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FUZE GEAR-TRAIN ANALYSIS

G. G. LOWEN
CITY COLLEGE OF NEW YORK

F. R. TEPPER
ARRADCOM

DECEMBER 1979

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
LARGE CALIBER
WEAPON SYSTEMS LABORATORY
DOVER, NEW JERSEY

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This report documents the development of the tools needed to compare the efficiency of fuze-related gear trains designed to operate in a spin environment. Computer models have been developed for two and three pass step-up designs with clock (ogival) and involute tooth shapes. Using appropriate moment input-output relationships, the computer programs develop point and cycle efficiency for each type of gear train. Pivot friction partially caused by centrifugal force on the gear and pinion combinations during spin, and

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20. Abstract (continued)

tooth-to-tooth contact friction are considered. All models allow a variety of parameter variations.

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INTRODUCTION

This project provides the computer programs needed to compare the efficiency of fuze-related gear trains operating in a spin environment. Specifically, two and three pass step-up computer models with both involute and ogival (clock) tooth shapes were developed.

By using appropriate moment input-output relationships, the computer programs allow the determination of point and cycle efficiencies. Pivot friction, partly due to the centrifugal forces on the gear and pinion combinations, is considered in addition to tooth-to-tooth contact friction. The models derived allow a wide variety of parameter variations.

The main body of this report consists of nine appendixes, each of which contains a detailed analysis of each combination of tooth forms, the number of passes, and the spin environment. The derivation of moment relationships, pivot friction, gear tooth geometry, and the direct-contact mechanism kinematics are also included. The computer programs used are listed and instructions in their use and in interpreting the results are given.

POINT EFFICIENCY AND CYCLE EFFICIENCY

The point efficiency ε_P is defined as

$$\varepsilon_P = \frac{M_{oREF}}{M_{in}} \tag{1}$$

where M_{in} represents the instantaneous input moment to the gear train. M_{oREF} stands for the instantaneous equilibrant output moment M_o after it has been referred to the input shaft by way of the instantaneous angular velocity ratio

$$K_{RATIO} = \left| \frac{\dot{\psi}}{\dot{\phi}} \right| . \tag{2}$$

In the above,

$\dot{\psi}$ = instantaneous angular velocity of the output gear

$\dot{\phi}$ = instantaneous angular velocity of the input gear

Equation 1 then becomes

$$\varepsilon_P = K_{RATIO} \frac{M_o}{M_{in}} \tag{3}$$

The cycle efficiency ε_C represents the ratio of the work available at the output shaft to that done by the input moment during one tooth cycle of the input gear. Thus,

$$\varepsilon_C = \frac{\int M_o \, d\psi}{\int M_{in} \, d\phi} \tag{4}$$

The quantities $d\psi$ and $d\phi$ represent infinitesimal rotations of the output and input gears, respectively.

DESCRIPTION OF STUDY

Appendix A

Appendix A furnishes the background, as well as the derivations, for the moment input-output expressions for two and three pass step-up gear trains (where in each mesh the gear is the driver) with involute teeth and unity contact ratio.

Section 1 shows the development of a sign convention for the direction of the contact point friction force. It is based on the direction of the relative velocity between the contact points on the gear and pinion teeth. Section 2 discusses how to deal with the normal and friction forces at the gear and pinion pivots of single and multiple mesh trains. Section 3 shows the application of the above results to the moment input-output analysis of a single mesh. The frame is assumed stationary for this case, and the external loads are confined to the driving input moment and the equilibrating output moment.

The basic geometry of the three pass step-up gear train, mounted on a rotating fuze body, is formulated in section 4 for use in the moment input-output analysis. Force and moment equilibria of the individual component gears, which also account for the centrifugal forces, lead to the desired expression. Section 5 includes a similar derivation for a two pass step-up gear train with

involute teeth. In order to be able to continuously compute the moment relationships for these trains, a method for determining the simultaneous locations of the contact points of all the meshes had to be worked out. Such a method is given in section 6 together with certain angular relationships of the pivot locations on the model fuze body.

The kinematic relationships in involute gear trains are relatively simple compared to those in ogival trains because of the constant transmission ratio and the invariant direction of the line-of-action in each individual mesh.

Appendix B

To avoid the severe undercutting of pinions which generally is associated with step-up gear meshes, it is necessary to use non-standard involute gearing. Appendix B shows both the theory and the necessary steps for the design of unequal addendum gears and pinions of unity contact ratio. In addition, a numerical example is given.

Appendix C

This appendix contains four computer programs which make it possible to determine the point and cycle efficiencies of three gear combinations containing unequal addendum involute meshes with

unity contact ratio. In each case, the structure of the program is thoroughly discussed and a sample run is used to interpret the results. The names of these programs and their relationship to work described in the other appendixes are given below:

1. Program INVOL 1 : Design of unequal addendum involute gear and pinion set with unity contact ratio.
   This program is based on the work in Appendix B.Five sample computations,which are used in other programs,are shown.

2. Program INVOL 2 : Point and cycle efficiencies for single pass involute step-up gear mesh with unity contact ratio.
   This program is based on the work in section 3 of Appendix A.

3. Program INVOL 3 : Point and cycle efficiencies for three pass involute step-up gear train in spin environment.(All meshes have unity contact ratio).
   This program is based on the work in sections 4 and 6 of Appendix A.

4. Program INVOL 4 : Point and cycle efficiencies for two pass involute step-up gear train in spin environment.(All meshes have unity contact ratio).
   This program is based on the work in sections 5 and 6 of Appendix A.

Appendix D

This appendix describes the geometry of an ogival tooth in which each side of the tooth profile has a circular arc blending tangentially into a radial straight line flank. The basic tooth nomenclature is defined and methods for determining the required tooth parameters are given.

Appendix E

Section 1 of Appendix E gives all necessary kinematic derivations for a single step-up mesh with ogival teeth. The motion of an ogival mesh consists of two phases. On first contact, the circular arc portion of the driving gear tooth makes contact with the circular arc portion of the driven pinion tooth. Later in the cycle, and up to the point of final disengagement, the circular arc portion of the gear tooth contacts the straight line portion of the pinion tooth. These phases of motion were named "round on round" and "round on flat," respectively. Equivalent four-link mechanism models were used for both regimes to obtain expressions for the pinion output angles, for transition angles, for output angular velocities and for contact-point relative velocities. In addition, a sensing expression was developed which allows the computer determination of that position of a given mesh at which the subsequent mesh comes into engagement. (Because of the variable transmission ratio of ogival meshes, there is only one set of teeth in contact at any one time).

Section 2 of this appendix shows derivations of moment input-output expressions for both phases of contact of a single ogival mesh. Again,while pivot friction is considered in addition to contact friction, the frame is assumed to be stationary for this single mesh.

Appendix F

Two computer programs which deal with the kinematics and the moment input-output relationships of a single pass step-up gear mesh with clock teeth are given in this appendix. The structure of each of these programs is again discussed in detail and sample runs are used to explain their input and output parameters. The names of these programs and their relationships to work in other sections are given below:

1. Program CLOCK 1 : Kinematics of a single step-up gear mesh with clock teeth.
   This program is based on work in Appendix D as well as on work in section 1 of Appendix E. In addition,it has been used to check the geometry of the ogival meshes which were used in programs CLOCK 2, CLOCK 3 and CLOCK 4 (See Appendix I for the latter two).

2. Program CLOCK 2 : Point and cycle efficiencies for single pass step-up gear mesh with clock teeth.
   This program is based on work in section 2 of Appendix E.

Appendix G

When one considers the kinematic relationships of ogival meshes which are mounted on a fuze body as parts of two or three pass step-up trains, it becomes necessary to account for the relative positions of the individual meshes on the fuze body. Appendix G gives the appropriate derivations for each of the three meshes of a three-pass train. The model of the fuze body is identical with that used for involute step-up trains.

Appendix H

This appendix shows the derivations of moment input-output expressions for two and three pass step-up gear trains with ogival teeth which must operate in a spin environment.

Because of the increase in rotational speed associated with each tooth mesh, increasingly more sets of teeth will come into engagement in the second and third meshes as one set of teeth moves through one complete contact cycle in the first (i.e., the input) mesh. With two phases of motion for each mesh, there will be eight contact combinations in a three-pass train. Section 1 of Appendix H gives a derivation for the moment input-output expression of each of these eight cases.

Section 2 shows similar work for the four contact combinations which are associated with two pass step-up gear trains with ogival teeth.

Both analyses account for the effects of the centrifugal forces.

Appendix I

This appendix contains two computer programs which allow the determination of point and cycle efficiencies for two and three pass step-up gear meshes with clock teeth. As for other programs, the origins of their mathematical formulations are thoroughly discussed. In addition, their input and output parameters are explained with the help of a sample run. The names of these programs as well as their relationships to work in other appendixes are given below:

1. Program CLOCK 3 : Point and cycle efficiencies for three pass clock (ogival) step-up gear train in spin environment. This program is based on work given in Appendix G as well as in section 1 of Appendix H. The general fuze geometry is that described in section 6 of Appendix A.

2. Program CLOCK 4 : Point and cycle efficiencies for two pass clock (ogival) step-up gear train in spin environment. The kinematics of this program is again based on work given in Appendix G. The moment input-output relationships are from section 2 of Appendix H.

APPENDIX A

STEP-UP GEAR TRAINS WITH INVOLUTE TEETH

1. DIRECTION OF FRICTION FORCE AT TOOTH-TO-TOOTH CONTACT

Figure A-1 uses the base circle and the line-of-action configuration of an involute mesh, in which the gear drives the pinion, to determine the direction of the friction forces at contact point C before and after the contact point passes through pitch point P. Distance d represents the length of the line-of-action between base circle tangent points L and L'. The distance between contact point C and point L along the line-of-action is measured by length a. Further:

θ = actual (rolling) pressure angle

R_b = gear base radius

r_b = pinion base radius

The friction force of the pinion tooth on the gear tooth, for example, will have the direction of the relative velocity, $\bar{V}_{C_p/C_g}$, of the contact point on the pinion tooth, C_p, with respect to the coincident contact point on the gear tooth, C_g. (The friction force of the gear tooth on the pinion tooth has the opposite direction.) This relative velocity changes direction at the pitch point, where it becomes instantaneously zero.

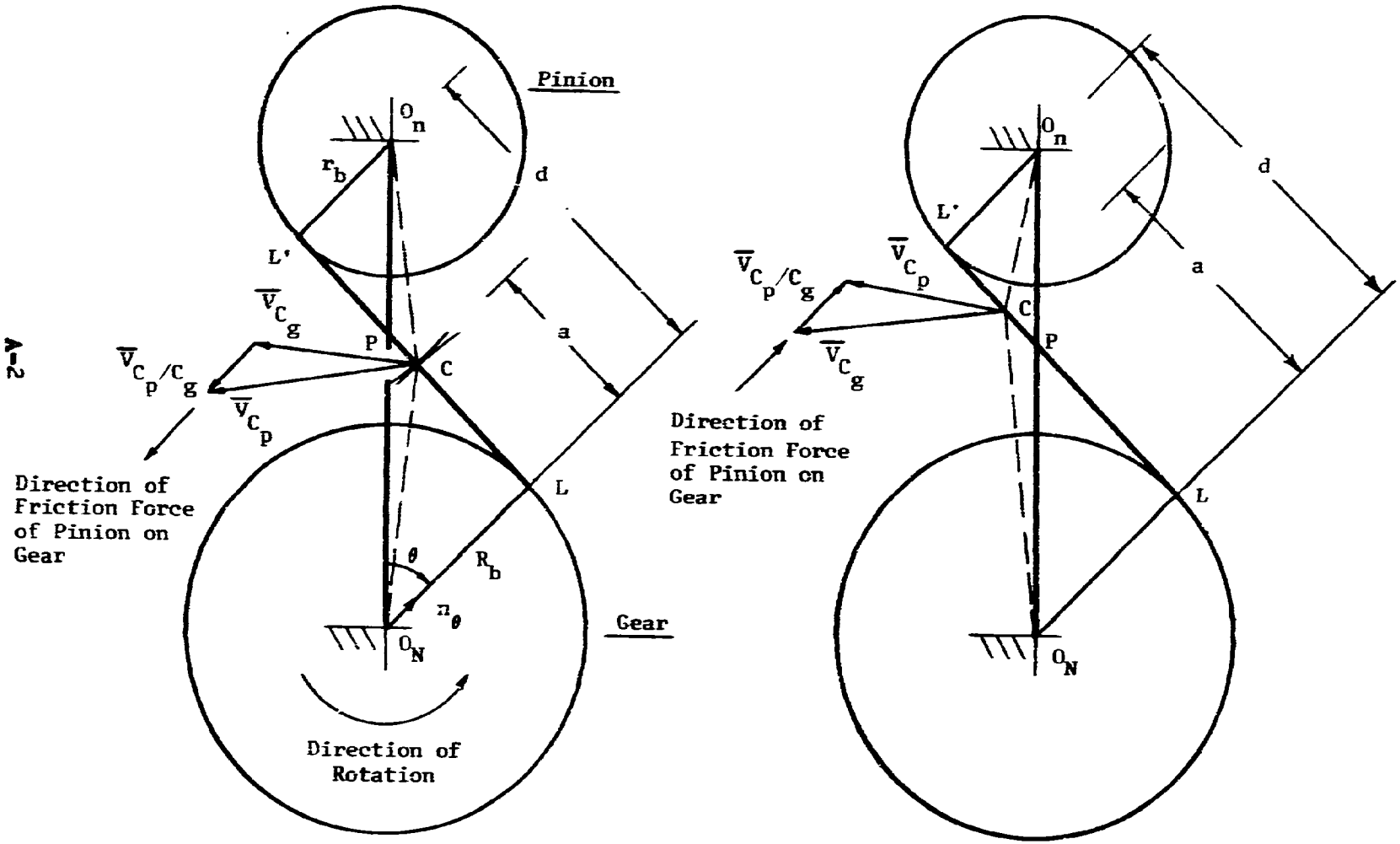


FIGURE A-1. DETERMINATION OF DIRECTION OF CONTACT FRICTION FORCES BY VELOCITY ANALYSIS

Figure A-1 shows contact before the pitch point (during approach). To obtain the direction of the relative velocity, $\overline{V}_{C_p/C_g}$, by a graphical analysis, one makes use of the velocity equation

$$\overline{V}_{C_p} = \overline{V}_{C_p/C_g} + \overline{V}_{C_g} \qquad \text{(A-1)}$$

where $\overline{V}_{C_p}$ = velocity of point C_p on pinion tooth with direction normal to line O_n- C

$\overline{V}_{C_p/C_g}$ = relative velocity between point C_p and point C_g. The direction of this velocity is normal to the line of action.

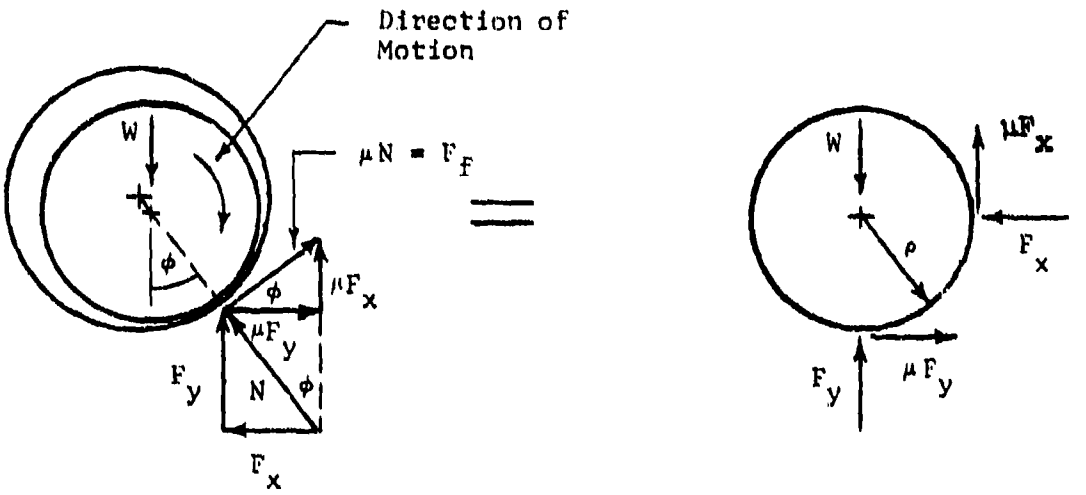
$\overline{V}_{C_g}$ = velocity of point C_g on gear tooth with direction normal to line O_N- C. The magnitude of this velocity was arbitrarily chosen.

The graphical construction, according to Eq (A1), shows that $\overline{V}_{C_p/C_g}$ has the direction opposite to that of the unit vector $\overline{n}_\theta$ shown at point O_N. As stated earlier, this represents

the friction force on the gear tooth during approach.

Figure A-1b shows the same graphical analysis for contact during recess. Once the pitch point is passed, both the relative velocity, $\overline{V}_{C_p/C_g}$, and the friction force of the pinion on the gear have the direction of the positive unit vector $\overline{n}_\theta$.

2. ASSUMPTIONS CONCERNING NORMAL AND FRICTION FORCES AT PIVOTS



a. b.

FIGURE A-2 FREE-BODY DIAGRAMS OF PIVOT SHAFTS

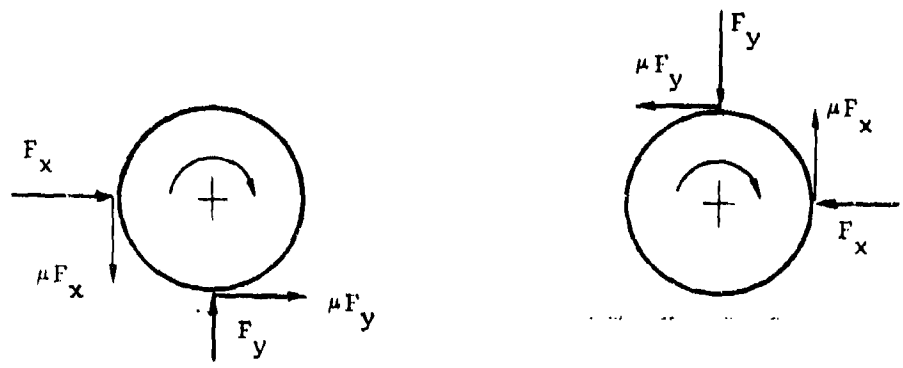
Figure A-2a shows a pivot shaft which is loaded by a known external force, W, and rotates in a clockwise direction. Due to friction between the shaft and the bearing, contact is made at an angle, $\phi = \tan^{-1}\mu$, where μ is the associated coefficient of friction. N is the normal contact

force. The friction force $F_f = \mu N$ opposes the clockwise rotation by creating a counterclockwise moment.

N may be resolved into the components F_x and F_y. The associated x and y components of the friction force are μF_y and μF_x, respectively.

The directions of the components of N and F_f are drawn in the same manner in Fig. A-2b in a somewhat more convenient representation. When the direction of the resultant external force, W, is not known, contact is possible anywhere on the periphery of the bearing and the components F_x and F_y of the normal contact force cannot be drawn with certainty in the free body diagram. The direction of the friction components must still oppose the motion.

Figure A-3 shows two general possibilities of drawing the free body diagram of a pivot which rotates in a clockwise direction. In either case, the moments of the friction components oppose the rotation while F_x and F_y may be positive or negative. Assume now, for example, that Fig. A-3a shows the wrong direction for F_x and that the solution of the applicable equilibrium equation will reverse the sign of F_x. This will automatically reverse the sign of the friction component μF_x also. Since contact is now made on the opposite



a. b.

FIGURE A-3 MOMENTS DUE TO FRICTION COMPONENTS ALWAYS OPPOSE MOTION

side of the pivot, the correct sign of the friction component is automatically assured.

The total friction moment then is expressed by

$$M_f = \pm \mu \rho \sqrt{F_x^2 + F_y^2} \qquad \text{(A-2)}$$

where ρ is the pivot radius. The sign of the above is chosen so that the rotation is opposed. In case F_x and F_y contain required terms that cannot be factored out of the square root of equation (A-2), the friction moment is conservatively overstated by the use of the absolute values of F_x and F_y

$$M_f = \pm \mu \rho \left(|F_x| + |F_y| \right) \qquad \text{(A-3a)}$$

If the expressions for F_x and F_y consist of sums of positive and negative terms, then F_x and F_y are presented as the sums of the absolute values of these terms. A conservative pivot friction moment becomes, similar to equation (A-3a),

$$M_f = \pm \mu \rho \left(\tilde{F}_x + \tilde{F}_y \right) \qquad \text{(A-3b)}$$

The tildes represent the sums of the absolute values of the component terms.

3. MOMENT INPUT-OUTPUT RELATIONSHIP FOR SINGLE STEP-UP GEAR MESH WITH INVOLUTE TEETH

Figure A-4 shows free body diagrams of the gear and the pinion of a single mesh where the gear is driven by a counterclockwise input moment M_{in}. It is desired to find the equilibrating output moment M_o.

a. UNIT VECTORS

The unit vector directed from point O_N to point L is given by

$$\bar{n}_\theta = \sin\theta\bar{i} + \cos\theta\bar{j} \qquad \text{(A-4)}$$

where θ represents the actual pressure angle, regardless of tooth modification. The unit vector directed along the line of action from point L to point L' is given by

$$\bar{n}_{\theta T} = -\cos\theta\bar{i} + \sin\theta\bar{j} \qquad \text{(A-5)}$$

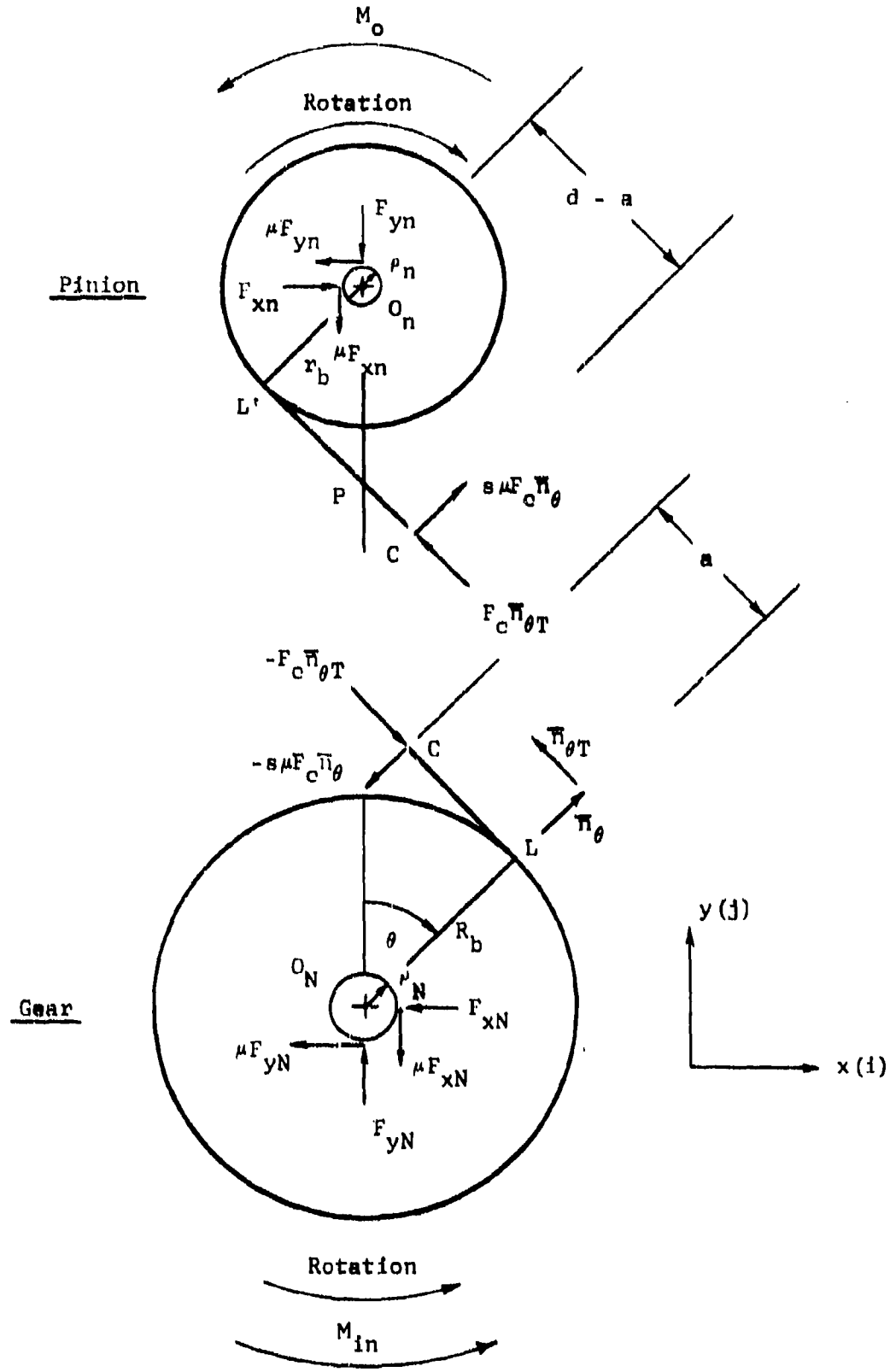


FIGURE A-4. FREE BODY DIAGRAM FOR SINGLE STEP-UP INVOLUTE GEAR MESH

5. NOMENCLATURE AND SIGNUM CONVENTION

F_{xN}, F_{yN} = x and y components of normal force acting on gear pivot

μF_{xN}, μF_{yN} = friction force components acting on gear pivot. Directions chosen to result in friction moments which oppose motion. (See part 2).

F_{xn}, F_{yn} = x and y components of normal force acting on pinion pivot

μF_{xn}, μF_{yn} = friction force components acting on pinion pivot

μ = coefficient of friction

F_c = normal contact force between gear and pinion. The force of the pinion on the gear is $(-)F_c\overline{n}_{\theta T}$, while the normal force of the gear on the pinion becomes $F_c\overline{n}_{\theta T}$.

μF_c = tooth contact friction force. The analysis in Section 1 shows that the friction force of the pinion on the gear, before the pitch point, acts in the direction of $(-)\overline{n}_\theta$. Therefore

$-s\mu F_c\overline{n}_\theta$ = friction force on gear with

s = +1 for $a < \overline{LP}$ (approach) (A-6)

and

$$s = -1 \quad \text{for } a > \overline{LP} \quad \text{(recess)} \tag{A-7}$$

while

$$s = 0 \quad \text{for } a = \overline{LP} \quad \text{(at pitch point)} \tag{A-8}$$

Further,

ρ_N, ρ_n = gear and pinion pivot radii

R_b, r_b = gear and pinion base circle radii

c. FORCE ANALYSIS OF THE GEAR

By inspecting Figure A-4, one sees that force equilibrium of the gear is expressed by

$$-F_c\bar{n}_{\theta T} - s\mu F_c\bar{n}_\theta - F_{xN}\bar{I} - \mu F_{xN}\bar{J} + F_{yN}\bar{J} - \mu F_{yN}\bar{I} = 0 \qquad (A\text{-}9)$$

Similarly, moment equilibrium of the gear is given by

$$M_{in}\bar{k} - \rho_N\mu\sqrt{F_{xN}^2 + F_{yN}^2}\,\bar{k} + R_b\bar{n}_\theta \times (-)F_c\bar{n}_{\theta T} + (R_b\bar{n}_\theta + a\bar{n}_{\theta T}) \times (-)s\mu F_c\bar{n}_\theta = 0 \qquad (A\text{-}10)$$

Note the use of equation (A-2) to express the friction moment at the gear pivot.

With the help of equations (A-4) and (A-5) one may write equations (A-9) and (A-10) in scalar form. Thus,

$$F_c\cos\theta - \mu sF_c\sin\theta - F_{xN} - \mu F_{yN} = 0 \qquad (A\text{-}11)$$

$$-F_c\sin\theta - \mu sF_c\cos\theta + F_{yN} - \mu F_{xN} = 0 \qquad (A\text{-}12)$$

Equation (A-10) becomes

$$M_{in} - \rho_N \mu \sqrt{F_{xN}^2 + F_{yN}^2} - R_b F_c + \mu s a F_c = 0 \qquad \text{(A-13)}$$

Simultaneous solution of equations (A-11) and (A-12) for F_{xN} and F_{yN} gives

$$F_{xN} = F_c \frac{(1 - \mu^2 s)\cos\theta - \mu(1 + s)\sin\theta}{1 + \mu^2} \qquad \text{(A-14)}$$

and

$$F_{yN} = F_c \frac{(1 - \mu^2 s)\sin\theta + \mu(1 + s)\cos\theta}{1 + \mu^2} \qquad \text{(A-15)}$$

When the above expressions are substituted into the moment equation (A-13) and if one notes that s^2 always equals +1, the following expression for F_c is obtained:

$$F_c = \frac{M_{in}}{R_b + \mu(\rho_N - as)} \qquad \text{(A-16)}$$

d. FORCE ANALYSIS OF THE PINION

Force equilibrium of the pinion is assured by

$$F_c\bar{n}_{\theta T} + \mu_s F_c \bar{n}_\theta + F_{xn}\bar{I} - F_{yn}\bar{J} - \mu F_{xn}\bar{J} - \mu F_{yn}\bar{I} = 0, \qquad \text{(A-17)}$$

while moment equilibrium is given by

$$M_o\bar{k} + \rho_n \mu \sqrt{F_{xn}^2 + F_{yn}^2}\,\bar{k} + \left[-r_b\bar{n}_\theta - (d-a)\bar{n}_{\theta T}\right] \times (F_c\bar{n}_{\theta T} + \mu_s F_c \bar{n}_\theta) = 0 \qquad \text{(A-18)}$$

In scalar form, the above become

$$-F_c\cos\theta + \mu_s F_c \sin\theta + F_{xn} - \mu F_{yn} = 0 \qquad \text{(A-19)}$$

$$F_c\sin\theta + \mu_s F_c \cos\theta - \mu F_{xn} - F_{yn} = 0 \qquad \text{(A-20)}$$

$$M_o + \mu\rho_n\sqrt{F_{xn}^2 + F_{yn}^2} - r_b F_c + \mu_s(d-a)F_c = 0 \qquad \text{(A-21)}$$

Equations (A-19) and (A-20) are now solved simultaneously for F_{xn} and F_{yn}. This gives

$$F_{xn} = F_c \frac{(1 + \mu^2 s)\cos\theta + \mu(1 - s)\sin\theta}{1 + \mu^2} \qquad \text{(A-22)}$$

and

$$F_{yn} = F_c \frac{(1 + \mu^2 s)\sin\theta - \mu(1 - s)\cos\theta}{1 + \mu^2} \qquad \text{(A-23)}$$

Equations (A-22) and (A-23) are then substituted into the moment equation (A-21). This furnishes the following expression for the normal contact force, F_c. (Again, s^2 always equals +1.):

$$F_c = \frac{M_o}{r_b - \mu\left[\rho_n + s(d - a)\right]} \qquad \text{(A-24)}$$

e. MOMENT INPUT-OUTPUT RELATIONSHIP

The equilibrant moment, M_o, may be expressed as a function of the input moment, M_{in}, after equations (A-16) and (A-24) have been set equal to each other. Thus,

$$M_o = \frac{M_{in}\left\{r_b - \mu\left[\rho_n + s(d - a)\right]\right\}}{R_b + \mu\left[\rho_N - sa\right]} \qquad \text{(A-25)}$$

The input-output relationship may also then be expressed in terms of

$$M_o = M_{in}\frac{r_b}{R_b}E_2 \qquad \text{(A-26)}$$

where $E_2 = \dfrac{1 - \dfrac{\mu\left[\rho_n + s(d - a)\right]}{r_b}}{1 + \dfrac{\mu(\rho_N - sa)}{R_b}}$,

and represents the efficiency of moment transmission of a single step-up mesh with involute teeth.

4. MOMENT INPUT-OUTPUT RELATIONSHIP FOR THREE STEP-UP GEAR TRAIN IN SPIN ENVIRONMENT

Figure A-5 shows the basic configuration of the three step-up gear train for which the relationship between the equilibrant output moment M_{o4}, acting on pinion 4, and the input moment M_{in}, acting on gear 1, is to be found.

The body-fixed x-y coordinate system has its origin at the spin axis, C, of the fuze body, and its x-axis coincides with the line $C\text{-}O_1$, where O_1 represents the pivot axis of the input gear-spin rotor combination. Points O_2, O_3 and O_4 represent the pivot axes of gear and pinion no. 2, gear and pinion no. 3, and pinion no. 4, respectively. Further,

$\mathcal{R}_i$ = distance from the spin axis to the various pivot axes

R_{bi} = base radii of gears

r_{bi} = base radii of pinions

β_1 = angle between positive x-axis and line of centers $O_1\text{-}O_2$

β_2 = angle between positive x-axis and line of centers $O_2\text{-}O_3$

β_3 = angle between positive x-axis and line of centers $O_3\text{-}O_4$

γ_1 = angle between positive x-axis and lines $C\text{-}O_1$

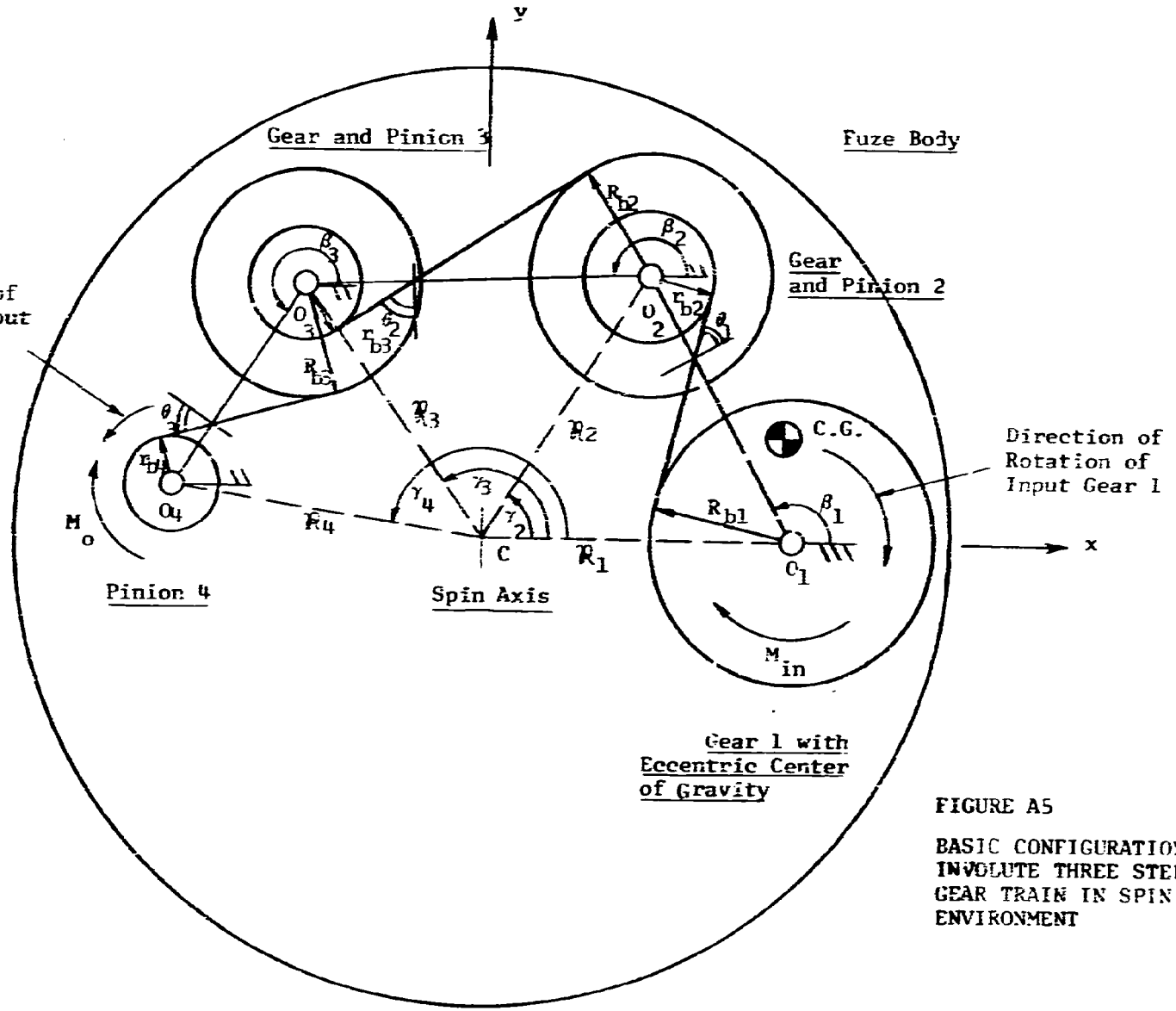


FIGURE A5

BASIC CONFIGURATION FOR INVOLUTE THREE STEP-UP GEAR TRAIN IN SPIN ENVIRONMENT

θ_1 = pressure angle of mesh between gear no. 1 and pinion no. 2

θ_2 = pressure angle of mesh between gear no. 2 and pinion no. 3

θ_3 = pressure angle of mesh between gear no. 3 and pinion no. 4

To obtain the moment input-output relationship of the total train, the input-output relationships of the individual components must first be obtained. The following equilibrium analyses include pivot as well as contact friction forces, in addition to loads due to the centrifugal forces on the individual components. The directions of the tooth-to-tooth friction forces are chosen according to the rules of Section 1 of this appendix, using an appropriate signum convention. The direction of the pivot friction forces is chosen according to Section 2 and, to avoid difficulties with the direction of the associated friction moment, equation (A-3b) will be used.

a. EQUILIBRIUM OF PINION 4

Figure A-6 shows a free body diagram of pinion 4. The contact between gear 3 and pinion 4 is shown before contact point C_3 has passed through pitch point P_3. The unit vector $\bar{n}_{34}$ is along the line-of-action in the direction of the contact force of the gear on the pinion.

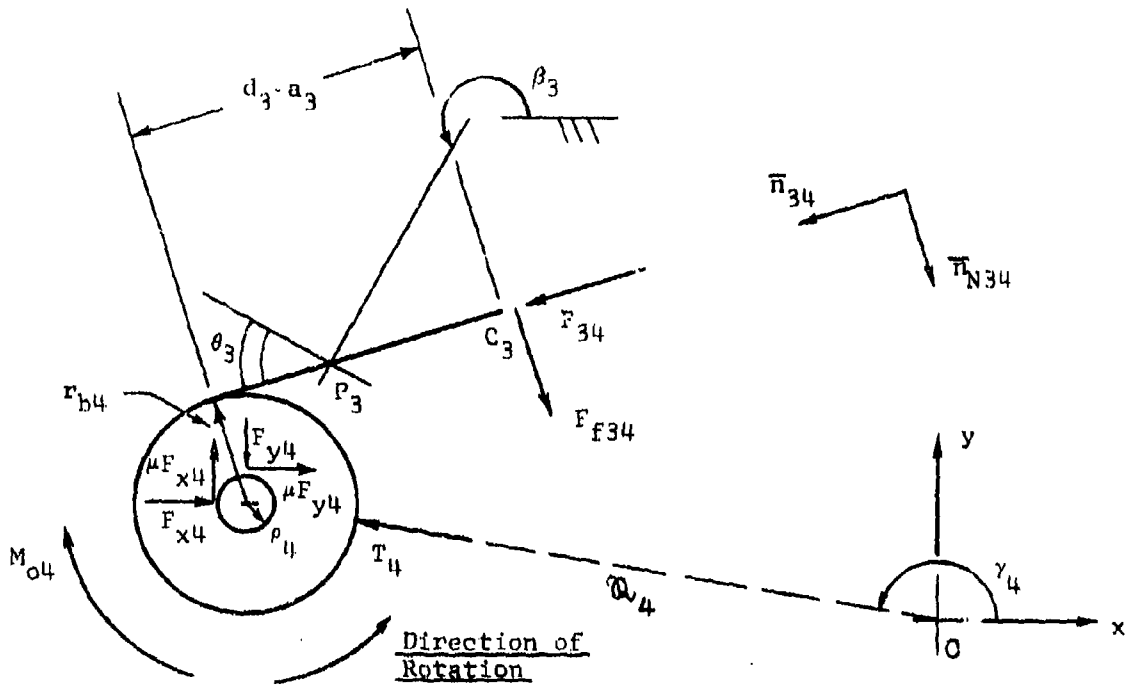


FIGURE A-6. FREE BODY DIAGRAM OF PINION 4

Thus,

$$\bar{n}_{34} = \sin(\beta_3 + \theta_3)\bar{I} - \cos(\beta_3 + \theta_3)\bar{J} \qquad \text{(A-27)}$$

The unit vector normal to the line-of-action is given by

$$\bar{n}_{N34} = \cos(\beta_3 + \theta_3)\bar{I} + \sin(\beta_3 + \theta_3)\bar{J} \qquad \text{(A-28)}$$

The contact force $\bar{F}_{34}$ then becomes

$$\bar{F}_{34} = F_{34}\,\bar{n}_{34} \qquad \text{(A-29)}$$

The friction force of gear 3 on pinion 4 is given by

$$\bar{F}_{f34} = \mu s_3 F_{34}\,\bar{n}_{N34} \qquad \text{(A-30)}$$

where $s_3 = +1$, for contact before the pitch point

$s_3 = 0$, for contact at the pitch point

$s_3 = -1$, for contact after the pitch point

(See also Section 1.)

The normal forces on the pivot shaft are given by

$$\bar{F}_{x4} = F_{x4}\,\bar{i} \tag{A-31}$$

and

$$\bar{F}_{y4} = -F_{y4}\,\bar{j} \tag{A-32}$$

The associated pivot friction forces are given by $\mu F_{x4}\,\bar{j}$ and $\mu F_{y4}\,\bar{i}$ for the indicated direction of rotation. The centrifugal force $\bar{T}_4$ on the pinion is represented by

$$\bar{T}_4 = T_4(\cos\gamma_4\,\bar{i} + \sin\gamma_4\,\bar{j}) \tag{A-33}$$

where

$$T_4 = \mathcal{R}_4\omega^2 m_4 \tag{A-34a}$$

with

$$\omega = \text{spin angular velocity} \tag{A-34b}$$

and

$$m_4 = \text{mass of pinion 4} \tag{A-34c}$$

The force equilibrium equation is given by

$$F_{34}\bar{n}_{34} + \mu s_3 F_{34}\bar{n}_{N34} + T_4(\cos\gamma_4\,\bar{i} + \sin\gamma_4\,\bar{j}) + F_{x4}\,\bar{i} + \mu F_{y4}\,\bar{i} + \mu F_{x4}\,\bar{j} - F_{y4}\,\bar{j} = 0 \tag{A-35}$$

Moment equilibrium is given by the following expression, in which the pivot friction moment is expressed according to Equation (A-3b):

$$-M_{o4} - \rho_4\mu(\tilde{F}_{x4} + \tilde{F}_{y4}) + r_{b4}F_{34} - \mu s_3(d_3 - a_3)F_{34} = 0 \qquad \text{(A-36)}$$

where ρ_4 represents the pivot radius. d_3 is the length of the line-of-action of the mesh from points of tangency to the base circles. a_3 is the distance on the line of action from the gear point of tangency to the contact point C_3.

Equation (A-35) gives the following component equations:

$$F_{34}\sin(\beta_3 + \theta_3) + \mu s_3 F_{34}\cos(\beta_3 + \theta_3) + T_4\cos\gamma_4 + F_{x4} + \mu F_{y4} = 0 \qquad \text{(A-37)}$$

$$-F_{34}\cos(\beta_3 + \theta_3) + \mu s_3 F_{34}\sin(\beta_3 + \theta_3) + T_4\sin\gamma_4 - F_{y4} + \mu F_{x4} = 0 \qquad \text{(A-38)}$$

Simultaneous solution of the above for F_{x4} and F_{y4} results in

$$F_{x4} = \frac{1}{1 + \mu^2}\left\{-T_4\left[\cos\gamma_4 + \mu\sin\gamma_4\right] - F_{34}\left[(1 + \mu^2 s_3)\sin(\beta_3 + \theta_3) - \mu(1 - s_3)\cos(\beta_3 + \theta_3)\right]\right\} \quad \text{(A-39)}$$

and

$$F_{y4} = \frac{1}{1 + \mu^2}\left\{T_4\left[\sin\gamma_4 - \mu\cos\gamma_4\right] - F_{34}\left[(1 + \mu^2 s_3)\cos(\beta_3 + \theta_3) + \mu(1 - s_3)\sin(\beta_3 + \theta_3)\right]\right\} \quad \text{(A-40)}$$

To obtain conservative values for the pivot friction moment in equation (A-36) according to equation (A-3b), one substitutes the largest possible values for F_{x4} and F_{y4}. This is accomplished by making the signs of T_4 and F_{34} positive and by using the absolute values of their respective coefficients in Equations (A-39) and (A-40). Equation (A-36) then becomes

$$-M_{o4} - \mu\rho_4(T_4A_1 + F_{34}A_2 + T_4A_3 + F_{34}A_4) - r_{b4}F_{34} - \mu s_3(d_3 - a_3)F_{34} = 0 \quad \text{(A-41)}$$

where

$$A_1 = \left|\frac{\sin\gamma_4 - \mu\cos\gamma_4}{1 + \mu^2}\right| \quad \text{(A-42)}$$

$$A_2 = \left| \frac{(1 + \mu^2 s_3)\cos(\beta_3 + \theta_3) + \mu(1 - s_3)\sin(\beta_3 + \theta_3)}{1 + \mu^2} \right| \quad \text{(A-43)}$$

$$A_3 = \left| \frac{\cos\gamma_4 + \mu\sin\gamma_4}{1 + \mu^2} \right| \quad \text{(A-44)}$$

$$A_4 = \left| \frac{(1 + \mu^2 s_3)\sin(\beta_3 + \theta_3) - \mu(1 - s_3)\cos(\beta_3 + \theta_3)}{1 + \mu^2} \right| \quad \text{(A-45)}$$

Finally equation (A-41) is solved for F_{34}

$$F_{34} = \frac{M_{04}}{D_1} + \frac{T_4 C_1}{D_1} \quad \text{(A-46)}$$

where

$$C_1 = \mu\rho_4(A_1 + A_3) \quad \text{(A-47)}$$

$$D_1 = r_{b4} - \mu\left[s_3(d_3 - a_3) + \rho_4(A_2 + A_4)\right] \quad \text{(A-48)}$$

b. EQUILIBRIUM OF GEAR AND PINION SET NO. 3

Figure A-7 shows the free body diagram of gear and pinion set no. 3. Contact point C_3, between pinion 4 and gear 3, is, as shown previously in Figure A-6, before pitch point P_3. The normal force, along the line-of-action, is given [see equation (A-29)] by

$$\bar{F}_{43} = -F_{34}\bar{n}_{34} \tag{A-49}$$

and the associated friction force $\bar{F}_{f43}$ is given [see equation (A-30)] by

$$\bar{F}_{f43} = -s_3\mu F_{34}\bar{n}_{N34} \tag{A-50}$$

The unit vectors along(and perpendicular to)the line-of-action of gear 2 and pinion 3 are given by

$$\bar{n}_{23} = -\sin(\beta_2 - \theta_2)\,\bar{I} + \cos(\beta_2 - \theta_2)\,\bar{J} \tag{A-51}$$

and

$$\bar{n}_{N23} = -\cos(\beta_2 - \theta_2)\,\bar{I} - \sin(\beta_2 - \theta_2)\,\bar{J} \tag{A-52}$$

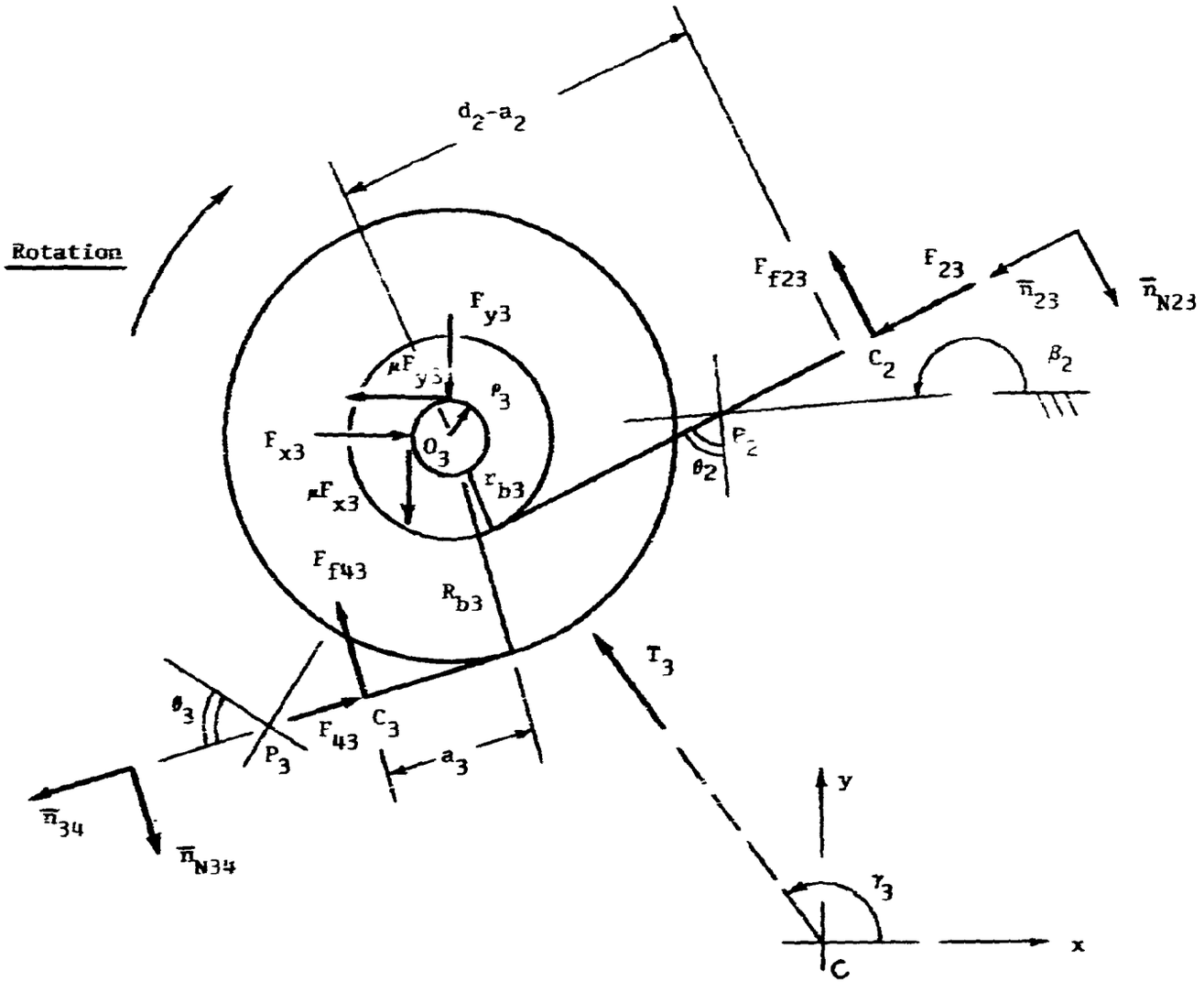


FIGURE A-7. EQUILIBRIUM OF GEAR AND PINION SET NO. 3

The contact point C_2 between gear 2 and pirion 3 is also shown before pitch point P_2 is passed.

The normal contact force between these teeth then becomes

$$\bar{F}_{23} = F_{23}\bar{n}_{23} \tag{A-53}$$

The associated friction force is given by

$$\bar{F}_{f23} = -\mu s_2 F_{23}\bar{n}_{N23} \tag{A-54}$$

where $s_2 = +1$ for contact before pitch point P_2

$s_2 = 0$ for contact at pitch point P_2

$s_2' = -1$ for contact after pitch point P_2

The normal forces on the pivot shaft are given by

$$\bar{F}_{x3} = F_{x3}\bar{I} \tag{A-55}$$

and

$$\bar{F}_{y3} = -F_{y3}\bar{J} \tag{A-56}$$

The associated pivot friction forces are represented by $(-)\mu F_{y3}\bar{I}$

and $(-)\mu F_{x3}\bar{J}$ for the indicated direction of gear rotation.

The centrifugal force, $\bar{T}_3$, on the assembly is given by

$$\bar{T}_3 = T_3(\cos\gamma_3\bar{i} + \sin\gamma_3\bar{j}) \qquad \text{(A-57)}$$

where

$$T_3 = R_3\omega^2 m_3 \qquad \text{(A-58a)}$$

with

$$m_3 = \text{mass of pinion and gear 3} \qquad \text{(A58b)}$$

Force equilibrium is given by

$$F_{23}\bar{n}_{23} - \mu s_2 F_{23}\bar{n}_{N23} - F_{34}\bar{n}_{34} - \mu s_3 F_{34}\bar{n}_{N34} + T_3(\cos\gamma_3\bar{I} + \sin\gamma_3\bar{J}) + F_{x3}\bar{I} - \mu F_{y3}\bar{I} - F_{y3}\bar{J} - \mu F_{x3}\bar{J} = 0 \qquad \text{(A-59)}$$

Moment equilibrium requires

$$R_{b3}F_{34} - \mu s_3 a_3 F_{34} - r_{b3}F_{23} + \mu s_2(d_2 - a_2)F_{23} + \mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3}) = 0 \qquad \text{(A-60)}$$

Note the use of equation (A-3b) for the pivot friction moment. ρ_3 represents the pivot radius and length d_2 is the length of the

line-of-action between the points of tangency to the base circles. a_2 is the distance along the line-of-action from the gear point of tangency to the contact point C_2.

The component form of equation (A-59) is represented by the following two expressions:

$$-F_{23}\sin(\beta_2 - \theta_2) + \mu s_2 F_{23}\cos(\beta_2 - \theta_2) - F_{34}\sin(\beta_3 + \theta_3) - \mu s_3 F_{34}\cos(\beta_3 + \theta_3) + T_3\cos\gamma_3 + F_{x3} - \mu F_{y3} = 0 \qquad (A\text{-}61)$$

and

$$F_{23}\cos(\beta_2 - \theta_2) + \mu s_2 F_{23}\sin(\beta_2 - \theta_2) + F_{34}\cos(\beta_3 + \theta_3) - \mu s_3 F_{34}\sin(\beta_3 + \theta_3) + T_3\sin\gamma_3 - F_{y3} - \mu F_{x3} = 0 \qquad (A\text{-}62)$$

Simultaneous solution of the above for F_{x3} and F_{y3} leads to

$$F_{x3} = \frac{1}{1+\mu^2}\Big\{F_{23}\left[(1+\mu^2 s_2)\sin(\beta_2 - \theta_2) + \mu(1 - s_2)\cos(\beta_2 - \theta_2)\right] + T_3\left[\mu\sin\gamma_3 - \cos\gamma_3\right] + F_{34}\left[(1 - \mu^2 s_3)\sin(\beta_3 + \theta_3) + \mu(1 + s_3)\cos(\beta_3 + \theta_3)\right]\Big\} \qquad (A\text{-}63)$$

and

$$F_{y3} = \frac{1}{1+\mu^2}\left\{F_{23}\left[(1+\mu^2 s_2)\cos(\beta_2-\theta_2) + \mu(s_2-1)\sin(\beta_2-\theta_2)\right] + T_3\left[\sin\gamma_3 + \cos\gamma_3\right] + F_{34}\left[(1-\mu^2 s_3)\cos(\beta_3+\theta_3) - \mu(s_3+1)\sin(\beta_3+\theta_3)\right]\right\} \quad \text{(A-64)}$$

Now, equations (A-63) and (A-64) are substituted into the moment equation (A-60) with consideration of the pivot friction moment according to equation (A-3b). This gives

$$R_{b3}F_{34} - \mu s_3 a_3 F_{34} - r_{b3}F_{23} + \mu s_2(d_2 - a_2)F_{23} + \mu r_3\left[F_{23}(A_5 + A_8) + T_3(A_6 + A_9) + F_{34}(A_7 + A_{10})\right] = 0 \quad \text{(A-65)}$$

where

$$A_5 = \left|\frac{(1+\mu^2 s_2)\cos(\beta_2-\theta_2) + \mu(s_2-1)\sin(\beta_2-\theta_2)}{1+\mu^2}\right| \quad \text{(A-66)}$$

$$A_6 = \left|\frac{\sin\gamma_3 + \mu\cos\gamma_3}{1+\mu^2}\right| \quad \text{(A-67)}$$

$$A_7 = \left| \frac{(1 - \mu^2 s_3)\cos(\beta_3 + \theta_3) - \mu(1 + s_3)\sin(\beta_3 + \theta_3)}{1 + \mu^2} \right| \quad \text{(A-68)}$$

$$A_8 = \left| \frac{(1 + \mu^2 s_2)\sin(\beta_2 - \theta_2) + \mu(1 - s_2)\cos(\beta_2 - \theta_2)}{1 + \mu^2} \right| \quad \text{(A-69)}$$

$$A_9 = \left| \frac{\mu \sin\gamma_3 - \cos\gamma_3}{1 + \mu^2} \right| \quad \text{(A-70)}$$

$$A_{10} = \left| \frac{(1 - \mu^2 s_3)\sin(\beta_3 + \theta_3) + \mu(1 + s_3)\cos(\beta_3 + \theta_3)}{1 + \mu^2} \right| \quad \text{(A-71)}$$

Finally equation (A-65) is solved for F_{23}

$$F_{23} = \frac{F_{34}C_2 + T_3C_3}{D_2} \quad \text{(A-72)}$$

where

$$C_2 = R_{b3} - \mu\left[s_3 a_3 - \rho_3(A_7 + A_{10})\right] \quad \text{(A-73)}$$

$$C_3 = \mu\rho_3(A_6 + A_9) \quad \text{(A-74)}$$

$$D_2 = r_{b3} - \mu\left[s_2(d_2 - a_2) + \rho_3(A_5 + A_8)\right] \quad \text{(A-75)}$$

c. EQUILIBRIUM OF GEAR AND PINION SET NO. 2

Figure A-8 shows the free body diagram of gear and pinion set no. 2. The contact point C_2, between gear 2 and pinion 3, is again shown before the pitch point P_2 is passed. (See also Figure A-7.) The normal force, along the line-of-action, becomes with equation (A-72)

$$\bar{F}_{32} = -F_{23}\bar{n}_{23} \tag{A-76}$$

The associated friction force, $\bar{F}_{f32}$, is given by

$$\bar{F}_{f32} = \mu_{s_2} F_{23}\bar{n}_{N23} \tag{A-77}$$

The unit vectors along(and perpendicular to)the line-of-action of gear 1 and pinion 2 are given by

$$\bar{n}_{12} = \sin(\beta_1 + \theta_1)\bar{I} - \cos(\beta_1 + \theta_1)\bar{J} \tag{A-78}$$

and

$$\bar{n}_{N12} = \cos(\beta_1 + \theta_1)\bar{I} + \sin(\beta_1 + \theta_1)\bar{J} \tag{A-79}$$

The contact point C_1 between gear 1 and pinion 2 is also shown before pitch point P_1 is passed.

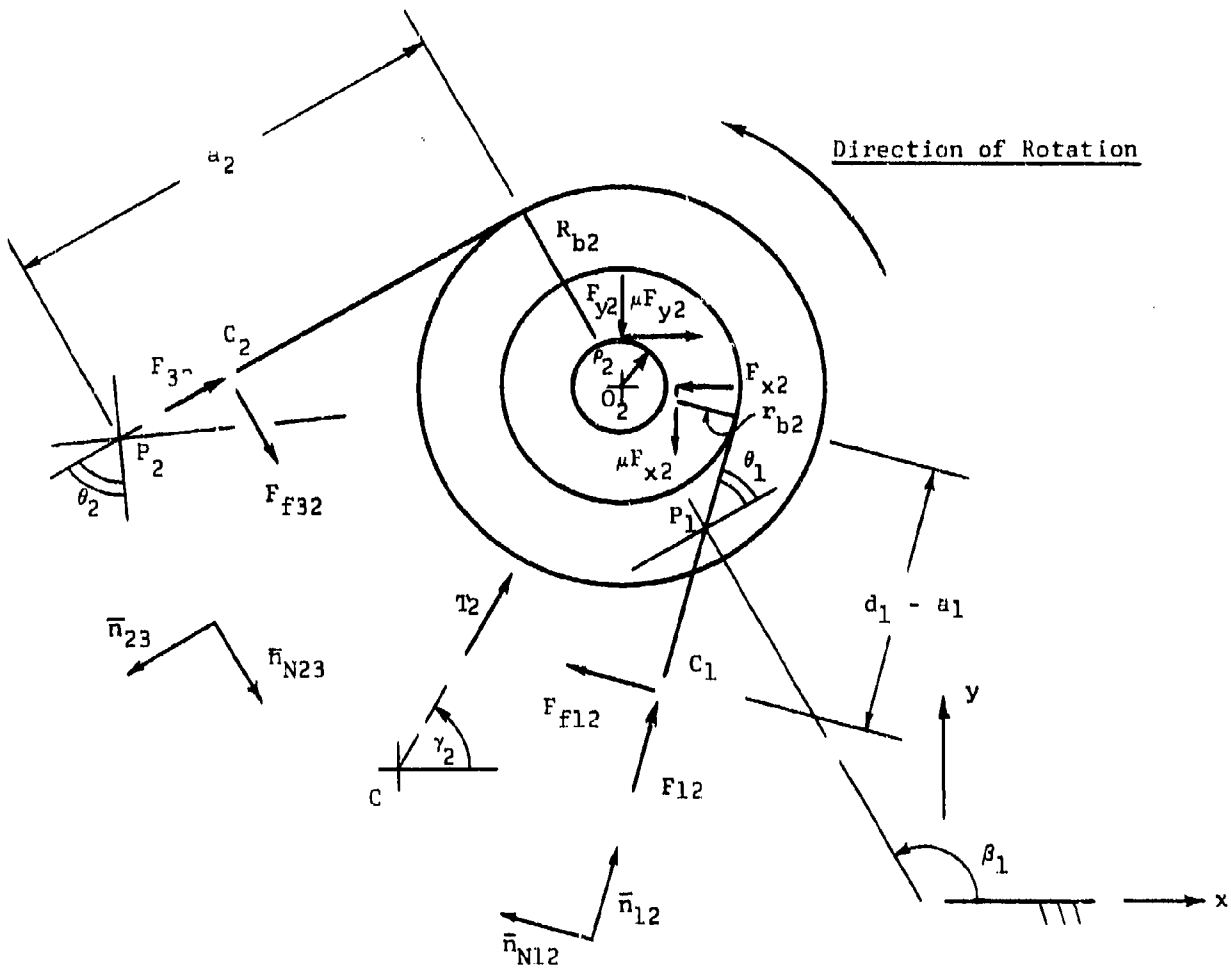


FIGURE A8, FREE BODY DIAGRAM OF GEAR AND PINION SET NO. 2

The normal contact force between the teeth of this mesh becomes

$$\bar{F}_{12} = F_{12}\bar{n}_{12} , \tag{A-80}$$

while the associated friction force is given by (See Section 1 of this Appendix.)

$$\bar{F}_{f12} = \mu s_1 F_{12}\bar{n}_{N12} \tag{A-81}$$

where $s_1 = +1$ for contact before pitch point P_1

$s_1 = 0$ for contact at pitch point P_1

$s_1 = -1$ for contact after pitch point P_1

The normal forces on the pivot shaft are

$$\bar{F}_{x2} = -F_{x2}\bar{I} \tag{A-82}$$

and

$$\bar{F}_{y2} = -F_{y2}\bar{J} \tag{A-83}$$

The associated pivot friction forces are again chosen such that

their friction moments oppose the indicated rotation.

The centrifugal force, $\bar{T}_2$, on this gear and pinion assembly is given by

$$\bar{T}_2 = T_2(\cos\gamma_2\bar{I} + \sin\gamma_2\bar{J}) \qquad \text{(A-84)}$$

where $T_2 = \mathcal{R}_2\omega^2 m_2$ (A-85a)

with

m_2 = mass of gear and pinion set no. 2 (A-85b)

Force equilibrium is given by

$$-F_{23}\bar{n}_{23} + \mu s_2 F_{23}\bar{n}_{N23} + F_{12}\bar{n}_{12} + \mu s_1 F_{12}\bar{n}_{N12} - T_2(\cos\gamma_2\bar{I} + \sin\gamma_2\bar{J}) - F_{x2}\bar{I} - F_{y2}\bar{J} + \mu F_{y2}\bar{I} - \mu F_{x2}\bar{J} = 0 \qquad \text{(A-86)}$$

Moment equilibrium is given by

$$-R_{b2}F_{23} + \mu s_2 a_2 F_{23} + r_{b2}F_{12} - \mu s_1(d_1 - a_1)F_{12} - \mu\rho_2(\tilde{F}_{x2} + \tilde{F}_{y2}) = 0 \qquad \text{(A-87)}$$

Again, equation (A-3b) is used to account for the pivot friction moment. ρ_2 represents the pivot radius. d_1 is the length of the line-of-action between the points of tangency to the base circles, and a_1 is the distance from the point of tangency of the gear to the contact point C_1.

