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SAMSO-TR 75-251 VOL II

STANFORD TELECOMMUNICATIONS, INC.
REPORT NO. STI TR 10255 VOL II

(2)

NAVSTAR GLOBAL POSITIONING SYSTEMS SPECIAL STUDIES AND ENGINEERING PROGRAM

AD B008412

VOLUME II

FINAL REPORT

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JAN 12 1976
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Prepared by:

James J. Spilker, Jr.
Horen Chang

October 25, 1975

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SPACE AND MISSILE SYSTEMS ORGANIZATION (AFSC)
LOS ANGELES AIR FORCE STATION
LOS ANGELES, CALIFORNIA

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NAVSTAR GLOBAL POSITIONING SYSTEMS SPECIAL STUDIES AND ENGINEERING PROGRAM.

VOLUME II.

~~FINAL REPORT~~

GENERALIZED CROSS-CORRELATION PROPERTIES OF GPS CODES IN CASES OF DIFFERENTIAL DOPPLER SHIFTS,

GPS/TRIDENT CODE DESIGN.

(9) Final rept. Mar 74-Oct 75,

Prepared by:

(10) James J. Spilker, Jr.
Horen Chang

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FOREWORD

This document is Volume II of a four volume report. The four volume report is titled, "NAVSTAR Global Positioning Systems Special Studies and Engineering Program". Volume II summarizes the work performed for the Air Force and addresses two subjects. The first subject deals with Generalized Cross-Correlation Projection of GPS Codes in Cases of Differential Doppler Shifts, and was submitted in draft form to SAMSO on 11 July 1975 as STI TR-7115-1.

The second subject deals with the GPS/TRIDENT Code Design and was submitted in draft form on August 6, 1975 as STI/GPS-051.

GENERALIZED CROSS-CORRELATION PROLERTIES OF GPS CODES IN CASES OF DIFFERENTIAL DOPPLER SHIFTS

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GENERALIZED CROSS-CORRELATION PROPERTIES OF GPS CODES IN CASES OF DIFFERENTIAL DOPPLER SHIFTS

I. INTRODUCTION

In the coruse of GPS (Global Positioning System) code design, 32 distinct Gold Codes from the same family of period 1023 bits were selected as the C/A (clear/acquisition) codes, each for one of the 32 NAVSTAR satellites. There are at least three main reasons for such a selection.

1. Gold code of period 1023 bits is a good ranging code: its (normalized) autocorrelation function has a maximum value (0 dB) when compared with itself and very small values (= -23.9, or -24.2, or -60.2 dB) when compared with its m-bit ($m \neq 0$) shifts such that false code-phase lock may be avoided during acquisition.

2. Gold codes from the same family of period 1023 bits are good multiple-access codes for spread-spectrum modulation: they have very small cross-correlation (= - 23.9, or -25.2, or -60.2 dB in the case of zero Doppler shift) between each other and therefore can be transmitted simultaneously in the same frequency band without severe mutual interference.

3. The 1023-bit period (= 1 m sec at bit rate 1.023 Mbps) appears to be a reasonable compromise between the correlation magnitude and the acquisition time: the code period has to be <u>long</u> in order to achieve

small autocorrelation ($m \neq 0$) and cross-correlation, whereas quick acquisition normally requires the code period to be short.

The good cross-correlation property of GPS C/A codes in the case of zero Doppler shift has been well known for years. Less well known are the cases when there are finite Doppler shifts involved. Sec. II defines the problem. Computer-simulation results on the statistics of the interference levels are presented in Sec. III. Finally, in Appendix A, the generation of the 32 C/A codes are depicted.

II. DEFINITION OF THE PROBLEM

Before proceeding with the details, it is convenient to agree on notation. Capital letter "G, X, Y,---" will usually be used to denote periodic sequences, while lower-case letters "g_i, x_i, y_i, ---"($i = 1, 2, 3$, ---) will generally stand for the i^{th} bits of the periodic sequences. To distinguish between a periodic sequence and its cyclic shift, let, for example, $G_1 = G$ be the periodic sequence starting with the first bit of G:

$$G_1 = G = g_1 \; g_2 \; g_3 \; --- \tag{1}$$

then G_{m+1} will designate the m-bit cyclic shift of G, starting with the $(m+1)^{th}$ bit of G_1:

$$G_{1+m} = g_{1+m} \quad g_{2+m} \quad g_{3+m} \; --- \tag{2}$$

With these preliminaries, consider the simplified GPS configuration in Figure 1, where only 6 of the 32 NAVSTAR satellites are shown. The C/A signal $C_j(t)$ transmitted by the j^{th} satellite at time zero can generally be expressed as a biphase-modulated carrier given by:

$$G_j(t) = D_j(t) \; G_1(k_j) \cos 2\pi f_c t; \; j=1, 2, --- \; 32 \tag{3}$$

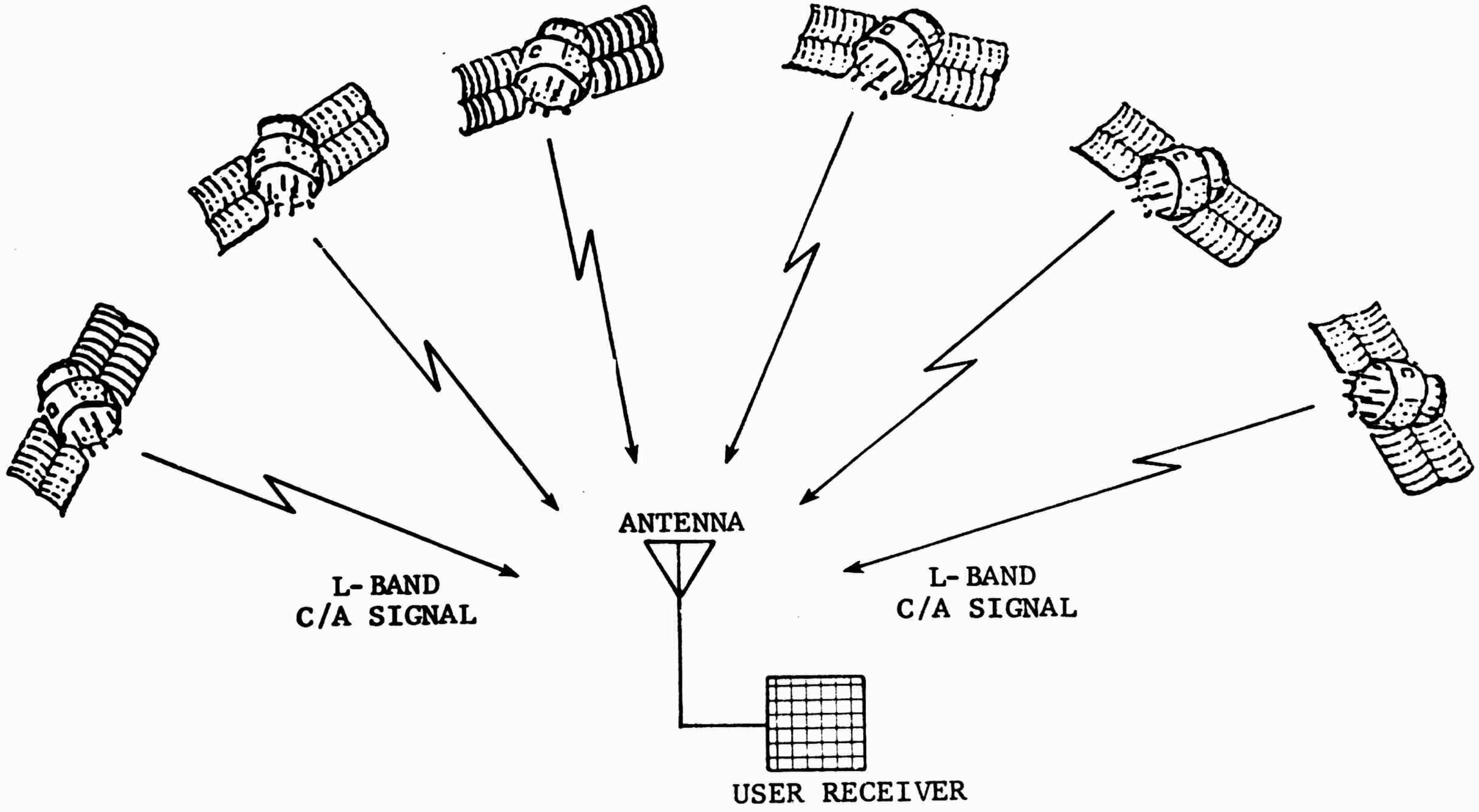

FIGURE 1 SIMPLIFIED GPS CONFIGURATION

All satellites have different distances and radial velocities with respect to the user receiver.

where

$D_j(t)$ = 50 bps data bit stream

f_c = 1575.42 MHz (L_1) or 1237.83 MHz (L_2) (4)

and

$G_1(k_j) = X_1 \oplus Y_{1+k_j}$

is the j^{th} C/A code generated by modulo-2 sum of two maximal-length sequences X_1 and Y_{1+k_j} (see Appendix A). Because normally all satellites have different distances and radial velocities with respect to the user receiver, the arriving C/A code phases (due to different distances or time delays) and carrier frequencies (due to different radial velocities or Doppler shifts) will be different. Symbolically, let $C_j(t)$ denote the j^{th} arriving C/A signal; then,

$$C_j(t) = D_j(t-\gamma_j)\, G_{m_j}(k_j) \cos 2\pi(f_c + \Delta f_j)(t-\gamma_j) \; ;$$

$$j = 1, 2, ---, 32 \qquad (5)$$

where

$G_{m_j}(k_j)$ = (m_j-1) - bit cyclic shift of $G_1(k_j)$

γ_j = the time delay between the j^{th} satellite and the user receiver.

$\Delta f_j = \frac{f_c}{c} v_j$ = the Doppler shift of the j^{th} carrier frequency.

and

$$m_j = 1 - 1.023 \text{ Mbps} \cdot \gamma_j \text{ (modulo 1023)}$$

v_j^r = the relative radial velocity between the j^{th} satellite and the user receiver

c = velocity of light

Because the data bit stream usually cannot worsen the cross-correlation properties to be discussed,[†] consider for simplicity of analysis only the remaining terms in (5). Then the input to the user receiver will be the summation of $C_j(t)$'s given by:

$$\sum_{j=1}^{32} C_j(t) = \sum_{j=1}^{32} G_{m_j}(k_j) \underbrace{\cos 2\pi(f_c + \Delta f_j)(t-\gamma_j)}_{R_j(t)} \qquad (6)$$

Refer to Figure 2, the simplified functional diagram of the user receiver channel constructed to receive the j^{th} C/A signal, and suppose, for ease of discussion, there is only one interferring signal (or undesired signal) $C_i(t)$ present (which can be readily generalized to cases when there are more than one interferring signals). The reference signal $R_j(t)$ may be obtained from the output of VCO (voltage-controlled oscillator) when the carrier PLL (phase-locked loop) is locked onto $C_j(t)$. The local code generator produces the k_j^{th} C/A Code $G_n(k_j)$, and the code phase selector shifts the code phase n until $n=m_j$ (or, equivalently, until the power meter exceeds about 0 dB). At this moment, the

---

[†] One of the reasons is that it has period (20 m sec) equal to integral multiple of the period of C/A codes (1 m sec) and hence has no effect on the spectra (or the cross-spectra) of the C/A codes to be discussed.

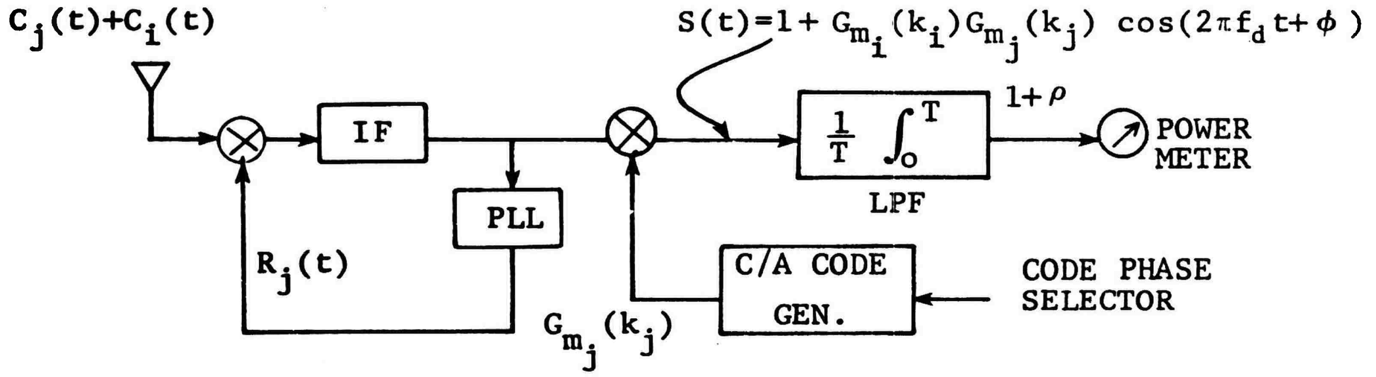

FIGURE 2

SIMPLIFIED FUNCTIONAL DIAGRAM OF THE RECEIVER CHANNEL TO DETECT $C_j(t)$

input S(t) to the integrator (or low-pass filter) becomes

$$S(t) = 1 + \underbrace{G_{m_j}(k_j)\, G_{m_i}(k_i) \cos(2\pi f_d t + \phi)}_{\delta} \tag{7}$$

where

$$f_d = \Delta f_i - \Delta f_j \tag{8}$$

is the <u>differential</u> Doppler shifts between the i^{th} and the j^{th} C/A signals arriving at the user receiver. Note that before the desired code phase is found (viz., before $n = m_j$), the δ term in (7) is given by

$$\begin{aligned} \delta &= G_n(k_j)\, G_{m_i}(k_i) \cos(2\pi f_d t + \phi) \\ &= G_\ell(k_\ell) \cos(2\pi f_d t + \phi) \end{aligned} \tag{9}$$

where

$$G_\ell(k_\ell) = G_n(k_j)\, G_{m_i}(k_i) \quad ; \quad k_\ell \neq k_j \text{ or } k_i \tag{10}$$

is a third Gold code from the same family.

