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MODEL Research CONTRACT NO. ________

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PREPARED BY D. L. Emmons 3/5/65

SUPERVISED BY J. A. Brousseau 3/8/65

APPROVED BY J. A. Brousseau 3/8/65

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ABSTRACT & KEY WORD

A theoretical analysis was developed for predicting the flow properties in the mixing region between a particle-laden, turbulent rocket exhaust and a surrounding air stream. It was assumed that the turbulent boundary layer equations, modified to account for particles, were valid within the mixing region. In treating the chemical aspects of the problem it was assumed that the flow was in equilibrium in accordance with three chemical reaction equations. The chemical species comprising the flow were limited to the following: H_2O, H_2, O_2, CO, CO_2, N_2, HCl, Al, Al_2O_3, and one additional inert species which remained as an input selection. The phenomenological model employed for the turbulent transport coefficients is discussed in detail and compared with various other models.

The solution of the partial differential equations was obtained by transforming the equations using the von Mises transformation, expressing the transformed equations in finite-difference form, and solving the resulting equations utilizing a computer program developed for the SRU 1107.

Results from the computer program were successfully compared with experimental results obtained from air-augmentation, free jet, and fuel injection experiments. Output data from the computer program comprises velocity, temperature, density, species concentration, and Mach number profiles at various axial locations along the mixing region.

Gas-Particle Flow
Mixing Analysis
Chemical Reactions
Turbulent Flow
Eddy Transport Coefficients
Air Augmented Rocket
Supersonic Combustion

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NOTATION

A	area, ft^2
b	width of mixing zone, ft.
c	constant in eddy viscosity expression, ft.
C_i	concentration of species i, lbm of i/lbm of mixture
C_p	specific heat at constant pressure, B/lbm-°R
D	molecular diffusion coefficient, ft^2/sec
e	internal energy, B/lbm
g	gravitational constant, 32.17 lbm-ft/lbf-sec^2
h	static enthalpy, B/lbm
Δh_f°	heat of formation, B/lbm
J	mechanical equivalent of heat, 778 ft-lb/B
K_p	equilibrium constant
k_f	forward reaction rate constant, ft^3/lb-mole sec
L	reference length, ft.
l	Prandtl's mixing length parameter, ft.
Le	turbulent Lewis number
$\dot{m}$	mass flux, lbm/ft^2-sec
m_p	particle density, lbm of P/ft^3 of
M	Mach number
n	exponent in eddy viscosity expression
P	static pressure, psfa
Pr	turbulent Prandtl number
R_u	universal gas constant, 1544 ft-lb/lb-mole °R

r	radius, ft.
Sc	turbulent Schmidt number
T	static temperature, °R
u	axial velocity component, ft/sec
v	radial velocity component, ft/sec
$\dot{W}_i$	net rate of production of species i, lbm/ft^3-sec
W_i	molecular weight of species i, lbm/lb-mole
x	axial distance, ft.
y	radial distance, ft.
δ	exponent for two-dimensional (δ =0) or axisymmetric flow (δ =1)
ϵ	eddy diffusivity, ft^2/sec
ρ_g	gas density, lbm of gas/ft^3 of gas
ρ_p	particle density, lbm of p/ft^3 of gas
σ	similarity parameter
τ	shear stress, lb/ft^2
ψ	stream function, lbm/sec

Subscripts

D	mass diffusivity
g	gas
H	heat diffusivity
i	species
m, n	grid point coordinates
p	particle
V	momentum diffusivity

1.0 INTRODUCTION

1.1 General Remarks

Analytical studies involving the turbulent mixing of multi-component gases are required in a number of flow problems. Examples of such problems are those pertaining to base heating, wakes behind bluff bodies, loss of communication due to plume attenuation, and most recently, air augmentation. Fundamental studies of the mixing process between two moving streams have been conducted over the past several years with varying degrees of success. However, it has been only recently, with the advent of high-speed computers, that significant progress has been achieved in analytically predicting the properties within turbulent mixing regions. An excellent monograph describing the recent theoretical developments pertaining to turbulent jet mixing has been compiled by Abramovich (1)*.

Recent air-augmentation studies indicate that accurate prediction of engine performance requires a detailed study of the mixing region between a rocket exhaust jet and a confined secondary air stream. Since rocket exhaust gases, in general contain excess fuel, not only must the mixing process be analyzed, but also chemical reactions must be taken into consideration. The problem is further complicated by the fact that the entire flow is confined within a duct, thus necessitating the consideration of axial pressure gradients. Analyses of flows similar to those found in this type of problem have recently been developed by Libby (2) and Vasiliu (3). Libby, for example, has analyzed the unconfined mixed flow field for supersonic combustion studies assuming equilibrium chemistry; whereas

*Numbers in parentheses refer to references listed in the Bibliography

Vasiliu assumed nonequilibrium chemistry in his analysis of the mixing between a rocket exhaust jet and a supersonic air stream. Mikhail (4), on the other hand, has treated the mixing of coaxial incompressible streams in a duct but did not consider chemical reactions.

1.2 Scope of Present Analysis

The analysis and associated computer program described herein were developed specifically for the air-augmentation problem. However, it should be noted that the computer program may be employed for other problems. The primary objective of the analysis was to provide a method for calculating the velocity, temperature, and composition profiles in the mixing region between a fuel-rich rocket exhaust and an air stream confined within a duct of specified geometry. Since the exhaust of solid propellant rockets often contains solid and/or liquid particles, the analysis was developed for either gas-particle systems, or systems consisting solely of gaseous species. In treating the chemical aspects of the problem it is assumed that the flow is in chemical equilibrium in accordance with three chemical reaction equations; or, if desired, the flow may be treated as frozen. The total number of chemical components comprising the flow is limited to a maximum of ten species.

The flow model employed for the analysis is presented in Figure 1. At the periphery of the rocket nozzle exit, where the air stream and rocket exhaust first come into contact, a mixing region is assumed to begin. As the flow moves downstream, the mixing region widens on both sides of the plume slipline - the inner boundary approaching the duct centerline and the outer boundary the duct wall. At some distance downstream from the nozzle

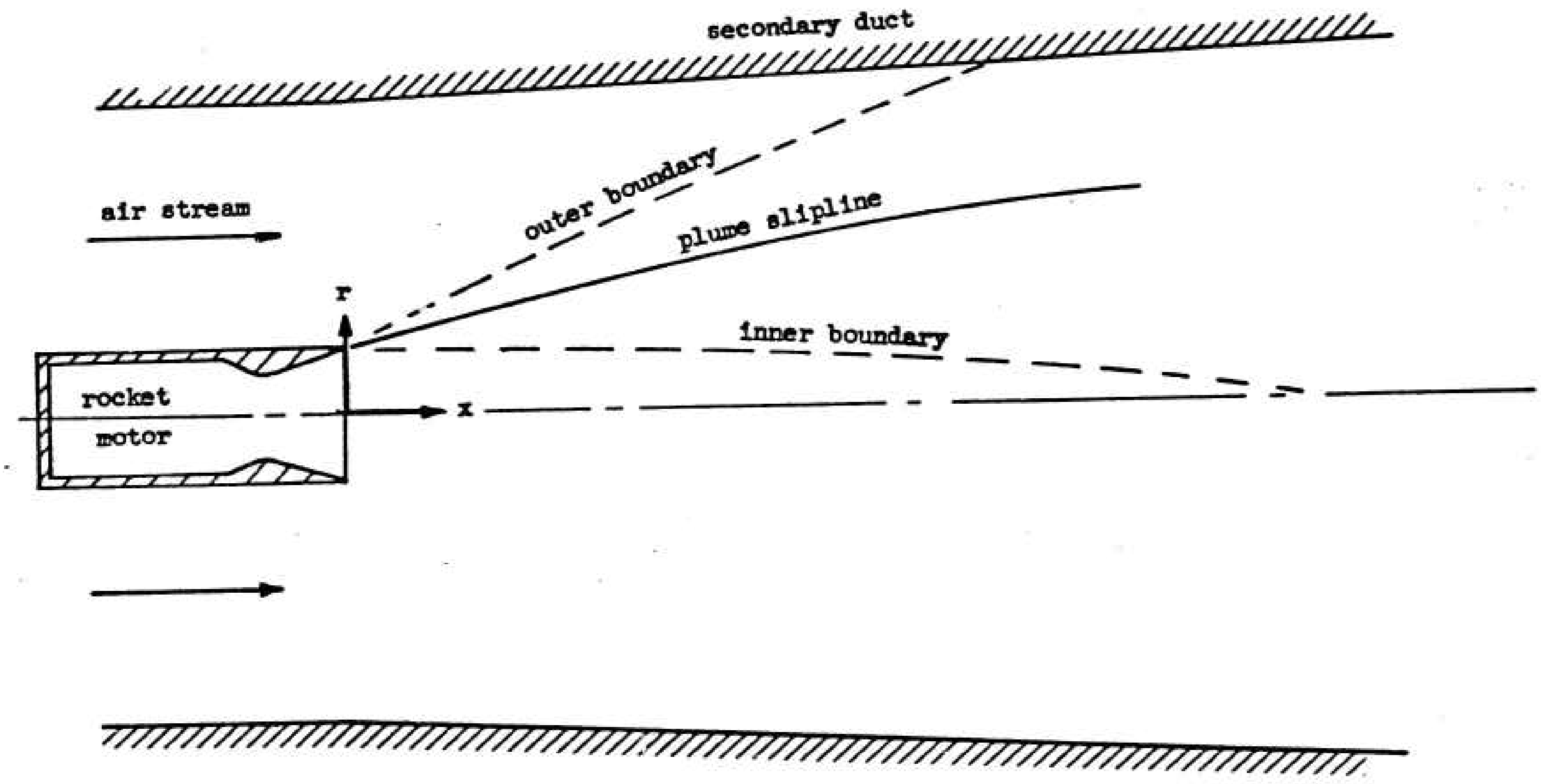

Figure 1 - Two Dimensional Mixing Flow Model

the mixing region may be considered to extend across the entire duct. In establishing the coordinates of the plume slipline the method of characteristics solution is employed (5). In the areas outside the mixing region, the flow is considered to be inviscid with uniform properties in the radial direction. Within the mixing region turbulent transport processes are assumed to prevail with all molecular transfer considered negligible. This assumption is justified for the problem under consideration since the flow is generally turbulent, and the turbulent transport coefficients are usually at least an order of magnitude greater than their molecular counterparts. The phenomenological model employed for the turbulent transport coefficients in the present study is discussed in detail and compared with various other models.

The major assumptions employed in the development of the analysis are as follows:

1. The flow is either axisymmetric or two-dimensional.
2. The entire flow field is turbulent.
3. The gas obeys the perfect gas law.
4. The radial pressure gradient across the mixing zone is negligible.
5. Mass transfer due to thermal and pressure gradients is negligible.
6. The turbulent Prandtl and Lewis numbers of the gas are constant but may have values other than unity.
7. The particles may have a turbulent Lewis number different than that of the gas.
8. The velocity and temperature lags between the gas and particles are negligible.

9. The volume occupied by the particles is negligible compared to that of the gas.
10. The boundary layer between the mixed flow and the duct wall is neglected. The boundary layers in the rocket exhaust and the air stream at the initial point of contact, however, may be considered.
11. The turbulent boundary layer equations are valid within the mixing region.

1.3 Computer Program

The solution of the partial differential equations is obtained by first transforming the equations using the von Mises transformation, expressing the transformed equations in finite-difference form, and then solving the resulting finite-difference equations utilizing a computer program developed for the SRU 1107. Details of the computer program are presented in Reference 6. Consideration is given to the stability and convergence of the solution utilizing the criteria established by Wu (7).

The parameters which must be input to the program are as follows: (a) the initial conditions of velocity, temperature, pressure, and species concentrations of the two streams, (b) axial step-size, (c) duct geometry, (d) plume slip-line coordinates, (e) the turbulent Lewis and Prandtl numbers, and (f) computer control information as noted in Reference 6. The output data comprises velocity, temperature, density, species concentration, and Mach number profiles within the mixing region at various axial locations; axial pressure distributions; and axial thrust acting on the duct.

The major limitations of the program are as follows: (a) the calculations are valid only in the temperature range 450R to 9950R, (b) the gaseous species which may considered are limited to H_2O, CO, CO_2, H_2, O_2, N_2, HCl Al; in addition, provision for one additional inert specie has been incorporated in the program, and (c) the particle specie which may be considered is Al_2O_3.

A

Typical run time of the program is 10 minutes for a case involving a rocket nozzle radius of 0.21 ft, a duct radius of 0.56 ft, axial step-size of 0.01 ft, and a total axial distance of 6.0 ft.

2.0 DERIVATION OF THE GOVERNING EQUATIONS

2.1 Fundamental Equations

2.1.1 General

The derivation of the fundamental equations contained herein parallels the derivation of the steady-state turbulent boundary layer equations for gaseous systems. The effects of particles are included with the assumption that the particles and gas are in dynamic and thermal equilibrium, i.e., no velocity or temperature lags. The particles, therefore, are not treated as discrete droplets, but as chemical species which have properties corresponding to their respective liquid or solid phases. In determining the mass transfer of the particles resulting from turbulent mixing, it is assumed that the particles do not follow the random motions of the gas, but due to their inertia diffuse more slowly than the gas. The derivations that follow pertain to a two-dimensional flow model and later are expanded to include an axisymmetric flow system.

2.1.2 Global Continuity Equation

Consider a rectangular control volume located within the mixing region as shown in Figure 2. It is assumed that variations in the z-direction are negligible. The flow entering the elemental volume in the x-direction consists of a gas-particle mixture with an average axial velocity u. All the flow properties are assumed to be time-averaged values consistent with turbulent theory (8). The total mass flow entering the volume in the axial direction is

$$\rho_g u A_g + m_p u A_p$$

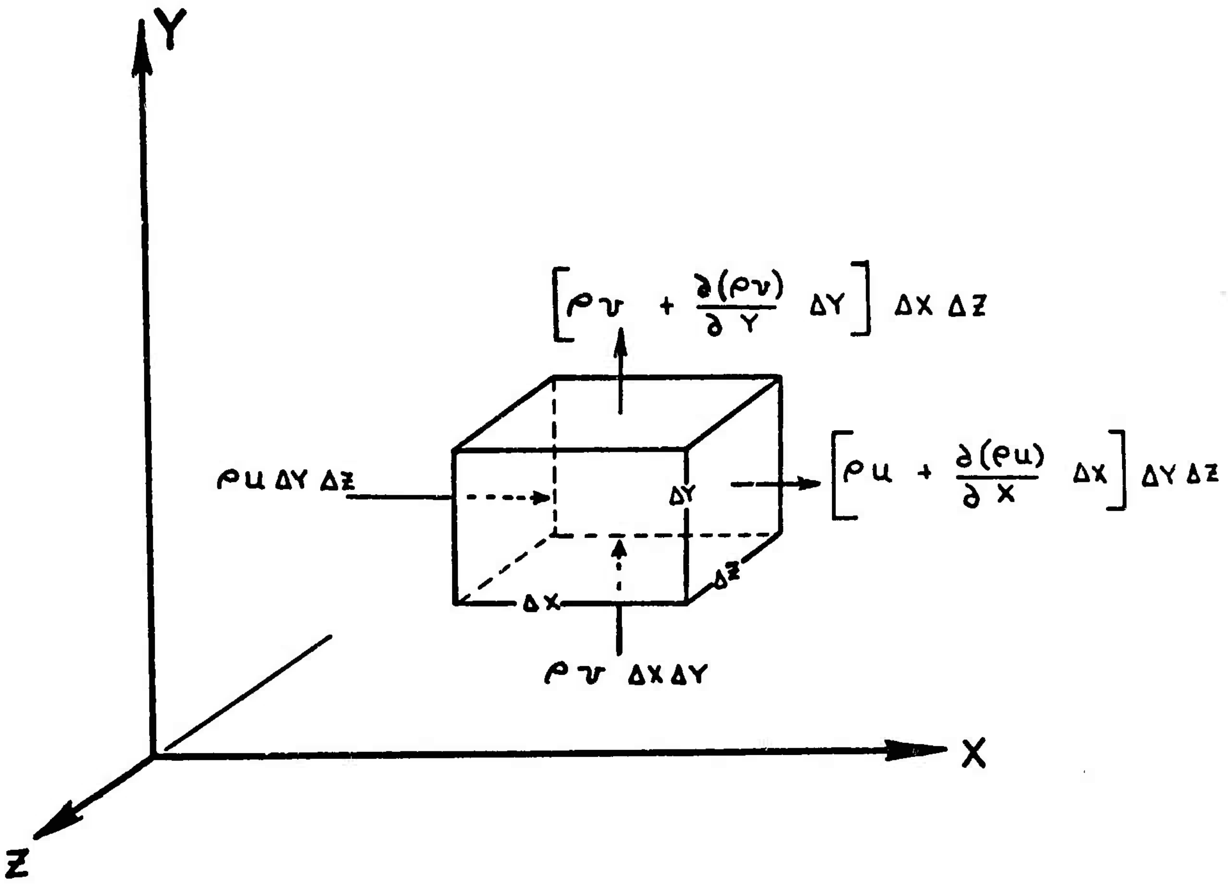

FIG. 2 Control Volume for Gas-Particle Mixtures

where ρ_g and m_p are the densities of the gas and particles, respectively, and A_g is the flow area occupied by the gas and A_p that by the particles. Then

$$A_g + A_p = \Delta y \Delta z$$

Assuming $A_g >> A_p$ and letting ρ_p equal the weight of particles per unit volume of gas, yields

$$(\rho_g + \rho_p) u \Delta y \Delta z = \rho u \Delta y \Delta z$$

for the total axial mass flow into the volume where

$$\rho = \rho_g + \rho_p$$

The axial mass flow leaving the volume is given by

$$\rho u \Delta y \Delta z + \frac{\partial}{\partial x}(\rho u \Delta y \Delta z) \Delta x$$

The net change in axial mass flow is

$$\frac{\partial}{\partial x}(\rho u) \Delta x \Delta y \Delta z$$

Similarly in the y-direction the net change is

$$\frac{\partial}{\partial y}(\rho v) \Delta x \Delta y \Delta z$$

Conservation of mass yields

$$\boxed{\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0} \qquad (1)$$

for the steady-state global continuity equation.

2.1.3 Species Continuity Equations

The species continuity equations are derived from mass balances for each of the chemical species. The mass balance for component "i" of the mixture flowing through the control volume consists of terms involving (a) the net mass change due to the flow entering and leaving the volume and (b) production (or depletion) of the species as a result of chemical reactions. The net transfer of component i in the x-direction due to the fluid motion is

$$-\frac{\partial}{\partial x}(\rho c_i u_i)\,\Delta x \Delta y \Delta z$$

where C_i is the mass fraction of component i (mass per unit mass of mixture) and U_i is its mean axial velocity. Similarly in the y-direction the net transfer is

$$-\frac{\partial}{\partial y}(\rho c_i v_i)\,\Delta x \Delta y \Delta z$$

The net rate of production of component i is given by

$$\dot{\omega}_i\,\Delta x \Delta y \Delta z$$

A mass balance for component i yields

$$\frac{\partial}{\partial x}(\rho c_i u_i) + \frac{\partial}{\partial y}(\rho c_i v_i) = \dot{\omega}_i \tag{2}$$

According to theory (9) the term $\rho c_i u_i$ may be expressed in terms of diffusion and convective fluxes. Thus

$$\rho c_i u_i = \dot{m}_{ix} + \rho c_i u \tag{3}$$

where m_{ix} is the diffusion flux resulting from concentration gradients and $\rho c_i u$ is the convective flux due to the bulk motion of the fluid. A By bulk motion is meant the average macroscopic motion of the flow in which the axial bulk velocity u, is defined by

$$u \equiv \sum_{i=1}^{n} c_i u_i \tag{4}$$

where h is the total number of species comprising the mixture. For molecular diffusion the flux $\dot{m}_{ix}$, may be expressed as

$$\dot{m}_{ix} = -\rho D_i \frac{\partial c_i}{\partial x} \tag{5}$$

where D_i is the molecular diffusion coefficient for component i. For turbulent diffusion it has been shown theoretically (9) that a similar expression is valid with the molecular diffusivity replaced by its turbulent counterpart. Thus,

$$\dot{m}_{ix} = -\rho \epsilon_{Di} \frac{\partial c_i}{\partial x} \tag{6}$$

where ϵ_{Di} is the eddy mass diffusivity of component i. Similarly in the y-direction

$$\dot{m}_{iy} = -\rho \epsilon_{Di} \frac{\partial c_i}{\partial y} \tag{7}$$

Substitution of equations 3, 6, and 7 into equation 2 yields

$$\frac{\partial (\rho c_i u)}{\partial x} + \frac{\partial (\rho c_i v)}{\partial y} = \frac{\partial \left(\rho \epsilon_{Di} \frac{\partial c_i}{\partial x}\right)}{\partial x} + \frac{\partial \left(\rho \epsilon_{Di} \frac{\partial c_i}{\partial y}\right)}{\partial y} + \dot{\omega}_i$$

Expanding the above equation and substituting the global continuity equation yields

$$\rho u \frac{\partial c_i}{\partial x} + \rho v \frac{\partial c_i}{\partial y} = \frac{\partial \left(\rho \epsilon_{Di} \frac{\partial c_i}{\partial x}\right)}{\partial x} + \frac{\partial \left(\rho \epsilon_{Di} \frac{\partial c_i}{\partial y}\right)}{\partial y} + \dot{\omega}_i$$

It is generally assumed, based on an order of magnitude analysis, that the first term on the right side of the above equation is negligible. Thus the species continuity equation for component i becomes

$$\boxed{\rho u \frac{\partial c_i}{\partial x} + \rho v \frac{\partial c_i}{\partial y} = \frac{\partial \left(\rho \epsilon_{Di} \frac{\partial c_i}{\partial y}\right)}{\partial y} + \dot{\omega}_i} \tag{8}$$

The net rates of production of the species are determined from a consideration of the chemical aspects of the problem. A detailed discussion of the determination of $\dot{w}_i$ is presented in Section 4 herein.

The species continuity equation developed above is generally assumed valid for gaseous species, with the eddy mass diffusivities taken to be identical for all species. However, for particles further consideration must be given to the turbulent transport mechanism. The turbulent motions within a gas are usually conceived as macroscopic "lumps or eddies" of fluid moving randomly from one region to another. For gas-particle systems it seems unlikely that the particles, due to their inertia, are capable of following the random fluctuations of the gas. Consequently, the net transfer of particles from one region to another is likely to be considerably less than that of the gas. This difference in the turbulent transport of gas and particles may, in theory, be accomplished by employing different values for the eddy mass diffusivities of the two components. Thus, in the present analysis two values of the eddy mass diffusivity are employed - one for the gas and the other for the particles. A detailed discussion of the numerical values to be employed for the eddy mass diffusivities is presented in Section 3 herein.

2.1.4 Momentum Equations

The axial momentum equation is derived by equating the net change in axial momentum of the fluid flowing through the control volume to the net axial forces acting on the control boundaries. Thus

$$\frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho u^2)}{\partial y} = -\frac{\partial p}{\partial x} - \frac{\partial \tau_{xy}}{\partial y} \tag{9}$$

where τ_{xy} is the shear stress which, for turbulent flow, may be expressed in terms of the velocity gradient and the eddy momentum diffusivity. Thus

$$\tau_{xy} = -\rho \epsilon_v \frac{\partial u}{\partial y}$$

Expanding equation 9 and substituting equation 1 yields

$$\boxed{\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}\left(\rho \epsilon_v \frac{\partial u}{\partial y}\right)} \qquad (10)$$

Employing the usual boundary layer approximations and an order of magnitude analysis, the y-direction momentum equation reduces to

$$\boxed{\frac{\partial P}{\partial y} = 0}$$

2.1.5 Energy Equation

The energy equation is derived following the methods of Schlichting (10) and Vasiliu (3). For an observer moving with the flow the First Law of Thermodynamics may be stated as follows:

$$\begin{bmatrix}\text{rate of increase}\\ \text{of internal energy}\end{bmatrix} = \begin{bmatrix}\text{rate of heat added}\\ \text{to the system}\end{bmatrix} + \begin{bmatrix}\text{work done by the system}\\ \text{boundaries on the}\\ \text{surroundings}\end{bmatrix}$$

The net change in internal energy of the gas-particle fluid flowing through the control volume may be expressed as

$$dE = \left(\rho u \frac{\partial e}{\partial x} + \rho v \frac{\partial e}{\partial y}\right) \Delta x \Delta y \Delta z \qquad (11)$$

or

$$dE = \rho \frac{De}{Dt} \Delta x \Delta y \Delta z \qquad (12)$$

where

$$\frac{D}{Dt} \equiv u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \tag{13}$$

is the substantial derivative for steady-state and e is the internal energy.