The component form of equation (A-86) is given by

$$F_{23}\sin(\beta_2 - \theta_2) - \mu s_2 F_{23}\cos(\beta_2 - \theta_2) + F_{12}\sin(\beta_1 + \theta_1) + \mu s_1 F_{12}\cos(\beta_1 + \theta_1) + T_2\cos\gamma_2 - F_{x2} + \mu F_{y2} = 0 \qquad \text{(A-88)}$$

$$-F_{23}\cos(\beta_2 - \theta_2) - \mu s_2 F_{23}\sin(\beta_2 - \theta_2) - F_{12}\cos(\beta_1 + \theta_1) + \mu s_1 F_{12}\sin(\beta_1 + \theta_1) + T_2\sin\gamma_2 - F_{y2} - \mu F_{x2} = 0 \qquad \text{(A-89)}$$

Simultaneous solution of the above furnishes

$$F_{x2} = \frac{1}{1 + \mu^2}\Big\{-F_{12}\left[\mu(1 - s_1)\cos(\beta_1 + \theta_1) - (1 + \mu^2 s_1)\sin(\beta_1 + \theta_1)\right] + T_2\left[\mu\sin\gamma_2 + \cos\gamma_2\right] + F_{23}\left[(1 - \mu^2 s_2)\sin(\beta_2 - \theta_2) - \mu(1 + s_2)\cos(\beta_2 - \theta_2)\right]\Big\} \qquad \text{(A-90)}$$

and

$$F_{y2} = \frac{1}{1+\mu^2}\Big\{-F_{12}\Big[\mu(1-s_1)\sin(\beta_1+\theta_1) + (1+\mu^2 s_1)\cos(\beta_1+\theta_1)\Big] + T_2\Big[\sin\gamma_2 - \mu\cos\gamma_2\Big] + F_{23}\Big[-\mu(1+s_2)\sin(\beta_2-\theta_2) - (1-\mu^2 s_2)\cos(\beta_2-\theta_2)\Big]\Big\} \quad \text{(A-91)}$$

Now, equations (A-90) and (A-91) are substituted into the moment equation (A-87). With consideration of the pivot friction moment according to equation (A-3b), this gives

$$-R_{b2}F_{23} + \mu s_2 a_2 F_{23} + r_{b2}F_{12} - \mu s_1(d_1 - a_1)F_{12} - \mu\rho_2\Big[F_{12}(A_{11}+A_{14}) + T_2(A_{12}+A_{15}) + F_{23}(A_{13}+A_{16})\Big] = 0 \quad \text{(A-92)}$$

In the above

$$A_{11} = \left|\frac{\mu(1-s_1)\sin(\beta_1+\theta_1) + (1+\mu^2 s_1)\cos(\beta_1+\theta_1)}{1+\mu^2}\right| \quad \text{(A-93)}$$

$$A_{12} = \left|\frac{\sin\gamma_2 - \mu\cos\gamma_2}{1+\mu^2}\right| \quad \text{(A-94)}$$

$$A_{13} = \left| \frac{\mu(1 + s_2)\sin(\beta_2 - \theta_2) + (1 - \mu^2 s_2)\cos(\beta_2 - \theta_2)}{1 + \mu^2} \right| \quad \text{(A-95)}$$

$$A_{14} = \left| \frac{\mu(1 - s_1)\cos(\beta_1 + \theta_1) - (1 + \mu^2 s_1)\sin(\beta_1 + \theta_1)}{1 + \mu^2} \right| \quad \text{(A-96)}$$

$$A_{15} = \left| \frac{\mu \sin\gamma_2 + \cos\gamma_2}{1 + \mu^2} \right| \quad \text{(A-97)}$$

$$A_{16} = \left| \frac{(1 - \mu^2 s_2)\sin(\beta_2 - \theta_2) - \mu(1 + s_2)\cos(\beta_2 - \theta_2)}{1 + \mu^2} \right| \quad \text{(A-98)}$$

Finally, equation (A-92) is solved for F_{12}

$$F_{12} = \frac{F_{23}C_4 + T_2C_5}{D_3} \quad \text{(A-99)}$$

where

$$C_4 = R_{b2} - \mu\left[s_2 a_2 - \rho_2(A_{13} + A_{16})\right] \quad \text{(A-100)}$$

$$C_5 = \mu\rho_2(A_{12} + A_{15}) \quad \text{(A-101)}$$

$$D_3 = r_{b2} - \mu\left[s_1(d_1 - a_1) + \rho_2(A_{11} + A_{14})\right] \quad \text{(A-102)}$$

d. EQUILIBRIUM OF GEAR NO. 1

Figure A-9 shows the free body diagram of gear no. 1, the input gear.

The contact point C_1 is identical with that shown in Figure A-8.

The normal force on gear no. 1 is given by (See equation (A-80).)

$$\bar{F}_{21} = -F_{12}\bar{n}_{12} \tag{A-103}$$

The associated friction force is

$$\bar{F}_{f21} = -\mu s_1 F_{12}\bar{n}_{N12} \tag{A-104}$$

The normal forces on the pivot shaft are given by

$$\bar{F}_{x1} = -F_{x1}\bar{I} \tag{A-105}$$

and

$$\bar{F}_{y1} = F_{y1}\bar{J} \tag{A-106}$$

The associated pivot friction forces are chosen in such a direction that their moments oppose the indicated rotation.

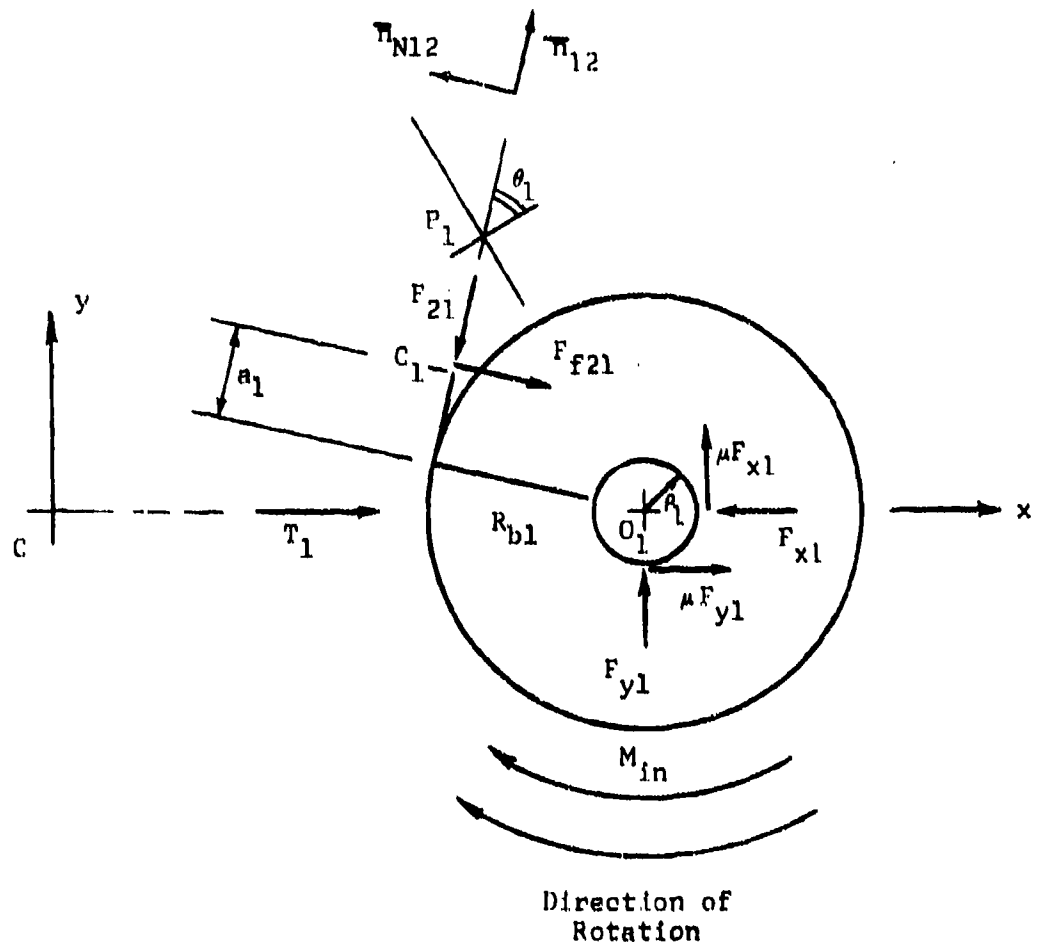


FIGURE A9. FREE BODY DIAGRAM OF GEAR 1

The centrifugal force $\bar{T}_1$ on gear 1 is given by

$$\bar{T}_1 = T_1\bar{I} \tag{A-107}$$

where

$$T_1 = \mathcal{R}_1\omega^2 m_1 \tag{A-108a}$$

$$m_1 = \text{mass of gear 1} \tag{A-108b}$$

Force equilibrium is given by

$$-F_{12}\bar{n}_{12} - \mu s_1 F_{12}\bar{n}_{N12} + T_1\bar{I} - F_{x1}\bar{I} + F_{y1}\bar{J} + \mu F_{y1}\bar{I} + \mu F_{x1}\bar{J} = 0 \tag{A-109}$$

Moment equilibrium is found from

$$R_{b1}F_{12} - \mu s_1 a_1 F_{12} - M_{in} + \mu\rho_1(\tilde{F}_{x1} + \tilde{F}_{y1}) = 0 \tag{A-110}$$

The form of equation (A-3b) is again utilized to obtain the pivot friction moment. ρ_1 represents the pivot radius of gear 1.

The component form of equation (A-109) becomes

$$-F_{12}\sin(\beta_1 + \theta_1) - \mu s_1 F_{12}\cos(\beta_1 + \theta_1) - F_{x1} + \mu F_{y1} + T_1 = 0 \qquad \text{(A-111)}$$

and

$$F_{12}\cos(\beta_1 + \theta_1) - \mu s_1 F_{12}\sin(\beta_1 + \theta_1) + F_{y1} + \mu F_{x1} = 0 \qquad \text{(A-112)}$$

Simultaneous solution of equations (A-111) and (A-112) furnishes the forces on the pivot, i.e.,

$$F_{x1} = \frac{-F_{12}\left[(1 - \mu^2 s_1)\sin(\beta_1 + \theta_1) + \mu(1 + s_1)\cos(\beta_1 + \theta_1)\right] + T_1}{1 + \mu^2} \qquad \text{(A-113)}$$

$$F_{y1} = \frac{F_{12}\left[\mu(1 + s_1)\sin(\beta_1 + \theta_1) - (1 - \mu^2 s_1)\cos(\beta_1 + \theta_1)\right] - \mu T_1}{1 + \mu^2} \qquad \text{(A-114)}$$

Equations (A-113) and (A-114) are now substituted into the moment equation (A-110) in the following manner: (Again, the method of equation (A-3b) is applied.)

$$R_{b1}F_{12} - \mu s_1 a_1 F_{12} - M_{in} + \mu\rho_1\left[F_{12}(A_{17} + A_{19}) + T_1(A_{18} + A_{20})\right] = 0 \qquad \text{(A-115)}$$

where

$$A_{17} = \left| \frac{(1 - \mu^2 s_1)\sin(\beta_1 + \theta_1) + \mu(1 + s_1)\cos(\beta_1 + \theta_1)}{1 + \mu^2} \right| \qquad \text{(A-116)}$$

$$A_{18} = \left| \frac{1}{1 + \mu^2} \right| \qquad \text{(A-117)}$$

$$A_{19} = \left| \frac{\mu(1 + s_1)\sin(\beta_1 + \theta_1) - (1 - \mu^2 s_1)\cos(\beta_1 + \theta_1)}{1 + \mu^2} \right| \qquad \text{(A-118)}$$

$$A_{20} = \left| \frac{\mu}{1 + \mu^2} \right| \qquad \text{(A-119)}$$

Finally, equation (A-115) is solved for F_{12}

$$F_{12} = \frac{M_{in}}{D_4} - \frac{T_1 C_6}{D_4} \qquad \text{(A-120)}$$

where

$$C_6 = \mu\rho_1(A_{18} + A_{20}) \qquad \text{(A-121)}$$

$$D_4 = R_{b1} - \mu\left[s_1 a_1 - \rho_1(A_{17} + A_{19})\right] \qquad \text{(A-122)}$$

e. INPUT-OUTPUT RELATIONSHIP

To obtain the input-output relationship for the complete gear train, equation (A-120) is now equated to equation (A-99). This furnishes

$$F_{23} = \frac{D_3}{C_4 D_4} (M_{in} - T_1 C_6) - T_2 \frac{C_5}{C_4} \qquad \text{(A-123)}$$

Further, the above is equated to equation (A-72). This results in the following expression for F_{34}

$$F_{34} = \frac{D_2 D_3}{C_3 C_4 D_4} (M_{in} - T_1 C_6) - T_2 \frac{C_5 D_2}{C_2 C_4} - T_3 \frac{C_3}{C_2} \qquad \text{(A-124)}$$

Finally, equation (A-124) is equated to equation (A-46). This establishes the input-output relationship

$$M_{04} = \frac{D_1 D_2 D_3}{C_2 C_4 D_4} (M_{in} - T_1 C_6) - T_2 \frac{C_5 D_1 D_2}{C_2 C_4} - T_3 \frac{C_3 D_1}{C_2} - T_4 C_1 \qquad \text{(A-125)}$$

5. MOMENT INPUT-OUTPUT RELATIONSHIP FOR INVOLUTE TWO STEP-UP GEAR TRAIN IN SPIN ENVIRONMENT

Figure A-10 shows the basic configuration of a two step-up gear train with involute teeth for which the relationship between the equilibrant output moment, M_{o3}, acting on pinion 3, and the input moment, M_{in}, acting on gear 1 is to be found. All nomenclature is identical with that used in Section 4 in connection with the three step-up gear train.

Again, the general relationship between input and output is found by assembling the input-output relationships of the individual component gears.

a. EQUILIBRIUM OF PINION 3

Figure A-11 shows the free body diagram of pinion 3. The contact point C_2 between gear 2 and pinion 3 is shown before the pitch point P_2 is passed.

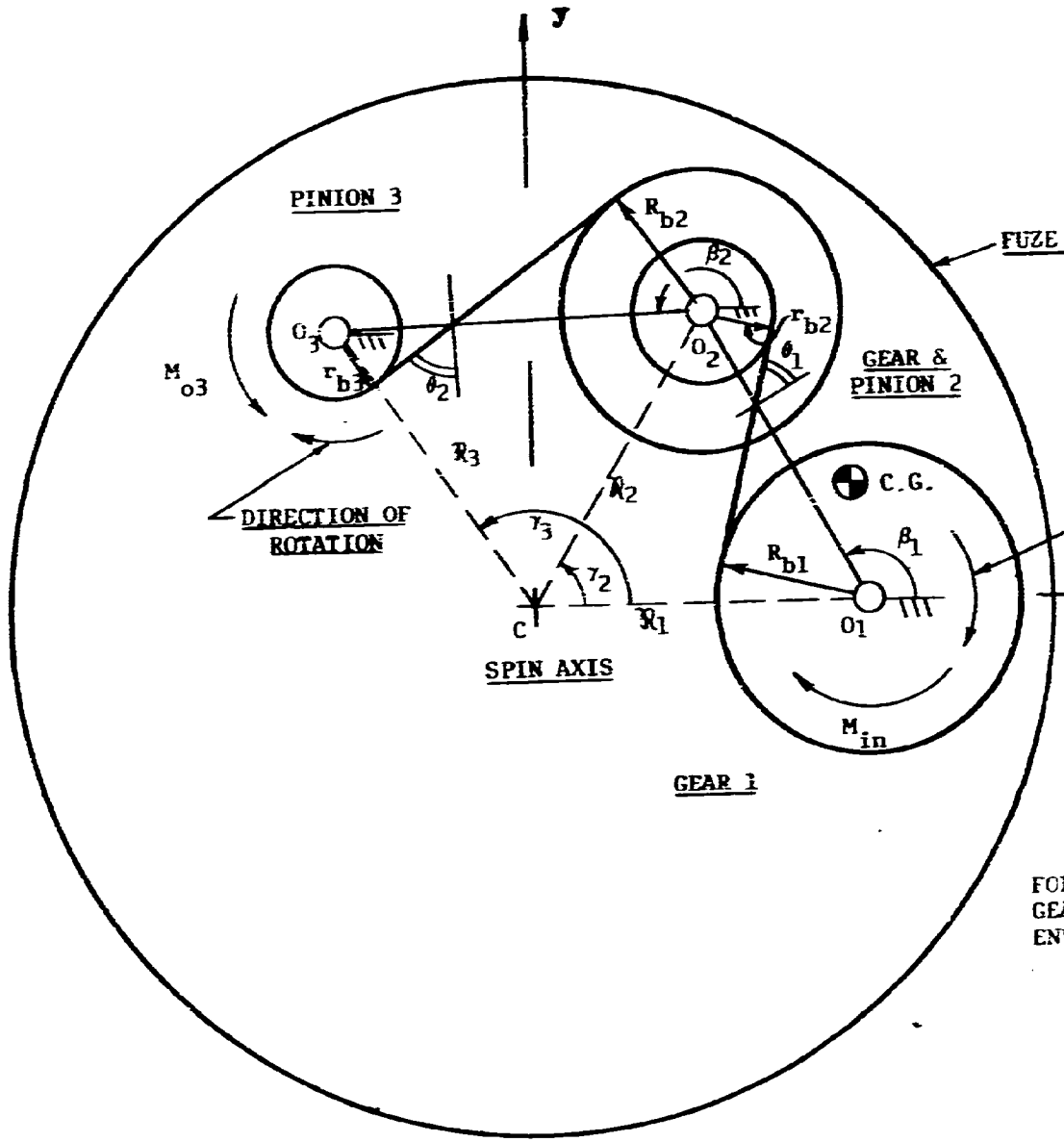


FIGURE A-10

BASIC CONFIGURATION FOR INVOLUTE TWO STEP-UP GEAR TRAIN IN SPIN ENVIRONMENT

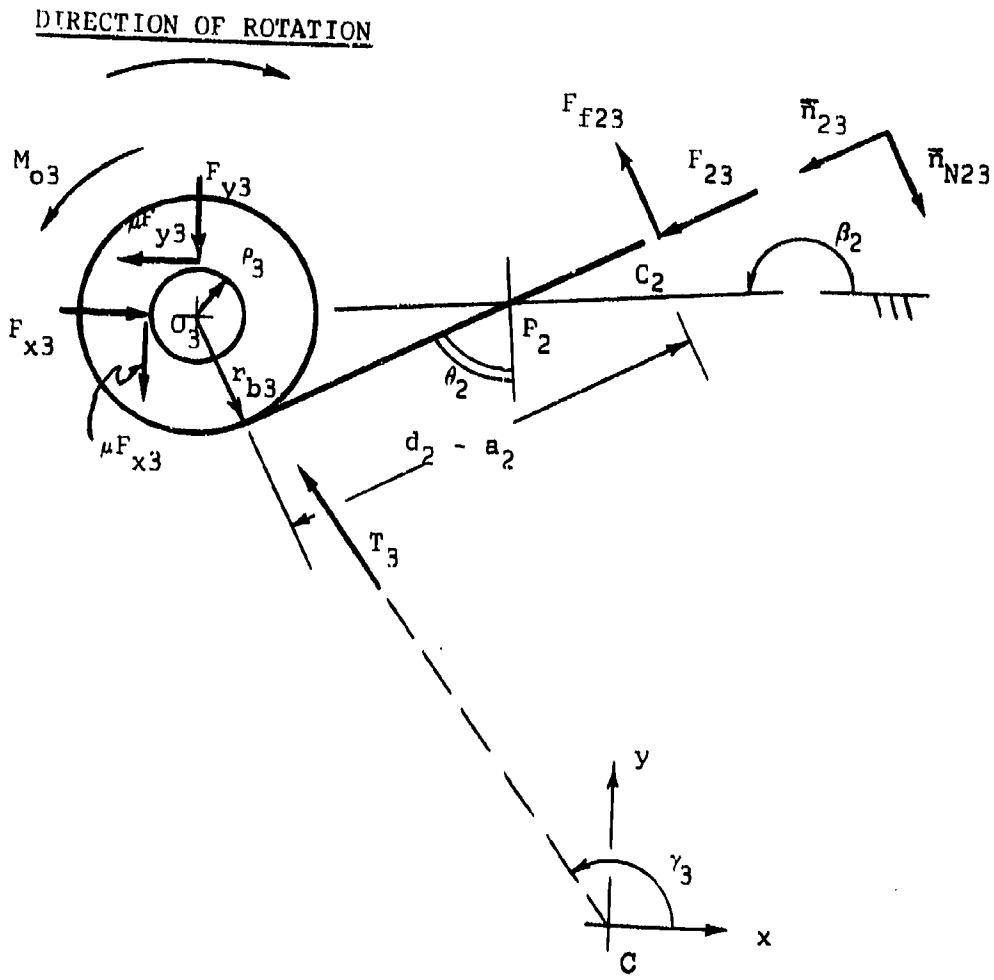


FIGURE A-11 FREE BODY DIAGRAM OF PINION 3

As in equation (A-53), the normal force between the teeth of gear 2 and pinion 3 is given by

$$\bar{F}_{23} = F_{23}\bar{n}_{23} \tag{A-126}$$

The associated friction force is given by equation (A-54), i.e.,

$$\bar{F}_{f23} = -\mu s_2 F_{23}\bar{n}_{N23} \tag{A-127}$$

where $s_2 = +1$, for contact before pitch point P_2

$s_2 = 0$, for contact at pitch point P_2

$s_2 = -1$, for contact after pitch point P_2

The unit vectors in equations (A-126) and (A-127) were defined by equations (A-51) and (A-52), respectively. The normal forces on the pivot shaft are given by

$$\bar{F}_{x3} = F_{x3}\bar{I} \tag{A-128}$$

and

$$\bar{F}_{y3} = -F_{y3}\bar{J} \tag{A-129}$$

The pivot friction forces become $(-)\mu F_{y3}\bar{I}$ and $(-)\mu F_{x3}\bar{J}$ for the indicated direction of rotation.

As in equation (A-57), the centrifugal force on the pinion is given by

$$\bar{T}_3 = T_3(\cos\gamma_3\bar{I} + \sin\gamma_3\bar{J}) \qquad \text{(A-130)}$$

where

$$T_3 = \mathcal{R}_3\omega^2 m_3 \qquad \text{(A-131)}$$

with

$$m_3 = \text{mass of pinion 3} \qquad \text{(A-132)}$$

Force equilibrium is given by

$$F_{23}\bar{n}_{23} - \mu s_2 F_{23}\bar{n}_{N23} + T_3(\cos\gamma_3\bar{I} + \sin\gamma_3\bar{J}) + F_{x3}\bar{I} - \mu F_{y3}\bar{I} - F_{y3}\bar{J} - \mu F_{x3}\bar{J} = 0 \qquad \text{(A-133)}$$

Moment equilibrium is obtained from

$$M_{o3} - r_{b3}F_{23} + \mu s_2(d_2 - a_2)F_{23} + \mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3}) = 0 \qquad \text{(A-134)}$$

Note the use of equation (A-3b) for the pivot friction moment. r_3 represents the pivot radius. d_2 is the length of the line of action between the points of tangency to the base circles of pinion 3 and gear 2. a_2 is the distance along the line-of-action from the gear point of tangency to contact point C_2.

The x and y components of equation (A-133) are given by

$$-F_{23}\sin(\beta_2 - \theta_2) + \mu s_3 F_{23}\cos(\beta_2 - \theta_2) + T_3\cos\gamma_3 + F_{x3} - \mu F_{y3} = 0 , \qquad \text{(A-135)}$$

and

$$F_{23}\cos(\beta_2 - \theta_2) + \mu s_3 F_{23}\sin(\beta_2 - \theta_2) + T_3\sin\gamma_3 - F_{y3} - \mu F_{x3} = 0 \qquad \text{(A-136)}$$

Simultaneous solution of these expressions for F_{x3} and F_{y3} yields

$$F_{x3} = \frac{1}{1 + \mu^2}\left\{ F_{23}\left[(1 + \mu^2 s_2)\sin(\beta_2 - \theta_2) + \mu(1 - s_2)\cos(\beta_2 - \theta_2)\right] + T_3\left[\mu\sin\gamma_3 - \cos\gamma_3\right]\right\} \qquad \text{(A-137)}$$

and

$$F_{y3} = \frac{1}{1+\mu^2}\left\{ F_{23}\left[(1+\mu^2 s_2)\cos(\beta_2-\theta_2) + \mu(s_2-1)\sin(\beta_2-\theta_2)\right] + T_3\left[\sin\gamma_3 + \mu\cos\gamma_3\right]\right\} \quad \text{(A-138)}$$

Now, equations (A-137) and (A-138) are substituted into the moment equation (A-134), with the pivot friction moment given according to the formulation of equation (A-3b):

$$M_{o3} - r_{b3}F_{23} + \mu s_2(d_2 - s_2)F_{23} + \mu\rho_3\left[F_{23}(A_1+A_3) + T_3(A_2+A_4)\right] = 0 \quad \text{(A-139)}$$

where

$$A_1 = \left|\frac{(1+\mu^2 s_2)\cos(\beta_2-\theta_2) + \mu(s_2-1)\sin(\beta_2-\theta_2)}{1+\mu^2}\right| \quad \text{(A-140)}$$

$$A_2 = \left|\frac{\sin\gamma_3 + \mu\cos\gamma_3}{1+\mu^2}\right| \quad \text{(A-141)}$$

$$A_3 = \left|\frac{(1+\mu^2 s_2)\sin(\beta_2-\theta_2) + \mu(1-s_2)\cos(\beta_2-\theta_2)}{1+\mu^2}\right| \quad \text{(A-142)}$$

$$A_4 = \left| \frac{\mu \sin\gamma_3 - \cos\gamma_3}{1 + \mu^2} \right| \qquad \text{(A-143)}$$

Finally, equation (A-139) is solved for F_{23}

$$F_{23} = \frac{M_{03}}{D_1} + T_3 \frac{C_1}{D_1} \qquad \text{(A-144)}$$

where

$$C_1 = \mu\rho_3(A_2 + A_4) \qquad \text{(A-145)}$$

$$D_1 = r_{b3} - \mu\left[s_2(d_2 - a_2) + \rho_3(A_1 + A_3)\right] \qquad \text{(A-146)}$$

b. EQUILIBRIUM OF GEAR AND PINION SET NO. 2

Figure A-12 shows the free body diagram of gear and pinion set no. 2. The contact point, C_2, between gear 2 and pinion 3 is again shown before the pitch point, P_2. With equation (A-126), the normal force between the teeth of this mesh becomes

$$\bar{F}_{32} = -F_{23}\bar{n}_{23} \tag{A-147}$$

The associated friction force is the negative of equation (A-127), i.e.,

$$\bar{F}_{f32} = \mu_{s_2} F_{23}\bar{n}_{N23} \tag{A-148}$$

Similar to equation (A-80), the normal force between gear 1 and pinion 2 is given by

$$\bar{F}_{12} = F_{12}\bar{n}_{12} \tag{A-149}$$

(See equation (A-78) for the definition of unit vector $\bar{n}_{12}$.)

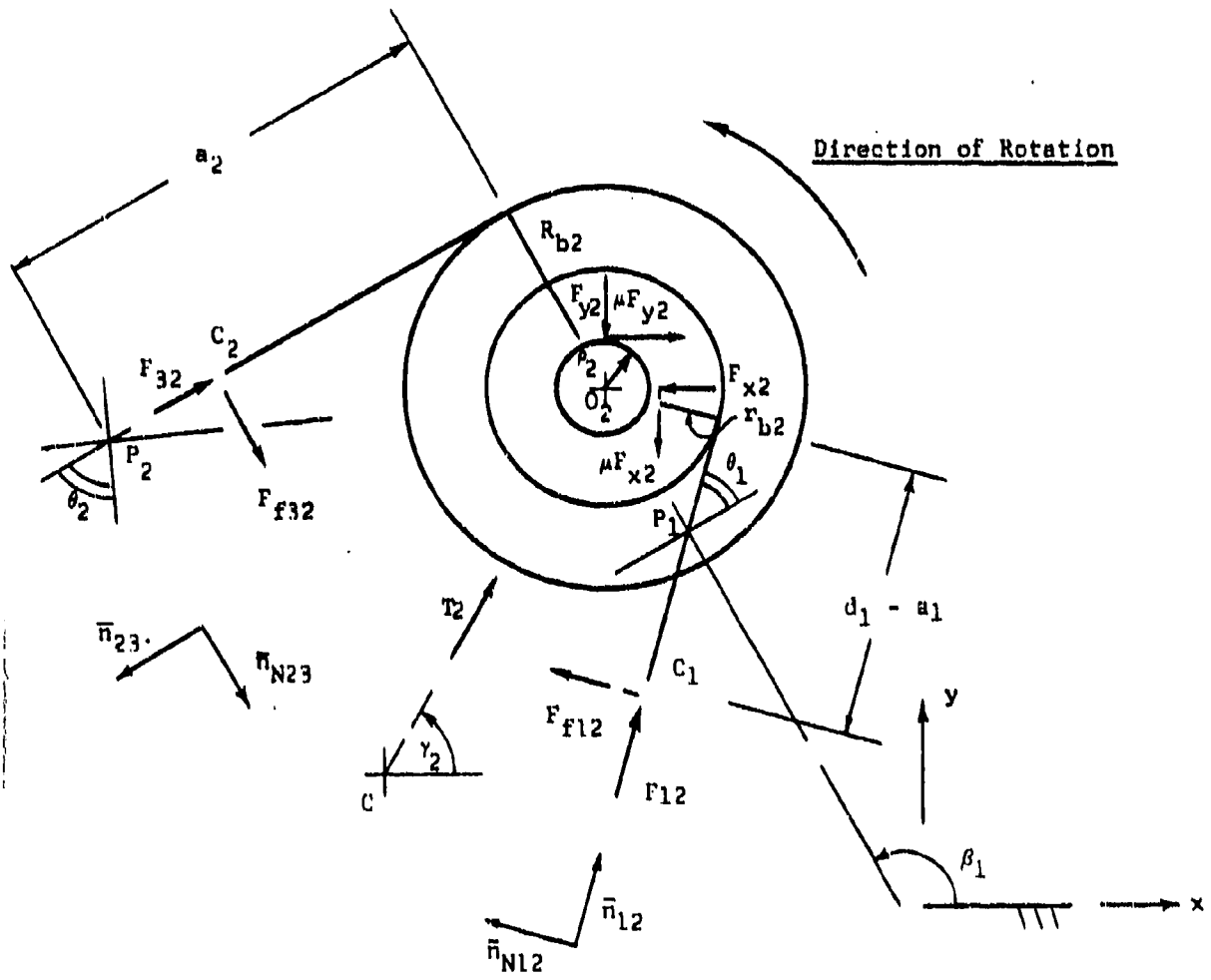


FIGURE A-12 FREE BODY DIAGRAM OF GEAR AND PINION SET NO. 2

Figure A-12 shows the contact point C_1 before the pitch point P_1 is passed.

The associated friction force is given by

$$\bar{F}_{f12} = \mu s_1 F_{12} \bar{n}_{N12} \qquad \text{(A-150)}$$

where $s_1 = +1$, for contact before pitch point P_1

$s_1 = 0$, for contact at pitch point P_1

$s_1 = -1$, for contact after pitch point P_1

The unit vector $\bar{n}_{N12}$ is given by equation (A-79).

The normal forces on the pivot shaft are given by

$$\bar{F}_{x2} = -F_{x2}\bar{I} \qquad \text{(A-151)}$$

and

$$\bar{F}_{y2} = -F_{y2}\bar{J} \qquad \text{(A-152)}$$

The associated friction forces $\mu F_{y2}\bar{I}$ and $(-)\mu F_{x2}\bar{J}$ were chosen so that their moments oppose the indicated rotation.

The centrifugal force on this gear and pinion assembly is expressed by

$$\bar{T}_2 = T_2(\cos\gamma_2\bar{I} + \sin\gamma_2\bar{J}) \qquad \text{(A-153)}$$

where $T_2 = \mathcal{R}_2\omega^2 m_2$ (A-154)

m_2 = mass of gear and pinion set no. 2 (A-155)

Force equilibrium is given by

$$-F_{23}\bar{n}_{23} + \mu s_2 F_{23}\bar{n}_{N23} + F_{12}\bar{n}_{12} + \mu s_1 F_{12}\bar{n}_{N12} + T_2(\cos\gamma_2\bar{I} + \sin\gamma_2\bar{J}) - F_{x2}\bar{I} + \mu F_{y2}\bar{I} - F_{y2}\bar{J} - \mu F_{x2}\bar{J} = 0 \qquad \text{(A-156)}$$

Moment equilibrium is given by

$$-R_{b2}F_{23} + \mu s_2 R_2 F_{23} + r_{b2}F_{12} - \mu s_1(d_1 - a_1)F_{12} - \mu\rho_2(\tilde{F}_{x2} + \tilde{F}_{y2}) = 0 \qquad \text{(A-157)}$$

Again, equation (A-3b) is used to obtain a conservative pivot friction moment. ρ_2 represents the pivot radius. d_1 and a_1 are similar to the previously used distances along the lines of action of the other meshes.

The component form of equation (A-156) becomes

$$F_{23}\sin(\beta_2 - \theta_2) - \mu s_2 F_{23}\cos(\beta_2 - \theta_2) + F_{12}\sin(\beta_1 + \theta_1)$$
$$+ \mu s_1 F_{12}\cos(\beta_1 + \theta_1) + T_2\cos\gamma_2 - F_{x2} + \mu F_{y2} = 0 \qquad \text{(A-158)}$$

and

$$-F_{23}\cos(\beta_2 - \theta_2) - \mu s_2 F_{23}\sin(\beta_2 - \theta_2) - F_{12}\cos(\beta_1 + \theta_1)$$
$$+ \mu s_1 F_{12}\sin(\beta_1 + \theta_1) + T_2\sin\gamma_2 - F_{y2} - \mu F_{x2} = 0 \qquad \text{(A-159)}$$

Simultaneous solution of the above expressions for F_{x2} and F_{y2} gives

$$F_{x2} = \frac{1}{1+\mu^2}\Big\{ -F_{12}\Big[\mu(1 - s_1)\cos(\beta_1 + \theta_1) - (1 + \mu^2 s_1)\sin(\beta_1 + \theta_1)\Big]$$
$$+ T_2\Big[\mu\sin\gamma_2 + \cos\gamma_2\Big]$$
$$+ F_{23}\Big[(1 - \mu^2 s_2)\sin(\beta_2 - \theta_2) - \mu(1 + s_2)\cos(\beta_2 - \theta_2)\Big]\Big\} \qquad \text{(A-160)}$$

$$F_{y2} = \frac{1}{1+\mu^2}\Big\{-F_{12}\Big[\mu(1-s_1)\sin(\beta_1+\theta_1)+(1+\mu^2 s_1)\cos(\beta_1+\theta_1)\Big] + T_2\Big[\sin\gamma_2 - \mu\cos\gamma_2\Big] + F_{23}\Big[-\mu(1+s_2)\sin(\beta_2-\theta_2)-(1-\mu^2 s_2)\cos(\beta_2-\theta_2)\Big]\Big\} \quad \text{(A-161)}$$

Now equations (A-160) and (A-161) are substituted into the moment equation (A-157) with the pivot friction moment formulated again according to equation (A-3b):

$$-R_{b2}F_{23} + \mu s_2 a_2 F_{23} + r_{b2}F_{12} - \mu s_1(d_1 - a_1)F_{12} - \mu P_2\left[F_{12}(A_5 + A_8) + T_2(A_6 + A_9) + F_{23}(A_7 + A_{10})\right] = 0 \tag{A-162}$$

where

$$A_5 = \left| \frac{\mu(1 - s_1)\sin(\beta_1 + \theta_1) + (1 + \mu^2 s_1)\cos(\beta_1 + \theta_1)}{1 + \mu^2} \right| \tag{A-163}$$

$$A_6 = \left| \frac{\sin\gamma_2 - \mu\cos\gamma_2}{1 + \mu^2} \right| \tag{A-164}$$

$$A_7 = \left| \frac{\mu(1 + s_2)\sin(\beta_2 - \theta_2) + (1 - \mu^2 s_2)\cos(\beta_2 - \theta_2)}{1 + \mu^2} \right| \tag{A-165}$$

$$A_8 = \left| \frac{\mu(1 - s_1)\cos(\beta_1 + \theta_1) - (1 + \mu^2 s_1)\sin(\beta_1 + \theta_1)}{1 + \mu^2} \right| \tag{A-166}$$

$$A_9 = \left| \frac{\mu\sin\gamma_2 + \cos\gamma_2}{1 + \mu^2} \right| \tag{A-167}$$

$$A_{10} = \left| \frac{(1 - \mu^2 s_2)\sin(\beta_2 - \theta_2) - \mu(1 + s_2)\cos(\beta_2 - \theta_2)}{1 + \mu^2} \right| \qquad \text{(A-168)}$$

Finally equation (A-162) is solved for F_{12}

$$F_{12} = \frac{F_{23}C_2}{D_2} + \frac{T_2 C_3}{D_2} \qquad \text{(A-169)}$$

where

$$C_2 = R_{b2} - \mu\left[s_2 a_2 - \rho_2(A_7 + A_{10})\right] \qquad \text{(A-170)}$$

$$C_3 = \mu\rho_2(A_6 + A_9) \qquad \text{(A-171)}$$

$$D_2 = r_{b2} - \mu\left[s_1(d_1 - a_1) + \rho_2(A_5 + A_8)\right] \qquad \text{(A172)}$$

c. EQUILIBRIUM OF GEAR NO. 1

Figure A-13 shows the free body diagram of gear no. 1, the input gear.

The contact point, C_1, corresponds to that shown in Figure A-12.

According to equation (A-149), the normal contact force between pinion 2 and gear 1 becomes

$$\bar{F}_{21} = -F_{12}\bar{n}_{12} \tag{A-173}$$

The associated friction force is the negative of equation (A-150), i.e.,

$$\bar{F}_{f21} = -\mu s_1 F_{12}\bar{n}_{N12} \tag{A-174}$$

The normal forces on the pivot shaft are given by

$$\bar{F}_{x1} = -F_{x1}\bar{I} \tag{A-175}$$

and

$$\bar{F}_{y1} = F_{y1}\bar{J} \tag{A-176}$$

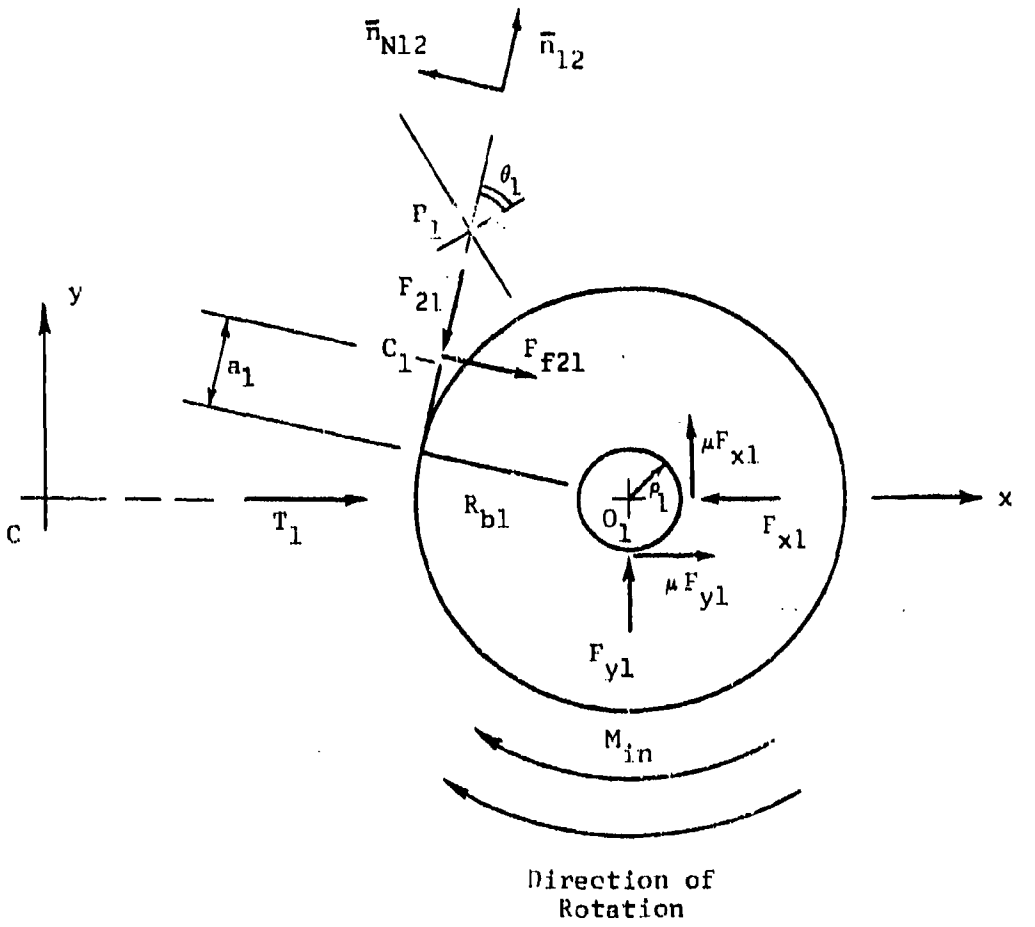


FIGURE A-13. FREE BODY DIAGRAM OF GEAR 1

The associated friction forces $\mu F_{y1}\bar{I}$ and $\mu F_{x1}\bar{J}$ were chosen with such directions that their moments oppose rotation due to input moment M_{in}.

Centrifugal force $\bar{T}_1$ on gear 1 is given by

$$\bar{T}_1 = T_1\bar{I} \tag{A-177}$$

where, as with equation (A-107)

$$T_1 = \mathcal{R}_1\omega^2 m_1 \tag{A-178}$$

and

$$m_1 = \text{mass of gear 1} \tag{A-179}$$

Force equilibrium of gear 1 is given by

$$-F_{12}\bar{n}_{12} - \mu s_1 F_{12}\bar{n}_{N12} + T_1\bar{I} - F_{x1}\bar{I} + F_{y1}\bar{J} + \mu F_{y1}\bar{I} + \mu F_{x1}\bar{J} = 0 \tag{A-180}$$

Moment equilibrium is given by

$$R_{b1}F_{12} - \mu s_1 a_1 F_{12} - M_{in} + \mu\rho_1(\hat{F}_{x1} + \hat{F}_{y1}) = 0 \tag{A-181}$$

Note that equations (A-180) and (A-181) have the same forms as equations (A-109) and (A-110).

The force component expressions are the same as given by equations (A-111) and (A-112), and their simultaneous solution for the pivot forces is identical to that given by equations (A-113) and (A-114), i.e.,

$$F_{x1} = \frac{-F_{12}\left[(1 - \mu^2 s_1)\sin(\beta_1 + \theta_1) + \mu(1 + s_1)\cos(\beta_1 + \theta_1)\right] + T_1}{1 + \mu^2} \tag{A-182}$$

and

$$F_{y1} = \frac{F_{12}\left[\mu(1 + s_1)\sin(\beta_1 + \theta_1) - (1 - \mu^2 s_1)\cos(\beta_1 + \theta_1)\right] - \mu T_1}{1 + \mu^2} \tag{A-183}$$

Equations (A-182) and (A-183) are now substituted into equation (A-181) according to the method of equation (A-3b)

$$R_{b1}F_{12} - \mu s_1 a_1 F_{12} - M_{in} + \mu\rho_1\left[F_{12}(A_{11} + A_{13}) + T_1(A_{12} + A_{14})\right] = 0 \tag{A-184}$$

where

$$A_{11} = \left| \frac{(1 - \mu^2 s_1)\sin(\beta_1 + \theta_1) + \mu(1 + s_1)\cos(\beta_1 + \theta_1)}{1 + \mu^2} \right| \quad \text{(A-185)}$$

$$A_{12} = \left| \frac{1}{1 + \mu^2} \right| \quad \text{(A-186)}$$

$$A_{13} = \left| \frac{\mu(1 + s_1)\sin(\beta_1 + \theta_1) - (1 - \mu^2 s_1)\cos(\beta_1 + \theta_1)}{1 + \mu^2} \right| \quad \text{(A-187)}$$

$$A_{14} = \left| \frac{\mu}{1 + \mu^2} \right| \quad \text{(A-188)}$$

Finally, equation (A-184) is solved for F_{12}

$$F_{12} = \frac{M_{in}}{D_3} - \frac{T_1 C_4}{D_3} \quad \text{(A-189)}$$

where

$$C_4 = \mu \rho_1 (A_{12} + A_{14}) \quad \text{(A-190)}$$

$$D_3 = R_{b1} - \mu\left[s_1 a_1 - \rho_1 (A_{11} + A_{13})\right] \quad \text{(A-191)}$$

d. INPUT-OUTPUT RELATIONSHIP

To obtain the input-output relationship for the complete gear train, equation (A-189) is now set equal to equation (A-169). This furnishes

$$F_{23} = \frac{D_2}{C_2 D_3}\left[M_{in} - T_1 C_4\right] - T_2 \frac{C_3}{C_2} \qquad \text{(A-192)}$$

The above is then set equal to equation (A-144). This results in the input-output moment relationship

$$M_{o3} = \frac{D_1 D_2}{C_2 D_3}\left[M_{in} - T_1 C_4\right] - T_2 \frac{C_3 D_1}{C_2} - T_3 C_1 \qquad \text{(A-193)}$$

6. AUXILIARY GEOMETRIC AND KINEMATIC EXPRESSIONS FOR TWO AND THREE STEP-UP GEAR TRAINS WITH INVOLUTE TEETH

a. NOMENCLATURE FOR INVOLUTE GEAR TEETH

R_{pi}, r_{pi} = pitch radii of gear and pinion of i^{th} gear and pinion set

R_{bi}, r_{bi} = base radii of gear and pinion of i^{th} gear and pinion set

R_{oi}, r_{oi} = outside radii of gear and pinion of i^{th} gear and pinion set

θ_j = effective pressure angle of j^{th} mesh

$\mathcal{R}_i$ = distance from spin axis to pivot of i^{th} gear and pinion set

b. ANGULAR RELATIONSHIPS BETWEEN PIVOT HOLES

Figure A-14 shows the angular relationships between the lines connecting the pivot holes as well as the spin center. (See also Figures A-5 and A-10.) The following serves to determine the angles γ_i and β_i for certain combinations of gears and pinions as well as spin radii $\mathcal{R}_i$.

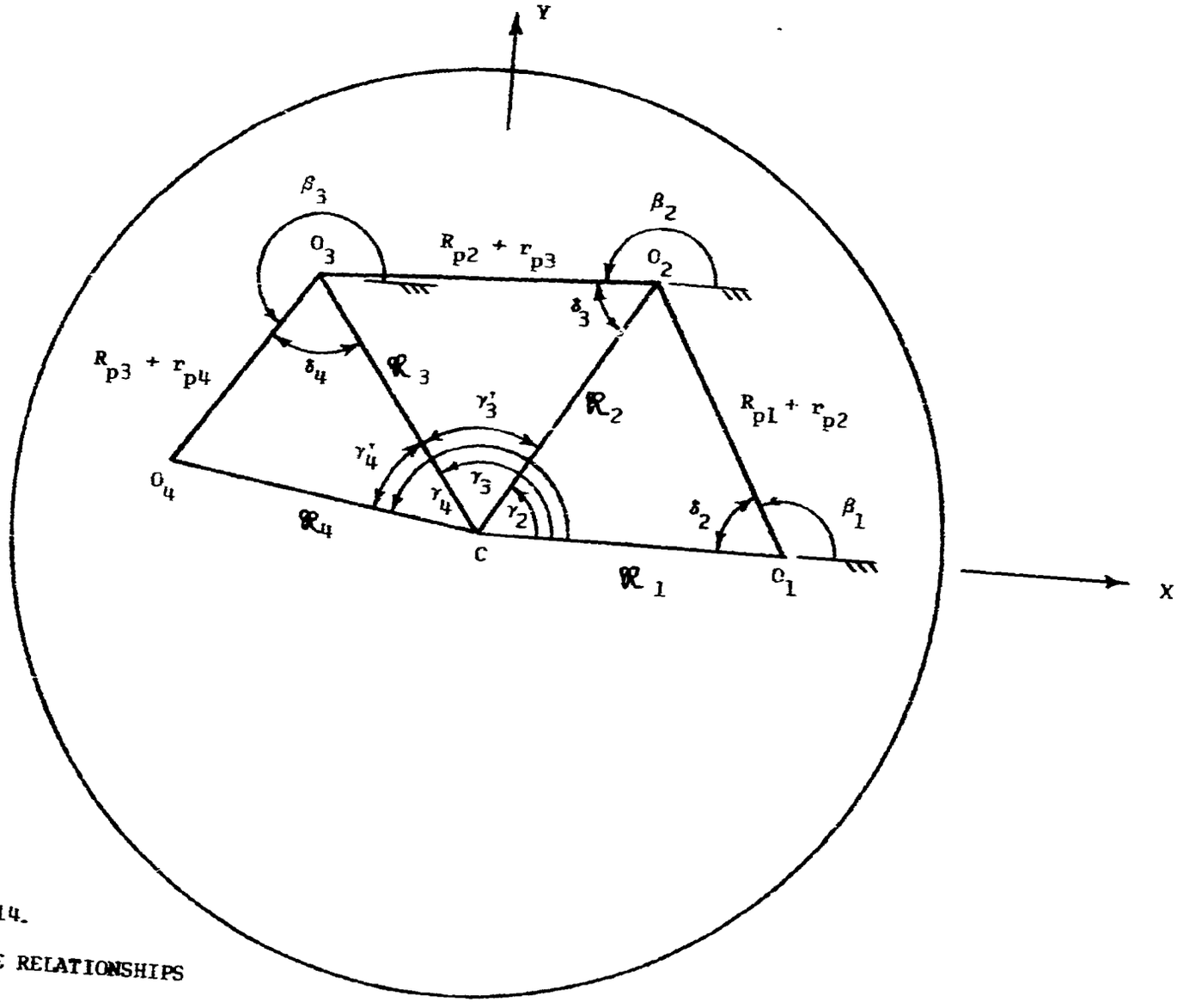


FIGURE A-14.

PIVOT HOLE RELATIONSHIPS

ANGLE γ_i

From

$$(R_{p1} + r_{p2})^2 = \ell_1^2 + \ell_2^2 - 2\ell_1\ell_2\cos\gamma_2$$

one obtains

$$\gamma_2 = \cos^{-1}\left[\frac{\ell_1^2 + \ell_2^2 - (R_{p1} + r_{p2})^2}{2\ell_1\ell_2}\right] \qquad \text{(A-194)}$$

Similarly, from

$$\gamma_3' = \cos^{-1}\left[\frac{\ell_2^2 + \ell_3^2 - (R_{p2} + r_{p3})^2}{2\ell_2\ell_3}\right]$$

one obtains

$$\gamma_3 = \gamma_2 + \gamma_3' \qquad \text{(A-195)}$$

Also with

$$\gamma_4' = \cos^{-1}\left[\frac{\mathcal{R}_3^2 + \mathcal{R}_4^2 - (R_{p3} + r_{p4})^2}{2\mathcal{R}_3\mathcal{R}_4}\right]$$

one obtains

$$\gamma_4 = \gamma_3 + \gamma_4' \qquad \text{(A-196)}$$

ANGLES δ_1

Since

$$\mathcal{R}_2^2 = (R_{p1} + r_{p2})^2 + \mathcal{R}_1^2 - 2(R_{p1} + r_{p2})\mathcal{R}_1\cos\delta_2$$

$$\delta_2 = \cos^{-1}\left[\frac{(R_{p1} + r_{p2})^2 + \mathcal{R}_1^2 - \mathcal{R}_2^2}{2\mathcal{R}_1(R_{p1} + r_{p2})}\right] \qquad \text{(A-197)}$$

Similarly,

$$\delta_3 = \cos^{-1}\left[\frac{(R_{p2} + r_{p3})^2 + \mathcal{R}_2^2 - \mathcal{R}_3^2}{2\mathcal{R}_2(R_{p2} + r_{p3})}\right] \qquad \text{(A-198)}$$

and

$$\delta_4 = \cos^{-1}\left[\frac{(R_{p3} + r_{p4})^2 + \mathcal{R}_3^2 - \mathcal{R}_4^2}{2\mathcal{R}_3(R_{p3} + r_{p4})}\right] \qquad \text{(A-199)}$$

ANGLES β_1

With Equation (A-197)

$$\beta_1 = \pi - \delta_2 \qquad \text{(A-200)}$$

Further, with Equations (A-194) and (A-198)

$$\beta_2 = \gamma_2 + (\pi - \delta_3) \qquad \text{(A-201)}$$

Finally, with Equations (A-195) and (A-199)

$$\beta_3 = \gamma_3 + (\pi - \delta_4) \qquad \text{(A-202)}$$

c. DETERMINATION OF CONTACT POINT C FOR VARIOUS MESHES

Figure A-15 shows the points of interest along the line of action of an involute gear which drives an involute pinion.

Points L and L' are the points of tangency to the base circles of radius R_b and r_b, respectively, and the distance $d = LL'$. Initial contact is made at point M, where the line of action intersects the pinion addendum circle of radius r_o. Final contact corresponds to point N. Here the line of action intersects the gear addendum circle of radius R_o.

The position of the instantaneous contact point C with respect to point L, i.e., the length, a, is expressed with the help of instantaneous angle α which has its origin at the line O_1L. Then,

$$a = LC = R_b\alpha \qquad \text{(A-203)}$$

A computer procedure for the determination of instantaneous angle α of any mesh must first find the associated initial and final angles of contact α_{in} and α_{fin}. In addition, it must contain a method for incrementing angle α. The following shows such a procedure for each of the meshes of a two pass and a three pass step-up gear train, together with a means of obtaining the signs of the signum terms.

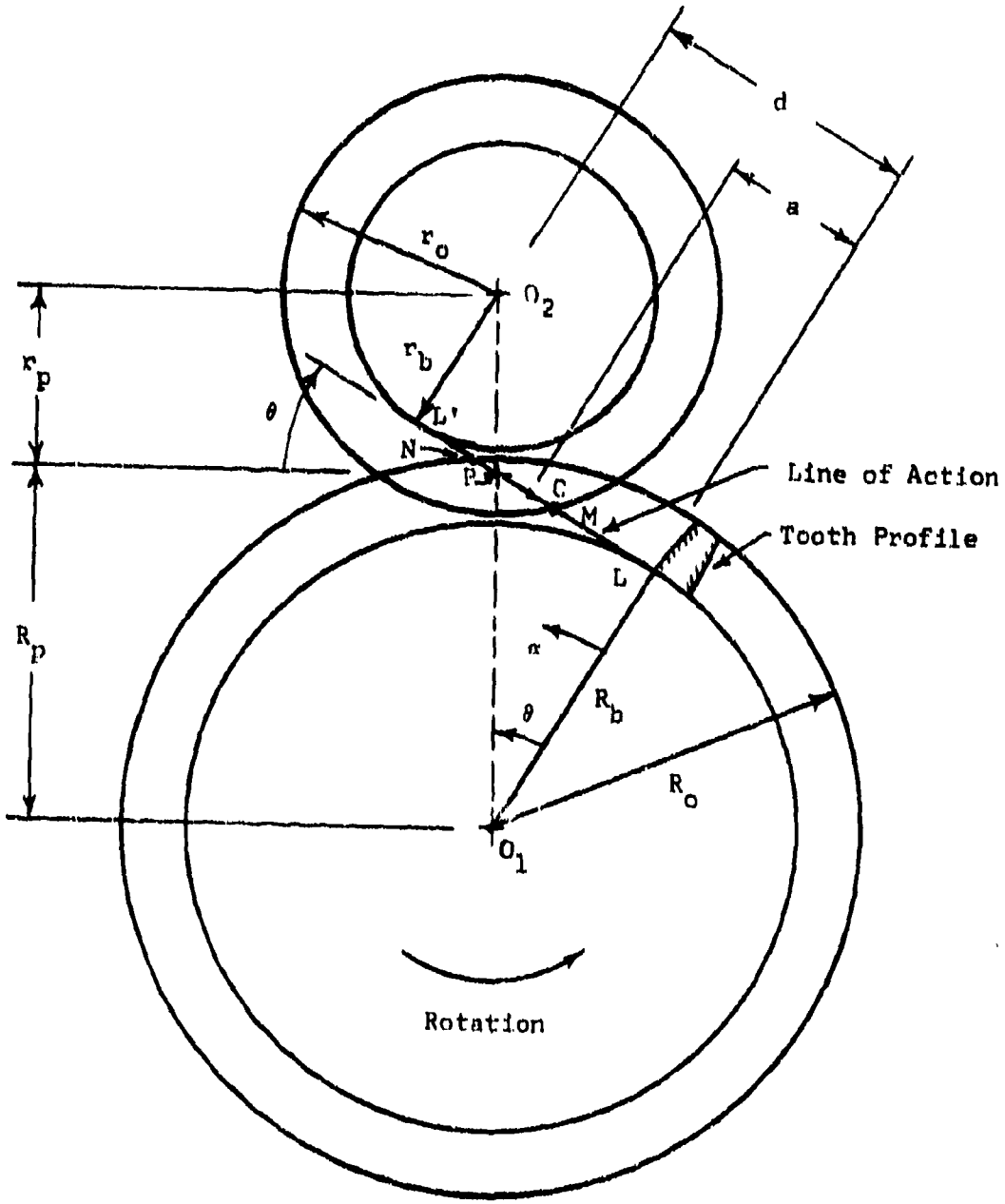


FIGURE A-15.

INVOLUTE MESH GEOMETRY

1.) MESH OF GEAR 1 AND PINION 2

The total length, $d_1 = LL'$, is given by

$$d_1 = (R_{b1} + r_{b1})\tan\theta_1 \qquad \text{(A-204)}$$

The initial angle of contact, α_{1IN}, is obtained from

$$\alpha_{1in} = \frac{ML}{R_{b1}} = \frac{(R_{b1} + r_{b2})\tan\theta_1 - \sqrt{r_{o2}^2 - r_{b2}^2}}{R_{b1}} \qquad \text{(A-205)}$$

Similarly, the final angle of contact, α_{1FIN}, is given by

$$\alpha_{1fin} = \frac{\sqrt{R_{o1}^2 - R_{b1}^2}}{R_{b1}} \qquad \text{(A-206)}$$

The magnitude of the increment $\Delta\alpha_1$ depends on whether one deals with a two step-up or a three step-up gear train.

Assuming that a two step-up train is involved and that one wishes to compute the length a_2 of mesh 2 K_2 times after the first contact, the angular increment, $\Delta\alpha_{22}$, has the magnitude

$$\Delta\alpha_{22} = \frac{\alpha_{2fin} - \alpha_{2in}}{K_2} \qquad \text{(A-207)}$$

(The second subscript refers to a two step-up configuration.) Because of the transmission ratio between gear sets 1 and 2, the associated angular increment of gear 1 will be smaller than $\Delta\alpha_{22}$, i.e.,

$$\Delta\alpha_{12} = \Delta\alpha_{22} \frac{r_{b2}}{R_{b1}} \qquad \text{(A-208)}$$

The instantaneous angle, α_1, will then be given by

$$\alpha_1 = \alpha_{1in} + j_{12}\Delta\alpha_{12} \qquad \text{(A-209)}$$

and, the instantaneous distance, a_1, becomes

$$a_1 = R_{b1}(\alpha_{1in} + j_{12}\Delta\alpha_{12}) \qquad \text{(A-210)}$$

In the above, j_{12} represents the number of times the angle α_1 has been incremented. While the total number of increments depends on the length of contact, the incrementing of α_1 comes to an end when $\alpha_1 \geq \alpha_{1fin}$. This also corresponds to a complete mechanism

cycle. Since mesh 2 goes through R_{b1}/r_{b2} times as many cycles as mesh no. 1, the angle α_2 has to be re-initialized to α_{2in} when $\alpha_2 \geq \alpha_{2fin}$, i.e., after K_2 increments. (For simplicity it is assumed that the motion starts when all meshes are at their initial contact angles, α_{in}.)

When a three step-up gear train is involved, one must make sure that enough computations are made for mesh 3. Assuming that K_3 increments are to be made, one obtains

$$\Delta\alpha_{33} = \frac{\alpha_{3fin} - \alpha_{3in}}{K_3} \qquad \text{(A-211)}$$

The associated angular increments for meshes 2 and 1 then become

$$\Delta\alpha_{23} = \Delta\alpha_{33} \frac{r_{b3}}{R_{b2}} \qquad \text{(A-212)}$$

and

$$\Delta\alpha_{13} = \Delta\alpha_{33} \frac{r_{b3}}{R_{b2}} \times \frac{r_{b2}}{R_{b1}} \qquad \text{(A-213)}$$

For this case the instantaneous distance a_1 becomes

$$a_1 = R_{b1}(\alpha_{1in} + j_{13}\Delta\alpha_{13}) \qquad \text{(A-214)}$$

j_{13} stands for the number of times α_1 has been incremented at any given instant. This incrementing again ends when $\alpha_1 \geq \alpha_{1fin}$. Meshes 2 and 3 are re-initialized to α_{2in} and α_{3in} as often as is necessary to complete one cycle for mesh no. 1.

The sign of s_1 is best obtained from the fact that at pitch point P_1

$$\alpha_{1p} = \frac{LP}{R_{b1}} = \frac{R_{b1}\tan\theta_1}{R_{b1}} = \tan\theta_1 \qquad \text{(A-215)}$$

then for

$$\begin{aligned} \alpha_1 < \tan\theta_1 &: \quad s_1 = +1 \\ \alpha_1 = \tan\theta_1 &: \quad s_1 = 0 \\ \alpha_1 > \tan\theta_1 &: \quad s_1 = -1 \end{aligned} \qquad \text{(A-216)}$$

2.) MESH OF GEAR 2 AND PINION 3

Similar to the previous section

$$d_2 = (R_{b2} + r_{b3})\tan\theta_2 \qquad \text{(A-217)}$$

$$\alpha_{2in} = \frac{(R_{b2} + r_{b3})\tan\theta_2 - \sqrt{r_{o3}^2 - r_{b3}^2}}{R_{b2}} \qquad \text{(A-218)}$$

$$\alpha_{2fin} = \frac{\sqrt{R_{o2}^2 - R_{b2}^2}}{R_{b2}} \qquad \text{(A-219)}$$

For a two step-up gear train, the instantaneous length, a_2, becomes

$$a_2 = R_{b2}(\alpha_{2in} + J_{22}\Delta\alpha_{22}) \qquad \text{(A-220)}$$

where $\Delta\alpha_{22}$ is given by Equation (A-207) and

$$J_{22} = 1, 2, \ldots, K_2.$$

When $\alpha_2 \geq \alpha_{2fin}$ it must be re-initialized to α_{2in} until mesh no. 1 has completed its full cycle.

For a three step-up gear train, the length, a_2, becomes

$$a_2 = R_{b2}(\alpha_{2in} + j_{23}\Delta\alpha_{23}) \qquad \text{(A-221)}$$

with $\Delta\alpha_{23}$ given by Equation (A-212). j_{23} stands for the number of times α_2 has been incremented and again depends on the length of contact. Re-initialization follows the same rule as given above.

The sign of s_2 is obtained as follows: (See Equation (A-215).)

For

$$\begin{aligned} \alpha_2 < \tan\theta_2 &: \quad s_2 = +1 \\ \alpha_2 = \tan\theta_2 &: \quad s_2 = 0 \\ \alpha_2 > \tan\theta_2 &: \quad s_2 = -1 \end{aligned} \qquad \text{(A-222)}$$

3.) MESH OF GEAR 3 AND PINION 4

Again, in the same vein as in Section 1,

$$d_3 = (R_{b3} + r_{b4})\tan\theta_3 \tag{A-223}$$

$$\alpha_{3in} = \frac{(R_{b3} + r_{b4})\tan\theta_3 - \sqrt{r_{o4}^2 - r_{b4}^2}}{R_{b3}} \tag{A-224}$$

$$\alpha_{3fin} = \frac{\sqrt{R_{o3}^2 - R_{b3}^2}}{R_{b3}} \tag{A-225}$$

The instantaneous length, a_3, is given by

$$a_3 = R_{b3}(\alpha_{3in} + j_{33}\Delta\alpha_{33}) \tag{A-226}$$

where $\Delta\alpha_{33}$ is given by Equation (A-211) and $j_{33} = 1, 2, \ldots, K_3$. Whenever $\alpha_3 \geq \alpha_{3fin}$ it must be re-initialized until mesh no. 1 completed its full cycle.

The sign of s_3 is again obtained with the help of an expression Equation (A-215):

$$\begin{aligned} \alpha_3 < \tan\theta_3 &: \quad s_3 = +1 \\ \alpha_3 = \tan\theta_3 &: \quad s_3 = 0 \\ \alpha_3 > \tan\theta_3 &: \quad s_3 = -1 \end{aligned} \qquad \text{(A-227)}$$

APPENDIX B

DESIGN OF UNEQUAL ADDENDUM INVOLUTE GEAR SETS WITH STANDARD CENTER DISTANCES

One of the ways of preventing undercutting in pinions with small numbers of teeth, which must mesh with gears of unequal and larger numbers of teeth, is to decrease the pinion dedendum together with the gear addendum by the necessary amount. To maintain standard working depth for such a mesh, the addendum of the pinion as well as the dedendum of the gear are increased by the same amount. (This, of course, presupposes that the gear is not undercut by this modification.) This can be accomplished without any change in the standard base and pitch radii or the associated standard center distance by "withdrawing" the hob during the cutting of the pinion and feeding it "deeper" when the gear is cut. In this way the pinion, which has its ouside radius increased by the hob withdrawal distance, will have a larger than standard circular tooth thickness at its standard pitch circle. The outside radius of the gear is decreased by the same amount. Because the cutter is fed to full depth, the gear tooth thickness at the standard pitch circle will be less than standard.

The following gives the design steps for this type of gearing and illustrates them by way of an example.