The contribution of δ to the power-meter readout can hence be written as

$$\begin{aligned} \rho &= \frac{1}{T}\int_0^T G_\ell(k_\ell) \cos(2\pi f_d t + \phi)\, dt \\ &\approx \frac{1}{T}\left| F_\ell(f_d) \right| \quad \text{for } \frac{1}{T} \lesssim 1 \text{ kHz and } f_d \ll 1 \text{ MHz} \end{aligned} \tag{11}$$

where

$$F_{\ell}(f) = \mathcal{F}\left\{G_{\ell}(k_{\ell})\right\} \qquad (12)$$

is the Fourier spectrum of $G_{\ell}(k_{\ell})$. Since $G_{\ell}(k_{\ell})$ is periodic with period = 1 msec, $F_{\ell}(f)$ is a line spectrum having components at $\pm$ N kHz, N being any integer.

III. CUMULATIVE PROBABILITY OF THE INTERFERENCE LEVEL ρ

As n in (9) shifts quickly bit-by-bit to search for the code phase of $G_{m_j}(k_j)$, m_i stays relatively unchanged; consequently, $G_{\ell}(k_{\ell})$ becomes the whole family of Gold codes. The cumulative probability of the interference level ρ can therefore be derived from the Fourier spectra of the whole family of Gold codes, as shown in Figure 3-8 for f_d = 0, 1, 2, 3, 4, and 5 kHz. (For most GPS user receivers, the differential Doppler shift $f_d \lesssim 5$ kHz). In addition, the maximum value of ρ for each case is tabulated in Table 2.

Differential Doppler Shift between Two Arriving C/A Signals at the User Receiver f_d	Max Value of INTERFERENCE LEVEL ρ
0 Hz	-23.9 dB
1 kHz	-21.1 dB
2 kHz	-21.1 dB
3 kHz	-21.6 dB
4 kHz	-21.1 dB
5 kHz	-21.9 dB