From the definition of enthalpy, $h = e + P/\rho$, the substantive derivative of the internal energy may be written as

$$\frac{De}{Dt} = \frac{Dh}{Dt} - \frac{D}{Dt}(P/\rho) \tag{14}$$

where the enthalpy of the mixture is given by

$$h = \sum_{i=1}^{n} C_i h_i = \sum_{i=1}^{n} C_i \left(\int_0^T c_{p_i} dT + \Delta h_{f_i}^{o} \right) \tag{15}$$

The heat of formation $\Delta h_{f_i}^{o}$, is evaluated at the reference temperature, 0°R. The substantive derivative of enthalpy is

$$\frac{Dh}{Dt} = \sum_{i=1}^{n} \left(C_i \frac{Dh_i}{Dt} + h_i \frac{DC_i}{Dt} \right) \tag{16}$$

Substitution of equations 14 and 16 into 12 yields

$$dE = \left[\rho \sum_{i=1}^{n} \left(C_i \frac{Dh_i}{Dt} + h_i \frac{DC_i}{Dt} \right) - \rho \frac{D(P/\rho)}{Dt} \right] \Delta x \Delta y \Delta z$$

or

$$dE = \left[\rho \sum_{i=1}^{n} \left(C_i \frac{dh_i}{dT} \frac{DT}{Dt} + h_i \frac{DC_i}{Dt} \right) - \rho \frac{D}{Dt} P/\rho \right] \Delta x \Delta y \Delta z$$

But

$$\frac{dh_i}{dT} = c_{p_i}$$

and

$$\sum_{i=1}^{n} C_i c_{p_i} = c_p$$

Thus, the net change of the gas-particle internal energy becomes

$$dE = \left[\rho c_p \frac{DT}{Dt} + \rho \sum_{i=1}^{n} h_i \frac{Dc_i}{Dt} - \rho \frac{D(P/\rho)}{Dt} \right] \Delta x \Delta y \Delta z \tag{17}$$

The heat transferred or added to the control volume is assumed to be that resulting from the following processes: (a) conduction and diffusion, (b) frictional heating, and (c) chemical reactions. The net heat transfer in the y-direction due to conduction and diffusion may be expressed as

$$dQ_y = \frac{\partial}{\partial y} \left[\left(\rho c_p \epsilon_H \frac{\partial T}{\partial y} \right) - \sum_{i=1}^{n} \rho v_i c_i h_i \right] \Delta x \Delta y \Delta z$$

where ϵ_H is the eddy diffusivity for heat and v_i is the y-direction diffusion velocity of component i defined by

$$v_i = - \frac{\epsilon_{Di}}{c_i} \frac{\partial c_i}{\partial y}$$

Thus the net heat transfer term becomes

$$dQ_y = \left[\frac{\partial}{\partial y} \left(\rho c_p \epsilon_H \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial y} \sum_{i=1}^{n} \rho \epsilon_{Di} h_i \frac{\partial c_i}{\partial y} \right] \Delta x \Delta y \Delta z$$

Expanding the second term in the above equation and substituting the species continuity equation yields

$$dQ_y = \left[\frac{\partial}{\partial y} \left(\rho c_p \epsilon_H \frac{\partial T}{\partial y} \right) + \sum_{i=1}^{n} \left(\rho \frac{Dc_i}{Dt} - \dot{w}_i \right) h_i + \sum_{i=1}^{n} \rho \epsilon_{Di} c_{Pi} \frac{\partial c_i}{\partial y} \frac{\partial T}{\partial y} \right] \Delta x \Delta y \Delta z \tag{18}$$

The frictional heating term is assumed identical to that generally employed in boundary layer analyses. Thus

$$dQ_f = \rho \epsilon_V \left(\frac{\partial u}{\partial y} \right)^2 \Delta x \Delta y \Delta z \tag{19}$$

The heat release due to chemical reaction is accounted for by including the heat of formation in the enthalpy term for each component. Thus the enthalpy of the mixture is

$$h = \sum_{i=1}^{n} C_i \left[\int_0^T C_{p_i} \, dT + \Delta h^\circ_{f_i} \right] \tag{20}$$

The net rate of work done by the control volume surface forces on the surroundings is shown by Schlichting to be given by

$$dW = - P \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \tag{21}$$

Employing equations 17, 18, 19 and 21 the energy equation becomes

$$\rho c_p \frac{DT}{Dt} = \frac{\partial}{\partial y} \left(\rho c_p \epsilon_H \frac{\partial T}{\partial y} \right) + \rho \frac{D(P/\rho)}{Dt} - P \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$
$$+ \rho \epsilon_V \left(\frac{\partial u}{\partial y} \right)^2 - \sum_{i=1}^{n} \dot{\omega}_i h_i + \sum_{i=1}^{n} \rho \epsilon_{D_i} C_{p_i} \frac{\partial C_i}{\partial y} \frac{\partial T}{\partial y}$$

The second term on the right side of the above equation may be simplified utilizing the global continuity equation. Thus

$$\rho \frac{D(P/\rho)}{Dt} = \frac{DP}{Dt} - \frac{P}{\rho} \frac{D\rho}{Dt} = \frac{DP}{Dt} + P \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$

Thus the energy equations becomes

$$\boxed{\begin{aligned} \rho c_p \left(u \frac{\partial T}{\partial y} + v \frac{\partial T}{\partial y} \right) &= \frac{\partial}{\partial y} \left(\rho c_p \epsilon_H \frac{\partial T}{\partial y} \right) + u \frac{\partial P}{\partial x} \\ &+ \rho \epsilon_v \left(\frac{\partial u}{\partial y} \right)^2 + \sum_{i=1}^{n} \left(\rho \epsilon_{D_i} C_{p_i} \frac{\partial C_i}{\partial y} \frac{\partial T}{\partial y} \right) - \sum_{i=1}^{n} \left(\dot{\omega}_i h_i \right) \end{aligned}} \tag{22}$$

2.1.6 Equation of State

The equation of state for the gas-particle mixture is developed based on the assumptions that (a) the gas obeys the perfect gas law and (b) the volume occupied by the particles is negligible. Consider a unit mass of mixture comprising a mass C_{ig}, of gaseous products, and a mass C_{ip} of particles. The total volume of the mixture is given by

$$\frac{C_{ip}}{m_p} + \frac{C_{ig}}{\rho_g} = \frac{1}{\rho}$$

where m_p and ρ_g are the densities of the particles and gas, respectively. Assuming that the partial volume occupied by the particles is negligible the expression for the total volume becomes

$$\frac{1}{\rho} = \frac{C_{ig}}{\rho_g} \tag{23}$$

The equation of state for the gaseous phase is

$$P = \rho_g R_g T \tag{24}$$

where Rg is the gas constant for the gaseous components, which may be expressed in terms of the universal gas constant and the gas molecular weight. Thus

$$R_g = \frac{R_v}{W_g} \tag{25}$$

where

$$W_g = \left[\sum_{i=1}^{n_g} C_i / W_i \right]^{-1} \sum_{i=1}^{n_g} C_i \tag{26}$$

The summation is performed only for the gaseous species - n_g being the total number of gaseous components. The total density of the mixture is obtained from equations 23 - 26. Thus

$$\boxed{\rho = \frac{P}{R_u T}\left[\sum_{i=1}^{n_g} C_i/W_i\right]^{-1}} \qquad (27)$$

2.1.7 Alternate Forms of the Fundamental Equations

The governing equations may be expressed in alternate forms employing dimensionless ratios involving the eddy diffusivities. By definition the turbulent Lewis and Prandtl numbers are

$$Le_i \equiv \frac{\epsilon_{Di}}{\epsilon_H} \qquad (28)$$

$$Pr \equiv \frac{\epsilon_v}{\epsilon_H} \qquad (29)$$

where ρ and C_p are the density and specific heat of the gas-particle mixture, respectively. Employing these dimensionless numbers the fundamental equations, in axisymmetric coordinates, become

Global Continuity:

$$\frac{\partial}{\partial x}(\rho u r^\delta) + \frac{\partial}{\partial r}(\rho v r^\delta) = 0 \qquad (30)$$

Species Continuity:

$$\rho u \frac{\partial C_i}{\partial x} + \rho v \frac{\partial C_i}{\partial r} = \frac{1}{r^\delta}\frac{\partial}{\partial r}\left(r^\delta \rho \epsilon_v \frac{Le_i}{Pr}\frac{\partial C_i}{\partial r}\right) + \dot{w}_i \qquad (31)$$

Momentum Equation:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = -\frac{\partial P}{\partial x} + \frac{1}{r^\delta}\frac{\partial}{\partial r}\left(\rho \epsilon_v r^\delta \frac{\partial u}{\partial r}\right) \qquad (32)$$

Energy Equation:

$$C_p\left(\rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial r}\right) = u\frac{\partial P}{\partial x} + \rho\epsilon_v\left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^\delta}\frac{\partial}{\partial r}\left(C_p r^\delta \rho \frac{\epsilon_v}{Pr}\frac{\partial T}{\partial r}\right) + \frac{\rho\epsilon_v}{Pr}\sum_{i=1}^{n_s}\left(C_{p_i} Le_i \frac{\partial T}{\partial r}\frac{\partial C_i}{\partial r}\right) - \sum_{i=1}^{n_s}(\dot{\omega}_i h_i) \tag{33}$$

Equation of State:

$$\rho = \frac{P}{R_u T}\left[\sum_{i=1}^{n_s} C_i/W_i\right]^{-1} \tag{34}$$

The exponent for the terms r^δ, makes the equations applicable for either two-dimensional $(\delta=0)$ or axisymmetric flow $(\delta=1)$. A system of $(4 + n)$ simultaneous equations are therefore available for determining the unknown parameters $u, v, T, \rho, C_i, \dot{\omega}_i$, and ϵ_v. Thus, a solution may be obtained if appropriate expressions for the parameters $\dot{\omega}_i$ and ϵ_v are available in terms of the remaining unknown parameters. Such expressions will be developed later herein; however, for the present these parameters will be retained in their present form with the assumption that suitable expressions are available.

2.2 Transformation of the Fundamental Equations

In obtaining a solution it is convenient to transform the above equations utilizing the von Mises transformation. Instead of the axisymmetric coordinates x and r (or the cartesian coordinates x and y) the stream function ψ, is introduced as one of the independent variables replacing the r-coordinate. The x-coordinate is not affected by the transformation. By definition the stream function is

$$\left(\frac{\partial \psi}{\partial r}\right)_x \equiv \rho u r^\delta \tag{35}$$

$$\left(\frac{\partial \psi}{\partial x}\right)_r \equiv -\rho v r^\delta \tag{36}$$

which when substituted into the global continuity equation (equation 30) yields

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial r}\right) - \frac{\partial}{\partial r}\left(\frac{\partial \psi}{\partial x}\right) = 0$$

Thus, the continuity equation is satisfied identically since

$$\frac{\partial^2 \psi}{\partial x \partial r} = \frac{\partial^2 \psi}{\partial r \partial x}$$

The transformation of the remaining equations is accomplished by noting, from the chain rule of the calculus, that

$$\left(\frac{\partial f}{\partial r}\right)_x = \left(\frac{\partial f}{\partial \psi}\right)_x \left(\frac{\partial \psi}{\partial r}\right)_x$$
$$= \rho u r^{\delta}\left(\frac{\partial f}{\partial \psi}\right)_x$$

and

$$\left(\frac{\partial f}{\partial x}\right)_r = \left(\frac{\partial f}{\partial \psi}\right)_x \left(\frac{\partial \psi}{\partial x}\right)_r + \left(\frac{\partial f}{\partial x}\right)_\psi$$
$$= -\rho v r^{\delta}\left(\frac{\partial f}{\partial \psi}\right)_x + \left(\frac{\partial f}{\partial x}\right)_\psi$$

where f represents any of the dependent variables. Hence, the transformed equations become

Species Continuity:

$$\frac{\partial c_i}{\partial x} = \frac{\partial}{\partial \psi}\left(\frac{\epsilon_v Le_i}{Pr} \rho^2 u r^{2\delta} \frac{\partial c_i}{\partial \psi}\right) + \frac{\dot{\omega}_i}{\rho u} \tag{37}$$

Momentum Equation:

$$\frac{\partial u}{\partial x} = -\frac{1}{\rho u}\frac{dP}{dx} + \frac{\partial}{\partial \psi}\left(\epsilon_v \rho^2 u r^{2\delta} \frac{\partial u}{\partial \psi}\right) \tag{38}$$

Energy Equation:

$$\frac{\partial T}{\partial x} = \frac{1}{\rho C_p}\frac{dP}{dx} + \frac{\epsilon_v \rho^2 u r^{2\delta}}{C_p}\left(\frac{\partial u}{\partial \psi}\right)^2 + \frac{1}{C_p}\frac{\partial}{\partial \psi}\left(C_p \frac{\epsilon_v \rho^2 u r^{2\delta}}{Pr}\frac{\partial T}{\partial \psi}\right) + \frac{\rho^2 u r^{2\delta}\epsilon_v}{C_p Pr}\sum_{i=1}^{n}\left(C_{p_i} Le_i \frac{\partial T}{\partial \psi}\frac{\partial c_i}{\partial \psi}\right) - \sum_{i=1}^{n}\left(\dot{\omega}_i h_i\right) \qquad (39)$$

A

In order to obtain the solution in the physical plane (x, r), it is necessary to transform the ψ-coordinate to the r-coordinate. From the definition of the stream function, equation 35, the r-value corresponding to any ψ-value is given by

$$r = \left(r_o^{\delta+1} + (\delta+1)\int_0^{\psi}\frac{d\psi}{\rho u}\right)^{1/\delta+1} \qquad (40)$$

where r is the r-value corresponding to the reference streamline, $\psi = 0$.

Shown in Figure 3 is the orientation of the x-r coordinate system and the location of streamlines for a typical problem.

2.3 Development of the Finite-Difference Equations

The finite difference technique of solving differential equations consists of replacing the partial derivatives by finite difference ratios, which is equivalent to physically replacing the continuous flow field by a network of finite elements. Consider the flow field illustrated in Figure 4 and assume it to be divided into a number of elements having grid sizes of Δx and $\Delta \psi$. If, in addition, it is assumed that values of all the flow properties are known along the vertical line designated m, referred to as "front m", it is then possible to calculate the flow properties at front m + 1 in the following manner.

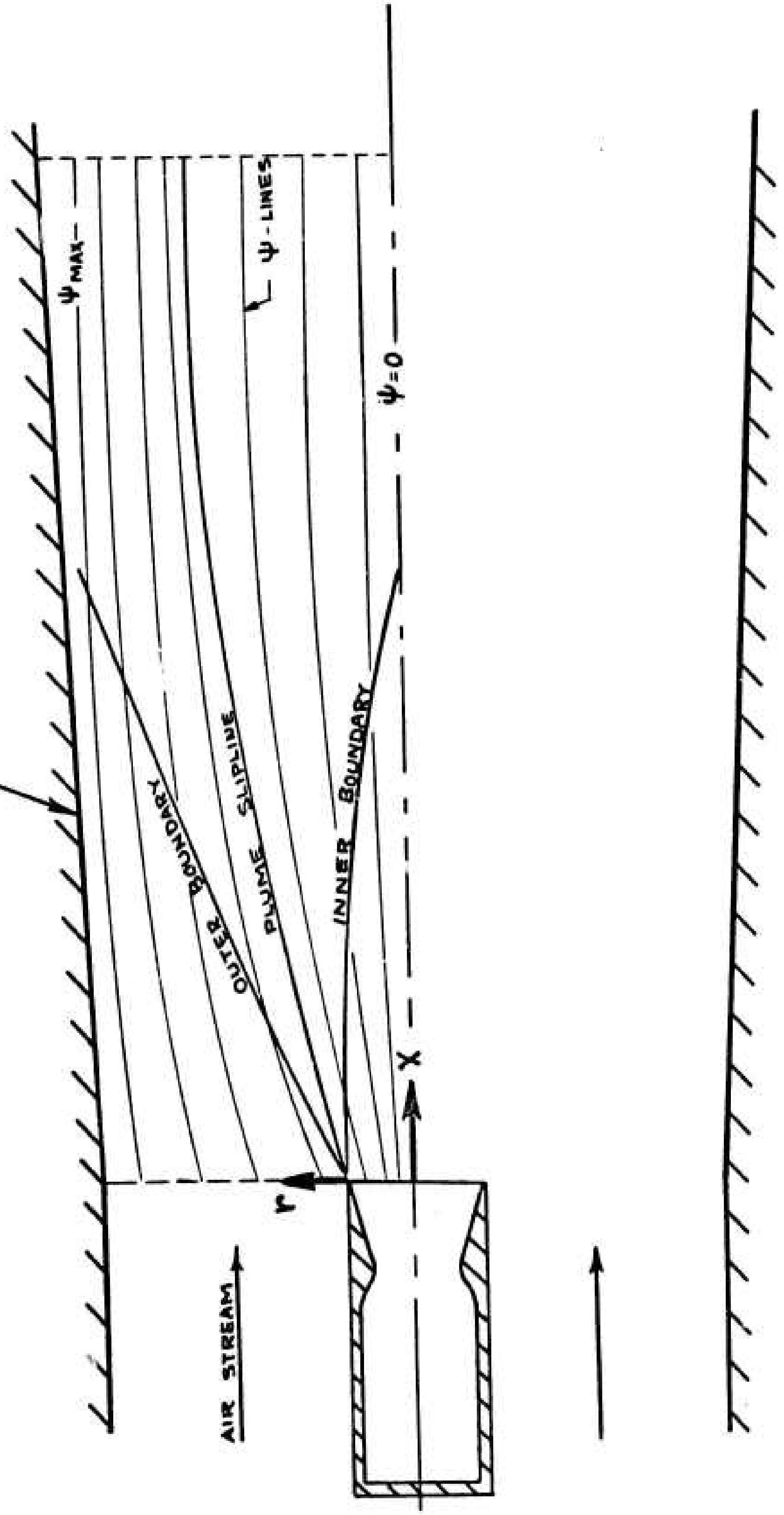

FIG. 3 Typical Flow Field

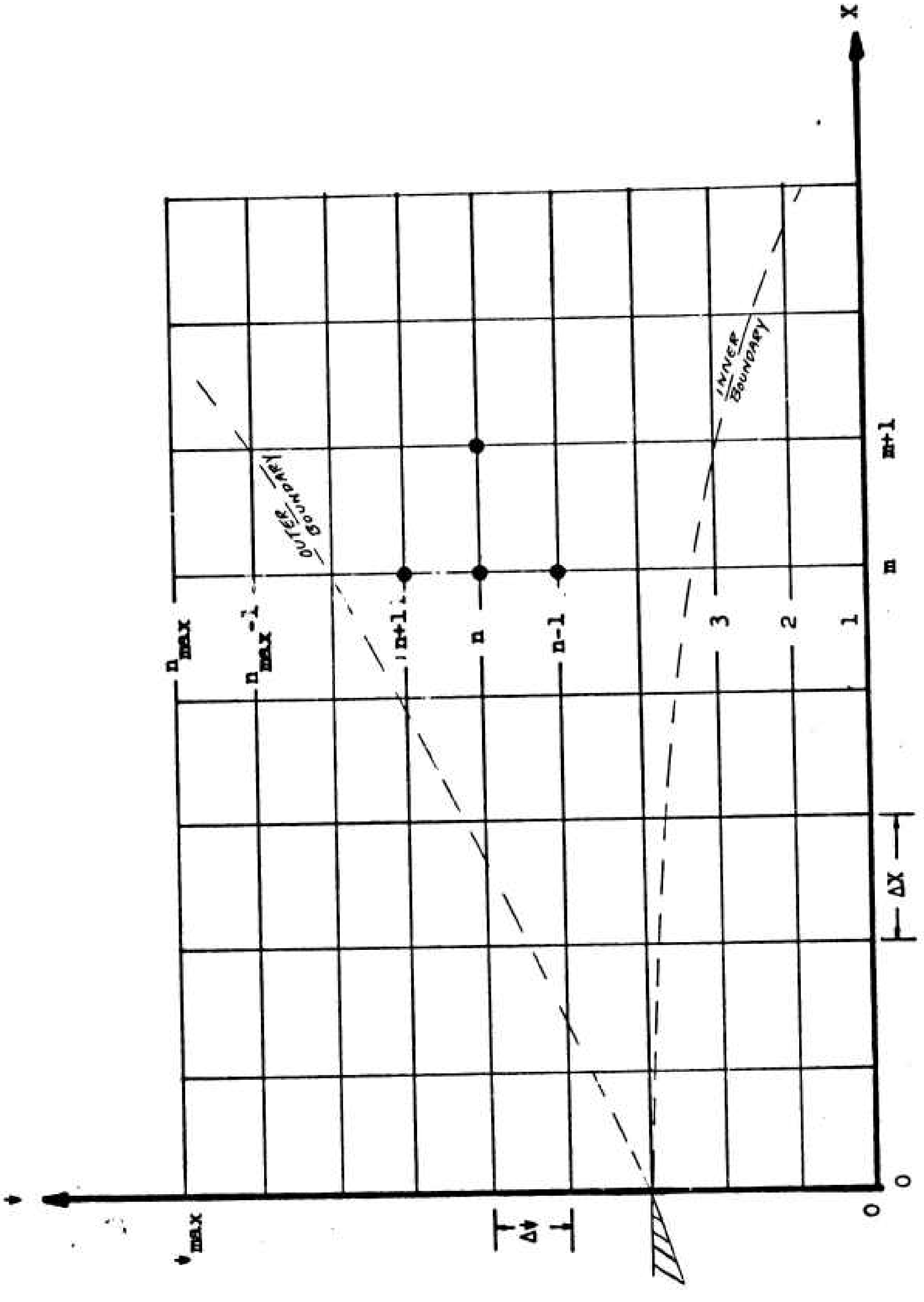

Figure 4 - Finite Difference Grid Network

For the grid point denoted (m + 1, n) a Taylor series expansion of the function f in terms of x-derivatives yields

$$f_{m+1,n} = f_{m,n} + \Delta x \left(\frac{\partial f}{\partial x}\right)_{m,n} + \frac{\Delta x^2}{2!}\left(\frac{\partial^2 f}{\partial x^2}\right)_{m,n} + ---$$

where f may be taken to be any dependent variable, e.g., u, t, e, etc. If Δx is taken sufficiently small the terms of higher order than unity may be neglected, thus the x-derivative becomes:

$$\left(\frac{\partial f}{\partial x}\right)_{m,n} = \frac{f_{m+1,n} - f_{m,n}}{\Delta x}$$

The above equation is termed the forward-difference approximation for the x-derivative. In a similar manner backward - difference and average - difference approximations may be derived (7). The resulting backward- and average-difference approximations are, respectively

$$\left(\frac{\partial f}{\partial x}\right)_{m,n} = \frac{f_{m,n} - f_{m-1,n}}{\Delta x}$$

$$\left(\frac{\partial f}{\partial x}\right)_{m,n} = \frac{f_{m+1,n} - f_{m-1,n}}{2\Delta x}$$

Following the analysis of Wu (7) the forward-difference approximations are used herein for the x-derivatives, and the average-difference approximation for the ψ-derivatives. The selection of these approximations is based on physical and stability considerations as discussed by Wu. In the present analysis, the following approximations are employed for the derivatives

$$\left(\frac{\partial f}{\partial X}\right)_{m,n} = \frac{f_{m+1,n} - f_{m,n}}{\Delta X} \tag{41}$$

$$\left(\frac{\partial f}{\partial \psi}\right)_{m,n} = \frac{f_{m,n+1} - f_{m,n-1}}{2\,\Delta \psi} \tag{42}$$

The second derivative with respect to ψ is given by the finite-difference approximation

$$\frac{\partial^2 f}{\partial \psi^2} = \frac{f_{m,n+1} - 2 f_{m,n} + f_{m,n-1}}{\Delta \psi^2} \tag{43}$$

The product terms appearing in the differential equations are approximated by

$$\frac{\partial f}{\partial \psi} \cdot \frac{\partial g}{\partial \psi} = \frac{(f_{m,n+1} - f_{m,n-1}) \cdot (g_{m,n+1} - g_{m,n-1})}{4\,\Delta \psi^2} \tag{44}$$

Substituting the above expressions for the derivatives appearing in equations 37-39 and rearranging terms, yields