1. STANDARD GEAR NOMENCLATURE

N = number of teeth of gear

n = number of teeth of pinion

θ = pressure angle

$P_d = \frac{N}{2R_p} = \frac{n}{2r_p}$ = diametral pitch

P_{cs} = standard circular pitch at pitch circle, where also

$$P_{cs} = \frac{\pi}{P_d}$$

R_p, r_p = pitch radii of gear and pinion, respectively

$R_b = R_p\cos\theta$, base circle radius of gear

$r_b = r_p\cos\theta$, base circle radius of pinion

T_{cs}, t_{cs} = $\frac{P_{cs}}{2}$, circular tooth thickness of gear and pinion, respectively, at standard pitch circles

$\frac{1}{P_d}$ = standard gear addendum

$\frac{1.157}{P_d}$ = standard gear dedendum

2. DETERMINATION OF HOB WITHDRAWAL DISTANCE C

Figure B-1 indicates the relationship between the pinion pitch radius, r_p, its root radius, r_r, and the rack cutter addendum A. The root radius is formed by the addendum line of the cutter tooth, so that

$$r_r = r_p - A \tag{B-1}$$

In order to avoid undercutting, the addendum line of a sharp cornered cutter must not pass below point L, the tangent point of the line of action and the base circle (see Figure B-2). The minimum root radius, r_{rm}, becomes for this case

$$r_{rm} = r_b \cos\theta = r_p \cos^2\theta \tag{B-2}$$

When the rack tooth corner is rounded off with a radius r_c, as shown in Figure B-3, the effective addendum line of the cutter tooth moves up the distance $r_c(1 - \sin\theta)$. To avoid undercutting with this type of cutter, the effective addendum line must not pass the base circle below point L in Figure B-2. This allows a reduction of the minimum allowable root radius to

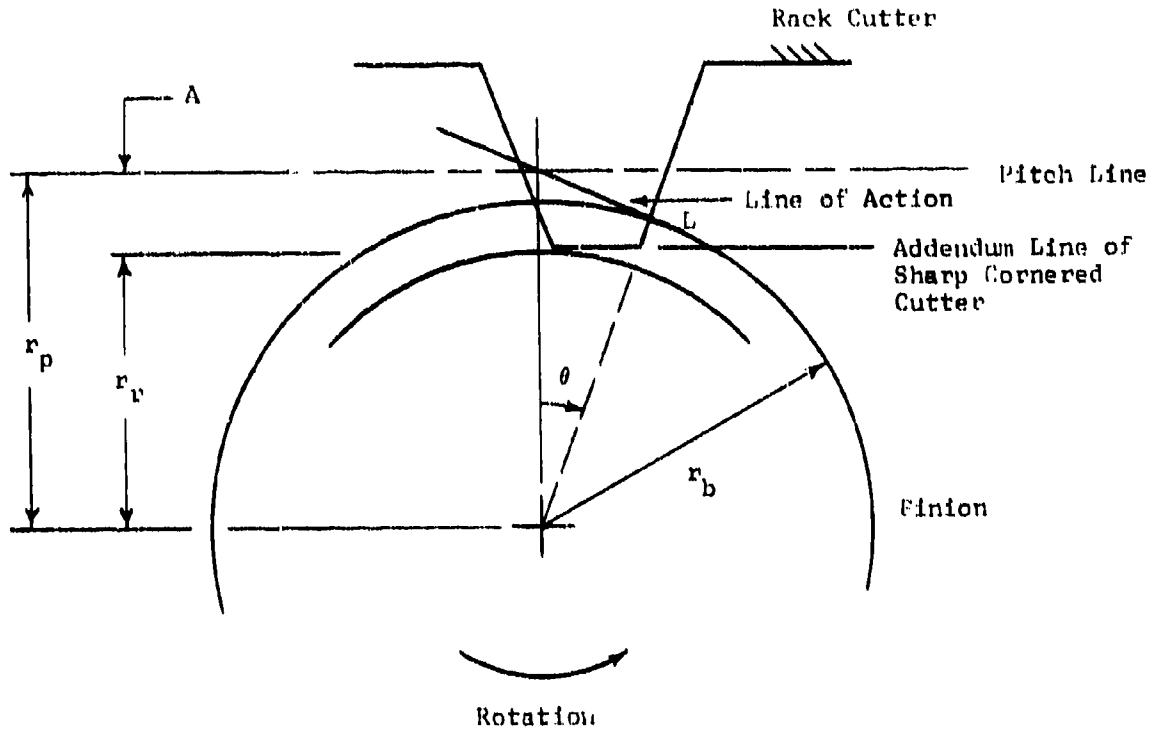


FIGURE B-1

RELATIONSHIP BETWEEN PINION PITCH RADIUS, r_p, RACK CUTTER ADDENDUM A AND RESULTING PINION ROOT RADIUS, r_r

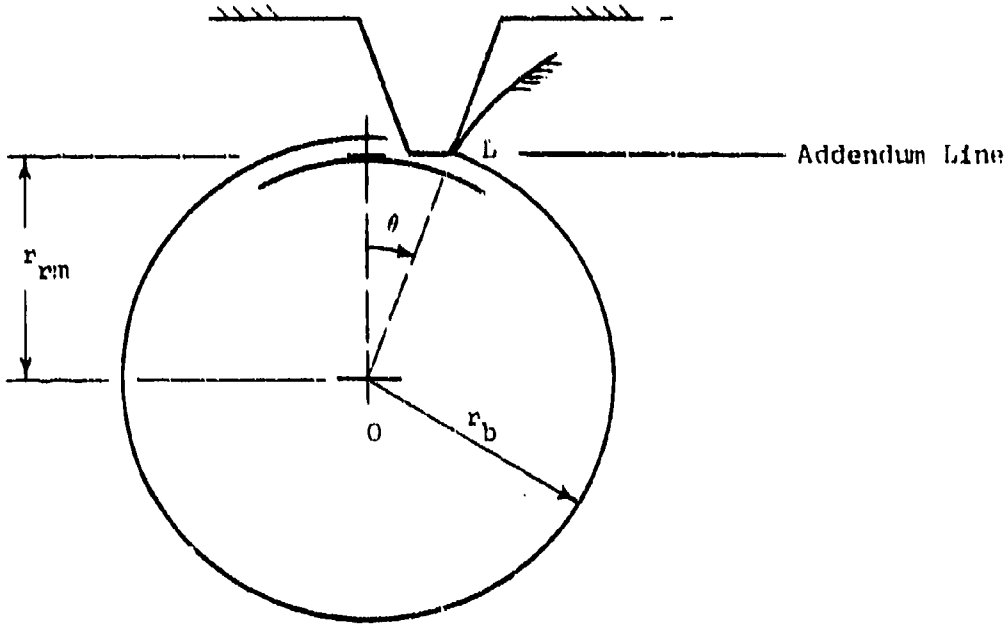


FIGURE B-2

MINIMUM ROOT RADIUS, r_{rm}, FOR RACK CUTTER WITH SHARP CORNER

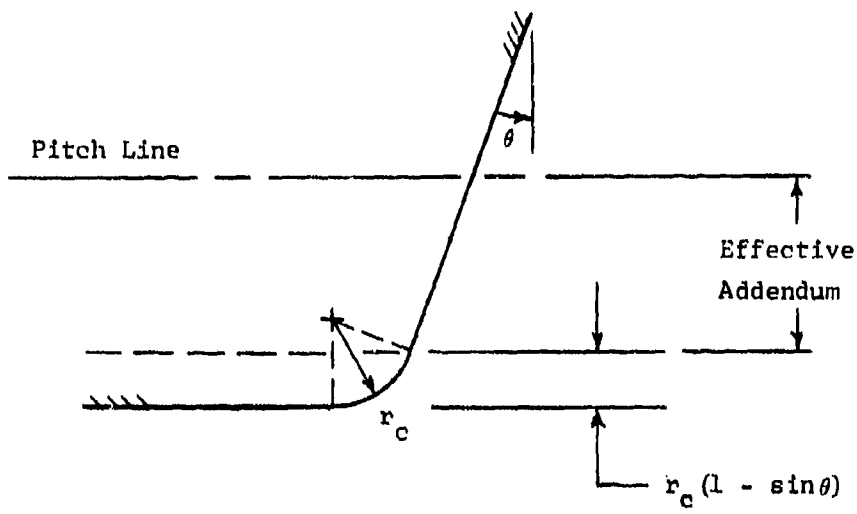


FIGURE B-3

RACK CUTTER WITH CORNER RADIUS, r_c , (EFFECTIVE ADDENDUM OF CUTTER IS DECREASED)

$$r_{rmc} = r_p\cos^2\theta - r_c(1 - \sin\theta) \qquad \text{(B-3)}$$

By common usage the corner radius may either be

$$r_c = .1\, t_s = \frac{.05\pi}{P_d}, \qquad \text{(B-4)}$$

or it is chosen such that the second term in Equation (B-3) becomes

$$r_c(1 - \sin\theta) \approx \frac{.157}{P_d} \qquad \text{(B-5)}$$

(This makes for an effective cutter addendum of $1/P_d$.)

Hob withdrawal becomes necessary if the root radius, obtained by setting the cutter to standard depth, is smaller than the minimum given by Equation (B-3). Thus, the hob withdrawal C is obtained from Equations (B-1) and (B-3), i.e.,

$$C = r_{rmc} - r_r \qquad \text{(B-6)}$$

With the cutter addendum $A = 1.157/P_d$, and using the expression of Equation (B-4) for the rack corner radius, Equation (B-6) becomes

$$C = \frac{1}{P_d}(1 + .157\sin\theta) - r_p\sin^2\theta \qquad \text{(B-7)}$$

If Equation (B-5) is used, with the same definition of the cutter addendum, one obtains for C

$$C = \frac{1}{P_d} - r_p\sin^2\theta \qquad \text{(B-7)}$$

3. OUTSIDE RADII OF PINION AND GEAR BLANKS

The outside radius of the pinion blank becomes

$$r_o = r_p + \frac{1}{P_d} + C \qquad \text{(B-8)}$$

The outside radius of the gear blank becomes

$$R_o = R_p + \frac{1}{P_d} - C \qquad \text{(B-9)}$$

4. TOOTH THICKNESS AT PITCH CIRCLES OF PINION AND GEAR

Since the thickness of the hobtooth at the pinion pitch circle will be reduced by the amount $2C\tan\theta$, due to the withdrawal C, the circular thickness of the pinion tooth at this location will be increased by this amount, i.e.,

$$t_c = t_{cs} + 2C\tan\theta \qquad \text{(B-10)}$$

The gear tooth thickness will be decreased to

$$T_c = T_{cs} - 2C\tan\theta \qquad \text{(B-11)}$$

at its pitch circle.

5. TOOTH THICKNESS AT OUTSIDE AND BASE RADII BY INVOLUTOMETRY

The circular tooth thickness at an arbitrary radius of an involute tooth may be obtained if the tooth thickness, radius, and pressure angle of any other location, such as the pitch circle, are known together with the pressure angle at the arbitrary radius. [See Equation (5-5), pg. 80, E. Buckingham: Analytical Mechanics of Gears, McGraw-Hill Book Co., Inc. New York, 1949.]

Accordingly, the circular tooth thickness, t_o, at the outside radius of the pinion, may be obtained from

$$t_o = t_c \frac{r_o}{r_p} - 2r_o(\text{INV}\theta_{OP} - \text{INV}\theta) \qquad \text{(B-12)}$$

where

$\text{INV}\theta = \tan\theta - \theta$, the involute function corresponding to the pressure angle θ

θ = pressure angle of mesh. This is also the pressure angle at the pitch circle.

$\theta_{OP} = \cos^{-1}\frac{r_b}{r_o}$, the pressure angle associated with the outside radius of the pinion

Similarly, the circular tooth thickness, T_o, at the outside radius of the gear is obtained from:

$$T_o = T_c \frac{R_o}{R_p} - 2R_o(INV\theta_{OG} - INV\theta) \qquad (B-13)$$

where

$\theta_{OG} = \cos^{-1}\frac{R_b}{R_o}$, the pressure angle associated with the outside radius of the gear

The tooth thickness at the base circle radius of the modified pinion is given by

$$t_b = t_c\cos\theta + 2r_b INV\theta \qquad (B-14)$$

This becomes for the gear

$$T_b = T_c\cos\theta + 2R_b INV\theta \qquad (B-15)$$

(The above is only of theoretical interest in case the inter-tooth space is not cut below the base circle radius.)

6. EXPRESSIONS FOR CONTACT RATIO AND PINION OUSIDE RADIUS FOR UNITY CONTACT RATIO

The expression for contact ratio [Equation 4-19, pg. 72, E. Buckingham: Analytical Mechanics of Gears] is given by

$$m_p = \frac{\sqrt{R_o^2 - R_b^2} + \sqrt{r_o^2 - r_b^2} - (R_b + r_b)\tan\theta}{p_{cs}\cos\theta} \qquad \text{(B-16)}$$

If the contact ratio of a certain mesh is larger than unity, and one wishes to reduce it to unity by reducing the outside radius of the pinion, one may find this new pinion outside radius with the help of

$$r_o' = \sqrt{r_b^2 + \left[p_{cs}\cos\theta + (R_b + r_b)\tan\theta - \sqrt{R_o^2 - R_b^2} \right]^2} \qquad \text{(B-17)}$$

7. EXAMPLE

Design a pinion and a gear with the following specifications:

$$P_d = 44, \quad N = 42, \quad n = 8, \quad \theta = 20^\circ$$

This gives

R_p = .47727 in. (1.212 cm) r_p = .09091 in. (.231 cm)

$R_b = R_p \cos 20^\circ$ = .44848 in. (1.139 cm) r_b = .08542 in. (.217 cm)

$p_{cs} = \frac{\pi}{44}$ = .07140 in. (.181 cm) $T_{cs} = t_{cs}$ = .03570 in. (.091 cm)

According to Equation (B-7), the hob withdrawal, C, is computed as

$$C = \frac{1}{44} - .09091 \sin^2 20^\circ = .012093 \text{ in. } (.031 \text{ cm})$$

Then, according to Equations (B-8) and (B-9)

$$r_o = .09091 + .022727 + .012093 = .12573 \text{ in. } (.319 \text{ cm})$$

$$R_o = .47727 + .022727 - .012093 = .48790 \text{ in. } (1.239 \text{ cm})$$

The new tooth thickness at the pitch radii is found with the help of Equations (B-10) and (B-11)

$$t_c = .03570 + 2(.012093)\tan 20^\circ = .04450 \text{ in. } (.113 \text{ cm})$$

$$T_c = .03570 - 2(.012093)\tan 20^\circ = .02689 \text{ in. } (.068 \text{ cm})$$

For the purposes of the present analysis, the pinion outside radius is now reduced to obtain a contact ratio of unity. With Equation (B-17)

r_o' = .110 in. (.279 cm) This is essentially the unmodified pinion radius of .113 in. (.287 cm).

Now compute the circular tooth thickness at the outside radii r_o' and R_o using the following data in Equations (B-12) and (B-13):

$$\theta = 20^\circ \qquad \text{INV } 20^\circ = .01490$$

$$\theta_{OP} = \cos^{-1}\frac{.08542}{.110} = 39.0509^\circ \qquad \text{INV } 39.0509^\circ = .12969$$

$$\theta_{OG} = \cos^{-1}\frac{.44848}{.48790} = 23.1889^\circ \qquad \text{INV } 23.1889^\circ = .02365$$

Then

$$t_o = .04450\left(\frac{.110}{.09091}\right) - 2(.110)(.12969 - .0149) = .02860 \text{ in. } (.073 \text{ cm})$$

and

$$T_o = .02689\left(\frac{.4879}{.47727}\right) - 2(.4879)(.02365 - .0149) = .01896 \text{ in. } (.048 \text{ cm})$$

These are sufficient to allow for rounding off the teeth.

Finally, check that the gear is not undercut. The actual root radius of the gear is

$$R_o = \frac{2.157}{P_d} = .4879 - .04902 = .4389 \text{ in. } (1.115 \text{ cm})$$

The minimum permissible root radius without undercutting, according to Equations (B-3) and (B-4), is computed from

$$R_{rm} = R_p\cos^2\theta - \frac{.05\pi}{P_d}(1 - \sin\theta) = .47727(.883) - .0023$$

$$= .419 \text{ in. } (1.064 \text{ cm})$$

Thus, the gear is not undercut.

APPENDIX C

COMPUTER MODELS FOR STEP-UP GEAR TRAINS WITH INVOLUTE TEETH

The present appendix contains descriptions, listings, and sample outputs of the following involute gear-train-related computer programs:

1. Program INVOL 1 : Design of unequal addendum involute gear and pinion set with unity contact ratio.

2. Program INVOL 2 : Point and cycle efficiencies for single pass involute step-up gear mesh with unity contact ratio.

3. Program INVOL 3 : Point and cycle efficiencies for three pass involute step-up gear train in spin environment.(All meshes have unity contact ratio).

4. Program INVOL 4 : Point and cycle efficiencies for two pass involute step-up gear train in spin environment.(All meshes have unity contact ratio).

The relevant background, the input parameters, the manner of the computations and the form of the output of each program are discussed in detail. The program proper forms the last part of each section.

1. Program INVOL 1: Design of Unequal Addendum Involute Gear and Pinion Set with Unity Contact Ratio.

The program INVOL 1 is based on Appendix B which shows the design equations for unequal addendum involute gear sets with standard center distances.

The nomenclature of the program is chosen to coincide as much as possible with that of Appendix B.

a. Input Parameters (See also Program INVOL 1, below)

The following parameters represent the input data of the program:

PDUBD = P_d, the diametral pitch

NG = N_G, the number of teeth of the gear

NP = N_P, the number of teeth of the pinion

THETAD = θ, the pressure angle of the mesh as well as of the hob (in degrees)

ISTOP = arbitrary single-digit integer for multiple data sets. Must be zero for last set of data.

b. Computations

The program computes the following quantities (where not otherwise indicated, consult Section 1 of Appendix B for nomenclature):

CAPRP = R_p

RP = r_p

CAPRB = R_b

RB = r_b

PSUBC	=	p_c
TSTAND	=	t_{cs}
C	=	hob withdrawal distance. See eq (B-7)
RO	=	r_o , See eq (B-8). This is the original pinion blank radius.
CAPRO	=	R_o , see eq (B-9)
TC	=	t_c , see eq (B-10)
CAPTC	=	T_c , see eq (B-11)
TB	=	t_b , see eq (B-14).Note also the computation of the involute function at the end of the program.
CAPTB	=	T_b, see eq (B-15)
ROFIN	=	r_o' , see eq (B-17). This is the pinion outside radius for unity contact ratio.
THETOG	=	θ_{OG}, according to expression associated with eq (B-13)
THETOP	=	θ_{OP}, according to expression associated with eq (B-12)
CAPTO	=	T_o , see eq (B-13)
TO	=	t_o , see eq (B-12). Computed with ROFIN.
CRATIO	=	m_p , see eq (B-16). This is the original contact ratio ,which uses the unmodified pinion blank radius RO.
CRFIN	=	represents the contact ratio when computed with the final pinion blank radius ROFIN.
ROOT	=	actual root radius of pinion
CAPROOT	=	actual root radius of gear
CAPRMIN	=	R_{rm} , see equations (B-3) and (B-4)

c. Output of Program

The output of the program is best explained by means of the first sample computations which are shown at the end of the program. This example is identical with that given in section 7 of Appendix B. The output lists the following:

I. The input parameters, PSUBD, NG, NP, and pressure angle, THETAD, are printed out.

II. The above is followed by computational results for

CAPRP

RP

CAPRB

TSTAND

C

CAPRO

RO, this is the original pinion blank radius before the unity contact ratio modification.

CRATIO, note that for the given case, this original contact ratio equals 1.3

ROFIN, the pinion outside radius which corresponds to unity contact ratio

CRFIN, this, of course, has to be unity because of the use of ROFIN. This computation serves as a check.

CAPTC, this tooth thickness, as well as the following one, is useful for strength computations

TC

THEOGD, this pressure angle, as well as the following one, corresponds to the final outside radii of the gear and pinion, respectively, and is needed for the computations of the tooth thicknesses at the outside radii

THEOPD

CAPTO, this tooth thickness at the outside radius of the gear, as well as the following one for the pinion, must be sufficiently large to allow for the presence of a tip radius

TO

CAPTB

TB

CAPROOT

ROOT

CAPRMIN

The values of the last five parameters are to some extent interconnected. First, it is important to see whether the gear is undercut. This does not occur as long as CAPROOT is larger than CAPRMIN, i.e.,the actual root radius is larger than the minimum allowable one. The gear tooth thickness at the base circle is of interest for strength purposes. Since, for the given case, CAPRB = .44849 in. (1.139 cm) and CAPROOT = .4388 in. (1.115 cm) the base circle lies above the root circle, and the greatest cross-section of the tooth is approximately equal to CAPTB. TB is called the theoretical

pinion tooth thickness at the base circle since it is possible that the base circle lies below the minimum allowable root circle of the pinion. In such a case, the actual tooth may be weaker than indicated by this dimension. For the present case, RB = .08543 in. (.217 cm) and ROOT = .07671 in. (.195 cm), and therefore, the base circle, which lies above the root circle, gives a good indication of the actual tooth cross-section at the root.

Program INVOL 1

This program contains five different sets of gears. This will be of use in programs INVOL 2, INVOL 3 and INVOL 4.

```
      1              PROGRAM INVOL 1(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
                     C
                     C  DESIGN OF UNEQUAL ADDENDUM INVOLUTE GEAR AND PINION SET
                     C  WITH UNITY CONTACT RATIO
      5              C
                           REAL INV
                        1  READ(5,2)PSUBD,NG,NP,THETAD,ISTOP
                        2  FORMAT(F10.4,2I10,F10.4,9X,I1)
                           PI = 3.14159
     10                    Z = PI/180.
                           THETA = THETAD*Z
                           CAPRP = NG/(2.*PSUBD)
                           RP = NP/(2.*PSUBD)
                           CAPRB = CAPRP*COS(THETA)
     15                    RB = RP*COS(THETA)
                           PSUBC = PI/PSUBD
                           TSTAND = PSUBC/2.
                           C = 1./PSUBD - RP*SIN(THETA)*SIN(THETA)
                           CAPRO = CAPRP + 1./PSUBD - C
     20                    RO = RP + 1./PSUBD + C
                           CAPTC = TSTAND - 2.*C*TAN(THETA)
                           TC = TSTAND + 2.*C*TAN(THETA)
                           CAPTB = CAPTC*COS(THETA) + 2.*CAPRB*INV(THETA)
                           TB = TC*COS(THETA) + 2.*RB*INV(THETA)
                           ROFIN = SQRT(RB*RB + (PSUBC*COS(THETA) + (CAPRB + RB)*TAN(THETA)
     25                 1  - SQRT(CAPRO*CAPRO - CAPRB*CAPRB))**2)
                           THETOG = ACOS(CAPRB/CAPRO)
                           THEOGD = THETOG/Z
                           THETOP = ACOS(RB/ROFIN)
                           THEOPD = THETOP/Z
     30                    CAPTO = CAPTC*CAPRO/CAPRP - 2.0*CAPRO*(INV(THETOG) - INV(THETA))
                           TO = TC*ROFIN/RP - 2.*ROFIN*(INV(THETOP) - INV(THETA))
                           CRATIO = (SQRT(CAPRO*CAPRO - CAPRB*CAPRB) + SQRT(RO*RO - RB*RB)
                        1  - (CAPRB + RB)*TAN(THETA))/(PSUBC*COS(THETA))
     35                    CRFIN = (SQRT(CAPRO*CAPRO - CAPRB*CAPRB) + SQRT(ROFIN*ROFIN
                        1  - RB*RB) - (CAPRB + RB)*TAN(THETA))/(PSUBC*COS(THETA))
                           ROOT = RO - 2.157/PSUBD
                           CAPROOT = CAPRO - 2.157/PSUBD
                           CAPRMIN = CAPRP*COS(THETA)*COS(THETA) - .05*PI*(1. - SIN(THETA))/
     40                 1PSUBD
                           WRITE(6,3)PSUBD,NG,NP,THETAD
                        3  FORMAT(*1DIAMETRAL PITCH (PSUBD) =*,F6.2/*0GEAR NUMBER OF TEETH (N
                        1G) =*,I3/*0PINION NUMBER OF TEETH (NP) =*,I3/*0PRESSURE ANGLE (THE
                        2TA) =*,F6.2)
     45                    WRITE(6,4)CAPRP,RP
                        4  FORMAT(*0GEAR PITCH RADIUS (CAPRP) =*,F9.5,3X,*PINION PITCH RADIUS
                        1 (RP) =*,F9.5)
                           WRITE(6,5)CAPRB,RB
                        5  FORMAT(*0GEAR BASE RADIUS (CAPRB) =*,F9.5,3X,*PINION BASE RADIUS (
     50                 1RB) =*,F9.5)
                           WRITE(6,6)TSTAND
                        6  FORMAT(*0STANDARD TOOTH THICKNESS AT PITCH RADII (TSTAND) =*,F9.5)
                           WRITE(7,7)C
```

```
      1              REAL FUNCTION INV(THETA)
                     INV = TAN(THETA) - THETA
                     RETURN
                     END
```

```
                7 FORMAT(*0HOB WITHDRAWAL DISTANCE (C) =*,F8.5)
       55         WRITE(6,8)CAPRO,RO                                          8
                8 FORMAT(*0GEAR BLANK RADIUS (CAPRO) =*,F9.5,3X,*ORIGINAL PINION BLA
                 1NK RADIUS (RO) =*,F9.5)
                  WRITE(6,9)CRATIO
                9 FORMAT(*0ORIGINAL CONTACT RATIO (CRATIO) =*,F6.3)
       60         WRITE(6,10)ROFIN
               10 FORMAT(*0PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROFIN) =*,
                 1F9.5)
                  WRITE(6,11)CRFIN
               11 FORMAT(*0FINAL CONTACT RATIO (CRFIN) =*,F6.3)
       65         WRITE(6,12)CAPTC,TC                                         I
               12 FORMAT(*0GEAR TOOTH THICKNESS AT PITCH CIRCLE (CAPTC) =*,F9.5,3X,
                 1*PINION TOOTH THICKNESS AT PITCH CIRCLE (TC) =*,F8.5)      +
                  WRITE(6,13)THEOGO,THEOPO
               13 FORMAT(*0GEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEOGO) =*,F9.5,
       70         13X,*PINION PRESSURE ANGLE AT FINAL OUTSIDE RADIUS (THEOPO) =*,
                 1F9.5)
                  WRITE(6,14)CAPTO,TO
               14 FORMAT(*0GEAR TOOTH THICKNESS AT OUTSIDE RADIUS (CAPTO) =*,F8.5,
                 13X,*PINION TOOTH THICKNESS AT FINAL OUTSIDE RADIUS (TO) =*,F8.5)
       75         WRITE(6,15)CAPTB,TB
               15 FORMAT(*0GEAR TOOTH THICKNESS AT BASE CIRCLE (CAPTB) =*,F8.5,3X,
                 1*THEORETICAL PINION TOOTH THICKNESS AT BASE CIRCLE (TB) =*,F8.5)  7
                  WRITE(6,16)CAPROOT,CAPRMIN
               16 FORMAT(*0RADIUS OF ROOT CIRCLE OF GEAR (CAPROOT) =*,F8.5,3X,
       80         1*MINIMUM ALLOWABLE RADIUS OF ROOT CIRCLE OF GEAR (CAPRMIN) =*,F8.5
                 2)
                  IF(CAPROOT .GE. CAPRMIN)WRITE(6,17)
                  IF(CAPROOT .LT. CAPRMIN)WRITE(6,18)
               17 FORMAT(*0THE GEAR IS NOT UNDERCUT*)
       85      18 FORMAT(*0THE GEAR IS UNDERCUT*)
                  WRITE(6,19)ROOT
               19 FORMAT(*0RADIUS OF ROOT CIRCLE OF PINION (ROOT) =*,F8.5)
                  IF(ISTOP .EQ. 0)GO TO 9999
                  GO TO 1
       90     9999 STOP
                  END
```

DIAMETRAL PITCH (PSUBD) = 44.00

GEAR NUMBER OF TEETH (NG) = 42

PINION NUMBER OF TEETH (NP) = 8

PRESSURE ANGLE (THETA) = 20.00

GEAR PITCH RADIUS (CAPRP) = .47727 PINION PITCH RADIUS (RP) = .09091

GEAR BASE RADIUS (CAPRB) = .44849 PINION BASE RADIUS (RB) = .08543

STANDARD TOOTH THICKNESS AT PITCH RADII (TSTAND) = .03570

HOB WITHDRAWAL DISTANCE (C) = .01209

GEAR BLANK RADIUS (CAPRO) = .48741 ORIGINAL PINION BLANK RADIUS (RO) = .12573

ORIGINAL CONTACT RATIO (CRATIO) = 1.342

PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROFIN) = .11000

FINAL CONTACT RATIO (CRFIN) = 1.000

GEAR TOOTH THICKNESS AT PITCH CIRCLE (CAPTC) = .02498 PINION TOOTH THICKNESS AT PITCH CIRCLE (TC) = .04458

GEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEOGO) = 23.18094 PINION PRESSURE ANGLE AT FINAL OUTSIDE RADIUS (THEOPO) = 39.85893

GEAR TOOTH THICKNESS AT OUTSIDE RADIUS (CAPTO) = .01994 PINION TOOTH THICKNESS AT FINAL OUTSIDE RADIUS (TO) = .02868

GEAR TOOTH THICKNESS AT BASE CIRCLE (CAPTB) = .03664 THEORETICAL PINION TOOTH THICKNESS AT BASE CIRCLE (TB) = .04437

RADIUS OF ROOT CIRCLE OF GEAR (CAPRGRT) = .43886 MINIMUM ALLOWABLE RADIUS OF ROOT CIRCLE OF GEAR (CAPRMIN) = .41909

THE GEAR IS NOT UNDERCUT

RADIUS OF ROOT CIRCLE OF PINION (RROOT) = .07671

DIAMETRAL PITCH (PSUBD) = 65.00

GEAR NUMBER OF TEETH (NG) = 27

PINION NUMBER OF TEETH (NP) = 9

PRESSURE ANGLE (THETA) = 20.00

GEAR PITCH RADIUS (CAPRP) = .20769 PINION PITCH RADIUS (RP) = .06923

GEAR BASE RADIUS (CAPRB) = .19517 PINION BASE RADIUS (RB) = .06506

STANDARD TOOTH THICKNESS AT PITCH RADII (TSTAND) = .02417

MOD WITHDRAWAL DISTANCE (C) = .00729

GEAR BLANK RADIUS (CAPRO) = .21579 ORIGINAL PINION BLANK RADIUS (RO) = .09199

ORIGINAL CONTACT RATIO (CRATIO) = 1.371

PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROFIN) = .08889

FINAL CONTACT RATIO (CRFIN) = 1.000

GEAR TOOTH THICKNESS AT PITCH CIRCLE (CAPTC) = .01686 PINION TOOTH THICKNESS AT PITCH CIRCLE (TC) = .02947

GEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEOGO) = 25.25342 PINION PRESSURE ANGLE AT FINAL OUTSIDE RADIUS (THEOPO) = 38.05982

GEAR TOOTH THICKNESS AT OUTSIDE RADIUS (CAPTO) = .01267 PINION TOOTH THICKNESS AT FINAL OUTSIDE RADIUS (TO) = .02024

GEAR TOOTH THICKNESS AT BASE CIRCLE (CAPTB) = .02354 THEORETICAL PINION TOOTH THICKNESS AT BASE CIRCLE (TB) = .02963

RADIUS OF ROOT CIRCLE OF GEAR (CAPROOT) = .18261 MINIMUM ALLOWABLE RADIUS OF ROOT CIRCLE OF GEAR (CAPRMIN) = .18181

THE GEAR IS NOT UNDERCUT

RADIUS OF ROOT CIRCLE OF PINION (ROOT) = .05472

DIAMETRAL PITCH (PSUBD) = 77.00

GEAR NUMBER OF TEETH (NG) = 27

PINION NUMBER OF TEETH (NP) = 9

PRESSURE ANGLE (THETA) = 20.00

GEAR PITCH RADIUS (CAPRP) = .17532 PINION PITCH RADIUS (RP) = .05844

GEAR BASE RADIUS (CAPRB) = .16479 PINION BASE RADIUS (RB) = .05492

STANDARD TOOTH THICKNESS AT PITCH RADII (TSTAND) = .02540

HOB WITHDRAWAL DISTANCE (C) = .00615

GEAR BLANK RADIUS (CAPRG) = .18218 ORIGINAL PINION BLANK RADIUS (R0) = .07750

ORIGINAL CONTACT RATIO (CRATIO) = 1.371

PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROFIN) = .04828

FINAL CONTACT RATIO (CRFIN) = 1.900

GEAR TOOTH THICKNESS AT PITCH CIRCLE (CAPTG) = .01592 PINION TOOTH THICKNESS AT PITCH CIRCLE (TG) = .02488

GEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEOGO) = 25.25382 PINION PRESSURE ANGLE AT FINAL OUTSIDE RADIUS (THEOPO) = 36.45942

GEAR TOOTH THICKNESS AT OUTSIDE RADIUS (CAPTO) = .01976 PINION TOOTH THICKNESS AT FINAL OUTSIDE RADIUS (TO) = .01710

GEAR TOOTH THICKNESS AT BASE CIRCLE (CAPTB) = .01907 THEORETICAL PINION TOOTH THICKNESS AT BASE CIRCLE (TB) = .02501

RADIUS OF ROOT CIRCLE OF GEAR (CAPROOT) = .15415 MINIMUM ALLOWABLE RADIUS OF ROOT CIRCLE OF GEAR (CAPRMIN) = .15347

THE GEAR IS NOT UNDERCUT

RADIUS OF ROOT CIRCLE OF PINION (ROOT) = .04957

DIAMETRAL PITCH (PSUBD) = 44.00

GEAR NUMBER OF TEETH (NG) = 56

PINION NUMBER OF TEETH (NP) = 8

PRESSURE ANGLE (THETA) = 20.00

GEAR PITCH RADIUS (CAPRP) = .63636 PINION PITCH RADIUS (RP) = .09091

GEAR BASE RADIUS (CAPRB) = .59799 PINION BASE RADIUS (RB) = .08543

STANDARD TOOTH THICKNESS AT PITCH RADII (TSTAND) = .03570

MOR WITHDRAWAL DISTANCE (C) = .01209

GEAR BLANK RADIUS (CAPRO) = .64700 ORIGINAL PINION BLANK RADIUS (RO) = .12573

ORIGINAL CONTACT RATIO (CRATIO) = 1.349

PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROFIN) = .10970

FINAL CONTACT RATIO (CNFIN) = 1.800

GEAR TOOTH THICKNESS AT PITCH CIRCLE (CAPTC) = .02698 PINION TOOTH THICKNESS AT PITCH CIRCLE (TC) = .04450

GEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEOGO) = 22.64485 PINION PRESSURE ANGLE AT FINAL OUTSIDE RADIUS (THEOPO) = 38.85337

GEAR TOOTH THICKNESS AT OUTSIDE RADIUS (CAPTO) = .01901 PINION TOOTH THICKNESS AT FINAL OUTSIDE RADIUS (TO) = .02901

GEAR TOOTH THICKNESS AT BASE CIRCLE (CAPTB) = .04310 THEORETICAL PINION TOOTH THICKNESS AT BASE CIRCLE (TB) = .04437

RADIUS OF ROOT CIRCLE OF GEAR (CAPROOT) = .59798 MINIMUM ALLOWABLE RADIUS OF ROOT CIRCLE OF GEAR (CAPRMIN) = .55957

THE GEAR IS NOT UNDERCUT

RADIUS OF ROOT CIRCLE OF PINION (ROOT) = .07671

DIAMETRAL PITCH (PSUBD) = 65.00

GEAR NUMBER OF TEETH (NG) = 56

PINION NUMBER OF TEETH (NP) = 8

PRESSURE ANGLE (THETA) = 20.00

GEAR PITCH RADIUS (CAPRP) = .43077 PINION PITCH RADIUS (RP) = .06154

GEAR BASE RADIUS (CAPRB) = .40479 PINION BASE RADIUS (RB) = .05783

STANDARD TOOTH THICKNESS AT PITCH RADII (ISTAND) = .02417

HOB WITHDRAWAL DISTANCE (C) = .00819

GEAR BLANK RADIUS (CAPRO) = .43797 ORIGINAL PINION BLANK RADIUS (RO) = .08511

ORIGINAL CONTACT RATIO (CRATIO) = 1.349

PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROFIN) = .07426

FINAL CONTACT RATIO (CRFIN) = 1.000

GEAR TOOTH THICKNESS AT PITCH CIRCLE (CAPTC) = .01821 PINION TOOTH THICKNESS AT PITCH CIRCLE (TC) = .03012

GEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEOGO) = 22.64484 PINION PRESSURE ANGLE AT FINAL OUTSIDE RADIUS (THEOPO) = 30.85337

GEAR TOOTH THICKNESS AT OUTSIDE RADIUS (CAPTO) = .01287 PINION TOOTH THICKNESS AT FINAL OUTSIDE RADIUS (TO) = .01964

GEAR TOOTH THICKNESS AT BASE CIRCLE (CAPTB) = .02918 THEORETICAL PINION TOOTH THICKNESS AT BASE CIRCLE (TB) = .03003

RADIUS OF ROOT CIRCLE OF GEAR (CAPROOT) = .40478 MINIMUM ALLOWABLE RADIUS OF ROOT CIRCLE OF GEAR (CAPRMIN) = .37879

THE GEAR IS NOT UNDERCUT

RADIUS OF ROOT CIRCLE OF PINION (ROOT) = .05192

2. Program INVOL 2: Point and Cycle Efficiencies for Single Pass Involute Step-Up Gear Mesh With Unity Contact Ratio

The program INVOL 2 is based on section 3 of Appendix A, which gives the moment input-output relationship for a single step-up gear mesh with involute teeth. The mesh has unity contact ratio. Again, the nomenclature of the program is chosen to coincide as much as possible with that of the original derivation. The contact geometry is adapted from section 6c of Appendix A.

a. Input Parameters (See also program in section d below)

The following parameters represent the input data of the program. Most of these are taken from the results of INVOL 1 since the moment expressions are for unity contact ratio only.

CAPRP = R_p, the pitch radius of the gear

RP = r_p, the pitch radius of the pinion

CAPRO = R_o, the outside radius of the gear

ROFIN = R_o', see eq. (B-17). This is the pinion outside radius for unity contact ratio.

RHOCAPN = ρ_N, the pivot radius of the gear

RHON = ρ_N, the pivot radius of the pinion

MU = μ, the coefficient of friction at both pivots as well as at the gear and pinion contact point

K, range divisor, i.e., it represents the number of times the output moment and the efficiency are computed between initial and final contact of the gear and the pinion

ISTOP, arbitrary single digit for multiple data sets. It must be zero for the last set of data.

b. Computations

Both point and cycle efficiencies are based on eq. (A-25). With the help of eq. (3), the point efficiency becomes

$$\eta_p = \frac{\{r_b - \mu[\rho_n + s(d-a)]\}}{\{R_b + \mu[\rho_N - sa]\}} \frac{\dot{\psi}}{\dot{\Phi}} \qquad (C-1)$$

Since the angular velocity is constant for involute gears, and may be expressed in terms of the base circle radii R_b and r_b,

$$K_{ratio} = \frac{\dot{\psi}}{\dot{\Phi}} = \frac{R_b}{r_b} \qquad (C-2)$$

Therefore,

$$\eta_p = E_2 \quad \text{(POINTEF)} \qquad (C-3)$$

as given by eq. (A-26).

The cycle efficiency expression is based on eq. (4). If one replaces both integrals by summations, one obtains

$$\eta_c = \frac{\Sigma\ M_o\ \Delta\psi}{\Sigma\ M_{in}\ \Delta\Phi} \qquad (C-4)$$

where $\Delta\Phi$ and $\Delta\psi$ now represent incremental changes in the input and output angles, respectively. The input moment is constant

over the total interval. Also

$$\Sigma\Delta\varphi = \alpha_{FIN} - \alpha_{IN} \tag{C-5}$$

(See eqs. (A205) and (A206).)

The increment $\Delta\psi$ is also constant and may therefore be taken outside the summation sign. It is expressed with the help of $\Delta\varphi$

$$\Delta\varphi = \Delta\alpha = \frac{\alpha_{FIN} - \alpha_{IN}}{K} = \text{DELALPH} \tag{C-6}$$

(This is an adaptation of eq. (A-207) to the single step-up mesh.)

With the above

$$\Delta\psi = K_{ratio}\,\Delta\alpha \tag{C-7}$$

Substitution into eq. (C-4) gives

$$\eta_o = \frac{K_{ratio}\,\Delta\alpha\,\Sigma M_o}{M_{in}\,(\alpha_{FIN} - \alpha_{IN})} \tag{C-8}$$

Since

$$\eta_p = K_{ratio}\,\frac{M_o}{M_{in}} \tag{C-9}$$

one obtains for the cycle efficiency

$$\eta_o = \frac{\Delta\alpha\,\Sigma\eta_p}{(\alpha_{FIN} - \alpha_{IN})} \quad \text{(CYCLEFF)} \tag{C-10}$$

To arrive at expressions for both POINTEF and CYCLEFF in the program, the following other important computations are necessary:

ALPHIN = α_{IN}, see eq. (A-205)

ALPHFIN = α_{FIN}, see eq. (A-206)

D = d, see eq. (A-204)

A = a, see eq. (A-203)

The signum value s is obtained according to eqs. (A-215) and (A-216), which are alternate ways of expressing eqs. (A-6) to (A-8).

c. Output of Program (see Program INVOL 2, below)

The output of the program is best explained by means of the single sample computation which is shown at the end of the program. This example uses the data of the first sample output of INVOL 1. The output lists the following:

I. Input Parameters

The input parameters CAPRP, RP, CAPRO, RO and THETAD are reproduced. In addition, the following dimensions, which were selected from a practical viewpoint, are shown:

RHOCAPN = ρ_N = .060 in. (.152 cm)

RHON = ρ_n = .030 in. (.076 cm)

MU = μ = .2

K = 25 (no substantial changes in cycle efficiency were encountered for larger values of K)

II. Computed Values

The point efficiency is listed as a function of the angle α, while the cycle efficiency is computed for the interval from α_{IN} to α_{FIN}. The signum parameter, s, is listed for checking purposes.

Program INVOL 2

```
      PROGRAM INVOL2 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C  POINT AND CYCLE EFFICIENCIES FOR SINGLE PASS INVOLUTE STEP-UP
C  GEAR MESH WITH UNITY CONTACT RATIO
C
      REAL MU,MTOT
    1 READ(5,2)CAPRP,RP,CAPRO,ROFIN,THETAD,RHOCAPN,RHON,MU,K,ISTOP
    2 FORMAT(5F10.4/3F10.4/7X,I3,9X,I1)
      WRITE(6,3)CAPRP,RP,CAPRO,ROFIN,THETAD,RHOCAPN,RHON,MU,K
    3 FORMAT(*1GEAR PITCH RADIUS (CAPRP) =*,F7.5,3X,*PINION PITCH RADIUS
     1 (RP) =*,F7.5/*0GEAR OUTSIDE RADIUS (CAPRO) =*,F7.5,3X,*PINION OUT
     2SIDE RADIUS FOR UNITY CONTACT RATIO (ROFIN) =*,F7.5/*0PRESSURE ANG
     3LE IN DEGREES (THETAD) =*,F6.2/*0GEAR PIVOT RADIUS (RHOCAPN) =*,
     4F5.3,3X,*PINION PIVOT RADIUS (RHON) =*,F5.3/*0COEFFICIENT OF FRICT
     5ION (MU) =*,F4.2/*0RANGE DIVISOR (K) =*,I4///)
      PI = 3.14159
      Z = PI/180.
      THETA = THETAD*Z
      CAPRB = CAPRP*COS(THETA)
      RB = RP*COS(THETA)
      ALPHIN = ((CAPRB*RB)*TAN(THETA) - SQRT(ROFIN*ROFIN - RB*RB))/CAPRB
      ALPHFIN = SQRT(CAPRO*CAPRO - CAPRB*CAPRB)/CAPRB
      DELALPH = (ALPHFIN-ALPHIN)/K
      MTOT = 0.
      D = (CAPRB*RB)*TAN(THETA)
      DO 5 I = 1,K
      ALPHA = ALPHIN + (I-1)*DELALPH
      IF(ALPHA .LT. TAN(THETA))S = 1.
      IF(ALPHA .EQ. TAN(THETA))S = 0.
      IF(ALPHA .GT. TAN(THETA))S = -1.
      A = CAPRB*ALPHA
      AA = 1. - MU*(RHON + S*(D-A))/RB
      BB = 1. + MU*(RHOCAPN - S*A)/CAPRB
      POINTEF = AA/BB
      MTOT = MTOT + POINTEF
      ALPHAD = ALPHA/Z
      WRITE(6,4)ALPHAD,S,POINTEF
    4 FORMAT(* ALPHAD =*,F6.2,3X,*S =*,F4.1,3X,*POINTEF =*,F6.4)
    5 CONTINUE
      CYCLEFF = DELALPH*MTOT/(ALPHFIN-ALPHIN)
      WRITE(6,6)CYCLEFF
    6 FORMAT(///* CYCLE EFFICIENCY =*,F6.4)
      IF(ISTOP .EQ. 0)GO TO 9999
      GO TO 1
 9999 STOP
      END
```

GEAR PITCH RADIUS (CAPRP) = .47727 PINION PITCH RADIUS (RP) = .09091

GEAR OUTSIDE RADIUS (CAPRO) = .49791 PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROSIN) = .11000

PRESSURE ANGLE IN DEGREES (THETAD) = 20.00

GEAR PIVOT RADIUS (RHOCAPN) = .060 PINION PIVOT RADIUS (RHON) = .030

COEFFICIENT OF FRICTION (MU) = .20

RANGE DIVISOR (K) = 25

AI PHAD = 15.97	S = 1.0	POINTEF = .7905
AI PHAD = 16.32	S = 1.0	POINTEF = .7979
AI PHAD = 16.66	S = 1.0	POINTEF = .8054
AI PHAD = 17.00	S = 1.0	POINTEF = .8129
AI PHAD = 17.34	S = 1.0	POINTEF = .8204
AI PHAD = 17.69	S = 1.0	POINTEF = .8279
AI PHAD = 18.03	S = 1.0	POINTEF = .8355
AI PHAD = 18.37	S = 1.0	POINTEF = .8430
AI PHAD = 18.72	S = 1.0	POINTEF = .8506
AI PHAD = 19.06	S = 1.0	POINTEF = .8582
AI PHAD = 19.40	S = 1.0	POINTEF = .8658
AI PHAD = 19.75	S = 1.0	POINTEF = .8735
AI PHAD = 20.09	S = 1.0	POINTEF = .8811
AI PHAD = 20.43	S = 1.0	POINTEF = .8888
AI PHAD = 20.77	S = 1.0	POINTEF = .8965
AI PHAD = 21.12	S =-1.0	POINTEF = .9067
AI PHAD = 21.46	S =-1.0	POINTEF = .9000
AI PHAD = 21.80	S =-1.0	POINTEF = .8933
AI PHAD = 22.15	S =-1.0	POINTEF = .8866
AI PHAD = 22.49	S =-1.0	POINTEF = .8800
AI PHAD = 22.83	S =-1.0	POINTEF = .8734
AI PHAD = 23.17	S =-1.0	POINTEF = .8667
AI PHAD = 23.52	S =-1.0	POINTEF = .8601
AI PHAD = 23.86	S =-1.0	POINTEF = .8535
AI PHAD = 24.20	S =-1.0	POINTEF = .8470

CYCLE EFFICIENCY = .8566

3. Program INVOL 3: Point and Cycle Efficiencies for Three Pass Involute Step-Up Gear Train in Spin Environment (All Meshes Have Unity Contact Ratio)

The program INVOL 3 is based on section 4 of Appendix A, which derives the moment input-output relationship for a three pass step-up gear train operating in a spin environment. Again, all meshes have unity contact ratio. As previously, the nomenclature of the program is chosen to coincide as closely as possible with that of the original derivations. The expressions for the contact geometry and other auxiliary geometric terms may be found in section 6 of Appendix A.

a. Input Parameters (see Program INVOL 3, below)

The following parameters represent the input data for the program. Those which involve gear dimensions only must be obtained from the results of INVOL 1 since the moment expressions are derived for unity contact ratio only.

MU = μ, the coefficient of friction at all pivots and at all tooth contact points

RPM, revolutions per minute of the fuze body

CAPRP1 = R_{p1}

CAPRP2 = R_{p2}

CAPRP3 = R_{p3}

RP2 = r_{p2}

RP3 = r_{p3}

RP4 = r_{p4}

THETA1 = θ_1

THETA2 = θ_2

THETA3 = θ_3

ISTOP, arbitrary single digit integer for multiple data set. It must be zero for last set of data.

R1 = $\mathcal{R}_1$

R2 = $\mathcal{R}_2$

R3 = $\mathcal{R}_3$

R4 = $\mathcal{R}_4$

RHO1 = ρ_1

RHO2 = ρ_2

RHO3 = ρ_3

RHO4 = ρ_4

CAPRB1 = R_{b1}

CAPRB2 = R_{b2}

CAPRB3 = R_{b3}

RB2 = r_{b2}

RB3 = r_{b3}

RB4 = r_{b4}

CAPRO1 = R_{o1}

CAPRO2 = R_{o2}

CAPRO3 = R_{o3}

RO2 = r'_{o2}

RO3 = r'_{o3}

RO4 = r'_{o4}

M1 = m_1, mass of input gear 1

M2 = m_2, mass of gear and pinion 2

M3 = m_3, mass of gear and pinion 3

M4 = m_4, mass of pinion 4

MD = md^2, the "mass-distance" product contained in the expression for the input moment M_{in}

K = K_3, the range divisor which is associated with gear 3, the driving gear of the last mesh (see eq. (A-211))

b. Computations (see COMMENT cards in program)

I. Computation of MIN, GAMMAS and BETAS

To start with, the program computes the input moment

$$MIN = M_{in} = md^2\omega^2 \qquad (C-11)$$

Subsequently, the angles γ_2, γ_3, γ_4 and β_1, β_2, β_3 are established according to the expressions given in section 6b of Appendix A.

II. Determination of Gear Train Constants

The determination of the gear train constants consists of the following:

RATIO = K_{ratio}, (see eq. (2)). Since the angular velocity is constant, this parameter may be expressed in terms of the applicable base radii, i.e.,

$$\frac{R_{b1} \times R_{b2} \times R_{b3}}{r_{b2} \times r_{b3} \times r_{b4}}$$

TEST1, TEST2 and TEST3 represent the tangent functions of the mesh pressure angles, which are used in conjunction with the values of the signum functions, s.

D1, D2 and D3 are given by eqs. (A-204), (A-217) and (A-223), respectively, and represent the distances between the points of tangency to the base circles along the lines-of-action of the three meshes.

MTOT = 0 represents the initialization of the sum of the output moments. This is used for the determination of the cycle efficiency.

III. Determination of Initial and Final Values of ALPHAS, Initialization of ALPHAS and Centrifugal Forces

The determination of the initial and final angles of rotation is accomplished with the help of subroutine ALPHA, at the end of the program, which makes use of eqs. (A-205), (A-206), (A-218), (A-219), (A-224) and (A-225). Thus, the initial values of the individual angles of rotation, ALPHA1, ALPHA2 and ALPHA3 are represented by AL1IN, AL2IN and AL3IN, while the final angles are given by AL1FIN, AL2FIN and AL3FIN.

The angular increments of gears 3, 2 and 1, i.e., DELAL3, DELAL2 and DELAL1, are determined with the help of eqs. (A-211) - (A-213), respectively.

The centrifugal forces, which act on the pivots of the various gear and/or pinion assemblies, are obtained by way of eqs. (A-33), (A-57), (A-84) and (A-107).

IV. Point and Cycle Efficiencies (See "output moment" in program)

Both point and cycle efficiencies are based on eq. (A-125) for the output moment MO4 = M_{o4}.

The point efficiency is computed directly in the manner of eq. (3), i.e.,

$$\eta_p = K_{ratio} \frac{M_{o4}}{M_{in}} = \text{POINTEF} \qquad \text{(C-12)}$$

The cycle efficiency is treated in the manner of eq. (C-8), i.e.,

$$\eta_p = \frac{K_{ratio}\, \Delta\alpha_1\, \Sigma M_{o4}}{M_{in}\, (\alpha_{1FIN} - \alpha_{1IN})} = \text{CYCLEFF} \qquad \text{(C-13)}$$

The program gives the summation as

$$\text{MTOT} = \Sigma M_{o4} \qquad \text{(C-14)}$$

V. Gear Train Motion Model

The simulation of the gear train motion, which is necessary for the computation of both POINTEF and CYCLEFF, is found in a loop which starts with statement label no. 14 (card no. 116) and ends with card no. 199. As discussed earlier, the motions of the individual driving gears are initialized at their respective angles, AL1IN, AL2IN and AL3IN. (This starting of the total train is arbitrary and is done only for convenience. There is an infinite number of other starting combinations each of which produces a different starting point efficiency.) The position of each mesh is subsequently incremented by the appropriate DELAL1, DELAL2 or DELAL3. When the angle ALPHA1 reaches the magnitude AL1FIN, CYCLEFF is determined, and the computation is ended. Since meshes 2 and 3 go through numerous cycles while mesh 1 goes through one cycle, they have

to be reset to their initial angles of rotation once their respective final angles have been reached. This is accomplished by the conditional statements on cards 116 and 117.

The values of the signum functions s_1, s_2 and s_3 are determined continuously according to eqs. (A-216), (A-222) and (A-227).

The instantaneous positions of the contact A1 = a_1, A2 = a_2, and A3 = a_3 are determined for each of the meshes by an appropriate adaptation of eq. (A-203). (See also eqs. (A-214), (A-220) and (A-226).)

The determination of the instantaneous output moment, MO4 = M_{o4}, requires the continuous computation of the variable quantities A_1 to A_{20}, C_1 to C_6 and D_1 to D_4, which are given originally in conjunction with the various equilibrium conditions in section 4 of Appendix A. The program uses the following nomenclature for these variables:

AA1 to AA20
CC1 to CC6
DD1 to DD4

c. Output (see Program INVOL 3, below)

Again, the output of the program is best explained by means of the sample computation which is shown at the end of the program. This example uses the gear data of the first three sample computations of program INVOL 1. The output lists the following:

I. Input Parameters

Mesh No. 1

CAPRP1 = R_{p1} = .47727 in. (1.212 cm)

CAPRB1 = R_{b1} = .44849 in. (1.139 cm)

CAPRO1 = R_{o1} = .48791 in. (1.239 cm)

RP2 = r_{p2} = .09091 in. (.231 cm)

RB2 = r_{b2} = .08543 in. (.217 cm)

RO2 = r'_{o2} = .11000 in. (.279 cm) (This is an ROFIN as given by INVOL 1.)

Also,

THETA1 = θ_1 = 20°

Mesh No. 2

CAPRP2 = R_{p2} = .20769 in. (.527 cm)

CAPRB2 = R_{b2} = .19517 in. (.496 cm)

CAPRO2 = R_{o2} = .21579 in. (.548 cm)

RP3 = r_{p3} = .06923 in. (.176 cm)

RB3 = r_{b3} = .06506 in. (.165 cm)

RO3 = r'_{o3} = .08089 in. (.205 cm)

Also,

THETA2 = θ_2 = 20°

Mesh No. 3

CAPRP3 = R_{p3} = .17532 in. (.445 cm)

CAPRB3 = R_{b3} = .16475 in. (.418 cm)

CAPRO3 = R_{o3} = .18216 in. (.463 cm)

RP4 = r_{p4} = .05844 in. (.148 cm)

RB4 = r_{b4} = .05492 in. (.139 cm)

RO4 = r'_{o4} = .06828 in. (.173 cm)

Also,

THETA3 = θ_3 = 20^o

In addition,

MU = μ = .2

RPM = 1000

M1 = m_1 = .51079 x 10^{-4} lb-sec^2/in. (8.943 g)

M2 = m_2 = .17413 x 10^{-4} lb-sec^2/in. (3.049 g)

M3 = m_3 = .69788 x 10^{-5} lb-sec^2/in. (1.222 g)

M4 = m_4 = .7745 x 10^{-6} lb-sec^2/in. (0.136 g)

R1 = $\mathcal{R}_1$ = .75 in. (1.905 cm)

R2 = $\mathcal{R}_2$ = .75 in. (1.905 cm)

R3 = $\mathcal{R}_3$ = .75 in. (1.905 cm)

R4 = $\mathcal{R}_4$ = .75 in. (1.905 cm)

RHO1 = ρ_1 = .060 in. (.152 cm)

RHO2 = ρ_2 = .030 in. (.076 cm)

RHO3 = ρ_3 = .025 in. (.064 cm)

RHO4 = ρ_4 = .020 in. (.051 cm)

MD = md^2 = .15 x 10^{-4} lb-sec^2 in. (16.944 g-cm^2)

K = 25

II. Computed Values

The point efficiency is given as a function of the angle α_1, together with the signum parameters s_1, s_2 and s_3 (given for checking purposes). The cycle efficiency is shown at the end of the output. In addition, the input moment, MIN, is printed out.