TABLE 2

MAXIMUM VALUE OF INTERFERENCE LEVEL ρ

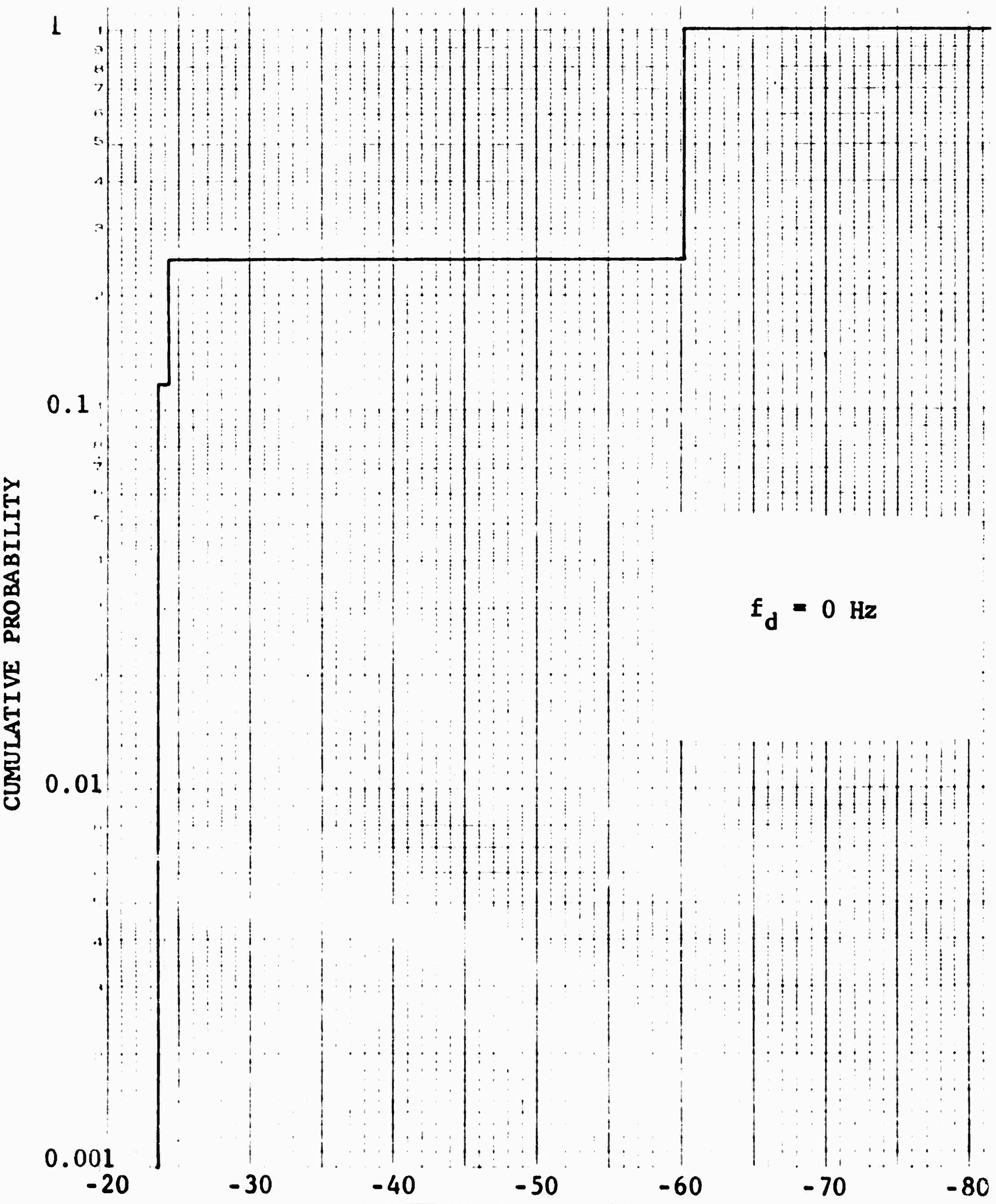

FIGURE 3
CUMULATIVE PROBABILITY OF INTERFERENCE
LEVEL FOR CPS C/A CODES

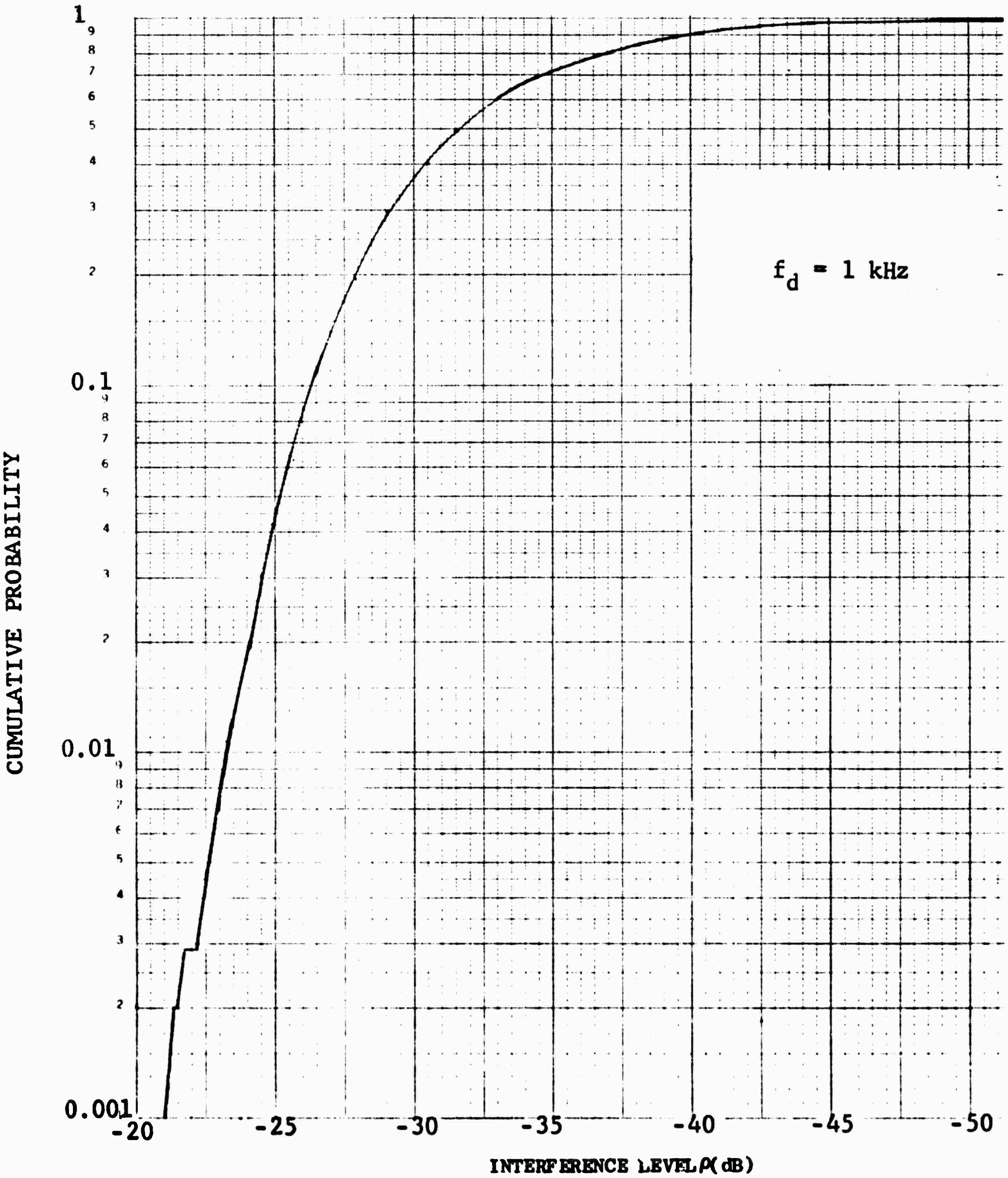

FIGURE 4
CUMULATIVE PROBABILITY OF INTERFERENCE LEVEL
FOR GPS C/A CODES

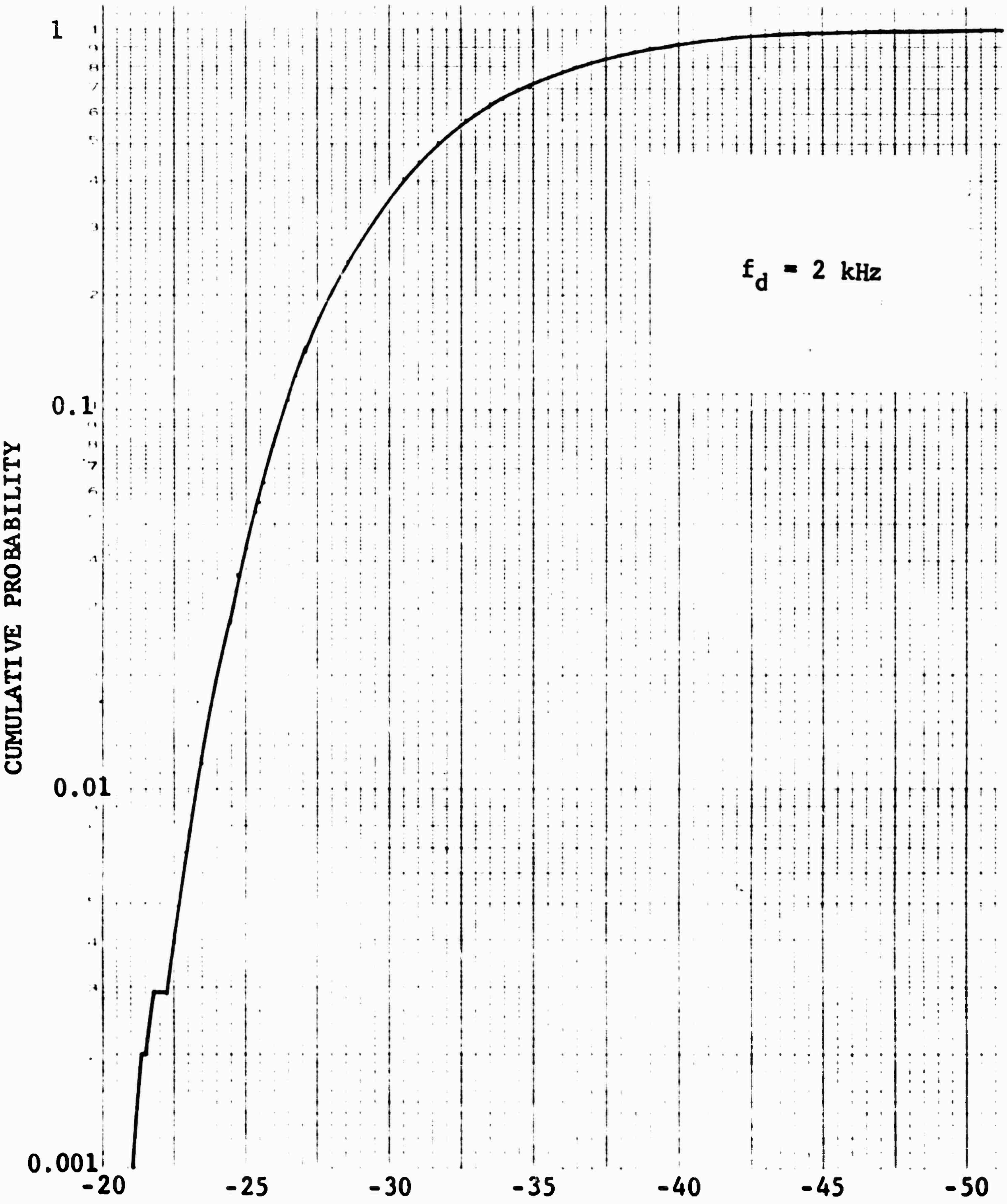

FIGURE 5
CUMULATIVE PROBABILITY OF INTERFERENCE
LEVEL FOR GPS C/A CODES

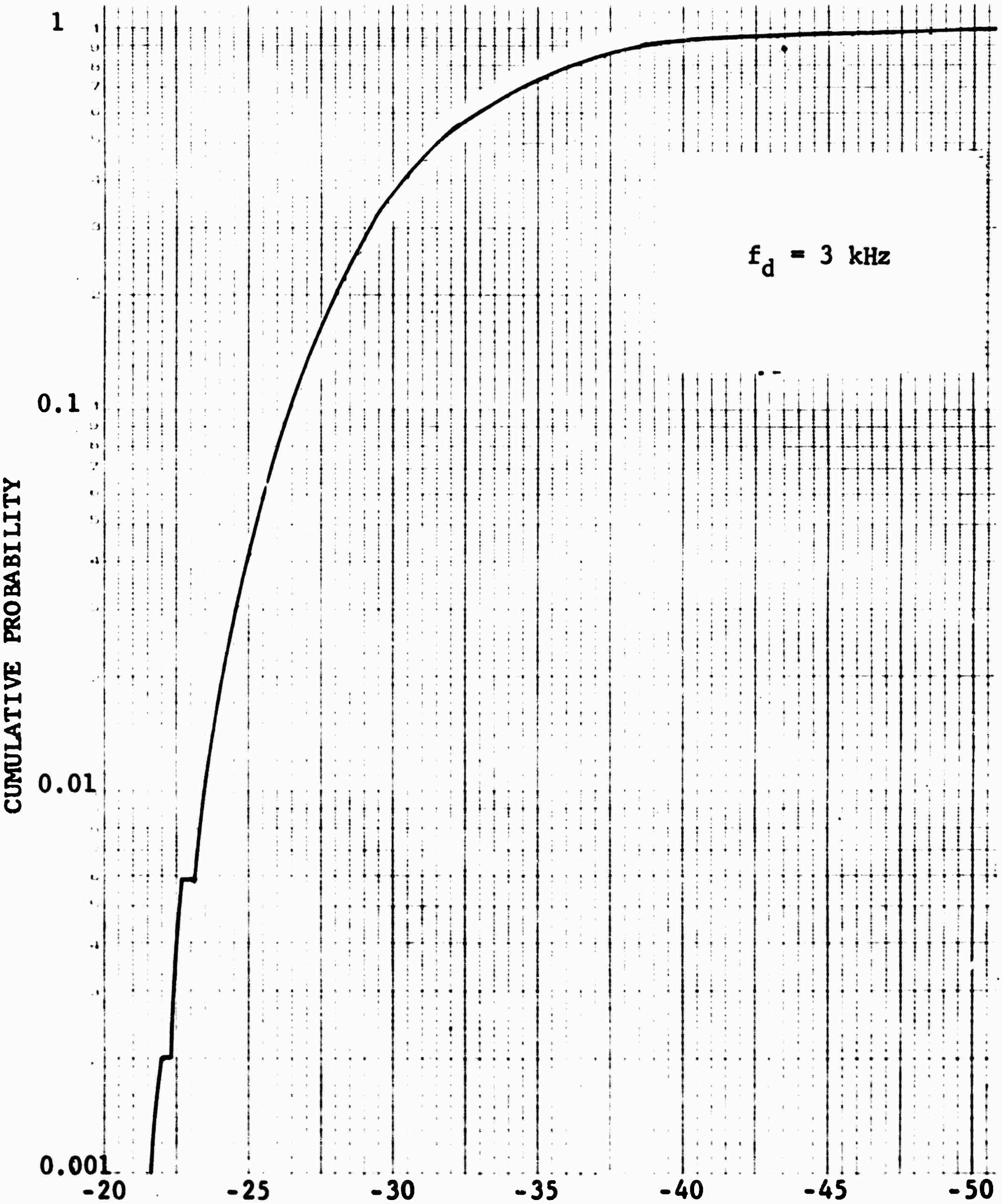

FIGURE 6
CUMULATIVE PROBABILITY OF INTERFERENCE
LEVEL FOR CPS C/A CODES

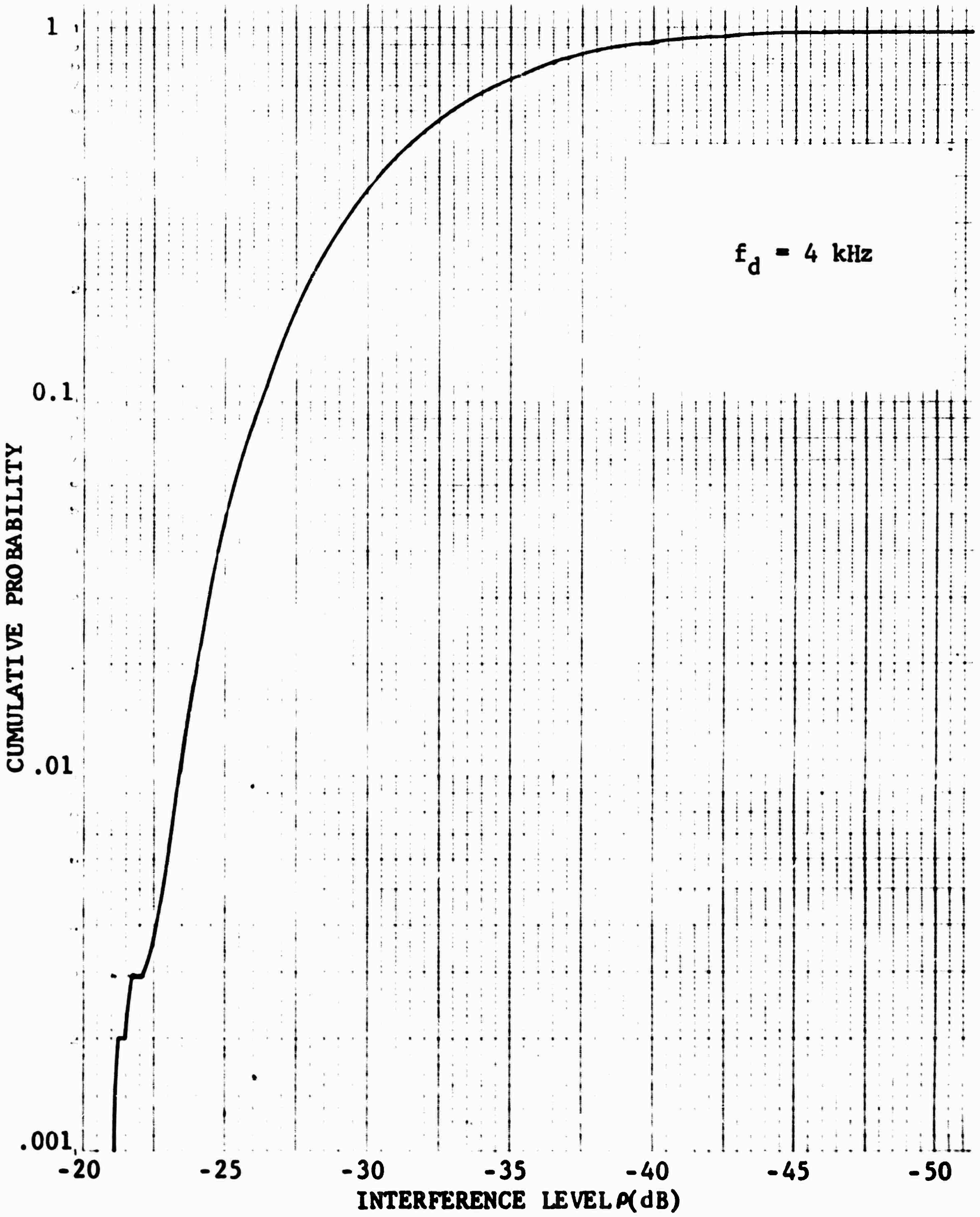

FIGURE 7
CUMULATIVE PROBABILITY OF INTERFERENCE
LEVEL FOR GPS C/A CODES

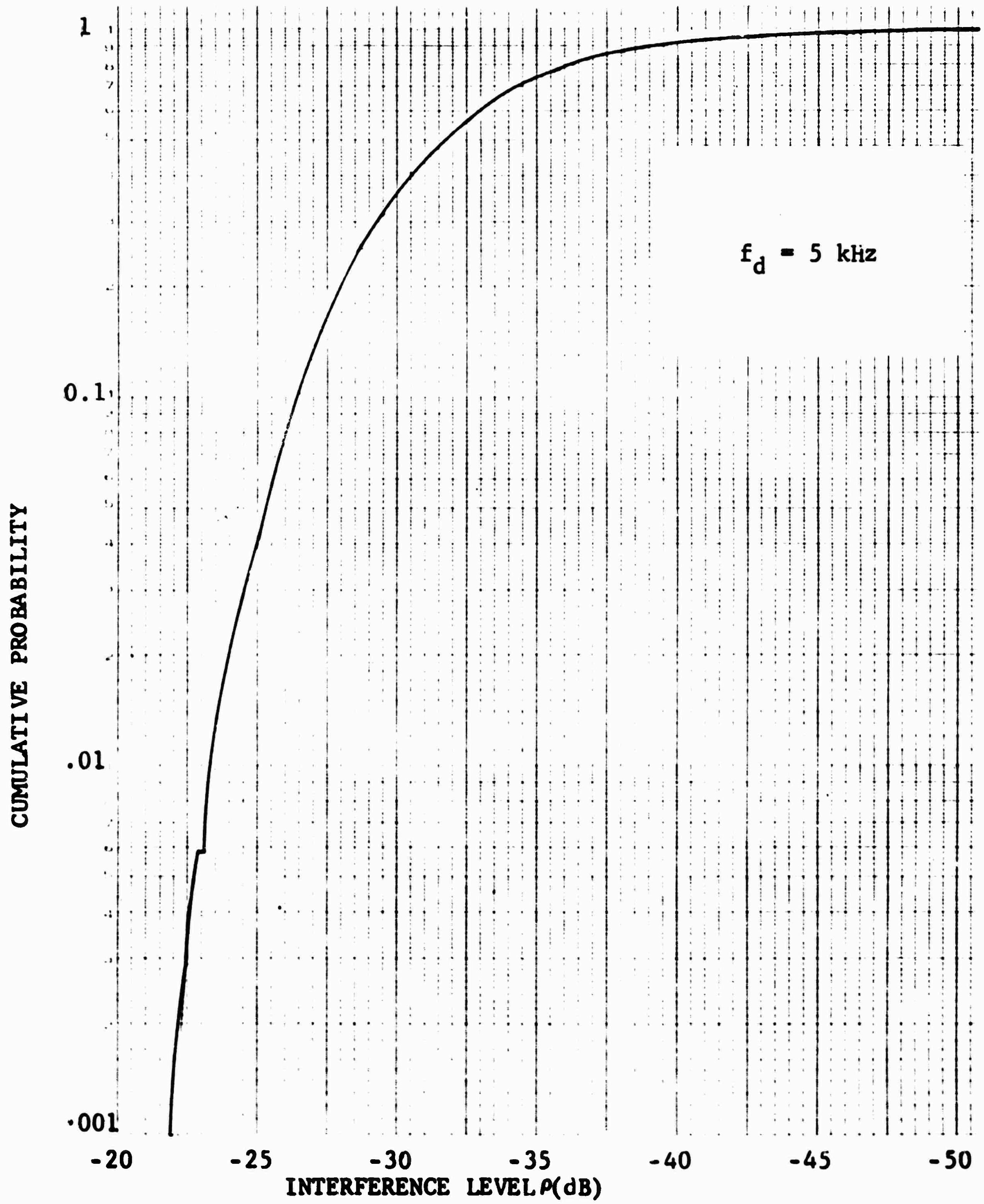

FIGURE 8
CUMULATIVE PROBABILITY OF INTERFERENCE LEVEL FOR CPS C/A CODES

APPENDIX A. GENERATION OF THE 32 GPS C/A CODES

Let X_1 and Y_1 , both of period 1023 bits, be the two maximal-length sequences that generate the 32 GPS C/A codes as illustrated in Figure A.1:

$$G_1(k_j) = X_1 \oplus Y_{1+k_j} \quad ; \quad j = 1, 2, ---, 32$$

Note that Y_{1+k_j}, the k_j-bit cyclic shift of Y_1, is obtained by taking the modulo-2 sum of S_1 and S_2, a pair of stages of the Y_1 generator. The values of S_1 and S_2 along with the first ten bits of the resulting GPS C/A codes are shown in Table A.1.