Species Continuity:

$$\begin{aligned} C_{i_{m+1,n}} = {} & C_{i_{m,n}} + \frac{\Delta X}{\Delta \psi^2} \frac{Le_i}{Pr} (\rho^2 u r^{2\delta} \epsilon_v)_{m,n} \left(C_{i_{m,n+1}} - 2 C_{i_{m,n}} + C_{i_{m,n-1}}\right) \\ & + \frac{\Delta X}{4 \Delta \psi^2} \frac{Le_i}{Pr} \left(C_{i_{m,n+1}} - C_{i_{m,n-1}}\right) \Big[(\rho \epsilon_v)_{m,n} \left\{ (\rho u r^{2\delta})_{m,n+1} - (\rho u r^{2\delta})_{m,n-1} \right\} \\ & + (\rho u r^{2\delta})_{m,n} \left\{ (\rho \epsilon_v)_{m,n+1} - (\rho \epsilon_v)_{m,n-1} \right\} \Big] + \left(\frac{\dot{w}_i \Delta X}{\rho u}\right)_{m,n} \end{aligned} \tag{45}$$

Momentum Equation:

$$\begin{aligned} u_{m+1,n} = {} & u_{m,n} - \left(\frac{\Delta X g}{\rho u}\right)_{m,n} \left(\frac{dP}{dX}\right)_m + \frac{\Delta X}{\Delta \psi^2} (\rho^2 u r^{2\delta} \epsilon_v)_{m,n} \left(u_{m,n+1} - 2 u_{m,n} + u_{m,n-1}\right) \\ & + \frac{\Delta X}{4 \Delta \psi^2} \Big[(\rho \epsilon_v)_{m,n} \left\{ (\rho u r^{2\delta})_{m,n+1} - (\rho u r^{2\delta})_{m,n-1} \right\} \\ & + (\rho u r^{2\delta})_{m,n} \left\{ (\rho \epsilon_v)_{m,n+1} - (\rho \epsilon_v)_{m,n-1} \right\} \Big] \left(u_{m,n+1} - u_{m,n-1}\right) \end{aligned} \tag{46}$$

Energy Equation:

$$
\begin{aligned}
T_{m+1,n} = {} & T_{m,n} + \left(\frac{\Delta x}{J\rho c_p} \frac{dp}{dx} \right)_{m,n} + \frac{\Delta x}{4\Delta\psi^2 P_r} \left(\rho u r^{2\sigma} \right)_{m,n} \Bigg[\frac{\left(u_{m,n+1} - u_{m,n-1} \right)^2}{Jg\,(c_p)_{m,n}} P_r \left(\rho \epsilon_v \right)_{m,n} \\
& + 4(\rho\epsilon_v)_{m,n} \left(T_{m,n+1} - 2T_{m,n} + T_{m,n-1} \right) + \left\{ (\rho\epsilon_v)_{m,n+1} - (\rho\epsilon_v)_{m,n-1} \right\} \left(T_{m,n+1} - T_{m,n-1} \right) \Bigg] \\
& + \frac{\Delta x}{4\Delta\psi^2 Pr} \left(\frac{\rho\epsilon_v}{c_p} \right)_{m,n} \left[\left\{ (\rho u r^{2\sigma} c_p)_{m,n+1} - (\rho u r^{2\sigma} c_p)_{m,n-1} \right\} \left(T_{m,n+1} - T_{m,n-1} \right) \right] \qquad (47) \\
& + \frac{\Delta x}{4\Delta\psi^2 Pr} \left(\frac{\rho^2 u r^{2\sigma} \epsilon_v}{c_p} \right)_{m,n} \sum_i c_{p_i} Le_i \left(c_{i_{m,n+1}} - c_{i_{m,n-1}} \right) \left(T_{m,n+1} - T_{m,n-1} \right) - \frac{\Delta x}{(\rho u c_p)_{m,n}} \sum_i \left(\dot{w}_i h_i \right)_{m,n}
\end{aligned}
$$

As can be seen, the above equations form a set of algebraic expressions in which the dependent variable at the m + 1 front is expressed explicitly in terms of the known properties at front m. The calculation of the flow properties throughout the grid network may then be carried out by successively applying the equations to each front. However, it is first necessary that the boundary conditions be specified.

2.4 Initial and Boundary Conditions

In starting the calculations for a specific problem, it is necessary that the properties along the initial front be known at each grid point - these properties are termed the initial conditions. The properties may be constant or variable dependent on the problem under consideration. For example, it may be desired to assume constant properties for the rocket exhaust jet except in the immediate neighborhood of the nozzle wall where the jet boundary layer exists. Similarly, the flow properties in the outer stream along the initial front may be variable corresponding to those existing within a boundary layer. In other problems, it may be desired to start the calculations at a point downstream where the mixing region has partially developed and the calculations continued for further development of the mixing region. For any of these situations, it is required that the properties be known at each grid point along the initial front before the calculations can begin.

Once the initial conditions are specified the flow properties at each grid point along the second front may be calculated except at the grid points corresponding to $\psi = 0$ and $\psi = \psi_{max}$. These streamlines are the boundaries of the grid network within which the mixing region lies. In order to obtain a complete solution for a given flow problem, it is necessary that the flow properties along the boundaries be known. These properties are termed boundary conditions. It is the usual practice in free jet problems to assume the flow to be inviscid in the areas outside the mixing region. The boundary conditions for such problems are therefore specified accordingly.

For problems in which the secondary flow is contained within a duct, however, three regimes of flow may be encountered each dictating different boundary conditions. For example, consider the flow field shown in Figure 5, illustrating the different flow regimes. The first regime begins at the exit plane of the nozzle and ends at the point where the inner edge of the mixing zone reaches the jet centerline. The boundary conditions for this regime may be taken to correspond to those for inviscid flow. The flow field, therefore, resembles that of free jet problems.

In the second regime, the mixing region widens until at some point the outer edge of the mixing zone reaches the duct wall, whereupon the third regime commences. The technique for determining the outer boundary condition in the second regime is identical to that in the first, i.e., the secondary flow outside the mixing region is assumed to be inviscid. The inner boundary conditions for the second and third flow regimes may be determined from the physical characteristics of the flow. Along the jet

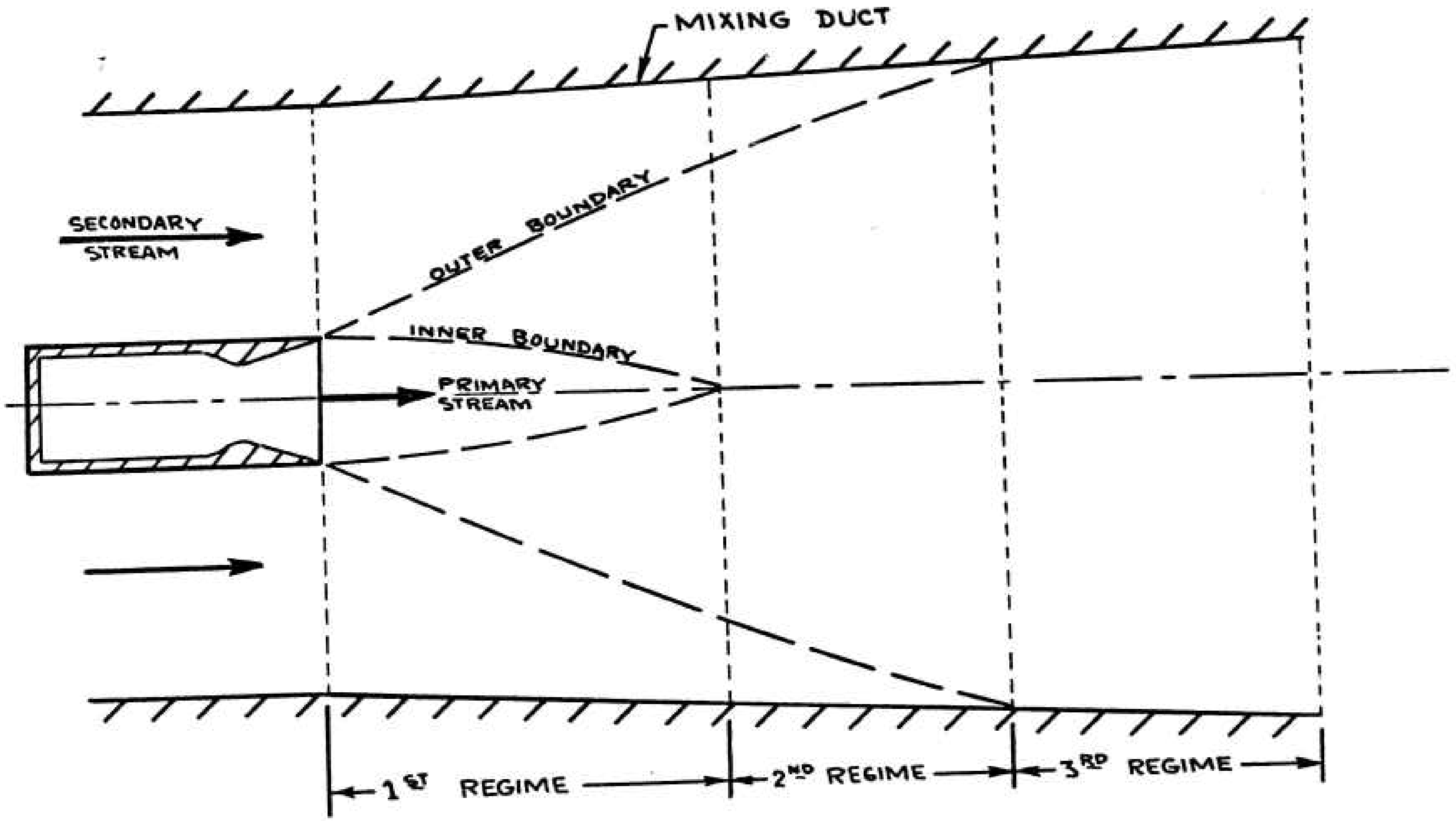

FIG. 5 Flow Regimes for Confined Mixing

centerline, the flow properties change as a result of the mixing process. However, from symmetry it may be concluded that on the centerline, the radial gradients of all properties must be zero. This condition may therefore be employed as the inner boundary condition for the second regime.

In the third regime, the inviscid outer boundary assumption is no longer valid since the mixing region extends across the entire duct. However, the outer boundary condition for this regime may be established by assuming that the heat transfer and shear stress at the duct wall are negligible. Thus, from the rate equations for shear stress and heat transfer it is seen that

$$\left(\frac{\partial u}{\partial \psi}\right)_{Wall} = 0$$

$$\left(\frac{\partial T}{\partial \psi}\right)_{Wall} = 0$$

In addition, since mass cannot be transferred across the duct wall, we have

$$\left(\frac{\partial c_i}{\partial \psi}\right)_{Wall} = 0$$

Thus, the inner and outer boundary conditions for the third regime are identical. Summarizing, the boundary conditions for the first flow regime may be expressed in the following form:

$$c_{i_{m+1}} = c_{i_m} + \left(\frac{\dot{\omega}_i \Delta X}{\rho u}\right)_m \tag{48}$$

$$u_{m+1} = u_m + \left(\frac{\Delta X g}{\rho u}\right)_m \left(\frac{dP}{dX}\right)_m \tag{49}$$

$$T_{m+1} = T_m + \left(\frac{\Delta X}{J \rho C_p} \frac{dP}{dX}\right)_m - \left(\frac{\Delta X}{\rho u C_p} \sum_{i=1}^{n} \dot{\omega}_i h_i\right)_m \tag{50}$$

The above equations are applicable for determining both the inner and outer boundary conditions. In the second regime, the boundary conditions for the inviscid region are as above; whereas, the inner boundary conditions are given by

$$\left(\frac{\partial C_i}{\partial \psi}\right)_{\psi=0} = \left(\frac{\partial u}{\partial \psi}\right)_{\psi=0} = \left(\frac{\partial T}{\partial \psi}\right)_{\psi=0} = 0 \tag{51}$$

For the third regime, the boundary conditions are expressed in the form

$$\begin{aligned} \left(\frac{\partial C_i}{\partial \psi}\right)_{\psi=0} &= \left(\frac{\partial C_i}{\partial \psi}\right)_{\psi=\psi_{Max}} = 0 \\ \left(\frac{\partial u}{\partial \psi}\right)_{\psi=0} &= \left(\frac{\partial u}{\partial \psi}\right)_{\psi=\psi_{Max}} = 0 \\ \left(\frac{\partial T}{\partial \psi}\right)_{\psi=0} &= \left(\frac{\partial T}{\partial \psi}\right)_{\psi=\psi_{Max}} = 0 \end{aligned} \tag{52}$$

It should be noted that in the above discussion it was assumed that the inner edge of the mixing zone reached the duct centerline before the outer edge reached the duct wall. In certain situations, however, these conditions may be reversed, the outer edge reaching the duct wall prior to the inner edge reaching the centerline, in which case the aforementioned boundary conditions are altered accordingly.

3.0 TURBULENT TRANSPORT COEFFICIENTS

3.1 General

In order to obtain a solution of the finite-difference equations, it is necessary that expressions for the eddy **diffusivities** be established in terms of known quantities. Unfortunately the eddy **diffusivities** are not fluid properties, as are their molecular counterparts, but parameters dependent upon the fluid motion, e.g., level of turbulence, velocity gradient, density gradient, etc. Since turbulent flow is characterized by random fluctuations, the derivation of accurate theoretical expressions for the eddy **diffusivities** requires a statistical analysis such as that initiated by Taylor (11). However, the statistical approach at the present time has not proven practical (12). Consequently, one must rely on the semi-empirical approach for determining expressions relating the eddy **diffusivities** to the flow properties.

3.2 Incompressible Theory

Most of the semi-empirical theories to date are extensions or modifications of Prandtl's mixing length concept. From Prandtl's initial hypothesis, it was found that the shear stress for incompressible turbulent flow can be determined from

$$\tau = \rho \ell^2 \left(\frac{\partial u}{\partial y}\right)^2 \tag{53}$$

where ℓ is Prandtl's mixing length parameter, assumed to be independent of the y-coordinate (1, 10). Broussinesq had earlier postulated that, in analogy with the shear stress law for laminar flow, the shear stress for turbulent flow could be expressed in the form:

$$\tau = \rho \epsilon_v \frac{\partial u}{\partial y} \tag{54}$$

where ϵ_v is the eddy diffusivity of momentum, often referred to as eddy viscosity, being analogous to molecular kinematic viscosity. From equations 53 and 54 it is found that

$$\epsilon_v = l^2 \frac{\partial u}{\partial y} \tag{55}$$

In an attempt to obtain more realistic values of ϵ_v when $\partial u/\partial y$ approaches zero, Prandtl modified the above expression; however, the resulting expression is difficult to use.

A simpler expression for ϵ_v was subsequently developed by Prandtl based on the experimental data of Reichardt. Assuming that ϵ_v was constant across the mixing zone, i.e., independent of the y-coordinate, Prandtl postulated that for free jets

$$\epsilon_v = k\, b\, (u_{max} - u_{min}) \tag{56}$$

where k is an empirical constant, b is the width of the mixing region, and $(U_{max}-U_{min})$ is the difference between the maximum and minimum velocities in the mixing region. Furthermore, it was found that

$$b = c\,x$$

so that Eqn. 56 becomes

$$\epsilon_v = K X (u_{max} - u_{min}) \qquad (57)$$

where the product kc, is replaced by another proportionately constant K. From Reichardt's experiments of incompressible free jets ($U_{min} = 0$), it was determined that the width of the mixing zone varied according to the relationship

$$b = 0.098 X \qquad (58)$$

The resulting expression for ϵ_v which best correlated the data was

$$\epsilon_v = 0.00137 X u_{max} \qquad (59)$$

An extension of the above expression is often employed for the case of two moving streams, thus

$$\epsilon_v = 0.00137 X (u_{max} - u_{min}) \qquad (60)$$

An alternate form of Equation 57 is frequently used in correlating experiment and theory in terms of the similarity parameter, thus

$$\epsilon_v = \frac{u_{max} + u_{min}}{4 \sigma^2} X \qquad (61)$$

where σ the similarity parameter, is a constant to be determined from experimental data. Physically, σ is a parameter related to the spreading rate of the mixing region - larger values of σ corresponding to smaller spreading rates. From the data of Reichardt, employed in obtaining Equation 60, the corresponding value of σ was determined to be 13.5.

Still another expression for ϵ_v is that given by Pai (12), which in its most general form is

$$\epsilon_v = \epsilon_0 \left(C + x/L \right)^n \tag{62}$$

where ϵ_0 is referred to as Reichardt's coefficient, C is a constant which accounts for a virtual origin of the mixing region, L is a reference length, and n is a number having a value between 0 and 1 depending on the degree of mixing. The constant C may be assumed to account for the boundary layers that build up in the two streams prior to the point of initial contact (x = 0) which produce, in effect, a mixing region of finite width at x = 0. For the case where C = 0 and n = 1, as generally is assumed, Equation 62 becomes

$$\epsilon_v = \epsilon_0 \, x/L \tag{63}$$

Equating Equations 61 and 63 and solving for ϵ_0 yields

$$\epsilon_0 = \frac{L \left(u_{max} + u_{min} \right)}{4 \sigma^2} \tag{64}$$

From extensive data of free jets exhausting into a quiescient atmosphere, the value of σ for jet Mach numbers less than unity has been relatively well established as being $\sigma = 12$. For cases where the receiving medium is also moving the value of σ is found to depend on the velocity and density ratio between the two streams as discussed below.

3.3 Compressible Theory

In the development of the ϵ_v expressions for incompressible flow it was assumed that ϵ_v was constant across the mixing zone, an assumption which has been substantiated by extensive experimental data. However, for compressible flow involving large temperature and velocity differences between the two streams, it has been postulated that ϵ_v is dependent not only on the x-coordinate, but the y-coordinate as well (13). Several mathematical models for the compressible eddy viscosity have been proposed by various investigators (14, 15, 16), which for the most part are more complicated and consequently more difficult to use than the incompressible models. In addition, since only limited experimental data are available, none of the proposed compressible models has been satisfactorily verified. Consequently it has been the general practice to employ the incompressible model for ϵ_v and adjust the empirical constants to account for the effects of compressibility.

The most frequently used parameter in correlating experiment and theory is the similarity parameter σ, defined in Equation 61. Experimentally determined values of σ for free jets exhausting into a quiescent atmosphere have been obtained for jet Mach numbers ranging up to approximately 3.0 (15). Figure 6 illustrates the variation of σ with jet Mach number for a jet exhausting into still air. The solid line faired through the experimental data was originally presented in Reference 15. The dashed line illustrates the theoretical variation of σ with Mach number as given by Korst (17). The majority of the experimental data were obtained with air jets having

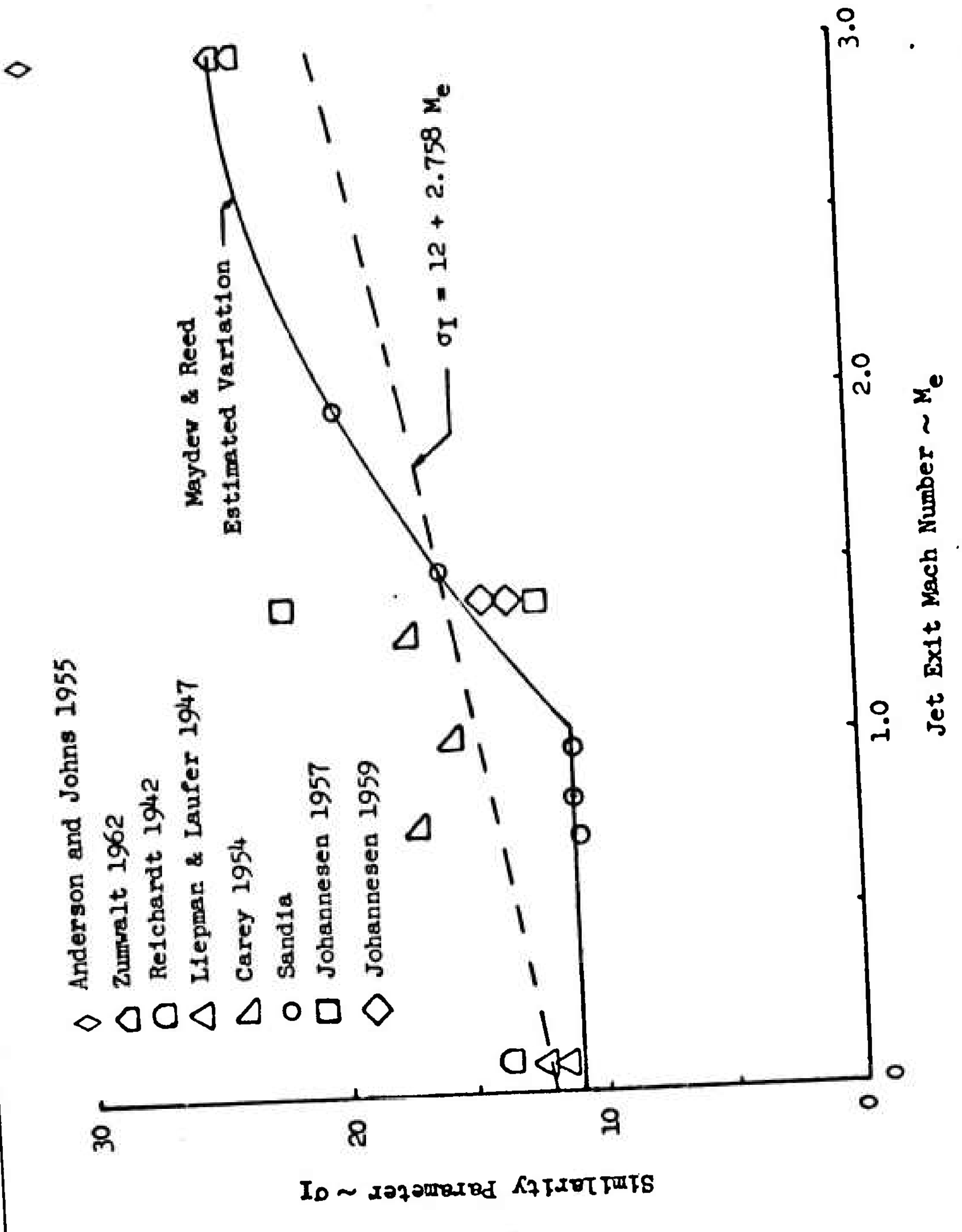

Figure 6 - Experimental and Theoretical σ Values for Free Jets

temperatures approximately equal to that of the ambient air.

One exception is the data of Anderson and Johns, which were obtained with a solid propellant rocket exhausting into still air. For Mach numbers less than unity, the value of σ has been relatively well established as being $\sigma = 12.0$. For Mach numbers greater than unity, the data illustrate that σ increases considerably.

For the turbulent mixing between two compressible coaxial streams, analytical attempts (18) have resulted in relating the two-stream σ-value and the velocity ratio between the two streams. The equivalent single stream value is obtained from Korst's empircal relationship

$$\sigma_I = 12 + 2.758 M_{Ia} \tag{65}$$

where M_{Ia} is an equivalent single stream Mach number related to the jet stream Crocco number as shown in Fig. 7. Also shown in Fig. 7 is the ratio of the aforementioned σ-values as a function of the velocity ratio between the two streams; indicating that the two stream σ-value increases with increasing velocity ratio. This trend has also been established from experimental data (12).

Since experimental data for predicting two stream σ-values are limited, the above technique for evaluating σ has been adopted herein with the exception that the equivalent single stream σ_I value is obtained from Fig. 6 corresponding to the equivalent single stream Mach number. The corresponding value of Reichard's coefficient is then determined from Equation 64.