Program INVOL 3

```
      PROGRAM INVOL 3(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C  POINT AND CYCLE EFFICIENCIES FOR THREE PASS INVOLUTE STEP-UP
C  IN SPIN ENVIRONMENT (ALL MESHES HAVE UNITY CONTACT RATIO)
C
      REAL MIN,MU,M1,M2,M3,M4,MO3,MO4,MTOT,MD
C
C   READ AND WRITE INPUT DATA
C
  100 READ(5,1)MU,RPM,CAPRP1,CAPRP2,CAPRP3,RP2,RP3,RP4,THETA1,
     1THETA2,THETA3,ISTOP
      READ(5,2)R1,R2,R3,R4
      READ(5,3)RHO1,RHO2,RHO3,RHO4
      READ(5,4)CAPRB1,CAPRB2,CAPRB3,RB2,RB3,RB4
      READ(5,5)CAPRO1,CAPRO2,CAPRO3,RO2,RO3,RO4
      READ(5,6)M1,M2,M3,M4
      READ(5,7)MD,K
      PI = 3.14159
      OMEGA = RPM*2.*PI/60.
      OM2 = OMEGA*OMEGA
    1 FORMAT(F10.3,F10.0/6F10.5/3F10.4/I1)
    2 FORMAT(4F10.4)
    3 FORMAT(4F10.4)
    4 FORMAT(6F10.5)
    5 FORMAT(6F10.5)
    6 FORMAT(4E10.2)
    7 FORMAT(E10.2/I3)
C
C  COMPUTATION OF MIN, GAMMAS AND BETAS
C
      MIN = MD*OM2
      GAMMA2 = ACOS((R1*R1 + R2*R2 - (CAPRP1+RP2)*(CAPRP1+RP2))/
     1(2.*R1*R2))
      GAMA3P = ACOS((R2*R2 + R3*R3 - (CAPRP2+PP3)*(CAPRP2-RP3))/
     1(2.*R2*R3))
      GAMMA3 = GAMMA2 + GAMA3P
      GAMA4P = ACOS((R3*R3 + R4*R4 - (CAPRP3+PP4)*(CAPRP3-RP4))/
     1(2.*R3*R4))
      GAMMA4 = GAMMA3 + GAMA4P
      DELTA2 = ACOS(((CAPRP1+RP2)*(CAPRP1+PP2) + R1*R1 - R2*R2)/
     1(2.*R1*(CAPRP1 + RP2)))
      DELTA3 = ACOS(((CAPRP2+RP3)*(CAPRP2+PP3) + R2*R2 - R3*R3)/
     1(2.*R2*(CAPRP2 + RP3)))
      DELTA4 = ACOS(((CAPRP3+RP4)*(CAPRP3+PP4) + R3*R3 - R4*R4)/
     1(2.*R3*(CAPRP3 + RP4)))
      BETA1 = PI - DELTA2
      BETA2 = GAMMA2 + PI - DELTA3
      BETA3 = GAMMA3 + PI - DELTA4
      WRITE(6,8)MIN,MU,RPM,CAPRP1,CAPRP2,CAPRP3,RP2,RP3,RP4,THETA1,
     1THETA2,THETA3
      WRITE(6,9)R1,R2,R3,R4,M1,M2,M3,M4
      WRITE(6,10)RHO1,RHO2,RHO3,RHO4
      WRITE(6,11)CAPRB1,CAPRB2,CAPRB3,RB2,RB3,RB4
```

```
      WRITE(6,12)CAPRO1,CAPRO2,CAPRO3,RO2,RO3,RO4
      WRITE(6,13)MD,K
 8 FORMAT(*1*,5X,*MIK =*,F12.5,3X,*MU =*,F6.3,3X,*PPM =*,F6.0//
     16X,*CAPRR1 =*,F8.5,3X,*CAPRR2 =*,F8.5,3X,*CAPRR3 =*,F8.5//6X,
     2*RP2 =*,F8.5,3X,*RP3 =*,F8.5,3X,*RP4 =*,F8.5//6X,
     3*THETA1 =*,F8.3,3X,*THETA2 =*,F8.3,3X,*THETA3 =*,F8.3/)
 9 FORMAT(6X,*R1 =*,F7.5,3X,*R2 =*,F7.5,3X,*R3 =*,F7.5,3X,*R4 =*,F7.5
     1//6X,*M1 =*,E15.5,3X,*M2 =*,E15.5,3X,*M3 =*,E15.5,3X,
     2*M4 =*,E15.5/)
10 FORMAT(6X,*RHO1 =*,F7.5,3X,*RHO2 =*,F7.5,3X,*RHO3 =*,F7.5,3X,
     1*RHO4 =*,F7.5/)
11 FORMAT(6X,*CAPRB1 =*,F7.5,3X,*CAPRB2 =*,F7.5,3X,*CAPRB2 =*,F7.5,
     13X,*RB2 =*,F7.5,3X,*RB3 =*,F7.5,3X,*RB4 =*,F7.5/)
12 FORMAT(6X,*CAPRO1 =*,F7.5,3X,*CAPRO2 =*,F7.5,3X,*CAPRO3 =*,F7.5,3X
     1,*RO2 =*,F7.5,3X,*RO3 =*,F7.5,3X,*RO4 =*,F7.5/)
13 FORMAT(6X,*MD =*,E10.3//6X,*RANGE DIVISOR =*,I4//)
C
C  CONVERSION TO RADIANS
C
      Z = PI/180.
      THETA1 = THETA1*Z
      THETA2 = THETA2*Z
      THETA3 = THETA3*Z
C
C  DETERMINATION OF GEAR TRAIN CONSTANTS
C
      RATIO = CAPRR3*CAPRB2*CAPRB1/(RB2*RB3*RB4)
      TEST1 = TAN(THETA1)
      TEST2 = TAN(THETA2)
      TEST3 = TAN(THETA3)
      D1 = (CAPRB1 + RB2)*TAN(THETA1)
      D2 = (CAPRP2 + RB3)*TAN(THETA2)
      D3 = (CAPRR3 + RB4)*TAN(THETA3)
      MTST = 0.
C
C  DETERMINATION OF INITIAL AND FINAL VALUES OF ALPHAS
C
      CALL ALPHA(CAPRB1,RB2,THETA1,CAPRO1,RO2,AL1IN,AL1FIN)
      CALL ALPHA(CAPRB2,RB3,THETA2,CAPRO2,RO3,AL2IN,AL2FIN)
      CALL ALPHA(CAPRB3,RB4,THETA3,CAPRO3,RO4,AL3IN,AL3FIN)
C
      DELAL3 = (AL3FIN - AL3IN)/K
      DELAL2 = DELAL3*RB3/CAPRB2
      DELAL1 = DELAL2*RB2/CAPRB1
C
C  INITIALIZATION OF ALPHAS
C
      ALPHA1 = AL1IN
      ALPHA2 = AL2IN
      ALPHA3 = AL3IN
C
C  CENTRIFUGAL FORCES
C
```

```
      T1 = M1*S1*OM2
      T2 = M2*R2*OM2
      T3 = M3*R3*OM2
      T4 = M4*R4*OM2
C
      DENOM = 1. + MU*MU
C
C  UPDATE VALUES OF ALPHAS
C
      IF(ALPHA2 .GT. AL2FIN)ALPHA2 = AL2IN
      IF(ALPHA3 .GT. AL3FIN)ALPHA3 = AL3IN
C
C  TEST TO DETERMINE IF CONTACT POINT IS IN APPROACH OR RECESS
C        IF APPROACH, S = 1.
C        IF RECESS, S = -1.
C        AT PITCH POINT, S = 0.
C
      IF(ALPHA1 .LT. TEST1)S1 = 1.
      IF(ALPHA2 .LT. TEST2)S2 = 1.
      IF(ALPHA3 .LT. TEST3)S3 = 1.
      IF(ALPHA1 .GT. TEST1)S1 = -1.
      IF(ALPHA2 .GT. TEST2)S2 = -1.
      IF(ALPHA3 .GT. TEST3)S3 = -1.
      IF(ALPHA1 .EQ. TEST1)S1 = 0.
      IF(ALPHA2 .EQ. TEST2)S2 = 0.
      IF(ALPHA3 .EQ. TEST3)S3 = 0.
C
C  DETERMINATION OF INPUT FOR MOMENT EXPRESSIONS
C
      A1 = ALPHA1*CAPRB1
      A2 = ALPHA2*CAPRB2
      A3 = ALPHA3*CAPRB3
      AA1 = ABS((SIN(GAMMA4) - MU*COS(GAMMA4))/DENOM)
      AA2 = ABS(((1.+S3*MU*MU)*COS(BETA3+THETA3) + MU*(1.-S3)
     1*SIN(BETA3+THETA3))/DENOM)
      AA3 = ABS((COS(GAMMA4) + MU*SIN(GAMMA4))/DENOM)
      AA4 = ABS(((1.+S3*MU*MU)*SIN(BETA3+THETA3) - MU*(1.-S3)
     1*COS(BETA3+THETA3))/DENOM)
      AA5 = ABS((1.+MU*MU*S3)*COS(BETA2-THETA2) + MU*(S2-1.)
     1*SIN(BETA2-THETA2))/DENOM)
      AA6 = ABS((SIN(GAMMA3) + MU*COS(GAMMA3))/DENOM)
      AA7 = ABS(((1.-MU*MU*S3)*COS(BETA3+THETA3) - MU*(1.+S3)
     1*SIN(BETA3+THETA3))/DENOM)
      AA8 = ABS((1.+MU*MU*S2)*SIN(BETA2-THETA2) + MU*(1.-S2)
     1*COS(BETA2-THETA2))/DENOM)
      AA9 = ABS((MU*SIN(GAMMA3) - COS(GAMMA3))/DENOM)
      AA10 = ABS(((1.-MU*MU*S3)*SIN(BETA3+THETA3) + MU*(1.+S3)
     1*COS(BETA3+THETA3))/DENOM)
      AA11 = ABS((MU*(1.+S1)*SIN(BETA1+THETA1) + (1.+S1*MU*MU)
     1*COS(BETA1+THETA1))/DENOM)
      AA12 = ABS((SIN(GAMMA2) - MU*COS(GAMMA2))/DENOM)
      AA13 = ABS((MU*(1.+S2)*SIN(BETA2-THETA2) + (1.-MU*MU*S2)
     1*COS(BETA2-THETA2))/DENOM)
```

```
      AA14 = ABS((MU*(1.-S1)*COS(BETA2+THETA1) - (1.+MU*MU*S1)
     1*SIN(BETA1+THETA1))/DENOM)
      AA15 = ABS((MU*SIN(GAMMA2) + COS(GAMMA2))/DENOM)
      AA16 = ABS(((1.-MU*MU*S2)*SIN(BETA2-THETA2) - MU*(1.+S2)
     1*COS(BETA2-THETA2))/DENOM)
      AA17 = ABS(((1.-MU*MU*S1)*SIN(BETA1+THETA1) + MU*(1.+S1)
     1*COS(BETA1+THETA1))/DENOM)
      AA18 = ABS(1./DENOM)
      AA19 = ABS((MU*(1.+S1)*SIN(BETA1+THETA1) - (1.-MU*MU*S1)
     1*COS(BETA1+THETA1))/DENOM)
      AA20 = ABS(MU/DENOM)
      DD1 = RR4 - MU*(S3*(D3-A3) + RHO4*(AA2+AA4))
      DD2 = -MU*(RHO3*(AA5+AA8) + S2*(D2-A2)) + RR3
      DD3 = RR2 - MU*(S1*(D1-A1) + RHO2*(AA11+AA14))
      DD4 = CAPR81 - MU*(S1*A1 - RHO1*(AA17+AA19))
      CC1 = MU*RHO4*(AA1+AA3)
      CC2 = CAPR83 - MU*(S3*A3 - RHO3*(AA7+AA10))
      CC3 = MU*RHO3*(AA6+AA9)
      CC4 = CAPR82 - MU*(S2*A2 - RHO2*(AA13+AA16))
      CC5 = MU*RHO2*(AA12+AA15)
      CC6 = MU*RHO1*(AA18+AA20)
C
C  OUTPUT MOMENT
C
      ALPH1D = ALPHA1*180./PI
      MO4 = DD1*DD2*DD3/(CC2*CC4+DD4*(MIN-T1*CC6) - T2*CC5*DD1*DD2
     1/(CC2*CC4) - T3*CC3*DD1/CC2 - T4*CC1
      POINTEF = RATIO*MO4/MIN
      WRITE(6,15)ALPH1D,S1,S2,S3,POINTEF
   15 FORMAT(6X,*ALPHA1 =*,F4.2,* (DEG)*,3X,*S1 =*,F5.1,3X,*S2 =*,F5.1,
     13X,*S3 =*,F5.1,3X,*POINT EFFICIENCY =*,F7.5)
      MTOT = MTOT + MO4
C
C  ADVANCE GEAR TRAIN TO NEXT POSITION
C
      ALPHA1 = ALPHA1 + DELA1
      IF(ALPHA1 .GT. ALIFIN)GO TO 16
      ALPHA2 = ALPHA2 + DELA2
      ALPHA3 = ALPHA3 + DELA3
      GO TO 14
   16 CYCLEFF = RATIO*DELA1*MTOT/(MIN*(ALIFIN-ALIIN))
      WRITE(6,17)CYCLEFF
   17 FORMAT(//8X,52,*CYCLE EFFICIENCY =*,F5.3)
      IF(ISTOP .NE. 0)GO TO 100
      STOP
      END
```

```
      SUBROUTINE ALPHA   74/74  OPT=1                    FTN 4.6+428     89/06/78  18.41.15       PAGE     1

    1                 SUBROUTINE ALPHA(CAPRB,RB,THETA,CAPRO,RO,ALIN,ALFIN)
                      C
                      C   THIS SUBROUTINE COMPUTES THE INITIAL AND FINAL VALUES OF ALPHAS
                      C
    5                 ALIN = ((CAPRB + RB)*TAN(THETA) - SQRT(RO*RO - RB*RB))/CAPRB
                      ALFIN = SQRT(CAPRO*CAPRO - CAPRB*CAPRB)/CAPRB
                      RETURN
                      END
```

```
MIN = -.1664E+00   MU = .200   RPM = 1000.

CAPRP1 = .47727   CAPRP2 = -.28760   CAPRP3 = -.17532

RP2 = -.00091   RP3 = .04823   RP4 = .05444

THETA1 = 20.000   THETA2 = 20.000   THETA3 = 20.000

R1 = .75000   R2 = .75000   R3 = .75000   R4 = .75000

M1 = -.5187E-04   M2 = -.1743E-04   M3 = -.6978E-05   M4 = -.7765E-06

RHO1 = .85000   RHO2 = .83000   RHO3 = .82500   RHO4 = .82000

CAPRB1 = .44869   CAPRB2 = -.19517   CAPRB3 = -.10475   RB2 = -.00543   RB3 = .06506   RB4 = .05492

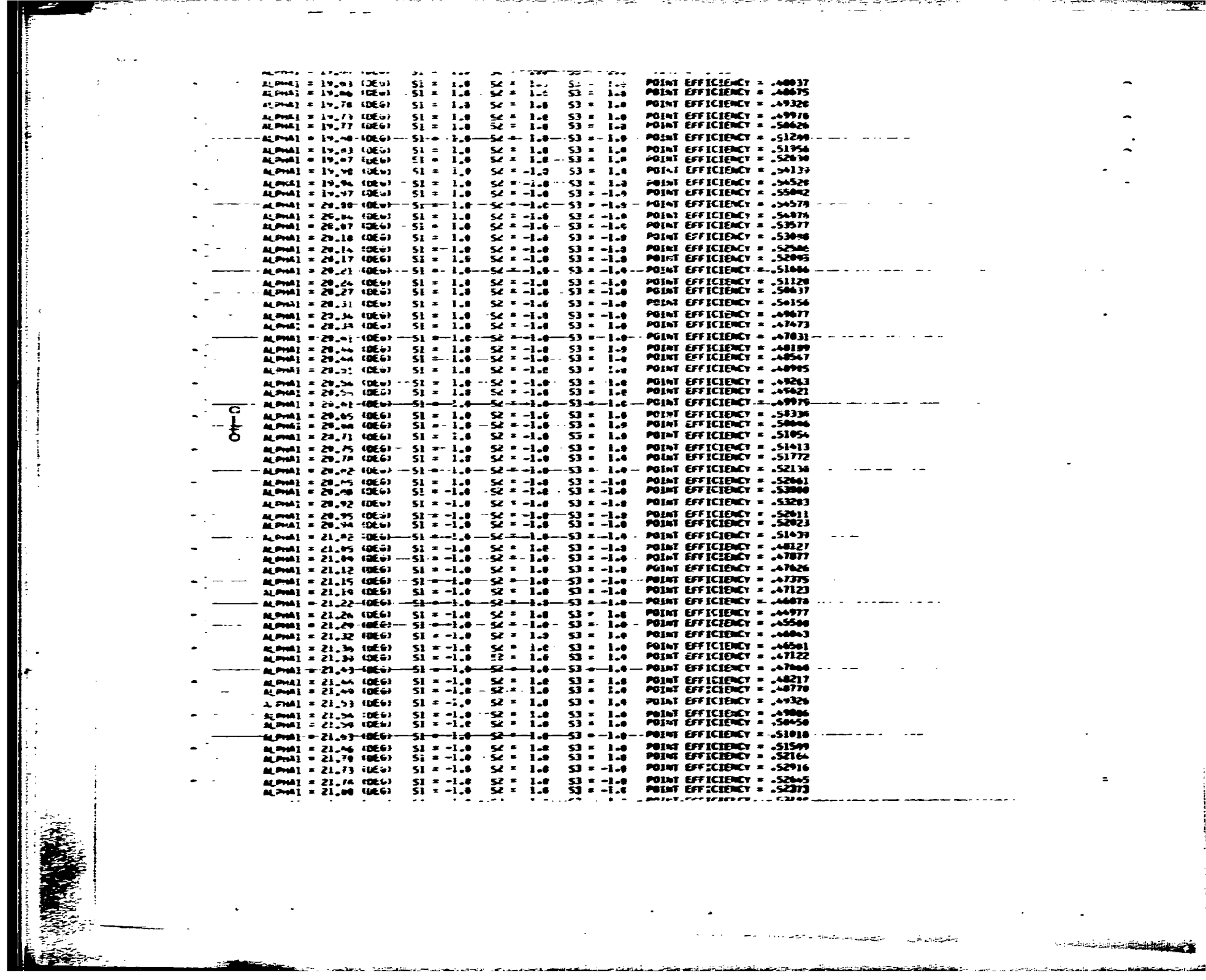
CAPRO1 = -.4470   CAPRO2 = -.21579   CAPRO3 = -.10216   RO2 = -.11000   RO3 = .04689   RO4 = .06828

MB = .150E-04

RANGE DivISOR = 2

ALPHA1 = 15.97 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .37862
ALPHA1 = 16.01 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .37597
ALPHA1 = 16.04 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .38117
ALPHA1 = 16.08 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .38642
ALPHA1 = 16.11 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .39172
ALPHA1 = 16.14 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .39707
ALPHA1 = 16.18 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .40247
ALPHA1 = 16.21 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .40791
ALPHA1 = 16.24 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .41341
ALPHA1 = 16.28 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .41896
ALPHA1 = 16.31 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .42456
ALPHA1 = 16.35 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .43021
ALPHA1 = 16.38 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .43591
ALPHA1 = 16.41 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .44166
ALPHA1 = 16.45 (DEG)   S1 = 1.0   S2 = 1.0   S3 = -1.0   POINT EFFICIENCY = .44895
ALPHA1 = 16.48 (DEG)   S1 = 1.0   S2 = 1.0   S3 = -1.0   POINT EFFICIENCY = .44752
ALPHA1 = 16.52 (DEG)   S1 = 1.0   S2 = 1.0   S3 = -1.0   POINT EFFICIENCY = .44409
ALPHA1 = 16.55 (DEG)   S1 = 1.0   S2 = 1.0   S3 = -1.0   POINT EFFICIENCY = .44464
ALPHA1 = 16.58 (DEG)   S1 = 1.0   S2 = 1.0   S3 = -1.0   POINT EFFICIENCY = .44318
ALPHA1 = 16.62 (DEG)   S1 = 1.0   S2 = 1.0   S3 = -1.0   POINT EFFICIENCY = .44177
ALPHA1 = 16.65 (DEG)   S1 = 1.0   S2 = 1.0   S3 = -1.0   POINT EFFICIENCY = .44822
ALPHA1 = 16.68 (DEG)   S1 = 1.0   S2 = 1.0   S3 = -1.0   POINT EFFICIENCY = .43872
ALPHA1 = 16.72 (DEG)   S1 = 1.0   S2 = 1.0   S3 = -1.0   POINT EFFICIENCY = .43721
ALPHA1 = 16.75 (DEG)   S1 = 1.0   S2 = 1.0   S3 = -1.0   POINT EFFICIENCY = .43569
ALPHA1 = 16.79 (DEG)   S1 = 1.0   S2 = 1.0   S3 = -1.0   POINT EFFICIENCY = .43415
ALPHA1 = 16.82 (DEG)   S1 = 1.0   S2 = 1.0   S3 = -1.0   POINT EFFICIENCY = .43260
ALPHA1 = 16.85 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .41505
ALPHA1 = 16.89 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .42153
ALPHA1 = 16.92 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .42726
ALPHA1 = 16.96 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .43304
ALPHA1 = 16.99 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .43888
ALPHA1 = 17.02 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .44476
ALPHA1 = 17.06 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .45071
ALPHA1 = 17.09 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .45670
ALPHA1 = 17.12 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .46275
ALPHA1 = 17.16 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .46886
ALPHA1 = 17.19 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .47502
ALPHA1 = 17.23 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .48123
ALPHA1 = 17.26 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .48750
ALPHA1 = 17.29 (DEG)   S1 = 1.0   S2 = 1.0   S3 = 1.0   POINT EFFICIENCY = .49383
ALPHA1 = 17.33 (DEG)   S1 = 1.0   S2 = 1.0   S3 = -1.0   POINT EFFICIENCY = .50196
```

ALPHA1 = 17.48 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .50335
ALPHA1 = 17.43 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .49873
ALPHA1 = 17.46 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .49413
ALPHA1 = 17.58 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .48956
ALPHA1 = 17.53 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .48501
ALPHA1 = 17.57 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .48048
ALPHA1 = 17.68 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .47598
ALPHA1 = 17.63 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .47150
ALPHA1 = 17.67 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .46704
ALPHA1 = 17.78 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .46261
ALPHA1 = 17.73 (DEG)	S1 = 1.0	S2 = -1.0	S3 = 1.0	POINT EFFICIENCY = .44284
ALPHA1 = 17.77 (DEG)	S1 = 1.0	S2 = -1.0	S3 = 1.0	POINT EFFICIENCY = .44545
ALPHA1 = 17.88 (DEG)	S1 = 1.0	S2 = -1.0	S3 = 1.0	POINT EFFICIENCY = .44981
ALPHA1 = 17.84 (DEG)	S1 = 1.0	S2 = -1.0	S3 = 1.0	POINT EFFICIENCY = .45218
ALPHA1 = 17.87 (DEG)	S1 = 1.0	S2 = -1.0	S3 = 1.0	POINT EFFICIENCY = .45555
ALPHA1 = 17.98 (DEG)	S1 = 1.0	S2 = -1.0	S3 = 1.0	POINT EFFICIENCY = .45891
ALPHA1 = 17.94 (DEG)	S1 = 1.0	S2 = -1.0	S3 = 1.0	POINT EFFICIENCY = .46228
ALPHA1 = 17.97 (DEG)	S1 = 1.0	S2 = -1.0	S3 = 1.0	POINT EFFICIENCY = .46565
ALPHA1 = 18.01 (DEG)	S1 = 1.0	S2 = -1.0	S3 = 1.0	POINT EFFICIENCY = .46903
ALPHA1 = 18.04 (DEG)	S1 = 1.0	S2 = -1.0	S3 = 1.0	POINT EFFICIENCY = .47245
ALPHA1 = 18.07 (DEG)	S1 = 1.0	S2 = -1.0	S3 = 1.0	POINT EFFICIENCY = .47577
ALPHA1 = 18.11 (DEG)	S1 = 1.0	S2 = -1.0	S3 = 1.0	POINT EFFICIENCY = .47915
ALPHA1 = 18.14 (DEG)	S1 = 1.0	S2 = -1.0	S3 = 1.0	POINT EFFICIENCY = .48252
ALPHA1 = 18.17 (DEG)	S1 = 1.0	S2 = -1.0	S3 = 1.0	POINT EFFICIENCY = .48598
ALPHA1 = 18.21 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .49098
ALPHA1 = 18.24 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .48635
ALPHA1 = 14.28 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .49184
ALPHA1 = 18.31 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .47735
ALPHA1 = 18.34 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .47299
ALPHA1 = 18.38 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .48845
ALPHA1 = 18.41 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .46404
ALPHA1 = 18.45 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .45964
ALPHA1 = 18.48 (DEG)	S1 = 1.0	S2 = -1.0	S3 = -1.0	POINT EFFICIENCY = .45527
ALPHA1 = 18.51 (DEG)	S1 = 1.0	S2 = 1.0	S3 = -1.0	POINT EFFICIENCY = .42652
ALPHA1 = 18.55 (DEG)	S1 = 1.0	S2 = 1.0	S3 = -1.0	POINT EFFICIENCY = .42585
ALPHA1 = 18.58 (DEG)	S1 = 1.0	S2 = 1.0	S3 = -1.0	POINT EFFICIENCY = .42356
ALPHA1 = 18.61 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .40722
ALPHA1 = 18.65 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .41292
ALPHA1 = 18.68 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .41847
ALPHA1 = 18.72 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .42419
ALPHA1 = 18.75 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .42994
ALPHA1 = 18.78 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .43575
ALPHA1 = 18.82 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .44162
ALPHA1 = 18.85 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .44754
ALPHA1 = 18.89 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .45351
ALPHA1 = 18.92 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .45954
ALPHA1 = 18.95 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .46562
ALPHA1 = 18.99 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .47176
ALPHA1 = 19.02 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .47796
ALPHA1 = 19.05 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .48421
ALPHA1 = 19.09 (DEG)	S1 = 1.0	S2 = 1.0	S3 = -1.0	POINT EFFICIENCY = .49213
ALPHA1 = 19.12 (DEG)	S1 = 1.0	S2 = 1.0	S3 = -1.0	POINT EFFICIENCY = .49852
ALPHA1 = 19.14 (DEG)	S1 = 1.0	S2 = 1.0	S3 = -1.0	POINT EFFICIENCY = .48890
ALPHA1 = 19.19 (DEG)	S1 = 1.0	S2 = 1.0	S3 = -1.0	POINT EFFICIENCY = .48727
ALPHA1 = 19.22 (DEG)	S1 = 1.0	S2 = 1.0	S3 = -1.0	POINT EFFICIENCY = .48563
ALPHA1 = 19.26 (DEG)	S1 = 1.0	S2 = 1.0	S3 = -1.0	POINT EFFICIENCY = .48397
ALPHA1 = 19.29 (DEG)	S1 = 1.0	S2 = 1.0	S3 = -1.0	POINT EFFICIENCY = .48230
ALPHA1 = 19.33 (DEG)	S1 = 1.0	S2 = 1.0	S3 = -1.0	POINT EFFICIENCY = .48061
ALPHA1 = 19.36 (DEG)	S1 = 1.0	S2 = 1.0	S3 = -1.0	POINT EFFICIENCY = .47891
ALPHA1 = 19.40 (DEG)	S1 = 1.0	S2 = 1.0	S3 = -1.0	POINT EFFICIENCY = .47720
ALPHA1 = 19.43 (DEG)	S1 = 1.0	S2 = 1.0	S3 = -1.0	POINT EFFICIENCY = .47547
ALPHA1 = 19.46 (DEG)	S1 = 1.0	S2 = 1.0	S3 = -1.0	POINT EFFICIENCY = .47373
ALPHA1 = 19.50 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .45538
ALPHA1 = 19.53 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .46154
ALPHA1 = 19.56 (DEG)	S1 = 1.0	S2 = 1.0	S3 = 1.0	POINT EFFICIENCY = .46776


ALPHA1 = 19.93 (DEG) S1 = 1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .48037
ALPHA1 = 19.96 (DEG) S1 = 1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .48675
ALPHA1 = 19.98 (DEG) S1 = 1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .49320
ALPHA1 = 20.01 (DEG) S1 = 1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .49970
ALPHA1 = 20.03 (DEG) S1 = 1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .50626
ALPHA1 = 20.06 (DEG) S1 = 1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .51290
ALPHA1 = 20.08 (DEG) S1 = 1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .51956
ALPHA1 = 20.11 (DEG) S1 = 1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .52630
ALPHA1 = 20.13 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .54137
ALPHA1 = 20.16 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .54528
ALPHA1 = 20.18 (DEG) S1 = 1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .55042
ALPHA1 = 20.21 (DEG) S1 = 1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .54579
ALPHA1 = 20.24 (DEG) S1 = 1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .54076
ALPHA1 = 20.27 (DEG) S1 = 1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .53577
ALPHA1 = 20.29 (DEG) S1 = 1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .53090
ALPHA1 = 20.32 (DEG) S1 = 1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .52586
ALPHA1 = 20.34 (DEG) S1 = 1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .52095
ALPHA1 = 20.37 (DEG) S1 = 1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .51606
ALPHA1 = 20.40 (DEG) S1 = 1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .51120
ALPHA1 = 20.42 (DEG) S1 = 1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .50637
ALPHA1 = 20.45 (DEG) S1 = 1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .50156
ALPHA1 = 20.47 (DEG) S1 = 1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .49677
ALPHA1 = 20.50 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .47473
ALPHA1 = 20.53 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .47031
ALPHA1 = 20.55 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .48189
ALPHA1 = 20.58 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .48567
ALPHA1 = 20.60 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .48985
ALPHA1 = 20.63 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .49263
ALPHA1 = 20.66 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .49621
ALPHA1 = 20.68 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .49979
ALPHA1 = 20.71 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .50336
ALPHA1 = 20.73 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .50046
ALPHA1 = 20.76 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .51054
ALPHA1 = 20.79 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .51413
ALPHA1 = 20.81 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .51772
ALPHA1 = 20.84 (DEG) S1 = 1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .52130
ALPHA1 = 20.86 (DEG) S1 = 1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .52661
ALPHA1 = 20.89 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .53080
ALPHA1 = 20.92 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .53283
ALPHA1 = 20.95 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .52611
ALPHA1 = 20.97 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .52023
ALPHA1 = 21.00 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .51439
ALPHA1 = 21.05 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .48127
ALPHA1 = 21.08 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .47877
ALPHA1 = 21.12 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .47626
ALPHA1 = 21.15 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .47375
ALPHA1 = 21.18 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .47123
ALPHA1 = 21.22 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .46870
ALPHA1 = 21.26 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .44977
ALPHA1 = 21.29 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .45508
ALPHA1 = 21.32 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .46063
ALPHA1 = 21.36 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .46561
ALPHA1 = 21.39 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .47122
ALPHA1 = 21.43 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .47668
ALPHA1 = 21.46 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .48217
ALPHA1 = 21.49 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .48778
ALPHA1 = 21.53 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .49326
ALPHA1 = 21.56 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .49886
ALPHA1 = 21.59 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .50450
ALPHA1 = 21.63 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .51018
ALPHA1 = 21.66 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .51599
ALPHA1 = 21.70 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .52164
ALPHA1 = 21.73 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .52916
ALPHA1 = 21.76 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .52665
ALPHA1 = 21.80 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .52373

ALPHA1 = 21.93 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .51827

ALPHA1 = 21.98 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .51553

ALPHA1 = 22.03 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .51278

ALPHA1 = 22.07 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .51863

ALPHA1 = 22.08 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .50728

ALPHA1 = 22.03 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .50452

ALPHA1 = 22.07 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .50175

ALPHA1 = 22.12 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .49898

ALPHA1 = 22.16 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .47877

ALPHA1 = 22.17 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .48436

ALPHA1 = 22.20 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .48994

ALPHA1 = 22.24 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .49553

ALPHA1 = 22.27 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .50125

ALPHA1 = 22.31 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .50696

ALPHA1 = 22.36 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .51271

ALPHA1 = 22.37 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .51850

ALPHA1 = 22.41 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .52432

ALPHA1 = 22.46 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .53041

ALPHA1 = 22.47 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .54128

ALPHA1 = 22.51 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .54414

ALPHA1 = 22.54 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .54698

ALPHA1 = 22.54 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .54981

ALPHA1 = 22.61 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .55465

ALPHA1 = 22.64 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .54836

ALPHA1 = 22.68 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .54231

ALPHA1 = 22.71 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .53631

ALPHA1 = 22.75 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .53036

ALPHA1 = 22.78 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .52445

ALPHA1 = 22.81 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .51859

ALPHA1 = 22.85 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .51277

ALPHA1 = 22.88 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .50700

ALPHA1 = 22.92 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .50127

ALPHA1 = 22.95 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .49558

ALPHA1 = 22.98 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .48995

ALPHA1 = 23.02 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .44733

ALPHA1 = 23.05 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .46908

ALPHA1 = 23.08 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .47261

ALPHA1 = 23.12 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .47524

ALPHA1 = 23.15 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .47786

ALPHA1 = 23.18 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .48046

ALPHA1 = 23.22 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .48306

ALPHA1 = 23.25 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .48564

ALPHA1 = 23.26 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .46822

ALPHA1 = 23.32 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .49078

ALPHA1 = 23.36 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .49333

ALPHA1 = 23.38 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .49587

ALPHA1 = 23.42 (DEG) S1 = -1.0 S2 = -1.0 S3 = 1.0 POINT EFFICIENCY = .49840

ALPHA1 = 23.44 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .50093

ALPHA1 = 23.48 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .50369

ALPHA1 = 23.52 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .49946

ALPHA1 = 23.56 (DEG) S1 = -1.0 S2 = -1.0 S3 = -1.0 POINT EFFICIENCY = .49340

ALPHA1 = 23.59 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .46199

ALPHA1 = 23.63 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .45962

ALPHA1 = 23.66 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .45724

ALPHA1 = 23.69 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .45486

ALPHA1 = 23.73 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .45247

ALPHA1 = 23.76 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .45008

ALPHA1 = 23.78 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .44768

ALPHA1 = 23.83 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .44527

ALPHA1 = 23.86 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .44286

ALPHA1 = 23.90 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .42993

ALPHA1 = 23.93 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .42993

ALPHA1 = 23.96 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .43497

ALPHA1 = 24.00 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .44003

ALPHA1 = 24.03 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .44514

```
ALPHA1 = 24.07 (DEG)   S1 = -1.0   S2 =  1.0   S3 =  1.0   POINT EFFICIENCY = .45022
ALPHA1 = 24.10 (DEG)   S1 = -1.0   S2 =  1.0   S3 =  1.0   POINT EFFICIENCY = .45544
ALPHA1 = 24.13 (DEG)   S1 = -1.0   S2 =  1.0   S3 =  1.0   POINT EFFICIENCY = .46064
ALPHA1 = 24.17 (DEG)   S1 = -1.0   S2 =  1.0   S3 =  1.0   POINT EFFICIENCY = .46588
ALPHA1 = 24.20 (DEG)   S1 = -1.0   S2 =  1.0   S3 =  1.0   POINT EFFICIENCY = .47115
ALPHA1 = 24.24 (DEG)   S1 = -1.0   S2 =  1.0   S3 =  1.0   POINT EFFICIENCY = .47646
ALPHA1 = 24.27 (DEG)   S1 = -1.0   S2 =  1.0   S3 =  1.0   POINT EFFICIENCY = .48180
ALPHA1 = 24.30 (DEG)   S1 = -1.0   S2 =  1.0   S3 =  1.0   POINT EFFICIENCY = .48718
ALPHA1 = 24.34 (DEG)   S1 = -1.0   S2 =  1.0   S3 =  1.0   POINT EFFICIENCY = .49260
ALPHA1 = 24.37 (DEG)   S1 = -1.0   S2 =  1.0   S3 = -1.0   POINT EFFICIENCY = .49968
ALPHA1 = 24.40 (DEG)   S1 = -1.0   S2 =  1.0   S3 = -1.0   POINT EFFICIENCY = .49709
ALPHA1 = 24.44 (DEG)   S1 = -1.0   S2 =  1.0   S3 = -1.0   POINT EFFICIENCY = .49449
ALPHA1 = 24.47 (DEG)   S1 = -1.0   S2 =  1.0   S3 = -1.0   POINT EFFICIENCY = .49149
ALPHA1 = 24.51 (DEG)   S1 = -1.0   S2 =  1.0   S3 = -1.0   POINT EFFICIENCY = .48928
ALPHA1 = 24.54 (DEG)   S1 = -1.0   S2 =  1.0   S3 = -1.0   POINT EFFICIENCY = .48667

CYCLE EFFICIENCY = .481
```

4. Program INVOL 4: Point and Cycle Efficiencies for Two Pass Involute Step-Up Gear Train in Spin Environment (All Meshes Have Unity Contact Ratio)

The program INVOL 4 is based on section 5 of Appendix A, which derives the moment input-output relationship for a two pass step-up gear train, operating in a spin environment. Here again, all meshes have unity contact ratio. INVOL 4 is very similar to INVOL 3 in its construction. Again, the expressions for the contact geometry and other auxiliary geometric terms may be found in section 6 of Appendix A.

a. Input Parameters (see Program INVOL 4, below)

The following parameters represent the input data for the program. Those which involve gear dimensions only must be obtained from the results of INVOL 1 since the moment expressions are again derived for unity contact ratio only.

MU = μ, coefficient of friction at all pivots and at all tooth contact points

RPM, revolutions per minute of the fuze body

CAPRP1 = R_{p1}

CAPRP2 = R_{p2}

RP2 = r_{p2}

RP3 = r_{p3}

THETA1 = θ_1

THETA2 = θ_2

ISTOP, arbitrary single-digit integer for multiple data sets.
It must be zero for last set of data.

R1 = $\mathcal{R}_1$

R2 = $\mathcal{R}_2$

R3 = $\mathcal{R}_3$

RHO1 = ρ_1

RHO2 = ρ_2

RHO3 = ρ_3

CAPRB1 = R_{b1}

CAPRB2 = R_{b2}

RB2 = r_{b2}

RB3 = r_{b3}

CAPRO1 = R_{o1}

CAPRO2 = R_{o2}

RO2 = r'_{o2}

RO3 = r'_{o3}

M1 = m_1, mass of input gear 1

M2 = m_2, mass of gear and pinion 2

M3 = m_3, mass of pinion 3

MD = md^2, "mass-distance" product contained in the following expression for the input moment M_{in}

K = K_2, the range divisor which is associated with gear 2, the driving gear of the last mesh for this case (see eq. (A-207))

b. Computations (see COMMENT cards in program)

I. Computation of MIN, Gammas and Betas

To start with, the program computes the input moment

$$MIN = M_{in} = md\omega^2 \qquad (C\text{-}15)$$

The program computes the angles γ_2, γ_3 and β_1, β_2 according to the expressions given in section 6b of Appendix A.

II. Determination of the Gear Train Constants

The determination of the gear train constants consists of the following:

RATIO = K_{ratio}, (see eq. (2)). Since the angular velocity is constant, this parameter may be expressed in terms of the applicable base radii, i.e.,

$$\frac{R_{b1} \times R_{b2}}{r_{b2} \times r_{b3}}$$

TEST1 and TEST2 represent the tangent functions of the mesh pressure angles, which are used in conjunction with the values of the signum functions s.

D1, and D2 are given by eqs. (A-204) and (A-217), respectively, and represent the distances between the points of tangency to the base circles along the lines-of-action of the two meshes.

MTOT = 0 represents the initialization of the sum of the output moments. This is used for the determination of the cycle efficiency.

III. Determination of Initial and Final Values of ALPHAS, Initialization of ALPHAS and Centrifugal Forces

The determination of the initial and final angles of rotation is accomplished with the help of subroutine ALPHA, at the end of the program, which makes use of eqs. (A-205), (A-206) as well as (A-218) and (A-219). Thus, the initial values of the individual angles of rotation, ALPHA1 and ALPHA2, are represented by AL1IN and AL2IN, respectively, while the final ones are given by AL1FIN and AL2FIN.

The angular increments of gears 2 and 1, i.e., DELAL2 and DELAL1, are det mined with the help of eqs. (A-207) and (A-208), respectively.

The centrifugal forces, which act on the pivots of the various gear and/or pinion assemblies, are obtained by way of eqs. (A-131), (A-154) and (A-178).

IV. Point and Cycle Efficiencies (See "output moment" in program)

Both point and cycle efficiencies are based on eq. (A-193) for the output moment MO3 = M_{o3}.

The point efficiency is computed directly in the manner of eq. (3), i.e.,

$$\eta_p = K_{ratio} \frac{M_{o3}}{M_{in}} = \text{POINTEF} \qquad \text{(C-16)}$$

The cycle efficiency is treated in the manner of eq. (C-13), i.e.,

$$\eta_o = \frac{K_{ratio}\,\Delta\alpha_1\,\Sigma M_{o3}}{M_{in}\,(\alpha_{1FIN} - \alpha_{1IN})} = \text{CYCLEFF} \qquad \text{(C-17)}$$

The pr.gram gives the summation as

$$MTOT = \Sigma M_{o3} \qquad \text{(C-18)}$$

V. Gear Train Motion Model

The simulation of the gear train motion, which is necessary for the computation of both POINTEF and CYCLEFF, is found in a loop which begins with statement label no. 14 (card no. 98) and ends with card no. 161.

As discussed earlier, the motions of the individual driving gears are initialized at their respective angles, AL1IN and AL2IN. (This starting of the total train is arbitrary and is done only for convenience. There is an infinite number of other starting combinations, each of which produces a different starting point efficiency.) The position of each mesh is subsequently incremented by the appropriate DELAL1 or DELAL2. When the angle ALPHA1 reaches the magnitude AL1FIN, CYCLEFF is determined, and the computation is ended. Since mesh 2 goes through a number of cycles while mesh 1 goes through one cycle, mesh 2 has to be reset to its starting position, AL2IN, once the angle AL2FIN has been reached. This is accomplished by the conditional statement on card no. 98.

The values of the signum functions s_1 and s_2 are determined continuously according to eqs. (A-216) and (A-222).

The instantaneous positions of the contact, A1 = a_1 and A2 = a_2, are determined for each of the meshes by appropriate

adaptations of eq. (A-203). (See also eqn. (A-214) and (A-220).)

The determination of the instantaneous output moment, MO3 = M_{o3}, requires the continuous computation of the variable quantities A_1 to A_{14}, C_1 to C_4 and D_1 to D_3, which are given originally in conjunction with the various equilibrium conditions in section 5 of Appendix A. The program uses the following nomenclature for these variables:

AA1 to AA14

CC1 to CC4

DD1 to DD3

c. Output (see Program INVOL 4, below)

The output of the program is again best explained with the help of the sample computation shown at the end of the program. This example uses the gear data of the fourth and fifth sample computations of program INVOL 1. The output lists the following:

I. Input Parameters

Mesh No. 1

CAPRP1 = R_{p1} = .63636 in. (1.616 cm)

CAPRB1 = R_{b1} = .59799 in. (1.519 cm)

CAPRO1 = R_{o1} = .64700 in. (1.643 cm)

RP2 = r_{p2} = .09091 in. (0.231 cm)

RB2 = r_{b2} = .08543 in. (0.217 cm)

RO2 = r'_{o2} = .1097 in. (0.279 cm) (This is a ROFIN as given by INVOL 1.)

Also

THETA1 = θ_1 = 20°

Mesh No. 2

CAPRP2 = R_{p2} = .43077 in. (1.094 cm)

CAPRB2 = R_{b2} = .40479 in. (1.028 cm)

CAPRO2 = R_{o2} = .43977 in. (1.117 cm)

RP3 = r_{p3} = .06154 in. (0.156 cm)

RB3 = r_{b3} = .05783 in. (0.147 cm)

RO3 = r'_{o3} = .07426 in. (0.189 cm)

Also

THETA2 = θ_2 = 20°

In addition,

MU = μ = .2

RPM = 1000

M1 = m_1 = .51079 x 10^{-4} lb-sec^2/in. (8.943 g)

M2 = m_2 = .17413 x 10^{-4} lb-sec^2/in. (3.049 g)

M3 = m_3 = .69788 x 10^{-5} lb-sec^2/in. (1.222 g)

R1 = $\mathcal{R}_1$ = .75 in. (1.905 cm)

R2 = $\mathcal{R}_2$ = .75 in. (1.905 cm)

R3 = $\mathcal{R}_3$ = .75 in. (1.905 cm)

RHO1 = ρ_1 = .060 in. (0.152 cm)

RHO2 = ρ_2 = .030 in. (0.076 cm)

RHO3 = ρ_3 = .025 in. (0.064 cm)

MD = md^2 = .15 x 10^{-4} lb-sec^2 in. (16.944 g-cm^2)

K = 25

II. Computed Values

The point efficiency is given as a function of the angle α_1, together with the signum parameters s_1 and s_2 (given for checking purposes). The cycle efficiency is shown at the end of the output. In addition, the input moment, MIN, is printed out.

Program INVOL 4

```
PROGRAM INVOL 4      74/74   OPT=1                    FTN 4.6+420        09/06/79  17.45.42       PAGE     1

      1              PROGRAM INVOL 4(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
                     C
                     C  POINT AND CYCLE EFFICIENCIES FOR TWO PASS INVOLUTE STEP-UP GEAR TRAIN
                     C  IN SPIN ENVIRONMENT (ALL WFSHFS HAVE UNITY CONTACT RATIO)
      5              C
                        REAL MIN,MU,M1,M2,M3,M33,MTOT,MD
                     C
                     C   READ AND WRITE INPUT DATA
                     C
     10              100 READ(5,1)MU,RPM,CAPRP1,CAPRP2,RP2,RP3,THETA1,THETA2,ISTOP
                        READ(5,2)R1,R2,R3
                        READ(5,3)RHO1,RHO2,RHO3
                        READ(5,4)CAPRB1,CAPRB2,RB2,RB3
                        READ(5,5)CAPRO1,CAPRO2,RO2,RO3
     15                 READ(5,6)M1,M2,M3
                        READ(5,7)MD,K
                      1 FORMAT(F10.3,F10.0/4F10.5/2F10.4/I1)
                      2 FORMAT(3F10.5)
                      3 FORMAT(3F10.5)
     20               4 FORMAT(4F10.5)
                      5 FORMAT(4F10.5)
                      6 FORMAT(3E10.4)
                      7 FORMAT(F10.4/I3)
                        PI = 3.14159
     25                 OMEGA = RPM*2.*PI/60.
                        OM2 = OMEGA*OMEGA
                     C
                     C  COMPUTATION OF MIN, GAMMAS AND BETAS
                     C
     30                 MIN = MD*OM2
                        GAMMA2 = ACOS((R1*R1 + R2*R2 - (CAPRP1+RP2)*(CAPRP1+RP2))/
                       1(2.*R1*R2))
                        GAMA3P = ACOS((R2*R2 + P3*P3 - (CAPRP2+RP3)*(CAPRP2-RP3))/
                       1(2.*R2*R3))
     35                 GAMMA3 = GAMMA2 + GAMA3P
                        DELTA2 = ACOS(((CAPRP1+RP2)*(CAPRP1+RP2) + R1*R1 - R2*R2)/
                       1(2.*R1*(CAPRP1 + RP2)))
                        DELTA3 = ACOS(((CAPRP2+RP3)*(CAPRP2+RP3) + R2*R2 - P3*R3)/
                       1(2.*P2*(CAPRP2 + RP3)))
     40                 BETA1 = PI - DELTA2
                        BETA2 = GAMMA2 + PI - DELTA3
                        WRITE(6,8)MIN,MU,RPM,CAPRP1,CAPRP2,RP2,RP3,THETA1,THETA2
                        WRITE(6,9)R1,R2,R3,M1,M2,M3
                        WRITE(6,10)RHO1,RHO2,RHO3
     45                 WRITE(6,11)CAPRB1,CAPRB2,RB2,RB3
                        WRITE(6,12)CAPRO1,CAPRO2,RO2,RO3
                        WRITE(6,13)MD,K
                      8 FORMAT(*1*,5X,*MIN =*,F12.5,3X,*MU =*,F6.3,3X,*RPM =*,F6.0//6X,
                       1*CAPRP1 =*,F8.5,3X,*CAPRP2 =*,F8.5//6X,*RP2 =*,F8.5,3X,*RP3 =*,
                       2F8.5//6X,*THETA1 =*,F8.3,3X,*THETA2 =*,F8.3/)
     50                 9 FORMAT(6X,*R1 =*,F7.5,3X,*R2 =*,F7.5,3X,*R3 =*,F7.5//6X,*M1 =*,
                       1E12.5,3X,*M2 =*,E12.5,3X,*M3 =*,E12.5/)
                       10 FORMAT(6X,*RHO1 =*,F7.5,3X,*RHO2 =*,F7.5,3X,*RHO3 =*,F7.5/)
```

```
                  11 FORMAT(6X,*CAPRB1 =*,F7.5,3X,*CAPRB2 =*,F7.5,3X,*RB2 =*,F7.5,3X,
                    1*RB3 =*,F7.5/)
                  12 FORMAT(6X,*CAPRO1 =*,F7.5,3X,*CAPRO2 =*,F7.5,3X,*RO2 =*,F7.5,3X,
                    1* RO3 =*,F7.5/)
                  13 FORMAT(6X,*MD =*,E10.3//6X,*RANGE DIVISOR =*,I4//)
                C
                C  CONVERSION TO RADIANS
                C
                     Z = PI/180.
                     THETA1 = THETA1*Z
                     THETA2 = THETA2*Z
                C
                C  DETERMINATION OF GEAR TRAIN CONSTANTS
                C
                     RATIO = (CAPRB2*CAPRB1)/(RB2*RB3)
                     TEST1 = TAN(THETA1)
                     TEST2 = TAN(THETA2)
                     D1 = (CAPRB1 + RB2)*TAN(THETA1)
                     D2 = (CAPRB2 + RB3)*TAN(THETA2)
                     DTOT = 0.
                C
                C  DETERMINATION OF INITIAL AND FINAL VALUES OF ALPHAS
                C
                     CALL ALPHA(CAPRB1,RB2,THETA1,CAPRO1,RO2,AL1IN,AL1FIN)
                     CALL ALPHA(CAPRB2,RB3,THETA2,CAPRO2,RO3,AL2IN,AL2FIN)
                C
                     DELAL2 = (AL2FIN - AL2IN)/K
                     DELAL1 = DELAL2*RB2/CAPRB1
                C
                C  INITIALIZATION OF ALPHAS
                C
                     ALPHA1 = AL1IN
                     ALPHA2 = AL2IN
                C
                C  CENTRIFUGAL FORCES
                C
                     T1 = M1*R1*OM2
                     T2 = M2*R2*OM2
                     T3 = M3*R3*OM2
                C
                     DENOM = 1. + MU*MU
                C
                C  UPDATE VALUES OF ALPHAS
                C
                  14 IF(ALPHA2 .GT. AL2FIN)ALPHA2 = AL2IN
                C
                C  TEST TO DETERMINE IF CONTACT POINT IS IN APPROACH OR RECESS
                C       IF APPROACH, S = 1.
                C       IF RECESS, S = -1.
                C       AT PITCH POINT, S = 0.
                C
                     IF(ALPHA1 .LT. TEST1)S1 = 1.
                     IF(ALPHA2 .LT. TEST2)S2 = 1.
```

```
      IF(ALPHA1 .GT. TEST1)S1 = -1.
      IF(ALPHA2 .GT. TEST2)S2 = -1.
      IF(ALPHA1 .EQ. TEST1)S1 = 0.
  110 IF(ALPHA2 .EQ. TEST2)S2 = 0.
C
C  DETERMINATION OF INPUT FOR MOMENT EXPRESSIONS
C
      A1 = ALPHA1*CAPRB1
  115 A2 = ALPHA2*CAPRB2
      AA1 = ABS((1.+MU*MU*S2)*COS(BETA2-THETA2) + MU*(S2-1.)*
     1SIN(BETA2-THETA2))/DENOM)
      AA2 = ABS((SIN(GAMMA3) + MU*COS(GAMMA3))/DENOM)
      AA3 = ABS(((1.+MU*MU*S2)*SIN(BETA2-THETA2) + MU*(1.-S2)*COS(BETA2
  120 1-THETA2))/DENOM)
      AA4 = ABS((MU*SIN(GAMMA3) - COS(GAMMA3))/DENOM)
      AA5 = ABS((MU*(1.-S1)*SIN(BETA1+THETA1) + (1.+S1*MU*MU)*
     1COS(BETA1+THETA1))/DENOM)
      AA6 = ABS((SIN(GAMMA2) - MU*COS GAMMA2))/DENOM)
  125 AA7 = ABS((MU*(1.+S2)*SIN(BETA2-THETA2) + (1.-MU*MU*S2)*
     1COS(BETA2-THETA2))/DENOM)
      AA8 = ABS((MU*(1.-S1)*COS(BETA1+THETA1) - (1.+MU*MU*S1)*
     1SIN(BETA1+THETA1))/DENOM)
      AA9 = ABS((MU*SIN(GAMMA2) + COS(GAMMA2))/DENOM)
  130 AA10 = ABS(((1.-MU*MU*S2)*SIN(BETA2-THETA2) - MU*(1.+S2)*
     1COS(BETA2-THETA2))/DENOM)
      AA11 = ABS(((1.-MU*MU*S1)*SIN(BETA1+THETA1) + MU*(1.+S1)*
     1COS(BETA1+THETA1))/DENOM)
      AA12 = 1./DENOM
  135 AA13 = ABS((MU*(1.+S1)*SIN(BETA1+THETA1) - (1.-MU*MU*S1)*
     1COS(BETA1+THETA1))/DENOM)
      AA14 = MU/DENOM
      CC1 = MU*RHO3*(AA2+AA4)
      CC2 = CAPRB2 - MU*(S2*A2-RHO2*(AA7+AA10))
  140 CC3 = MU*RHO2*(AA6+AA9)
      CC4 = MU*RHO1*(AA12+AA14)
      DD1 = RB3 - MU*(S2*(02-A2)+RHO3*(AA1+AA3))
      DD2 = RB2 - MU*(S1*(01-A1)+RHO2*(AA5+AA8))
      DD3 = CAPRB1 - MU*(S1*A1-RHO1*(AA11+AA13))
  145 ALPHID = ALPHA1/Z
C
C  OUTPUT MOMENT
C
      MO3 = (DD1*DD2)/(CC2*DD3)*(MIN-T1*CC4) - T2*CC3*DD1/CC2 - T3*CC1
  150 POINTEF = RATIO*MO3/MIN
      WRITE(6,15)ALPHID,S1,S2,POINTEF
   15 FORMAT(6X,*ALPHA1 =*,F6.2,3X,*S1 =*,F5.1,3X,*S2 =*,F5.1,3X,
     1*POINT EFFICIENCY =*,F7.5)
      MTOT = MTOT + MO3
  155 C
C  ADVANCE GEAR TRAIN TO NEXT POSITION
C
      ALPHA1 = ALPHA1 + DELAL1
      IF(ALPHA1 .GT. AL1FIN)GO TO 16
```

```
      160          ALPHA2 = ALPHA2 + DELAL1*CAPRB1/RB2
                   GO TO 14
                16 CYCLEFF = (RATIO*DELAL1*NTOT)/(MIN*(ALIFIN-ALIIN))
                   WRITE(6,17)CYCLEFF
                17 FORMAT(*0*,6X,*CYCLE EFFICIENCY =*,F5.3)
      165          IF(J .NE. 0)GO TO 100
                   STOP
                   END
```

```
      SUBROUTINE ALPHA(CAPRB,RB,THETA,CAPRO,RO,ALIN,ALFIN)
C
C     THIS SUBROUTINE COMPUTES THE INITIAL AND FINAL VALUES OF ALPHAS
C
      ALIN = ((CAPRB + RB)*TAN(THETA) - SQRT(RO**2 - RB**2))/CAPRB
      ALFIN = SQRT(CAPRO**2 - CAPRB**2)/CAPRB
      RETURN
      END
```

MIN = .1544OE+00 MU = .200 RPM = 1000.

CAPRP1 = .63630 CAPRP2 = .43077

RP2 = .99001 RP3 = .96156

THETA1 = 20.030 THETA2 = 20.000

R1 = .75000 R2 = .75000 R3 = .75000

M1 = .51879E-04 M2 = .17613E-04 M3 = .69768E-05

RHO1 = .86886 RHO2 = .83988 RHO3 = .82500

CAPR81 = .59799 CAPRB2 = .48479 RB2 = .80543 RB3 = .85783

CAPHO1 = .56700 CAPHO2 = .43797 RO2 = .10978 RO3 = .87426

MO = .150E-04

RANGE DIVISION = 25

ALPHA1 = 17.24	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.41855
ALPHA1 = 17.28	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.41630
ALPHA1 = 17.31	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.42267
ALPHA1 = 17.35	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.42786
ALPHA1 = 17.39	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.43360
ALPHA1 = 17.42	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.43952
ALPHA1 = 17.46	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.44539
ALPHA1 = 17.50	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.45126
ALPHA1 = 17.53	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.45720
ALPHA1 = 17.57	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.46314
ALPHA1 = 17.61	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.46918
ALPHA1 = 17.64	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.47509
ALPHA1 = 17.68	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.48110
ALPHA1 = 17.72	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.48714
ALPHA1 = 17.75	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.49321
ALPHA1 = 17.79	S1 = 1.0	S2 = -1.0	POINT EFFICIENCY =	.51625
ALPHA1 = 17.83	S1 = 1.0	S2 = -1.0	POINT EFFICIENCY =	.51258
ALPHA1 = 17.86	S1 = 1.0	S2 = -1.0	POINT EFFICIENCY =	.50890
ALPHA1 = 17.90	S1 = 1.0	S2 = -1.0	POINT EFFICIENCY =	.50522
ALPHA1 = 17.94	S1 = 1.0	S2 = -1.0	POINT EFFICIENCY =	.50153
ALPHA1 = 17.97	S1 = 1.0	S2 = -1.0	POINT EFFICIENCY =	.49784
ALPHA1 = 18.01	S1 = 1.0	S2 = -1.0	POINT EFFICIENCY =	.49415
ALPHA1 = 18.05	S1 = 1.0	S2 = -1.0	POINT EFFICIENCY =	.49044
ALPHA1 = 18.08	S1 = 1.0	S2 = -1.0	POINT EFFICIENCY =	.48674
ALPHA1 = 18.12	S1 = 1.0	S2 = -1.0	POINT EFFICIENCY =	.48302
ALPHA1 = 18.16	S1 = 1.0	S2 = -1.0	POINT EFFICIENCY =	.47930
ALPHA1 = 18.19	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.43000
ALPHA1 = 18.23	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.43594
ALPHA1 = 18.27	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.44191
ALPHA1 = 18.31	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.44790
ALPHA1 = 18.34	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.45391
ALPHA1 = 18.38	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.45995
ALPHA1 = 18.42	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.46601
ALPHA1 = 18.45	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.47210
ALPHA1 = 18.49	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.47821
ALPHA1 = 18.53	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.48435
ALPHA1 = 18.56	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.49051
ALPHA1 = 18.60	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.49670
ALPHA1 = 18.64	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.50291
ALPHA1 = 18.67	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.50915
ALPHA1 = 18.71	S1 = 1.0	S2 = 1.0	POINT EFFICIENCY =	.51541

```
ALPHA1 = 18.78   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .53565
ALPHA1 = 18.82   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .53161
ALPHA1 = 18.86   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .52777
ALPHA1 = 18.90   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .52392
ALPHA1 = 18.93   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .52087
ALPHA1 = 18.97   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .51621
ALPHA1 = 19.00   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .51234
ALPHA1 = 19.04   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .50847
ALPHA1 = 19.08   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .50460
ALPHA1 = 19.11   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .50072
ALPHA1 = 19.15   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .49959
ALPHA1 = 19.19   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .49573
ALPHA1 = 19.22   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .46189
ALPHA1 = 19.26   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .46887
ALPHA1 = 19.30   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .47428
ALPHA1 = 19.33   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .44852
ALPHA1 = 19.37   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .48678
ALPHA1 = 19.41   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .44306
ALPHA1 = 19.44   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .49937
ALPHA1 = 19.48   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .50571
ALPHA1 = 19.52   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .51287
ALPHA1 = 19.55   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .51846
ALPHA1 = 19.59   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .52467
ALPHA1 = 19.63   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .53131
ALPHA1 = 19.66   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .53777
ALPHA1 = 19.70   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .54248
ALPHA1 = 19.74   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .54843
ALPHA1 = 19.77   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .55448
ALPHA1 = 19.81   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .55847
ALPHA1 = 19.85   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .56446
ALPHA1 = 19.88   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .56244
ALPHA1 = 19.92   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .53842
ALPHA1 = 19.96   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .53443
ALPHA1 = 20.00   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .53037
ALPHA1 = 20.03   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .52633
ALPHA1 = 20.07   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .52229
ALPHA1 = 20.11   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .46932
ALPHA1 = 20.14   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .47565
ALPHA1 = 20.18   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .46201
ALPHA1 = 20.22   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .49839
ALPHA1 = 20.25   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .49479
ALPHA1 = 20.29   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .50123
ALPHA1 = 20.33   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .50789
ALPHA1 = 20.36   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .51417
ALPHA1 = 20.40   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .52060
ALPHA1 = 20.44   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .52722
ALPHA1 = 20.47   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .53378
ALPHA1 = 20.51   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .54037
ALPHA1 = 20.55   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .54698
ALPHA1 = 20.58   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .55362
ALPHA1 = 20.62   S1 =  1.0   S2 =  1.0   POINT EFFICIENCY = .56029
ALPHA1 = 20.66   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .56584
ALPHA1 = 20.69   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .56167
ALPHA1 = 20.73   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .57751
ALPHA1 = 20.77   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .57334
ALPHA1 = 20.80   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .56916
ALPHA1 = 20.84   S1 =  1.0   S2 = -1.0   POINT EFFICIENCY = .56498
ALPHA1 = 20.88   S1 = -1.0   S2 = -1.0   POINT EFFICIENCY = .57563
ALPHA1 = 20.91   S1 = -1.0   S2 = -1.0   POINT EFFICIENCY = .56973
ALPHA1 = 20.95   S1 = -1.0   S2 = -1.0   POINT EFFICIENCY = .56384
ALPHA1 = 20.99   S1 = -1.0   S2 = -1.0   POINT EFFICIENCY = .55798
ALPHA1 = 21.02   S1 = -1.0   S2 = -1.0   POINT EFFICIENCY = .55214
ALPHA1 = 21.06   S1 = -1.0   S2 =  1.0   POINT EFFICIENCY = .49519
ALPHA1 = 21.10   S1 = -1.0   S2 =  1.0   POINT EFFICIENCY = .50033
```

ALPHA1 = 21.17 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .51705

ALPHA1 = 21.21 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .51575

ALPHA1 = 21.24 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .52048

ALPHA1 = 21.28 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .52682

ALPHA1 = 21.32 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .53115

ALPHA1 = 21.35 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .53628

ALPHA1 = 21.39 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .54141

ALPHA1 = 21.43 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .54654

ALPHA1 = 21.47 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .55166

ALPHA1 = 21.50 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .55678

ALPHA1 = 21.54 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .56190

ALPHA1 = 21.58 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .56702

ALPHA1 = 21.61 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .56114

ALPHA1 = 21.65 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .56527

ALPHA1 = 21.69 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .57342

ALPHA1 = 21.72 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .57359

ALPHA1 = 21.76 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .56778

ALPHA1 = 21.80 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .56199

ALPHA1 = 21.83 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .55622

ALPHA1 = 21.87 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .55847

ALPHA1 = 21.91 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .56474

ALPHA1 = 21.94 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .53963

ALPHA1 = 21.98 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .53334

ALPHA1 = 22.02 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .47808

ALPHA1 = 22.05 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .48296

ALPHA1 = 22.09 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .48796

ALPHA1 = 22.13 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .49294

ALPHA1 = 22.16 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .49791

ALPHA1 = 22.20 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .50288

ALPHA1 = 22.24 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .50785

ALPHA1 = 22.27 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .51282

ALPHA1 = 22.31 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .51779

ALPHA1 = 22.35 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .52275

ALPHA1 = 22.38 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .52772

ALPHA1 = 22.42 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .53268

ALPHA1 = 22.46 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .53763

ALPHA1 = 22.49 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .54259

ALPHA1 = 22.53 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .54754

ALPHA1 = 22.57 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .57894

ALPHA1 = 22.60 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .56523

ALPHA1 = 22.64 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .55953

ALPHA1 = 22.68 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .55385

ALPHA1 = 22.71 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .54819

ALPHA1 = 22.75 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .54255

ALPHA1 = 22.79 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .53694

ALPHA1 = 22.82 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .53134

ALPHA1 = 22.86 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .52575

ALPHA1 = 22.90 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .52019

ALPHA1 = 22.93 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .51465

ALPHA1 = 22.97 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .46692

ALPHA1 = 23.01 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .46574

ALPHA1 = 23.05 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .47855

ALPHA1 = 23.08 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .47537

ALPHA1 = 23.12 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .44818

ALPHA1 = 23.16 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .46699

ALPHA1 = 23.19 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .44980

ALPHA1 = 23.23 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .49491

ALPHA1 = 23.27 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .49941

ALPHA1 = 23.30 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .50421

ALPHA1 = 23.34 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .50901

ALPHA1 = 23.38 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .51388

ALPHA1 = 23.41 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .51868

ALPHA1 = 23.45 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .52339

ALPHA1 = 23.48 S1 = -1.0 S2 = 1.0 POINT EFFICIENCY = .52818

ALPHA1 = 23.52 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .50807

[illegible] = [illegible] S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .54538

ALPHA1 = 23.00 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .53573

ALPHA1 = 23.63 S1 = -1.0 S2 = -1.0 POINT EFFICIENCY = .53423

CYCLE EFFICIENCY = .513

APPENDIX D

GEOMETRY OF GENERAL CLOCK GEAR TOOTH

The general ogival tooth of thickness t and outside radius r_o consists of a circular arc of radius ρ which blends tangentially into a radial tooth flank, as shown in Figure D-1 (only one center of curvature is indicated). The center of curvature, C, is located at a distance a from the center of the gear or pinion. Frequently this distance a equals the pitch radius r_p. The tooth geometry may either be described with the help of the parameters t, ρ and a, or with the combination t, ρ and r_o. Both approaches are shown below.

1. TOOTH GEOMETRY WITH HELP OF PARAMETERS t, ρ AND a

C_x and C_y represent the coordinates of the center of curvature C. C_x is defined by

$$C_x = \rho - \frac{t}{2} \qquad \text{(D-1)}$$

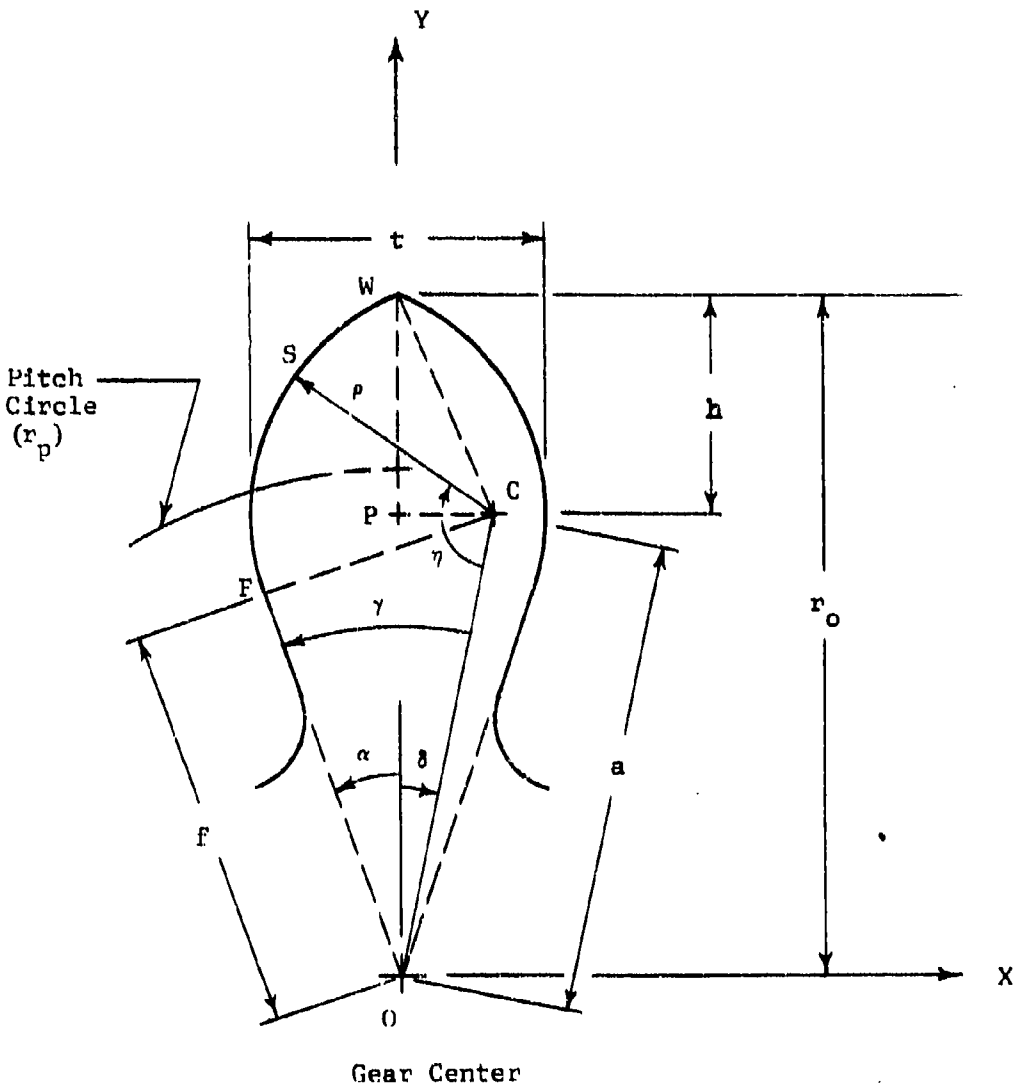


FIGURE D-1

GEOMETRY OF OGIVAL TOOTH (ONLY ONE CENTER OF CURVATURE SHOWN)

The angle δ is given by

$$\delta = \sin^{-1} \frac{C_x}{a} \qquad \text{(D-2)}$$

With the above,

$$C_y = a \cos\delta \qquad \text{(D-3)}$$

Further, the outside radius r_o may be computed from

$$r_o = C_y + \sqrt{\rho^2 - C_x^2} \qquad \text{(D-4)}$$

The angle γ is obtained from

$$\gamma = \sin^{-1} \frac{\rho}{a} \quad , \qquad \text{(D-5)}$$

and the flank angle α from

$$\alpha = \gamma - \delta \qquad \text{(D-6)}$$

The distance, f, from the center of rotation, O, to the blend point, F, of the flank of the tooth, is given by

$$f = a \cos\gamma \qquad \text{(D-7)}$$

The angle η defines the tooth contact point S on the ogival, i.e., circular, portion of the tooth with the lines OC and CS. The minimum and maximum angles η_{min} and η_{max} are, respectively,

$$\eta_{min} = \frac{\pi}{2} - \gamma \qquad \text{(D-8)}$$

and

$$\eta_{max} = \sin^{-1} \frac{r_o \sin\delta}{\rho} \qquad \text{(D-9)}$$

(This angle extends into the second quadrant)

2. TOOTH GEOMETRY FROM PARAMETERS ρ, t AND r_o

If the outside radius, r_o, is given, distance a must be computed. The length, C_x, is still given by Equation (D-1), while

$$a = \sqrt{(r_o - h)^2 + C_x^2} \qquad \text{(D-10)}$$

where, according to Figure D-1,

$$h = \sqrt{\rho^2 - C_x^2} \qquad \text{(D-11)}$$

All other quantities of interest remain as before, i.e.,

$$\delta = \sin^{-1} \frac{C_x}{a} \qquad \text{See Equation (D-2)}$$

$$\gamma = \sin^{-1} \frac{\rho}{a} \qquad \text{See Equation (D-5)}$$

$$\alpha = \gamma - \delta \qquad \text{See Equation (D-6)}$$

$$f = a \cos\gamma \qquad \text{See Equation (D-7)}$$

$$\eta_{min} = \frac{\pi}{2} - \gamma \qquad \text{See Equation (D-8)}$$

$$\eta_{max} = \sin^{-1} \frac{r_o \sin \delta}{\rho} \qquad \text{See Equation (D-9)}$$

APPENDIX E

KINEMATICS AND MOMENT INPUT-OUTPUT RELATIONSHIP FOR SINGLE STEP-UP GEAR MESH WITH CLOCK TEETH

This appendix shows the kinematics of a single step-up gear mesh with ogival teeth and derives the moment input-output relationship. Both contact and pivot friction are included.

1. KINEMATICS OF AN OGIVE MESH

Figure E-1 indicates the condition of initial contact in ogival meshes when the circular arc portion of the gear tooth drives the circular arc portion of the pinion tooth.[1] This type of direct contact will be called "round on round". As the motion progresses, the circular arc of the gear tooth moves into contact with the straight flank of the pinion tooth (Figure E-2). During this "round on flat" phase, distance g at first decreases and then increases again. Before round on round contact can again occur for a given mesh, i.e., g has become equal to f_p again, the subsequent mesh comes into engagement with round on round contact. (See below.)

[1] See section b-VII of this appendix for a check that indicates whether the initial contact is round on round or whether possibly the round portion of the pinion touches the flat portion of the gear.