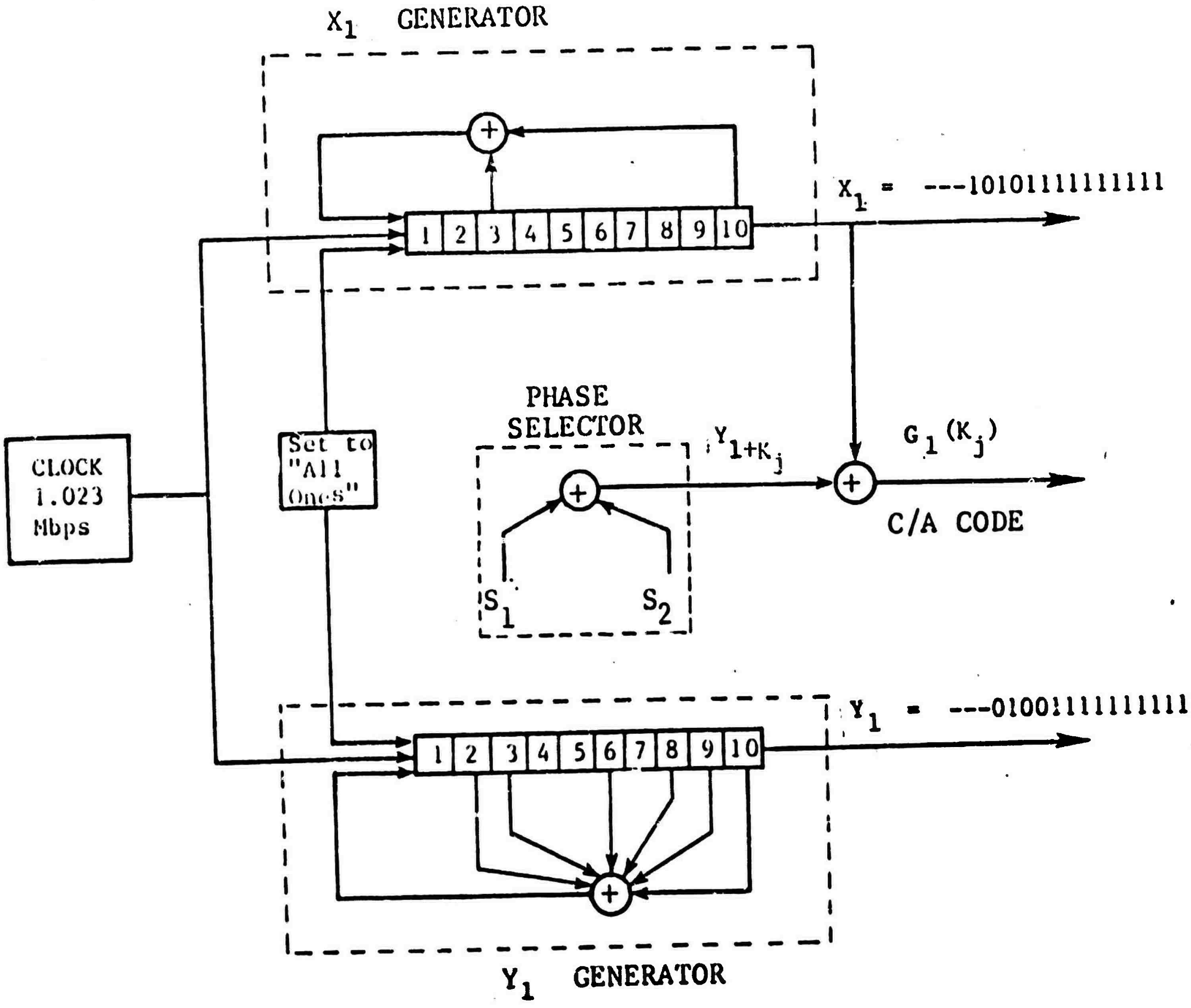

FIGURE A.1
C/A CODE GENERATION

TABLE A.1

Code Phase Selections and the First 10 Bits
of the Resulting C/A Codes

j	S_1	S_2	First Ten Bits of $\{G_n(K_j)\}$	
			Decimal	Binary (From Left to Right)
1	2	6	800	1100100000
2	3	7	912	1110010000
3	4	8	968	1111001000
4	5	9	996	1111100100
5	1	9	603	1001011011
6	2	10	813	1100101101
7	1	8	601	1001011001
8	2	9	812	1100101100
9	3	10	918	1110010110
10	2	3	836	1101000100
11	3	4	930	1110100010
12	5	6	1000	1111101000
13	6	7	1012	1111110100
14	7	8	1018	1111111010
15	8	9	1021	1111111101
16	9	10	1022	1111111110
17	1	4	622	1001101110
18	2	5	823	1100110111
19	3	6	923	1110011011
20	4	7	973	1111001101
21	5	8	998	1111100110
22	6	9	1011	1111110011
23	1	3	563	1000110011
24	4	6	966	1111000110
25	5	7	995	1111100011
26	6	8	1009	1111110001
27	7	9	1016	1111111000
28	8	10	1020	1111111100
29	1	6	599	1001010111
30	2	7	811	1100101011
31	3	8	917	1110010101
32	4	9	970	1111001010

GPS/TRIDENT CODE DESIGN

1. Introduction

This report summarizes the results of a GPS/TRIDENT code design under contract to GPS (Global Positioning System), Joint Program Office (F04701-74-C-0301). The main purpose of this study is to devise a ranging code, called H code, for trajectory determination of the TRIDENT missile employing NAVSTAR GPS. In addition to being a good ranging code, the H code is further required, for spread-spectrum multiple-access (SSMA) operation in GPS/TRIDENT system, to be a good spectrum-spreading, multiplexing code in the sense that it has very small interference or cross-correlation ($\leq$ -30 dB) with any one of the 32 C/A (clear/acquisition) codes transmitted by the GPS NAVSTAR satellites.

Figure 1 illustrates the basic concept of GPS/TRIDENT system. All six C/A codes (which are also good ranging and multiplexing codes, each from one of the six NAVSTAR satellites) and the H code [from the down range support ship (DRSS)] are transmitted synchronously (with atomic standards on all transmitter clocks) to the TRIDENT missile in the same frequency band L_1. Because all seven sources normally have different distances and different radial velocities with respect to the TRIDENT, the arriving code phases and carrier frequencies will be different from one access to another and; furthermore, from one instant to another as all vehicles are moving with time.

The TRIDENT transponder translates the received signals from L_1- to S-band by multiplying the received signals with an auxiliary sinusoid and then coherently retransmits these frequency-translated signals via the down-link to the DRSS. Basically each signal treats the other six signals as additive interference because each signal will be received by one of the seven independent channels of the DRSS receiver. A receiver channel

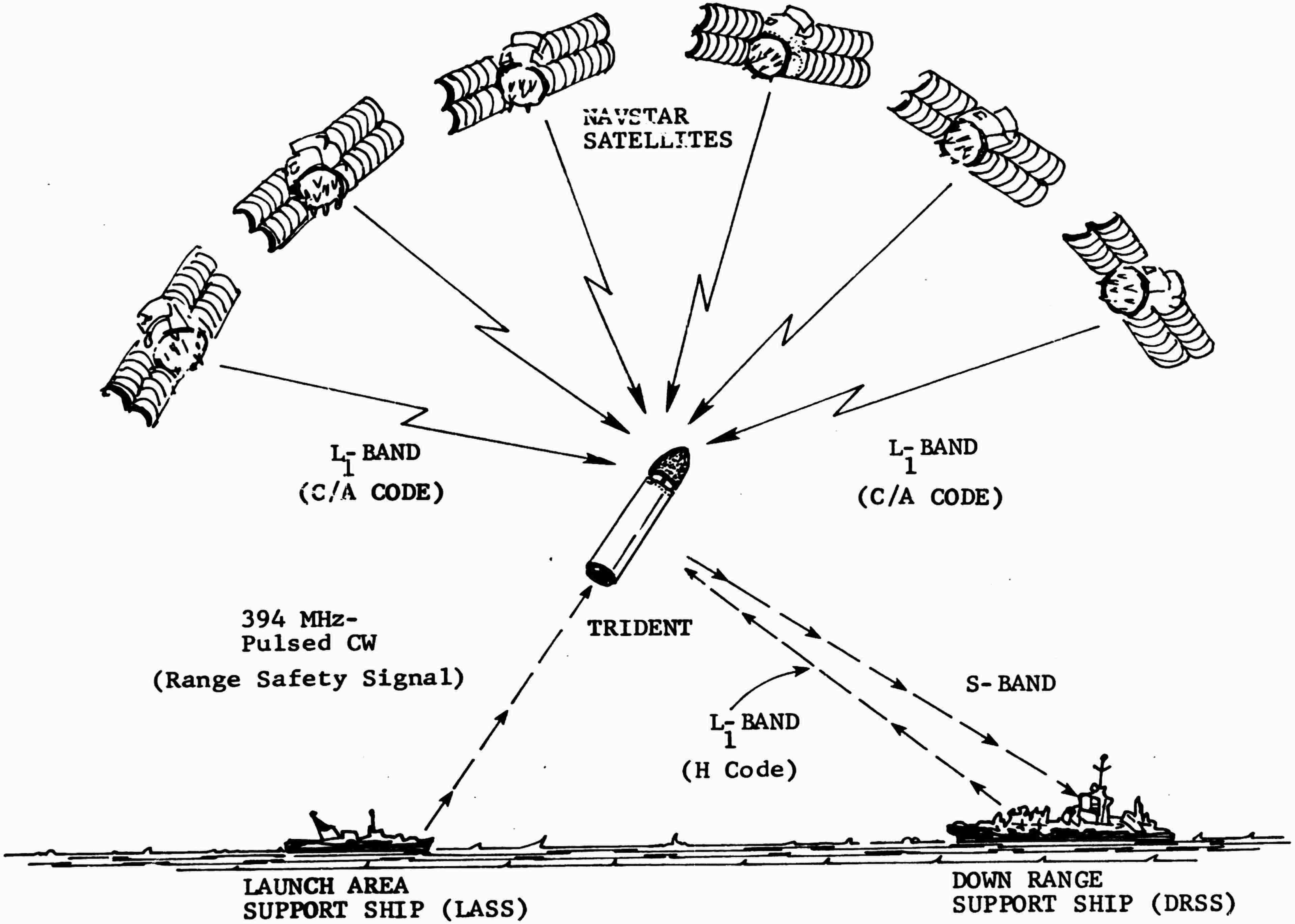

FIGURE 1

GPS/TRIDENT SYSTEM CONCEPT

consists mainly of a phase-locked carrier tracking loop, a delayed-locked code tracking loop, and a correlation detector. Theoretically, the phase of a desired code may be found by the following steps:

1. Cross-correlate the received signals with a locally generated replica of the desired code.

2. Shift the phase of the local code until maximum correlation, which in the absence of interference and noise has a normalized value of 1 or 0 dB at the correlator output, is reached. At this moment, the local code is said to be locked onto (i.e., in phase synchronization with) the desired code.