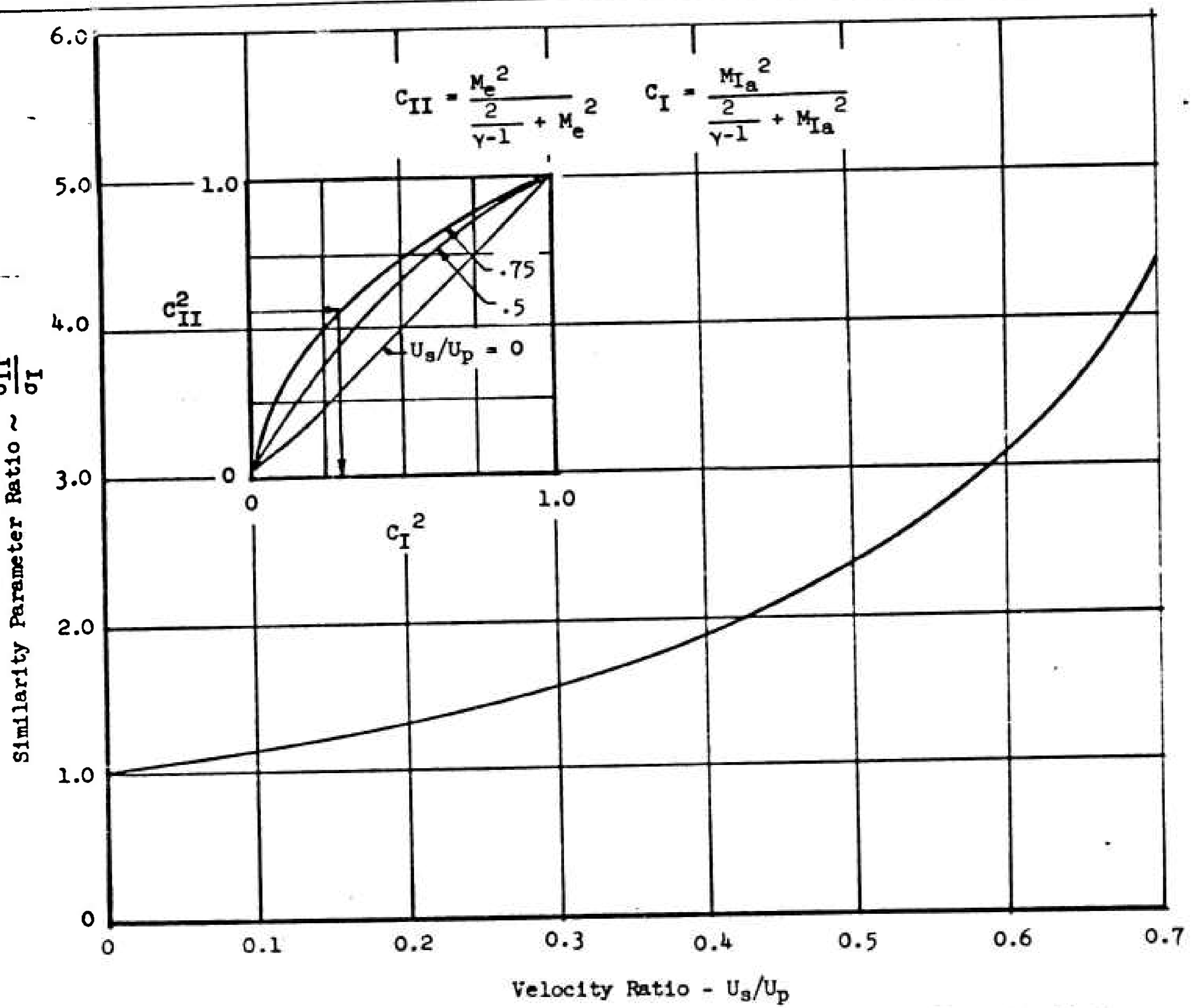

Figure 7 - Theoretical Effect of Velocity Ratio on Similarity Parameter Ratio

It should be noted that in utilizing any of the above ϵ_V expressions, difficulties may arise resulting from the method employed in correlating experiment and theory. The accepted method of establishing the validity of a particular model is to employ a specific theoretical analysis and several trial values of the empirical constant in the ϵ_V expression, until agreement is obtained between the measured and calculated velocity, temperature, and concentration profiles. Unfortunately, the resulting ϵ_V expressions reflect any peculiarities of the particular theoretical analysis used in the correlation, and consequently should be used with reservation for other analyses.

3.4 Turbulent Lewis and Prandtl Numbers

The turbulent Lewis and Prandtl numbers are the turbulent counterparts of the corresponding numbers based on molecular properties. By definition

$$Le_i = \frac{\epsilon_{Di}}{\epsilon_H} \qquad (66)$$

$$Pr = \frac{\epsilon_V}{\epsilon_H} \qquad (67)$$

Another dimensionless ratio frequently employed is the Schmidt number defined by

$$Sc_i = \frac{Pr}{Le_i} = \frac{\epsilon_V}{\epsilon_{Di}} \qquad (68)$$

where the subscript i infers that the various species may have different eddy mass diffusivities. . In early mixing analyses it was often assumed, in analogy with the molecular ratios, that Pr = Sc = Le = 1; an assumption which simplifies the governing equations considerably. However, recent experimental data indicate that these parameters may differ markedly from unity (14, 19). For example, from the experimental results of Forstall and Shapiro (19), it was found that the turbulent Pr and Sc numbers were equal and constant throughout the mixing region, with a value of approximately 0.7. From the experimental results of Zakkay, et al, (14) it was concluded that the turbulent Schmidt and Lewis numbers ranged from 0.3 to 2.3 and 0.4 to 1.0, respectively.

In the present analysis, values of the turublent Prandtl and Lewis numbers may be employed other than unity, however they remain as constants independent of the x-y coordinates. For gaseous components the values of Pr and Sc most frequently cited in the literature vary between 0.5 and 1.2. It was shown in Ref. 20 that variations of Sc have negligible effects on the radial profiles, so that any value within the above range may be used with reasonable accuracy.

The Lewis and Prandtl numbers for the solid and/or liquid particles in gas-particle mixtures are at the present time unknown. The experimental data of Longwell and Weiss (21), however, indicate that the ratio of gas to liquid eddy mass diffusivities is approximately 2. In their studies liquid diesel fuel was injected into a turbulent air stream and concentrations measured at various axial locations. The results were compared to similar data obtained with naphtha as the injected fuel. As noted by the authors the naphtha evaporated readily and was assumed to be mainly in the gaseous phase. The results showed that the ratio of the gas to liquid ϵ_D varied from approximately 1.2 to 2.0 for gas stream velocities ranging from 200 to 500 fps, respectively.

In view of the lack of experimental data it is difficult to ascertain values that should be employed for the turbulent Lewis and Prandtl numbers in order to accurately predict flow profiles. However, until reliable experimental data are available, the values cited above may be employed with results within the accuracy required in most engineering problems.

4.0 CHEMICAL MODEL

4.1 General

In considering the chemical aspects of problems involving the mixing between reactive components, two choices of chemical behavior are available; namely, equilibrium or nonequilibrium chemistry. Because of the high velocities encountered in rocket exhaust jets, it appears that in such problems nonequilibrium chemistry would prevail throughout most of the flow field. The most accurate analysis should, therefore, be based on nonequilibrium flow including the complicated chemical equations describing the reaction mechanism. However, in view of the complexity of such an analysis and the resulting lengthy calculations, it is advantageous to use equilibrium chemistry whenever that choice leads to accurate results.

In the discussion that follows the equations for both equilibrium and nonequilibrium chemistry are developed for the hydrogen-oxygen system and the results compared. From the comparison of temperature profiles, it was ascertained that the assumption of equilibrium chemistry was valid for hydrogen-oxygen systems. This assumption was therefore employed for such systems and extended to other systems as discussed below.

4.2 Nonequilibrium Chemistry

For nonequilibrium chemistry the net rate of production of the species w_k, is determined using the methods of theoretical reaction kinetics. For example, consider the chemical reaction between hydrogen and oxygen forming water vapor. It is well known that the reaction mechanism between these molecules actually consists of several chain-type reactions including the following:

$$\begin{aligned}
H_2 + OH &\rightleftharpoons H_2O + H \\
H + O_2 &\rightleftharpoons OH + O \\
H_2 + O &\rightleftharpoons OH + H \\
H + H + M &\rightleftharpoons H_2 + M \\
H + OH + M &\rightleftharpoons H_2O + M \\
O + O + M &\rightleftharpoons O_2 + M
\end{aligned} \tag{70}$$

where M is a third body usually taken to represent all the molecules in the mixture.

In order to determine the net rates of production of the various species entering the above reactions, it is necessary to apply the reaction kinetics equations to each of the reaction equations. In order to simplify the present analysis, however, it is assumed that the above complex chain reactions for the H_2-O_2 system may be replaced by the following one-step reaction equation

$$2H_2 + O_2 \rightleftharpoons 2H_2O \tag{71}$$

where k_f is an "overall" reaction rate constant.

Following Penner (22) it can be shown that the net rate of production of H_2O, in accordance with reaction equation 71, may be expressed as

$$\dot{w}_1 = 2 W_1 k_f' \rho^3 \frac{C_3}{W_3} \left(\frac{C_2}{W_2} \right)^2 \left[1 - \frac{W_3}{K_p' R T \rho C_3} \left(\frac{C_1}{W_1} \frac{W_2}{C_2} \right)^2 \right] \tag{72}$$

where k_f' is the forward reaction rate constant, K_p' is the equilibrium constant, and W is the molecular weight. The subscripts 1, 2, 3 correspond to the species H_2O, H_2 and O_2, respectively.

From further application of the reaction rate equations it may be shown that the net rate of production of H_2 is related to that of H_2O. Thus

$$\dot{w}_2 = -\dot{w}_1 \frac{W_2}{W_1} \tag{73}$$

From conservation of mass the net rate of production of O_2 becomes

$$\dot{w}_3 = -(\dot{w}_1 + \dot{w}_2) \tag{74}$$

The expression for the reaction rate constant k_f is assumed to be of the form

$$k_f = BT^{\gamma} \exp\left(-\frac{E}{RT}\right) \tag{75}$$

where B is the frequency factor, E is the activation energy, and γ is a constant.

Equations (72) through (74) may be employed in solving the finite difference energy and species conservation equations provided the empirical constants in equation (75) may be established. The primary difficulty in all non-equilibrium problems is that of establishing the reaction mechanism and associated expressions for the reaction rate constants. For the H_2-O_2 system the complex chain reaction equations and rate constants are relatively well established. It is possible therefore, to establish an expression for an "overall" reaction rate constant in the following manner. Utilizing the chain reactions and the associated reaction rate constants, the temperature distribution throughout a stream tube may be calculated for a range of initial conditions of the flow parameters employing the one-dimensional analysis of Ref. 23. Then, using the same analytical technique and the one-step chemical reaction a value of the overall reaction rate constant may be determined which best reproduces the previously determined temperature distribution.

The above technique was employed for a range of initial conditions corresponding to those anticipated for the problem being considered herein. Typical results are shown in Fig. 8 wherein are presented the calculated temperature distributions resulting from the use of chain reactions (the solid line) and the one-step reaction using various expressions for the overall reaction rate constant (the broken lines). The reaction equations employed in calculating the solid line were those presented in equation 70 herein. The value of the overall reaction rate constant was varied by altering the frequency factor; the activation energy, obtained from Ref. 3, was assumed constant at 1.6 x 10^4 cal/gm-mole. The average value of the frequency factor which best correlated the results obtained from the complex reactions was $B = 10^{17}$. Thus, the expression employed for the overall reaction rate constant for the H_2O_2 reaction was determined to be

$$k_f = 10^{17} T^{0.5} \exp\left(\frac{-1.45 \times 10^4}{T}\right) \tag{76}$$

The w_i terms appearing in the finite difference equations are then determined from equations 72-74.

4.3 Equilibrium Chemistry

Employing the assumption of equilibrium chemistry the net rate of production of each species may be determined from the equation

$$\dot{\omega}_i = \rho u (c_i - \bar{c}_i) \tag{77}$$

where $\bar{C}_K$ and C_K are the concentrations of species K immediately before and after the chemical reaction, respectively. The problem, therefore, reduces to that of calculating the equilibrium composition C_K , for initial concentrations $\bar{C}_K$ at a given pressure. In the discussion that follows the equilibrium equations are developed not only for the hydrogen-oxygen system, but in addition, for systems comprising the following species: H_2O, H_2, O_2, CO_2, CO, N_2, HCl, Al, and Al_2O_3. The chemical reaction equations are assumed to be as follows:

$$2H_2 + O_2 \rightleftharpoons 2H_2O \tag{78}$$

$$2CO + O_2 \rightleftharpoons 2CO_2 \tag{79}$$

$$2Al + 3/2\ O_2 \rightleftharpoons Al_2O_3 \tag{80}$$

Consider now the elemental control volume shown in Fig. 2. As the fluid traverses the distance Δx, the concentrations change as a result of the mixing process. If it is assumed that the fluid entering the volume is in chemical equilibrium, then at $x + \Delta x$ the fluid is not in equilibrium. Since the assumption of equilibrium flow implies that the reactions occur at an infinite rate, the composition at $x + \Delta x$ may be determined as follows. From an atom balance the following relations may be obtained which are valid for both reactants and products:

$$\tilde{C}_2 = \bar{C}_2 + \bar{C}_1 \frac{W_2}{W_1} \tag{81}$$

$$\tilde{C}_3 = \bar{C}_3 + \bar{C}_1 \frac{W_3}{2W_1} + \bar{C}_4 \frac{W_3}{W_4} + \bar{C}_5 \frac{W_3}{2W_5} + \frac{3}{2} \frac{W_3}{W_{10}} \bar{C}_{10} \qquad (82)$$

$$\tilde{C}_4 + \tilde{C}_5 \frac{W_4}{W_5} = \bar{C}_4 + \bar{C}_5 \frac{W_4}{W_5} \qquad (83)$$

$$\tilde{C}_6 = \bar{C}_6 \qquad (84)$$

$$\tilde{C}_7 = \bar{C}_7 \qquad (85)$$

$$\tilde{C}_8 = \bar{C}_8 \qquad (86)$$

$$\tilde{C}_9 = \bar{C}_9 + \frac{2W_9}{W_{10}} \bar{C}_{10} \qquad (87)$$

where $\tilde{c}_i$ is a pseudo-mass fraction defined by the above equations and the subscript $i = 1, 2, ---, 10$ refers to the species H_2O, H_2, O_2, CO_2, CO, N_2, HCl, any inert component, Al, Al_2O_3, respectively. Equations 81-87 are also valid for the products - the $\bar{c}_i$ on the right side replaced by c_i.

The equilibrium constants for the reaction equations 78-80, expressed in mass fractions, are

$$K_P^1 = \frac{W_3}{C_3 P W_M} \left(\frac{C_1 W_2}{W_1 C_2} \right)^2 \qquad (88)$$

$$K_P^2 = \frac{W_3}{C_3 P W_M} \left(\frac{C_4 W_5}{W_4 C_5} \right)^2 \qquad (89)$$

$$K_P^3 = \left(\frac{W_9}{C_9} \right)^2 \left(\frac{W_3}{C_3} \right)^{3/2} (P W_M)^{-7/2} \qquad (90)$$

where the fugacity of Al_2O_3 is assumed to be unity (24), and W_M is the average molecular weight of the gas mixture. Values of the equilibrium constants are obtained from tables of K_p at various temperatures (25). Equations 81-90 are seen to form a system of ten equations for the ten unknown concentrations c_i. The solution for the concentrations comprises an iteration technique discussed in Ref. 6. The net rate of production terms are then calculated from Equation 77.

4.4 Comparison of Equilibrium and Nonequilibrium Chemistry

In order to illustrate the difference between equilibrium and nonequilibrium chemistry, solutions were obtained for a typical problem employing both chemical models for the hydrogen-oxygen system. Shown in Fig. 9 is the temperature profile throughout the mixed flow field at an axial distance of four feet downstream from the exhaust nozzle of a LO_2/LH_2 rocket motor. The conditions of the rocket exhaust and secondary air stream at the exit plane of the nozzle are noted in the figure. As can be seen, the difference between the two cases is not great, thus demonstrating that the selection of equilibrium chemistry provides sufficiently accurate results for the given conditions. As a point of interest, it is worthwhile to note that the machine computation times for equilibrium and nonequilibrium flow were approximately 2 and 110 minutes, respectively. In view of the long computation times required for nonequilibrium flow, and since the assumption of equilibrium flow yields reasonably accurate results, the latter chemical model was assumed for the problem under study herein.

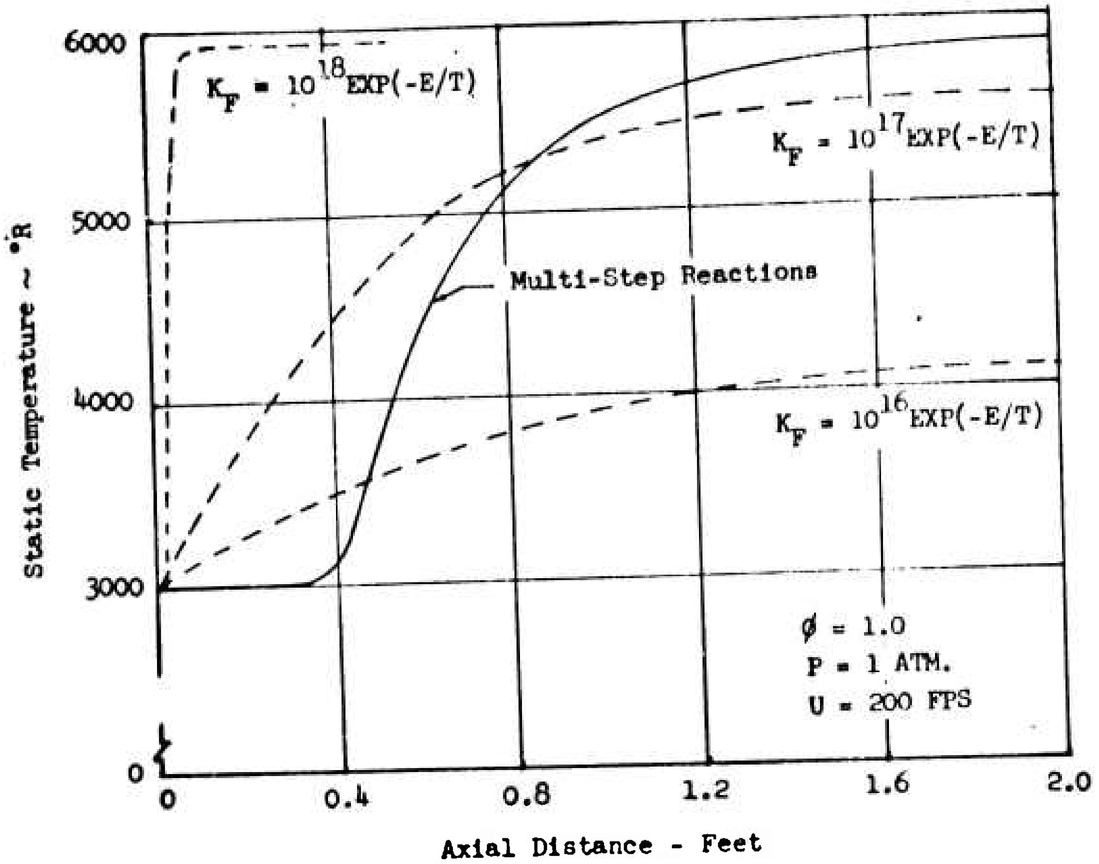

Figure 8 - Effect of Reaction Mechanism and Rate Constants on Theoretical Static Temperature Distribution

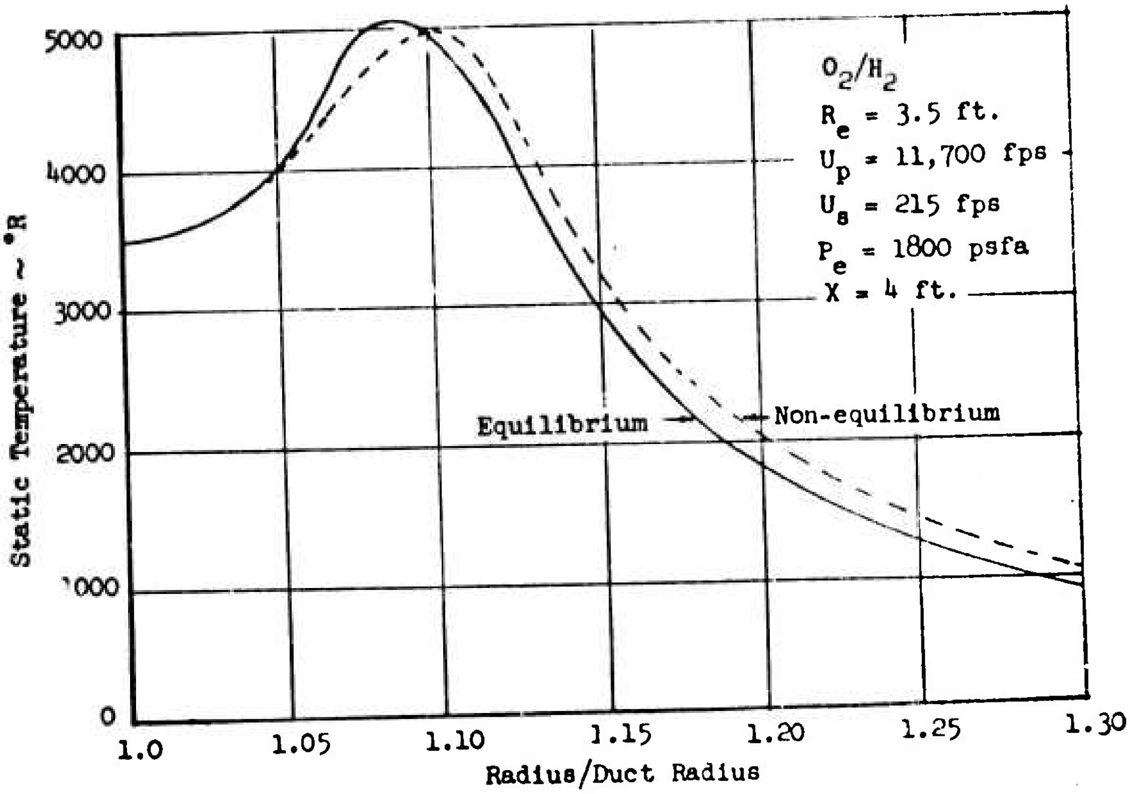

Figure 9 - Comparison of Theoretical Temperature Profiles for Non-Equilibrium and Equilibrium Flow

5.0 COMPUTER PROGRAM

5.1 Capability

Although the computer program described herein was developed specifically for the air-augmentation problem it may, however, be employed for other related problems. For example, the program may be utilized in calculating the mixing region for free jet problems, where the ambient atmosphere is either moving or at rest. In addition, calculations may be performed for problems involving the mixing and combustion of fuel injected into an air stream, with application to ramjet combustor studies. Problems involving chemically reactive species may be solved employing either equilibrium or frozen chemistry. It is noted that the program is limited to the calculation of turbulent flow fields - laminar flow problems being excluded.

In the following sections will be described the input options available to the user, the important features of the program, and the procedure to be followed in establishing the input data for typical problems. For convenience in the discussion that follows a list of the pertinent computer program notation is presented in Appendix 7.1.

5.2 Pressure or Duct Profile Option

In air-augmentation problems it is generally desirable to specify the duct geometry; whereas, in free jet problems it is more convenient to specify the pressure distribution. In the computer program the option is available to specify either the duct profile in the form $r_D = f(x)$, or the pressure distribution in the form $p = g(x)$. The program control for this option is termed INDUCT - having values of 0 or 1 for pressure distribution or duct geometry input, respectively. The input of $p = g(x)$, or $r_D = f(x)$, is in the form of a table. The x-values are termed XTAB and the p- or r_D- values termed PTAB in program notation. It should be noted that, from the viewpoint of machine time, it is advantageous to input the pressure distribution whenever possible.

If the pressure input option is chosen the duct radius is calculated from the initial specified duct radius, denoted RDZERO, and conservation of mass of the flow through the duct. Conversely, if the duct input option is chosen, the pressure at each front is calculated from the initial specified pressure PZERO, and conservation of mass. Problems involving free jets are treated by specifying RDZERO very large compared to the nozzle exit radius RE.