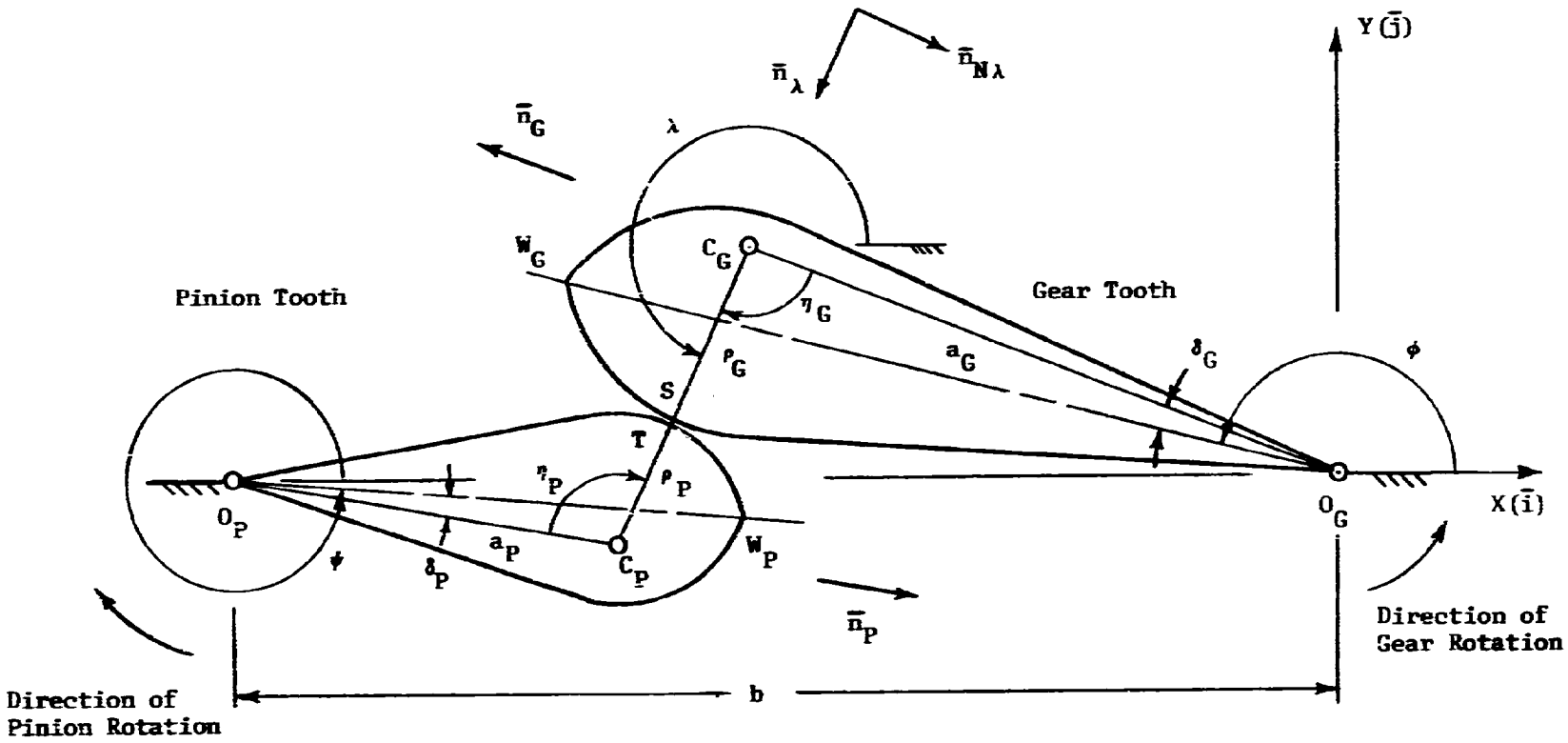


FIGURE E-1

ROUND ON ROUND PHASE OF CONTACT (GEAR DRIVES PINION)

The kinematics of both round on round and round on flat phases will be given first. Further it will be shown how to determine those gear angles, ϕ, at which the regime changes from round on round to round on flat, and at which contact is taken over by the subsequent set of teeth. Since the ratio of angular velocities between gear and pinion varies with contact point, and the pinion moves faster for a given gear speed for round on round than for round on flat, the original set of teeth disengages as soon as the subsequent one makes contact. Thus, the contact ratio is unity for this type of gearing.

The gear tooth nomenclature given in Appendix D is used throughout. The additional subscripts G and P refer to gear and pinion, respectively.

a. ROUND ON ROUND PHASE OF MOTION

Figure E-1 shows the input angle ϕ as the angle between the x-axis and the center line, $O_G W_G$, of the gear tooth. The output angle, ψ, is the angle between the x-axis and the centerline $O_P W_P$. During this phase of motion, the line connecting the centers of curvature C_G and C_P is of constant length $L = \rho_G + \rho_P$ and passes

through the instantaneous contact points S on the gear and T on the pinion. Because of this constant length one may obtain both the output angle ψ and the "coupler" angle λ from the equivalent four-bar linkage O_G-C_C-C_P-O_P, with ground pivot distance b.

I. UNIT VECTORS AND ANGLES η_G AND η_P

The unit vectors associated with round on round phase are given now.

In direction O_G-C_G

$$\bar{n}_G = \cos(\phi - \delta_G)\bar{I} + \sin(\phi - \delta_G)\bar{J} \qquad \text{(E-1)}$$

In direction C_G-C_P

$$\bar{n}_\lambda = \cos\lambda\bar{I} + \sin\lambda\bar{J} \qquad \text{(E-2)}$$

The unit vector normal to $\bar{n}_\lambda$ (in the right hand sense) becomes

$$\bar{n}_{N\lambda} = -\sin\lambda\bar{I} + \cos\lambda\bar{J} \qquad \text{(E-3)}$$

In direction O_p-C_p,

$$\bar{n}_p = \cos(\psi - \delta_p)\bar{I} + \sin(\psi - \delta_p)\bar{J} \qquad \text{(E-4)}$$

Since $\lambda - (\phi - \delta_G) = \pi - \eta_G$

$$\eta_G = \pi + \phi - \lambda - \delta_G \qquad \text{(E-5)}$$

The angle η_p is obtained from

$$\eta_p = (\psi - \delta_p) - \lambda \quad \text{for } \psi - \delta_p > 90^\circ \qquad \text{(E-6a)}$$

and

$$\eta_p = 2\pi + (\psi - \delta_p) - \lambda \quad \text{for } \psi - \delta_p < 90^\circ \qquad \text{(E-6b)}$$

The above values for η_G and η_p are only good for the round on round phase of the motion. For the round on flat phase the angle η_{GF}, of the gear, may be of interest.(See Figure E-2). This angle is given by:

$$\eta_{GF} = (\phi - \delta_G) - (\psi + \alpha_p) + \frac{3\pi}{2} \quad \text{for } \psi + \alpha_p > 90^\circ \qquad \text{(E-6c)}$$

[Pinion flank in 4th quadrant]

and

$$\eta_{GF} = (\phi - \delta_G) - (\psi + \alpha_p) - \frac{\pi}{2} \quad \text{for } \psi + \alpha_p < 90^\circ \qquad \text{(E-6d)}$$

[Pinion flank in first quadrant]

II. DETERMINATION OF OUTPUT ANGLE ψ AND "COUPLER" ANGLE λ

The loop equation of the equivalent four-bar linkage is given by

$$a_G\bar{n}_G + (\rho_G + \rho_P)\bar{n}_\lambda - a_P\bar{n}_P + b\bar{i} = 0 \qquad \text{(E-7)}$$

With appropriate substitutions for the unit vectors, according to Equations (E-1), (E-2), and (E-4), one obtains the following component equation

$$a_G\cos(\phi - \delta_G) + L\cos\lambda + b - a_P\cos(\psi - \delta_P) = 0 \qquad \text{(E-8)}$$

$$a_G\sin(\phi - \delta_G) + L\sin\lambda - a_P\sin(\psi - \delta_P) = 0 \qquad \text{(E-9)}$$

where

$$L = \rho_G + \rho_P \qquad \text{(E-10)}$$

To eliminate λ, let

$$\sin^2\lambda + \cos^2\lambda = 1 \qquad \text{(E-11)}$$

Substitution of expressions for $\sin\lambda$ and $\cos\lambda$ from Equations (E-9) and (E-8), respectively, leads to

$$A_R \sin\psi + B_R \cos\psi = C_R \qquad \text{(E-12)}$$

where

$$A_R = \sin(\phi - \delta_G) + \cos(\phi - \delta_G)\tan\delta_P + \frac{b}{a_G}\tan\delta_P$$

$$B_R = \cos(\phi - \delta_G) + \frac{b}{a_G} - \sin(\phi - \delta_G)\tan\delta_P$$

$$C_R = \frac{a_P^2 + a_G^2 + b^2 - L^2}{2a_P a_G \cos\delta_P} + \frac{b\cos(\phi - \delta_G)}{a_P \cos\delta_P}$$

Equation(E-12) is solved for ψ by a method similar to the one given on pg. 296 of R. S. Hartenberg and J. Denavit; Kinematic Synthesis of Linkages, McGraw-Hill Book Co., New York, 1964. Thus,

$$\psi = 2\tan^{-1}\frac{A_R \pm \sqrt{A_R^2 + B_R^2 - C_R^2}}{B_R + C_R} \qquad \text{(E-13)}$$

The correct sign in Equation (E-13) must be found from the geometric conditions of the equivalent four-bar linkage.

The "coupler" angle λ may now be determined either from Equation (E-8) or from Equation (E-9), i.e.,

$$\lambda = \cos^{-1}\left[\frac{a_P\cos(\psi - \delta_P) - a_G\cos(\phi - \delta_G) - b}{L}\right] \qquad \text{(E-14)}$$

or

$$\lambda = \sin^{-1}\left[\frac{a_P\sin(\psi - \delta_P) - a_G\sin(\phi - \delta_G)}{L}\right] \qquad \text{(E-15)}$$

III. DETERMINATION OF OUTPUT PINION ANGULAR VELOCITY $\dot{\psi}$

Differentiation of Equation (E-12) with respect to time leads to

$$\dot{\psi} = \dot{\phi}\left[\frac{A_{RD}\sin\psi + B_{RD}\cos\psi - C_{RD}}{A_R\cos\psi - B_R\sin\psi}\right] \qquad \text{(E-16)}$$

where A_R and B_R are given with Equation (E-12), and where

$$A_{RD} = \tan\delta_P\sin(\phi - \delta_G) - \cos(\phi - \delta_G)$$

$$B_{RD} = \sin(\phi - \delta_G) + \tan\delta_P\cos(\phi - \delta_G)$$

$$C_{RD} = \frac{b}{a_P\cos\delta_P}\sin(\phi - \delta_G)$$

IV. RELATIVE VELOCITY AT CONTACT POINT BETWEEN GEAR AND PINION

With point S on the gear and point T on the pinion, as shown by Figure E-1, the relative velocity between gear and pinion at the contact point is given by

$$\bar{V}_{S/T} = \bar{V}_{S/O_G} - \bar{V}_{T/O_P} \tag{E-17}$$

This relative velocity is tangent to the contacting surfaces and can therefore be expressed in terms of the unit vector $\bar{n}_{N\lambda}$ (see Figure E-1). Then, the above becomes

$$\bar{V}_{S/T} = \left[(\bar{V}_{S/O_G} \cdot \bar{n}_{N\lambda}) - (\bar{V}_{T/O_F} \cdot \bar{n}_{N\lambda})\right] \bar{n}_{N\lambda} \tag{E-18}$$

or

$$\bar{V}_{S/T} = \left\{ \left[\dot{\psi}\bar{k} \times (a_G\bar{n}_G + \rho_G\bar{n}_\lambda)\right] \cdot \bar{n}_{N\lambda} - \left[\dot{\psi}\bar{k} \times (a_P\bar{n}_P - \rho_P\bar{n}_\lambda)\right] \cdot \bar{n}_{N\lambda} \right\} \bar{n}_{N\lambda} \tag{E-19}$$

Appropriate substitution of unit vectors and simplification results in

$$\bar{V}_{S/T} = \left\{ \dot{\phi}\left[a_G\cos(\phi - \delta_G - \lambda) + \rho_G\right] - \dot{\psi}\left[a_P\cos(\psi - \delta_P - \lambda) - \rho_P\right] \right\} \bar{n}_{N\lambda} \tag{E-20}$$

b. ROUND ON FLAT PHASE OF MOTION

Figure E-2 gives the details of the round on flat contact between the driving gear and the driven pinion. The input angle ϕ and the output angle ψ are again defined counter-clockwise between the x-axis and the respective tooth center lines $O_G W_G$ and $O_P W_P$. Since contact is always made on the straight radial flank of the pinion, the line SC_G of the gear is always normal to the flank of the pinion. The contact point is at a distance g from the pinion center, O_P, and this distance is always smaller than, or equal to the distance f_P (which is defined by Equation (D-7) in Appendix D). Again, the subscripts G and P are used for gear and pinion tooth parameters, respectively.

I. UNIT VECTORS

As before, the unit vector in direction $O_G C_G$ is given by

$$\bar{n}_G = \cos(\phi - \delta_G)\bar{I} + \sin(\phi - \delta_G)\bar{J} \qquad \text{(E-21)}$$

The unit vector in direction $O_P T$, along the flank of the pinion is given by

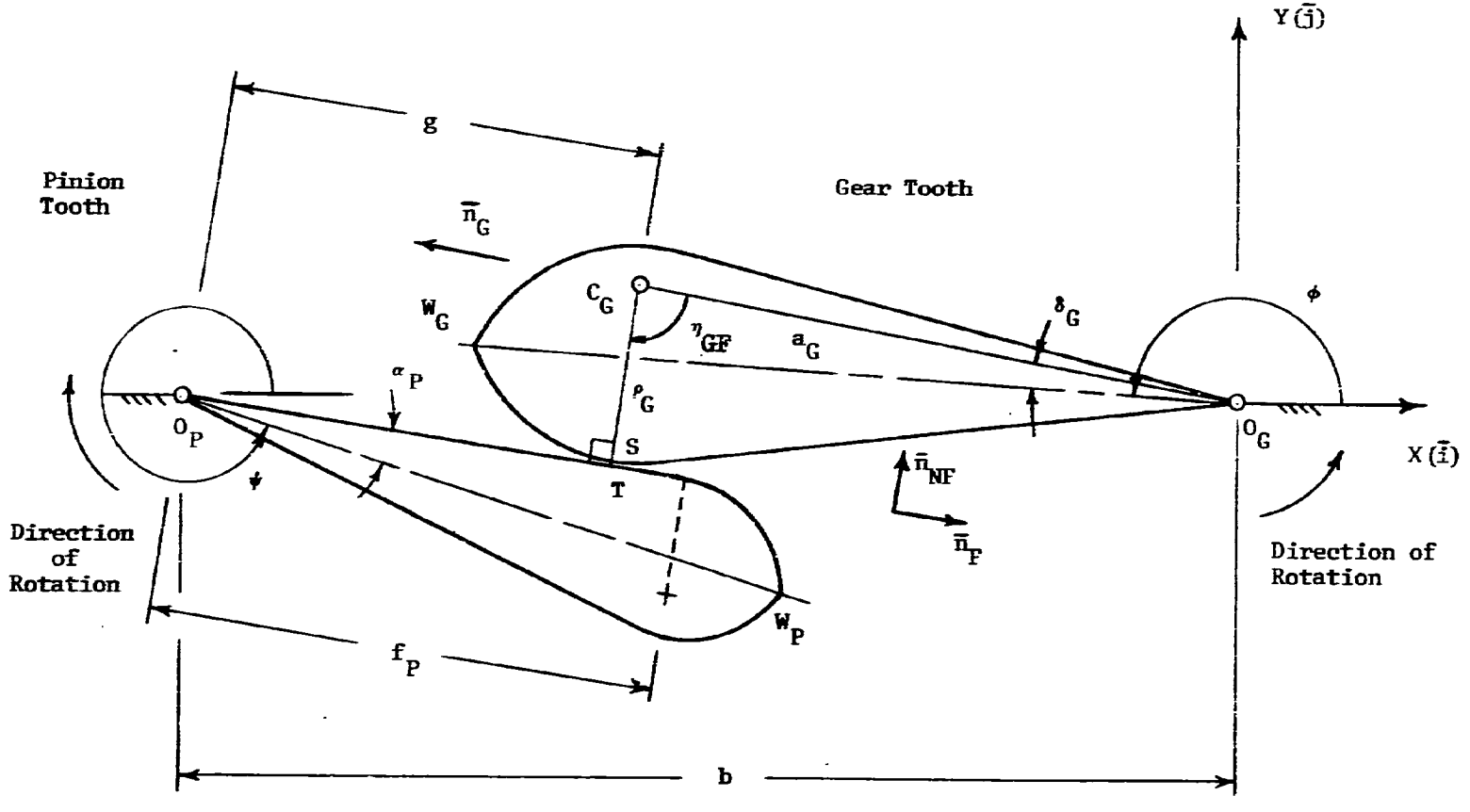


FIGURE E-2

ROUND ON FLAT PHASE OF CONTACT (GEAR DRIVES PINION)

$$\bar{n}_F = \cos(\psi + \alpha_p)\bar{I} + \sin(\psi + \alpha_p)\bar{J} \qquad \text{(E-22)}$$

The unit vector in direction SC_G is normal to $\bar{n}_F$ in the right hand sense

$$\bar{n}_{NF} = -\sin(\psi + \alpha_p)\bar{I} + \cos(\psi + \alpha_p)\bar{J} \qquad \text{(E-23)}$$

II. DETERMINATION OF OUTPUT ANGLE ψ AND DISTANCE g

The vector equation for the mechanism loop O_G-C_G-S-T-O_P, which forms the basis of the desired solution, has the following form:

$$a_G\bar{n}_G - \rho_G\bar{n}_{NF} - g\bar{n}_F + b\bar{I} = 0 \qquad \text{(E-24)}$$

Substitution of Equations (E-21) to (E-23) furnishes the component equations

$$a_G\cos(\phi - \delta_G) + \rho_G\sin(\psi + \alpha_p) - g\cos(\psi + \alpha_p) + b = 0 \qquad \text{(E25)}$$

$$a_G\sin(\phi - \delta_G) - \rho_G\cos(\psi + \alpha_p) - g\sin(\psi + \alpha_p) = 0 \qquad \text{(E26)}$$

From Equation (E-26) one obtains for g

$$g = \frac{a_G \sin(\phi - \delta_G) - \rho_G \cos(\psi + \alpha_P)}{\sin(\psi + \alpha_P)} \qquad \text{(E-27)}$$

This expression for g is now substituted into Equation(E-25). Rearrangement and simplification lead to

$$A_F \sin\psi + B_F \cos\psi = C_F \qquad \text{(E-28)}$$

where

$$A_F = b\cos\alpha_P + a_G\cos(\phi - \delta_G - \alpha_P)$$

$$B_F = b\sin\alpha_P - a_G\sin(\phi - \delta_G - \alpha_P)$$

$$C_F = -\rho_G$$

The solution of Equation (E-28) for ψ is obtained in the same way as that of Equation (E-13), i.e.,

$$\psi = 2\tan^{-1}\frac{A_F \pm \sqrt{A_F^2 + B_F^2 - C_F^2}}{B_F + C_F} \qquad \text{(E-29)}$$

The correct sign in Equation (E-29) depends on the geometric conditions of the mechanism position as in all four-bar linkage solutions of this type.

III. DETERMINATION OF PINION ANGULAR VELOCITY $\dot{\psi}$

Differentiation of Equation (E-28) with respect to time gives

$$\dot{\psi} = \dot{\phi}\left[\frac{A_{FD}\sin\psi + B_{FD}\cos\psi}{A_F\cos\psi - B_F\sin\psi}\right] \qquad \text{(E-30)}$$

where A_F and B_F are given with Equation (E-28) and where

$$A_{FD} = a_G\sin(\phi - \delta_G - \alpha_P)$$

$$B_{FD} = a_G\cos(\phi - \delta_G - \alpha_P)$$

IV. RELATIVE VELOCITY AT CONTACT POINT BETWEEN GEAR AND PINION

As for round on round contact, the relative velocity between point S on the gear tooth and point T on the pinion tooth is tangent to the contacting surfaces. In this case it will have the direction of unit vector $\bar{n}_F$ (see Figure E-2). Then,

$$\bar{V}_{S/T} = \bar{V}_{S/O_G} - \bar{V}_{T/O_P} = \left[(\bar{V}_{S/O_G} \cdot \bar{n}_F) - (\bar{V}_{T/O_P} \cdot \bar{n}_F) \right] \bar{n}_F \tag{E-31}$$

Since, because of the radial flank of the pinion, the velocity of the contact point T is normal to unit vector $\bar{n}_F$.

$$\bar{V}_{T/O_P} \cdot \bar{n}_F = 0 \tag{E-32}$$

Therefore,

$$\bar{V}_{S/T} = \left[\bar{V}_{S/O_G} \cdot \bar{n}_F \right] \bar{n}_F \tag{E-33}$$

or

$$\bar{V}_{S/T} = \left\{ \left[\dot{\phi} \bar{k} \times (a_G \bar{n}_P - \rho_G \bar{n}_{NF}) \right] \cdot \bar{n}_F \right\} \bar{n}_F \tag{E-34}$$

Appropriate substitution of unit vectors gives

$$\bar{V}_{S/T} = \dot{\phi} \left[\rho_G + a_G \sin(\psi - \phi + \alpha_P + \delta_G) \right] \bar{n}_F \tag{E-35}$$

V. DETERMINATION OF TRANSITION ANGLES FROM ROUND ON ROUND TO ROUND ON FLAT MOTION

The transition angles ϕ_T and ψ_T, which occur when the round on flat phase follows the round on round one, may be determined with the help of the modified component equations (E-25) and (E-26), i.e., one lets $\phi = \phi_T$, $\psi = \psi_T$ and $g = f_P$. This results in

$$a_G\cos(\phi_T - \delta_G) + \rho_G\sin(\psi_T + \alpha_P) - f_P\cos(\psi_T + \alpha_P) + b = 0 \qquad \text{(E-36)}$$

and

$$a_G\sin(\phi_T - \delta_G) - \rho_G\cos(\psi_T + \alpha_P) - f_P\sin(\psi_T + \alpha_P) = 0 \qquad \text{(E-37)}$$

From the above,

$$\cos(\phi_T - \delta_G) = \frac{1}{a_G}\left[f_P\cos(\psi_T + \alpha_P) - b - \rho_G\sin(\psi_T + \alpha_P)\right] \qquad \text{(E-38)}$$

and

$$\sin(\phi_T - \delta_G) = \frac{1}{a_G}\left[f_P\sin(\psi_T + \alpha_P) + \rho_G\cos(\psi_T + \alpha_P)\right] \qquad \text{(E-39)}$$

The angle ψ_T may now be obtained by first eliminating ϕ_T from Equations (E-38) and (E-39). This is accomplished with

$$\sin^2(\phi_T - \delta_G) + \cos^2(\phi_T - \delta_G) = 1 \qquad \text{(E-40)}$$

Substitution into the above leads to the following expression in ψ_T:

$$A_T \sin\psi_T + B_T \cos\psi_T = C_T \qquad \text{(E-41)}$$

where

$$A_T = \rho_G \cos\alpha_P + f_P \sin\alpha_P$$

$$B_T = \rho_G \sin\alpha_P - f_P \cos\alpha_P$$

$$C_T = \frac{a_G^2 - f_P^2 - b^2 - \rho_G^2}{2b}$$

Equation (E-41) is again solved in the manner of Equation (E-13)

$$\psi_T = 2 \tan^{-1} \frac{A_T \pm \sqrt{A_T^2 + B_T^2 - C_T^2}}{B_T + C_T} \qquad \text{(E-42)}$$

The correct sign must again be determined from the geometric conditions.

The associated transition angle ϕ_T can now be obtained either with the help of Equation (E-38) or Equation (E-39):

$$\phi_T = \cos^{-1}\left[\frac{f_P\cos(\psi_T + \alpha_P) - \rho_G\sin(\psi_T + \alpha_P) - b}{a_G}\right] + \delta_G \qquad \text{(E-43)}$$

or

$$\phi_T = \sin^{-1}\left[\frac{f_P\sin(\psi_T + \alpha_P) + \rho_G\cos(\psi_T + \alpha_P)}{a_G}\right] + \delta_G \qquad \text{(E-44)}$$

VI. SENSING GEOMETRY FOR THE DETERMINATION OF CONTACT OF SUBSEQUENT TOOTH MESH

The following derives a computer sensing equation which indicates when contact is transferred from one tooth mesh to the succeeding one. Figure E-3 illustrates the case. The active mesh is in the round on flat mode and the subsequent mesh will make its initial contact in the round on round mode. This

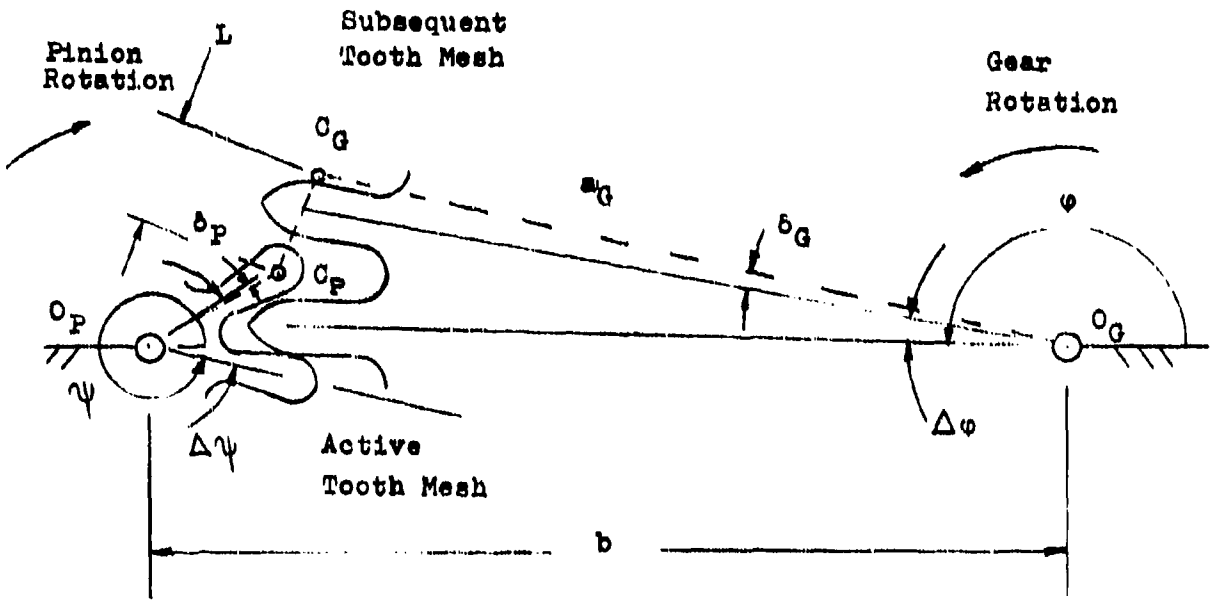


Figure E3
Sensing Geometry for Contact of Subsequent Tooth Mesh

assertion is based on experience with the three gear and pinion combinations of the M125A1 booster. In each of these instances, the subsequent mesh makes contact before the round of the gear has left the flat of the pinion in the active mesh, i.e., $g < f_p$. (See work in section V.) It has also been found that initial contact of the subsequent mesh is always in the round on round mode. Generally, contact between the flat of the gear and the round of the pinion does not occur, and it has not been considered in this report. Section VII gives a criterion for the existence of this inverted round on flat mode of contact.

Once contact has been made by the subsequent mesh it becomes the new active mesh. This can be proven by the fact that, for a given angular velocity $\dot{\phi}$ of the gear, the angular velocity of the pinion is always larger in the initial stages of the round on round mode than in the final stages of the round on flat one. Thus, once the new mesh has made contact, the old one separates rapidly and the "contact ratio" is always unity.

The above may be shown theoretically by the position of the instant center of rotation between the gear and the pinion on line $O_G O_P$.

If $\Delta\phi$ and $\Delta\psi$ represent the angles between the individual tooth center lines of the gear and pinion, respectively, (see Figure E-3), the

closure equation for the subequent mesh may be written in terms of the active mesh as follows:

$$a_p\left[\cos(\psi - \delta_p + \Delta\psi)\bar{I} + \sin(\psi - \delta_p + \Delta\psi)\bar{J}\right] + L_x\bar{I} + L_y\bar{J}$$

$$= b\bar{I} + a_G\left[\cos(\phi - \delta_G - \Delta\phi)\bar{I} + \sin(\phi - \delta_G - \Delta\phi)\bar{J}\right] \qquad \text{(E-45)}$$

where L_x, L_y = components of the distance $L = C_pC_G$

ψ = pinion angle determined from round on flat mode according to Equation (E-29)

The components L_x and L_y may be obtained from Equation (E-45):

$$L_x = b + a_G\cos(\phi - \delta_G - \Delta\phi) - a_p\cos(\psi - \delta_p + \Delta\psi) \qquad \text{(E-46)}$$

and

$$L_y = a_G\sin(\phi - \delta_G - \Delta\phi) - a_p\sin(\psi - \delta_p + \Delta\psi) \qquad \text{(E-47)}$$

Contact will have occurred when the distance L becomes equal to, or slightly smaller than, the sum of the two radii of curvature ρ_G and ρ_p. Thus, the criterion of contact becomes

$$\sqrt{L_x^2 + L_y^2} \leq \rho_G + \rho_p \qquad \text{(E-48)}$$

VII. POSSIBILITY OF FLAT OF GEAR MAKING CONTACT WITH THE ROUND PORTION OF THE PINION

Figure E-4 shows a transition configuration in which the flat part of the gear tooth makes contact with the circular arc of the pinion, i.e., $\overline{O_G S} = f_G$. The radius of curvature ρ_p of of the pinion will only then be normal to the flat of the gear

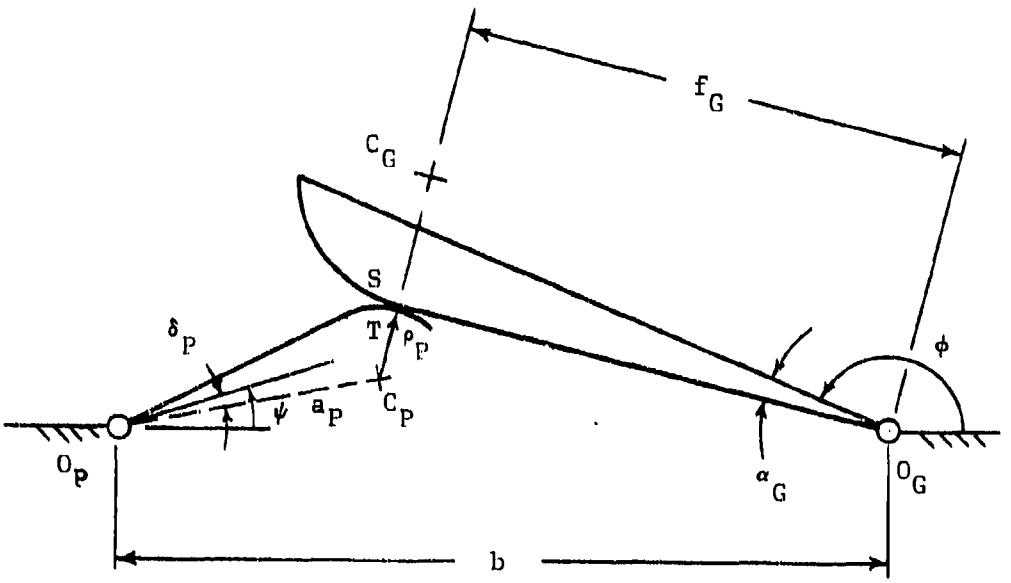


FIGURE E-4

FLAT OF GEAR CONTACTS ROUND OF PINION

when $\overline{O_G S} \leq f_G$. Accordingly, in order to avoid this type of contact, the following criterion must be met:

$$b > \left[a_p + \sqrt{f_G^2 + \rho_P^2} \right] \qquad \text{(E-49)}$$

The three gear meshes of the M125A1 booster always satisfy equation (E-49), and therefore the initial contact is made in the round on round mode.

2. MOMENT INPUT-OUTPUT RELATIONSHIP FOR SINGLE STEP-UP GEAR MESH WITH OGIVAL TEETH

The present section concerns itself with the determination of the equilibrant moment M_o acting on the output pinion of a single ogival mesh, when the input moment M_{in}, which acts on the gear, is given. This relationship must be derived both for the round on round and for the round on flat phases of the motion. The directions of the pivot friction forces are again chosen such that the resulting moments oppose the motion of the respective component. (See Appendix A.) The magnitude of these friction moments is expressed in the manner of equation (A-2) of the aforementioned appendix. The direction of the friction force of the gear on the pinion is the same as the direction of the relative velocity $\bar{V}_{S/T}$, of the gear contact point S with respect to the pinion contact point T. This will be expressed by the appropriate signum convention. It must also be remembered that the kinematic expressions must conform to the motion phase under consideration.

a. INPUT-OUTPUT RELATIONSHIP FOR THE ROUND ON ROUND PHASE OF MOTION

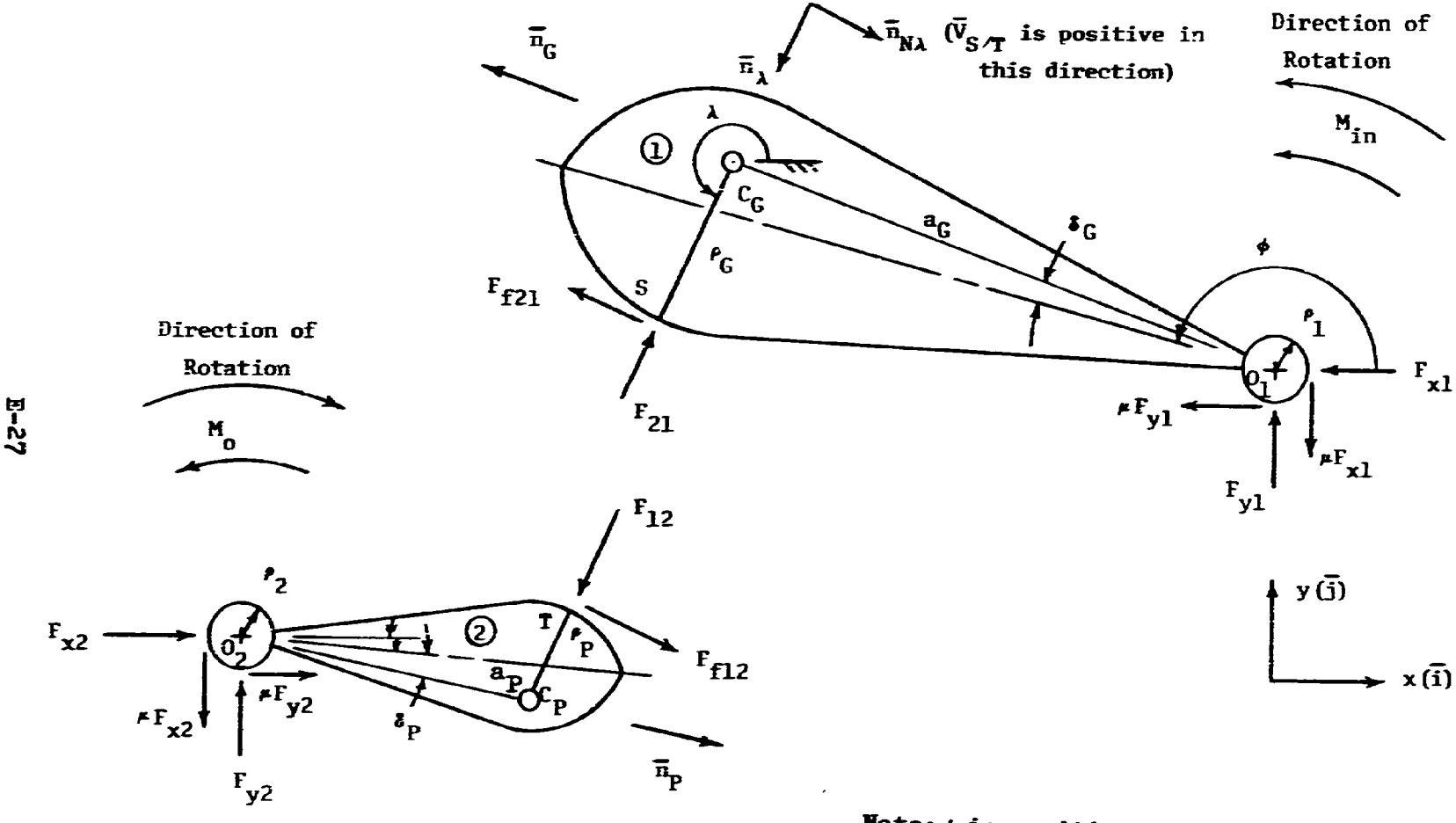
Figure E-5 shows the free body diagrams for the round on round phase of motion with the gear considered to be component no. 1 and the pinion defined as component no. 2. The contact forces F_{12} and F_{21} are expressed with the help of the unit vector $\bar{n}_\lambda$ [see equation (E-2)] . This unit vector is always normal to both contacting surfaces at points S and T. The unit vector $\bar{n}_{N\lambda}$ is used to describe the direction of the friction forces at the contact point. The sense of these friction forces is determined with the help of

$$s = \frac{V_{S/T}}{|V_{S/T}|} \tag{E-50}$$

(See equation (E-20) for $\bar{V}_{S/T}$.) Since the friction force F_{f12} of the gear on the pinion has the same direction as the relative velocity $V_{S/T}$, one may write

$$\bar{F}_{f12} = \mu s F_{12} \bar{n}_{N\lambda} \tag{E-51}$$

where μ represents the coefficient of friction at the contact point.



Note: ϕ is positive in ccw direction

FIGURE E-5

FREE BODY DIAGRAM FOR ROUND ON ROUND PHASE

Since

$$\bar{F}_{21} = - F_{12}\bar{n}_{\lambda} \qquad \text{(E-52)}$$

the friction force of the pinion on the gear becomes

$$\bar{F}_{f21} = - \mu_s F_{12}\bar{n}_{N\lambda} \qquad \text{(E-53)}$$

I. FORCE AND MOMENT EQUILIBRIUM OF THE GEAR

The force equilibrium of the gear is assured when

$$-F_{12}\bar{n}_{\lambda} - \mu_s F_{12}\bar{n}_{N\lambda} - F_{x1}\bar{i} + F_{y1}\bar{j} - \mu F_{y1}\bar{i} - \mu F_{x1}\bar{j} = 0 \qquad \text{(E-54)}$$

In the above μ also stands for the pivot coefficient of friction.

Moment equilibrium is given by

$$M_{in}\bar{k} - \mu\rho_1\sqrt{F_{x1}^2 + F_{y1}^2}\,\bar{k} + \left[a_G\bar{n}_G + \rho_G\bar{n}_{\lambda}\right] \times \left[-F_{12}\bar{n}_{\lambda} - \mu_s F_{12}\bar{n}_{N\lambda}\right] = 0 \qquad \text{(E-55)}$$

The unit vector $\bar{n}_G$ is defined by equation (E-1).

The following component equations may be obtained from equation (E-54):

$$-F_{12}\cos\lambda + \mu s F_{12}\sin\lambda - F_{x1} - \mu F_{y1} = 0 \qquad \text{(E-56)}$$

and

$$-F_{12}\sin\lambda - \mu s F_{12}\cos\lambda + F_{y1} - \mu F_{x1} = 0 \qquad \text{(E-57)}$$

After substitution and cross-multiplication, one obtains the moment expression, equation (E-55), in scalar form:

$$M_{in} - \mu\rho_1\sqrt{F_{x1}^2 + F_{y1}^2} + F_{12}\left[-\mu s\rho_G + a_G\sin(\phi - \delta_G - \lambda) - \mu s a_G\cos(\phi - \delta_G - \lambda)\right] = 0 \qquad \text{(E-58)}$$

Simultaneous solution of equations (E-56) and (E-57) for F_{x1} and F_{y1} results in

$$F_{x1} = -F_{12}\left[\frac{(1 + \mu^2 s)\cos\lambda + \mu(1 - s)\sin\lambda}{1 + \mu^2}\right] \qquad \text{(E-59)}$$

and

$$F_{y1} = F_{12}\left[\frac{(1 + \mu^2 s)\sin\lambda - \mu(1 - s)\cos\lambda}{1 + \mu^2}\right] \qquad \text{(E-60)}$$

These results are now substituted in equation (E-58). Since s^2 is unity and always positive, the resulting expression for F_{12} has the following form:

$$F_{12} = \frac{M_{in}}{\mu(\rho_1 + s\rho_G) + a_G[\mu s\cos(\phi - \delta_G - \lambda) - \sin(\phi - \delta_G - \lambda)]} \qquad \text{(E-61)}$$

II. FORCE AND MOMENT EQUILIBRIUM OF THE PINION

Force equilibrium of the pinion is assured by

$$F_{12}\bar{n}_\lambda + \mu s F_{12}\bar{n}_{N\lambda} + F_{x2}\bar{i} + F_{y2}\bar{j} + \mu F_{y2}\bar{i} - \mu F_{x2}\bar{j} = 0 \qquad \text{(E-62)}$$

Moment equilibrium must satisfy

$$M_o\bar{k} + \mu\rho_2\sqrt{F_{x2}^2 + F_{y2}^2}\,\bar{k} + \left[a_p\bar{n}_p - \rho_p\bar{n}_\lambda\right] \times \left[F_{12}\bar{n}_\lambda + \mu s F_{12}\bar{n}_{N\lambda}\right] = 0 \qquad \text{(E-63)}$$

The unit vector $\bar{n}_p$ is defined by equation (E-4).

Equation (E-62) gives rise to the following component equations:

$$F_{12}\cos\lambda - \mu s F_{12}\sin\lambda + F_{x2} + \mu F_{y2} = 0 \qquad \text{(E-64)}$$

and

$$F_{12}\sin\lambda + \mu s F_{12}\cos\lambda + F_{y2} - \mu F_{x2} = 0 \qquad \text{(E-65)}$$

The moment equation (E-63) becomes, in scalar form,

$$M_o + \mu\rho_2\sqrt{F_{x2}^2 + F_{y2}^2} + F_{12}\left[-\mu s\rho_p - a_p\sin(\psi - \delta_p - \lambda) + \mu s a_p\cos(\psi - \delta_p - \lambda)\right] = 0 \qquad \text{(E-66)}$$

Simultaneous solution of equations (E-64) and (E-65) for F_{x2} and F_{y2} results in

$$F_{x2} = F_{12}\left[\frac{\mu(1 + s)\sin\lambda - (1 - \mu^2 s)\cos\lambda}{1 + \mu^2}\right] \qquad \text{(E-67)}$$

and

$$F_{y2} = -F_{12}\left[\frac{\mu(1 + s)\cos\lambda + (1 + \mu^2 s)\sin\lambda}{1 + \mu^2}\right] \qquad \text{(E-68)}$$

The above expressions are now substituted into equation (E-66). Again, s^2 is unity and always positive, and the following expression may be obtained for the contact force F_{12} in terms of the equilibrant moment M_o

$$F_{12} = \frac{M_o}{\mu(s\rho_P - \rho_2) - a_P[\mu\cos(\psi - \delta_P - \lambda) - \sin(\psi - \delta_P - \lambda)]} \qquad \text{(E-69)}$$

III. MOMENT INPUT-OUTPUT RELATIONSHIP

When equations (E-61) and (E-69) are set equal to each other, one obtains the following input-output relationship:

$$M_o = M_{in}\left[\frac{\mu(s_R\rho_P - \rho_2) - a_P[\mu s_R\cos(\psi - \delta_P - \lambda) - \sin(\psi - \delta_P - \lambda)]}{\mu(\rho_1 + s_R\rho_G) + a_G[\mu s_R\cos(\phi - \delta_G - \lambda) - \sin(\phi - \delta_G - \lambda)]}\right] \qquad \text{(E-70)}$$

To account for the fact that equ.(E-70) is only valid for the round on round phase of motion, the signum symbol has now been changed to s_R.

b. INPUT-OUTPUT RELATIONSHIP FOR ROUND ON FLAT PHASE OF MOTION

Figure E-6 gives the free body diagrams for the round on flat phase of the motion. Again, the gear is considered to be component no. 1, while the pinion is component no. 2. Using the unit vector $\bar{n}_{NF}$, of equation (E-22), which is normal to the flat side of the pinion to express the force $\bar{F}_{12}$ of the gear on the pinion, one obtains

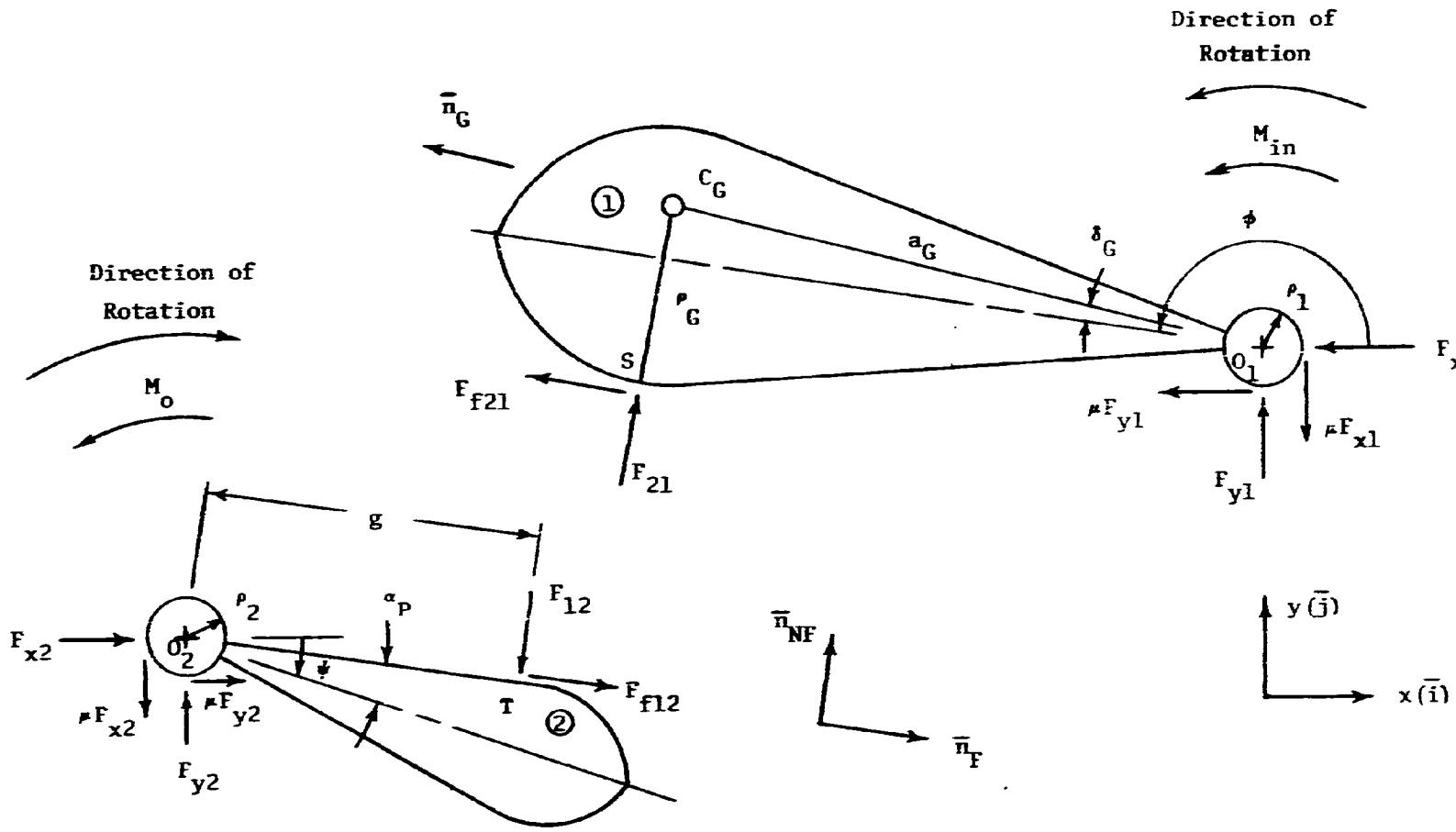
$$\bar{F}_{12} = - F_{12}\bar{n}_{NF} \tag{E-71}$$

The friction force of the gear on the pinion again has the same direction as the now applicable relative velocity $\bar{V}_{S/T}$ of equation (E-35). With

$$s = \frac{V_{S/T}}{|V_{S/T}|} \tag{E-72}$$

as the applicable signum convention, the friction force $\bar{F}_{f12}$ becomes

$$\bar{F}_{f12} = \mu s F_{12}\bar{n}_F \tag{E-73}$$



Note: ψ is positive in ccw direction

FIGURE E-6

FREE BODY DIAGRAM FOR ROUND ON FLAT PHASE

The contact force $\bar{F}_{21}$ of the pinion on the gear and the associated friction force are equal and opposite to the forces given by equations (E-71) and (E-73) respectively. Thus,

$$\bar{F}_{21} = F_{12}\bar{n}_{NF} \tag{E-74}$$

and

$$\bar{F}_{f21} = -\mu_s F_{12}\bar{n}_F \tag{E-75}$$

Note that the kinematics of the round on flat phase must now be used.

I. FORCE AND MOMENT EQUILIBRIUM OF THE GEAR

Force equilibrium of the gear is given by

$$F_{12}\bar{n}_{NF} - \mu_s F_{12}\bar{n}_F - F_{x1}\bar{I} + F_{y1}\bar{J} - \mu F_{y1}\bar{I} - \mu F_{x1}\bar{J} = 0 \tag{E-76}$$

Moment equilibrium requires

$$M_{in}\bar{k} - \mu\rho_1\sqrt{F_{x1}^2 + F_{y1}^2}\,\bar{k} + \left[a_G\bar{n}_G - \rho_G\bar{n}_{NF}\right] \times \left[F_{12}\bar{n}_{NF} - \mu_s F_{12}\bar{n}_F\right] = 0 \tag{E-77}$$

Equation (E-76) furnishes the following component expressions

$$-F_{12}\sin(\psi + \alpha_P) - \mu s F_{12}\cos(\psi + \alpha_P) - F_{x1} - \mu F_{y1} = 0 \tag{E-78}$$

and

$$F_{12}\cos(\psi + \alpha_P) - \mu s F_{12}\sin(\psi + \alpha_P) + F_{y1} - \mu F_{x1} = 0 \tag{E-79}$$

The scalar form of equation (E-77) is given by

$$M_{in} - \mu\rho_1\sqrt{F_{x1}^2 + F_{y1}^2} + F_{12}\left[-\mu s\rho_G + a_G\cos(\phi - \delta_G - \psi - \alpha_P) + \mu s a_G \sin(\phi - \delta_G - \psi - \alpha_P)\right] = 0 \tag{E-80}$$

Simultaneous solution of equations (E-78) and (E-79) for F_{x1} and F_{y1} furnishes

$$F_{x1} = F_{12}\left[\frac{\mu(1 - s)\cos(\psi + \alpha_P) - (1 + \mu^2 s)\sin(\psi + \alpha_P)}{1 + \mu^2}\right] \tag{E-81}$$

and

$$F_{y1} = -F_{12}\left[\frac{\mu(1 - s)\sin(\psi + \alpha_P) + (1 + \mu^2 s)\cos(\psi + \alpha_P)}{1 + \mu^2}\right] \tag{E-82}$$

The above results are now substituted into equation (E-80). Since s^2 is again unity and positive at all times, the resulting expression for F_{12} becomes, in terms of M_{in},

$$F_{12} = \frac{M_{in}}{\mu(\rho_1 + s\rho_G) - a_G\left[\cos(\phi - \delta_G - \psi - \alpha_P) + \mu s \sin(\phi - \delta_G - \psi - \alpha_P)\right]} \qquad \text{(E-83)}$$

II . FORCE AND MOMENT EQUILIBRIUM OF THE PINION

Force equilibrium of the pinion is expressed by

$$-F_{12}\bar{n}_{NF} + \mu s F_{12}\bar{n}_F + F_{x2}\bar{i} + F_{y2}\bar{j} + \mu F_{y2}\bar{i} - \mu F_{x2}\bar{j} = 0 \qquad \text{(E-84)}$$

Moment equilibrium requires that

$$M_o\bar{k} + \mu\rho_2\sqrt{F_{x2}^2 + F_{y2}^2}\,\bar{k} + g\bar{n}_F \times (-F_{12}\bar{n}_{NF}) = 0 \qquad \text{(E-85)}$$

Equation (E-84) furnishes the following component equations

$$F_{12}\sin(\psi + \alpha_p) + \mu s F_{12}\cos(\psi + \alpha_p) + F_{x2} + \mu F_{y2} = 0 \qquad \text{(E-86)}$$

and

$$-F_{12}\cos(\psi + \alpha_p) + \mu s F_{12}\sin(\psi + \alpha_p) + F_{y2} - \mu F_{x2} = 0 \qquad \text{(E-87)}$$

The scalar form of the moment equation (E-85) becomes

$$M_o + \mu\rho_2\sqrt{F_{x2}^2 + F_{y2}^2} - sF_{12} = 0 \qquad \text{(E-88)}$$

Simultaneous solution of equations (E-86) and (E-87) for F_{x2} and F_{y2} leads to

$$F_{x2} = -F_{12}\left[\frac{(1 - \mu^2 s)\sin(\psi + \alpha_p) + \mu(1 + s)\cos(\psi + \alpha_p)}{1 + \mu^2}\right] \qquad \text{(E-89)}$$

and

$$F_{y2} = F_{12}\left[\frac{(1 - \mu^2 s)\cos(\psi + \alpha_p) - \mu(1 + s)\sin(\psi + \alpha_p)}{1 + \mu^2}\right] \qquad \text{(E-90)}$$

The above expressions are now substituted into equation (E-88). Again, s^2 is unity and positive. The following expression for F_{12} is now obtained

$$F_{12} = \frac{M_o}{\mathcal{E} - \mu\rho_2} \qquad \text{(E-91)}$$

III. MOMENT INPUT-OUTPUT RELATIONSHIP

When equations (E-83) and (E-91) are set equal to each other, one obtains the following input-output relationship:

$$M_o = M_{in}\left[\frac{\mathcal{E} - \mu\rho_2}{\mu(\rho_1 + s_F\rho_G) - a_G[\cos(\phi - \delta_G - \psi - \alpha_p) + \mu s_F \sin(\phi - \delta_G - \psi - \alpha_p)]}\right]$$

(E-92)

Note that the signum symbol has been changed to s_F in the above expression to account for the fact that the expression is only valid in the round on flat phase of motion.

APPENDIX F

COMPUTER MODELS FOR A SINGLE STEP-UP GEAR MESH WITH CLOCK TEETH

The present appendix contains descriptions, listings and sample outputs of the following computer models relating to single step-up gear meshes containing clock (ogival) teeth

1. Program CLOCK 1: Kinematics of single pass step-up gear mesh with clock (ogival) teeth
2. Program CLOCK 2: Point and cycle efficiencies for single pass step-up gear mesh with clock (ogival) teeth

The relevant background, the input parameters, the manner of the computations, and the form of the output of each program are discussed in detail. The program proper forms the last part of each section.

1. Program CLOCK 1: Kinematics of Single Pass Step-Up Gear Mesh With Clock (Ogival) Teeth

The program CLOCK 1 is based upon that portion of Appendix E which deals with the kinematics of single mesh step-up clock gearing. It may be used to check on the geometric performance of clock type meshes.

The nomenclature of the program is chosen to coincide as much as possible with that of Appendix D as well as that of Appendix E.

a. Input Parameters (see Program CLOCK 1, below)

The following parameters represent the input data for the program:

CAPRP = R_P, the pitch radius of the gear

RP = r_P, the pitch radius of the pinion

AG = a_G, the distance from the center of rotation of the gear to the center of curvature of the circular arc portion of the gear tooth

AP = a_P, the distance from the center of rotation of the pinion to the center of curvature of the circular arc portion of the pinion tooth

RHOG = ρ_G, the radius of curvature of the circular arc portion of the gear tooth

RHOP = ρ_P, the radius of curvature of the circular arc portion of the pinion tooth

TG = t_G, the maximum thickness of the gear tooth

TP = t_p, the maximum thickness of the pinion tooth

NG = n_G, the number of teeth of the gear

NP = n_p, the number of teeth of the pinion

K = range divisor

PHIDOT = $\dot{\varphi}$ = 1, all velocity computations are based on this value

b. Computations (see also COMMENT cards in program)

I. Computation of Gear Tooth Parameters

The following tooth parameters of the gear as well as of the pinion are first computed:

For the gear

CXG = c_{xG} (see eq. (D-1))

DELG = δ_G (see eq. (D-2))

CYG = c_{yG} (see eq. (D-3))

ROG = r_{oG} (see eq. (D-4))

GAMMG = γ_G (see eq. (D-5))

ETMING = η_{min_G} (see eq. (D-8))

ETMAXG = η_{max_G} (see eq. (D-9))

For the pinion

CXP = c_{xp} (see eq. (D-1))

DELP = δ_p (see eq. (D-2))

CYP = c_{yp} (see eq. (D-3))

ROP = r_{op} (see eq. (D-4))

GAMMP = γ_p (see eq. (D-5))

ALPHP = α_p (see eq. (D-6))

ETMINP = η_{min_p} (see eq. (D-8))

ETMAXP = η_{max_p} (see eq. (D-9))

FP = f_p (see eq. (D-7))

In general

B = b, the center distance between the gear and the pinion

DPHI = $\Delta\varphi$, the angle between the center lines of adjacent gear teeth

DPSI = $\Delta\psi$, the angle between the center lines of adjacent pinion teeth

L (see eq. (E-10))

II. Determination of Transition Angle

In order to define the ranges for the round on round and the round on flat phases of motion, it is first necessary to locate the transition angles φ_T and ψ_T. This is accomplished with the help of the expression contained in section 1b-V, Appendix E. The transition angle ψ_T is first solved according to eq. (E-42). Since the solution furnishes two answers, i.e., the angles ψ_{T1} and ψ_{T2}, it is necessary to decide which of these is the desired one. The principal criterion for selecting the correct value of ψ_T is based on the motion of the contact point T with respect to the pinion. The first transition angles ψ_{T1} and φ_{T1} occur as the contact point moves from the round portion of the pinion to the flat one (for increasing values of φ). As this motion is continued, the distance g becomes smaller than the transition parameter f_p. Once g has reached its minimum, the contact point moves outward on the

pinion flat until g theoretically equals f_p once again. This part of the motion is never completed in actuality since the subsequent set of teeth takes over before g reaches this transition value. It is this second transition situation which corresponds to the transition angles ψ_{T2} and φ_{T2}. Since the round on flat equation does not recognize any limitation on the length of the pinion flat, an increase of φ over the value of φ_{T2} will have associated with it a theoretical value for g which is larger than f_p. Thus, one may identify the correct value of φ_T and ψ_T by noting that an increase in the angle φ must lead to an associated value of g which is smaller than f_p.

The program uses this criterion in the following manner after ψ_{T1} and ψ_{T2} are known:

A. Subroutine TRANS is called and the angle φ_{T1}, which is associated with ψ_{T1}, is computed with the help of eqs. (E-43) and (E-44).

B. The angle φ is made slightly larger than φ_{T1} to produce PHINEXT, and eq. (E-29) is used to find the associated angle PSINEX. Since there are two such angles, the subroutine selects the one which is closest to the value of transition angle ψ_{T1}. Subsequently, the associated value of g_1 is computed according to eq. (E-27).

C. Steps A and B are then repeated identically for the second transition angle ψ_{T2}. This results in the determination of g_2.

D. Control remains with the main program, and that value of ψ_T is chosen for which the associated value of g is smaller

than f_p.

For checking purposes, a subsidiary test for the determination of the transition angle was added to the program. It is based on the idea that, for the correct transition angle ψ_T, the line representing the flat portion of the pinion will make a smaller angle with the center line $O_G O_P$ than will be the case for the incorrect one. To this end, TEST1 and TEST2 find both angles with the help of

$$TEST = 360^\circ - (\psi_T + \alpha_p) \qquad (F\text{-}1)$$

III. Determination of Correct Sign for Round on Flat Regime

The sign preceding the square root in eq. (E-29), for the round on flat regime, is determined with the help of the angle φ_T. The condition which yields that angle ψ which is closest to ψ_T governs. The variable SIGNF is used for the sign under consideration.

IV. Computation of Final and Initial Values of φ and ψ

The final and initial values of the gear and pinion angles are found by continuously evaluating the round on flat equation (E-29), using the previously determined value of SIGNF, and simultaneously checking the contact condition for the subsequent set of teeth, as given by eq. (E-48). This loop is initiated at the transition angle and terminated once the condition of eq. (E-48) is met. This allows the simultaneous determination

of both the angles at which the first set of teeth loses contact and at which the second set of teeth comes into engagement. The latter is accomplished by subtracting $\Delta\varphi$ from the "loss of contact" angle φ_F and adding $\Delta\psi$ to the "loss of contact" angle ψ_F (see computations following statement label no. 70). PHIF and PSIFF represent the angles when contact is lost for a given mesh, while PHII and PSII stand for the angles when initial contact is made.

V. Determination of Correct Sign for Round on Round Regime

Eq. (E-13) is used to determine the angle ψ while the gear and pinion are in the round on round regime. The correct sign in eq. (E-13) is obtained by comparing the value of the angle ψ, as computed with PHII, with the value of PSII. SIGNR is the variable which furnishes this sign.

VI. Kinematics

Since the limits as well as the correct signs for both the round on round and the round on flat regimes are known, the various kinematic properties of interest may now be computed.

The angular increment DDPHI of the input angle is found by dividing the range from PHII to PHIF into K parts. (K is the range divisor.) The computational loop begins with statement label no. 110 and ends with the third card from the end of the main program. The computation is terminated when the angle φ exceeds PHIF.

A. Round on Round Regime

As long as PHI is smaller than the transition angle PHIT, the kinematics of the round on round phase of motion is computed. This results in the following:

PSI = ψ (see eq. (E-13))

LAMDA = λ (see eqs. (E-14) and (E-15))

PSIDOT = $\dot{\psi}$ (see eq. (E-16))

VSTR = $\bar{V}_{S/T}$ for round on round (see eq. (E-20))

SR = s (see eq. (E-50) for later use)

ETAPR = η_P (see eqs. (E-6a) and (E-6b))

ETAGR = η_G (see eq. (E-5). For discussion of the usefulness of η_P and η_G, see output in section c below.)

B. Round on Flat Regime

When PHI is larger than the transition angle PHIT, the kinematics of the round on flat phase is computed. This computation includes the following:

PSI = ψ (see eq. (E-29))

G = g (see eq. (E-27))

PSIDOT = $\dot{\psi}$ (see eq. (E-30))

VSTF = $\bar{V}_{S/T}$ for round on flat (see eq. (E-35))

SF = s (see eq. (E-72) for later use)

ETAGF = η_{GF} (see eqs. (E-6c) and (E-6d))

c. Output (see Program CLOCK 1, below)

As previously, the output of the program is best explained with the help of the sample problem at the end of the program. It contains the following:

I. Input Parameters

CAPRP = R_p = .47725 in. (1.212 cm)

RP = r_p = .09085 in. (0.231 cm)

AG = a_G = .47725 in. (1.212 cm)

AP = a_p = .09085 in. (0.231 cm)

RHOG = ρ_G = .03870 in. (0.098 cm)

RHOP = ρ_p = .01740 in. (0.044 cm)

TG = t_G = .03480 in. (0.088 cm)

TP = t_p = .02800 in. (0.071 cm)

NG = n_G = 42

NP = n_p = 8

K = 50

II. Computed Values

The following parameters are printed out:

DELGD = δ_G = 2.5880°

DELPD = δ_p = 2.1448°

GAMMPD = γ_p = 11.0418°

ALPHPD = α_p = 8.8970°

ETMINGD = η_{min_G} = 85.3488°

ETMAXGD = η_{max_G} = 144.0484°

ETMINPD = η_{min_P} = 78.9582°

ETMAXPD = η_{max_P} = 166.5870°

FP = f_p = .08917 in. (0.226 cm)

The printout concerning the transition angles consists of two lines. While the program automatically picks that transition configuration which leads to a decreasing value of g as φ is increased, both transition angles are printed out for checking purposes. Thus, one finds

φ_{T1} = 186.36° ψ_{T1} = 308.63° g_1 = .0895 in. (0.227 cm) TEST1 = 42.47°

φ_{T2} = 178.75° ψ_{T2} = 346.65° g_2 = .0888 in. (0.226 cm) TEST2 = 4.45°

Clearly, the second line represents the solution since g_2 = .0888 for a slight increase in the angle φ over φ_T. This is less than the value of f_p = .08917. In addition, TEST2 furnishes the smaller number of degrees. Recall that TEST represents the angle between the flat of the pinion and the line connecting the pivots.

Furthermore, the program shows the initial and final angles of contact:

PHIID = the angle φ at initial contact

PSIID = the angle ψ at initial contact

PHIFD = the angle φ at final contact

PSIFD = the angle ψ at final contact

Note that angle φ increases while the angle ψ decreases from the beginning to the end of the contact.

The computational loop begins with PHIID and ends with PHIFD. Further, when PHID reaches the approximate value of PHITD, the output shifts from round on round parameters to round on flat ones. The purpose of the multiple output throughout the motion of the mesh is to gain insight concerning the behavior of the mesh as well as to be able to check geometric values.

The following conclusions may be drawn for either of the phases as well as in general for the gears under consideration.

A. Round on Round Phase ($176.2623^\circ < \varphi < 178.6623^\circ$)

PSIDOT, the angular velocity of the pinion, is negative, and at all times near the "gear ratio" of 42/8 = 5.25.

SR is printed here only for checking purposes. It becomes important in the moment input-output analyses of program CLOCK 2.

The angles ETAGRD and ETAPRD are of interest because one needs to make sure that the contact between gear and pinion does not occur too close to the respective tooth tips. This is necessary since the present mathematical model has a pointed tip while in a real situation the tips are rounded. Thus, if contact is sufficiently far from the tips of the teeth, the present model will give valid answers.

The range of ETAGRD is approximately between 86° and 90°. This is considerably smaller than the ETMAXGD of approximately 144° and larger than the ETMINGD of approximately 85°. (The

latter shows that the round of the pinion tooth does not touch the flat of the gear tooth.)

The angle ETAPRD of the pinion tooth is always considerably smaller than ETMAXPD. Since contact is transferred to the flat portion of the pinion at the end of this phase of motion, ETAPRD almost equals ETMINPD at that point.

LAMDAD is given for general checking purposes only.

B. Round on Flat Phase ($178.6623^\circ < \varphi < 184.834^\circ$)

The angular velocity $\dot{\psi}$ continues relatively smoothly after the transition. The distance g remains smaller than f_p, as expected, and it reaches a minimum of .0822 in. at $\varphi \approx 182.5^\circ$. It is further to be noted that g never reaches the value of f_p again since the subsequent set of teeth takes over when g = .08439 in. (0.214 cm). For the present program to be valid, it is necessary that contact ends before the round on flat phase is completed.