Once the phase and hence the propagation delay of every received code is known, the position of TRIDENT can then be easily calculated.

Evidently, to avoid false code-phase lock during acquisition, it is desired that each of the seven codes (= six C/A codes plus the H code) has large correlation when compared with itself and very small correlation when compared with (1) its m-bit shift (m being any integer ≠ 0 modulo the period of the code) or (2) any of the other six codes.

In the course of GPS code design, 32 distinct Gold codes from the same family of period 1023 bits (= 1 m sec at bit rate 1.023 Mbps) were selected as the C/A codes (see Appendix A for their generations), each for one of the 32 NAVSTAR satellites. As can be seen from Appendix B, all 32 (and hence any subset of six) C/A codes are good ranging and multiplexing codes and have the desired properties stated in the previous paragraph;

only the H code remains to be designed. Sec. 2 presents four promising candidates of the H code and their implementations. Based upon a simplified functional diagram of the correlation receiver, Sec. 3 analyzes the effects of interfering codes from other accesses at the correlator output. Computer-simulation results of the generalized correlation properties of these four H codes are briefly described in Sec. 4; further details of these results may be found in Appendices C through E in the form of computer listings. All numbers in the listings are in dB and read from left to right then top to bottom. Section 5 gives some general remarks on H1/H2-type codes. A CC* code (a code succeeded by its conjugate) which was examined (but not recommended) is discussed in Section 6.

2. FOUR RECOMMENDED GPS/TRIDENT CODES AND THEIR IMPLEMENTATIONS

Before proceeding with the details, it is convenient to agree on notation. Capital letter "G, H, X, Y. ---" will usually be used to denote periodic sequences, while lower-case letters "g_i, h_i, x_i, y_i, ---"(i = 1, 2, 3, ---) will generally stand for the i^{th} bits of the periodic sequences. To distinquish between a periodic sequence and its cyclic shift, let, for example, $G_1 = G$ be the periodic sequence starting with the first bit of G:

$$G_1 = G = g_1 \; g_2 \; g_3 \; - - - \qquad (1)$$

then G_{m+1} will designate the m-bit cyclic shift of G, starting with the $(m+1)^{th}$ bit of G_1:

$$G_{1+m} = g_{1+m} \;\; g_{2+m} \;\; g_{3+m} \;\; - - - \qquad (2)$$

Further, unless otherwise specified, the following conventions will be used throughout this report:

X, Y = the two psuedo-noise (PN) sequences that generate the 32 GPS C/A codes

$G(k_j)$ = the j^{th} GPS C/A code ($j = 1, 2, ---, 32$)

H = the GPS/TRIDENT code

and for convenience of discussion, the four recommended GPS/TRIDENT codes will be denoted by H1, H2, H3, and H4.

Figure 2-5 depict the implementations of the four GPS/TRIDENT codes with their parameters of interest shown in Table 1. Explicity,

$$H1 = X_1 \; X_2 \; X_3 - - - X_{32} \qquad (3)$$

$$H2 = X_1 \; X_2 \; X_3 - - - X_{64} \qquad (4)$$

are sequences of cyclic shifts of X, and H3 and H4 are shortened (or short-cyled) PN sequences:

H3 = a periodic sequence having period equal to the first 32 x 1023 bits of a PN sequence of period $2^{15} - 1$ (= 32 x 1023 +31) bits

H3 = a periodic sequence having period equal to the first 64 x 1023 bits of a PN sequence of period $2^{16} - 1$ (= 64 x 1023 +63) bits

To further manifest the short-cycling operation, Figures 6 and 7 illustrate the state diagrams of the H3 and H4 generators; the initial state (or Sate 1) is defined as the state when the contents of the sequence generator are "all ones".