5.3 Reference Streamline

In performing the finite-difference calculations it is necessary that the coordinates of a reference streamline be specified, i.e., $\psi_{ref} = f(x,r)$. Thus the inverse transformation, given by Eqn. 40, may be performed. In the present program the reference ψ-line may be specified as coincident with either the rocket exhaust plume boundary or the centerline of the flow field as shown in Fig. 10. In the former case the coordinates of the plume slipline may be determined utilizing the method of characteristics program of Ref. 5. The slipline coordinates are input to the program in tabular form, denoted as XTAB and RTAB. The input value of RZERO1 is

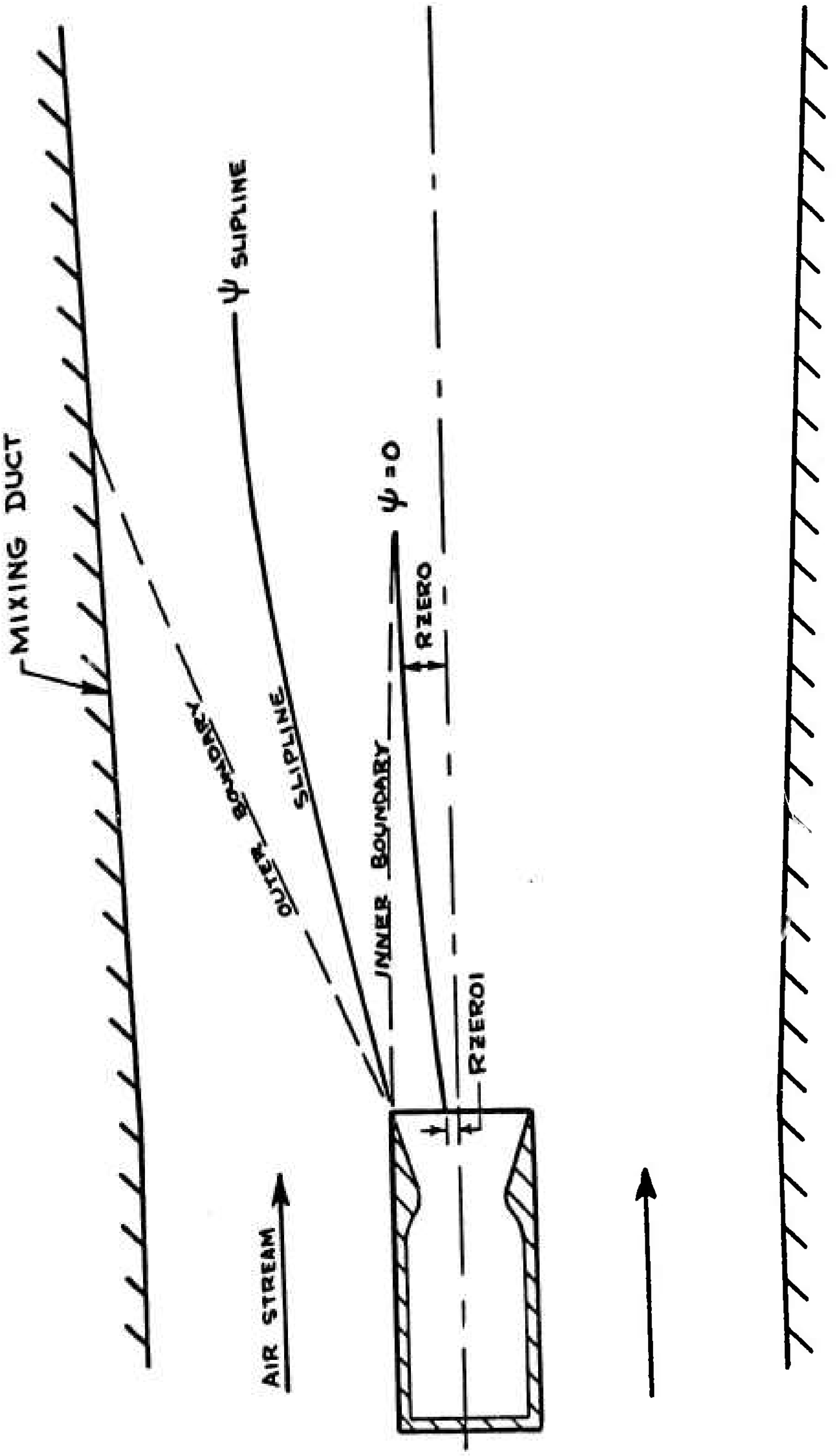

FIG. 10 Location of Reference Streamline Coincident with Plume Slipline

accordingly specified as being slightly larger than zero, e.g., RZERO1 = 0.001 is usually adequate. One problem which is generally encountered using the slipline as ψ_{ref}, is as follows. As the calculations proceed in the x-direction the mixing zone widens, until at a certain distance the mixing zone crosses the $\psi = 0$ streamline. Once this condition occurs the reference ψ-line is shifted to the jet centerline and the calculations continued, or if desired the program is stopped. The control parameter for this option is termed ISET - having values of 0 or 1 for stopping or continuing the program, respectively. For cases where it is desired to specify the centerline as the reference ψ-line the value of RZERO1 and RTAB are set equal to zero and the radius of the jet, respectively.

5.4 Mixing Zone Definition

The width of the mixing zone, as defined herein, is determined from the species concentrations. At a given "front", the calculation of all flow properties begins in the mixing region and proceeds inward and outward until the edges of the mixing region, as established from the following criteria, are determined. At each grid point the difference is calculated between the concentration of species i at the grid point in question and the outer boundary condition of C_i, denoted as $C_{i_{NMAX}}$. This quantity is calculated for each of the species, i = 1-10, and the maximum difference determined. The maximum difference is then compared with an input quantity termed TESMAX. The outer edge of the mixing region is established when

$$\left(C_{iN} - C_{iNMAX}\right)_{MAX} \leq TESMAX \qquad (91)$$

Similarly, the inner edge of the mixing zone is established when

$$(C_{iN} - C_{iNMIN})_{MAX} \leq TESMIN \tag{92}$$

where C_{iNMIN} is the inner boundary condition of C_i, and TESMIN is an input quantity. The values of TESMIN and TESMAX should be sufficiently small so that the mixing zone boundaries determined from species concentrations are larger than the corresponding boundaries determined from velocity or temperature profiles. A value of 0.00001 for both TESMIN and TESMAX is sufficient for most problems - smaller values yield wider mixing zones.

5.5 Stability Criteria

The major difficulties in solving differential equations utilizing the finite-difference technique involve the problems of stability and convergence of the solution. A stable solution is defined as one where small errors, which are inherent in the calculations, do not increase in size. In unstable solutions the errors increase in size during the calculations and eventually become so large that the results are noticeably inaccurate. A solution is defined as convergent if by decreasing the grid size (Δx and $\Delta \psi$) the calculated velocity, temperature, and concentration profiles are not significantly altered. It has been established that if a solution is stable it is also convergent (7, 26).

Employing Karplus' theory of stability it was determined that for the present problem the species continuity equation dictated the most stringent stability criteria (5). Thus in order to insure a stable solution, and consequently a convergent solution, the stability criteria dictates that

$$\Delta\psi \geq \left(2\rho^2 u r^{2\delta} \epsilon_v \Delta x \frac{Le}{Pr}\right)^{1/2} \tag{93}$$

where

$$\epsilon_v = \epsilon_o \left(C + x/L\right)^n \tag{94}$$

In application, the right side of Eqn. 93 is evaluated at each grid point in the mixing region along a given front, and the maximum value determined. For stability the value of $\Delta\psi$ must be greater than or equal to the established maximum value. As can be seen the value of the right side of Eqn. 93 is related to the axial distance. Thus, it becomes evident that if constant values of $\Delta\psi$ and Δx are selected, special consideration must be given to the selection of their values so as to insure stability throughout the calculations.

In order to remove the above difficulty the value of Δx in the present program is assumed to be constant and the value of $\Delta\psi$ altered whenever required during the calculations. The most convenient method of changing $\Delta\psi$ is to merely double its value whenever Eqn. 93 indicates that a change is required. Thus in the present program the stability test is performed at each front, and if required, $\Delta\psi$ is doubled-all the calculations being performed within the computer.

As a result of doubling $\Delta\psi$ it is apparent that certain streamlines, which were calculated prior to the change of $\Delta\psi$, are omitted in subsequent calculations. For example, in the program the ψ-lines are numbered consecutively with the reference ψ-line, which is retained throughout the calculations, assigned the number 1. Then as a result of doubling $\Delta\psi$ the even numbered ψ-lines are omitted in the remaining calculations. After the doubling process the ψ-lines are again numbered consecutively. Thus the ψ-lines numbered N = 3, 5, 7, --- before the doubling process, become N = 2, 3, 4 --- in subsequent fronts. The general equation for tracing an odd numbered ψ-line is

$$n' = \frac{n+1}{2}$$

where n and n′ are the numbered ψ-lines immediately before and after doubling $\Delta\psi$, respectively.

5.6 Determination of Initial Grid Size

The initial grid size, Δx and $\Delta\psi$, may be established from the stability criteria and the maximum possible number of grid points along the initial front (x = 0). Because of computer storage limitations the maximum number of radial grid points is limited to 196. In order to trace the ψ-line originating at the outer periphery of the nozzle it is advantageous to set KSLIP = 129. Thus, for given initial conditions of the jet, the initial value of $\Delta\psi$ may be calculated from the expression

$$\Delta\psi_{INITIAL} = \frac{(\rho u)_{NMIN}\,(RE)^{\delta+1}}{(\delta+1)\ KSLIP} \qquad (95)$$

Once $\Delta\psi$ is established the value of Δx may then be determined from the stability criteria. Thus from Eqn. 93

$$\Delta x \leq \frac{\Delta\psi^2 Pr}{2\rho^2 u r^{2\delta} \epsilon_v Le} \tag{96}$$

where, initially, the value of x in Eqn. 94 may be taken as Δx, and the product $\rho^2 u r^{2\delta}$ is evaluated at the radius of the jet RE. This value of Δx is sufficient to insure stability at the first front.

However, for subsequent fronts it is generally not sufficient, so that a value somewhat smaller than that given above should be employed. From a consideration of machine time the value of Δx generally should be such that

$$\frac{x_{MAX}}{\Delta x} < 3000$$

A smaller value of Δx than that given above may be employed if required; however, it should be noted that machine time may be increased significantly.

5.7 Input Procedure

In order to describe the input procedure it is convenient to refer to Fig. 11 wherein is presented the standard input form for the computer program. The method of selecting the values for each parameter on the input form is discussed below.

NMIN (Two Options)

The two options available for NMIN are 0 or the number assigned to the ψ-line corresponding to the inner edge of the mixing region. The

12A6	TITLE									
1514	NMIN	NMAX	KSLIP	NS	NCHEM	NCP8	NXTAB	INDUCT	ISET	(Keypunch - right adjust)
8E10.0	DX	XMAX	XPRINT	RZERO1	RE	RDZERO	PZERO	DPDXO		
8E10.0	DELTA	TESMAX	TESMIN	A	FL	EPSON	FCON	PRT		
2E10.0	FLETG	FLETP								
3E10.0	XTAB	RTAB	PTAB							
A6,4XE10.0	GENN	W_8								Input if and only if NCP8 > 0.
2E10.0	T_{CP8}	C_{P8}								
8E10.0	C1	C2	C3	C4	C5	C6	C7	C8		Input if and only if NMIN is input > 0.
	Use a separate sheet to input the species concentration for the first eight species with N running from NMIN to NMAX.									
4E10.0	C9	C10	T	U						
	Use a separate sheet to input the last two species concentration and the temperature and the X-component of velocity for N running from NMIN to NMAX.									
1E10.0	X_{ZERO}									
8E10.0	$C1_1$	$C2_1$	$C3_1$	$C4_1$	$C5_1$	$C6_1$	$C7_1$	$C8_1$		Input if and only if NMIN is input ≤ 0.
8E10.0	$C9_1$	$C10_1$	$C1_{NMAX}$	$C2_{NMAX}$	$C3_{NMAX}$	$C4_{NMAX}$	$C5_{NMAX}$	$C6_{NMAX}$		
8E10.0	$C7_{NMAX}$	$C8_{NMAX}$	$C9_{NMAX}$	$C10_{NMAX}$	T_1	T_{NMAX}	U_1	U_{NMAX}		

CALC			REVISED	DATE	INPUT SHEET Air Augmented Rocket Mixing Program	
CHECK						
APR					THE BOEING COMPANY	PAGE
APR						

U3 4038 8000

Figure 11 - Input Form for Computer Program

REV LTR

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purpose of the latter option is to permit the input of variable flow properties at the exit plane of the nozzle ($x = 0$), or at any x-location if desired.

NMAX (Two Options)

The two options are NMAX = 196 or the number assigned to the ψ-line corresponding to the outer edge of the mixing region. The purpose of the latter option is discussed above.

KSLIP

KSLIP is the number assigned to the ψ-line originating at the outer periphery of the nozzle ($r = RE$). In most problems it is desirable to set KSLIP = 129 for purposes of tracing that ψ-line; i.e., not losing the ψ-line when $\Delta\psi$ is doubled. Other values which may be employed for KSLIP are 65, 33, 17, 9, or 5.

NS

The parameter NS is employed to permit a curve fit of the flow properties between the rocket exhaust and secondary streams at $x = 0$, thereby removing the discontinuity of properties at $r = RE$. The value of NS may be NS = 0 or any small integer, e.g., NS = 3 is generally employed.

NCHEM (Two Options)

The two options are NCHEM = 0 or 1 corresponding to equilibrium or frozen chemistry, respectively.

NCP8

The value of NCP8 is the number of temperature values in the table of

TCP8 vs C_{P8} for the general species.

NXTAB

The value of NXTAB is the number of x-values in the table of XTAB vs. PTAB.

INDUCT (Two Options)

The two options are INDUCT = 0 or 1 corresponding to the selection of pressure distribution or duct geometry input, respectively.

ISET (Two Options)

The two options are ISET = 0 or 1 corresponding to the following conditions:

ISET = 0; if RZERO1 $\neq$ 0 the program will stop when the inner edge of the mixing zone reaches the N=1 ψ-line.

ISET = 1; the program will not stop as above, but will shift the reference ψ-line to the centerline and continue the calculations.

DX

The axial step size may be determined using the method outlined in Section 5.5.

XMAX

The maximum axial distance over which the calculations are desired is to be expressed in feet.

XPRINT

The value of XPRINT should be selected so that the ratio XPRINT/DX is an integer.

RZERO1

The initial r-value of the reference ψ-line may be selected as 0 or any positive value depending on whether the reference ψ-line is desired to be located on the jet centerline or the plume slipline, respectively (see Section 5.3).

RE

The value of the nozzle radius is to be expressed in feet.

RDZERO

The value of the initial duct radius is to be expressed in feet.

PZERO

The initial pressure in the secondary stream is to be expressed in PSFA.

DPDXO

The initial value of the pressure gradient may be input if known. Otherwise, the value 0 may be employed.

DELTA (Two Options)

The two options are 0 to 1 corresponding to a two-dimensional or axisymmetric flow model, respectively.

TESMAX and TESMIN

The values employed for TEXMAX and TESMIN determine the width of the mixing zone defined by species concentration - smaller values yielding wider mixing zones. A value of 0.00001 is generally satisfactory for both parameters (see Section 5.4).

A, FL, EPSON, and FCON

A complete discussion of these parameters is presented in Section 3. Values generally employed for the above parameters are

$$A = FL = 1$$

$$FCON = 0$$

The value of EPSON must be determined for each problem from empirical expressions as discussed in Section 3.3.

PRT, FLETG, and FLETP

The turbulent Prandtl and Lewis numbers of the gas may be specified as 0.8 and 1.2, respectively. A particle Lewis number of 0.6 may be employed until data become available for more accurately establishing its value.

XTAB, RTAB, and PTAB

The data for the plume slipline coordinates and the pressure distribution (or duct radius) is input in tabular form. For cases where the slipline geometry is unknown a value of RTAB equal to the nozzle exit radius RE, should be employed for all XTAB values. It should be noted that the input of pressure or duct radius at various axial distance XTAB, must be compatible with the input of the control parameter INDUCT.

GENN and W_8

If the user desires to incorporate an additional chemical species, other than any of those specified in the program, the chemical symbol of the component is input for GENN and the corresponding molecular weight for W_8.

T_{CP8} and C_{P8}

If an additional chemical species is included the specific heat of the component as a function of temperature is input in tabular form. It should be noted that this table, as well as GENN and W_8, are input only if an additional species is desired. Otherwise, these parameters may be omitted.

G_i, T, and U (Two Options)

The two options available to the user correspond to whether or not it is desired to input variable properties as the initial conditions. If the variable property option is selected NMIN must be different than zero. If the constant property option is selected the value of NMIN must be zero.

The variable properties are input as a function of N. The constant properties are input for the rocket exhaust (C_{i_1}, T_1, u_1,) and the secondary stream ($C_{i_{NMAX}}$, T_{NMAX}, u_{NMAX}).

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7.0 APPENDICES

7.1 Computer Program Notation

A	program notation for the exponent n, in Eqn. 94
C_{iNMAX}	outer boundary condition for the concentration of species i
C_{iNMIN}	inner boundary condition for the concentration of species i
CP8	specific heat of the general species (i = 8)
DELTA	$\delta = 0$ for two-dimensional model $\delta = 1$ for axisymmetric model
DPDXO	initial pressure gradient dp/dx
DX	axial step size Δx
EPSON	Reichardt's constant ϵ_o, in Eqn. 94
FCON	constant C, in Eqn. 94
FL	reference length L, in Eqn. 94
FLETG	turbulent Lewis number of the gas
FLETP	turbulent Lewis number of the particles
GENN	molecular formula for the general species (i = 8)
INDUCT	INDUCT = 0 for pressure distribution input; INDUCT = 1 for duct profile input
ISET	ISET = 0 stops program when NMIN = 1 and $r_o \neq 0$ ISET = 1 program continues after shifting r_o to the centerline
KSLIP	number assigned to the ψ-line originating at the lip of the nozzle ($r = r_e$)
NCHEM	NCHEM = 0 corresponds to equilibrium flow; NCHEM = 1 corresponds to frozen flow

7.1 Computer Program Notation (cont'd)

NCP8	number of temperature values in the table of CP8 vs T
NMAX	maximum number of ψ-lines desired (NMAX = 196)
NMIN	NMIN = 0 for constant property input; NMIN ≠ 0 for variable property input
NS	number of ψ-lines within which the flow properties between the inner and outer streams are curve-fitted
NXTAB	number of values in the table of X vs PTAB
PRT	turbulent Prandtl number
PTAB	value of r_D (or P) in XTAB table
PZERO	initial static pressure in the outer stream
RDZERO	initial radius of the duct
RE	radius of the nozzle
RTAB	radius of the slipline in the XTAB table
RZERO1	initial radius of the reference ψ-line
T	static temperature
T_{CP8}	temperature values in the table of T vs CP8
TESMAX	parameter used in determining NMAX
TESMIN	parameter used in determining NMIN
V	axial velocity
W_8	molecular weight of the general species (i = 8)
XMAX	maximum axial distance over which the calculations are performed
XPRINT	axial increments for print-out of data

7.1 Computer Program Notation (cont'd)

XTAB	values of x in the XTAB table
XZERO	initial x-value

7.2 Comparison of Theory and Experiment

Theoretical results obtained from the computer program are compared herein with available experimental data. In establishing the correlations the model employed for the eddy viscosity, as described in Section 4, was employed. In certain cases the turbulent transport coefficients were varied about their predicted values in order to obtain better correlations. In such instances the variation from the predicted values is discussed. It is noted that analytical results were compared with data obtained from air-augmentation experiments; however, due to the classified nature of the experimental data the correlation is not presented herein (see Ref. 28).

In the experiments conducted by Maydew and Reed (15) radial static and total pressure surveys were made at various axial locations downstream of an air jet exhausting into a quiescent atmosphere. Shown in Figure 12 are the theoretically and experimentally determined velocity profiles at 11.5 inches downstream of an air jet having a Mach number of 1.96. The parameter y' is the radial distance measured from the point where the velocity has decayed to one-half its value at the inner edge of the mixing zone, σ is the similarity parameter, X_e is the effective length of the mixing zone, and V_1 is the velocity at the inner edge of the mixing zone. The solid curve represents the theoretical results obtained from the present analysis employing a value of Reichardt's coefficients corresponding to the value of σ determined experimentally by Maydew and Reed. As can be seen the predicted velocities agree quite well with the experimental values over the entire mixing region. Similar agreement was obtained between the experimental and theoretical velocity profiles at axial locations nearer the nozzle exit.

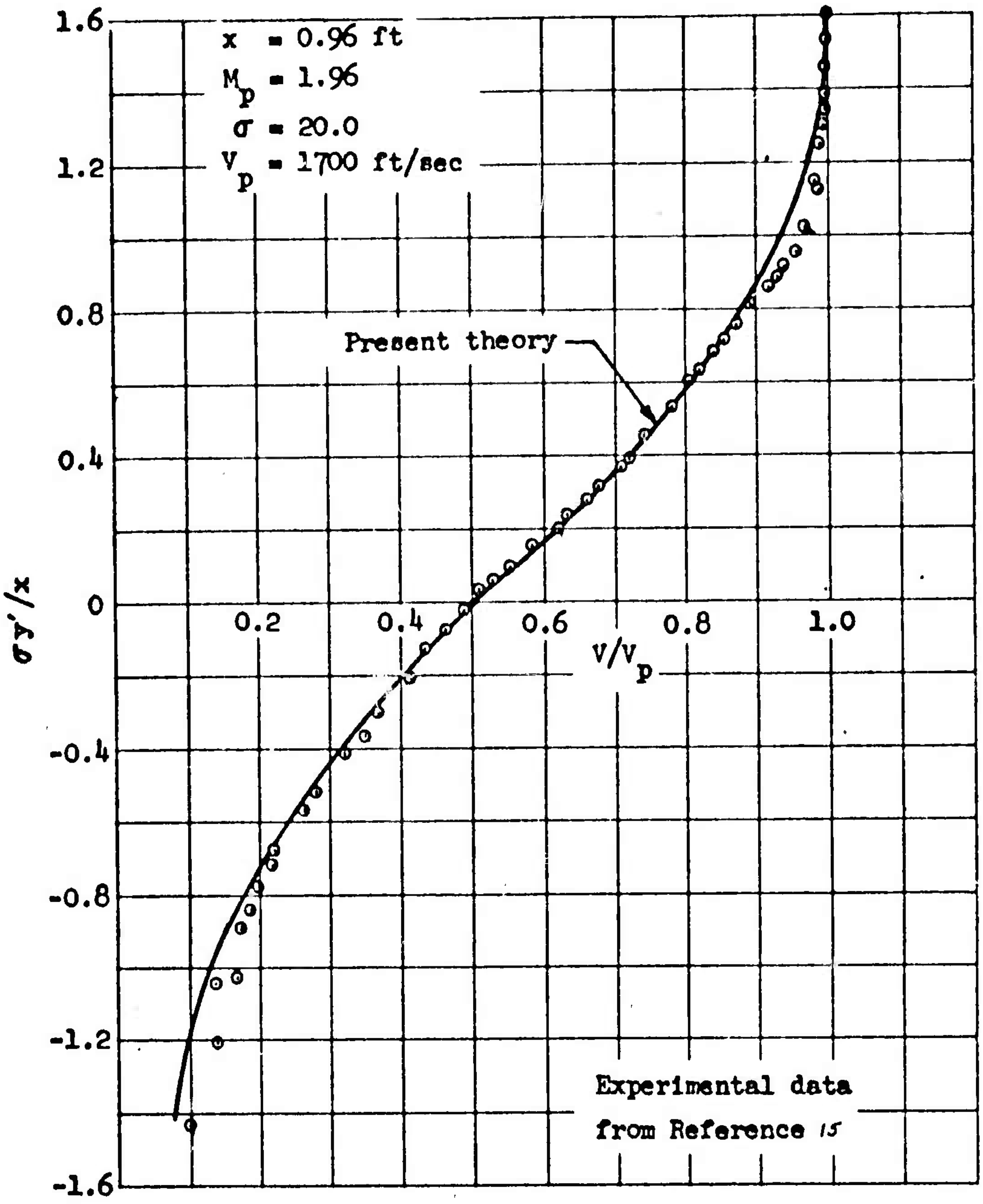

Figure 12 - Comparison of theory and experiment for free jet velocity profiles

In the experiments of Forstall and Shapiro (19) the mixing region between a circular jet and an annular coaxial stream was investigated. By using 10 percent by volume of helium as a tracer in the jet, the profiles of concentration and velocity in the mixing region were determined. Shown in Figure 13 is a comparison of the theoretical and experimental velocity and concentration profiles at an axial distance of X/D = 8. As can be seen reasonable agreement exists between the theoretical and experimental profiles. As noted in Ref. 19 the Prandtl and Schmidt numbers were both determined to be approximately 0.7, which corresponds to a Lewis number of unity. Theoretical results for two values of the Lewis number are shown for the concentration profiles in Figure 13. Small variations of the Lewis number are seen to effect little change in the profiles.

The experimental investigation of Zakkay, et al., was conducted to determine the turbulent mixing of coaxial jets for both subsonic and supersonic flow. In establishing a correlation, the data obtained with argon as the central stream was employed. The exit Mach number of the argon jet was approximately unity. The Mach number of the annular air stream was 1.6. Shown in Figure 14 is the comparison of the theoretical and measured velocity and concentration profiles at various axial locations along the jet centerline. In establishing the correlation the value employed for Reichardt's coefficient was $\epsilon_o = 0.5$ ft^2/sec from X = 0 - 0.42 ft, and $\epsilon_o = 1.0$ ft^2/sec from X = 0.42 - 0.75. As can be seen good agreement exists between the theoretical and measured concentration profiles at each axial location. The velocity profile at X = 0.75 ft indicates that theory predicts slightly higher velocity ratios than those measured experimentally, which possibly may be attributed to an incorrect form for the eddy viscosity expression.