As before, s_F is given for checking purposes only, and its value has been confirmed, just as was done for s_R, by graphical analysis (not shown).

The angle ETAGFD of the gear reaches a maximum of approximately 130° at the end of this phase. Since this is well below the maximum value ETMAXGD = 144°, there is enough margin for a tip radius on the gear tooth.

C. General Considerations

As expected for a direct contact mechanism of this type, the angular velocity ratio, as represented by PSIDOT, is not constant and has a greater absolute value at initial contact than at final contact. This indicates that the pinion will speed up somewhat as the subsequent set of teeth comes into contact, and that therefore, the original set of teeth loses contact at that instant. This means that the "effective contact ratio" is unity.

Program CLOCK 1

```
PROGRAM CLOCK1     73/74  OPT=1                FTN 4.6+420     89/12/78  16.25.30     PAGE  1

      1            PROGRAM CLOCK 1(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
                   C
                   C  KINEMATICS OF SINGLE PASS STEP-UP GEAR MESH WITH CLOCK (OGIVAL) TEETH
                   C
      5            REAL NP,NG,L,LX,LY,X,LAMDA,LAMDAD
                   READ(5,1)CAPRP,RP
                 1 FORMAT(2F10.5)
                   READ(5,1)AG,AP
                   READ(5,1)RHOG,RHOP
     10            READ(5,1)TG,TP
                   READ(5,2)NG,NP
                 2 FORMAT(2F10.0)
                   READ(5,3)K
                 3 FORMAT(F10.0)
     15            PI = 3.14159
                   Z = PI/180.
                   PHIDOT = 1.
                   WRITE(6,4)CAPRP,RP,AG,AP
                 4 FORMAT('1',5X,'CAPRP =',F7.5,3X,'RP =',F7.5,3X,'AG =',F7.5,3X,
     20           1'AP =',F7.5/)
                 5 FORMAT(6X,'RHOG =',F7.5,3X,'RHOP =',F7.5/)
                   WRITE(6,5)RHOG,RHOP
                   WRITE(6,6)TG,TP
                 6 FORMAT(6X,'TG =',F7.5,3X,'TP =',F7.5/)
     25            WRITE(6,7)NG,NP
                 7 FORMAT(6X,'NG =',F4.0,3X,'NP =',F4.0/)
                   WRITE(6,8)PHIDOT
                 8 FORMAT(6X,'PHIDOT =',F5.1/)
                   WRITE(6,9)K
     30          9 FORMAT(6X,'K =',F5.0///)
                   C
                   C  COMPUTATION OF GEAR TOOTH PARAMETERS
                   C
                   CXG = RHOG - TG/2.
     35            DELG = ASIN(CXG/AG)
                   DELGD = DELG/Z
                   CYG = AG*COS(DELG)
                   RUG = CYG + SQRT(RHOG*RHOG - CXG*CXG)
                   GAMMG = ASIN(RHOG/AG)
     40            GAMMGD = GAMMG/Z
                   ETAINGD = 90. - GAMMGD
                   ETAMAXGD = 180. - ASIN(HOG*SIN(DELG)/RHOG)/Z
                   CXP = RHOP - TP/2.
                   DELP = ASIN(CXP/AP)
     45            DELPD = DELP/Z
                   CYP = AP*COS(DELP)
                   RUP = CYP + SQRT(RHOP*RHOP - CXP*CXP)
                   GAMMP = ASIN(RHOP/AP)
                   GAMMPD = GAMMP/Z
     50            ALPHP = GAMMP - DELP
                   ALPHPD = ALPHP/Z
                   ETAINPD = 90. - GAMMPD
                   ETAMAXPD = 180. - ASIN(RGP*SIN(DELP)/RHOP)/Z
```

```
      FP = AP*COS(GAMMP)
   55 B = CAPMP + AP
      DPHI = 360./NG*Z
      DPSI = 360./NP*Z
      L = RMOG + RMOP
      WRITE(6,10)DELGD,DELPD,GAMMPD,ALPHPD,ETMINGD,ETMAXGD,ETMINPD,
     1ETMAXPD,FP
   60 10 FORMAT(6X,*DELGD =*,F9.4,3X,*DELPD =*,F9.4//6X,*GAMMPD =*,F9.4,3X
     1,*ALPHPD =*,F9.4//6X,*ETMINGD =*,F9.4,3X,*ETMAXGD =*,F9.4,3X,*ETMI
     2NPD =*,F9.4,3X,*ETMAXPD =*,F9.4//6X,*FP =*,F7.5///)
      C
   65 C  DETERMINATION OF TRANSITION ANGLE
      C
      AT = RMOG*COS(ALPHP) - FP*SIN(ALPHP)
      BT = RMOG*SIN(ALPHP) - FP*COS(ALPHP)
      CT = (AG*AG - FP**2 - B*B - RMOG*RMOG)/(2.*B)
   70 ROOTT = AT*AT + BT*BT - CT*CT
      YT1 = AT + SQRT(ROOTT)
      YT2 = AT - SQRT(ROOTT)
      XT = BT + CT
      PSIT1 = 2.*ATAN2(YT1,XT)
   75 PSIT2 = 2.*ATAN2(YT2,XT)
      IF(PSIT1 .LT. 0.)PSIT1 = PSIT1 + 2.*PI
      IF(PSIT2 .LT. 0.)PSIT2 = PSIT2 + 2.*PI
      PSIT1D = PSIT1/Z
      PSIT2D = PSIT2/Z
   80 CALL TRANS(RMOG,ALPHP,FP,AG,B,DELG,Z,PSIT1,PHIT1,G1)
      CALL TRANS(RMOG,ALPHP,FP,AG,B,DELG,Z,PSIT2,PHIT2,G2)
      PHIT1D = PHIT1/Z
      PHIT2D = PHIT2/Z
      TEST1 = 360. - (PSIT1D + ALPHPD)
   85 TEST2 = 360. - (PSIT2D + ALPHPD)
      WRITE(6,11)PHIT1D,PSIT1D,G1,TEST1
      WRITE(6,12)PHIT2D,PSIT2D,G2,TEST2
   11 FORMAT(5X,*PHIT1D =*,F9.4,3X,*PSIT1D =*,F9.4,3X,*G1 =*,F7.4,3X,
     1*TEST1 =*,F7.2/)
   90 12 FORMAT(6X,*PHIT2D =*,F9.4,3X,*PSIT2D =*,F9.4,3X,*G2 =*,F7.4,3X,
     1*TEST2 =*,F7.2///)
      IF(G1 .LT. FP)GO TO 20
      PHIT = PHIT2
      PSIT = PSIT2
   95 GO TO 30
   20 PHIT = PHIT1
      PSIT = PSIT1
   30 PHITD = PHIT/Z
      PSITD = PSIT/Z
  100 C
      C  DETERMINATION OF CORRECT SIGN FOR ROUND ON FLAT REGIME
      C
      AF = B*COS(ALPHP) - AG*COS(PHIT-DELG-ALPHP)
      BF = B*SIN(ALPHP) - AG*SIN(PHIT-DELG-ALPHP)
  105 CF = -RMOG
      ROOTF = AF*AF + BF*BF - CF*CF
```

```
      YF1 = AF + SQRT(RDOTF)
      YF2 = AF - SQRT(RDOTF)
      XF = BF + CF
      PSIF1 = 2.*ATAN2(YF1,XF)
      PSIF2 = 2.*ATAN2(YF2,XF)
      IF(PSIF1 .LT. 0.)PSIF1 = PSIF1 + 2.*PI
      IF(PSIF2 .LT. 0.)PSIF2 = PSIF2 + 2.*PI
      IF(ABS(PSIF1-PSIT) .LT. ABS(PSIF2-PSIT))GO TO 40
      SIGNF = -1.
      GO TO 50
   40 SIGNF = 1.
C
C  COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI
C
   50 DO 60 I=1,1000
      PHID = PHITD + (I-1.)/1000.
      PHI = PHID*Z
      AF = B*COS(ALPHP) + AG*COS(PHI-DELG-ALPHP)
      BF = B*SIN(ALPHP) - AG*SIN(PHI-DELG-ALPHP)
      CF = -RHOG
      RDOTF = AF*AF + BF*BF - CF*CF
      YF = AF + SIGNF*SQRT(RDOTF)
      XF = BF + CF
      PSIF = 2.*ATAN2(YF,XF)
      IF(PSIF .LT. 0.)PSIF = PSIF + 2.*PI
      CX = B + AG*COS(PHI-DELG-DPHI) - AP*COS(PSIF-DPSI-DELP)
      CY = AG*SIN(PHI-DELG-DPHI) - AP*SIN(PSIF-DELP-DPSI)
      IF(SQRT(CX*CX + CY*CY) .LE. 1)GO TO 70
   60 CONTINUE
   70 PHIF = PHI
      PSIFF = PSIF
      PHII = PHIF - DPHI
      PSII = PSIFF - DPSI
      IF(PSII.GT.2.*PI)PSII=PSII-2.*PI
      PHIID = PHII/Z
      PSIID = PSII/Z
      PHIFD = PHIF/Z
      PSIFD = PSIFF/Z
      WRITE(6,80)PHIID,PSIID,PHIFD,PSIFD
   80 FORMAT(6X,'PHIID =',F9.4,3X,'PSIID =',F9.4,3X,'PHIFD =',F9.4,3X,
     1'PSIFD =',F9.4///)
C
C  DETERMINATION OF CORRECT SIGN FOR POUND ON ROUND REGIME
C
      AR = SIN(PHII-DELG) + COS(PHII-DELG)*TAN(DELP) + B*TAN(DELP)/AG
      BR = COS(PHII-DELG) - B/AG - SIN(PHII-DELG)*TAN(DELP)
      CR = (AP*AP + AG*AG + B*B - L*L)/(2.*AP*AG*COS(DELP))
     1 + B*COS(PHII-DELG)/(AP*COS(DELP))
      RDOTR = AR*AR + BR*BR - CR*CR
      YR1 = BR + SQRT(RDOTR)
      YR2 = AR - SQRT(RDOTR)
      XR = BR + CR
      PSIR1 = 2.*ATAN2(YR1,XR)
```

```
      PSIN2 = 2.*ATAN2(YN2,XN)
      IF(PSIN1 .LT. 0.)PSIN1 = PSIN1 + 2.*PI
      IF(PSIN2 .LT. 0.)PSIN2 = PSIN2 + 2.*PI
      IF(ABS(PSII-PSIN1) .LT. ABS(PSII-PSIN2))GO TO 90
      SIGNN = -1.
      GO TO 100
   90 SIGNN = 1.
C
C  KINEMATICS
C
  100 DOPHI = (PHIF-PHII)/K
      PHI = PHII - DOPHI
  110 PHI = PHI + DOPHI
      IF(PHI .GE. PHIF) GO TO 200
      PHIO = PHI/Z
      IF(PHI .GE. PHIT)GO TO 120
      AN = SIN(PHI-DELG) + COS(PHI-DELG)*TAN(DELP) + B*TAN(DELP)/AG
      BN = COS(PHI-DELG) + B/AG - TAN(DELP)*SIN(PHI-DELG)
      CN = (AP*AP + AG*AG + B*B - L*L)/(2.*AP*AG*COS(DELP))
     1+ B*COS(PHI-DELG)/(AP*COS(DELP))
      ROOTN = AN*AN + BN*BN - CN*CN
      YN = AN+SIGNN*SQRT(ROOTN)
      XN = BN + CN
      PSI = 2.*ATAN2(YN,XN)
      IF(PSI .LT. 0.)PSI = PSI + 2.*PI
      PSIO = PSI/Z
      SLAN = (AP*SIN(PSI-DELP) - AG*SIN(PHI-DELG))/L
      CLAN = (AP*COS(PSI-DELP) - AG*COS(PHI-DELG) - B)/L
      LAMDA = ATAN2(SLAN,CLAN)
      IF(LAMDA .LT. 0.)LAMDA = LAMDA + 2.*PI
      LAMDAD = LAMDA/Z
      ANO = TAN(DELP)*SIN(PHI-DELG) - COS(PHI-DELG)
      BNO = SIN(PHI-DELG) + TAN(DELP)*COS(PHI-DELG)
      CNO = B*SIN(PHI-DELG)/(AP*COS(DELP))
      PSIDOT = (ANO*SIN(PSI) + BNO*COS(PSI) - CNO)/(AN*COS(PSI) - BN*
     1SIN(PSI))*PHIDOT
      VSTN = PHIDOT*(AG*COS(PHI-DELG-LAMDA) + HNOG) - PSIDOT*(AP*
     1COS(PSI-DELP-LAMDA) - HNOP)
      SN = VSTN/ABS(VSTN)
      IF(PSID-DELPD .LT. 90.)ETAPRD = (2.*PI + PSI - LAMDA - DELP)/Z
      IF(PSID-DELPD .GE. 90.)ETAPRD = (PSI-LAMDA-DELP)/Z
      ETAGRD = (PI + PHI - LAMDA - DELG)/Z
      WRITE(6,115)PHIO,PSIO,PSIDOT,SN,ETAGRD,ETAPRD,LAMDAD
  115 FORMAT(6X,*PHIO =*,F9.4,3X,*PSIO =*,F9.4,3X,*PSIDOT =*,F8.4,3X,
     1*SN =*,F6.1,3X,*ETAGRD =*,F9.4,3X,*ETAPRD =*,F9.4,3X,*LAMDAD =*,
     2F9.4)
      GO TO 110
  120 AF = B*COS(ALPHP) + AG*COS(PHI-DELG-ALPHP)
      BF = B*SIN(ALPHP) - AG*SIN(PHI-DELG-ALPHP)
      CF = -RHOG
      ROOTF = AF*AF + BF*BF - CF*CF
      YF = AF + SIGNF*SQRT(ROOTF)
      XF = BF + CF
```

```
      PSI = 2.*ATAN2(YF,XF)
      IF(PSI .LT. 0.)PSI = PSI + 2.*PI
  215 PSID = PSI/Z
      B = (AG*SIN(PHI-DELG) - RHOG*COS(PSI*ALPHP))/SIN(PSI*ALPHP)
      AFD = AG*SIN(PHI-DELG-ALPHP)
      BFD = AG*COS(PHI-DELG-ALPHP)
      PSIDOT = PHIDOT*(AFD*SIN(PSI) + BFD*COS(PSI))/(AF*COS(PSI) - BF*
  220 1SIN(PSI))
      VSTF = PHIDOT*(RHOG + AG*SIN(PSI-PHI+ALPHP+DELG))
      SF = VSTF/ABS(VSTF)
      IF(PSID.ALPHPD.GE.90.)ETAGFD=(PHI-DELG-PSI-ALPHP+3.*PI/2.)/Z
      IF(PSID.ALPHPD.LT.90.)ETAGFD=(PHI-DELG-PSI-ALPHP+.5*PI)/Z
  225 WRITE(6,13)PHID,PSID,PSIDOT,B,SF,ETAGFD
   13 FORMAT(5X,*PHID =*,F9.4,3X,*PSID =*,F9.4,3X,*PSIDOT =*,F8.4,3X,
     1*B =*,F7.5,3X,*SF =*,F4.1,3X,*ETAGFD =*,F9.4)
      GO TO 110
   20 STOP
  230 END
```

```
      SUBROUTINE TRANS(RHOG,ALPHP,FP,AG=0,DELG=0,PSIT,PHIT,G)
      PI = 3.1+159
      ST = (FP*SIN(PSIT+ALPHP) + RHOG*COS(PSIT+ALPHP))/AG
      CT = (FP*COS(PSIT+ALPHP) - RHOG*SIN(PSIT+ALPHP) - B)/AG
      PHIT = ATAN2(ST,CT) + DELG
      IF(PHIT .LT. 0.)PHIT = PHIT + 2.*PI
      PHINXT = PHIT + .1*Z
      AF = B*COS(ALPHP) + AG*COS(PHINXT - DELG - ALPHP)
      BF = B*SIN(ALPHP) - AG*SIN(PHINXT - DELG - ALPHP)
      CF = -RHOG
      ROOTF = AF*AF + BF*BF - CF*CF
      YF1 = AF + SQRT(ROOTF)
      YF2 = AF - SQRT(ROOTF)
      BF = BF + CF
      PSINXT1 = 2.*ATAN2(YF1,BF)
      PSINXT2 = 2.*ATAN2(YF2,BF)
      IF(PSINXT1 .LT. 0.)PSINXT1 = PSINXT1 + 2.*PI
      IF(PSINXT2 .LT. 0.)PSINXT2 = PSINXT2 + 2.*PI
      IF(ABS(PSINXT1-PSIT) .LT. ABS(PSINXT2-PSIT))GO TO 1
      PSINXT = PSINXT2
      GO TO 2
    1 PSINXT = PSINXT1
    2 G = (AG*SIN(PHINXT-DELG) - RHOG*COS(PSINXT+ALPHP))/SIN(PSINXT
     1-ALPHP)
      RETURN
      END
```

CA^DP = -.47725 dp = -.09985 AB = .47725 AP = .09885

PHIS = .83878 PHIP = .81748

TS = .83488 TP = .82889

NG = 47. NP = 8.

PHIDOT = 1.0

K = 58.

DELGD = 2.5540 DELPD = 2.1446

GA^PD = 11.8913 ALPHPD = 8.8979

ETHIGD = 85.3485 ETHAGD = 144.8484 ETHIPD = 78.9582 ETHAPD = 146.0878

FP = .8891?

PHITID = 146.3623 PSITID = 348.0298 G1 = .0095 TEST1 = 42.47

PHIT2D = 178.7537 PSIT2D = 346.0548 G2 = .0488 TEST2 = 4.45

PHIID = 176.2623 PSIID = 359.8278 PHIFD = 184.8337 PSIFD = 314.9278

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PHID = 176.2623	PSID = 359.8195	PSIDOT = -5.2458	SR = -1.0	ETAGRD = 84.7219	ETAPRD = 90.6923	LAMDAD = 266.9824
PHID = 176.4237	PSID = 358.9204	PSIDOT = -5.2451	SR = -1.0	ETAGRD = 84.8959	ETAPRD = 89.7958	LAMDAD = 267.0698
PHID = 176.6852	PSID = 358.0211	PSIDOT = -5.2448	SR = -1.0	ETAGRD = 85.0172	ETAPRD = 88.7484	LAMDAD = 267.1388
PHID = 176.7700	PSID = 357.1214	PSIDOT = -5.2501	SR = -1.0	ETAGRD = 87.0557	ETAPRD = 87.8137	LAMDAD = 267.1629
PHID = 176.9488	PSID = 356.2218	PSIDOT = -5.2546	SR = -1.0	ETAGRD = 87.2214	ETAPRD = 86.9877	LAMDAD = 267.1686
PHID = 177.1194	PSID = 355.3198	PSIDOT = -5.2695	SR = -1.0	ETAGRD = 87.4345	ETAPRD = 86.0281	LAMDAD = 267.1469
PHID = 177.2989	PSID = 354.4174	PSIDOT = -5.2674	SR = -1.0	ETAGRD = 87.6351	ETAPRD = 85.1748	LAMDAD = 267.0978
PHID = 177.4623	PSID = 353.5138	PSIDOT = -5.2752	SR = -1.0	ETAGRD = 87.8831	ETAPRD = 84.3478	LAMDAD = 267.0212
PHID = 177.6337	PSID = 352.6087	PSIDOT = -5.2839	SR = -1.0	ETAGRD = 88.1587	ETAPRD = 83.5469	LAMDAD = 266.9178
PHID = 177.6652	PSID = 351.7021	PSIDOT = -5.2933	SR = -1.0	ETAGRD = 88.4628	ETAPRD = 82.7722	LAMDAD = 266.7852
PHID = 177.9788	PSID = 350.7938	PSIDOT = -5.3033	SR = 1.0	ETAGRD = 88.7931	ETAPRD = 82.0235	LAMDAD = 266.6255
PHID = 178.1488	PSID = 349.8838	PSIDOT = -5.3137	SR = 1.0	ETAGRD = 89.1521	ETAPRD = 81.3011	LAMDAD = 266.4388
PHID = 178.3194	PSID = 348.9728	PSIDOT = -5.3244	SR = 1.0	ETAGRD = 89.5398	ETAPRD = 80.6048	LAMDAD = 266.2224
PHID = 178.4889	PSID = 348.0583	PSIDOT = -5.3353	SR = 1.0	ETAGRD = 89.9541	ETAPRD = 79.9347	LAMDAD = 265.9788
PHID = 178.6623	PSID = 347.1427	PSIDOT = -5.3462	SR = 1.0	ETAGRD = 90.3973	ETAPRD = 79.2909	LAMDAD = 265.7078
PHID = 178.8337	PSID = 346.2252	PSIDOT = -5.3568	G = .00289	SF = 1.0	ETAGFD = 91.1536	
PHID = 179.0052	PSID = 345.3021	PSIDOT = -5.4032	G = .00831	SF = 1.0	ETAGFD = 92.2488	
PHID = 179.1788	PSID = 344.3738	PSIDOT = -5.4294	G = .00775	SF = 1.0	ETAGFD = 93.3477	
PHID = 179.3488	PSID = 343.4488	PSIDOT = -5.4554	G = .00722	SF = 1.0	ETAGFD = 94.4522	
PHID = 179.5194	PSID = 342.5036	PSIDOT = -5.4781	G = .00671	SF = 1.0	ETAGFD = 95.5608	
PHID = 179.6984	PSID = 341.5629	PSIDOT = -5.4974	G = .00623	SF = 1.0	ETAGFD = 96.6731	
PHID = 179.8623	PSID = 340.6199	PSIDOT = -5.5131	G = .00577	SF = 1.0	ETAGFD = 97.7883	
PHID = 180.0337	PSID = 339.6729	PSIDOT = -5.5252	G = .00534	SF = 1.0	ETAGFD = 98.9059	
PHID = 180.2052	PSID = 338.7248	PSIDOT = -5.5335	G = .00493	SF = 1.0	ETAGFD = 100.0253	
PHID = 180.3788	PSID = 337.7758	PSIDOT = -5.5380	G = .00455	SF = 1.0	ETAGFD = 101.1458	
PHID = 180.5488	PSID = 336.8263	PSIDOT = -5.5386	G = .00420	SF = 1.0	ETAGFD = 102.2667	
PHID = 180.7194	PSID = 335.8771	PSIDOT = -5.5353	G = .00388	SF = 1.0	ETAGFD = 103.3874	
PHID = 180.8989	PSID = 334.9281	PSIDOT = -5.5288	G = .00359	SF = 1.0	ETAGFD = 104.5071	
PHID = 181.0623	PSID = 333.9829	PSIDOT = -5.5168	G = .00331	SF = 1.0	ETAGFD = 105.6253	
PHID = 181.2337	PSID = 333.0375	PSIDOT = -5.5015	G = .00307	SF = 1.0	ETAGFD = 106.7412	

PHID = 181.4834 PSID = 332.0960 PSIDOT = -5.4823 G = .00286 SF = 1.0 ETAGFD = 107.8542
PHID = 181.5700 PSID = 331.1581 PSIDOT = -5.4591 G = .00268 SF = 1.0 ETAGFD = 108.8635
PHID = 181.7488 PSID = 330.2245 PSIDOT = -5.4329 G = .00252 SF = 1.0 ETAGFD = 110.0605
PHID = 181.9194 PSID = 329.2959 PSIDOT = -5.4011 G = .00240 SF = 1.0 ETAGFD = 111.1606
PHID = 182.0909 PSID = 328.3729 PSIDOT = -5.3663 G = .00230 SF = 1.0 ETAGFD = 112.2629
PHID = 182.2623 PSID = 327.4562 PSIDOT = -5.3279 G = .00224 SF = 1.0 ETAGFD = 113.3511
PHID = 182.4337 PSID = 326.5464 PSIDOT = -5.2859 G = .00220 SF = 1.0 ETAGFD = 114.4323
PHID = 182.6052 PSID = 325.6441 PSIDOT = -5.2403 G = .00220 SF = 1.0 ETAGFD = 115.5068
PHID = 182.7766 PSID = 324.7499 PSIDOT = -5.1915 G = .00222 SF = 1.0 ETAGFD = 116.5717
PHID = 182.9480 PSID = 323.8644 PSIDOT = -5.1394 G = .00227 SF = 1.0 ETAGFD = 117.6286
PHID = 183.1194 PSID = 322.9880 PSIDOT = -5.0843 G = .00235 SF = 1.0 ETAGFD = 118.6704
PHID = 183.2909 PSID = 322.1213 PSIDOT = -5.0263 G = .00246 SF = 1.0 ETAGFD = 119.7145
PHID = 183.4623 PSID = 321.2648 PSIDOT = -4.9656 G = .00260 SF = 1.0 ETAGFD = 120.7425
PHID = 183.6337 PSID = 320.4198 PSIDOT = -4.9023 G = .00277 SF = 1.0 ETAGFD = 121.7597
PHID = 183.8052 PSID = 319.5842 PSIDOT = -4.8367 G = .00297 SF = 1.0 ETAGFD = 122.7660
PHID = 183.9766 PSID = 318.7608 PSIDOT = -4.7689 G = .00320 SF = 1.0 ETAGFD = 123.7608
PHID = 184.1480 PSID = 317.9492 PSIDOT = -4.6990 G = .00346 SF = 1.0 ETAGFD = 124.7438
PHID = 184.3194 PSID = 317.1498 PSIDOT = -4.6274 G = .00374 SF = 1.0 ETAGFD = 125.7146
PHID = 184.4909 PSID = 316.3628 PSIDOT = -4.5542 G = .00405 SF = 1.0 ETAGFD = 126.6731
PHID = 184.6623 PSID = 315.5885 PSIDOT = -4.4795 G = .00439 SF = 1.0 ETAGFD = 127.6188
PHID = 184.8337 PSID = 314.8270 PSIDOT = -4.4036 G = .00476 SF = 1.0 ETAGFD = 128.5517

2. Program Clock 2: Point and Cycle Efficiencies for Single Pass Step-Up Gear Mesh With Clock (Ogival) Teeth

The program CLOCK 2 furnishes the moment input-output relationship for a single step-up gear mesh with clock (ogival) teeth as derived in section 2 of Appendix E. This program uses the same kinematics as program CLOCK 1, and it differs from that program only in that it also determines both point and cycle efficiencies. Because of the above, sections a to c below will only contain discussions on those portions of the program which are different from CLOCK 1. The last section shows the complete program CLOCK 2.

a. Input Parameters (see Program CLOCK 2, below)

In addition to the input parameters of CLOCK 1, the following data must be supplied to the program:

MU = μ, the coefficient of friction at the gear and pinion pivots as well as at the contact point between the gear and the pinion

RHO1 = ρ_1, the pivot radius of the gear

RHO2 = ρ_2, the pivot radius of the pinion

b. Computations (See also COMMENT cards in program)

I. Computation of Gear Tooth Parameters

The computation of the gear tooth parameters is identical

with that given in program CLOCK 1.

II. Determination of Transition Angle

The transition angle is determined in the same manner as described in program CLOCK 1.

III. Determination of Correct Sign for Round on Flat Regime

This computation is also identical with that given in program CLOCK 1.

IV. Computation of Final and Initial Values of φ and ψ

This computation is also identical with that given in program CLOCK 1.

V. Determination of Correct Sign for Round on Round Regime

This computation is also identical with that given in program CLOCK 1.

VI. Kinematics, Point and Cycle Efficiencies

The angular increment DDPHI of the input angle is found in the same manner as shown in program CLOCK 1. While the initial, transition and final angles of the mesh are obtained in the same manner as shown in CLOCK 1, the computational loop of the program ranges from the initial angle PHII to one increment before the final angle PHIF. This is necessary in

order to accommodate the numerical integration for the cycle efficiency.

A. Round on Round Regime

As before for CLOCK 1, as long as PHI is smaller than the transition angle PHIT, the kinematics of the round on round phase of the motion is computed for each increment. These kinematic computations are identical with those given in CLOCK 1. The point efficiency in this phase of motion is obtained with the help of eq. (E-70) and takes the form of eq. (3), i.e.,

$$\eta_p = K_{ratio} \frac{M_o}{M_{in}} \tag{F-2}$$

Since the transmission ratio depends on the angular velocity ratio, and the input angular velocity is unity, eq. (F-2) becomes

$$\eta_p = \frac{M_o}{M_{in}} \left|\dot{\psi}\right| \quad \text{(POINTEF)} \tag{F-3}$$

B. Round on Flat Regime

When PHI is larger than the transition angle PHIT, the kinematics of the round on flat phase becomes applicable. While the computed kinematic quantities are identical with those of CLOCK 1, the present program contains an expression for the point efficiency for the round on flat phase of the motion. This expression is obtained with the help of eq. (E-92), and it

is computed in the manner of eq. (F-3) with the now current values of the output angular velocity PSIDOT.

C. Computation of Cycle Efficiency

Once the computational loop is terminated, the cycle efficiency is computed in the manner of eq. (C-10) of Appendix C, i.e.,

$$\eta_C = \frac{\Delta\varphi \; \Sigma \eta_p}{(\varphi_{FIN} - \varphi_{IN})} \quad \text{(CYCLEFF)} \qquad \text{(F-4)}$$

The summation was accomplished inside the loop in terms of

$$\text{MTOT} = \text{MTOT} + \text{POINTEF} \qquad \text{(F-5)}$$

Further,

$\Delta\varphi$ = DDPHI in the program, and similarly

φ_{FIN} = PHIF and

φ_{IN} = PHII

c. Output (See section d below)

The output of CLOCK 2 is identical with that of CLOCK 1 with the exception that the point and cycle efficiencies are printed out. The identical geometric parameters are used, and therefore, the same kinematic output is obtained.

Program CLOCK 2

```
      PROGRAM CLOCK 2(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C  POINT AND CYCLE EFFICIENCIES FOR SINGLE PASS STEP-UP
C  GEAR-MESH WITH CLOCK LOGIVAL TEETH
C
      REAL NP,NG,L,LA,LF,K,ML,MTOT,LAMDA
      READ(5,1)CAPRP,RP
    1 FORMAT(2F10.4)
      READ(5,1)AG,AP
      READ(5,1)RHOG,RHOP
      READ(5,1)IG,TP
      READ(5,2)NG,NP
    2 FORMAT(2F10.0)
      READ(5,1)RHO1,RHO2
      READ(5,3)K
    3 FORMAT(F10.0)
      READ(5,3)MU
      PI = 3.1459
      Z = PI/180.
      PHIDOT = 1.
      WRITE(6,4)CAPPP,RP,AG,AP
    4 FORMAT('1',5X,'CAPRP =',F7.5,3X,'RP =',F7.5,3X,'AG =',F7.5,3X,
     1'AP =',F7.5/)
      WRITE(6,5)RHOG,RHOP
    5 FORMAT(6X,'RHOG =',F7.5,3X,'RHOP =',F7.5/)
      WRITE(6,6)IG,TP
    6 FORMAT(6X,'IG =',F7.5,3X,'TP =',F7.5/)
      WRITE(6,7)NG,NP
    7 FORMAT(6X,'NG =',F4.0,3X,'NP =',F4.0/)
      WRITE(6,8)RHO1,RHO2
    8 FORMAT(6X,'RHO1 =',F6.4,3X,'RHO2 =',F6.4/)
      WRITE(6,9)PHIDOT,K,MU
    9 FORMAT(6X,'PHIDOT =',F5.1//6X,'K =',F5.0//6X,'MU =',F5.2///)
      MTOT = 0.
C
C  COMPUTATION OF GEAR TOOTH PARAMETERS
C
      CXG = RHOG - IG/2.
      DELG = ASIN(CXG/AG)
      CYG = AG*COS(DELG)
      ROG = CYG + SQRT(RHOG*RHOG - CXG*CXG)
      DELGD = DELG/Z
      GAMMG = ASIN(RHOG/AG)
      GAMMGD = GAMMG/Z
      ETMINGD = 90. - GAMMGD
      ETMAXGD = 180. - ASIN(ROG*SIN(DELG)/RHOG)/Z
      CXP = RHOP - TP/2.
      DELP = ASIN(CXP/AP)
      CYP = AP*COS(DELP)
      ROP = CYP + SQRT(RHOP*RHOP - CXP*CXP)
      DELPD = DELP/Z
      GAMMP = ASIN(RHOP/AP)
      GAMMPD = GAMMP/Z
```

```
                      ALPHP = GAMMP - DELP
                      ALPHPD = ALPHP/Z
                      ETMINPD = 90. - GAMMPD
                      ETMAXPD = 180. - ASIN(ROP*SIN(DELP)/RHOP)/Z
                      FP = AP*COS(GAMMP)
                      B = CAPRP - RP
                      DPHI = 360./HG*Z
                      DPSI = 360./HP*Z
                      L = RHOG + RHOP
                      WRITE(6,10)DELGD,DELPD,GAMMPD,ALPHPD,ETMINGD,ETMAXGD,
                     1ETMINPD,ETMAXPD,FP
                   10 FORMAT(6X,*DELGD =*,F9.4,3X,*DELPD =*,F9.4//6X,*GAMMPD =*,F9.4,3X,
                     1*ALPHPD =*,F9.4//6X,*ETMINGD =*,F9.4,3X,*ETMAXGD =*,F9.4
                     2//6X,*ETMINPD =*,F9.4,3X,*ETMAXPD =*,F9.4//6X,*FP =*,F7.5///)
                C
                C  DETERMINATION OF TRANSITION ANGLE
                C
                      AT = RHOG*COS(ALPHP) + FP*SIN(ALPHP)
                      BT = RHOG*SIN(ALPHP) - FP*COS(ALPHP)
                      CT = (AG*AG - FP*FP - B*B - RHOG*RHOG)/(2.*B)
                      ROOTT = AT*AT + BT*BT - CT*CT
                      YT1 = AT + SQRT(ROOTT)
                      YT2 = AT - SQRT(ROOTT)
                      XT = BT + CT
                      PSIT1 = 2.*ATAN2(YT1,XT)
                      PSIT2 = 2.*ATAN2(YT2,XT)
                      IF(PSIT1 .LT. 0.)PSIT1 = PSIT1 + 2.*PI
                      IF(PSIT2 .LT. 0.)PSIT2 = PSIT2 + 2.*PI
                      PSIT1D = PSIT1/Z
                      PSIT2D = PSIT2/Z
                      CALL TRANS(RHOG,ALPHP,FP,AG,B,DELG,Z,PSIT1,PHIT1,G1)
                      CALL TRANS(RHOG,ALPHP,FP,AG,B,DELG,Z,PSIT2,PHIT2,G2)
                      PHIT1D = PHIT1/Z
                      PHIT2D = PHIT2/Z
                      TEST1 = 360. - (PSIT1D + ALPHPD)
                      TEST2 = 360. - (PSIT2D + ALPHPD)
                      WRITE(6,11)PHIT1D,PSIT1D,G1,TEST1
                      WRITE(6,12)PHIT2D,PSIT2D,G2,TEST2
                   11 FORMAT(6X,*PHIT1D =*,F9.4,3X,*PSIT1D =*,F9.4,3X,*G1 =*,F7.4,3X,
                     1*TEST1 =*,F7.2/)
                   12 FORMAT(6X,*PHIT2D =*,F9.4,3X,*PSIT2D =*,F9.4,3X,*G2 =*,F7.4,3X,
                     1*TEST2 =*,F7.2///)
                      IF(G1 .LT. FP)GO TO 20
                      PHIT = PHIT2
                      PSIT = PSIT2
                      GO TO 30
                   20 PHIT = PHIT1
                      PSIT = PSIT1
                   30 PHITD = PHIT/Z
                      PSITD = PSIT/Z
                C
                C  DETERMINATION OF CORRECT SIGN FOR ROUND ON FLAT REGIME
                C
```

```
      AF = B*COS(ALPHP) + AG*COS(PHIT-DELG-ALPHP)
      BF = B*SIN(ALPHP) - AG*SIN(PHIT-DELG-ALPHP)
      CF = -RHOG
      ROOTF = AF*AF + BF*BF - CF*CF
  110 YF1 = AF + SQRT(ROOTF)
      YF2 = AF - SQRT(ROOTF)
      XF = BF + CF
      PSIF1 = 2.*ATAN2(YF1,XF)
      PSIF2 = 2.*ATAN2(YF2,XF)
  115 IF(PSIF1 .LT. 0.)PSIF1 = PSIF1 + 2.*PI
      IF(PSIF2 .LT. 0.)PSIF2 = PSIF2 + 2.*PI
      IF(ABS(PSIF1-PSIT) .LT. ABS(PSIF2-PSIT))GO TO 50
      SIGNF = -1.
      GO TO 60
  120 50 SIGNF = 1.
C
C  COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI
C
      DO 60 I=1,1000
  125 PHID = PHITD + (I-1.)/100.
      PHI = PHID*Z
      AF = B*COS(ALPHP) - AG*COS(PHI-DELG-ALPHP)
      BF = B*SIN(ALPHP) - AG*SIN(PHI-DELG-ALPHP)
  130 CF = -RHOG
      ROOTF = AF*AF + BF*BF - CF*CF
      YF = AF + SIGNF*SQRT(ROOTF)
      XF = BF + CF
      PSIF = 2.*ATAN2(YF,XF)
  135 IF(PSIF .LT. 0.)PSIF = PSIF + 2.*PI
      LX = B + AG*COS(PHI-DELG-DPHI) - AP*COS(PSIF+DPSI-DELP)
      LY = AG*SIN(PHI-DELG-DPHI) - AP*SIN(PSIF-DELP+DPSI)
      IF(SQRT(LX*LX + LY*LY) .LE. L)GO TO 70
   60 CONTINUE
  140 70 PHIF = PHI
      PSIFF = PSIF
      PHII = PHIF - DPHI
      PSII = PSIFF + DPSI
      IF(PSII.GT.2.*PI)PSII=PSII-2.*PI
  145 PHIID = PHII/2
      PSIID = PSII/2
      PHIFD = PHIF/2
      PSIFD = PSIFF/2
      WRITE(6,80)PHIID,PSIID,PHIFD,PSIFD
  150 80 FORMAT(4X,'PHIID =',F9.4,3X,'PSIID =',F9.4,3X,'PHIFD =',F9.4,3X,
     1'PSIFD =',F9.4///)
C
C  DETERMINATION OF CORRECT SIGN FOR ROUND ON ROUND REGIME
      AR = SIN(PHII-DELG) + COS(PHII-DELG)*TAN(DELP) - B*TAN(DELP)/AG
      BR = COS(PHII-DELG) + B/AG - SIN(PHII-DELG)*TAN(DELP)
  155 CR = (AP*AP + AG*AG + B*B - L*L)/(2.*AP*AG*COS(DELP))
      1. B*COS(PHII-DELG)/(AP*COS(DELP))
      ROOTR = AR*AR + BR*BR - CR*CR
      YR1 = AR + SQRT(ROOTR)
```

```
      100               YR2 = AR - SQRT(ROOTR)
                        AR = BR + CR
                        PSIR1 = 2.*ATAN2(YR1,AR)
                        PSIR2 = 2.*ATAN2(YR2,AR)
                        IF(PSIR1 .LT. 0.)PSIR1 = PSIR1 + 2.*PI
      105               IF(PSIR2 .LT. 0.)PSIR2 = PSIR2 + 2.*PI
                        IF(ABS(PSI1-PSIR1) .LT. ABS(PSI1-PSIR2))GO TO 90
                        SIGNR = -1.
                        GO TO 100
                     90 SIGNR = 1.
      170            C
                     C  KINEMATICS
                     C
                    100 DOPHI = (PHIF-PHII)/K
                        PHI = PHII - DOPHI
      175           110 PHI = PHI + DOPHI
                        IF(PHI .GE. PHIF - DOPHI)GO TO 200
                        PHID = PHI/Z
                        IF(PHI .GE. PHIT)GO TO 120
                        AR = SIN(PHI-DELG) + COS(PHI-DELG)*TAN(DELP) + B*TAN(DELP)/AG
      180               BR = COS(PHI-DELG) + B/AG - TAN(DELP)*SIN(PHI-DELG)
                        CR = -AP**2 + AG**2 + B**2 - L**2/(2.*AP*AG*COS(DELP))
                       1+ B*COS(PHI-DELG)/(AP*COS(DELP))
                        ROOTR = ARYAR + BR*BR - CR*CR
                        YR = AR+SIGNR*SQRT(ROOTR)
      185               AR = BR + CR
                        PSI = 2.*ATAN2(YR,AR)
                        IF(PSI .LT. 0.)PSI = PSI + 2.*PI
                        PSID = PSI/Z
                        SGAP = (AP*SIN(PSI-DELP) - AG*SIN(PHI-DELG))/L
      190               CLAP = (AP*COS(PSI-DELP) - AG*COS(PHI-DELG) - B)/L
                        LAMDA = ATAN2(SLAM,CLAM)
                        IF(LAMDA .LT. 0.)LAMDA = LAMDA + 2.*PI
                        ARD = TAN(DELP)*SIN(PHI-DELG) - COS(PHI-DELG)
                        BRD = SIN(PHI-DELG) + TAN(DELP)*COS(PHI-DELG)
      195               CRD = B*SIN(PHI-DELG)/(AP*COS(DELP))
                        PSIDOT = (ARD*SIN(PSI) + BRD*COS(PSI) - CRD)/(AR*COS(PSI) - BR*
                       1SIN(PSI))*PHIDOT
                        VSTR = PHIDOT*(AV*COS(PHI-DELG-LAMDA) + RHOG) - PSIDOT*(AP*
                       1COS(PSI-DELP-LAMDA) - RHOP)
      200               SR = VSTR/ABS(VSIN)
                        IF(PSID-DELPD .LT. -90.)ETAP=0 = (2.*PI + PSI - LAMDA - DELP)/Z
                        IF(PSID-DELPD .GE. -90.)ETAP=0 = (PSI-LAMDA-DELP)/Z
                        ETAGC = (PI + PHI - LAMDA - DELG)/Z
                        EFRR = MU*(SR*RHOP-RHO2) - AP*(MU*SR*COS(PSI-DELP-LAMDA) - SIN(PSI
                       1-DELP-LAMDA))
      205               EFRD = MU*(RHO1+ SR*RHOG) + AG*(MU*SR*COS(PHI-DELG-LAMDA) - SIN(
                       1PHI-DELG-LAMDA))
                        POINTEF = EFRR/(MU*ABS(PSIDOT)
                        WRITE(61,110)PHIO,PSIO,PSIDOT,SR,ETAGRD,ETAPRD,POINTEF
      210           115 FORMAT(6X,*PHIO =*,F9.4,3X,*PSIO =*,F9.4,3X,*PSIDOT =*,F8.4,3X,
                       1*SR =*,F8.1,3X,*ETAGRD =*,F9.4,3X,*ETAPRD =*,F9.4,3X,*POINTEF =*,
                       2F6.3)
```

```
                      GO TO 150
                12a   AF = B*COS(ALPHP) + AG*COS(PHI-DELG-ALPHP)
          21c         BF = B*SIN(ALPHP) - AG*SIN(PHI-DELG-ALPHP)
                      CF = -RHOG
                      RHOTF = AF*AF + BF*BF - CF*CF
                      VF = AF + SIGNF*SQRT(RHOTF)
                      AF = BF + CF
          22a         PSI = 2.*ATAN2(VF,AF)
                      IF(PSI .LT. 0.)PSI = PSI + 2.*PI
                      PSID = PSI/Z
                      G = (AG*SIN(PHI-DELG) - RHOG*COS(PSI+ALPHP))/SIN(PSI+ALPHP)
                      AFD = AG*SIN(PHI-DELG-ALPHP)
          23c         BFD = AG*COS(PHI-DELG-ALPHP)
                      PSIDOT = PHIDOT*(AFD*SIN(PSI) + BFD*COS(PSI))/(AF*COS(PSI) - BF*
                     1SIN(PSI))
                      VSTF = PHIDOT*(RHOG + AG*SIN(PSI-PHI+ALPHP-DELG))
                      SF = VSTF/ABS(VSTF)
          23a         IF(PSID+ALPHPD.GE.90.)ETAGFD=(PHI-DELG-PSI-ALPHP+3.*PI/2.)/Z
                      IF(PSID+ALPHPD.LT.90.)ETAGFD=(PHI-DELG-PSI-ALPHP-.5*PI)/Z
                      EFFM = G - MU*RHO**2
                      EFFU = MU*(RHO) + SF*RHOG) - AG*(COS(PHI-DELG-PSI-ALPHP)
                     1+ MU*SF*SIN(PHI-DELG-PSI-ALPHP))
          23c         POINTEF = EFFM/EFFU*ABS(PSIDOT)
                      WRITE(6,13)PHID,PSID,PSIDOT,G,SF,ETAGFD,POINTEF
                13a   FORMAT(0X,4PHID =*,F9.4,3X,4*PSID =*,F9.4,3X,4*PSIDOT =*,F8.4,3X,
                     1*G =*,F7.5,3X,*SF =*,F4.1,3X,*ETAGFD =*,F9.4,3X,*POINTEF =*,F6.3)
                15a   RTOT = RTOT + POINTEF
          24a         GO TO 11*
                20a   CYCLEFF = RTOT*90*PHI/(PHIF-PHII)
                      WRITE(6,21)CYCLEFF
                21a   FORMAT(*0*,5X,*CYCLE EFFICIENCY =*,F6.3)
                      STOP
          24c         END
```

```
      SUBROUTINE TRANS(RHOG,ALPHP,FP,AG,B,DELG,Z,PSIT,PHIT,G)
      PI = 3.14159
      SI = (FP*SIN(PSIT+ALPHP) + RHOG*COS(PSIT+ALPHP))/AG
      CI = (FP*COS(PSIT+ALPHP) - RHOG*SIN(PSIT+ALPHP) - B)/AG
      PHIT = ATAN2(ST,CT) + DELG
      IF(PHIT .LT. 0.)PHIT = PHIT + 2.*PI
      PHINEXT = PHIT + .1**7
      AF = B*COS(ALPHP) + AG*COS(PHINEXT - DELG - ALPHP)
      BF = B*SIN(ALPHP) - AG*SIN(PHINEXT - DELG - ALPHP)
      CF = -RHOG
      ROOTF = AF*AF + BF*BF - CF*CF
      YF1 = AF + SQRT(ROOTF)
      YF2 = AF - SQRT(ROOTF)
      XF = BF + CF
      PSINEX1 = 2.*ATAN2(YF1,XF)
      PSINEX2 = 2.*ATAN2(YF2,XF)
      IF(PSINEX1 .LT. 0.)PSINEX1 = PSINEX1 + 2.*PI
      IF(PSINEX2 .LT. 0.)PSINEX2 = PSINEX2 + 2.*PI
      IF(ABS(PSINEX1-PSI1) .LT. ABS(PSINEX2-PSI1))GO TO 1
      PSINEXT = PSINEX2
      GO TO 2
    1 PSINEXT = PSINEX1
    2 G = (AG*SIN(PHINEXT-DELG) - RHOG*COS(PSINEXT+ALPHP))/SIN(PSINEXT
     1+ALPHP)
      RETURN
      END
```

CA[illegible] = .[illegible]725 ED = .9[illegible]45 AG = .9[illegible]725 AP = .89805

PH[illegible] = .83979 R[illegible] = .21748

T[illegible] = .[illegible] TP = .82[illegible]

WG = 42. WP = 8.

[illegible]1 = .9[illegible] RHO2 = .9399

PHIDOT = 1.9

K = 5%.

E = .2[illegible]

MEL90 = 2.5548 DELP0 = 2.1648

CA[illegible]0 = 11.8418 ALPHA0 = 8.8078

ET[illegible] = [illegible] ET[illegible] = 149.7484

ET[illegible] = 78.4582 ETMAP0 = 148.5878

FP = .99917

PHIT10 = 184.3823 PSIT10 = 348.6734 G1 = .8695 TEST1 = 42.67

PHIT20 = 178.7537 PSIT20 = 346.[illegible] G2 = .8688 TEST2 = 4.45

PHII0 = 178.2823 PSII0 = 359.8276 PHIF0 = 184.8337 PSIF0 = 314.8276

PHI0 = 174.2623	PSI0 = 359.8195	PSIDOT = -5.2458	SF =-1.0	ETAGR0 = 84.7219	ETAmn0 = 93.6923	POINTEF = .875			
PHI0 = 174.6337	PSI0 = 358.9244	PSIDOT = -5.2451	SF =-1.0	ETAGR0 = 84.8958	ETAmn0 = 89.7858	POINTEF = .879			
PHI0 = 174.6852	PSI0 = 358.8211	PSIDOT = -5.2468	SF =-1.0	ETAGR0 = 86.9172	ETAmn0 = 88.7484	POINTEF = .883			
PHI0 = 174.7786	PSI0 = 357.1214	PSIDOT = -5.2581	SF =-1.0	ETAGR0 = 87.0557	ETAmn0 = 87.8137	POINTEF = .886			
PHI0 = 174.9483	PSI0 = 356.2218	PSIDOT = -5.2546	SF =-1.0	ETAGR0 = 87.2214	ETAmn0 = 86.9077	POINTEF = .890			
PHI0 = 177.1194	PSI0 = 355.3194	PSIDOT = -5.2635	SF =-1.0	ETAGR0 = 87.4145	ETAmn0 = 86.0201	POINTEF = .893			
PHI0 = 177.2984	PSI0 = 354.4174	PSIDOT = -5.2674	SF =-1.0	ETAGR0 = 87.6351	ETAmn0 = 85.1768	POINTEF = .897			
PHI0 = 177.4623	PSI0 = 353.5138	PSIDOT = -5.2752	SF =-1.0	ETAGR0 = 87.8831	ETAmn0 = 84.3478	POINTEF = .900			
PHI0 = 177.6737	PSI0 = 352.6187	PSIDOT = -5.2838	SF =-1.0	ETAGR0 = 88.1587	ETAmn0 = 83.5449	POINTEF = .904			
PHI0 = 177.8852	PSI0 = 351.7321	PSIDOT = -5.2933	SF =-1.0	ETAGR0 = 88.4620	ETAmn0 = 82.7722	POINTEF = .907			
PHI0 = 177.9766	PSI0 = 350.7934	PSIDOT = -5.3933	SF = 1.0	ETAGR0 = 88.7931	ETAmn0 = 82.0235	POINTEF = .910			
PHI0 = 178.1484	PSI0 = 349.8838	PSIDOT = -5.3137	SF = 1.0	ETAGR0 = 89.1521	ETAmn0 = 81.3011	POINTEF = .907			
PHI0 = 178.3194	PSI0 = 348.9726	PSIDOT = -5.3244	SF = 1.0	ETAGR0 = 89.5390	ETAmn0 = 80.6068	POINTEF = .903			
PHI0 = 178.4989	PSI0 = 348.0583	PSIDOT = -5.3352	SF = 1.0	ETAGR0 = 89.9541	ETAmn0 = 79.9367	POINTEF = .899			
PHI0 = 178.6623	PSI0 = 347.1427	PSIDOT = -5.3462	SF = 1.0	ETAGR0 = 90.3973	ETAmn0 = 79.2906	POINTEF = .896			
PHI0 = 178.8337	PSI0 = 346.2252	PSIDOT = -5.3588	E = .88689	SF = 1.0	ETAGF0 = 91.1514	POINTEF = .892			
PHI0 = 179.0852	PSI0 = 345.3821	PSIDOT = -5.4082	E = .88831	SF = 1.0	ETAGF0 = 92.2486	POINTEF = .888			
PHI0 = 179.1786	PSI0 = 344.3734	PSIDOT = -5.4294	E = .88775	SF = 1.0	ETAGF0 = 93.3477	POINTEF = .885			
PHI0 = 179.3484	PSI0 = 343.4484	PSIDOT = -5.4554	E = .89722	SF = 1.0	ETAGF0 = 94.4522	POINTEF = .881			
PHI0 = 179.5194	PSI0 = 342.5836	PSIDOT = -5.4781	E = .89671	SF = 1.0	ETAGF0 = 95.5698	POINTEF = .877			
PHI0 = 179.6989	PSI0 = 341.5629	PSIDOT = -5.4974	E = .89623	SF = 1.0	ETAGF0 = 96.6731	POINTEF = .874			
PHI0 = 179.8623	PSI0 = 340.6198	PSIDOT = -5.5131	E = .89577	SF = 1.0	ETAGF0 = 97.7883	POINTEF = .870			
PHI0 = 180.0337	PSI0 = 339.6728	PSIDOT = -5.5252	E = .89534	SF = 1.0	ETAGF0 = 98.9059	POINTEF = .866			
PHI0 = 180.2852	PSI0 = 338.7264	PSIDOT = -5.5335	E = .89493	SF = 1.0	ETAGF0 = 100.0253	POINTEF = .863			
PHI0 = [illegible]	PSI0 = 337.77[illegible]	PSIDOT = -5.5388	E = .89455	SF = 1.0	ETAGF0 = 101.1458	POINTEF = .859			

PHID = 180.5480 PSID = 336.8263 PSIDOT = -5.5386 G = .08428 SF = 1.0 ETAGFD = 142.2667 POINTEF = .855

PHID = 180.7194 PSID = 335.8771 PSIDOT = -5.5353 G = .08388 SF = 1.0 ETAGFD = 143.3874 POINTEF = .852

PHID = 180.8949 PSID = 334.9287 PSIDOT = -5.5286 G = .08358 SF = 1.0 ETAGFD = 144.5071 POINTEF = .848

PHID = 181.0623 PSID = 333.9828 PSIDOT = -5.5168 G = .08331 SF = 1.0 ETAGFD = 145.6253 POINTEF = .844

PHID = 181.2337 PSID = 333.0375 PSIDOT = -5.5015 G = .08307 SF = 1.0 ETAGFD = 146.7412 POINTEF = .841

PHID = 181.4052 PSID = 332.0969 PSIDOT = -5.4823 G = .08286 SF = 1.0 ETAGFD = 147.8542 POINTEF = .837

PHID = 181.5766 PSID = 331.1581 PSIDOT = -5.4591 G = .08268 SF = 1.0 ETAGFD = 148.9635 POINTEF = .834

PHID = 181.7480 PSID = 330.2245 PSIDOT = -5.4320 G = .08252 SF = 1.0 ETAGFD = 150.0685 POINTEF = .830

PHID = 181.9194 PSID = 329.2959 PSIDOT = -5.4011 G = .08248 SF = 1.0 ETAGFD = 151.1685 POINTEF = .827

PHID = 182.0909 PSID = 328.3729 PSIDOT = -5.3663 G = .08238 SF = 1.0 ETAGFD = 152.2629 POINTEF = .823

PHID = 182.2623 PSID = 327.4552 PSIDOT = -5.3279 G = .08224 SF = 1.0 ETAGFD = 153.3511 POINTEF = .819

PHID = 182.4337 PSID = 326.5464 PSIDOT = -5.2859 G = .08228 SF = 1.0 ETAGFD = 154.4323 POINTEF = .816

PHID = 182.6052 PSID = 325.6441 PSIDOT = -5.2403 G = .08228 SF = 1.0 ETAGFD = 155.5068 POINTEF = .812

PHID = 182.7766 PSID = 324.7496 PSIDOT = -5.1915 G = .08222 SF = 1.0 ETAGFD = 156.5717 POINTEF = .809

PHID = 182.9480 PSID = 323.8644 PSIDOT = -5.1394 G = .08227 SF = 1.0 ETAGFD = 157.6286 POINTEF = .805

PHID = 183.1195 PSID = 322.9886 PSIDOT = -5.0843 G = .08235 SF = 1.0 ETAGFD = 158.6764 POINTEF = .802

PHID = 183.2909 PSID = 322.1213 PSIDOT = -5.0263 G = .08246 SF = 1.0 ETAGFD = 159.7145 POINTEF = .798

PHID = 183.4623 PSID = 321.2648 PSIDOT = -4.9656 G = .08260 SF = 1.0 ETAGFD = 160.7425 POINTEF = .795

PHID = 183.6337 PSID = 320.4198 PSIDOT = -4.9023 G = .08277 SF = 1.0 ETAGFD = 161.7597 POINTEF = .791

PHID = 183.8052 PSID = 319.5842 PSIDOT = -4.8367 G = .08297 SF = 1.0 ETAGFD = 162.7668 POINTEF = .788

PHID = 183.9766 PSID = 318.7609 PSIDOT = -4.7689 G = .08320 SF = 1.0 ETAGFD = 163.7608 POINTEF = .784

PHID = 184.1480 PSID = 317.9492 PSIDOT = -4.6990 G = .08346 SF = 1.0 ETAGFD = 164.7438 POINTEF = .781

PHID = 184.3194 PSID = 317.1498 PSIDOT = -4.6274 G = .08374 SF = 1.0 ETAGFD = 165.7146 POINTEF = .777

PHID = 184.4909 PSID = 316.3628 PSIDOT = -4.5542 G = .08405 SF = 1.0 ETAGFD = 166.6731 POINTEF = .774

PHID = 184.6623 PSID = 315.5885 PSIDOT = -4.4795 G = .08439 SF = 1.0 ETAGFD = 167.6188 POINTEF = .770

CYCLE EFFICIENCY = .850

APPENDIX G

KINEMATICS OF TWO AND THREE STEP-UP GEAR TRAINS WITH CLOCK TEETH

Figure G-1 shows the basic configuration of a three step-up gear train used in certain fuze applications. The general layout is identical to that shown in Figure A-5 of Appendix A. Now, ogival type gear teeth are used instead of involute type ones. Again, it is required to find the equilibrant moment, M_{04}, acting on pinion no. 4 which holds the input moment, M_{in}, acting on gear no. 1, in equilibrium when both pivot and contact friction are taken into account, and when the fuze body spins. Appendix H gives the force and moment analyses for the determination of this moment input-output relationship. The same appendix also shows the derivation of such an input-output relationship for a two step-up gear train with ogival gear teeth which must operate in a spin environment. (Figure A-10 of Appendix A shows this type of configuration with involute teeth.) The present appendix lays the groundwork for the moment relationships of Appendix H by providing the kinematics of the three ogival gear meshes involved.

In each case both round on round and round on flat phases

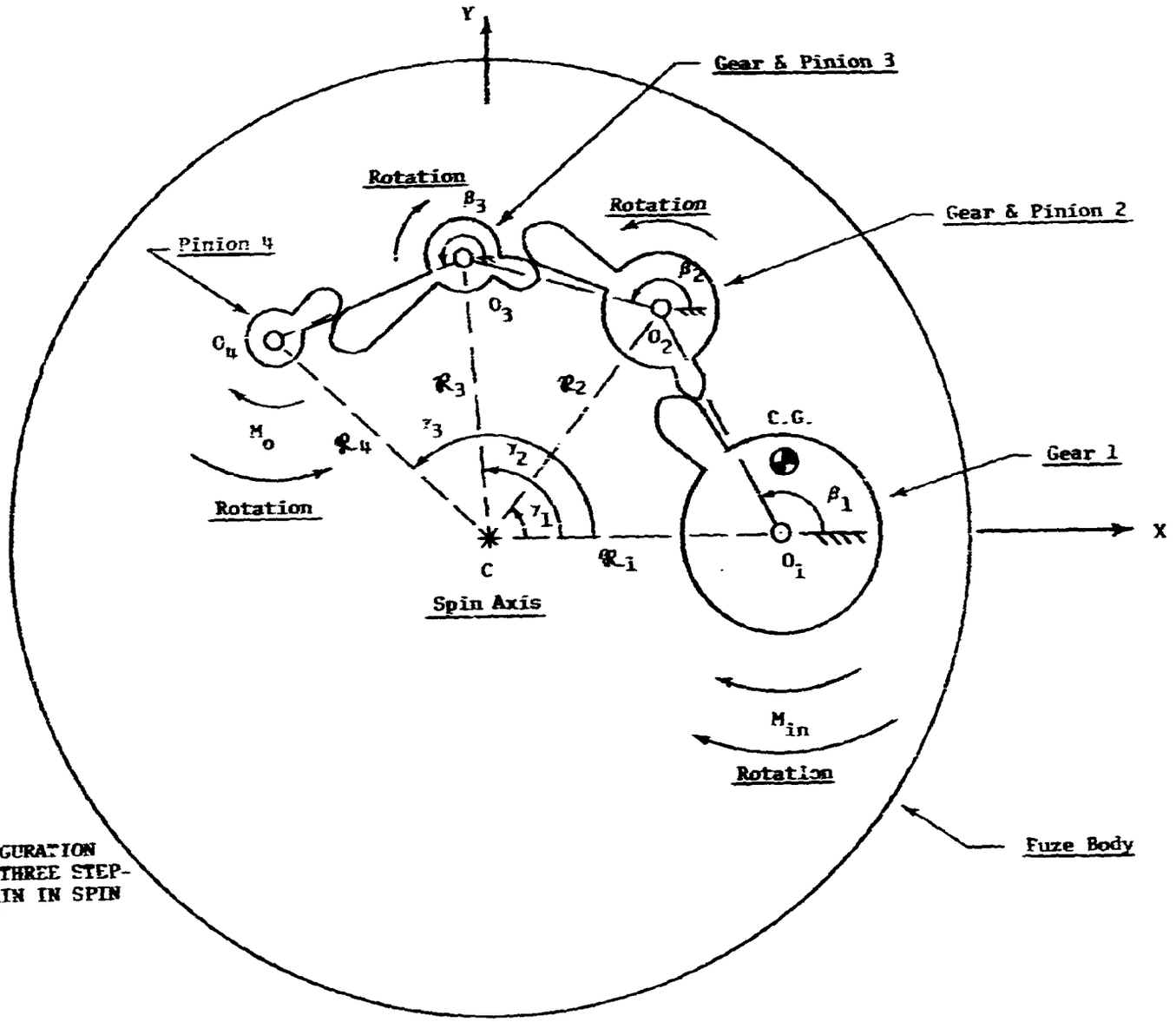

FIGURE G-1

BASIC CONFIGURATION FOR OGIVAL THREE STEP-UP GEAR TRAIN IN SPIN ENVIRONMENT

of motion have to be considered. All derivations follow the pattern set in Section 1 of Appendix E. The derivations must take into account that the driving gears of meshes no. 1 and no. 3, i.e., between gear no. 1 and pinion no. 2 and between gear no. 3 and pinion no. 4, respectively, have clockwise rotations. The driving gear of mesh no. 2, i.e., between gear no. 2 and pinion no. 3, has counterclockwise rotation.

Finally, the inclinations of the various pivot to pivot centerlines with respect to the body-fixed X-axes, as represented by the angles β_1, β_2 and β_3, must be considered.

For the sake of simplicity, the notation will in most cases not differentiate between round on round and round on flat phases of motion. For example, the output angle, ψ_i, and its derivatives will have the same symbol for both phases.

For definitions of angles β_i and γ_i as well as the distances ℓ_i, see Appendix A-6.

1. KINEMATICS OF MESH NO. 1 (GEAR NO. 1 AND PINION NO. 2)

a. ROUND ON ROUND PHASE OF MOTION

Figure G-2 shows the round on round phase of the motion of mesh no. 1 in a schematic manner. Only the contacting faces of the gear and the pinion are indicated.