Because both the 32 and 64 m sec clock pulses (which control

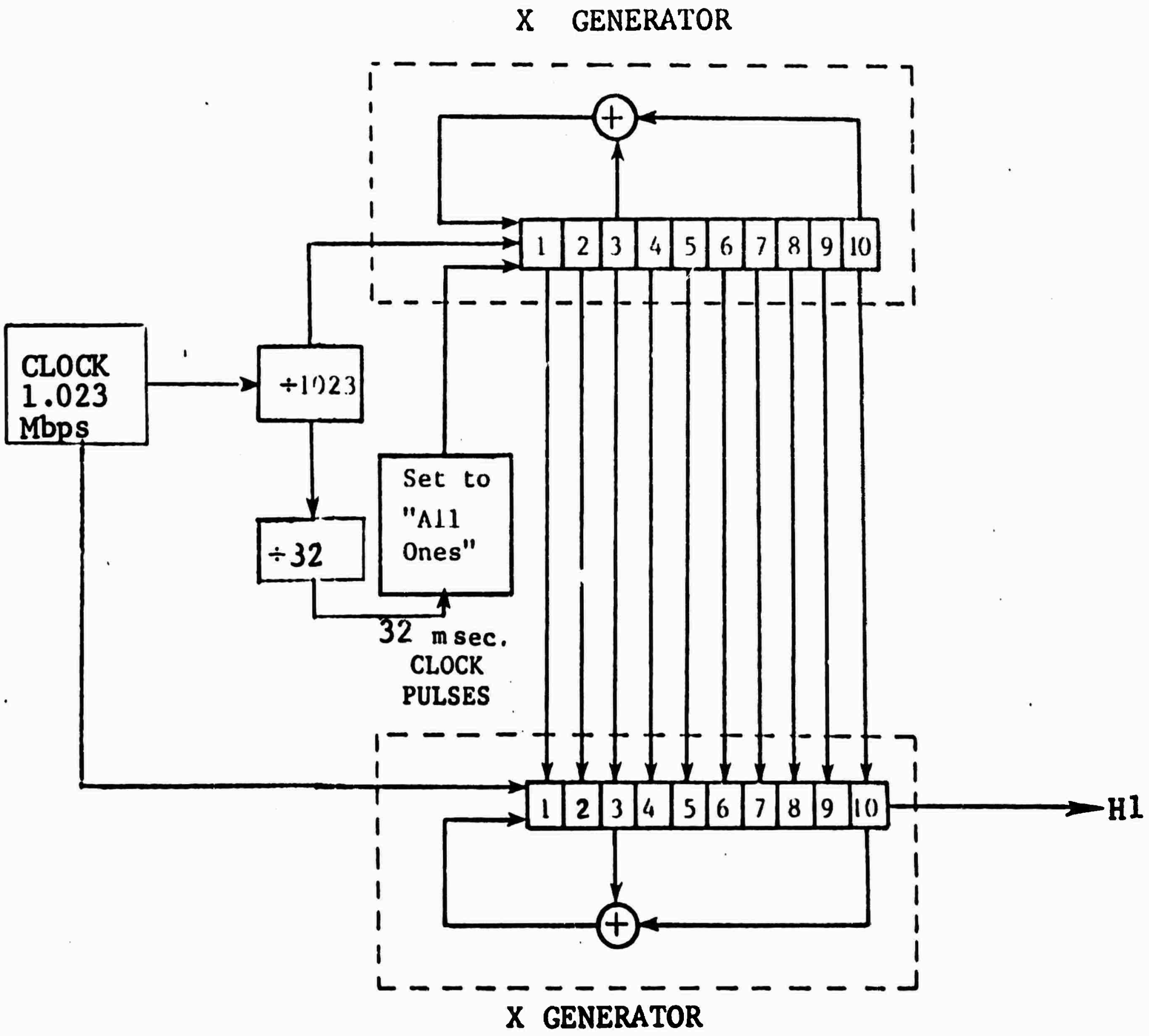

FIGURE 2

H1 CODE GENERATOR

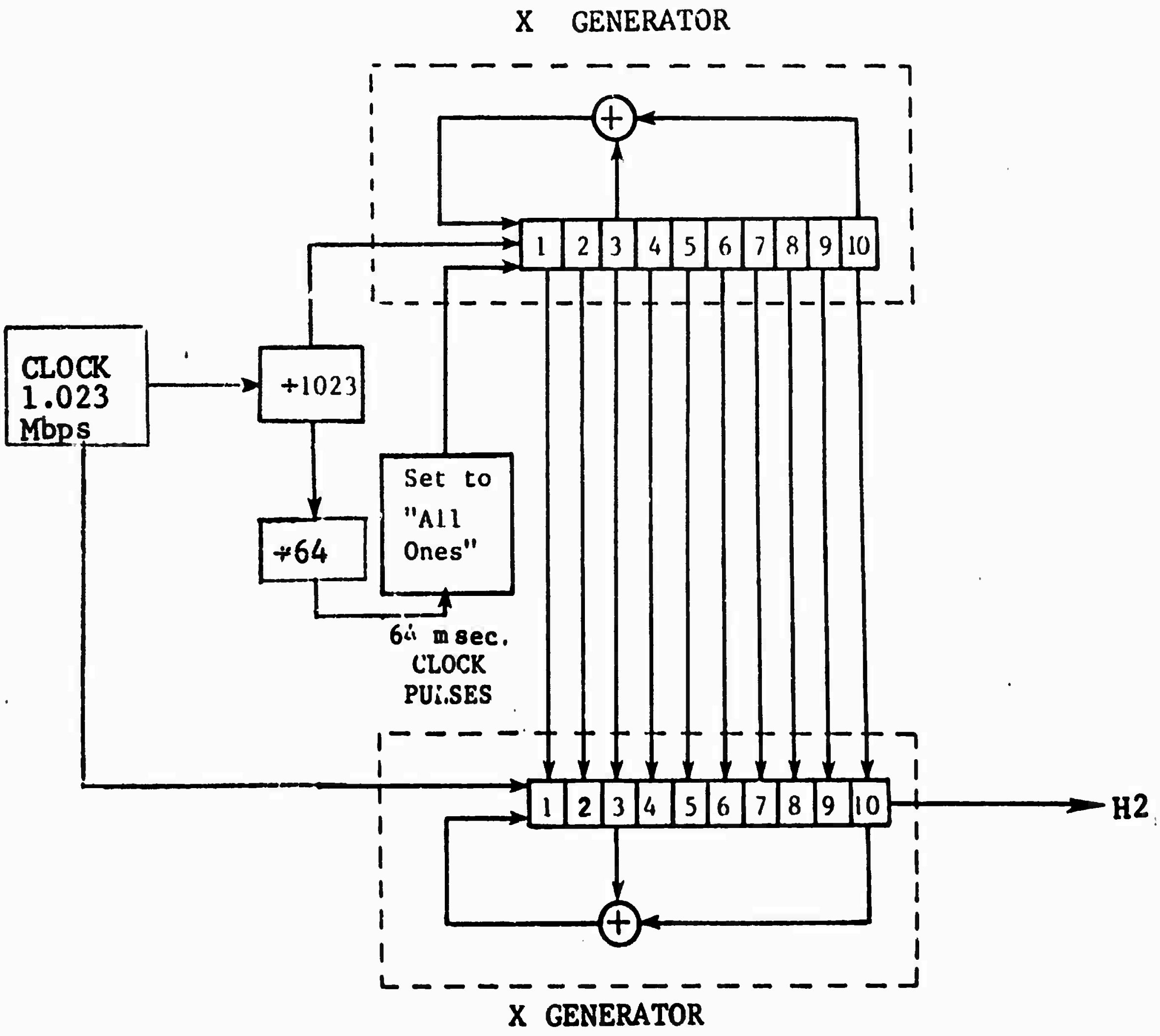

FIGURE 3

H2 CODE GENERATOR

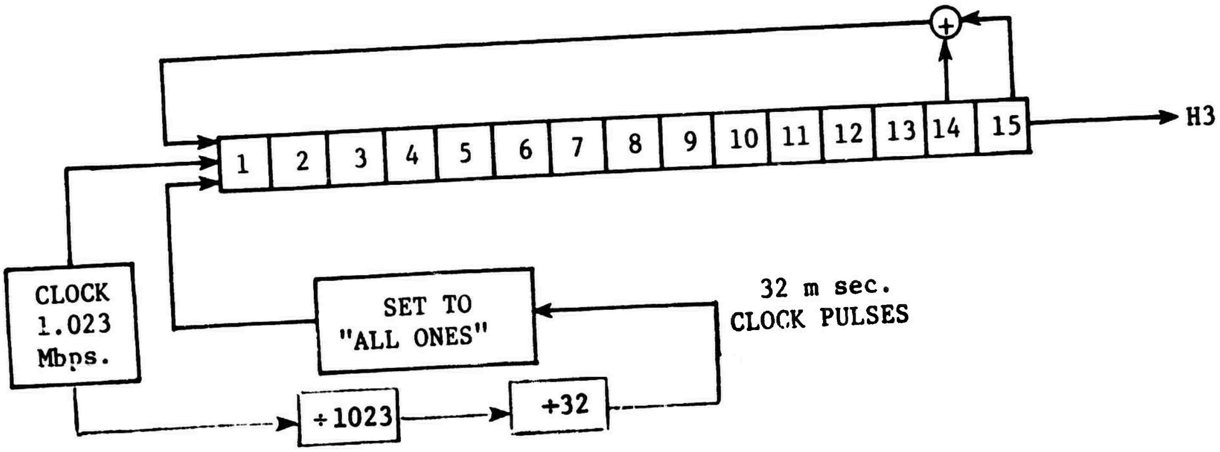

FIGURE 4 H3 CODE GENERATOR

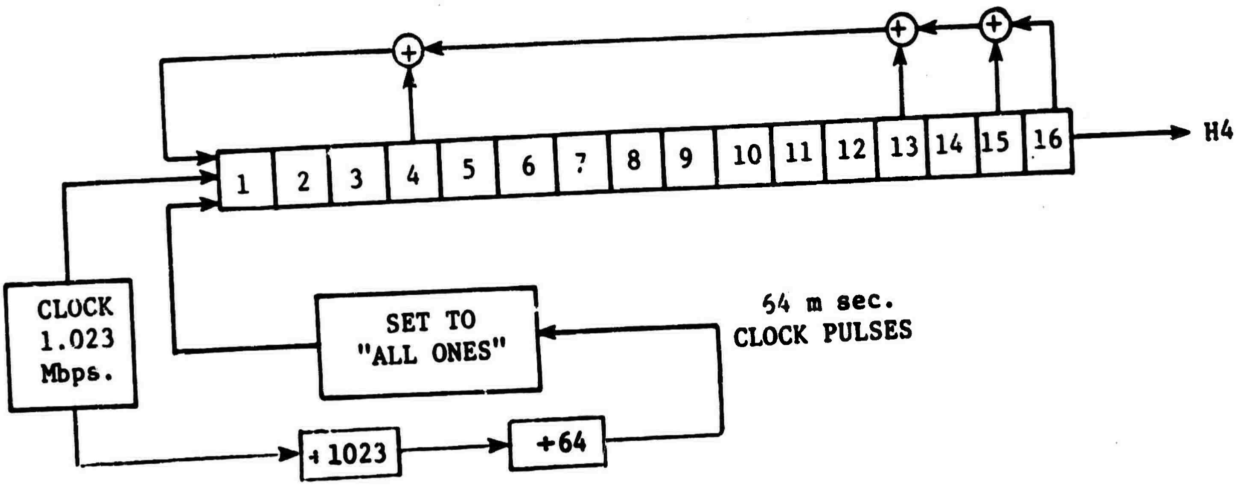

FIGURE 5 H4 CODE GENERATOR

H Codes	Period, Bits	Chip Rate	No. of Zeros in One Period	No. of Ones in One Period	First 30 Bits (From Left to Right Then Top to Bottom)
H 1	32x1023	1.023 Mbps	16,352	16,384	1 1 1 1 1 1 1 1 1 1 0 0 0 1 1 1 0 0 0 1 0 0 1 1 1 0 1 1 0 0
H 2	64x1023	1.023 Mbps	32,704	32,768	1 1 1 1 1 1 1 1 1 1 0 0 0 1 1 1 0 0 0 1 0 0 1 1 1 0 1 1 0 0
H 3	32x1023	1.023 Mbps	16,360	16,376	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
H 4	64x1023	1.023 Mbps	32,736	32,736	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 1 0

TABLE 1 PARAMETERS OF THE FOUR RECOMMENDED GPS/TRIDENT CODES

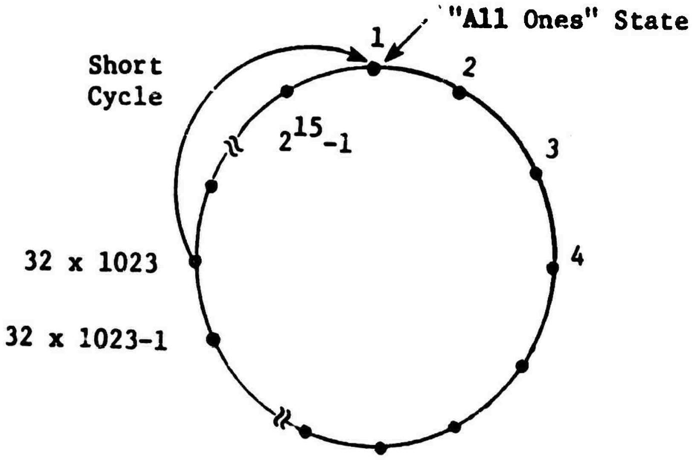

Figure 6 State Diagram of the H3 Generator

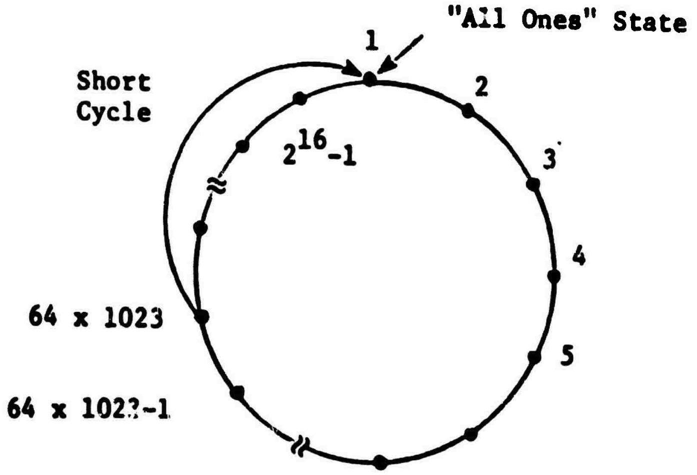

Figure 7 State Diagram of the H4 Generator

the periods or short-cyclings of the H codes by initializing the states of the code generators at every 32 or 64 m sec) can be easily derived from the 1.023 Mbps chip rate pulses as shown in Figures 2-5, they do not constitute a separate frequency-standard problem. Note that the periods of all four recommended H codes are chosen to be equal to integral multiples of the period (=1023 bits) of the GPS C/A codes, thereby, to a certain degree, simplifying the subsequent computation of the cross-correlation between the H codes and the GPS C/A codes. Finally, it is worth mentioning that if the 32 and 64 m sec clock pulses in Figures 4 and 5 are removed, then the H3 and H4 generators will become the simplest (in the sense that they have the minimum number of feedback taps) generators capable of implementing PN sequences of periods $2^{15}-1$ and $2^{16}-1$ bits, respectively.

3.0 Correlation Receiver Analysis

According to the discussions presented in Section 1, the H code is desired to have very small interference with any one of 32 GPS C/A codes so that false code phase lock may be avoided during acquisition. To further understand the effect of such an interference, consider Figure 8, a simplified functional diagram of the DRSS receiver intended to detect the H code. Let the interfering signal $C_j(t)$ be only one of the six C/A signals and of the same power as the desired signal D(t). If for simplicity of analysis the absolute time and code-phase references are neglected (which will not change the result we are going to derive), then the input to the receiver can be simply written as

$$
\begin{aligned}
S(t) &= D(t) + C_j(t) \\
&= H_1 \cos 2\pi f_H t + G_{1+m}(k_j) \cos 2\pi (f_H + f_d)t
\end{aligned}
\qquad (5)
$$

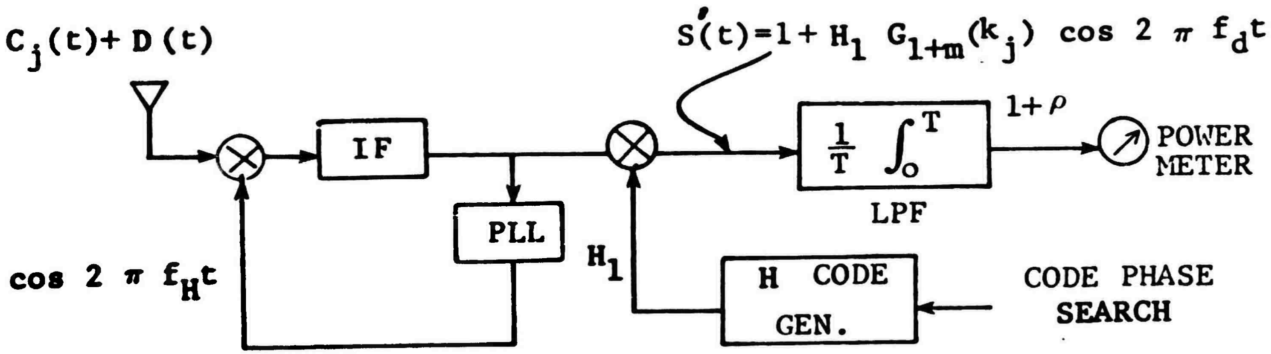

FIGURE 8

SIMPLIFIED FUNCTIONAL DIAGRAM OF THE RECEIVER CHANNEL TO DETECT H CODE

where

f_H = carrier frequency of D(t)

m = the relative code phase between the H code carried by D(t) and the j^{th} C/A code G (k_j) carried by $C_j(t)$

f_d = the differential Doppler shift or the carrier-frequency difference between D(t) and $C_j(t)$

Note that m and f_d are caused by the fact that both sources have different distances and radial velocities with respect to the TRIDENT, and not by coherent retransmission from the TRIDENT to the DRSS. As sources are moving with time, m and f_d will also change from one instant to another.

When both the phase-locked loop (PLL) and the local H code generator are locked onto D(t), the input to the finite-time integrator becomes

$$S(t) = 1 + \underbrace{H_1 G_{1+m}(k_j) \cos 2\pi f_d t}_{\delta} \tag{6}$$

Because the (0 to T) finite-time integrator can normally be approximated as a low-pass filter of bandwidth $\frac{1}{T}$, the contribution of the interference term δ in (6) to the power-meter readout can hence be expressed as

$$\rho \approx \frac{1}{T} \left| F_{GH}(k_j, m, f_d) \right| \tag{7}$$

where

$$\frac{1}{T} F_{GH}(k_j, f) = \frac{1}{T} \mathcal{F} \left\{ H_1 G_{1+m}(k_j) \right\} \tag{8}$$

is defined as the cross-spectrum between H_1 and $G_{1+m}(k_j)$, m being the relative code phase between H and G (k_j).

As evident from (7), as m and f_d are changing with time, the requirement of small interference level ρ at the correlator output is equivalent to the requirement of small cross-spectrum between H_1 and $G_{1+m}(K_j)$ for all possible relative code phases

$$m = 0, 1, 2, ----, 1022 \text{ bits}$$

and all possible differential Doppler offset f_d (ranging from 0 to 5 kHz in most cases of practical interest). (As will be seen later in Sections 4.2 and 4.3, all four recommended H codes appear to have $\rho \leq -30$ dB required.)

4.0 GENERALIZED CORRELATION PROPERTIES OF GPS/TRIDENT CODES

4.1 Autocorrelation Properties

Define the autocorrelation function of the H code as

$$A(m) = \frac{1}{p} \sum_{i=1}^{p} h_i h_{i+m} \tag{9}$$

where

p = the period of the H code, bits
h_i = the i^{th} bit of the H code = 1 or -1
h_{i+m} = the $(i + m)^{th}$ bit of the H code = 1 or -1
m = the relative code phase, bits

then it is desired that (Section 1)

$$A(m) \ll A(0) \tag{10}$$

for any $m \neq 0$ (module p). All four recommended H codes have such a desired property as evidenced in Figures 9-12, where A(m) in dB is defined as 20 $\log_{10}$ A(m) and only cases when m = 0 to 1000 bits are shown because the receiver normally will have some knowledge about the position of the TRIDENT, say within accuracy of ±300 km (≈±1000-bit code-phase delay at chip rate 1.023 Mbps) or so.

