
      PHIUB(ILE-1)=PHI(L)
      PHIUB(ITE+1)=PHI(IJW)+2.*PART
C  COMPUTE CP LOWER (B) AND CP UPPER (D)
      DO 297 I=ILE,ITE
      M=(I-1)*JM
      L=M+JW
      B(I)=-2.*(AX1(I)*(PHI(L+JM)-PHI(L))+AX2(I)*(PHI(L)-PHI(L-JM)))
      D(I)=-2.*(AX1(I)*(PHIUB(I+1)-PHIUB(I))+AX2(I)*(PHIUB(I)-
     1  PHIUB(I-1)))
  297 CONTINUE
      PHI(IJW)=PHI(IJW)+PART
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      WRITE (6,911) NITERG
      WRITE (6,903) (D(I),I=ILE,ITE)
      WRITE (6,912) NITERG
      WRITE (6,903) (B(I),I=ILE,ITE)
  903 FORMAT (10E13.5)
  911 FORMAT (1H ,/,* AT ITERATION*I5* SCALED PRESSURE COEFFICIENT, UPPE
     1R (ILE TO ITE) =*)
  912 FORMAT (1H ,/,* AT ITERATION*I5* SCALED PRESSURE COEFFICIENT, LOWE
     1R (ILE TO ITE) =*)
      RETURN
      END
      SUBROUTINE TRI (I)
      COMPLEX PHIUB,B,D,PHI,P
      REAL K,M8
      COMMON /DELTA/ DX(99),DY(99),AX1(99),AX2(99),BX1(99),BX2(99),
     1  CX(99),AY1(99),AY2(99),X(100),Y(100),FPU(99),FPL(99),PHIUB(99),
     2  IM,IM1,JM,JM1,JW,JWP1,JWM1,ILE,ITE,DYBU1,DYBU2,DYBL1,DYBL2,
     3  K,SMALLK,OMEG,XH,NDOUB,CPCPB,TITLE(8),M8,DEL,ALPHA,ALPHAF
      COMMON /COEFF/ A(99),B(99),C(99),D(99),PHI(10000)
      DO 10 KK=3,JM1
      J=JM1-KK+3
      P=A(J-1)/B(J)
      B(J-1)=B(J-1)-P*C(J)
      D(J-1)=D(J-1)-P*D(J)
   10 CONTINUE
      M=(I-1)*JM
      PHI(M+2)=D(2)/B(2)
      DO 20 J=3,JM1
      L=M+J
      PHI(L)=(D(J)-PHI(L-1)*C(J))/B(J)
   20 CONTINUE
      RETURN
      END
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