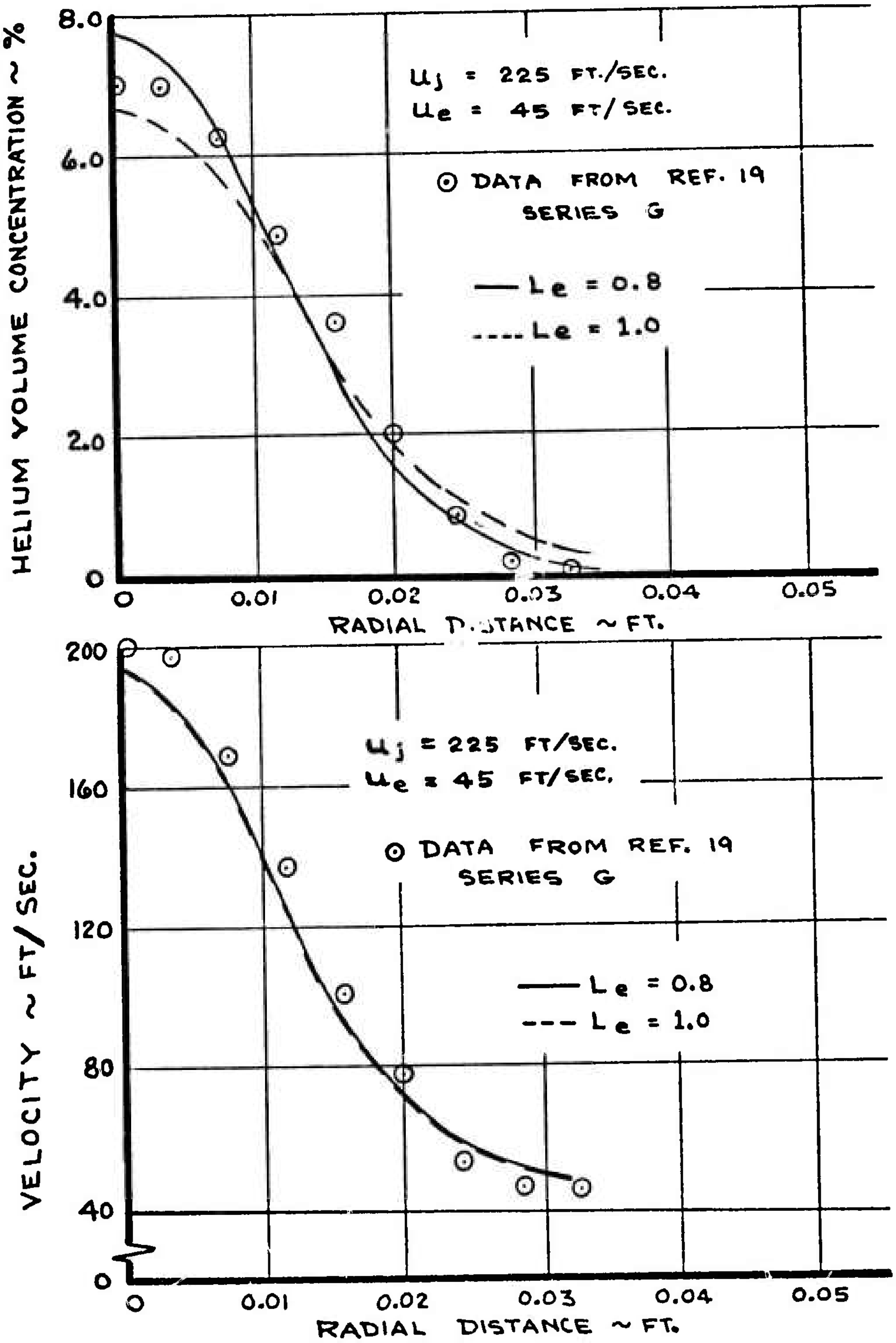

Fig. 13 COMPARISON OF THEORY AND EXPERIMENT

However, judging from the accuracy of the predicted concentration profiles and the fair agreement of the velocity profiles, it is concluded that the present eddy viscosity model is adequate for most engineering problems.

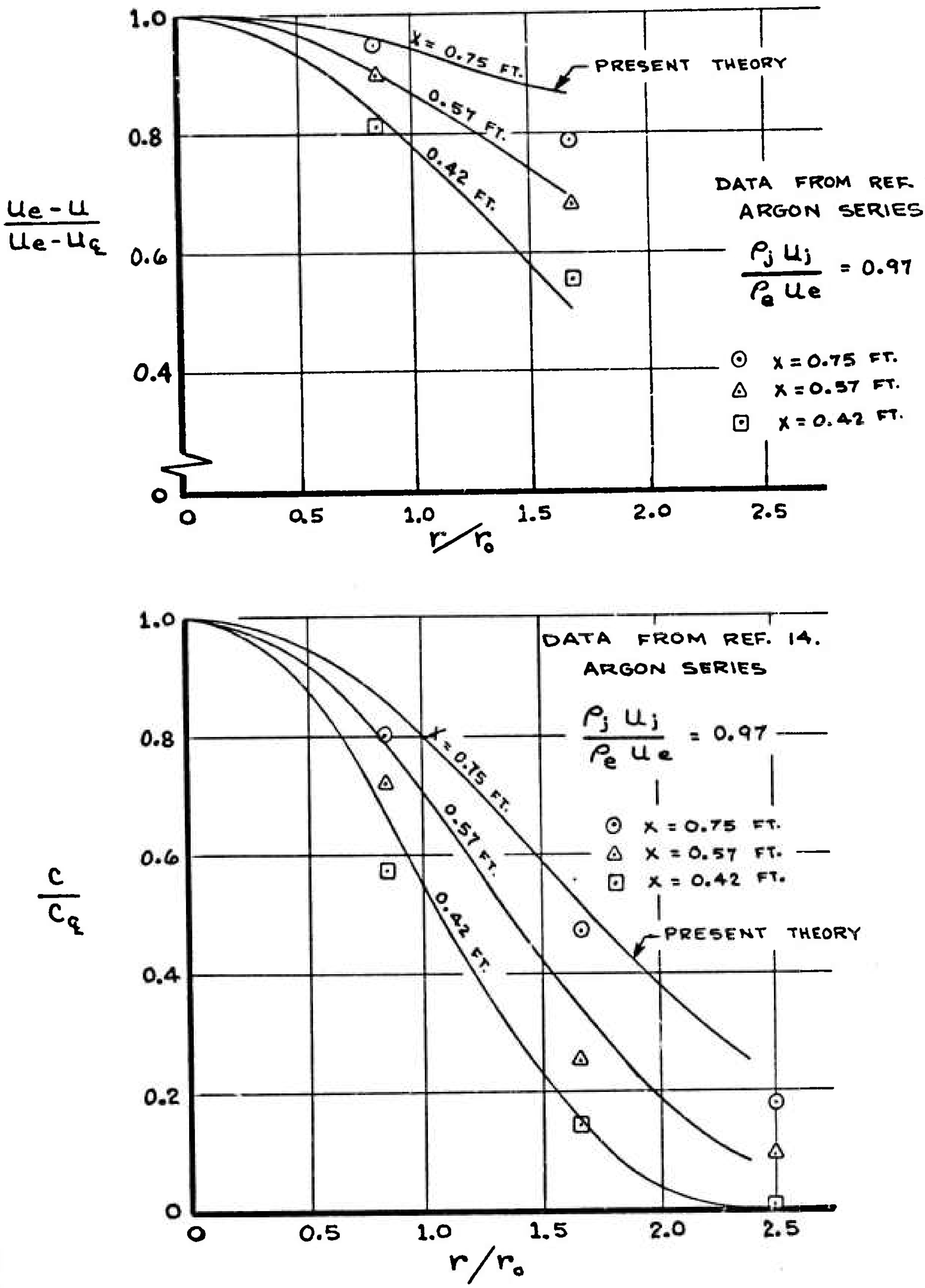

Fig. 14 COMPARISON OF THEORY AND EXPERIMENT

7.3 Sample Problem

In order to illustrate the utility of the computer program the solution of a typical air-augmentation problem is presented. Consider a rocket motor with the following operating conditions:

propellant - solid (16% Al)
nozzle exit radius - 0.63 ft.
exit velocity - 5570 ft/sec
exit static pressure - 45,000.0 psfa
exit static temperature - 4385°R
density - 0.035 lbm/ft^3
Mach number - 1.5

The secondary stream is assumed to be air with the following flow conditions at the exit plane of the nozzle:

static temperature - 1094°R
static pressure - 11992 psfa
velocity - 856 ft/sec
duct diameter - 2.52 ft.

It is desired to perform the calculations over an axial distance of 1.575 ft. The duct is assumed to be constant area. The preparation of the input data, presented in Fig. 15, is as follows.

NMIN

Since the flow properties of the jet are assumed constant at the nozzle exit plane, the value of NMIN is zero.

NMAX

NMAX is set equal to 196.

KSLIP

The value of KSLIP is taken to be

KSLIP = 129

NS

The value of NS is arbitrarily taken to be

NS = 3

NCHEM

It is desired to perform the calculations for equilibrium flow. Thus

NCHEM = 0

NCP8

Since it is not required to employ an additional species, other than those specified in the program, the value of NCP8 is

NCP8 = 0

NXTAB

The number of x-values in the XTAB table is

NXTAB = 6

INDUCT

It is desired to input the duct geometry, thus

INDUCT = 1

ISET

It is desired to continue the run beyond the point where the mixing zone crosses the reference streamline, thus

$$ISET = 1$$

DX

The value of DX is established from the stability criteria in the following manner. From Eqn. 94, 95 and 96 the initial value of ΔX required for stability is A

$$\Delta X \leq \frac{RE\,(U_{NMIN})^{1/2}}{2\ KSLIP\,(2\,\epsilon_0\,Le/P_r)^{1/2}}$$

$$= \frac{0.63\,(5570)^{1/2}}{2 \cdot 129 \cdot (2 \cdot 4.95 \cdot 1.2/0.8)^{1/2}}$$

$$= 0.0475 \text{ ft}$$

The value of ΔX is taken somewhat smaller than the above value in order to maintain a sufficiently large number of grid points in the mixing region (see Section 5.5). Thus

$$DX = 0.005 \text{ ft.}$$

XMAX

The value of XMAX is

$$XMAX = 1.575 \text{ ft.}$$

XPRINT

The desired increment between printout of data is

XPRINT = 0.25 ft.

RZERO1

Since the static pressure of the secondary stream is considerably less than that of the rocket exhaust gases it is desired that the reference streamline be located along the plume slipline. Thus

RZERO1 = 0.001 ft.

RE

The radius of the nozzle at the exit is

RE = 0.63 ft.

RDZERO

The initial value of the duct radius is

RDZERO = 2.52 ft.

PZERO

The initial pressure of the secondary stream is

PZERO = 11992.0 psfa

DPDXO

The initial pressure gradient is obtained from the method of characteristics program employed to determine the slipline coordinates (5). Thus

DPDXO = -800.0 psfa/ft.

DELTA

The flow is axisymmetric, thus

DELTA = 1.0

TESMAX and TESMIN

The values of TESMAX and TESMIN are taken to be

$$\text{TESMAX} = \text{TESMIN} = 0.00001$$

A, FL, EPSON, FCON

It is assumed that the eddy viscosity expression is of the form

$$\epsilon_v = \epsilon_o X$$

Thus, from Eqn. 94 it is seen that

$$A = 1.0$$

$$FL = 1.0$$

$$FCON = 0$$

The value of EPSON is determined from the equation

$$\text{EPSON} = \epsilon_o = \frac{\bar{U}}{2\sigma^2}$$

where σ is determined from Figures 6 and 7, and $\bar{U}$ is the average velocity between the two streams. Thus

$$\bar{U} = \frac{856 + 5570}{2}$$

$$= 3213 \text{ ft/sec}$$

From Figures 6 and 7 the values of σ_{II}/σ_I and σ_{II} are determined to be

$$\sigma_{II}/\sigma_I = 1.2$$

$$\sigma_{II} = 15 \times 1.2 = 18 \text{ ft}^{-\frac{1}{2}}$$

which yields

$$\epsilon_o = \frac{3213}{2 \times 18^2}$$

$$= 4.95 \text{ ft}^2/\text{sec}$$

PRT, FLETG, FLETP

These values are assumed to be

$$PRT = 0.8$$

$$FLETG = 1.2$$

$$FLETP = 0.6$$

XTAB, RTAB, PTAB

The values of XTAB and RTAB are determined from the method of characteristics program (5). The values of PTAB are the duct radii at the various XTAB's. For the present problem PTAB is assumed constant at

$$PTAB = 2.52 \text{ ft.}$$

GENN, W8, TCP8, CP8

Since a general species is not required the input for these parameters is omitted.

C_{i1} and C_{iNMAX}

The concentration of the species in the rocket exhaust are determined from an equilibrium thermochemical program (27). The secondary stream is assumed to be air.

T_1 and V

The temperatures and velocities are as follows:

$$T_1 = 4385°R$$

$$T_{NMAX} = 1094°R$$

$$U_1 = 5570 \text{ ft/sec}$$

$$U_{NMAX} = 856 \text{ ft/sec}$$

The input form for the above sample problem is presented in Fig. 15.

12A6

TITLE	
BC-3276-2	☒☒☒

1514

NMIN	NMAX	KSLIP	NS	NCHEM	NCP8	NXTAB	INDUCT	ISET
	196	129				6	1	1

(Keypunch - right adjust)

8E10.0

DX	XMAX	XPRINT	RZERO1	RE	RDZERO	PZERO	DPDXO
0.005	1.575	0.25	0.001	0.63	2.52	11982.0	-800.0

8E10.0

DELTA	TESMAX	TESMIN	A	FL	EPSON	FCON	PRT
1.0	0.00001	0.00001	1.0	1.0	4.95		0.8

2E10.0

FLETG	FLETP
1.2	0.6

3E10.0

XTAB	RTAB	PTAB
0.0	0.63	2.52
0.01	0.64	↓
0.08	0.68	
0.5	0.87	
1.58	1.21	
2.5	1.31	2.52

A6,4XE10.0

GENN		W_8
	☒	

2E10.0

T_{CP8}	C_{P8}

Input if and only if NCP8 > 0.

8E10.0

C1	C2	C3	C4	C5	C6	C7	C8

Use a separate sheet to input the species concentration for the first eight species with N running from NMIN to NMAX.

4E10.0

C9	C10	T	U

Use a separate sheet to input the last two species concentration and the temperature and the X-component of velocity for N running from NMIN to NMAX.

1E10.0

X_{ZERO}

Input if and only if NMIN is input > 0.

8E10.0

$C1_1$	$C2_1$	$C3_1$	$C4_1$	$C5_1$	$C6_1$	$C7_1$	$C8_1$
0.0914	0.0226		0.0239	0.282	0.0851	0.193	

8E10.0

$C9_1$	$C10_1$	$C1_{NMAX}$	$C2_{NMAX}$	$C3_{NMAX}$	$C4_{NMAX}$	$C5_{NMAX}$	$C6_{NMAX}$
	0.302			0.233			0.767

8E10.0

$C7_{NMAX}$	$C8_{NMAX}$	$C9_{NMAX}$	$C10_{NMAX}$	T_1	T_{NMAX}	U_1	U_{NMAX}
				4385.0	1094.0	5570.0	856.0

Input if and only if NMIN is input ≤ 0.

			REVISED	DATE	
CALC					INPUT SHEET
CHECK					Air Augmented Rocket Mixing Program
APR					THE BOEING COMPANY
APR					PAGE

U3 4038 8000

Figure 15 - Input Form for Sample Problem

```
                                     INITIAL INPUT
  BC-3276-2
     NMIN     NMAX    KSLIP       NS    NCHEM     NCPA    NXTAB  INDUCT     MMAX  LFRONT     ISET
      123      135      129        3      -0        0        6       1      501      50        1
         DX         XMAX          YPRINT         RZERO1             RE         RDZERO        PZERO        DPDXO
  .5000000-02   1.5750000     .25000005      .1000000-02     .63000004      2.5200000     11991.999    -800.00000
      DELTA         TESMAX         TESMIN             A              FL          EPSON         FCON          PRT
  1.0000000     .1000000-04    .1000000-04    1.0000000       1.0000000      4.9499999    -.0000000     .60000004
      FLETG         FLETP            ETA
  1.2000000     .60000004      1.0000000
       XTAB          RTAB           PTAB
   .0000000     .63000004      2.5200000
   .1000000-01  .64000005      2.5200000
   .8000000-01  .68000004      2.5200000
  .50000005     .87000003      2.5200000
  1.5800000     1.2100000      2.5200000
  2.5000000     1.3100000      2.5200000
       DPSI             X            PSTD
  .30223922     .0000000       550.99927


  PROPERTIES GIVEN BELOW ARE AT FRONT      1     DISTANCE DOWNSTREAM =          .0000FEET     PRESSURE=    11991.9999PSF

  INITIAL FLOW IN SECONDARY STREAM =        3275.4638 LBS/SEC
  RADIUS OF DUCT =           2.5200 FT.    THRUST =             .0000 LB.    DUCT AREA =          19.9504 SQ FT.

     N        Y         X-COMP     TEMPERATURE   DENSITY      MACH
                       VELOCITY                              NUMBER
     1     .0000       5570.0       4384.9     .3496-01    1.4804
   123    .61549       5570.0       4384.9     .3496-01    1.4804
   124    .61801       5570.0       4384.9     .3496-01    1.4864
   125    .62051       5570.0       4384.9     .3496-01    1.4804
   126    .62301       5570.0       4384.9     .3496-01    1.4804
   127    .62547       5220.8       4141.2     .3825-01    1.4549
   128    .62783       4347.8       3531.7     .4650-01    1.3582
   129    .63005       3213.0       2739.5     .6869-01    1.1809
   130    .63221       2078.1       1947.2     .1054+00    .94304
   131    .63450       1205.1       1337.7    .16334       .67377
   132    .63708       856.00       1094.0    .20458       .53228
   133    .63980       856.00       1094.0    .20458       .53228
   134    .64240       856.00       1094.0    .20458       .53228
   135    .64517       856.00       1094.0    .20458       .53228

     N  - - - - - - - - - - - - - - - - C O N C E N T R A T I O N   O F   S P E C I E S - - - - - - - - - - - - - - - - -
```

FIG. 16 SAMPLE OUTPUT DATA

```
          H2O          H2          O2            CO2         CO          N2         HCL                     AL         AL2O3
  1     .9140-01    .2260-01    -.0000       .2390-01    .28204     .8510-01    .19304      .0000     -.0000     .30205
123     .9140-01    .2260-01    -.0000       .2390-01    .28204     .8510-01    .19304      .0000     -.0000     .30205
124     .9140-01    .2260-01    -.0000       .2390-01    .28204     .8510-01    .19304      .0000     -.0000     .30205
125     .9140-01    .2260-01    -.0000       .2390-01    .28204     .8510-01    .19304      .0000     -.0000     .30205
126     .9140-01    .2260-01    -.0000       .2390-01    .28204     .8510-01    .19304      .0000     -.0000     .30205
127     .8463-01    .2093-01     .1726-01    .2213-01    .26116    .13566       .17975      .0000     -.0000     .27967
128     .6770-01    .1674-01     .6041-01    .1770-01    .20893    .26193       .14301      .0000     -.0000     .22375
129     .4570-01    .1130-01     .1165+00    .1195-01    .14104    .42609        .9650-01   .0000     -.0000     .15105
130     .2370-01    .5859-02    .17264       .6196-02     .7311-01 .59026        .5004-01   .0000     -.0000      .7830-01
131     .6770-02    .1674-02    .21579       .1770-02     .2089-01 .71653        .1430-01   .0000     -.0000      .2237-01
132    -.0000      -.0000       .23304      -.0000      -.0000     .76704      -.0000       .0000     -.0000     -.0000
133    -.0000      -.0000       .23304      -.0000      -.0000     .76704      -.0000       .0000     -.0000     -.0000
134    -.0000      -.0000       .23304      -.0000      -.0000     .76704      -.0000       .0000     -.0000     -.0000
135    -.0000      -.0000       .23304      -.0000      -.0000     .76704      -.0000       .0000     -.0000     -.0000

DELTA PSI WAS SET =   .60452

DELTA PSI WAS SET =   1.2090



PROPERTIES GIVEN BELOW ARE AT FRONT    51       DISTANCE DOWNSTREAM =          .2500FEET     PRESSURE=    11730.5382PSF
RADIUS OF DUCT =           2.5202 FT.     THRUST =         31.6754 LB.     DUCT AREA =          13.9531 SQ FT.


   N        Y         X-COMP    TEMPERATURE   DENSITY      MACH
                     VELOCITY                             NUMBER
   1     .43375      5613.4      4362.5      .3437-01    1.5017
  26     .70815      5613.4      4362.5      .3437-01    1.5017
  27     .71695      5613.4      4362.5      .3437-01    1.5017
  28     .72564      5613.4      4362.5      .3437-01    1.5017
  29     .73422      5613.1      4362.8      .3437-01    1.5016
  30     .74270      5607.3      4367.4      .3437-01    1.4999
  31     .75117      5536.5      4414.5      .3436-01    1.4803
  32     .75975      5024.9      4707.8      .3453-01    1.3485
  33     .77083      3072.7      5285.5      .3900-01     .88578
  34     .78476      1445.2      2898.6      .7645-01     .57441
  35     .79739      1067.8      1733.8     .12735        .53893
  36     .80760      960.84      1345.3     .16352        .54454
  37     .81668      923.55      1191.5     .18420        .55293
  38     .82515      909.90      1127.3     .19443        .55845
  39     .83331      905.17      1101.4     .19886        .56130
  40     .84130      903.71      1091.9     .20053        .56252
  41     .84918      903.32      1088.8     .20107        .56297
  42     .85699      903.23      1088.0     .20127        .56311
  43     .86471      903.21      1087.8     .20127        .56314
  44     .87237      903.21      1087.7     .20127        .56314
  45     .87997      903.21      1087.7     .20127        .56314

   N  - - - - - - - - - - - - - - - - C O N C E N T R A T I O N   O F   S P E C I E S - - - - - - - - - - - - - - - -
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FIG. 16 (CONT'D)

```
        H2O         H2          O2          CO2         CO          N2          HCL                     AL          AL203
  1   .9140-01   .2260-01   -.0000       .2390-01   .28204      .8510-01   .19304      .0000      -.0000      .30205
 26   .9140-01   .2260-01   -.0000       .2390-01   .28204      .8510-01   .19304      .0000      -.0000      .30205
 27   .9140-01   .2260-01   -.0000       .2390-01   .28204      .8510-01   .19304      .0000      -.0000      .30205
 28   .9140-01   .2260-01   -.0000       .2390-01   .28204      .8510-01   .19304      .0000      -.0000      .30205
 29   .9144-01   .2258-01    .0000       .2395-01   .28191      .8533-01   .19298      .0000       .0000      .30204
 30   .9191-01   .2240-01    .2665-06    .2448-01   .28039      .8600-01   .19221      .0000       .0000      .30189
 31   .9562-01   .2101-01    .2421-05    .2624-01   .26913      .1084+00   .16645      .0000       .4146-07   .29923
 32   .1138+00   .1446-01    .1697-04    .4415-01   .21787      .20280     .15974      .0000       .5132-06   .27256
 33   .1187+00   .1427-02    .2292-01    .1166+00   .5843-01    .46278     .8612-01    .0000       .2308-06   .14024
 34   .4122-01   .3883-07    .15295      .6530-01   .1127-05    .67170     .2698-01    .0000       .0000      .2820-01
 35   .1562-01   .1469-06    .20265      .2474-01   .6758-05    .73091     .1023-01    .0000       .0000      .6903-02
 36   .6845-02   .1128-06    .21072      .1084-01   .5626-05    .75121     .4482-02    .0000       .0000      .1939-02
 37   .3085-02   .5435-07    .22704      .4683-02   .3461-05    .75991     .2020-02    .0000       .0000      .5388-03
 38   .1338-02   .0000       .23044      .2118-02   .1837-05    .76395     .8765-03    .0000       .0000      .1358-03
 39   .5370-03   .0000       .23200      .8496-03   .8529-06    .76580     .3517-03    .0000       .0000      .2963-04
 40   .1929-03   .0000       .23267      .3052-03   .3465-06    .76660     .1263-03    .0000       .0000      .5451-05
 41   .5826-04   .0000       .23293      .9216-04   .1155-06    .76691     .3816-04    .0000       .0000      .8206-06
 42   .1154-04   .0000       .23302      .1825-04   .2427-07    .76702     .7558-05    .0000       .0000      .7428-07
 43  -.0000     -.0000       .23304     -.0000     -.0000       .76704    -.0000       .0000      -.0000     -.0000
 44  -.0000     -.0000       .23304     -.0000     -.0000       .76704    -.0000       .0000      -.0000     -.0000
 45  -.0000     -.0000       .23304     -.0000     -.0000       .76704    -.0000       .0000      -.0000     -.0000

 DELTA PSI WAS SET =    2.4179


 PROPERTIES GIVEN BELOW ARE AT FRONT   101     DISTANCE DOWNSTREAM =        .5000FEET     PRESSURE=    11444.9935PSF
 RADIUS OF DUCT =          2.5200 FT.    THRUST =          7.5641 LB.    DUCT AREA =         19.9510 SQ FT.