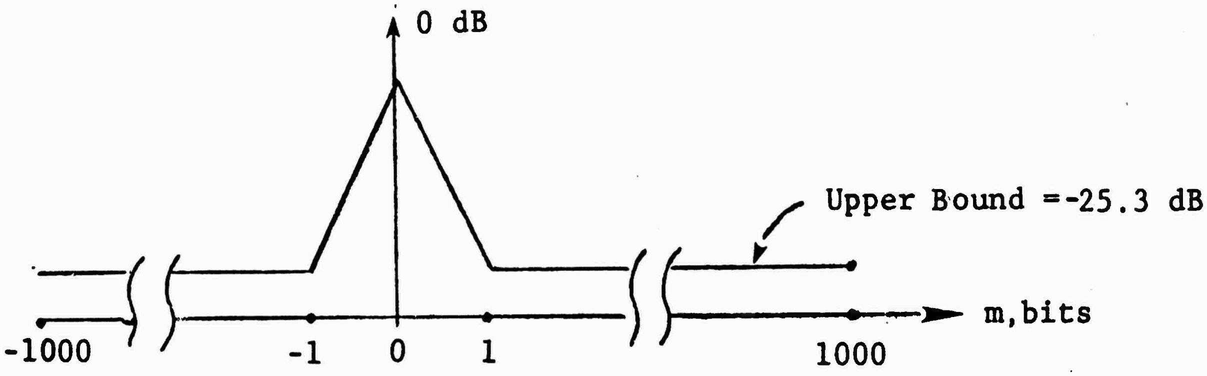

Figure 9 Autocorrelation Function of H1

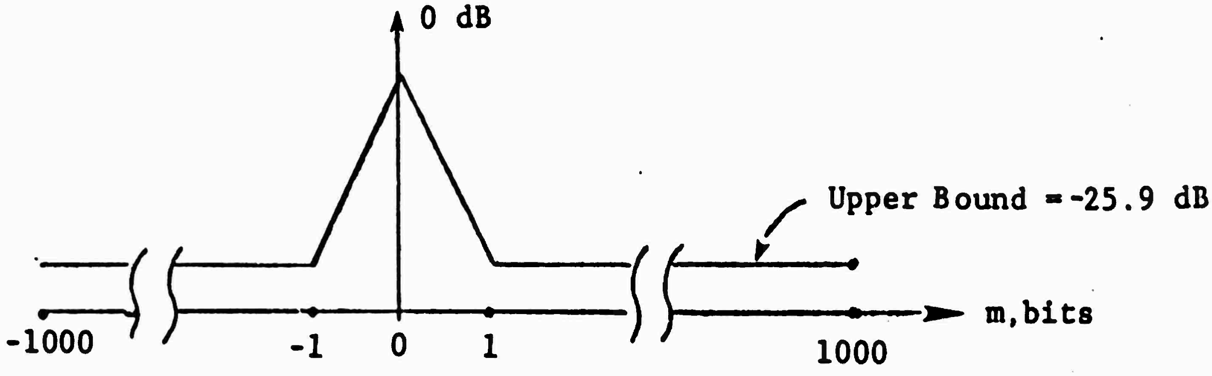

Figure 10 Autocorrelation Function of H2

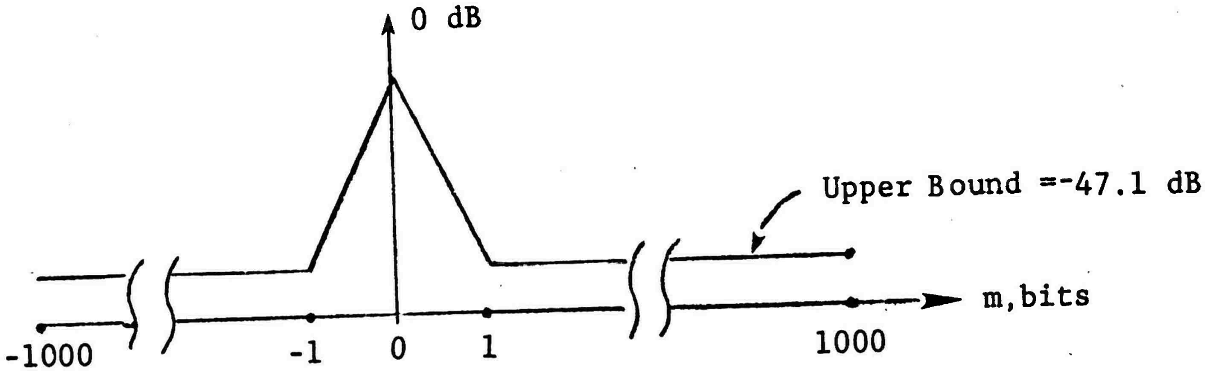

Figure 11 Autocorrelation Function of H 3

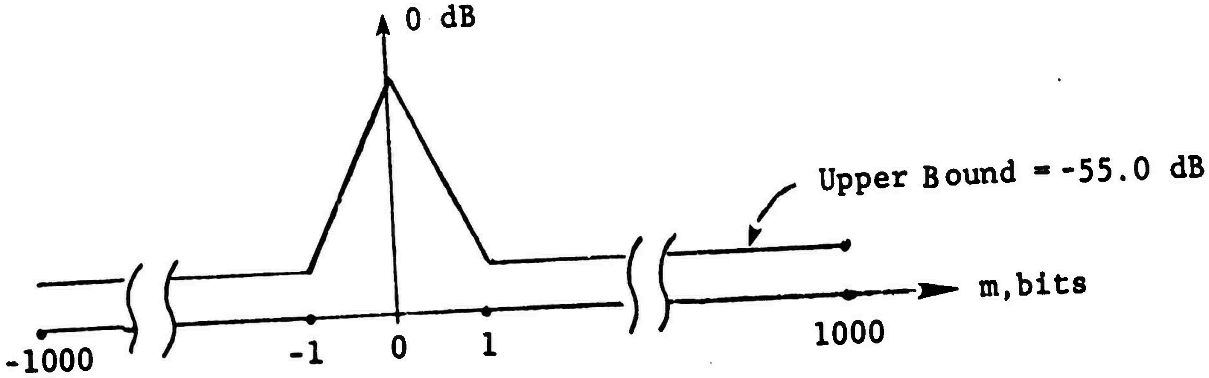

Figure 12 Autocorrelation Function of H 4

Detailed listing of these four autocorrelation functions may be found in Appendix A.

4.2 Cross-Correlation Properties in the Case of Zero Differential Doppler Offset

In the special case when the differential Doppler offset $f_d = 0$, the interference level ρ in (7) becomes the cross-correlation function $C_{GH}(m)$between H_1 and $G_{1+m}(k_j)$

$$\rho = C_{GH}(m) = \frac{1}{p} \sum_{i-1}^{p} h_i \; g_{i+m} \; (k_j) \tag{11}$$

if

$$p = 1023 \times n \tag{12}$$

$$T = n \times \ell \text{ msec} \tag{13}$$

n, ℓ being any positive integers. Computer-simulation study indicates that for H = H1

$$\max \left\{ C_{GH1} \; (m) \right\} \leq -33.8 \text{ dB} \tag{14}$$

for all m = 0, 1, -----, 1022 and all 32 GPS C/A codes, and for H = H2 the corresponding upper bound is given by

$$\max \left\{ C_{GH2}(m) \right\} \leq -37.3 \text{ dB} \tag{15}$$

Further details of these results are shown in Appendix D.

4.3 Cross-Spectra Between GPS/TRIDENT Codes and GPS C/A Codes

As was demonstrated in Section 3, the interference level ρ for $f_d \neq 0$ is equal to the cross-spectrum $\frac{1}{T} \left|F_{GH}(k_j, m, f)\right|$ at $f = f_d$. Although FFT (fast Fourier transformation) subroutine had been employed to reduce the computation time required during the simulation study of the cross-spectrum, it still takes about 0.35 min CPU time of IBM 360/67, for each m and j selected. For each H code there are a total of 1023 m's (m = 0, 1, -----, 1022) and 32 j's (j = 1,2,---,32), or a total of 1023 x 32 cross-spectra to be calculated, thereby requiring about

$$0.35 \times 1023 \times 32 \approx 10^4 \text{ min}$$

CPU time to run all possible cases. Because of such a time-consuming nature, only several randomly chosen cases are studied with the results summarized in Table 2. All four recommended H codes appear to have maximum magnitudes of cross-spectra (or maximum interference level ρ) less than about -35 dB. Details of these results, again, may be found in Appendix E. Note that all cross-spectra are line-spectra with frequency difference between adjacent lines equal to 31.25 Hz for H = H1 and H2 or 15.625 Hz for H = H2 and H4. To give a better feeling about the characteristics of these spectra, two particular spectra are illustrated in Figures 13 and 14.

Table 2 Maximum Magnitudes of Cross-Spectra Between GPS/TRIDENT Codes and GPS C/A Codes for f = 0 to 5 kHz

H Codes	GPS C/A Codes $G(k_j)$	Relative Code Phase m, bits	Maximum Magnitudes of Cross-Spectra, dB
H1	$G(k_{16})$	108	-34.6
		642	-36.8
	$G(k_{18})$	49	-37.8
		859	-36.9
H2	$G(k_1)$	0	-41.2
		1	-40.8
		2	-40.0
		3	-40.1
		4	-40.3
	$G(k_8)$	167	-40.9
		168	-41.0
		169	-40.5
		498	-40.0
		499	-40.4
		500	-40.6
	$G(k_{16})$	663	-37.3
		755	-41.1
	$G(k_{18})$	793	-37.3
H3	$G(k_1)$	0	-38.0
		1	-35.6
	$G(k_{16})$	108	-38.0
H4	$G(k_1)$	0	-41.4
		1	-41.3
		899	-41.1
		900	-41.7