I. UNIT VECTORS

The unit vector in the direction O_1C_{G1} of the gear is given by

$$\bar{n}_{G1} = \cos(\phi_1 + \delta_{G1})\bar{I} + \sin(\phi_1 + \delta_{G1})\bar{J} \qquad (G-1)$$

The unit vector in the direction $C_{G1}C_{P1}$ is given by

$$\bar{n}_{\lambda 1} = \cos\lambda_1\bar{I} + \sin\lambda_1\bar{J} \qquad (G-2)$$

The unit vector normal to $\bar{n}_{\lambda 1}$ (in the right hand sense) becomes

$$\bar{n}_{N\lambda 1} = -\sin\lambda_1\bar{I} + \cos\lambda_1\bar{J} \qquad (G-3)$$

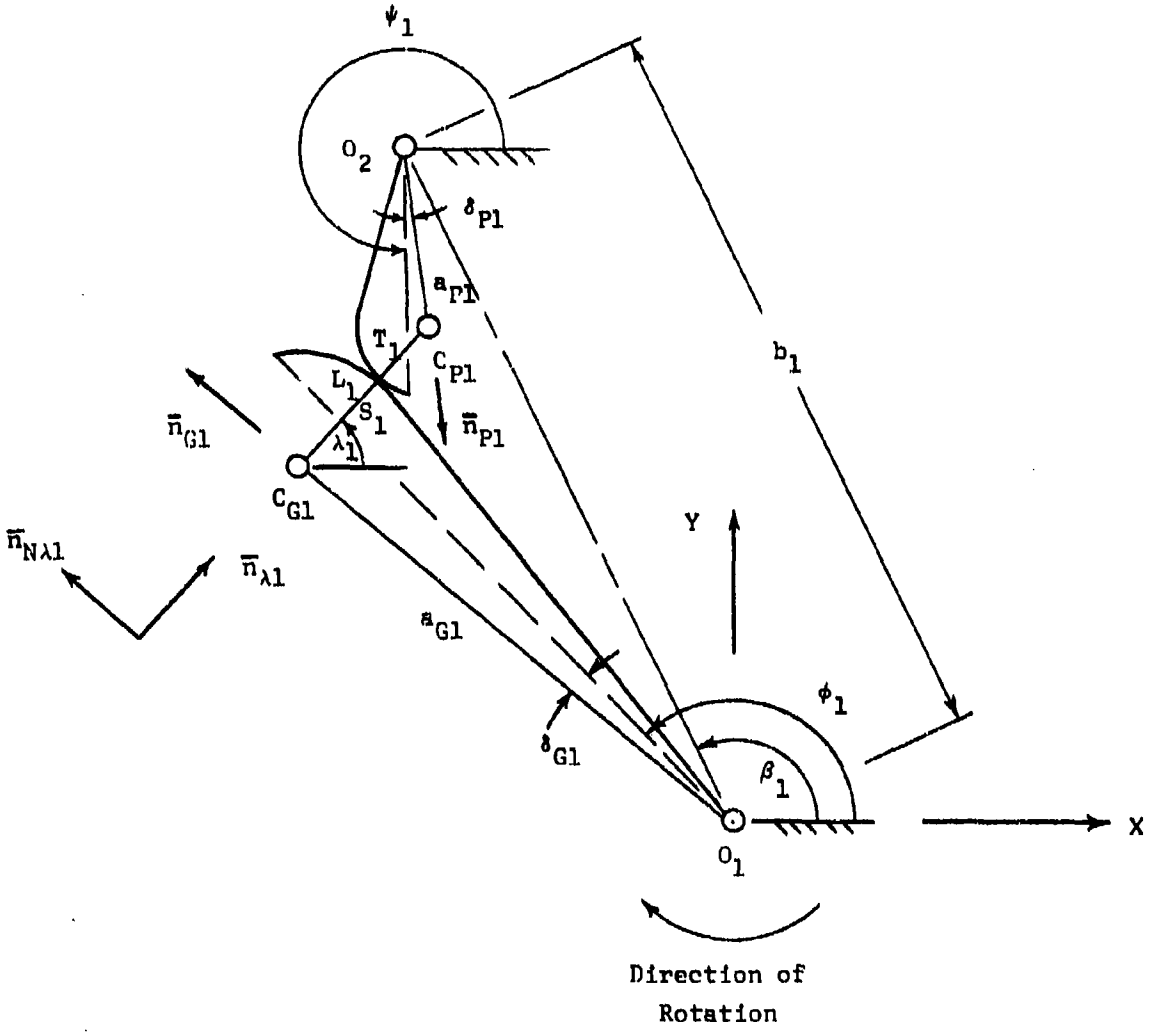

FIGURE G-2

ROUND ON ROUND PHASE FOR MESH NO. 1

The unit vector in the direction O_2C_{P1} is given by

$$\bar{n}_{P1} = \cos(\psi_1 + \delta_{P1})\bar{I} + \sin(\psi_1 + \delta_{P1})\bar{J} \qquad (G\text{-}4)$$

Finally, the unit vector along the centerline, O_1O_2, is given by

$$\bar{n}_{\beta 1} = \cos\beta_1\bar{I} + \sin\beta_1\bar{J} \qquad (G\text{-}5)$$

II. DETERMINATION OF OUTPUT ANGLE ψ_1 AND "COUPLER" ANGLE λ_1

The loop equation of the equivalent four-bar linkage is given by

$$a_{G1}\bar{n}_{G1} + L_1\bar{n}_{\lambda 1} - a_{P1}\bar{n}_{P1} - b_1\bar{n}_{\beta 1} = 0 \qquad (G\text{-}6)$$

where $L_1 = \rho_{G1} + \rho_{P1}$ (G-7)

With the appropriate substitution for the unit vectors, one obtains the following component equations:

$$a_{G1}\cos(\phi_1 + \delta_{G1}) + L_1\cos\lambda_1 - b_1\cos\beta_1 - a_{P1}\cos(\psi_1 + \delta_{P1}) = 0 \qquad \text{(G-8)}$$

and

$$a_{G1}\sin(\phi_1 + \delta_{G1}) + L_1\sin\lambda_1 - b_1\sin\beta_1 - a_{P1}\sin(\psi_1 + \delta_{P1}) = 0 \qquad \text{(G-9)}$$

To eliminate λ_1, let

$$\sin^2\lambda_1 + \cos^2\lambda_1 = 1 \qquad \text{(G-10)}$$

The above trigonometric functions are obtained from equations (G-8) and (G-9), respectively. Substitution into equation (G-10) furnishes

$$A_{1R}\sin\psi_1 + B_{1R}\cos\psi_1 = C_{1R} \qquad \text{(G-11)}$$

where

$$A_{1R} = a_{G1}\sin(\phi_1 + \delta_{G1} - \delta_{P1}) - b_1\sin(\beta_1 - \delta_{P1})$$

$$B_{1R} = a_{G1}\cos(\phi_1 + \delta_{G1} - \delta_{P1}) - b_1\cos(\beta_1 - \delta_{P1})$$

$$C_{1R} = \frac{a_{P1}^2 + a_{G1}^2 + b_1^2 - L_1^2 - 2a_{G1}b_1\cos(\phi_1 + \delta_{G1} - \beta_1)}{2a_{P1}}$$

Equation (G-11) is now solved for ψ_1 in the manner described in Appendix E in connection with equation (E-12), i.e.,

$$\psi_1 = 2 \tan^{-1} \frac{A_{1R} \pm \sqrt{A_{1R}^2 + B_{1R}^2 - C_{1R}^2}}{B_{1R} + C_{1R}} \qquad \text{(G-12)}$$

The correct sign on equation (G-12) must be found from geometric considerations.

The coupler angle λ_1 may now be determined either from equation (G-8) or from equation (G-9). Thus,

$$\lambda_1 = \cos^{-1} \left[\frac{b_1 \cos\beta_1 + a_{P1} \cos(\psi_1 + \delta_{P1}) - a_{G1} \cos(\phi_1 + \delta_{G1})}{L_1} \right] \qquad \text{(G-13)}$$

or

$$\lambda_1 = \sin^{-1} \left[\frac{b_1 \sin\beta_1 + a_{P1} \sin(\psi_1 + \delta_{P1}) - a_{G1} \sin(\phi_1 + \delta_{G1})}{L_1} \right] \qquad \text{(G-14)}$$

III. DETERMINATION OF ANGULAR VELOCITY $\dot{\psi}_1$ OF PINION NO. 2

Differentiation of equation (G-11) with respect to time gives

$$\dot{\psi}_1 = \dot{\phi}_1 \left[\frac{B_{1RD}\cos\psi_1 - A_{1RD}\sin\psi_1 + C_{1RD}}{A_{1R}\cos\psi_1 - B_{1R}\sin\psi_1} \right] \qquad \text{(G-15)}$$

where

$$A_{1RD} = a_{G1}\cos(\phi_1 + \delta_{G1} - \delta_{P1})$$

$$B_{1RD} = a_{G1}\sin(\phi_1 + \delta_{G1} - \delta_{P1})$$

$$C_{1RD} = \frac{a_{G1}b_1\sin(\phi_1 + \delta_{G1} - \beta_1)}{a_{P1}}$$

IV. RELATIVE VELOCITY AT THE CONTACT POINT

The relative velocity $\bar{V}_{S1/T1_R}$ of the contact point S_1 on gear no. 1 with respect to point T_1 on pinion no. 2, represents the vectorial difference between the absolute velocities of these points. Thus,

$$\bar{V}_{S1/T1_R} = \bar{V}_{S1/C} - \bar{V}_{T1/C} \qquad (G\text{-}16)$$

where C represents the spin center of the fuze body. If $\bar{\omega}$ stands for the angular velocity of the fuze body, then appropriate substitution into equation (G-16) gives (See also Figures G-1 and G-2)

$$\bar{V}_{S1/T1_R} = \left[\bar{\omega} \times \bar{R}_1 + (\bar{\omega} + \dot{\bar{\phi}}_1) \times (\bar{a}_{G1} + \bar{\rho}_{G1})\right] - \left[\bar{\omega} \times \bar{R}_2 + (\bar{\omega} + \dot{\bar{\psi}}_1) \times (\bar{a}_{P1} + \bar{\rho}_{P1})\right] \qquad (G\text{-}17)$$

Since

$$\bar{\omega} \times \left[\bar{R}_1 + \bar{a}_{G1} + \bar{\rho}_{G1}\right] = \bar{\omega} \times \left[\bar{R}_2 + \bar{a}_{P1} + \bar{\rho}_{P1}\right] \qquad (G\text{-}18)$$

because the position vectors in brackets are equal, equation (G-17) may be written

$$\bar{V}_{S1/T1_R} = \bar{V}_{S1/O_1} - \bar{V}_{T1/O_2}$$

$$= \bar{\dot{\phi}}_1 \times (\bar{a}_{G1} + \bar{\rho}_{G1}) - \bar{\dot{\psi}}_1 \times (\bar{a}_{P1} + \bar{\rho}_{P1}) \qquad \text{(G-19)}$$

Note that $\bar{V}_{S1/T1_R}$ becomes the vectorial difference of the contact point velocities with respect to the fuze body.

Since this relative velocity is tangent to the contacting surfaces, it may be written as the vectorial difference of the velocity components along these surfaces. Accordingly, equation (G-19) becomes, with the help of the unit vector $\bar{n}_{N\lambda 1}$,

$$\bar{V}_{S1/T1_R} = \left\{ \left[\dot{\phi}_1 \bar{k} \times (a_{G1} \bar{n}_{G1} + \rho_{G1} \bar{n}_{\lambda 1}) \right] \cdot \bar{n}_{N\lambda 1} - \left[\dot{\psi}_1 \bar{k} \times (a_{P1} \bar{n}_{P1} - \rho_{P1} \bar{n}_{\lambda 1}) \right] \cdot \bar{n}_{N\lambda 1} \right\} \bar{n}_{N\lambda 1} \qquad \text{(G-20)}$$

Appropriate substitution of unit vectors given earlier in Section I and simplification results in

$$\bar{V}_{S1/T1_R} = \left\{ \dot{\phi}_1 \left[a_{G1} \cos(\phi_1 + \delta_{G1} - \lambda_1) + \rho_{G1} \right] - \dot{\psi}_1 \left[a_{P1} \cos(\psi_1 + \delta_{P1} - \lambda_1) - \rho_{P1} \right] \right\} \bar{n}_{N\lambda 1} \qquad \text{(G-21)}$$

b. ROUND ON FLAT PHASE OF MOTION

Figure G-3 shows a schematic view of mesh no. 1 in the round on flat phase of the motion.

I. UNIT VECTORS

The unit vector in the direction O_2T_1 is given by

$$\bar{n}_{F1} = \cos(\psi_1 - \alpha_{P1})\bar{I} + \sin(\psi_1 - \alpha_{P1})\bar{J} \qquad \text{(G-22)}$$

The unit vector $\bar{n}_{NF1}$, in the direction $C_{G1}S_1$, is always normal to $\bar{n}_{F1}$

$$\bar{n}_{NF1} = -\sin(\psi_1 - \alpha_{P1})\bar{I} + \cos(\psi_1 - \alpha_{P1})\bar{J} \qquad \text{(G-23)}$$

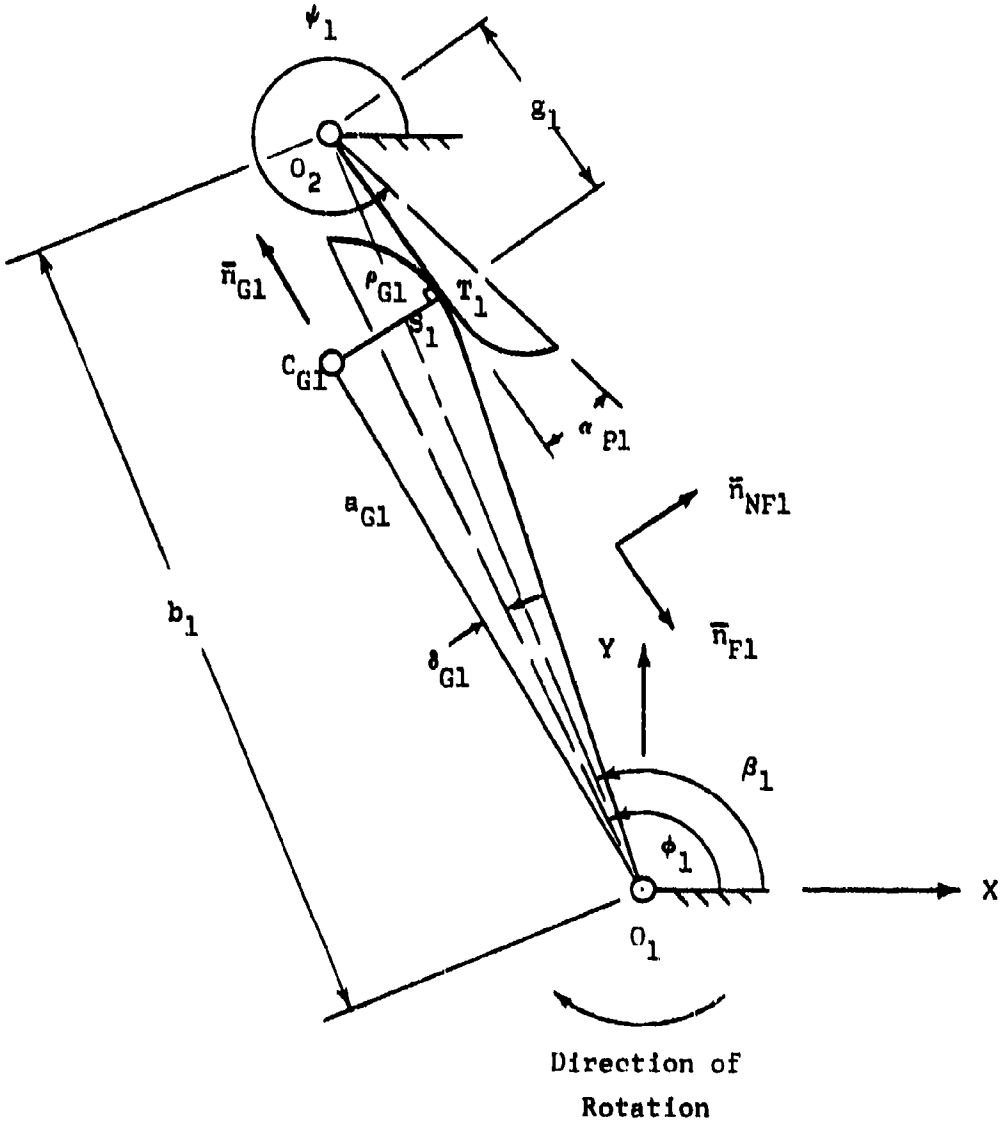

FIGURE G-3

ROUND ON FLAT PHASE FOR MESH NO. 1

II. DETERMINATION OF OUTPUT ANGLE ψ_1 AND DISTANCE s_1

The vector equation for the mechanism loop O_1-C_{G1}-S_1-T_1-O_2 has the form:

$$a_{G1}\bar{n}_{G1} + \rho_{G1}\bar{n}_{NF1} - s_1\bar{n}_{F1} - b_1\bar{n}_{\beta 1} = 0 \qquad (G\text{-}24)$$

Substitution of equations (G-1), (G-5), (G-22) and (G-23) leads to the following component equations

$$a_{G1}\cos(\phi_1 + \delta_{G1}) - \rho_{G1}\sin(\psi_1 - \alpha_{P1}) - b_1\cos\beta_1 - s_1\cos(\psi_1 - \alpha_{P1}) = 0 \qquad (G\text{-}25)$$

and

$$a_{G1}\sin(\phi_1 + \delta_{G1}) + \rho_{G1}\cos(\psi_1 - \alpha_{P1}) - b_1\sin\beta_1 - s_1\sin(\psi_1 - \alpha_{P1}) = 0 \qquad (G\text{-}26)$$

From equation (G-26) one obtains for the quantity s_1

$$s_1 = \frac{a_{G1}\sin(\phi_1 + \delta_{G1}) + \rho_{G1}\cos(\psi_1 - \alpha_{P1}) - b_1\sin\beta_1}{\sin(\psi_1 - \alpha_{P1})} \qquad (G\text{-}27)$$

This expression is now substituted in equation (G-25). This leads to the following:

$$A_{1F}\sin\psi_1 + B_{1F}\cos\psi_1 = C_{1F} \qquad \text{(G-28)}$$

where

$$A_{1F} = a_{G1}\cos(\phi_1 + \delta_{G1} + \alpha_{P1}) - b_1\cos(\beta_1 + \alpha_{P1})$$

$$B_{1F} = -a_{G1}\sin(\phi_1 + \delta_{G1} + \alpha_{P1}) + b_1\sin(\beta_1 + \alpha_{P1})$$

$$C_{1F} = \rho_{G1}$$

Equation (G-28) is solved for ψ_1 in the now customary manner:

$$\psi_1 = 2\tan^{-1}\frac{A_{1F} \pm \sqrt{A_{1F}^2 + B_{1F}^2 - C_{1F}^2}}{B_{1F} + C_{1F}} \qquad \text{(G-29)}$$

The appropriate sign is again found from geometric considerations.

III. DETERMINATION OF ANGULAR VELOCITY $\dot{\psi}_1$ DURING ROUND ON FLAT PHASE OF MOTION

Implicit differentiation of equation (G-28) with respect to time gives, for $\dot{\psi}_1$:

$$\dot{\psi}_1 = \dot{\phi}_1 \left[\frac{A_{1FD}\sin\psi_1 + B_{1FD}\cos\psi_1}{A_{1F}\cos\psi_1 - B_{1F}\sin\psi_1} \right] \qquad (G\text{-}30)$$

where

$$A_{1FD} = a_{G1}\sin(\phi_1 + \delta_{G1} + \alpha_{P1})$$

$$B_{1FD} = a_{G1}\cos(\phi_1 + \delta_{G1} + \alpha_{P1})$$

IV. RELATIVE VELOCITY $\bar{V}_{S1/T1_F}$ AT CONTACT POINT DURING ROUND ON FLAT PHASE OF MOTION

For the round on flat phase, the relative velocity $\bar{V}_{S1/T1_F}$ may be expressed by

$$\bar{V}_{S1/T1_F} = \bar{V}_{S1/O_1} - \bar{V}_{T1/O_2} \tag{G-31}$$

Now, this velocity has the direction of the unit vector $\pm \bar{E}_{F1}$. Since there is no velocity component along the pinion flank, equation (G-31) becomes

$$\begin{aligned} \bar{V}_{S1/T1_f} &= \bar{V}_{S1/O_1} \cdot \bar{n}_{F1} \\ &= \left\{ \left[\dot{\phi}_1 x \left(a_{G1} \bar{E}_{G1} + \rho_{G1} \bar{K}_{NF1} \right) \right] \cdot \bar{K}_{F1} \right\} \bar{n}_{F1} \end{aligned} \tag{G-32}$$

Appropriate substitution of unit vectors furnishes

$$\bar{V}_{S1/T1_F} = \dot{\phi}_1 \left[a_{G1} \sin(\psi_1 - \alpha_{P1} - \phi_1 - \delta_{G1}) - \rho_{G1} \right] \bar{K}_{F1} \tag{G-33}$$

V. DETERMINATION OF TRANSITION ANGLES

The transition angle, ϕ_{1T}, and the corresponding angle, ψ_{1T}, are reached when the round or round phase is followed by the round on flat one. They are obtained by letting $g_1 = f_{P1}$ in the component equations (G-25) and (G-26). This gives

$$a_{G1}\cos(\phi_{1T} + \delta_{G1}) - \rho_{G1}\sin(\psi_{1T} - \alpha_{P1}) - b_1\cos\beta_1 - f_{P1}\cos(\psi_{1T} - \alpha_{P1}) = 0 \qquad \text{(G-34)}$$

and

$$a_{G1}\sin(\phi_{1T} + \delta_{G1}) + \rho_{G1}\cos(\psi_{1T} - \alpha_{P1}) - b_1\sin\beta_1 - f_{P1}\sin(\psi_{1T} - \alpha_{P1}) = 0 \qquad \text{(G-35)}$$

From the above, one obtains

$$\cos(\phi_{1T} + \delta_{G1}) = \frac{1}{a_{G1}}\left[\rho_{G1}\sin(\psi_{1T} - \alpha_{P1}) + b_1\cos\beta_1 + f_{P1}\cos(\psi_{1T} - \alpha_{P1})\right] \qquad \text{(G-36)}$$

$$\sin(\phi_{1T} + \delta_{G1}) = \frac{1}{a_{G1}}\left[-\rho_{G1}\cos(\psi_{1T} - \alpha_{P1}) + b_1\sin\beta_1 + f_{P1}\sin(\psi_{1T} - \alpha_{P1})\right] \qquad \text{(G-37)}$$

The angle ψ_{1T} is now obtained by letting

$$\sin^2(\phi_{1T} + \delta_{G1}) + \cos^2(\phi_{1T} + \delta_{G1}) = 1$$

This results in

$$A_{1T}\sin\psi_{1T} + B_{1T}\cos\psi_{1T} = C_{1T} \tag{G-38}$$

where

$$A_{1T} = \rho_{G1}\cos(\beta_1 + \alpha_{P1}) + f_{P1}\sin(\beta_1 + \alpha_{P1})$$

$$B_{1T} = -\rho_{G1}\sin(\beta_1 + \alpha_{P1}) + f_{P1}\cos(\beta_1 + \alpha_{P1})$$

$$C_{1T} = \frac{a_{G1}^2 - \rho_{G1}^2 - b_1^2 - f_{P1}^2}{2b_1}$$

Finally,

$$\psi_{1T} = 2\tan^{-1}\frac{A_{1T} \pm \sqrt{A_{1T}^2 + B_{1T}^2 - C_{1T}^2}}{B_{1T} + C_{1T}} \tag{G-39}$$

The appropriate sign must be found from geometric considerations. The associated angle ϕ_{1T} may be found with the help of either equation (G-36) or equation (G-37):

$$\phi_{1T} = \cos^{-1}\left[\frac{\rho_{G1}\sin(\psi_{1T} - \alpha_{P1}) + f_{P1}\cos(\psi_{1T} - \alpha_{P1}) + b_1\cos\beta_1}{a_{G1}}\right] - \delta_{G1} \qquad \text{(G-40)}$$

or

$$\phi_{1T} = \sin^{-1}\left[\frac{-\rho_{G1}\cos(\psi_{1T} - \alpha_{P1}) + f_{P1}\sin(\psi_{1T} - \alpha_{P1}) + b_1\sin\beta_1}{a_{G1}}\right] - \delta_{G1} \qquad \text{(G-41)}$$

VI. SENSING EQUATION FOR THE DETERMINATION OF CONTACT ON SUBSEQUENT TOOTH MESH

The following contact sensing equation is derived with the assumption that subsequent contact is made in the round on round phase of the motion, in the manner shown in Section VI of Appendix E. Now, the configuration is that of Figure G-2 where gear no. 1 rotates in a clockwise direction.

Before contact is made, the distance between the centers of curvature C_{G1} and C_{P1} is given by

$$\overline{C_{G1}C_{P1}} = L_{x1}\bar{I} + L_{y1}\bar{J} \qquad (G\text{-}42)$$

If $\Delta\phi_1$ and $\Delta\psi_1$ represent the tooth spacing angles of gear no. 1 and pinion no. 2, respectively, the associated loop equation becomes (see Figures E-3 and G-2).

$$a_{G1}\left[\cos(\phi_1 + \Delta\phi_1 + \delta_{G1})\bar{I} + \sin(\phi_1 + \Delta\phi_1 + \delta_{G1})\bar{J}\right] + L_{x1}\bar{I} + L_{y1}\bar{J}$$
$$= b_1(\cos\beta_1\bar{I} + \sin\beta_1\bar{J}) + a_{P1}\left[\cos(\psi_1 - \Delta\psi_1 + \delta_{P1})\bar{I} + \sin(\psi_1 - \Delta\psi_1 + \delta_{P1})\bar{J}\right] \qquad (G\text{-}43)$$

where

ψ_1 = angle of pinion no. 2 as determined for the round on flat mode with equation (G-29)

The magnitudes of L_{x1} and L_{y1} are determined from the component form of equation (G-43), i.e.,

$$L_{x1} = b_1\cos\beta_1 + a_{P1}\cos(\psi_1 - \Delta\psi_1 + \delta_{P1}) - a_{G1}\cos(\phi_1 + \Delta\phi_1 + \delta_{G1}) \qquad (G\text{-}44)$$

and

$$L_{y1} = b_1\sin\beta_1 + a_{P1}\sin(\psi_1 - \Delta\psi_1 + \delta_{P1}) - a_{G1}\sin(\phi_1 + \Delta\phi_1 + \delta_{G1}) \qquad (G\text{-}45)$$

Contact will have occurred as soon as

$$\sqrt{L_{x1}^2 + L_{y1}^2} \leq \rho_{G1} + \rho_{P1} \qquad \text{(G-46)}$$

2. KINEMATICS OF MESH NO. 2 (GEAR NO. 2 AND PINION NO. 3)

a. ROUND ON ROUND PHASE OF MOTION

Figure G-4 gives a schematic representation of the round on round phase of the motion. Only the contacting faces of the gear and the pinion are shown. It is to be noted that the input gear rotates in the counter-clockwise direction, and that the output angle ψ_1 of mesh no. 1 is identical to the input angle ϕ_2 of mesh no. 2.

I. UNIT VECTORS

The unit vector in the direction O_2C_{G2} of the gear is given by

$$\bar{n}_{G2} = \cos(\phi_2 - \delta_{G2})\bar{I} + \sin(\phi_2 - \delta_{G2})\bar{J} \qquad (G\text{-}47)$$

The unit vector in the direction $C_{G2}C_{P2}$ is given by

$$\bar{n}_{\lambda 2} = \cos\lambda_2\bar{I} + \sin\lambda_2\bar{J} \qquad (G\text{-}48)$$

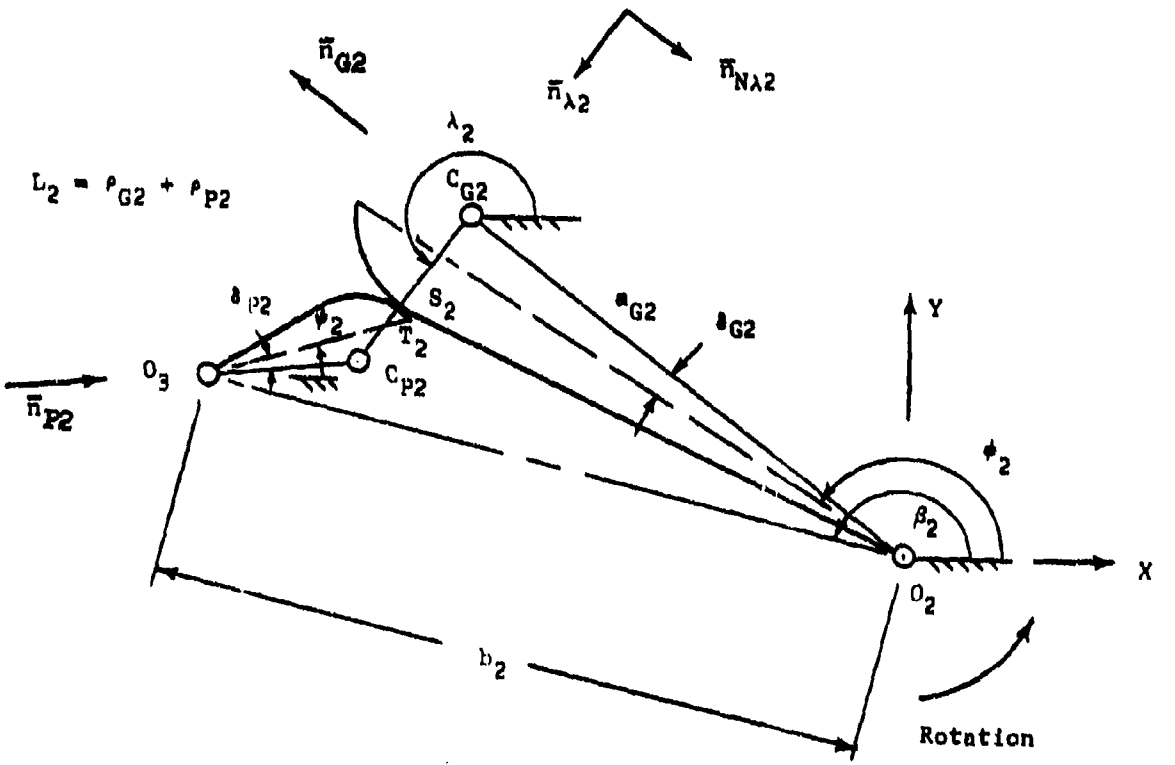

FIGURE G-4

ROUND ON ROUND PHASE FOR MESH NO. 2

The unit vector normal to $\bar{n}_{\lambda 2}$ in the right hand sense becomes

$$\bar{n}_{N\lambda 2} = -\sin\lambda_2\bar{I} + \cos\lambda_2\bar{j} \qquad \text{(G-49)}$$

The pinion unit vector $\bar{n}_{P2}$, in the direction O_3C_{P2}, is represented by

$$\bar{n}_{P2} = \cos(\psi_2 - \delta_{P2})\bar{I} + \sin(\psi_2 - \delta_{P2})\bar{j} \qquad \text{(G-50)}$$

Finally, the unit vector along the centerline O_2O_3 is given by

$$\bar{n}_{\beta 2} = \cos\beta_2\bar{I} + \sin\beta_2\bar{j} \qquad \text{(G-51)}$$

II. DETERMINATION OF OUTPUT ANGLE ψ_2 AND "COUPLER" ANGLE λ_2

The loop equation of the equivalent four-bar linkage is given by

$$a_{G2}\bar{n}_{G2} + L_2\bar{n}_{\lambda 2} - a_{P2}\bar{n}_{P2} - b_2\bar{n}_{\beta 2} = 0 \qquad \text{(G-52)}$$

where

$$L_2 = \rho_{G2} + \rho_{P2} \qquad \text{(G-53)}$$

After substitution of the unit vector, as given earlier, one obtains the following component equations

$$a_{G2}\cos(\phi_2 - \delta_{G2}) + L_2\cos\lambda_2 - a_{P2}\cos(\psi_2 - \delta_{P2}) - b_2\cos\beta_2 = 0 \qquad \text{(G-54)}$$

and

$$a_{G2}\sin(\phi_2 - \delta_{G2}) + L_2\sin\lambda_2 - a_{P2}\sin(\psi_2 - \delta_{P2}) - b_2\sin\beta_2 = 0 \qquad \text{(G-55)}$$

To solve for the output angle ψ_2 in terms of the input angle ϕ_2, substitute the expressions for $\sin\lambda_2$ and $\cos\lambda_2$, as obtained from the component equations (G-54) and (G-55), into

$$\sin^2\lambda_2 + \cos^2\lambda_2 = 1 \qquad \text{(G-56)}$$

This leads to

$$A_{2R}\sin\psi_2 + B_{2R}\cos\psi_2 = C_{2R} \qquad \text{(G-57)}$$

where

$$A_{2R} = b_2\sin(\beta_2 + \delta_{P2}) - a_{G2}\sin(\phi_2 - \delta_{G2} + \delta_{P2})$$

$$B_{2R} = b_2\cos(\beta_2 + \delta_{P2}) - a_{G2}\cos(\phi_2 - \delta_{G2} + \delta_{P2})$$

$$C_{2R} = \frac{L_2^2 - b_2^2 - a_{G2}^2 - a_{P2}^2 + 2a_{G2}b_2\cos(\phi_2 - \delta_{G2} - \beta_2)}{2a_{P2}}$$

Equation (G-57) is then solved for ψ_2 in the manner discussed in Appendix E

$$\psi_2 = 2\tan^{-1}\frac{A_{2R} \pm \sqrt{A_{2R}^2 + B_{2R}^2 - C_{2R}^2}}{B_{2R} + C_{2R}} \qquad \text{(G-58)}$$

The correct sign must again be determined from geometric considerations

The angle λ_2 may now be determined either from equation (G-54) or equation (G-55)

$$\lambda_2 = \cos^{-1}\left[\frac{b_2\cos\beta_2 + a_{P2}\cos(\psi_2 - \delta_{P2}) - a_{G2}\cos(\phi_2 - \delta_{G2})}{L_2}\right] \qquad \text{(G-59)}$$

or

$$\lambda_2 = \sin^{-1}\left[\frac{b_2\sin\beta_2 + a_{P2}\sin(\psi_2 - \delta_{P2}) - a_{G2}\sin(\phi_2 - \delta_{G2})}{L_2}\right] \qquad \text{(G-60)}$$

III. DETERMINATION OF OUTPUT ANGULAR VELOCITY $\dot{\psi}_2$

Implicit differentiation of equation (G-57) with respect to time leads to

$$\dot{\psi}_2 = \dot{\phi}_2 \left[\frac{A_{2RD}\sin\psi_2 - B_{2RD}\cos\psi_2 - C_{2RD}}{A_{2R}\cos\psi_2 - B_{2R}\sin\psi_2} \right] \qquad \text{(G-61)}$$

where

$$A_{2RD} = a_{G2}\cos(\phi_2 - \delta_{G2} + \delta_{P2})$$

$$B_{2RD} = a_{G2}\sin(\phi_2 - \delta_{G2} + \delta_{P2})$$

$$C_{2RD} = \frac{a_{G2}b_2\sin(\phi_2 - \delta_{G2} - \beta_2)}{a_{P2}}$$

IV. RELATIVE VELOCITY AT THE CONTACT POINT

The relative velocity $\bar{V}_{S2/T2_R}$, of point S_2 on gear no. 2 with respect to point T_2 on pinion no. 3 has the direction of the unit vector $\bar{n}_{N\lambda 2}$. Thus, in the manner of equation (G-20).

$$\bar{V}_{S2/T2_R} = \left\{ \left[\dot{\phi}_2 \bar{k} \times (a_{G2}\bar{n}_{G2} + \rho_{G2}\bar{n}_{\lambda 2}) \right] \cdot \bar{n}_{N\lambda 2} - \left[\dot{\psi}_2 \bar{k} \times (a_{P2}\bar{n}_{P2} - \rho_{P2}\bar{n}_{\lambda 2}) \right] \cdot \bar{n}_{N\lambda 2} \right\} \bar{n}_{N\lambda 2} \tag{G-62}$$

Substitution of unit vectors yields

$$\bar{V}_{S2/T2_R} = \left\{ \dot{\phi}_2 \left[a_{G2}\cos(\phi_2 - \delta_{G2} - \lambda_2) + \rho_{G2} \right] - \dot{\psi}_2 \left[a_{P2}\cos(\psi_2 - \delta_{P2} - \lambda_2) - \rho_{P2} \right] \right\} \bar{n}_{N\lambda 2} \tag{G-63}$$

b. ROUND ON FLAT PHASE OF MOTION

Figure G-5 shows a schematic view of mesh no. 2 in the round on flat phase of the motion. Again, only the contacting sides of the gear teeth are indicated.

I. UNIT VECTORS

The unit vector in the direction O_3T_2, along the flank of pinion no. 3, is given by

$$\bar{n}_{F2} = \cos(\psi_2 + \alpha_{P2})\bar{I} + \sin(\psi_2 + \alpha_{P2})\bar{J} \qquad (G\text{-}64)$$

The unit vector $\bar{n}_{NF2}$, in the direction S_2C_{G2}, is normal to $\bar{n}_{F2}$ in the right hand sense

$$\bar{n}_{NF2} = -\sin(\psi_2 + \alpha_{P2})\bar{I} + \cos(\psi_2 + \alpha_{P2})\bar{J} \qquad (G\text{-}65)$$

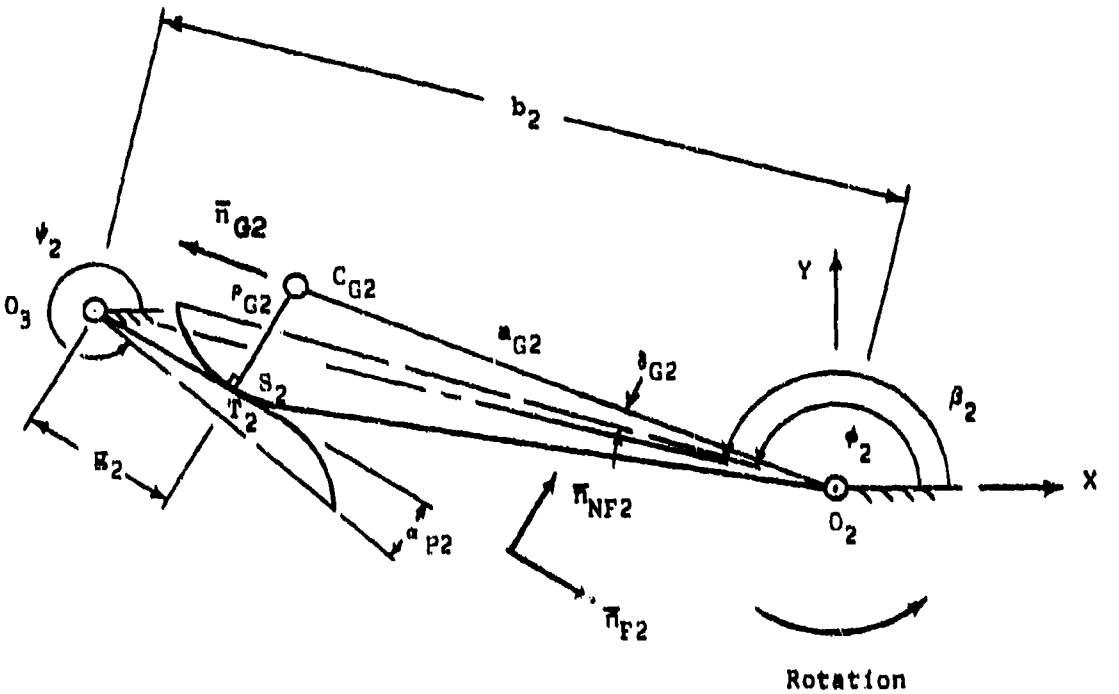

FIGURE G-5

ROUND ON FLAT PHASE OF MOTION OF MESH NO. 2

II. DETERMINATION OF OUTPUT ANGLE ψ_2 AND DISTANCE g_2

The vector equation for the mechanism loop O_2-C_{G2}-S_2-T_2-O_3 has the form

$$a_{G2}\bar{n}_{G2} - \rho_{G2}\bar{n}_{NF2} - g_2\bar{n}_{F2} - b_2\bar{n}_{\beta 2} = 0 \qquad \text{(G-66)}$$

Appropriate substitutions for the unit vectors furnish the following component equations.

$$a_{G2}\cos(\phi_2 - \delta_{G2}) + \rho_{G2}\sin(\psi_2 + \alpha_{P2}) - b_2\cos\beta_2 - g_2\cos(\psi_2 + \alpha_{P2}) = 0 \qquad \text{(G-67)}$$

and

$$a_{G2}\sin(\phi_2 - \delta_{G2}) - \rho_{G2}\cos(\psi_2 + \alpha_{P2}) - b_2\sin\beta_2 - g_2\sin(\psi_2 + \alpha_{P2}) = 0 \qquad \text{(G-68)}$$

From equation (G-68) one obtains the following expression for g_2

$$g_2 = \frac{a_{G2}\sin(\phi_2 - \delta_{G2}) - \rho_{G2}\cos(\psi_2 + \alpha_{P2}) - b_2\sin\beta_2}{\sin(\psi_2 + \alpha_{P2})} \qquad \text{(G-69)}$$

This expression is now substituted into equation (G-67), and one obtains the following:

$$A_{2F}\sin\psi_2 + B_{2F}\cos\psi_2 = C_{2F} \qquad \text{(G-70)}$$

where

$$A_{2F} = a_{G2}\cos(\phi_2 - \delta_{G2} - \alpha_{P2}) - b_2\cos(\beta_2 - \alpha_{P2})$$

$$B_{2F} = -a_{G2}\sin(\phi_2 - \delta_{G2} - \alpha_{P2}) + b_2\sin(\beta_2 - \alpha_{P2})$$

$$C_{2F} = -\rho_{G2}$$

Equation (G-70) is solved in the customary manner:

$$\psi_2 = 2\tan^{-1}\frac{A_{2F} \pm \sqrt{A_{2F}^2 + B_{2F}^2 - C_{2F}^2}}{B_{2F} + C_{2F}} \qquad \text{(G-71)}$$

The appropriate sign is again found from geometric considerations.

III. DETERMINATION OF ANGULAR VELOCITY $\dot{\psi}_2$ DURING ROUND ON FLAT PHASE OF MOTION

Implicit differentiation of equation (G-70) with respect to time gives the following expression for $\dot{\psi}_2$:

$$\dot{\psi}_2 = \dot{\phi}_2 \left[\frac{A_{2FD}\sin\psi_2 + B_{2FD}\cos\psi_2}{A_{2F}\cos\psi_2 - B_{2F}\sin\psi_2} \right] \qquad (G\text{-}72)$$

where

$$A_{2FD} = a_{G2}\sin(\phi_2 - \delta_{G2} - \alpha_{P2})$$

$$B_{2FD} = a_{G2}\cos(\phi_2 - \delta_{G2} - \alpha_{P2})$$

IV. RELATIVE VELOCITY $\bar{V}_{S2/T2_F}$ AT CONTACT POINT DURING ROUND ON FLAT PHASE OF MOTION

Again, the relative velocity $\bar{V}_{S2/T2_F}$ consists only of that component of $\bar{V}_{S2/O_2}$ which is directed along the pinion flank. Thus,

$$\bar{V}_{S2/T2_F} = \left[\bar{V}_{S2/O_2} \cdot \bar{n}_{F2} \right] \bar{n}_{F2}$$

$$= \left\{ \left[\dot{\phi}_2 \bar{k} \times (a_{G2}\bar{n}_{G2} - \rho_{G2}\bar{n}_{NF2}) \right] \cdot \bar{n}_{F2} \right\} \bar{n}_{F2} \qquad (G-73)$$

Appropriate substitution of unit vectors gives

$$\bar{V}_{S2/T2_F} = \dot{\phi}_2 \left[a_{G2}\sin(\psi_2 + \alpha_{P2} - \phi_2 + \delta_{G2}) + \rho_{G2} \right] \bar{n}_{F2} \qquad (G-74)$$

V. DETERMINATION OF TRANSITION ANGLES

The transition angle ϕ_{2T} and the angle ψ_{2T}, which correspond to the transition from the round on round to the round on flat phase of motion, are obtained by letting $g_2 = f_{P2}$ in the component equations (G-67) and (G-68). From this one finds the following with $\phi_2 = \phi_{2T}$ and $\psi_2 = \psi_{2T}$:

$$\cos(\phi_{2T} - \delta_{G2}) = \frac{1}{a_{G2}} \left[-\rho_{G2}\sin(\psi_{2T} + \alpha_{P2}) + b_2\cos\beta_2 + f_{P2}\cos(\psi_{2T} + \alpha_{P2}) \right] \qquad (G-75)$$

and

$$\sin(\phi_{2T} - \delta_{G2}) = \frac{1}{a_{G2}}\left[\rho_{G2}\cos(\psi_{2T} + \alpha_{P2}) + b_2\sin\beta_2 + f_{P2}\sin(\psi_{2T} + \alpha_{P2})\right] \quad \text{(G-76)}$$

The angle ψ_{2T} is now found by substituting the above expressions into

$$\sin^2(\phi_{2T} - \delta_{G2}) + \cos^2(\phi_{2T} - \delta_{G2}) = 1 \quad \text{(G-77)}$$

This results in

$$A_{2T}\sin\psi_{2T} + B_{2T}\cos\psi_{2T} = C_{2T} \quad \text{(G-78)}$$

where

$$A_{2T} = -\rho_{G2}\cos(\beta_2 - \alpha_{P2}) + f_{P2}\sin(\beta_2 - \alpha_{P2})$$

$$B_{2T} = \rho_{G2}\sin(\beta_2 - \alpha_{P2}) + f_{P2}\cos(\beta_2 - \alpha_{P2})$$

$$C_{2T} = \frac{a_{G2}^2 - \rho_{G2}^2 - b_2^2 - f_{P2}^2}{2\,b_2}$$

Finally, in the usual way

$$\psi_{2T} = 2\tan^{-1}\frac{A_{2T} \pm \sqrt{A_{2T}^2 + B_{2T}^2 - C_{2T}^2}}{B_{2T} + C_{2T}} \qquad \text{(G-79)}$$

Again, the sign must be decided from geometric considerations.

The associated angle ϕ_{2T} may be found with the help of either equation (G-75) or equation (G-76), i.e.,

$$\phi_{2T} = \cos^{-1}\left[\frac{-\rho_{G2}\sin(\psi_{2T} + \alpha_{P2}) + b_2\cos\beta_2 + f_{P2}\cos(\psi_{2T} + \alpha_{P2})}{a_{G2}}\right] + \delta_{G2} \qquad \text{(G-80)}$$

or

$$\phi_{2T} = \sin^{-1}\left[\frac{\rho_{G2}\cos(\psi_{2T} + \alpha_{P2}) + b_2\sin\beta_2 + f_{P2}\sin(\psi_{2T} + \alpha_{P2})}{a_{G2}}\right] + \delta_{G2} \qquad \text{(G-81)}$$

VI. SENSING EQUATION FOR THE DETERMINATION OF CONTACT ON SUBSEQUENT TOOTH MESH

The contact sensing equation for mesh no. 2 is derived similarly to that for mesh no. 1 before contact is made in the round on round mode. The distance between the centers of curvature C_{G2} and C_{P2} is given by

$$\overline{C_{G2}C_{P2}} = L_{x2}\bar{i} + L_{y2}\bar{j} \qquad (G\text{-}82)$$

If $\Delta\phi_2$ and $\Delta\psi_2$ represent the tooth spacing angles of gear no. 2 and pinion no. 3 respectively, the associated loop equation becomes (see Figures E-3 and G-4)

$$a_{G2}\left[\cos(\phi_2 - \Delta\phi_2 - \delta_{G2})\bar{i} + \sin(\phi_2 - \Delta\phi_2 - \delta_{G2})\bar{j}\right] + L_{x2}\bar{i} + L_{y2}\bar{j}$$
$$- a_{P2}\left[\cos(\psi_2 + \Delta\psi_2 - \delta_{P2})\bar{i} + \sin(\psi_2 + \Delta\psi_2 - \delta_{P2})\bar{j}\right]$$
$$- b_2\left[\cos\beta_2\bar{i} + \sin\beta_2\bar{j}\right] = 0 \qquad (G\text{-}83)$$

Note that for mesh no. 2, the angular increment $\Delta\phi_2$ is negative while $\Delta\psi_2$ is positive. Further, as before, the angle ψ_2 must be

determined for the round on flat phase of the motion.

The magnitudes of L_{x2} and L_{y2} are determined from the components of equation (G-83), i.e.,

$$L_{x2} = b_2\cos\beta_2 + a_{P2}\cos(\psi_2 + \Delta\psi_2 - \delta_{P2}) - a_{G2}\cos(\phi_2 - \Delta\phi_2 - \delta_{G2}) \tag{G-84}$$

while

$$L_{y2} = b_2\sin\beta_2 + a_{P2}\sin(\psi_2 + \Delta\psi_2 - \delta_{P2}) - a_{G2}\sin(\phi_2 - \Delta\phi_2 - \delta_{G2}) \tag{G-85}$$

Contact will occur as soon as

$$\sqrt{L_{x2}^2 + L_{y2}^2} \leq \rho_{G2} + \rho_{P2} \tag{G-86}$$

3. KINEMATICS OF MESH NO. 3 (GEAR NO. 3 AND PINION NO. 4)

Since mesh no. 3 is kinematically equivalent to mesh no. 1, the kinematic equations for mesh no. 3 may be obtained from those for mesh no. 1. The angle β_3 must replace the angle β_1 and the center distance b_3 is used instead of b_1. All parameters of gear no. 1 are replaced by those of gear no. 3 and the pinion parameters of pinion no. 4 are substituted for those of pinion no. 2.

It is to be noted that the input angle ϕ_3 of mesh no. 3 is identical to the output angle ψ_2 of mesh no. 2.

a. ROUND ON ROUND PHASE OF MOTION

The ouput angle ψ_3 is obtained with the help of equation (G-12)

$$\psi_3 = 2 \tan^{-1} \frac{A_{3R} \pm \sqrt{A_{3R}^2 + B_{3R}^2 - C_{3R}^2}}{B_{3R} + C_{3R}} \qquad \text{(G-87)}$$

where

$$A_{3R} = a_{G3}\sin(\phi_3 + \delta_{G3} - \delta_{P3}) - b_3\sin(\beta_3 - \delta_{P3})$$

$$B_{3R} = a_{G3}\cos(\phi_3 + \delta_{G3} - \delta_{P3}) - b_3\cos(\beta_3 - \delta_{P3})$$

$$C_{3R} = \frac{a_{P3}^2 + a_{G3}^2 + b_3^2 - L_3^2 - 2a_{G3}b_3\cos(\phi_3 + \delta_{G3} - \beta_3)}{2\,a_{P3}}$$

and

$$L_3 = \rho_{G3} + \rho_{P3} \qquad \text{(G-88)}$$

The angle λ_3 may be found with the help of equation (G-13) or equation (G-14)

$$\lambda_3 = \cos^{-1}\left[\frac{b_3\cos\beta_3 + a_{P3}\cos(\psi_3 + \delta_{P3}) - a_{G3}\cos(\phi_3 + \delta_{G3})}{L_3}\right] \qquad \text{(G-89)}$$

or

$$\lambda_3 = \sin^{-1}\left[\frac{b_3\sin\beta_3 + a_{P3}\sin(\psi_3 + \delta_{P3}) - a_{G3}\sin(\phi_3 + \delta_{G3})}{L_3}\right] \qquad \text{(G-90)}$$

The angular velocity $\dot{\psi}_3$ is obtained from equation (G-15)

$$\dot{\psi}_3 = \dot{\phi}_3\left[\frac{B_{3RD}\cos\psi_3 - A_{3RD}\sin\psi_3 + C_{3RD}}{A_{3R}\cos\psi_3 - B_{3R}\sin\psi_3}\right] \qquad \text{(G-91)}$$

where

$$A_{3RD} = a_{G3}\cos(\phi_3 + \delta_{G3} - \delta_{P3})$$

$$B_{3RD} = a_{G3}\sin(\phi_3 + \delta_{G3} - \delta_{P3})$$

$$C_{3RD} = \frac{a_{G3}b_3\sin(\phi_3 + \delta_{G3} - \beta_3)}{a_{P3}}$$

The relative velocity $\bar{V}_{S3/T3_R}$ becomes, according to Equation (G-21),

$$\bar{V}_{S3/T3_R} = \left\{ \dot{\phi}_3\left[a_{G3}\cos(\phi_3 + \delta_{G3} - \lambda_3) + \rho_{G3}\right] - \dot{\psi}_3\left[a_{P3}\cos(\psi_3 + \delta_{P3} - \lambda_3) - \rho_{P3}\right] \right\} \bar{n}_{N\lambda 3} \qquad \text{(G-92)}$$

where, according to equation (G-3),

$$\bar{n}_{N\lambda 3} = -\sin\lambda_3\bar{I} + \cos\lambda_3\bar{J} \qquad \text{(G-93)}$$

b. ROUND ON FLAT PHASE OF MOTION

The output angle ψ_3 is obtained from equation (G-29)

$$\psi_3 = 2 \tan^{-1} \frac{A_{3F} \pm \sqrt{A_{3F}^2 + B_{3F}^2 - C_{3F}^2}}{B_{3F} + C_{3F}} \qquad \text{(G-94)}$$

where

$$A_{3F} = a_{G3}\cos(\phi_3 + \delta_{G3} + \alpha_{P3}) - b_3\cos(\beta_3 + \alpha_{P3})$$

$$B_{3F} = -a_{G3}\sin(\phi_3 + \delta_{G3} + \alpha_{P3}) + b_3\sin(\beta_3 + \alpha_{P3})$$

$$C_{3F} = \rho_{G3}$$

The distance g_3 becomes, according to equation (G-27),

$$g_3 = \frac{a_{G3}\sin(\phi_3 + \delta_{G3}) + \rho_{G3}\cos(\psi_3 - \alpha_{P3}) - b_3\sin\beta_3}{\sin(\psi_3 - \alpha_{P3})} \qquad \text{(G-95)}$$

The angular velocity $\dot{\psi}_3$ for the round on flat phase of the motion is found from equation (G-30)

$$\dot{\psi}_3 = \dot{\phi}_3 \left[\frac{A_{3FD}\sin\psi_3 + B_{3FD}\cos\psi_3}{A_{3F}\cos\psi_3 - B_{3F}\sin\psi_3} \right] \qquad \text{(G-96)}$$

where

$$A_{3FD} = a_{G3}\sin(\phi_3 + \delta_{G3} + \alpha_{P3})$$

$$B_{3FD} = a_{G3}\cos(\phi_3 + \delta_{G3} + \alpha_{P3})$$

The relative velocity $\bar{V}_{S3/T3_F}$ for the round on flat phase comes from equation (G-33)

$$\bar{V}_{S3/T3_F} = \dot{\phi}_3 \left[a_{G3}\sin(\psi_3 - \alpha_{P3} - \phi_3 - \delta_{G3}) - \rho_{G3} \right] \bar{n}_{F3} \qquad \text{(G-97)}$$

where

$$\bar{n}_{F3} = \cos(\psi_3 - \alpha_{P3})\bar{i} + \sin(\psi_3 - \alpha_{P3})\bar{j} \qquad \text{(G-98)}$$

according to equation (G-22).

The transition angle ψ_{3T} is obtained by way of equation (G-39)

$$\psi_{3T} = 2 \tan^{-1} \frac{A_{3T} \pm \sqrt{A_{3T}^2 + B_{3T}^2 - C_{3T}^2}}{B_{3T} + C_{3T}} \qquad \text{(G-99)}$$

where

$$A_{3T} = \rho_{G3}\cos(\beta_3 + \alpha_{P3}) + f_{P3}\sin(\beta_3 + \alpha_{P3})$$

$$B_{3T} = -\rho_{G3}\sin(\beta_3 + \alpha_{P3}) + f_{P3}\cos(\beta_3 + \alpha_{P3})$$

$$C_{3T} = \frac{a_{G3}^2 - \rho_{G3}^2 - b_3^2 - f_{P3}^2}{2\, b_3}$$

The associated angle ϕ_{3T} may be obtained from equation (G-40) or from equation (G-41)

$$\phi_{3T} = \cos^{-1}\left[\frac{\rho_{G3}\sin(\psi_{3T} - \alpha_{P3}) + f_{P3}\cos(\psi_{3T} - \alpha_{P3}) + b_3\cos\beta_3}{a_{G3}}\right] - \delta_{G3} \qquad \text{(G-100)}$$

or

$$\phi_{3T} = \sin^{-1}\left[\frac{-\rho_{G3}\cos(\psi_{3T} - \alpha_{P3}) + f_{P3}\sin(\psi_{3T} - \alpha_{P3}) + b_3\sin\beta_3}{a_{G3}}\right] - \delta_{G3} \qquad \text{(G-101)}$$

Finally, the contact sensing equation is based on equations (G-44) - (G-46). Contact will occur, when

$$\sqrt{L_{x3}^2 + L_{y3}^2} \leq \rho_{G3} + \rho_{P3} \tag{G-102}$$

where with the tooth spacing angles $\Delta\phi_3$ and $\Delta\psi_3$,

$$L_{x3} = b_3\cos\beta_3 + a_{P3}\cos(\psi_3 - \Delta\psi_3 + \delta_{P3}) - a_{G3}\cos(\phi_3 + \Delta\phi_3 + \delta_{G3}) \tag{G-103}$$

and

$$L_{y3} = b_3\sin\beta_3 + a_{P3}\sin(\psi_3 - \Delta\psi_3 + \delta_{P3}) - a_{G3}\sin(\phi_3 + \Delta\phi_3 + \delta_{G3}) \tag{G-104}$$

APPENDIX H

MOMENT INPUT-OUTPUT RELATIONSHIPS FOR TWO AND THREE STEP-UP GEAR TRAINS WITH TEETH OPERATING IN A SPIN ENVIRONMENT

The following gives the derivations for the moment input-output relationships of two and three step-up gear trains which operate in a spin environment.

Figure G-1 of Appendix G shows the basic configuration of a three step-up gear train. The input moment, M_{in}, which acts on gear no. 1, is held in equilibrium by the moment, M_{o4}, which acts on pinion no. 4.

Since in all three meshes there may either be round on round or round on flat type of contact, the force and moment analyses must account for various contact combinations. Table H-1 shows the eight different phase combinations which may occur in a three step-up gear train, and for which input-output relationships must be found. The two step-up gear train, which is shown in Figure A-10 of Appendix A for involute gearing, does not contain pinion 4 and gear no. 3. Here, the input moment M_{in}, which acts on gear no. 1, is held in equilibrium by moment M_{o3}, which acts on pinion no. 3.

Case no.	Mesh No. 3 (Gear 3 & Pinion 4)	Mesh No. 2 (Gear 2 & Pinion 3)	Mesh No. 1 (Gear 1 & Pinion 2)
1	R	R	R
2	R	R	F
3	R	F	F
4	R	F	R
5	F	F	F
6	F	F	R
7	F	R	R
8	F	R	F

TABLE H-1

POSSIBLE COMBINATIONS OF PHASES FOR THREE STEP-UP GEAR TRAIN AS SHOWN IN FIGURE G-1
R = Round on Round
F = Round on Flat

When ogival teeth are involved, there are four possible phase combinations of the two remaining meshes. These are shown in Table H-2. Again input-output relationships must be obtained for each of them.

Case No.	Mesh No. 2 (Gear 2 & Pinion 3)	Mesh No. 1 (Gear 1 & Pinion 2)
1	R	R
2	R	F
3	F	F
4	F	R

TABLE H-2

POSSIBLE COMBINATIONS OF PHASES FOR TWO STEP-UP GEAR TRAIN AS SHOWN IN FIGURE A-10
R = Round on Round
F = Round on Flat

The unit vectors, mechanism angles and kinematic terms necessary for the following analyses were derived in Appendix G. (See also Appendix D for a description of the geometry of ogival teeth.) Certain terms used in connection with mesh no. 3 may be obtained from expressions derived for mesh no. 1 in Appendix G by the replacement of the appropriate subscript numbers, since the kinematics of these meshes are identical. The following additional nomenclature is used:

$\mathcal{R}_i$ = distance from spin axis C to pivot points O_i of individual gears. (i = 1, 2, 3, 4 as applicalbe)

γ_i = angle of lines $\mathcal{R}_i = CO_i$ with respect to the body-fixed X-axis

ω = spin velocity of fuze body

m_i = mass of various gears, pinions and gear-pinion combinations

Q_i = $m_i \mathcal{R}_i \omega^2$, centrifugal force acting on individual gear components. (Now called Q_i to differentiate it from the pinion contact point T_i.)

ρ_i = pivot radius

ρ_{Pi} = radius of curvature of pinion tooth (ogival)

ρ_{Gi} = radius of curvature of gear tooth (ogival)

μ = coefficient of friction at pivots as well as at contact point between gears and pinions

The pivot friction moments are obtained according to equation (A-3b) of Appendix A. They always oppose motion regardless of the assumption of direction of the pivot reactions F_{xi} and F_{yi}. To this end the pivot forces $\tilde{F}_{xi}$ and $\tilde{F}_{yi}$, which represent the sums of the absolute values of their component parts, are added

algebraically. The algebraic addition of such modified reactions provides a conservative, i.e., a somewhat overstated friction moment.

The directions of the friction forces of the gears on the pinions are always those of the relative velocities $\bar{V}_{Si/Ti}$, where points S_i and T_i are located at the contact points of the gears and pinions, respectively. This allows the introduction of a signum convention. For the round on round phases,

$$s_{iR} = \frac{V_{Si/Ti_R}}{\left|V_{Si/Ti_R}\right|} \qquad \text{(H-1)}$$

For the round on flat phase, the convention becomes

$$s_{iF} = \frac{V_{Si/Ti_F}}{\left|V_{Si/Ti_F}\right|} \qquad \text{(H-2)}$$

The expressions for the above relative velocities, which are different for round on round and for round on flat, are given in Appendix G.