                       X-COMP    TEMPERATURE    DENSITY     MACH
  N       Y           VELOCITY                              NUMBER
  1    .59348         5061.3       4337.5      .3373-01    1.5187
 10    .76170         5061.3       4337.5      .3373-01    1.5187
 11    .77815         5061.3       4337.5      .3373-01    1.5187
 12    .79426         5061.3       4337.5      .3373-01    1.5187
 13    .81005         5061.1       4337.8      .3373-01    1.5186
 14    .82552         5050.2       4343.6      .3372-01    1.5169
 15    .84087         5580.1       4404.9      .3361-01    1.4949
 16    .85656         5061.7       4763.2      .3335-01    1.3512
 17    .87555         3125.5       5348.7      .3756-01     .89508
 18    .90110         1494.6       2912.2      .7433-01     .59311
 19    .92280         1125.0       1763.5      .1222+00     .56301
 20    .94018         1018.3       1374.1      .15630       .57109
 21    .95552         979.80       1211.6      .17683       .58173
 22    .96967         962.91       1137.8      .18603       .58803
 23    .98318         955.51       1104.2      .19359       .59257
 24    .99630         954.09       1089.7      .19609       .59435
 25   1.0091          953.27       1083.8      .19710       .59543
 26   1.0218          953.02       1081.7      .19747       .59577
 27   1.0343          952.95       1081.1      .19758       .59588
```

FIG. 16 (CONT'D)

```
28  1.0466       952.94      1080.9      .19761      .59591
29  1.0588       952.94      1080.9      .19762      .59591
30  1.0709       952.94      1080.9      .19762      .59591

N  - - - - - - - - - - - - - - - C O N C E N T R A T I O N   O F   S P E C I E S - - - - - - - - - - - - - - -
       H2O         H2          O2          CO2         CO          N2          HCL                     AL          AL2O3
  1  .9140-01    .2260-01    -.0000      .2390-01    .28204      .8510-01    .19304      .0000      -.0000      .30205
 10  .9140-01    .2260-01    -.0000      .2390-01    .28204      .8510-01    .19304      .0000      -.0000      .30205
 11  .9140-01    .2260-01    -.0000      .2390-01    .28204      .8510-01    .19304      .0000      -.0000      .30205
 12  .9140-01    .2260-01    -.0000      .2390-01    .28204      .8510-01    .19304      .0000      -.0000      .30205
 13  .9144-01    .2259-01     .0000      .2394-01    .28193      .8531-01    .19299      .0000       .0000      .30204
 14  .9190-01    .2238-01     .3081-06   .2453-01    .28026      .8829-01    .19214      .0000       .0000      .30194
 15  .9632-01    .2081-01     .3172-05   .2876-01    .26770      .1109+00    .18574      .0000       .8576      .29963
 16  .1160+00    .1389-01     .2568-04   .4532-01    .21423      .20945      .15786      .0000       .5838-06   .27303
 17  .1173+00    .1521-02     .2588-01   .1127+00    .6033-01    .46406      .8575-01    .0000       .3670-06   .13904
 18  .4079-01    .4235-07     .15391     .6463-01    .1243-05    .67268      .2671-01    .0000       .0000      .2649-01
 19  .1609-01    .1324-05     .20182     .2540-01    .6306-04    .72981      .1054-01    .0000       .0000      .6480-02
 20  .7524-02    .4163-06     .21842     .1189-01    .2057-04    .74963      .4929-02    .0000       .0000      .1921-02
 21  .3708-02    .9943-07     .22583     .5068-02    .5968-05    .75846      .2429-02    .0000       .0000      .5925-03
 22  .1811-02    .0000        .22952     .2866-02    .1943-05    .76286      .1186-02    .0000       .0000      .1741-03
 23  .8456-03    .0000        .23140     .1339-02    .7064-06    .76509      .5537-03    .0000       .0000      .4639-04
 24  .3690-03    .0000        .23233     .5042-03    .2718-06    .76619      .2416-03    .0000       .0000      .1088-04
 25  .1470-03    .0000        .23276     .2328-03    .1005-06    .76670      .9625-04    .0000       .0000      .2205-05
 26  .5116-04    .0000        .23295     .8101-04    .3001-07    .76693      .3350-04    .0000       .0000      .3740-06
 27  .1384-04    .0000        .23302     .2191-04    .0000       .76701      .9062-05    .0000       .0000      .3921-07
 28  -.0000      -.0000       .23304     -.0000      -.0000      .76704      -.0000      .0000      -.0000      -.0000
 29  -.0000      -.0000       .23304     -.0000      -.0000      .76704      -.0000      .0000      -.0000      -.0000
 30  -.0000      -.0000       .23304     -.0000      -.0000      .76704      -.0000      .0000      -.0000      -.0000

PROPERTIES GIVEN BELOW ARE AT FRONT   151      DISTANCE DOWNSTREAM =        .7500FEET     PRESSURE=    11062.7586PSF

RADIUS OF DUCT =          2.5198 FT.    THRUST =        -31.7370 LB.    DUCT AREA =        19.9475 SQ FT.

  N       Y          X-COMP     TEMPERATURE    DENSITY     MACH
                     VELOCITY                              NUMBER
  1   .72517       5726.2       4303.2       .3286-01    1.5419
  8   .84011       5726.2       4303.2       .3286-01    1.5419
  9   .85527       5726.2       4303.3       .3286-01    1.5419
 10   .87017       5726.2       4303.3       .3286-01    1.5419
 11   .88482       5725.8       4304.0       .3286-01    1.5417
 12   .89923       5723.1       4307.8       .3285-01    1.5407
 13   .91344       5706.9       4326.3       .3281-01    1.5350
 14   .92757       5631.9       4397.1       .3264-01    1.5104
 15   .94200       5365.0       4611.3       .3228-01    1.4313
 16   .95770       4630.7       5116.3       .3213-01    1.2383
 17   .97674       3181.9       5306.6       .3670-01    .91599
 18   .99930       1931.1       3841.7       .5435-01    .67180
 19   1.0208       1425.3       2473.2       .8453-01    .61053
 20   1.0389       1224.8       1847.6       .1128+00    .60060
```

FIG. 16 (CONT'D)

```
 21  1.0548     1131.4      1528.7      .13606      .60515
 22  1.0690     1082.5      1349.8      .15377      .61274
 23  1.0821     1055.0      1243.0      .16672      .61988
 24  1.0945     1039.0      1177.1      .17586      .62506
 25  1.1063     1029.5      1135.7      .18211      .63000
 26  1.1178     1024.0      1109.8      .18625      .63310
 27  1.1290     1020.8      1093.8      .18890      .63521
 28  1.1401     1019.0      1084.1      .19054      .63660
 29  1.1509     1018.0      1078.3      .19152      .63748
 30  1.1616     1017.5      1075.0      .19209      .63801
 31  1.1722     1017.2      1073.2      .19241      .63832
 32  1.1827     1017.1      1072.2      .19257      .63849
 33  1.1931     1017.0      1071.7      .19266      .63857
 34  1.2034     1017.0      1071.5      .19270      .63861
 35  1.2136     1017.0      1071.4      .19271      .63863
 36  1.2237     1017.0      1071.4      .19271      .63863
 37  1.2338     1017.0      1071.4      .19271      .63863

 N  - - - - - - - - - - - - - - - - C O N C E N T R A T I O N   O F   S P E C I E S - - - - - - - - - - - - - - - - -
        H2O          H2          O2          CO2         CO          N2          HCL                      AL          AL2O3
  1   .9140-01   .2260-01   -.0000      .2390-01   .28204     .8510-01   .19304     .0000      -.0000      .30205
  8   .9140-01   .2260-01   -.0000      .2390-01   .28204     .8510-01   .19304     .0000      -.0000      .30205
  9   .9140-01   .2260-01   -.0000      .2390-01   .28204     .8510-01   .19304     .0000      -.0000      .30205
 10   .9140-01   .2260-01   -.0000      .2390-01   .28204     .8510-01   .19304     .0000      -.0000      .30205
 11   .9147-01   .2257-01    .0000      .2398-01   .28182     .8549-01   .19294     .0000       .0000      .30204
 12   .9160-01   .2245-01    .2050-06   .2430-01   .28078     .8730-01   .19242     .0000       .0000      .30199
 13   .9312-01   .2196-01    .9846-06   .2591-01   .27672     .9437-01   .19042     .0000       .0000      .30156
 14   .9739-01   .2043-01    .3013-05   .3055-01   .26417     .1164+00   .18419     .0000       .8867-07   .29887
 15   .1090+00   .1649-01    .7885-05   .4108-01   .23343     .17160     .16858     .0000       .4842-06   .28620
 16   .12802     .8909-02    .3778-03   .5893-01   .17240     .29556     .13632     .0000       .1400-05   .24176
 17   .1153+00   .1367-02    .2764-01   .1152+00   .5632-01   .46960     .6419-01   .0000       .2641-06   .13687
 18   .6117-01   .6899-05    .1149+00   .9652-01   .3079-03   .62541     .4009-01   .0000       .0000      .4787-01
 19   .3103-01   .1317-05    .17289     .4906-01   .7965-04   .69524     .2032-01   .0000       .0000      .1761-01
 20   .1760-01   .7086-06    .19848     .2616-01   .3163-04   .72585     .1166-01   .0000       .0000      .7484-02
 21   .1101-01   .3566-06    .21166     .1742-01   .1711-04   .74157     .7209-02   .0000       .0000      .3476-02
 22   .7083-02   .2018-06    .21929     .1121-01   .1077-04   .75066     .4638-02   .0000       .0000      .1686-02
 23   .4639-02   .1131-06    .22403     .7339-02   .7244-05   .75631     .3039-02   .0000       .0000      .8291-03
 24   .3048-02   .5094-07    .22712     .4822-02   .4947-05   .75999     .1996-02   .0000       .0000      .4051-03
 25   .1990-02   .0000       .22919     .3148-02   .3342-05   .76244     .1303-02   .0000       .0000      .1940-03
 26   .1252-02   .0000       .23055     .2024-02   .2203-05   .76408     .8394-03   .0000       .0000      .9017-04
 27   .8101-03   .0000       .23147     .1282-02   .1405-05   .76517     .5305-03   .0000       .0000      .4042-04
 28   .5000-03   .0000       .23207     .7911-03   .8623-06   .76589     .3275-03   .0000       .0000      .1739-04
 29   .2998-03   .0000       .23246     .4743-03   .5059-06   .76635     .1963-03   .0000       .0000      .7152-05
 30   .1733-03   .0000       .23271     .2743-03   .2809-06   .76664     .1135-03   .0000       .0000      .2802-05
 31   .9542-04   .0000       .23286     .1510-03   .1440-06   .76682     .6249-04   .0000       .0000      .1038-05
 32   .4845-04   .0000       .23295     .7660-04   .6409-07   .76693     .3173-04   .0000       .0000      .3557-06
 33   .1959-04   .0000       .23301     .3147-04   .2244-07   .76700     .1303-04   .0000       .0000      .1032-06
 34   .4587-05   .0000       .23304     .7258-05   .0000      .76703     .3004-05   .0000       .0000      .1111-07
 35  -.0000     -.0000       .23304    -.0000     -.0000      .76704    -.0000      .0000      -.0000     -.0000
 36  -.0000     -.0000       .23304    -.0000     -.0000      .76704    -.0000      .0000      -.0000     -.0000
 37  -.0000     -.0000       .23304    -.0000     -.0000      .76704    -.0000      .0000      -.0000     -.0000
```

```
PROPERTIES GIVEN BELOW ARE AT FRONT  701     DISTANCE DOWNSTREAM =      1.0000FEET     PRESSURE=   10584.8160PSF

RADIUS OF DUCT =          2.5200 FT.    THRUST =          -3.7467 LB.    DUCT AREA =        19.9500 SQ FT.


   N      Y         X-COMP    TEMPERATURE    DENSITY     MACH
                   VELOCITY                              NUMBER
   1   .83199      5808.8       4259.0      .3177-01   1.5719
   5   .89277      5808.8       4259.0      .3177-01   1.5719
   6   .90733      5808.8       4259.0      .3177-01   1.5719
   7   .92166      5808.8       4259.1      .3177-01   1.5718
   8   .93577      5808.7       4259.2      .3176-01   1.5718
   9   .94967      5808.3       4260.0      .3176-01   1.5717
  10   .96338      5806.5       4262.9      .3176-01   1.5709
  11   .97690      5799.3       4273.0      .3173-01   1.5681
  12   .99030      5774.1       4302.1      .3165-01   1.5569
  13  1.0036       5700.2       4375.2      .3146-01   1.5338
  14  1.0172       5515.4       4535.7      .3113-01   1.4706
  15  1.0315       5115.6       4851.7      .3074-01   1.3639
  16  1.0473       4373.8       5246.8      .3146-01   1.1800
  17  1.0654       3302.2       5264.5      .3540-01    .95445
  18  1.0859       2313.4       4346.0      .4570-01    .75712
  19  1.1061       1759.0       3077.3      .6501-01    .67990
  20  1.1241       1486.3       2333.5      .8367-01    .65412
  21  1.1401       1341.0       1903.1      .1048+00    .64871
  22  1.1544       1256.6       1638.3      .1216+00    .65121
  23  1.1676       1204.0       1466.0      .13567      .65647
  24  1.1799       1169.6       1349.0      .14723      .66231
  25  1.1915       1146.4       1267.3      .15655      .66764
  26  1.2025       1130.4       1208.9      .16396      .67269
  27  1.2132       1119.2       1166.7      .16976      .67677
  28  1.2235       1111.3       1136.1      .17424      .68012
  29  1.2336       1105.8       1113.7      .17765      .68279
  30  1.2435       1102.0       1097.5      .18020      .68488
  31  1.2532       1099.3       1085.8      .18209      .68650
  32  1.2629       1097.5       1077.4      .18347      .68772
  33  1.2723       1096.2       1071.5      .18446      .68862
  34  1.2816       1095.4       1067.3      .18515      .68929
  35  1.2909       1094.9       1064.4      .18563      .68976
  36  1.3000       1094.5       1062.4      .18596      .69009
  37  1.3091       1094.3       1061.1      .18618      .69032
  38  1.3182       1094.1       1060.3      .18633      .69047
  39  1.3271       1094.1       1059.7      .18642      .69057
  40  1.3360       1094.0       1059.4      .18647      .69064
  41  1.3449       1094.0       1059.2      .18650      .69067
  42  1.3537       1094.0       1059.1      .18652      .69069
  43  1.3624       1094.0       1059.1      .18653      .69070
  44  1.3711       1094.0       1059.1      .18653      .69070
  45  1.3797       1094.0       1059.1      .18653      .69070

 N  - - - - - - - - - - - - - - - - C O N C E N T R A T I O N   O F   S P E C I E S - - - - - - - - - - - - - - - - - -
       H2O          H2          O2           CO2        CO          N2         HCL                    AL        AL2O3
  1   .9140-01   .2260-01   -.0000       .2390-01   .28204     .8510-01   .19304      .0000     -.0000     .30205
  5   .9140-01   .2260-01   -.0000       .2390-01   .28204     .8510-01   .19304      .0000     -.0000     .30205
  6   .9140-01   .2260-01   -.0000       .2390-01   .28204     .8510-01   .19304      .0000     -.0000     .30205
```

FIG. 16 (CONT'D)

```
 7  .9140-01  .2260-01  -.0000    .2390-01  .23204     .8510-01  .19304     .0000  -.0000     .30205
 8  .9141-01  .2260-01   .0000    .2391-01  .28201     .8517-01  .19303     .0000   .0000     .30204
 9  .9150-01  .2256-01   .0000    .2401-01  .28176     .8561-01  .19290     .0000   .0000     .30204
10  .9177-01  .2247-01   .1391-06 .2429-01  .28096     .8700-01  .19251     .0000   .0000     .30201
11  .9255-01  .2219-01   .5094-06 .2511-01  .27873     .9094-01  .19139     .0000   .0000     .30185
12  .9446-01  .2152-01   .1712-05 .2713-01  .27324     .1006+00  .18866     .0000   .0000     .30112
13  .9858-01  .2006-01   .3797-05 .3160-01  .26135     .1214+00  .18278     .0000   .1309-06  .29838
14  .1065+00  .1728-01   .7819-05 .4001-01  .23878     .16089    .17161     .0000   .4069-06  .28972
15  .1214+00  .1236-01   .5808-04 .4936-01  .20323     .22679    .15239     .0000   .6792-06  .26688
16 .13254     .6086-02   .1527-02 .7376-01  .14192     .33386     .1226+00  .0000   .1510-05  .21706
17  .1170+00  .1321-02   .2651-01 .1173+00   .5534-01  .46881    .8441-01  .0000   .2093-06  .13743
18  .7556-01  .5163-04   .8859-01 .1162+00   .2698-02  .59120    .4977-01  .0000   .0000      .6569-01
19  .4477-01  .4796-05  .14681    .7060-01   .2470-03  .66340    .2934-01  .0000   .0000      .3047-01
20  .2847-01  .3298-05  .17811    .4490-01   .1647-03  .70111    .1866-01  .0000   .0000      .1541-01
21  .1928-01  .2303-05  .19580    .3041-01   .1136-03  .72239    .1264-01  .0000   .0000      .8409-02
22  .1363-01  .1480-05  .20669    .2150-01   .7206-04  .73548    .8933-02  .0000   .0000      .4837-02
23  .9908-02  .9574-06  .21367    .1564-01   .4725-04  .74411    .6492-02  .0000   .0000      .2877-02
24  .7327-02  .6177-06  .21886    .1157-01   .3107-04  .75008    .4901-02  .0000   .0000      .1745-02
25  .5474-02  .3929-06  .22244    .8646-02   .2051-04  .75437    .3586-02  .0000   .0000      .1068-02
26  .4109-02  .2446-06  .22508    .6492-02   .1361-04  .75753    .2692-02  .0000   .0000      .6547-03
27  .3067-02  .1468-06  .22705    .4879-02   .9067-05  .75990    .2022-02  .0000   .0000      .3998-03
28  .2313-02  .8156-07  .22856    .3658-02   .6109-05  .76169    .1515-02  .0000   .0000      .2420-03
29  .1726-02  .3963-07  .22970    .2729-02   .4137-05  .76305    .1131-02  .0000   .0000      .1448-03
30  .1279-02  .1463-07  .23056    .2023-02   .2920-05  .76408    .8378-03  .0000   .0000      .8535-04
31  .9401-03  .0000     .23122    .1487-02   .1932-05  .76487    .6157-03  .0000   .0000      .4947-04
32  .6841-03  .0000     .23172    .1092-02   .1328-05  .76546    .4481-03  .0000   .0000      .2814-04
33  .4922-03  .0000     .23209    .7786-03   .9122-06  .76591    .3223-03  .0000   .0000      .1569-04
34  .3495-03  .0000     .23237    .5529-03   .6238-06  .76624    .2289-03  .0000   .0000      .8567-05
35  .2448-03  .0000     .23257    .3866-03   .4227-06  .76648    .1600-03  .0000   .0000      .4572-05
36  .1677-03  .0000     .23272    .2654-03   .2819-06  .76660    .1099-03  .0000   .0000      .2382-05
37  .1125-03  .0000     .23283    .1781-03   .1835-06  .76678    .7371-04  .0000   .0000      .1207-05
38  .7322-04  .0000     .23290    .1158-03   .1146-06  .76688    .4795-04  .0000   .0000      .5911-06
39  .4553-04  .0000     .23296    .7203-04   .6583-07  .76694    .2982-04  .0000   .0000      .2753-06
40  .2597-04  .0000     .23299    .4109-04   .3441-07  .76698    .1701-04  .0000   .0000      .1177-06
41  .1169-04  .0000     .23302    .1849-04   .1320-07  .76702    .7655-05  .0000   .0000      .4116-07
42  .4657-05  .0000     .23304    .7368-05   .0000     .76703    .3059-05  .0000   .0000      .0000
43 -.0000    -.0000     .23304   -.0000     -.0000     .76704   -.0000     .0000  -.0000     -.0000
44 -.0000    -.0000     .23304   -.0000     -.0000     .76704   -.0000     .0000  -.0000     -.0000
45 -.0000    -.0000     .23304   -.0000     -.0000     .76704   -.0000     .0000  -.0000     -.0000

DELTA PSI WAS SET =   4.8358

THE MINIMUM SUBSCRIPT= 1

THE MINIMUM SUBSCRIPT= 1



PROPERTIES GIVEN BELOW ARE AT FRONT   216     DISTANCE DOWNSTREAM =      1.0750FEET     PRESSURE=   10397.4958PSF

RADIUS OF DUCT =          2.5201 FT.    THRUST =          14.5853 LB.    DUCT AREA =          19.9517 SQ FT.
```

```
 N      Y      X-COMP    TEMPERATURE   DENSITY    MACH
               VELOCITY                           NUMBER
 1   .86405    5841.6      4241.2     .3134-01   1.5841
 2   .89410    5841.6      4241.2     .3133-01   1.5839
 3   .92317    5841.6      4241.2     .3133-01   1.5839
 4   .95136    5841.5      4241.4     .3133-01   1.5839
 5   .97875    5840.3      4243.7     .3133-01   1.5833
 6   1.0054    5823.7      4266.0     .3127-01   1.5769
 7   1.0318    5691.0      4399.1     .3092-01   1.5316
 8   1.0595    5059.7      4902.4     .3027-01   1.3514
 9   1.0935    3341.5      5251.9     .3497-01   .96707
10   1.1322    1869.5      3240.6     .6064-01   .70523
11   1.1562    1419.8      2019.5     .9712-01   .66825
12   1.1936    1262.8      1541.1    .12688      .67303
13   1.2174    1194.0      1316.2    .14618      .68377
14   1.2369    1159.9      1198.1    .16249      .69311
15   1.2590    1142.1      1132.8    .17165      .69990
16   1.2762    1132.7      1096.2    .17725      .70445
17   1.2968    1127.8      1075.8    .18051      .70731
18   1.3149    1125.4      1064.8    .18232      .70900
19   1.3327    1124.2      1059.0    .18328      .70995
20   1.3502    1123.7      1056.2    .18375      .71044
21   1.3674    1123.5      1054.8    .18397      .71068
22   1.3844    1123.4      1054.3    .18406      .71079
23   1.4012    1123.4      1054.1    .18409      .71082
24   1.4178    1123.4      1054.0    .18410      .71083
25   1.4342    1123.4      1054.0    .18410      .71083

 N  - - - - - - - - - - - - - - - - C O N C E N T R A T I O N   O F   S P E C I E S - - - - - - - - - - - - - - - -
       H2O         H2          O2          CO2         CO          N2          HCL                    AL          AL2O3
 1  .9140-01    .2260-01    .0000       .2390-01    .28204      .8510-01    .19304      .0000      .0000       .30205
 2  .9140-01    .2260-01    .0000       .2390-01    .28204      .8510-01    .19304      .0000      .0000       .30205
 3  .9140-01    .2260-01    .0000       .2390-01    .28204      .8510-01    .19304      .0000      .0000       .30205
 4  .9140-01    .2260-01    .0000       .2390-01    .28204      .8511-01    .19304      .0000      .0000       .30204
 5  .9164-01    .2252-01    .0000       .2415-01    .28135      .8632-01    .19270      .0000      .0000       .30203
 6  .9328-01    .2193-01    .1012-05    .2588-01    .27605      .9460-01    .19036      .0000      .0000       .30163
 7  .1008+00    .1932-01    .7217-05    .3351-01    .25505      .13167      .17988      .0000      .1900-06    .29655
 8  .1245+00    .1132-01    .9339-04    .5244-01    .19497      .24326      .14829      .0000      .7362-06    .26105
 9  .1169+00    .1286-02    .2695-01    .1180+00    .5436-01    .47005      .8406-01    .0000      .1925-06    .13714
10  .4859-01    .3097-06    .13911      .7713-01    .1332-04    .65441      .3188-01    .0000      .0000       .3448-01
11  .2195-01    .2196-05    .19069      .3463-01    .1106-03    .71023      .1438-01    .0000      .0000       .1029-01
12  .1168-01    .1202-05    .21048      .1642-01    .6037-04    .74001      .7651-02    .0000      .0000       .3754-02
13  .6713-02    .5362-06    .22005      .1060-01    .3080-04    .75150      .4399-02    .0000      .0000       .1501-02
14  .3976-02    .2317-06    .22534      .6280-02    .1549-04    .75784      .2605-02    .0000      .0000       .6160-03
15  .2362-02    .1203-06    .22847      .3732-02    .7796-05    .76158      .1547-02    .0000      .0000       .2495-03
16  .1383-02    .4001-07    .23036      .2187-02    .3891-05    .76384      .9062-03    .0000      .0000       .9739-04
17  .7901-03    .0000       .23151      .1249-02    .1924-05    .76522      .5175-03    .0000      .0000       .3608-04
18  .4359-03    .0000       .23220      .6693-03    .9395-06    .76604      .2855-03    .0000      .0000       .1256-04
19  .2298-03    .0000       .23260      .3635-03    .4463-06    .76651      .1505-03    .0000      .0000       .4075-05
20  .1138-03    .0000       .23282      .1600-03    .2035-06    .76678      .7451-04    .0000      .0000       .1220-05
21  .5069-04    .0000       .23295      .8019-04    .8260-07    .76693      .3320-04    .0000      .0000       .3260-06
22  .1778-04    .0000       .23301      .2613-04    .2638-07    .76700      .1165-04    .0000      .0000       .6882-07
23  -.0000      -.0000      .23304      -.0000      -.0000      .76704      -.0000      .0000      -.0000      -.0000
24  -.0000      -.0000      .23304      -.0000      -.0000      .76704      -.0000      .0000      -.0000      -.0000
25  -.0000      -.0000      .23304      -.0000      -.0000      .76704      -.0000      .0000      -.0000      -.0000

    .10935+01   .25201+01   .10397+05   .50000-02   .10409+05   .31342-01   .86401-00   .58416+04
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FIG. 16 (CONT'D)

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```
 .48358+01             14        15        39        23
.424+04   .424+04   .424+04   .424+04   .424+04   .427+04   .440+04   .490+04   .525+04   .324+04
.202+04   .154+04   .132+04   .120+04   .424+04   .424+04   .424+04   .424+04   .424+04   .427+04
.440+04   .490+04   .525+04   .324+04   .202+04   .154+04   .132+04   .120+04   .113+04   .110+04
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                         TRANSFORMATION COMPLETED


PROPERTIES GIVEN BELOW ARE AT FRONT   216      DISTANCE DOWNSTREAM =        1.0750FEET      PRESSURE=    10397.4958PSF

RADIUS OF DUCT =          2.5201 FT.    THRUST =          14.5853 LB.    DUCT AREA =          19.9517 SQ FT.