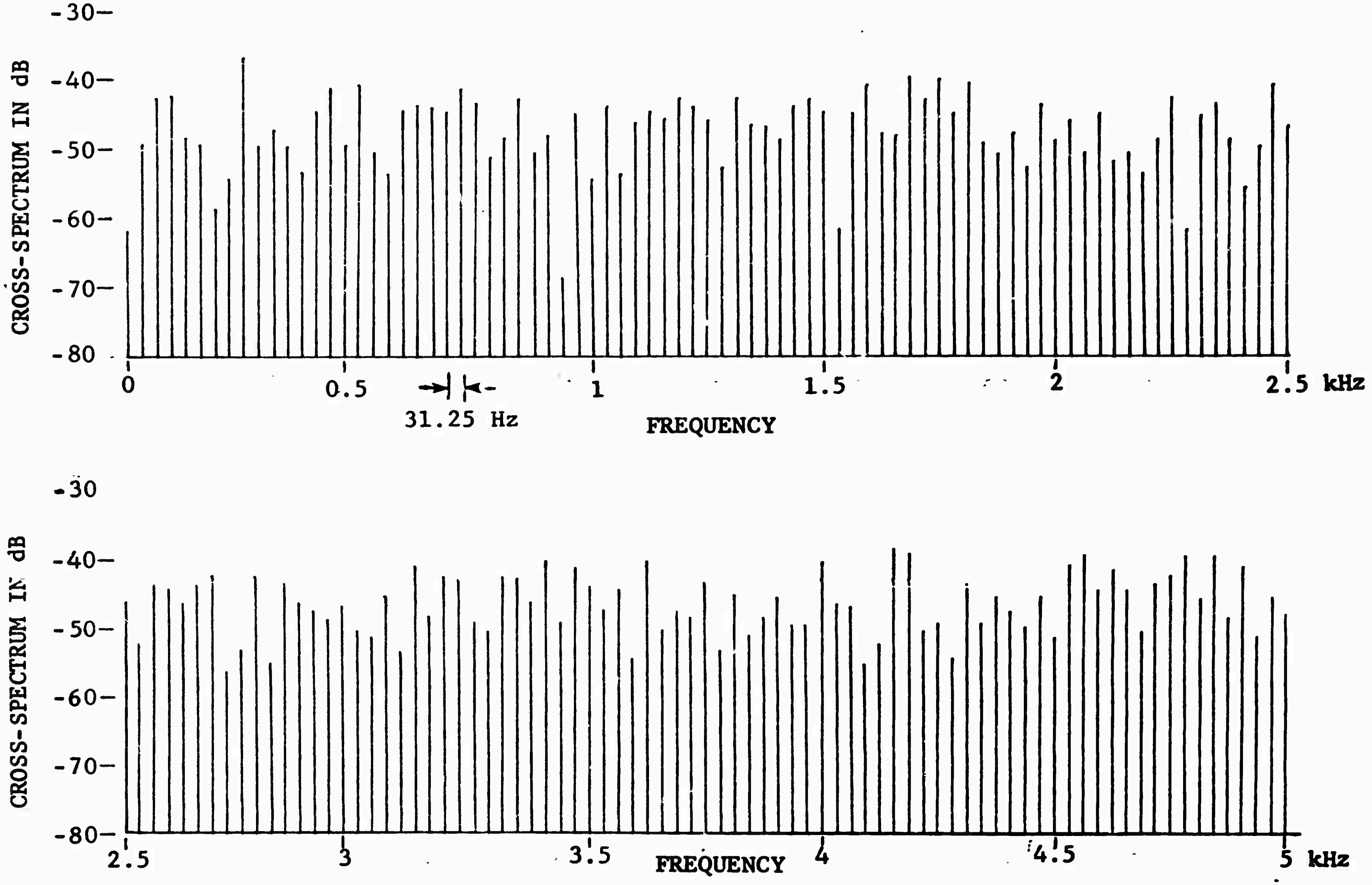

Figure 13 Cross-Spectrum Between H_1 and $G(k_{16})$ For m = 108 bit

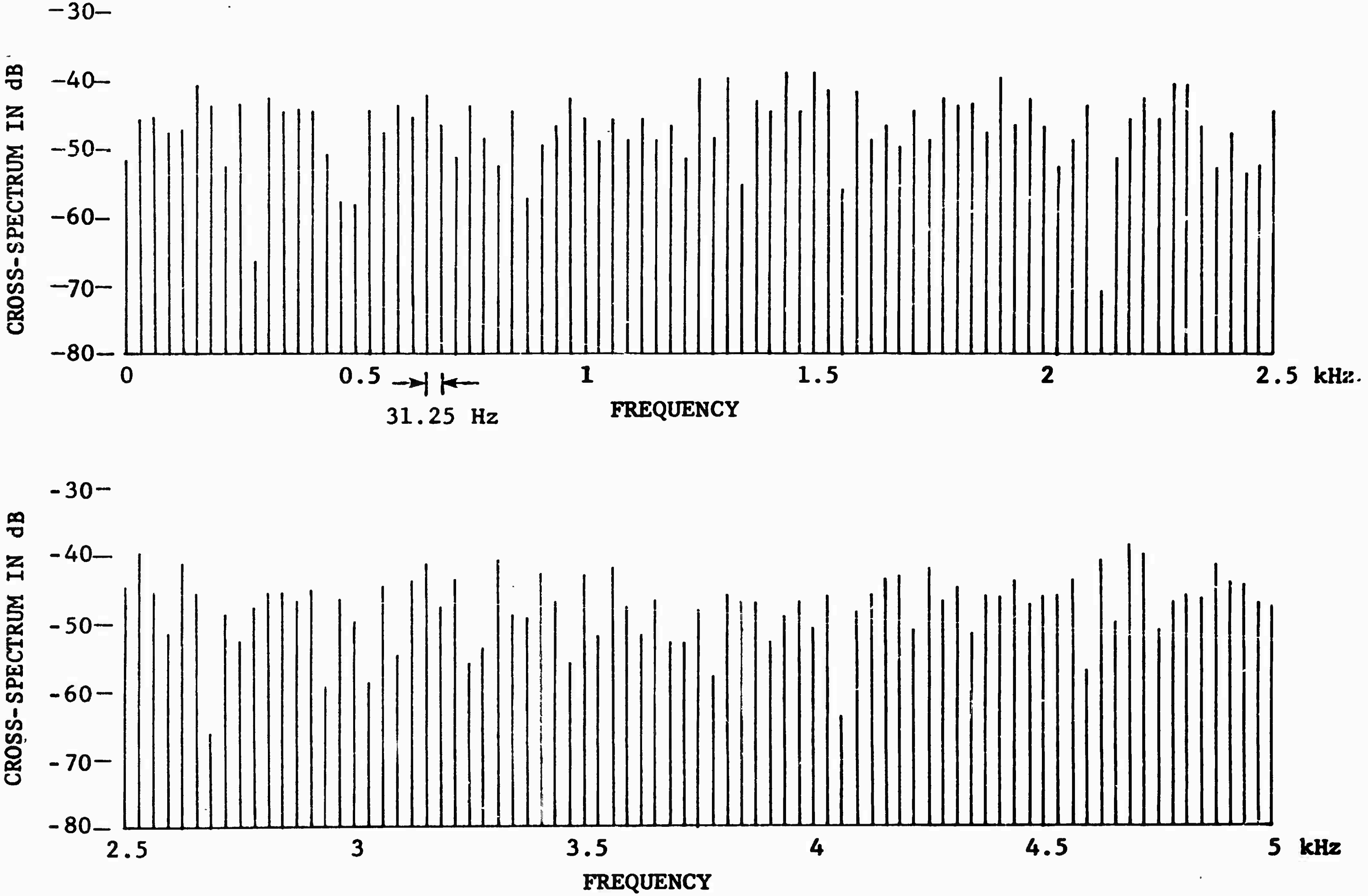

Figure 14 Cross-Spectrum Between H3 and $G(k_1)$ for $m = 0$ bit

5. Some Remarks on H1/H2-Type Codes

An interesting property of the H1/H2-type codes is that the maximum value of the interference level ρ decreases as the code period increases. Stated another way, the H1/H2-type codes have very good partial cross-correlation property. To demonstrate this point, Figure 15 plots ρ_{max} the maximum value of ρ (including cases of any differential Doppler shifts less than ~ 100 kHz) in dB versus the period p of the codes from 1 x 1023 to 512 x 1023 bits Again, because of the excessive amount of computation time required for an exhaustive computer-simulation study, this curve is derived from only limited computer runs. Based upon the statistics of these limited computer runs, it is believed, however, that evan an exhaustive study would not yield ρ more than ~ 3 dB higher than what is predicted by Figure 15. A -3.5 dB/octave dotted line is also shown for comparison. As can be seen clearly from this figure, for code periods between 4 x 1023 and 512 x 1023 bits, every increase of the code period by a factor of 2 decreases ρ_{max} by a factor of ~ 3.5 dB

Another important property of the H1/H2-type code which should be pointed out is that its autocorrelation functions A(m) has spikes at code phase shifts $m = n \times 1022$ and $n \times 1022 + p/1023$ bits, where n = 1,2,3,---, and p is the period of the code. As an example, Table 3 lists the spikes of A(m) of H2. Notice that A(m) is symmetrical with respect to $m = \frac{p}{2}$ (= 32 x 1023 for H2). To avoid ambiguities created by these spikes during code phase tracking, it is required that the receiver has prior knowledge of the received code phase within accuracy of ± 1022 bits. Note that a similar (but not the same) ambiguity also exists in the autocorrelation function of a single Gold code of period 1023 bits.

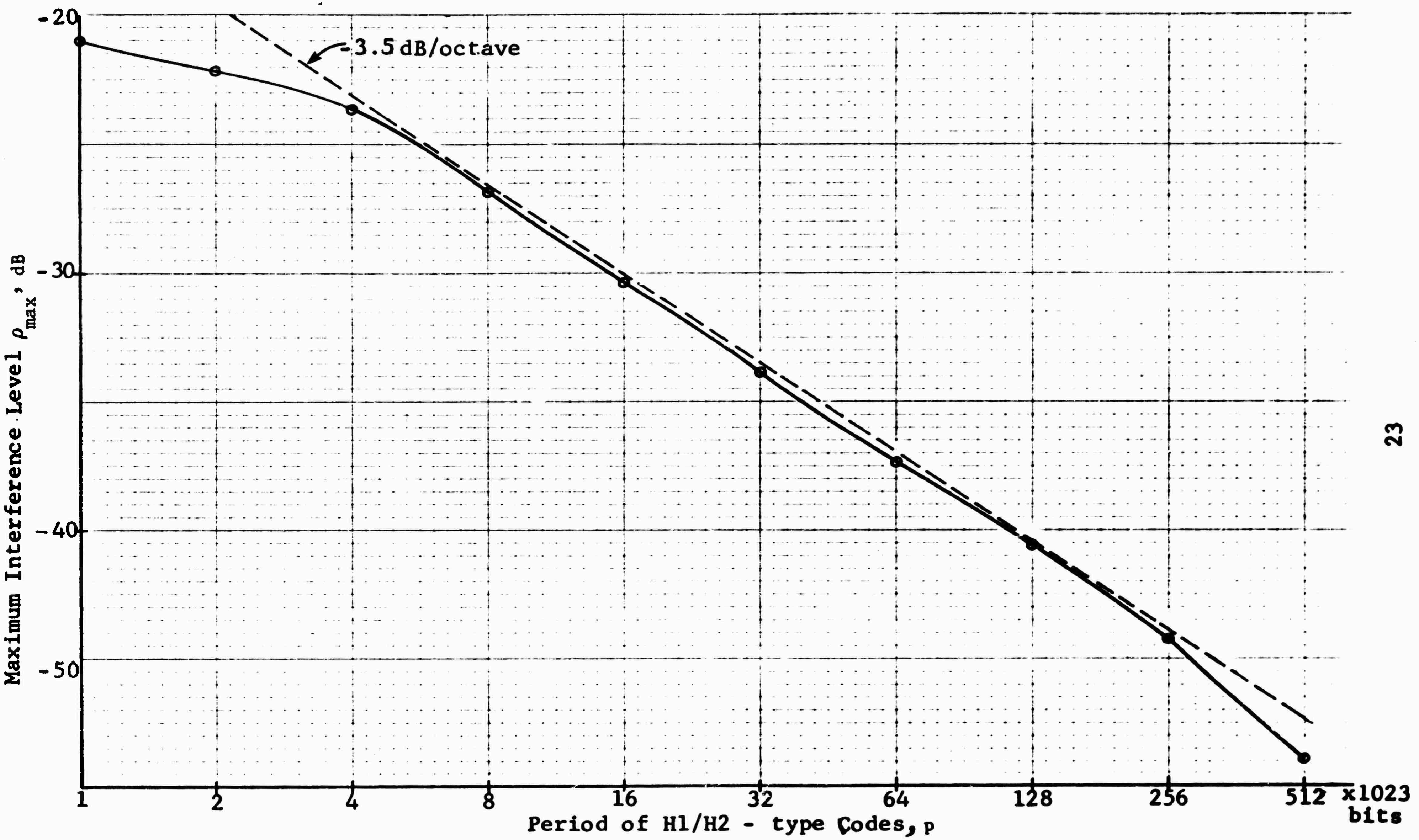

Figure 15 MAXIMUM INTERFERENCE LEVEL ρ_{max} VERSUS PERIOD OF H1/H2-TYPE CODES

Table 3 SPIKES IN THE AUTOCORRELATION FUNCTION A(m) of H2

Relative Code Phase m, bits	Autocorrelation Magnitude $\lvert A(m)\rvert$, dB
1 x 1022	-0.1
1 x 1022 + 64	-32.3
2 x 1022	-0.3
2 x 1022 + 64	-28.3
3 x 1022	-0.4
3 x 1022 + 64	-25.5
⋮	⋮
31 x 1022	-6.0
31 x 1022 + 64	-6.5
32 x 1022	-6.3
32 x 1022 + 64	-6.3
33 x 1022	-6.5
33 x 1022 + 64	-6.0
⋮	⋮
61 x 1022	-25.5
61 x 1022 + 64	-0.4
62 x 1022	-28.3
62 x 1022 + 64	-0.3
63 x 1022	-32.3
63 x 1022 + 64	-0.1

6. A CC* Code

For a code $C = c_1\ c_2\ c_3\ \text{---}\ c_p$ consisting of p bits, define its conjugate $C^* = c_1^*\ c_2^*\ c_3^*\ \text{---} c_p^*$ as follows:

$$c_i^* = \begin{cases} 1 & \text{if } c_i = -1 \\ -1 & \text{if } c_i = 1 \end{cases}$$

where c_i^* and c_i are the i^{th} bits of C^* and C, respectively. Further, let

$$CC^* = c_1\ c_2\ c_3 \text{---} c_p\ c_1^*\ c_2^*\ c_3^*\ \text{---}\ c_p^*$$

be a periodic sequence of period 2p. Then it can be easily proved that CC^* always has zero (or $-\infty$ dB) cross-correlation with any periodic sequence of period p.

In the case of no differential Doppler shift, the interference ρ between CC^* and G (a GPS C/A code), at the output of the coherent correlation receiver, is just the cross-correlation (or the DC term of the cross-spectrum) between CC^* and G, which is $-\infty$ dB for any C having period p = 1023 bits. Therefore, in this particular case, a CC^*-type code would be an ideal choice for the GPS/TRIDENT code. However, in practical cases, there usually exists certain differential Doppler shifts among various SSMA channels such that the magnitude of ρ becomes the magnitude of the spectral line of the cross-spectrum between CC^* and G, at a frequency nearest to the differential Doppler shift. Unfortunately, as one would expect from the structure of the CC^* code, the cross-spectrum between CC^* and G, denoted by F(f), has relatively spiky magnitudes at odd-numbered lines, i.e., at frequencies $f = \frac{1}{2p}, \frac{3}{2p}, \frac{5}{2p}$, --- Hz. (Note that F(f) is a line spectrum with frequency difference between adjacent lines equal to $\frac{1}{2p} = 0.5$ kHz). For example, when $C = G = G(k_1)$ and the relative code phase between CC^* and G is $m = 1$ bit, computer simulation results indicate that the magnitudes of the first few spectral lines of F, in dB, appear as follows:

- ∞,	-28.5,	-78.4,	-22.7,	-72.5,	-31.2,	-68.6,	-33.1 dB,---
↑	↑	↑	↑	↑	↑	↑	↑
0 Hz	0.5kHz	1kHz	1.5kHz	2 kHz	2.5kHz	3kHz	3.5kHz

As a consequence, CC^* is not recommended to be a good candidate of the GPS/TRIDENT code.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This volume deals with the generalized cross-correlation of properties of the GPS Clear signals in the case of differential doppler shifts. It also addresses modified GPS-like codes that were designed for use for the GPS/TRIDENT SATRACK system.

The cross-correlation properties of the GPS C/A in the case of zero doppler shift is well known. This report addresses the case where finite Doppler shifts are involved. → OVER

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The SATRACK system requires a good ranging code which also is a good spectrum-spreading multiple access code that has very small interference or cross-correlation with the GPS codes. This report addresses such codes.