   N      Y         X-COMP     TEMPERATURE    DENSITY     MACH
                   VELOCITY                              NUMBER
   1   .6300-04    5841.6       4241.2      .3133-01    1.5839
  13   .79816      5841.6       4241.2      .3133-01    1.5839
  14   .83075      5841.6       4241.2      .3133-01    1.5839
  15   .86211      5841.6       4241.2      .3133-01    1.5839
  16   .89237      5841.6       4241.2      .3133-01    1.5839
  17   .92163      5841.6       4241.2      .3133-01    1.5839
  18   .95000      5841.5       4241.4      .3133-01    1.5839
  19   .97754      5840.3       4243.7      .3133-01    1.5833
  20  1.0043       5823.7       4266.0      .3127-01    1.5709
  21  1.0309       5091.0       4399.1      .3092-01    1.5316
  22  1.0589       5059.7       4902.4      .3027-01    1.3514
  23  1.0929       3341.5       5251.9      .3497-01    .96707
  24  1.1319       1669.5       3240.6      .6064-01    .70523
  25  1.1560       1419.8       2019.5      .9712-01    .66825
  26  1.1935       1262.8       1541.1      .12688      .67303
  27  1.2174       1194.0       1316.2      .14818      .68377
  28  1.2390       1159.9       1198.1      .16249      .69311
  29  1.2592       1142.1       1132.8      .17165      .69990
  30  1.2785       1132.7       1096.2      .17725      .70445
  31  1.2972       1127.8       1075.8      .18051      .70731
  32  1.3154       1125.4       1064.8      .18232      .70900
  33  1.3332       1124.2       1059.0      .18328      .70995
  34  1.3508       1123.7       1056.2      .18375      .71044
  35  1.3681       1123.5       1054.8      .18397      .71068
  36  1.3851       1123.4       1054.3      .18406      .71079
  37  1.4020       1123.4       1054.1      .18409      .71082
  38  1.4187       1123.4       1054.0      .18410      .71083
  39  1.4351       1123.4       1054.0      .18410      .71083
  40  1.4514       1123.4       1054.0      .18410      .71083
  41  1.4675       1123.4       1054.0      .18410      .71083

   N  - - - - - - - - - - - - - - - - - C O N C E N T R A T I O N   O F   S P E C I E S - - - - - - - - - - - - - - - - - -
          H2O          H2         O2          CO2        CO           N2        HCL                         AL        AL2O3
   1   .9140-01   .2260-01   .0000      .2390-01   .28204      .8510-01   .19304     .0000          .0000     .30205
  13   .9140-01   .2260-01   .0000      .2390-01   .28204      .8510-01   .19304     .0000          .0000     .30205
  14   .9140-01   .2260-01   .0000      .2390-01   .28204      .8510-01   .19304     .0000          .0000     .30205
  15   .9140-01   .2260-01   .0000      .2390-01   .28204      .8510-01   .19304     .0000          .0000     .30205
  16   .9140-01   .2260-01   .0000      .2390-01   .28204      .8510-01   .19304     .0000          .0000     .30205
  17   .9140-01   .2260-01   .0000      .2390-01   .28204      .8510-01   .19304     .0000          .0000     .30205
  18   .9140-01   .2260-01   .0000      .2390-01   .28204      .8511-01   .19304     .0000          .0000     .30204
  19   .9164-01   .2252-01   .0000      .2415-01   .28135      .8632-01   .19270     .0000          .0000     .30203
```

FIG. 16 (CONT'D)

```
20  .9328-01  .2193-01  .1012-05  .2588-01  .27665    .9460-01  .19036    .0000     .0000     .30163
21  .1008+00  .1932-01  .7217-05  .3351-01  .25565    .13167    .17988    .0000     .1900-06  .29655
22  .1245+00  .1132-01  .9339-04  .5244-01  .19497    .24326    .14829    .0000     .7362-06  .26105
23  .1168+00  .1286-02  .2695-01  .1180+00  .5436-01  .47005    .8406-01  .0000     .1925-06  .13714
24  .4869-01  .3697-06  .13011    .7713-01  .1332-04  .65441    .3188-01  .0000     .0000     .3448-01
25  .2195-01  .2198-05  .19069    .3463-01  .1106-03  .71623    .1438-01  .0000     .0000     .1029-01
26  .1168-01  .1202-05  .21048    .1842-01  .6037-04  .74001    .7651-02  .0000     .0000     .3754-02
27  .6713-02  .5962-06  .22005    .1060-01  .3060-04  .75150    .4399-02  .0000     .0000     .1501-02
28  .3976-02  .2817-06  .22534    .6280-02  .1549-04  .75784    .2605-02  .0000     .0000     .6160-03
29  .2362-02  .1203-06  .22847    .3732-02  .7796-05  .76158    .1547-02  .0000     .0000     .2495-03
30  .1383-02  .4001-07  .23036    .2187-02  .3891-05  .76384    .9962-03  .0000     .0000     .9739-04
31  .7901-03  .0000     .23151    .1249-02  .1924-05  .76522    .5175-03  .0000     .0000     .3608-04
32  .4359-03  .0000     .23220    .6893-03  .9395-06  .76604    .2855-03  .0000     .0000     .1256-04
33  .2298-03  .0000     .23260    .3635-03  .4483-06  .76651    .1505-03  .0000     .0000     .4075-05
34  .1138-03  .0000     .23282    .1600-03  .2035-06  .76678    .7451-04  .0000     .0000     .1220-05
35  .5069-04  .0000     .23295    .8019-04  .8260-07  .76693    .3320-04  .0000     .0000     .3260-06
36  .1778-04  .0000     .23301    .2813-04  .2638-07  .76700    .1165-04  .0000     .0000     .6882-07
37 -.0000    -.0000     .23304   -.0000    -.0000     .76704   -.0000     .0000    -.0000    -.0000
38 -.0000    -.0000     .23304   -.0000    -.0000     .76704   -.0000     .0000    -.0000    -.0000
39 -.0000    -.0000     .23304   -.0000    -.0000     .76704   -.0000     .0000    -.0000    -.0000
40 -.0000    -.0000     .23304   -.0000    -.0000     .76704   -.0000     .0000    -.0000    -.0000
41 -.0000    -.0000     .23304   -.0000    -.0000     .76704   -.0000     .0000    -.0000    -.0000


PROPERTIES GIVEN BELOW ARE AT FRONT   266      DISTANCE DOWNSTREAM =        1.3250FEET      PRESSURE=     9686.0388PSF

RADIUS OF DUCT =        2.5201 FT.    THRUST =        10.8137 LB.    DUCT AREA =        19.9515 SQ FT.


N       Y       X-COMP    TEMPERATURE   DENSITY    MACH
                VELOCITY                           NUMBER
13   .34423     5968.7      4171.1      .2968-01   1.6313
14   .91169     5968.7      4171.1      .2968-01   1.6313
15   .94149     5968.7      4171.1      .2968-01   1.6313
16   .97018     5968.7      4171.2      .2968-01   1.6313
17   .99805     5968.6      4171.6      .2968-01   1.6312
18  1.0251      5967.0      4174.4      .2967-01   1.6305
19  1.0516      5955.9      4190.4      .2963-01   1.6259
20  1.0776      5895.4      4257.5      .2945-01   1.6038
21  1.1039      5652.8      4462.2      .2909-01   1.5288
22  1.1317      4936.6      4902.4      .2930-01   1.3457
23  1.1651      3485.2      5251.9      .3250-01   1.0086
24  1.2025      2226.1      3679.4      .4962-01    .78977
25  1.2360      1703.0      2393.0      .7644-01    .74289
26  1.2636      1490.3      1803.5      .1012+00    .73909
27  1.2875      1382.6      1498.4      .1215+00    .74655
28  1.3038      1323.0      1324.2      .13726      .75575
29  1.3286      1287.8      1218.3      .14897      .76397
30  1.3471      1266.2      1151.4      .15744      .77056
31  1.3649      1252.8      1108.4      .16342      .77554
32  1.3820      1244.4      1080.6      .16752      .77918
33  1.3986      1239.2      1062.7      .17026      .78174
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FIG. 16 (CONT'D)

```
34  1.4149     1236.1     1051.3     .17205     .78350
35  1.4309     1234.2     1044.2     .17318     .78406
36  1.4467     1233.1     1039.9     .17388     .78541
37  1.4623     1232.5     1037.3     .17430     .78588
38  1.4777     1232.2     1035.8     .17453     .78615
39  1.4929     1232.0     1035.0     .17466     .78631
40  1.5079     1232.0     1034.6     .17473     .78639
41  1.5228     1231.9     1034.4     .17476     .78643
42  1.5376     1231.9     1034.4     .17477     .78644
43  1.5522     1231.9     1034.4     .17477     .78644

N  - - - - - - - - - - - - - - - - - C O N C E N T R A T I O N   O F   S P E C I E S - - - - - - - - - - - - - - - -
       H2O         H2          O2          CO2         CO          N2          HCL                     AL          AL2O3
 1   .9140-01    .2260-01    .0000       .2390-01    .28204      .8510-01    .19304      .0000       .0000       .30205
14   .9140-01    .2260-01    .0000       .2390-01    .28204      .8510-01    .19304      .0000       .0000       .30205
15   .9140-01    .2260-01    .0000       .2390-01    .28204      .8510-01    .19304      .0000       .0000       .30205
16   .9140-01    .2260-01    .0000       .2390-01    .28204      .8510-01    .19304      .0000       .0000       .30205
17   .9142-01    .2259-01    .0000       .2393-01    .28197      .8523-01    .19301      .0000       .0000       .30204
18   .9168-01    .2250-01    .0000       .2423-01    .28120      .8656-01    .19263      .0000       .0000       .30202
19   .9280-01    .2209-01    .6740-06    .2559-01    .27778      .9243-01    .19097      .0000       .0000       .30174
20   .9671-01    .2068-01    .2559-05    .3028-01    .26593      .1128+00    .18522      .0000       .0000       .29961
21   .1074+00    .1686-01    .7501-05    .4200-01    .23403     .16751       .16973      .0000       .3119-06    .28800
22  .12858       .8892-02    .1707-03    .6069-01    .17170     .28400       .13659      .0000       .5171-06    .24336
23   .1159+00    .1231-02    .3062-01    .1172+00      .5252-01 .47546        .8253-01   .0000       .1986-06    .13518
24   .5900-01    .3715-05    .1200+00    .9327-01      .1576-03 .63050        .3865-01   .0000       .0000         .4627-01
25   .3042-01    .1530-05   .17451       .4611-01      .6870-04 .69665        .1992-01   .0000       .0000         .1727-01
26   .1761-01    .9671-06   .19870       .2816-01      .4631-04 .72583        .1167-01   .0000       .0000         .7490-02
27   .1124-01    .6797-06   .21135       .1776-01      .3409-04 .74103        .7362-02   .0000       .0000         .3557-02
28   .7387-02    .4749-06   .21877       .1167-01      .2501-04 .74995        .4839-02   .0000       .0000         .1771-02
29   .4954-02    .3186-06   .22347       .7626-02      .1794-04 .75558        .3245-02   .0000       .0000         .8998-03
30   .3346-02    .1986-06   .22657       .5285-02      .1253-04 .75930        .2192-02   .0000       .0000         .4579-03
31   .2256-02    .1192-06   .22868       .3563-02      .8510-05 .76182        .1479-02   .0000       .0000         .2304-03
32   .1508-02    .4731-07   .23013       .2383-02      .5617-05 .76355        .9882-03   .0000       .0000         .1137-03
33   .9954-03    .0000      .23112       .1572-02      .3604-05 .76474        .6521-03   .0000       .0000         .5466-04
34   .6453-03    .0000      .23179       .1020-02      .2247-05 .76555        .4328-03   .0000       .0000         .2549-04
35   .4090-03    .0000      .23225       .6462-03      .1360-05 .76610        .2679-03   .0000       .0000         .1149-04
36   .2516-03    .0000      .23256       .3975-03      .7963-06 .76646        .1648-03   .0000       .0000         .4986-05
37   .1464-03    .0000      .23276       .2345-03      .4475-06 .76670        .9718-04   .0000       .0000         .2070-05
38   .8184-04    .0000      .23289       .1294-03      .2304-06 .76680        .5361-04   .0000       .0000         .8077-06
39   .4022-04    .0000      .23297       .6357-04      .1123-06 .76695        .2535-04   .0000       .0000         .2803-06
40   .1546-04    .0000      .23301       .2444-04      .4219-07 .76701        .1013-04   .0000       .0000         .7216-07
41  -.0000      -.0000      .23304      -.0000       -.0000     .76704      -.0000       .0000      -.0000       -.0000
42  -.0000      -.0000      .23304      -.0000       -.0000     .76704      -.0000       .0000      -.0000       -.0000
43  -.0000      -.0000      .23304      -.0000       -.0000     .76704      -.0000       .0000      -.0000       -.0000

PROPERTIES GIVEN BELOW ARE AT FRONT   316      DISTANCE DOWNSTREAM =        1.5750FEET     PRESSURE=     8461.7002PSF

RADIUS OF DUCT =          2.5198 FT.    THRUST =        -23.2979 LB.    DUCT AREA =        19.9476 SQ FT.
```

FIG. 16 (CONT'D)

13

```
  N      Y        X-COMP    TEMPERATURE    DENSITY     MACH
                 VELOCITY                             NUMBER
  1   .41559     6198.3       4040.1      .2677-01    1.7200
 13   .93566     6198.3       4040.1      .2677-01    1.7200
 14   .96644     6198.3       4040.1      .2677-01    1.7200
 15   .99628     6198.3       4040.2      .2677-01    1.7200
 16  1.0252      6198.1       4040.6      .2677-01    1.7199
 17  1.0534      6196.8       4043.2      .2676-01    1.7193
 18  1.0809      6189.7       4054.3      .2673-01    1.7161
 19  1.1078      6157.9       4092.9      .2664-01    1.7036
 20  1.1345      6044.9       4200.7      .2641-01    1.6649
 21  1.1618      5725.1       4454.4      .2611-01    1.5696
 22  1.1911      5005.5       4937.8      .2618-01    1.3820
 23  1.2248      3793.0       5141.4      .2909-01    1.1111
 24  1.2610      2691.4       3908.8      .4068-01    .92317
 25  1.2936      2111.7       2658.6      .6008-01    .87491
 26  1.3209      1634.6       2015.6      .7918-01    .86435
 27  1.3446      1693.0       1657.3      .9611-01    .86800
 28  1.3658      1592.1       1441.0      .1103+00    .87550
 29  1.3853      1534.1       1302.1      .1219+00    .88344
 30  1.4035      1495.5       1209.2     .13118       .89062
 31  1.4207      1469.1       1145.3     .13636       .89608
 32  1.4372      1450.9       1100.6     .14386       .90161
 33  1.4531      1433.2       1069.0     .14802       .90551
 34  1.4686      1429.4       1046.7     .15110       .90853
 35  1.4838      1423.3       1031.0     .15335       .91082
 36  1.4987      1419.1       1019.9     .15496       .91253
 37  1.5133      1416.3       1012.2     .15610       .91379
 38  1.5277      1414.4       1007.0     .15699       .91469
 39  1.5420      1413.2       1003.4     .15743       .91532
 40  1.5561      1412.4       1001.1     .15779       .91576
 41  1.5700      1411.9       999.57     .15802       .91606
 42  1.5838      1411.6       998.60     .15817       .91625
 43  1.5975      1411.5       998.00     .15826       .91637
 44  1.6111      1411.4       997.66     .15831       .91644
 45  1.6245      1411.3       997.48     .15833       .91647
 46  1.6378      1411.3       997.40     .15835       .91649
 47  1.6511      1411.3       997.40     .15835       .91649


 N  - - - - - - - - - - - - - - - - C O N C E N T R A T I O N   O F   S P E C I E S - - - - - - - - - - - - - - - -
       H2O        H2         O2          CO2        CO          N2        HCL                     AL         AL2O3
  1  .9140-01  .2260-01   .0000      .2390-01   .28204     .8510-01   .19304       .0000      .0000      .30205
 13  .9140-01  .2260-01   .0000      .2390-01   .28204     .8510-01   .19304       .0000      .0000      .30205
 14  .9140-01  .2260-01   .0000      .2390-01   .28204     .8510-01   .19304       .0000      .0000      .30205
 15  .9140-01  .2260-01   .0000      .2390-01   .28204     .8510-01   .19304       .0000      .0000      .30205
 16  .9143-01  .2259-01   .0000      .2394-01   .28194     .8528-01   .19299       .0000      .0000      .30204
 17  .9167-01  .2250-01   .0000      .2422-01   .28123     .8650-01   .19265       .0000      .0000      .30202
 18  .9251-01  .2220-01   .3929-06   .2519-01   .27872     .9084-01   .19142       .0000      .0000      .30185
 19  .9502-01  .2130-01   .1472-05   .2802-01   .27130     .1037+00   .16777       .0000      .0000      .30080
 20  .1013+00  .1976-01   .3638-05   .3476-01   .25321    .13545      .17881       .0000      .0000      .29586
 21  .1148+00  .1445-01   .6368-05   .4643-01   .21606    .20071      .16034       .0000      .2459-06   .27797
 22 .13169     .7079-02   .3759-03   .6910-01   .15303    .31517      .12794       .0000      .4777-06   .22861
 23  .1155+00  .1084-02   .2896-01   .1225+00    .4833-01 .47735        .8199-01   .0000      .1005-06   .13453
 24  .6649-01  .1134-04   .1065+00   .1046+00    .5162-03 .61321        .4354-01   .0000      .0000       .5566-01
 25  .3744-01  .4411-05  .16145      .5907-01    .1797-03 .68034        .2454-01   .0000      .0000       .2402-01
 26  .2341-01  .2177-05  .18916      .3697-01    .9628-04 .71285        .1534-01   .0000      .0000       .1169-01
 27  .1567-01  .1179-05  .20293      .2476-01    .5534-04 .73077        .1027-01   .0000      .0000       .6178-02
```

FIG. 16 (CONT'D)

```
28   .1094-01   .7310-06  .21199    .1729-01  .3579-04  .74171    .7169-02  .0000    .0000    .3440-02
29   .7839-02   .4794-06  .21794    .1239-01  .2491-04  .74890    .5136-02  .0000    .0000    .1973-02
30   .5697-02   .3204-06  .22206    .9004-02  .1907-04  .75386    .3732-02  .0000    .0000    .1149-02
31   .4170-02   .2098-06  .22500    .6589-02  .1339-04  .75739    .2732-02  .0000    .0000    .6723-03
32   .3057-02   .1266-06  .22714    .4831-02  .9997-05  .75997    .2003-02  .0000    .0000    .3923-03
33   .2236-02   .5914-07  .22872    .3533-02  .7452-05  .76187    .1465-02  .0000    .0000    .2270-03
34   .1627-02   .0000     .22990    .2571-02  .5515-05  .76328    .1066-02  .0000    .0000    .1297-03
35   .1174-02   .0000     .23077    .1856-02  .4035-05  .76433    .7694-03  .0000    .0000    .7294-04
36   .8392-03   .0000     .23142    .1326-02  .2911-05  .76510    .5498-03  .0000    .0000    .4028-04
37   .5921-03   .0000     .23190    .9355-03  .2066-05  .76567    .3879-03  .0000    .0000    .2180-04
38   .4115-03   .0000     .23225    .6501-03  .1438-05  .76609    .2696-03  .0000    .0000    .1155-04
39   .2806-03   .0000     .23250    .4433-03  .9797-06  .76640    .1839-03  .0000    .0000    .5970-05
40   .1857-03   .0000     .23268    .2949-03  .6436-06  .76661    .1223-03  .0000    .0000    .3005-05
41   .1200-03   .0000     .23281    .1875-03  .4157-06  .76677    .7863-04  .0000    .0000    .1463-05
42   .7334-04   .0000     .23290    .1159-03  .2527-06  .76688    .4905-04  .0000    .0000    .6793-06
43   .4096-04   .0000     .23297    .6471-04  .1404-06  .76695    .2683-04  .0000    .0000    .2904-06
44   .1805-04   .0000     .23301    .2651-04  .6107-07  .76700    .1182-04  .0000    .0000    .1016-06
45  -.0000     -.0000     .23304   -.0000    -.0000     .76704   -.0000     .0000   -.0000   -.0000
46  -.0000     -.0000     .23304   -.0000    -.0000     .76704   -.0000     .0000   -.0000   -.0000
47  -.0000     -.0000     .23304   -.0000    -.0000     .76704   -.0000     .0000   -.0000   -.0000
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FIG. 16 (CONT'D)

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13 ABSTRACT

A theoretical analysis was developed for predicting the flow properties in the mixing region between a particle-laden, turbulent rocket exhaust and a surrounding air stream. It was assumed that the turbulent boundary layer equations, modified to account for particles were valid within the mixing region. In treating the chemical aspects of the problem it was assumed that the flow was in equilibrium. The chemical species comprising the flow were limited to the following: H_2O, H_2, O_2, CO, CO_2, N_2, HCl, Al, Al_2O_3, and one additional inert species. The phenomenological model employed for the turbulent transport coefficients is discussed in detail.

The solution of the partial differential equations was obtained using the von Mises transformation, expressing the equations in finite-difference form, and solving the resulting equations utilizing a computer program developed for the SRU 1107.

Results from the computer program were successfully compared with experimental results obtained from air-augmentation, free jet, and fuel injection experiments.

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