1. INPUT-OUTPUT ANALYSIS OF THREE STEP-UP GEAR TRAIN

a. CASE NO. 1: RRR

I. FORCE AND MOMENT EQUILIBRIA OF PINION NO. 4

Figure H-1 shows a schematic free body diagram of pinion no. 4 in the round on round mode of contact. The equivalent four-bar linkage associated with mesh no. 3 is also indicated. The pinion is acted upon by the equilibrant moment M_{04} in the direction opposite to its counterclockwise rotation. The contact force of gear no. 3 on the pinion is given by

$$\bar{F}_{34} = F_{34}\bar{n}_{\lambda 3} \qquad \text{(H-3)}$$

The associated friction force exerted by the gear on the pinion has the direction of the relative velocity $\bar{V}_{S3/T3_R}$, as given by equation (G-92). With the use of the signum convention of equation (H-1) the friction force $\bar{F}_{f34}$ becomes

$$\bar{F}_{f34} = \mu s_{3R} F_{34} \bar{n}_{N\lambda 3} \qquad \text{(H-4)}$$

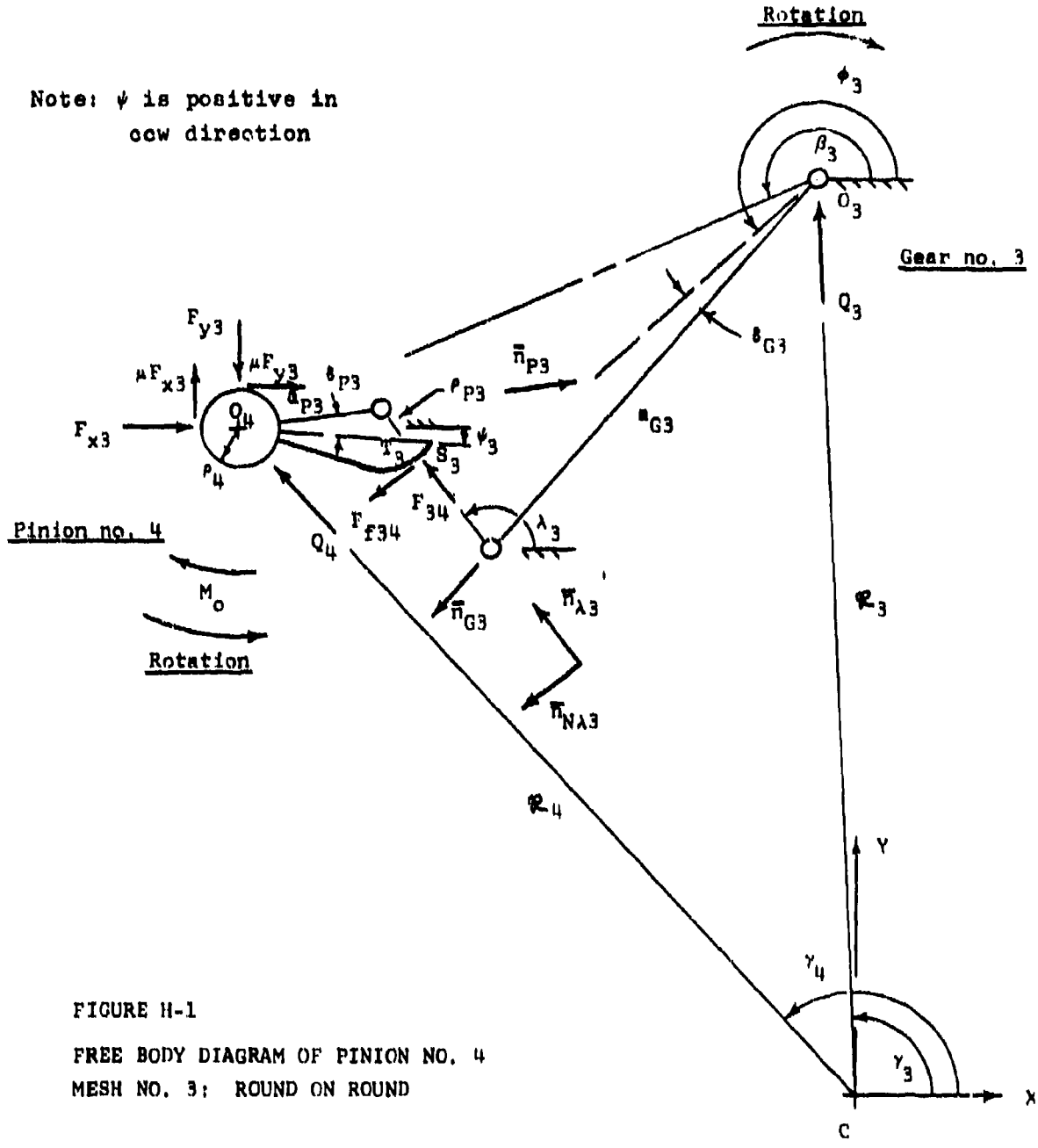

FIGURE H-1

FREE BODY DIAGRAM OF PINION NO. 4
MESH NO. 3: ROUND ON ROUND

The centrifugal force, due to the pinion mass, is given by

$$\bar{Q}_4 = Q_4(\cos\gamma_4 \bar{i} + \sin\gamma_4 \bar{j}) \qquad \text{(H-5)}$$

where

$$Q_4 = \mathcal{R}_4 m_4 \omega^2 \qquad \text{(H-6)}$$

Force equilibrium is given by

$$F_{34}\bar{n}_{\lambda 3} + \mu s_{3R} F_{34}\bar{n}_{N\lambda 3} + F_{x4}\bar{I} + \mu F_{y4}\bar{I} - F_{y4}\bar{J} + \mu F_{x4}\bar{J} + \bar{Q}_4 = 0 \qquad \text{(H-7)}$$

Moment equilibrium requires the following:

$$-M_{o4}\bar{k} - \mu\rho_4(\tilde{F}_{x4} + \tilde{F}_{y4})\bar{k} + (a_{p3}\bar{n}_{p3} - \rho_{p3}\bar{n}_{\lambda 3}) \times (F_{34}\bar{n}_{\lambda 3} + \mu s_{3R} F_{34}\bar{n}_{N\lambda 3}) = 0 \qquad \text{(H-8)}$$

Equation (H-7) furnishes the following component equations:

$$F_{34}\cos\lambda_3 - \mu s_{3R} F_{34}\sin\lambda_3 + F_{x4} + \mu F_{y4} + Q_4\cos\gamma_4 = 0 \qquad \text{(H-9)}$$

and

$$F_{34}\sin\lambda_3 + \mu s_{3R} F_{34}\cos\lambda_3 - F_{y4} + \mu F_{x4} + Q_4\sin\gamma_4 = 0 \qquad \text{(H-10)}$$

The scalar form of the moment equation becomes

$$-M_{o4} - \mu\rho_4(\tilde{F}_{x4} + \tilde{F}_{y4}) - F_{34}\Big\{a_{P3}\big[\sin(\psi_3 + \delta_{P3} - \lambda_3) - \mu s_{3R}\cos(\psi_3 + \delta_{P3} - \lambda_3)\big] + \mu s_{3R}\rho_{P3}\Big\} = 0 \qquad \text{(H-11)}$$

Simultaneous solution of equations (H-9) and (H-10) for F_{x4} and F_{y4} gives

$$F_{x4} = \frac{F_{34}\big[\mu(s_{3R} - 1)\sin\lambda_3 - (1 + \mu^2 s_{3R})\cos\lambda_3\big] - Q_4(\mu\sin\gamma_4 + \cos\gamma_4)}{1 + \mu^2} \qquad \text{(H-12)}$$

and

$$F_{y4} = \frac{F_{34}\big[(1 + \mu^2 s_{3R})\sin\lambda_3 + \mu(s_{3R} - 1)\cos\lambda_3\big] + Q_4(\sin\gamma_4 - \mu\cos\gamma_4)}{1 + \mu^2} \qquad \text{(H-13)}$$

The sum $\tilde{F}_{x4} + \tilde{F}_{y4}$ of equation (H-11) is now made up of equations (H-12) and (H-13) in the sense of equation (A-3b) of Appendix A

$$\tilde{F}_{x4} + \tilde{F}_{y4} = Q_4A_1 + F_{34}A_2 + Q_4A_3 + F_{34}A_4 \qquad \text{(H-14)}$$

$$A_1 = \left| \frac{\mu \sin\gamma_4 + \cos\gamma_4}{1 + \mu^2} \right| \qquad \text{(H-15)}$$

$$A_2 = \left| \frac{\mu(s_{3R} - 1)\sin\lambda_3 - (1 + \mu^2 s_{3R})\cos\lambda_3}{1 + \mu^2} \right| \qquad \text{(H-16)}$$

$$A_3 = \left| \frac{\sin\gamma_4 - \mu\cos\gamma_4}{1 + \mu^2} \right| \qquad \text{(H-17)}$$

$$A_4 = \left| \frac{(1 + \mu^2 s_{3R})\sin\lambda_3 + \mu(s_{3R} - 1)\cos\lambda_3}{1 + \mu^2} \right| \qquad \text{(H-18)}$$

Equation (H-14) is now substituted into the moment equation (H-11) and the latter is solved for the contact force F_{34}

$$F_{34} = \frac{M_{o4}}{C_2} + \frac{Q_4 C_1}{C_2} \qquad \text{(H-19)}$$

where

$$C_1 = \mu \rho_4 (A_1 + A_3)$$

$$C_2 = s_{P3}\left[\mu s_{3R}\cos(\psi_3 + \delta_{P3} - \lambda_3) - \sin(\psi_3 + \delta_{P3} - \lambda_3)\right]$$

$$- \mu\left[\rho_{P3} s_{3R} + \rho_4 (A_2 + A_4)\right]$$

II. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 3

Figure H-2 gives a schematic free body diagram of the gear and pinion combination no. 3. Mesh no. 2 is also indicated to obtain the directions of the forces of gear no. 2 on pinion no. 3. The forces of pinion no. 4 on gear no. 3 are opposite to those given by equations (H-3) and (H-4) respectively. Thus, the contact force becomes

$$\bar{F}_{43} = -\bar{F}_{34} = -F_{34}\bar{n}_{\lambda 3} \qquad \text{(H-20)}$$

The friction force of pinion no. 4 on gear no. 3 becomes:

$$\bar{F}_{f43} = -\bar{F}_{f34} = -\mu s_{3R} F_{34}\bar{n}_{N\lambda 3} \qquad \text{(H-21)}$$

The contact force of gear no. 2 on pinion no. 3 is given by

$$\bar{F}_{23} = F_{23}\bar{n}_{\lambda 2} \qquad \text{(H-22)}$$

while the associated friction force of gear no. 2 on pinion no. 3 becomes

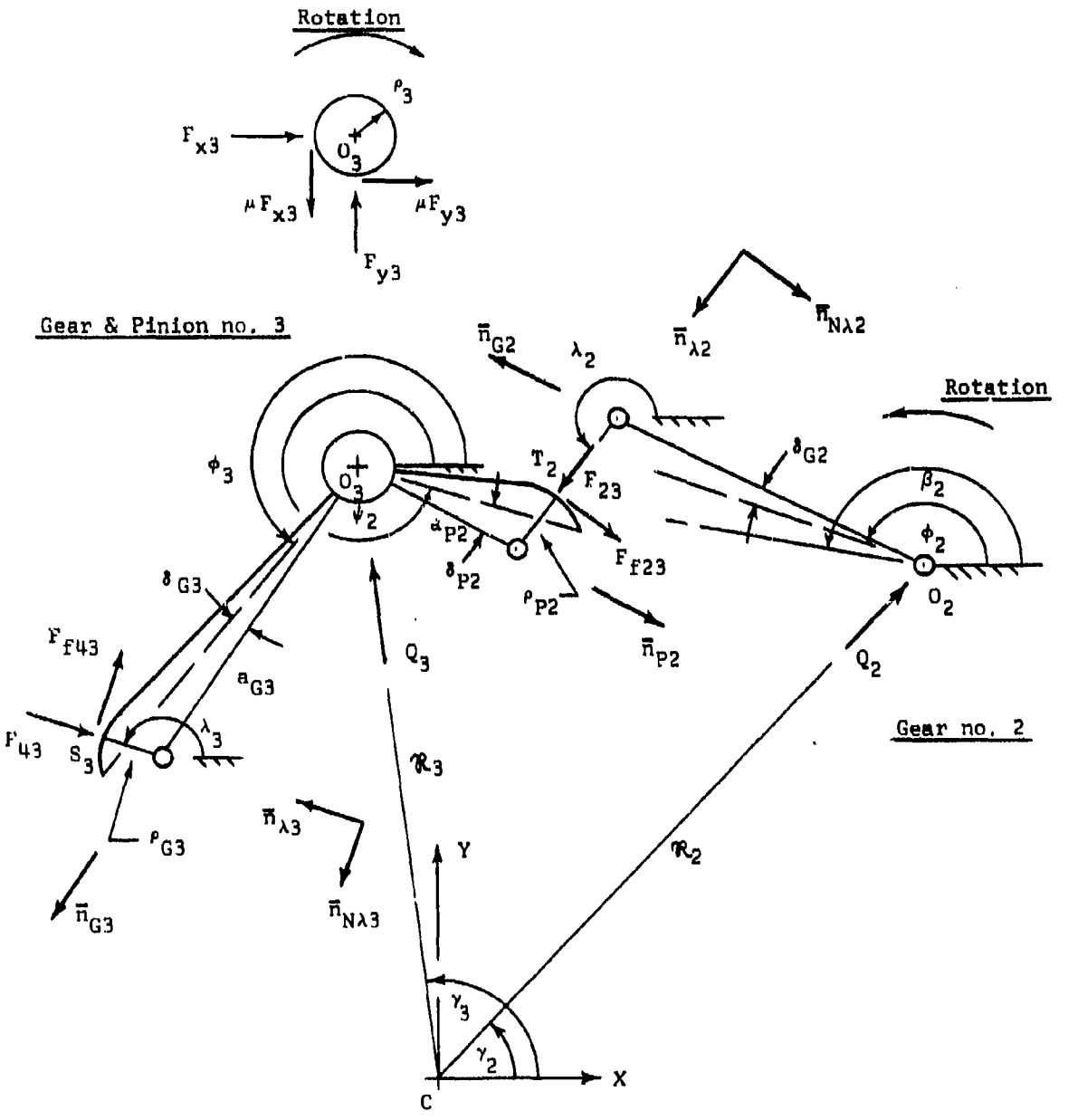

FIGURE H-2

FREE BODY DIAGRAM OF GEAR & PINION NO. 3
MESH NO. 3: ROUND ON ROUND
MESH NO. 2: ROUND ON ROUND

$$\bar{F}_{f23} = \mu s_{2R} F_{23} \bar{n}_{N\lambda 2} \tag{H-23}$$

The pivot reactions F_{x3} and F_{y3} as well as the associated friction forces are drawn in a separate diagram in Figure H-2. As was the case earlier, the friction moment due to the friction forces again opposes rotation.

The centrifugal force, due to the mass of the combined gear and pinion no. 3, is given by

$$\bar{Q}_3 = Q_3(\cos\gamma_3 \bar{I} + \sin\gamma_3 \bar{J}) \tag{H-24}$$

where

$$Q_3 = \kappa_3 m_3 \omega^2 \tag{H-25}$$

The force equilibrium of the combination is given by

$$-F_{34}\bar{n}_{\lambda 3} - \mu s_{3R} F_{34} \bar{n}_{N\lambda 3} + F_{23}\bar{n}_{\lambda 2} + \mu s_{2R} F_{23} \bar{n}_{N\lambda 2} + F_{x3}\bar{I} + \mu F_{y3}\bar{I}$$

$$+ F_{y3}\bar{J} - \mu F_{x3}\bar{J} + \bar{Q}_3 = 0 \tag{H-26}$$

The moment equation is given by

$$\mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3})\bar{k} + \left[a_{G3}\bar{n}_{G3} + \rho_{G3}\bar{n}_{\lambda 3}\right] \times \left[-F_{34}\bar{n}_{\lambda 3} - \mu s_{3R}F_{34}\bar{n}_{N\lambda 3}\right]$$

$$+ \left[a_{P2}\bar{n}_{P2} - \rho_{P2}\bar{n}_{\lambda 2}\right] \times \left[F_{23}\bar{n}_{\lambda 2} + \mu s_{2R}F_{23}\bar{n}_{N\lambda 2}\right] = 0 \qquad \text{(H-27)}$$

Equation (H-26) gives the following component expressions:

$$-F_{34}\cos\lambda_3 + \mu s_{3R}F_{34}\sin\lambda_3 + Q_3\cos\gamma_3 + F_{x3} + \mu F_{y3} + F_{23}\cos\lambda_2$$

$$- \mu s_{2R}F_{23}\sin\lambda_2 = 0 \qquad \text{(H-28)}$$

and

$$-F_{34}\sin\lambda_3 - \mu s_{3R}F_{34}\cos\lambda_3 + Q_3\sin\gamma_3 + F_{y3} - \mu F_{x3} + F_{23}\sin\lambda_2$$

$$+ \mu s_{2R}F_{23}\cos\lambda_2 = 0 \qquad \text{(H-29)}$$

The scalar form of the moment equation (H-27) becomes

$$\mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3}) - \mu s_{3R}\rho_{G3}F_{34} - \mu s_{2R}\rho_{P2}F_{23} + a_{G3}F_{34}\left[\sin(\phi_3 + \delta_{G3} - \lambda_3)\right.$$

$$\left. - \mu s_{3R}\cos(\phi_3 + \delta_{G3} - \lambda_3)\right] + a_{P2}F_{23}\left[-\sin(\psi_2 - \delta_{P2} - \lambda_2)\right.$$

$$\left. + \mu s_{2R}\cos(\psi_2 - \delta_{P2} - \lambda_2)\right] = 0 \qquad \text{(H-30)}$$

Simultaneous solution of equations (H-28) and (H-29) for the pivot reactions F_{x3} and F_{y3} gives

$$F_{x3} = \frac{1}{1+\mu^2}\left\{F_{34}\left[(1-\mu^2 s_{3R})\cos\lambda_3 - \mu(1+s_{3R})\sin\lambda_3\right] + F_{23}\left[\mu(1+s_{2R})\sin\lambda_2 - (1-\mu^2 s_{2R})\cos\lambda_2\right] + Q_3\left[\mu\sin\gamma_3 - \cos\gamma_3\right]\right\} \quad \text{(H-31)}$$

and

$$F_{y3} = \frac{1}{1+\mu^2}\left\{F_{34}\left[(1-\mu^2 s_{3R})\sin\lambda_3 + \mu(1+s_{3R})\cos\lambda_3\right] + F_{23}\left[(\mu^2 s_{2R}-1)\sin\lambda_2 - \mu(1+s_{2R})\cos\lambda_2\right] - Q_3\left[\sin\gamma_3 + \mu\cos\gamma_3\right]\right\} \quad \text{(H-32)}$$

The sum $\tilde{F}_{x3} + \tilde{F}_{y3}$ of equation (H-30) is now made up of equations (H-31) and (H-32) in the sense of equation (A-3b).

$$\tilde{F}_{x3} + \tilde{F}_{y3} = F_{34}A_5 + F_{23}A_6 + Q_3A_7 + F_{34}A_8 + F_{23}A_9 + Q_3A_{10} \quad \text{(H-33)}$$

where

$$A_5 = \left| \frac{(1 - \mu^2 s_{3R})\cos\lambda_3 - \mu(1 + s_{3R})\sin\lambda_3}{1 + \mu^2} \right| \qquad \text{(H-34)}$$

$$A_6 = \left| \frac{\mu(1 + s_{2R})\sin\lambda_2 - (1 - \mu^2 s_{2R})\cos\lambda_2}{1 + \mu^2} \right| \qquad \text{(H-35)}$$

$$A_7 = \left| \frac{\mu\sin\gamma_3 - \cos\gamma_3}{1 + \mu^2} \right| \qquad \text{(H-36)}$$

$$A_8 = \left| \frac{(1 - \mu^2 s_{3R})\sin\lambda_3 + \mu(1 + s_{3R})\cos\lambda_3}{1 + \mu^2} \right| \qquad \text{(H-37)}$$

$$A_9 = \left| \frac{(\mu^2 s_{2R} - 1)\sin\lambda_2 - \mu(1 + s_{2R})\cos\lambda_2}{1 + \mu^2} \right| \qquad \text{(H-38)}$$

$$A_{10} = \left| \frac{\sin\gamma_3 + \mu\cos\gamma_3}{1 + \mu^2} \right| \qquad \text{(H-39)}$$

Equation (H-33) is now substituted into equation (H-30) and the resulting expression is solved for the contact force F_{23}. Thus,

$$F_{23} = \frac{- F_{34}C_3 - Q_3C_4}{C_5} \qquad \text{(H-40)}$$

where

$$C_3 = \mu\rho_3(A_5 + A_8) - \mu s_{3R}\rho_{G3} + s_{G3}\left[\sin(\phi_3 + \delta_{G3} - \lambda_3) - \mu s_{3R}\cos(\phi_3 + \delta_{G3} - \lambda_3)\right]$$

$$C_4 = \mu\rho_3(A_7 + A_{10})$$

$$C_5 = \mu\rho_3(A_6 + A_9) - \mu s_{2R}\rho_{P2} + s_{P2}\left[\mu s_{2R}\cos(\psi_2 - \delta_{P2} - \lambda_2) - \sin(\psi_2 - \delta_{P2} - \lambda_2)\right]$$

III. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 2

Figure H-3 gives the free body diagram of the gear and pinion combination no. 2. In addition, mesh no. 1 is indicated to obtain the directions of the forces of gear no. 1 on pinion no. 2.

The forces of pinion no. 3 on gear no. 2 have directions opposite to those given by equation (H-22) and (H-23) respectively. Thus, the normal force is given by

$$\bar{F}_{32} = - F_{23}\bar{n}_{\lambda 2} \tag{H-41}$$

The friction force of pinion no. 3 on gear no. 2 becomes

$$\bar{F}_{f32} = - \mu s_{2R} F_{23}\bar{n}_{N\lambda 2} \tag{H-42}$$

The contact force of gear no. 1 on pinion no. 2 is given by

$$\bar{F}_{12} = F_{12}\bar{n}_{\lambda 1} \tag{H-43}$$

while the associated friction force of gear no. 1 on pinion no. 2 becomes

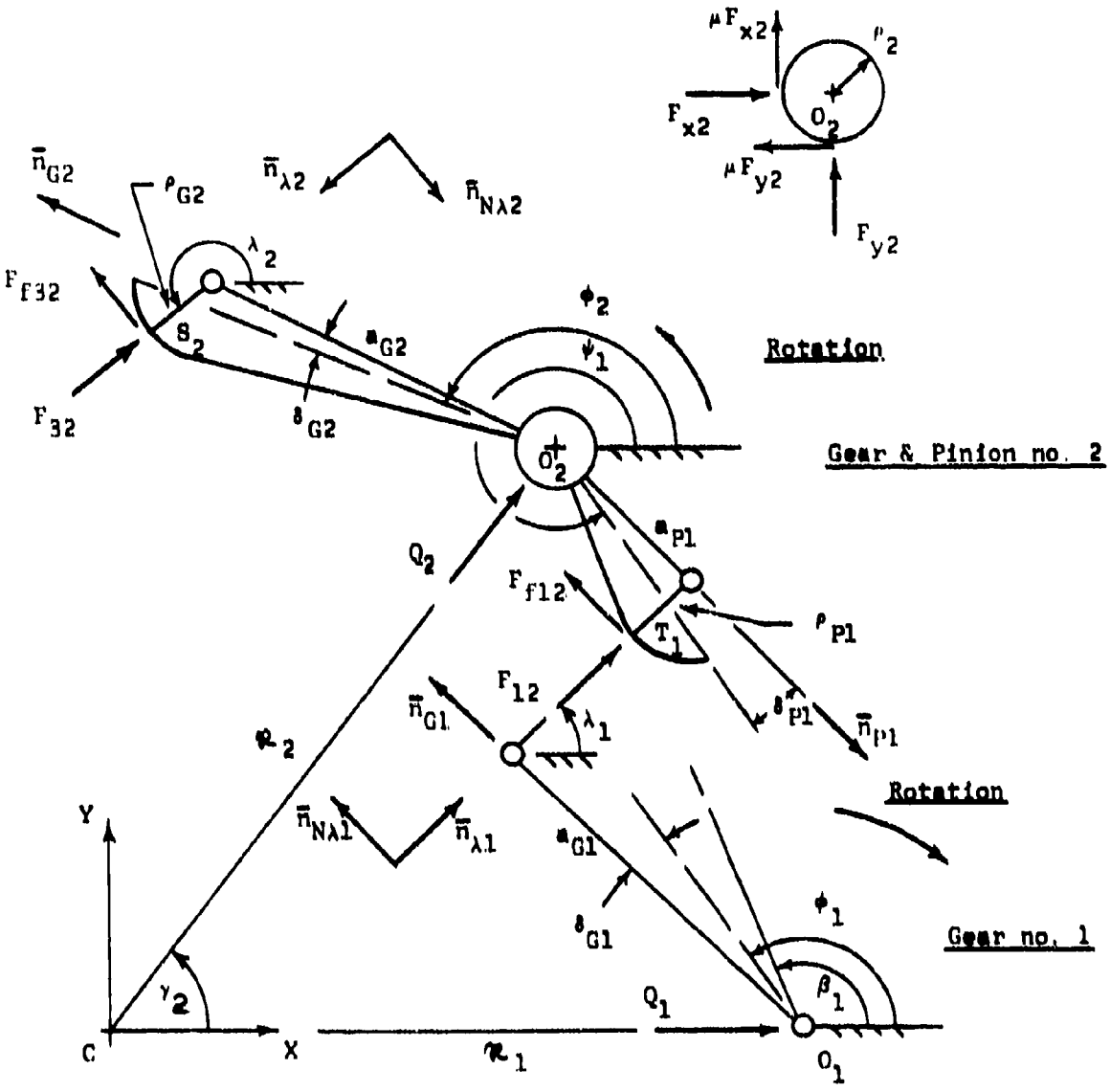

FIGURE H-3

FREE BODY DIAGRAM OF GEAR & PINION NO. 2

MESH NO. 2: ROUND ON ROUND

MESH NO. 1: ROUND ON ROUND

$$\bar{F}_{f12} = \mu s_{1R} F_{12} \bar{n}_{N\lambda 1} \qquad \text{(H-44)}$$

The pivot reactions, of the fuse body on the pivot shaft, together with the pivot friction forces, are shown in a separate diagram in Figure H-3. The centrifugal force, due to the mass of gear and pinion assembly no. 2, is given by

$$\bar{Q}_2 = Q_2(\cos\gamma_2 \bar{I} + \sin\gamma_2 \bar{J}) \qquad \text{(H-45)}$$

where

$$Q_2 = \mathcal{R}_2 m_2 \omega^2 \qquad \text{(H-46)}$$

Force equilibrium is assured by

$$-F_{23}\bar{n}_{\lambda 2} - \mu s_{2R} F_{23} \bar{n}_{N\lambda 2} + F_{12}\bar{n}_{\lambda 1} + \mu s_{1R} F_{12} \bar{n}_{N\lambda 1} + \bar{Q}_2 + F_{x2}\bar{I} - \mu F_{y2}\bar{I} + F_{y2}\bar{J} + \mu F_{x2}\bar{J} = 0 \qquad \text{(H-47)}$$

Moment equilibrium must satisfy

$$-\mu\rho_2(\tilde{F}_{x2} + \tilde{F}_{y2})\bar{k} + [a_{G2}\bar{n}_{G2} + \rho_{G2}\bar{n}_{\lambda 2}] \times [-F_{23}\bar{n}_{\lambda 2} - \mu s_{2R}F_{23}\bar{n}_{N\lambda 2}]$$

$$+ [a_{P1}\bar{n}_{P1} - \rho_{P1}\bar{n}_{\lambda 1}] \times [F_{12}\bar{n}_{\lambda 1} + \mu s_{1R}F_{12}\bar{n}_{N\lambda 1}] = 0 \qquad \text{(H-48)}$$

Equation (H-47) gives the following component expressions

$$-F_{23}\cos\lambda_2 + \mu s_{2R}F_{23}\sin\lambda_2 + F_{12}\cos\lambda_1 - \mu s_{1R}F_{12}\sin\lambda_1$$

$$+ Q_2\cos\gamma_2 + F_{x2} - \mu F_{y2} = 0 \qquad \text{(H-49)}$$

and

$$-F_{23}\sin\lambda_2 - \mu s_{2R}F_{23}\cos\lambda_2 + F_{12}\sin\lambda_1 + \mu s_{1R}F_{12}\cos\lambda_1$$

$$+ Q_2\sin\gamma_2 + F_{y2} + \mu F_{x2} = 0 \qquad \text{(H-50)}$$

The scalar form of the moment equation (H-48) becomes

$$-\mu\rho_2(\tilde{F}_{x2} + \tilde{F}_{y2}) + s_{G2}F_{23}\left[\sin(\phi_2 - \delta_{G2} - \lambda_2) - \mu s_{2R}\cos(\phi_2 - \delta_{G2} - \lambda_2)\right]$$
$$- \mu s_{2R}\rho_{G2}F_{23} + s_{P1}F_{12}\left[-\sin(\psi_1 + \delta_{P1} - \lambda_1) + \mu s_{1R}\cos(\psi_1 + \delta_{P1} - \lambda_1)\right]$$
$$- \mu s_{1R}\rho_{P1}F_{12} = 0 \qquad \text{(H-51)}$$

Simultaneous solution of equations (H-49) and (H-50) for F_{x2} and F_{y2} leads to

$$F_{x2} = \frac{1}{1+\mu^2}\left\{F_{23}\left[(1 + \mu^2 s_{2R})\cos\lambda_2 - \mu(s_{2R} - 1)\sin\lambda_2\right]\right.$$
$$+ F_{12}\left[\mu(s_{1R} - 1)\sin\lambda_1 - (1 + \mu^2 s_{1R})\cos\lambda_1\right]$$
$$\left. + Q_2\left[-\cos\gamma_2 - \mu\sin\gamma_2\right]\right\} \qquad \text{(H-52)}$$

and

$$F_{y2} = \frac{1}{1+\mu^2}\left\{F_{23}\left[(1 + \mu^2 s_{2R})\sin\lambda_2 - \mu(1 - s_{2R})\cos\lambda_2\right]\right.$$
$$+ F_{12}\left[\mu(1 - s_{1R})\cos\lambda_1 - (1 + \mu^2 s_{1R})\sin\lambda_1\right]$$
$$\left. + Q_2\left[\mu\cos\gamma_2 - \sin\gamma_2\right]\right\} \qquad \text{(H-53)}$$

The sum of $\tilde{F}_{x2}$ and $\tilde{F}_{y2}$ of equation (H-51) is now made up of equations (H-52) and (H-53) in the sense of equation (A-3b) of Appendix A

$$\tilde{F}_{x2} + \tilde{F}_{y2} = F_{23}A_{11} + F_{12}A_{12} + Q_2A_{13} + F_{23}A_{14} + F_{12}A_{15} + Q_2A_{16} \qquad \text{(H-54)}$$

where

$$A_{11} = \left| \frac{(1 + \mu^2 s_{2R})\cos\lambda_2 - \mu(s_{2R} - 1)\sin\lambda_2}{1 + \mu^2} \right| \qquad \text{(H-55)}$$

$$A_{12} = \left| \frac{\mu(s_{1R} - 1)\sin\lambda_1 - (1 + \mu^2 s_{1R})\cos\lambda_1}{1 + \mu^2} \right| \qquad \text{(H-56)}$$

$$A_{13} = \left| \frac{-\cos\gamma_2 - \mu\sin\gamma_2}{1 + \mu^2} \right| \qquad \text{(H-57)}$$

$$A_{14} = \left| \frac{(1 + \mu^2 s_{2R})\sin\lambda_2 - \mu(1 - s_{2R})\cos\lambda_2}{1 + \mu^2} \right| \qquad \text{(H-58)}$$

$$A_{15} = \left| \frac{\mu(1 - s_{1R})\cos\lambda_1 - (1 + \mu^2 s_{1R})\sin\lambda_1}{1 + \mu^2} \right| \qquad \text{(H-59)}$$

$$A_{16} = \left| \frac{\mu \cos\gamma_2 - \sin\gamma_2}{1 + \mu^2} \right| \qquad \text{(H-60)}$$

Equation (H-54) is now substituted into the moment equation (H-51). The resulting expression is then solved for the contact force F_{12}

$$F_{12} = \frac{- F_{23}C_6 + Q_2C_7}{C_8} \qquad \text{(H-61)}$$

where

$$C_6 = a_{G2}\left[\sin(\phi_2 - \delta_{G2} - \lambda_2) - \mu s_{2R}\cos(\phi_2 - \delta_{G2} - \lambda_2)\right] - \mu\rho_2(A_{11} + A_{14}) - \mu s_{2R}\rho_{G2}$$

$$C_7 = \mu\rho_2(A_{13} + A_{16})$$

$$C_8 = -\left\{a_{P1}\left[\sin(\psi_1 + \delta_{P1} - \lambda_1) - \mu s_{1R}\cos(\psi_1 + \delta_{P1} - \lambda_1)\right] + \mu\rho_2(A_{12} + A_{15}) + \mu s_{1R}\rho_{P1}\right\}$$

IV. FORCE AND MOMENT EQUILIBRIA OF INPUT GEAR NO. 1

Figure H-4 represents the free body diagram of the input gear no. 1 which has the input moment M_{in} acting on it.

The forces of pinion no. 2 on gear no. 1 are given according to equations (H-43) and (H-44),

$$\bar{F}_{21} = - F_{12}\bar{n}_{\lambda 1} \tag{H-62}$$

and

$$\bar{F}_{f21} = - \mu s_{1R} F_{12}\bar{n}_{N\lambda 1} \tag{H-63}$$

The moments due to the friction forces on the pivot oppose rotation as indicated.

The centrifugal force, due to the mass of gear no. 1, is given by

$$\bar{Q}_1 = Q_1\bar{i} \tag{H-64}$$

where

$$Q_1 = \mathcal{R}_1 m_1 \omega^2 \tag{H-65}$$

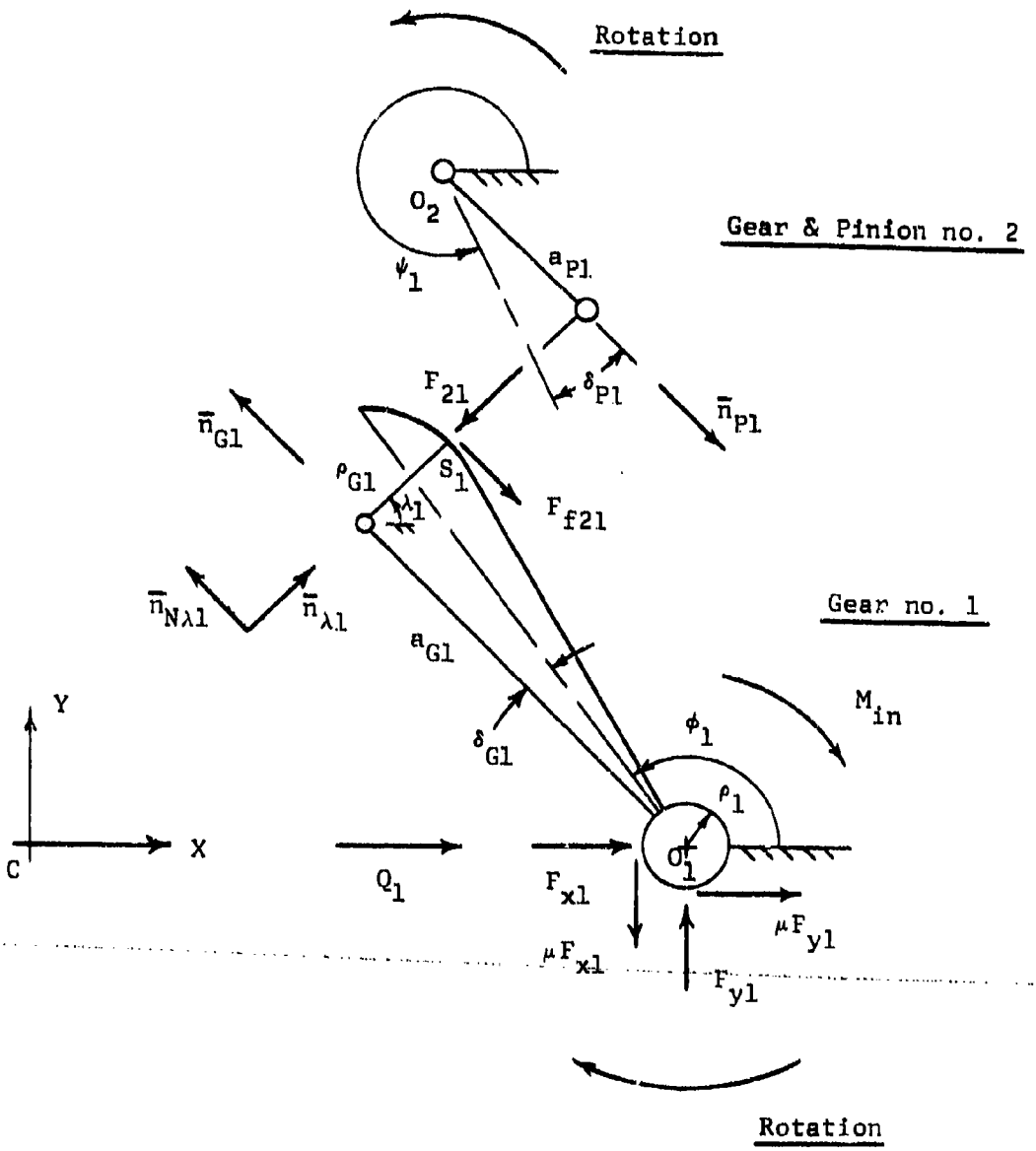

FIGURE H-4

FREE BODY DIAGRAM OF GEAR NO. 1
MESH NO. 1: ROUND ON ROUND

Force equilibrium requires, that

$$-F_{12}\bar{n}_{\lambda 1} - \mu s_{1R}F_{12}\bar{n}_{N\lambda 1} + \bar{Q}_1 + F_{x1}\bar{I} + \mu F_{y1}\bar{I} + F_{y1}\bar{J} - \mu F_{x1}\bar{J} = 0 \qquad \text{(H-66)}$$

Moment equilibrium is given by

$$\mu\rho_1(\tilde{F}_{x1} + \tilde{F}_{y1})\bar{k} - M_{in}\bar{k} + \left[a_{G1}\bar{n}_{G1} + \rho_{G1}\bar{n}_{\lambda 1}\right] \times \left[-F_{12}\bar{n}_{\lambda 1} - \mu s_{1R}F_{12}\bar{n}_{N\lambda 1}\right] = 0 \qquad \text{(H-67)}$$

Equation (H-66) furnishes the following component equations:

$$-F_{12}\cos\lambda_1 + \mu s_{1R}F_{12}\sin\lambda_1 + Q_1 + F_{x1} + \mu F_{y1} = 0 \qquad \text{(H-68)}$$

and

$$-F_{12}\sin\lambda_1 - \mu s_{1R}F_{12}\cos\lambda_1 + F_{y1} - \mu F_{x1} = 0 \qquad \text{(H-69)}$$

The scalar form of equation (H-67) becomes

$$\mu\rho_1(\tilde{F}_{x1} + \tilde{F}_{y1}) - M_{in} + a_{G1}F_{12}\left[\sin(\phi_1 + \delta_{G1} - \lambda_1) - \mu s_{1R}\cos(\phi_1 + \delta_{G1} - \lambda_1)\right] - \mu s_{1R}\rho_{G1}F_{12} = 0 \qquad \text{(H-70)}$$

The simultaneous solution of equations (H-68) and (H-69) furnishes

$$F_{x1} = \frac{1}{1+\mu^2}\left\{F_{12}\left[(1-\mu^2 s_{1R})\cos\lambda_1 - \mu(1+s_{1R})\sin\lambda_1\right] - Q_1\right\} \tag{H-71}$$

and

$$F_{y1} = \frac{1}{1+\mu^2}\left\{F_{12}\left[(1-\mu^2 s_{1R})\sin\lambda_1 + \mu(1+s_{1R})\cos\lambda_1\right] - \mu Q_1\right\} \tag{H-72}$$

Then, with the same reasoning as before,

$$\tilde{F}_{x1} + \tilde{F}_{y1} = F_{12}A_{17} + Q_1 A_{18} + F_{12}A_{19} + Q_1 A_{20} \tag{H-73}$$

where

$$A_{17} = \left|\frac{(1-\mu^2 s_{1R})\cos\lambda_1 - \mu(1+s_{1R})\sin\lambda_1}{1+\mu^2}\right| \tag{H-74}$$

$$A_{18} = \left|\frac{1}{1+\mu^2}\right| \tag{H-75}$$

$$A_{19} = \left| \frac{(1 - \mu^2 s_{1R})\sin\lambda_1 + \mu(1 + s_{1R})\cos\lambda_1}{1 + \mu^2} \right| \qquad \text{(H-76)}$$

$$A_{20} = \left| \frac{\mu}{1 + \mu^2} \right| \qquad \text{(H-77)}$$

Equation (H-73) is now substituted into equation (H-70) and the result is solved for the contact force F_{12}

$$F_{12} = \frac{M_{in} - Q_1 C_9}{C_{10}} \qquad \text{(H-78)}$$

where

$$C_9 = \mu\rho_1(A_{18} + A_{20})$$

$$C_{10} = \mu\rho_1(A_{17} + A_{19}) + a_{G1}\left[\sin(\phi_1 + \delta_{G1} - \lambda_1) - \mu s_{1R}\cos(\phi_1 + \delta_{G1} - \lambda_1)\right] - \mu s_{1R}\rho_{G1}$$

V. MOMENT INPUT-OUTPUT RELATIONSHIP

Equations (H-61) and (H-78), which are both expressions in F_{12}, are now set equal to each other and the result is solved for F_{23}

$$F_{23} = \frac{-C_8}{C_6C_{10}}(M_{in} - Q_1C_9) + \frac{C_7}{C_6}Q_2 \qquad \text{(H-79)}$$

The above is now equated to equation (H-40) and the result is solved for F_{34}

$$F_{34} = \frac{C_5C_8(M_{in} - Q_1C_9) - C_5C_{10}C_7Q_2 - C_4C_6C_{10}Q_3}{C_3C_6C_{10}} \qquad \text{(H-80)}$$

Finally equation (H-80) is equated to equation (H-19). This permits the determination of the equilibrant moment M_{041} (for case no. 1: RRR)

$$M_{041} = M_{in}\frac{C_2C_5C_8}{C_3C_6C_{10}} - Q_1\frac{C_2C_5C_8C_9}{C_3C_6C_{10}} - Q_2\frac{C_2C_5C_7}{C_3C_6} - Q_3\frac{C_2C_4}{C_3} - Q_4C_1 \qquad \text{(H-81)}$$

b. CASE NO. 2: RRF

Since only mesh 1 is now assumed to be in the round on flat phase of motion, the forces F_{34} and F_{23} remain as given by equations (H-19) and (H-40), respectively.

I. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 2

Figure H-5 shows the free body diagram of the gear and pinion set with the necessary portions of mesh no. 1.

The forces of pinion no. 3 on gear no. 2 are given by equations (H-41) and (H-42), i.e.,

$$\bar{F}_{32} = - F_{23}\bar{n}_{\lambda 2} \tag{H-82}$$

and

$$\bar{F}_{f32} = - \mu s_{2R} F_{23}\bar{n}_{N\lambda 2} \tag{H-83}$$

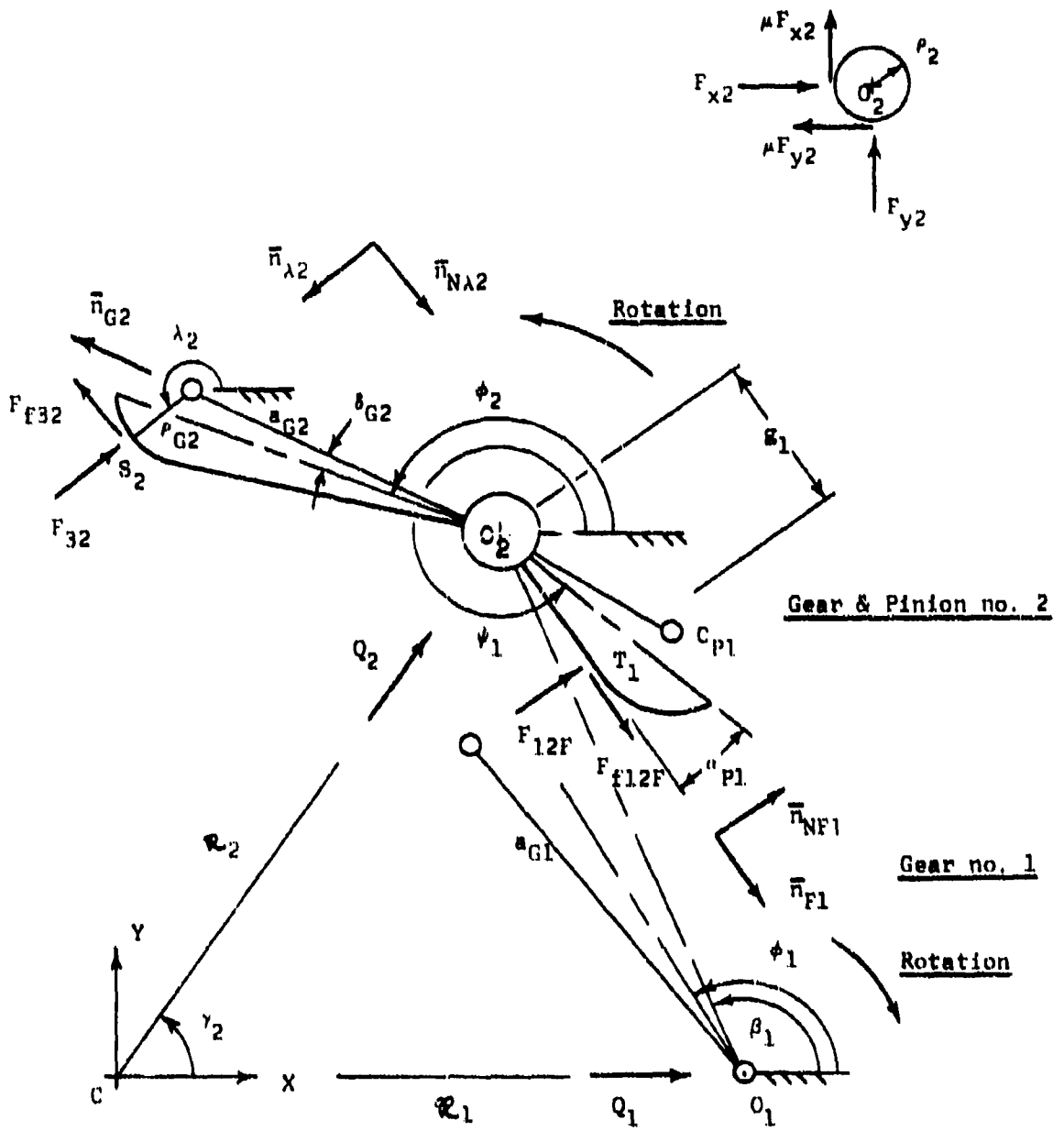

FIGURE H-5

FREE BODY DIAGRAM OF GEAR & PINION NO. 2

MESH NO. 2: ROUND ON ROUND

MESH NO. 1: ROUND ON FLAT

The contact force of gear no. 1 on pinion no. 2 is now given by

$$\bar{F}_{12F} = F_{12F}\bar{n}_{NF1} \tag{H-84}$$

(Note that the additional subscript F is introduced to distinguish round on flat from round on round contact.)

The associated friction force is given by

$$\bar{F}_{f12F} = \mu s_{1F} F_{12F}\bar{n}_{F1} \tag{H-85}$$

(See equation (H-2) for s_{1F}.)

The pivot reactions, together with the pivot friction forces, are shown in a separate diagram in Figure H-5. As before, the pivot friction moments oppose rotation.

The centrifugal force $\bar{Q}_2$ is again given by equations (H-45) and (H-46).

Force equilibrium is given by

$$-F_{23}\bar{n}_{\lambda 2} - \mu s_{2R} F_{23}\bar{n}_{N\lambda 2} + F_{12F}\bar{n}_{NF1} + \mu s_{1F} F_{12F}\bar{n}_{F1} + \bar{Q}_2 + F_{x2}\bar{I} - \mu F_{y2}\bar{I} + F_{y2}\bar{J} + \mu F_{x2}\bar{J} = 0 \tag{H-86}$$

Moment equilibrium about point O_2 requires

$$-\mu\rho_2(\bar{F}_{x2} + \bar{F}_{y2})\bar{k} + \left[a_{G2}\bar{n}_{G2} + \rho_{G2}\bar{n}_{\lambda 2}\right] \times \left[-F_{23}\bar{n}_{\lambda 2} - \mu s_{2R}F_{23}\bar{n}_{N\lambda 2}\right]$$

$$+ s_1\bar{n}_{F1} \times F_{12F}\bar{n}_{NF1} = 0 \qquad \text{(H-87)}$$

Note that since the line of action of the friction force F_{f12F} passes through point O_2, this friction force exerts no moment about point O_2.

Equation (H-86) furnishes the following component equations:

$$-F_{23}\cos\lambda_2 + \mu s_{2R}F_{23}\sin\lambda_2 - F_{12F}\sin(\psi_1 - \alpha_{P1}) + \mu s_{1F}F_{12F}\cos(\psi_1 - \alpha_{P1})$$

$$+ F_{x2} - \mu F_{y2} + Q_2\cos\gamma_2 = 0 \qquad \text{(H-88)}$$

and

$$-F_{23}\sin\lambda_2 - \mu s_{2R}F_{23}\cos\lambda_2 + F_{12F}\cos(\psi_1 - \alpha_{P1}) + \mu s_{1F}F_{12F}\sin(\psi_1 - \alpha_{P1})$$

$$+ F_{y2} + \mu F_{x2} + Q_2\sin\gamma_2 = 0 \qquad \text{(H-89)}$$

The scalar form of the moment equation (H-87) becomes

$$-\mu\rho_2(\tilde{F}_{x2} + \tilde{F}_{y2}) + s_{G2}F_{23}[\sin(\phi_2 - \delta_{G2} - \lambda_2) - \mu s_{2R}\cos(\phi_2 - \delta_{G2} - \lambda_2)]$$

$$- \mu s_{2R}r_{G2}F_{23} + F_{12F}\delta_1 = 0 \qquad \text{(H-90)}$$

Simultaneous solution of these component equations for F_{x2} and F_{y2} leads to

$$F_{x2} = \frac{1}{1+\mu^2}\Big\{F_{23}[\mu(1 - s_{2R})\sin\lambda_2 + (1 + \mu^2 s_{2R})\cos\lambda_2]$$

$$+ F_{12F}[(1 - \mu^2 s_{1F})\sin(\psi_1 - \alpha_{p1}) - \mu(1 + s_{1F})\cos(\psi_1 - \alpha_{p1})]$$

$$- Q_2[\mu\sin\gamma_2 + \cos\gamma_2]\Big\} \qquad \text{(H-91)}$$

and

$$F_{y2} = \frac{1}{1+\mu^2}\Big\{F_{23}[(1 + \mu^2 s_{2R})\sin\lambda_2 - \mu(1 - s_{2R})\cos\lambda_2]$$

$$+ F_{12F}[-\mu(1 + s_{1F})\sin(\psi_1 - \alpha_{p1}) + (\mu^2 s_{1F} - 1)\cos(\psi_1 - \alpha_{p1})]$$

$$+ Q_2[-\sin\gamma_2 + \mu\cos\gamma_2]\Big\} \qquad \text{(H-92)}$$

The sum $(\tilde{F}_{x2} + \tilde{F}_{y2})$ of equation (H-90) is now made up of equations (H-91) and (H-92) in the sense of equation (A-3b) of Appendix A

$$\tilde{F}_{x2} + \tilde{F}_{y2} = F_{23}A_{21} + F_{12F}A_{22} + Q_2A_{23} + F_{23}A_{24} + F_{12F}A_{25} + Q_2A_{26} \tag{H-93}$$

where

$$A_{21} = \left| \frac{\mu(1 - s_{2R})\sin\lambda_2 + (1 + \mu^2 s_{2R})\cos\lambda_2}{1 + \mu^2} \right| \tag{H-94}$$

$$A_{22} = \left| \frac{(1 - \mu^2 s_{1F})\sin(\psi_1 - \alpha_{P1}) - \mu(1 + s_{1F})\cos(\psi_1 - \alpha_{P1})}{1 + \mu^2} \right| \tag{H-95}$$

$$A_{23} = \left| \frac{\mu\sin\gamma_2 + \cos\gamma_2}{1 + \mu^2} \right| \tag{H-96}$$

$$A_{24} = \left| \frac{(1 + \mu^2 s_{2R})\sin\lambda_2 - \mu(1 - s_{2R})\cos\lambda_2}{1 + \mu^2} \right| \tag{H-97}$$

$$A_{25} = \left| \frac{-\mu(1 + s_{1F})\sin(\psi_1 - \alpha_{P1}) + (\mu^2 s_{1F} - 1)\cos(\psi_1 - \alpha_{P1})}{1 + \mu^2} \right| \tag{H-98}$$

$$A_{26} = \left| \frac{-\sin\gamma_2 + \mu\cos\gamma_2}{1 + \mu^2} \right| \qquad \text{(H-99)}$$

Equation (H-93) is now substituted into equation (H-90), and the resulting expression is solved for the contact force F_{12F}

$$F_{12F} = \frac{-F_{23}C_{11} + Q_2C_{12}}{C_{13}} \qquad \text{(H-100)}$$

where

$$C_{11} = a_{G2}[\sin(\phi_2 - \delta_{G2} - \lambda_2) - \mu\cos(\phi_2 - \delta_{G2} - \lambda_2)] - \mu\rho_2(A_{21} + A_{24}) - \mu a_{2R}\rho_{G2}$$

$$C_{12} = \mu\rho_2(A_{23} + A_{26})$$

$$C_{13} = e_1 - \mu\rho_2(A_{22} + A_{25})$$

II. FORCE AND MOMENT EQUILIBRIA OF INPUT GEAR NO. 1

Figure H-6 represents the free body diagram of input gear no. 1.

The forces of pinion no. 2 on this gear are given, according to equations (H-84) (H-85).

$$\bar{F}_{21F} = -F_{12F}\bar{n}_{NF1} \tag{H-101}$$

and

$$\bar{F}_{f21F} = -\mu s_{1F} F_{12F} \bar{n}_{F1} \tag{H-102}$$

The moments due to the pivot friction forces oppose the indicated rotation due to the moment M_{in}.

The centrifugal force $\bar{Q}_1$ has been defined by equations (H-64) and (H-65).

The force equilibrium equation is given by

$$-F_{12F}\bar{n}_{NF1} - \mu s_{1F} F_{12F} \bar{n}_{F1} + Q_1\bar{I} + F_{x1}\bar{I} + \mu F_{y1}\bar{I} + F_{y1}\bar{J} - \mu F_{x1}\bar{J} = 0 \tag{H-103}$$

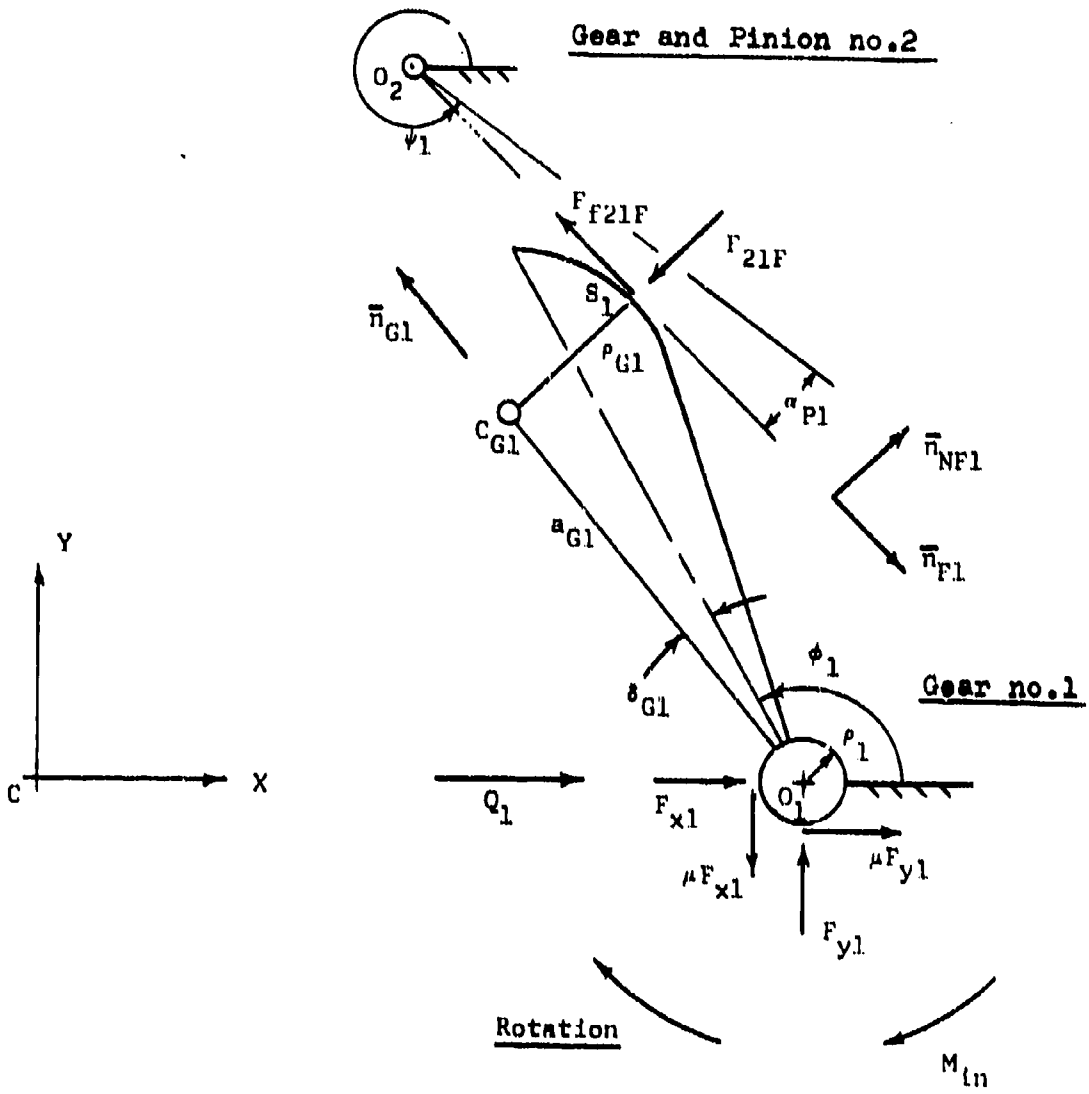

FIGURE H-6

FREE BODY DIAGRAM OF GEAR NO. 1
MECH NO. 1: ROUND ON FLAT

The moment equilibrium equation becomes

$$-M_{in}\bar{k} + \mu\rho_1(\tilde{F}_{x1} + \tilde{F}_{y1})\bar{k} + [a_{G1}\bar{n}_{G1} + \rho_{G1}\bar{n}_{NF1}] \times [-F_{12F}\bar{n}_{NF1} - \mu s_{1F}F_{12F}\bar{n}_{F1}] = 0 \quad \text{(H-104)}$$

Equation (H-103) furnishes the following component expressions

$$F_{12F}\sin(\psi_1 - \alpha_{P1}) - \mu s_{1F}F_{12F}\cos(\psi_1 - \alpha_{P1}) + Q_1 + F_{x1} + \mu F_{y1} = 0 \quad \text{(H-105)}$$

and

$$-F_{12F}\cos(\psi_1 - \alpha_{P1}) - \mu s_{1F}F_{12F}\sin(\psi_1 - \alpha_{P1}) + F_{y1} - \mu F_{x1} = 0 \quad \text{(H-106)}$$

The scalar form of the moment equation (H-104) becomes

$$-M_{in} + \mu\rho_1(\tilde{F}_{x1} + \tilde{F}_{y1}) + \mu s_{1F}\rho_{G1}F_{12F} + a_{G1}F_{12F}[-\cos(\phi_1 + \delta_{G1} - \psi_1 + \alpha_{P1}) + \mu s_{1F}\sin(\phi_1 + \delta_{G1} - \psi_1 + \alpha_{P1})] = 0 \quad \text{(H-107)}$$

Simultaneous solution of equations (H-105) and (H-106) for F_{x1} and F_{y1} furnishes

$$F_{x1} = \frac{F_{12F}[-(1 + \mu^2 s_{1F})\sin(\psi_1 - \alpha_{P1}) + \mu(s_{1F} - 1)\cos(\psi_1 - \alpha_{P1})] - Q_1}{1 + \mu^2} \quad \text{(H-108)}$$

and

$$F_{y1} = \frac{F_{12F}\left[(1 + \mu^2 s_{1F})\cos(\psi_1 - \alpha_{P1}) + \mu(s_{1F} - 1)\sin(\psi_1 - \alpha_{P1})\right] - \mu Q_1}{1 + \mu^2} \qquad \text{(H-109)}$$

Now let

$$\tilde{F}_{x1} + \tilde{F}_{y1} = F_{12F}A_{27} + Q_1 A_{28} + F_{12F}A_{29} + Q_1 A_{30} \qquad \text{(H-110)}$$

where

$$A_{27} = \left|\frac{-(1 + \mu^2 s_{1F})\sin(\psi_1 - \alpha_{P1}) + \mu(s_{1F} - 1)\cos(\psi_1 - \alpha_{P1})}{1 + \mu^2}\right| \qquad \text{(H-111)}$$

$$A_{28} = \left|\frac{1}{1 + \mu^2}\right| \qquad \text{(H-112)}$$

$$A_{29} = \left|\frac{\mu(s_{1F} - 1)\sin(\psi_1 - \alpha_{P1}) + (1 + \mu^2 s_{1F})\cos(\psi_1 - \alpha_{P1})}{1 + \mu^2}\right| \qquad \text{(H-113)}$$

$$A_{30} = \left|\frac{\mu}{1 + \mu^2}\right| \qquad \text{(H-114)}$$

Equation (H-110) is now substituted into the moment equation (H-107). This furnishes

$$F_{12F} = \frac{M_{1n} - Q_1 C_{14}}{C_{15}} \qquad \text{(H-115)}$$

where

$$C_{14} = \mu\rho_1(A_{28} + A_{30})$$

$$C_{15} = \mu\rho_1(A_{27} + A_{29}) + \mu s_{1F}\rho_{G1}$$

$$+ a_{G1}[\mu s_{1F}\sin(\phi_1 + \delta_{G1} - \psi_1 + \alpha_{P1}) - \cos(\phi_1 + \delta_{G1} - \psi_1 + \alpha_{P1})]$$

III. MOMENT INPUT-OUTPUT RELATIONSHIP

Equations (H-100) and (H-115), which are both expressions in F_{12F}, are now set equal to each other and the result is solved for F_{23}

$$F_{23} = \frac{-C_{13}}{C_{11}C_{15}} (M_{in} - Q_1 C_{14}) + Q_2 \frac{C_{12}}{C_{11}} \qquad \text{(H-116)}$$

The above expression is now equated to equation (H-40) and solved for F_{34}

$$F_{34} = \frac{C_5 C_{13}}{C_3 C_{11} C_{15}} (M_{in} - Q_1 C_{14}) - Q_2 \frac{C_5 C_{12}}{C_3 C_{11}} - Q_3 \frac{C_4}{C_3} \qquad \text{(H-117)}$$

Finally, this expression is set equal to equation (H-19) and the result is solved for the equilibrant moment M_{042} (for case 2: RRF)

$$M_{042} = M_{in} \frac{C_2 C_5 C_{13}}{C_3 C_{11} C_{15}} - Q_1 \frac{C_2 C_5 C_{13} C_{14}}{C_3 C_{11} C_{15}} - Q_2 \frac{C_2 C_5 C_{12}}{C_3 C_{11}} - Q_3 \frac{C_2 C_4}{C_3} - Q_4 C_1 \qquad \text{(H-118)}$$

c. CASE NO. 3: RFT

With both meshes no. 1 and no. 2 in the round on flat phase of motion, only force F_{34} of the round on round phase can be incorporated for the present case. The equilibrium equations for gear and pinion set no. 3, gear and pinion set no. 2 and the input gear no. 1 must be newly derived.

I. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 3

Figure H-7 shows the free body diagram of gear and pinion set no. 3, together with the necessary outline of mesh no. 2.

The forces of pinion no. 4 on gear no. 3 are given by equations (H-20) and (H-21)

$$\bar{F}_{43} = - F_{34}\bar{n}_{\lambda 3} \tag{H-119}$$

and

$$\bar{F}_{f43} = - \mu_{3R} F_{34}\bar{n}_{N\lambda 3} \tag{H-120}$$

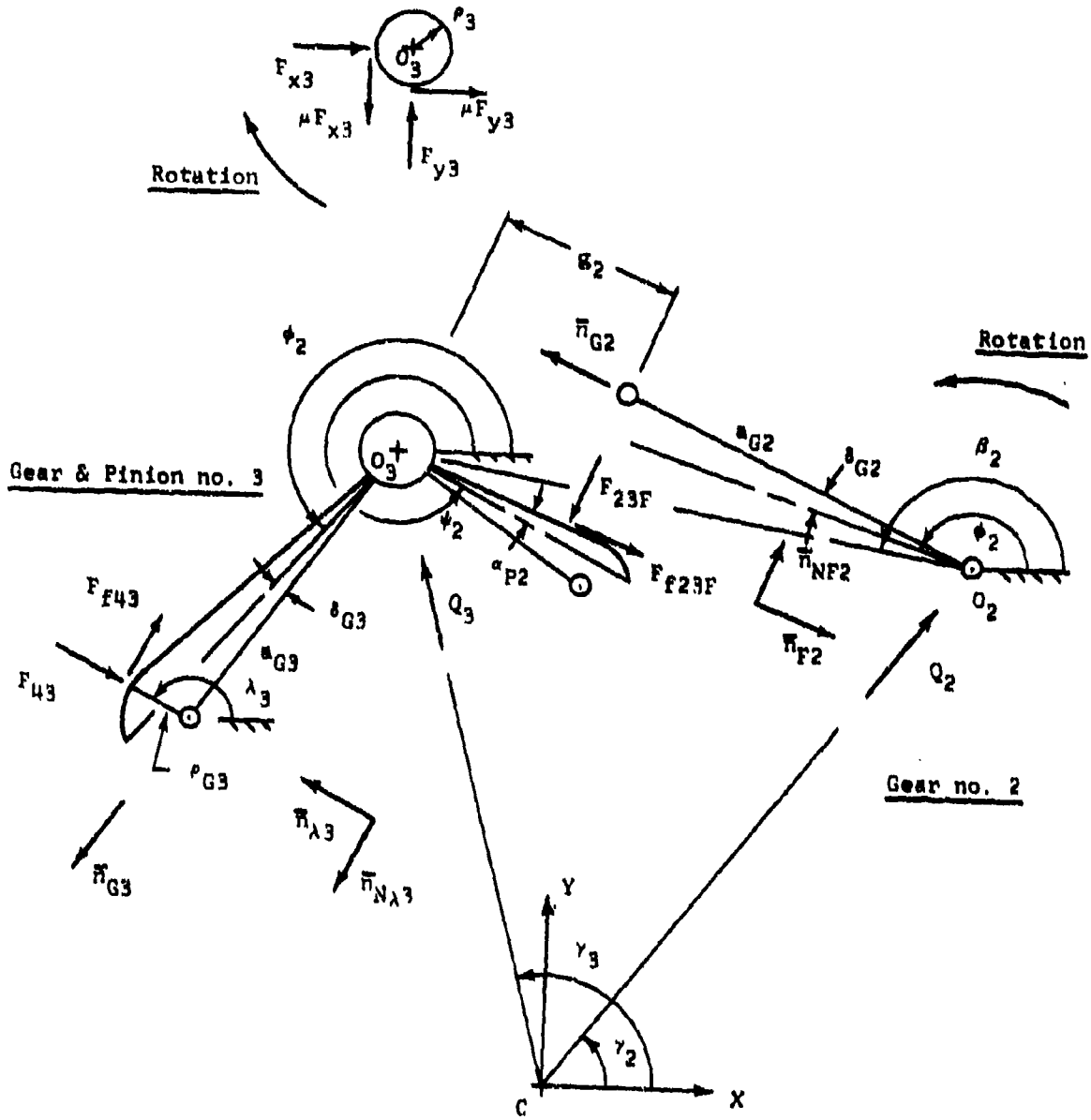

FIGURE H-7

FREE BODY DIAGRAM OF GEAR & PINION NO. 3
MESH NO. 3: ROUND ON ROUND
MESH NO. 2: ROUND ON FLAT

The normal contact force of gear no. 2 on pinion no. 3 is given by

$$\bar{F}_{23F} = - F_{23F}\bar{n}_{NF2} \qquad \text{(H-121)}$$

The associated friction force is given by

$$\bar{F}_{f23F} = \mu s_{2F} F_{23F}\bar{n}_{F2} \qquad \text{(H-122)}$$

The pivot friction forces are chosen so that the resulting friction moments oppose the indicated rotation. The centrifugal force $\bar{Q}_3$ is that of equations (H-24) and (H-25).

Force equilibrium is given by

$$-F_{34}\bar{n}_{\lambda 3} - \mu s_{3R} F_{34}\bar{n}_{N\lambda 3} - F_{23F}\bar{n}_{NF2} + \mu s_{2F} F_{23F}\bar{n}_{F2} + \bar{Q}_3 + F_{x3}\bar{i} + \mu F_{y3}\bar{i} + F_{y3}\bar{j} - \mu F_{x3}\bar{j} = 0 \qquad \text{(H-123)}$$

Moment equilibrium about point O_3 is given by

$$\mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3})\bar{k} + [a_{G3}\bar{n}_{G3} + \rho_{G3}\bar{n}_{\lambda 3}] \times [-F_{34}\bar{n}_{\lambda 3} - \mu s_{3R} F_{34}\bar{n}_{N\lambda 3}] + s_2\bar{n}_{F2} \times \bar{F}_{23F} = 0 \qquad \text{(H-124)}$$

Note that the friction force F_{f23F} exerts no moment about point O_3.

Equation (H-123) furnishes the following component equations:

$$-F_{34}\cos\lambda_3 + \mu s_{3R}F_{34}\sin\lambda_3 + Q_3\cos\gamma_3 + F_{x3} + \mu F_{y3} + F_{23F}\sin(\psi_2 + \alpha_{P2}) + \mu s_{2F}F_{23F}\cos(\psi_2 + \alpha_{P2}) = 0 \qquad \text{(H-125)}$$

and

$$-F_{34}\sin\lambda_3 - \mu s_{3R}F_{34}\cos\lambda_3 + Q_3\sin\gamma_3 + F_{y3} - \mu F_{x3} - F_{23F}\cos(\psi_2 + \alpha_{P2}) + \mu s_{2F}F_{23F}\sin(\psi_2 + \alpha_{P2}) = 0 \qquad \text{(H-126)}$$

The scalar form of the moment equation (H-124) becomes

$$\mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3}) + s_{G3}F_{34}[\sin(\phi_3 + \delta_{G3} - \lambda_3) - \mu s_{3R}\cos(\phi_3 + \delta_{G3} - \lambda_3)] - \mu s_{3R}\rho_{G3}F_{34} - s_2F_{23F} = 0 \qquad \text{(H-127)}$$

Simultaneous solution of equations (H-125) and (H-126) for F_{x3} and F_{y3} leads to

$$F_{x3} = \frac{1}{1+\mu^2}\left\{F_{34}\left[(1-\mu^2 s_{3R})\cos\lambda_3 - \mu(1+s_{3R})\sin\lambda_3\right] + F_{23F}\left[-\mu(1+s_{2F})\cos(\psi_2+\alpha_{P2}) - (1-\mu^2 s_{2F})\sin(\psi_2+\alpha_{P2})\right] + Q_3\left[\mu\sin\gamma_3 - \cos\gamma_3\right]\right\} \quad \text{(H-128)}$$

and

$$F_{y3} = \frac{1}{1+\mu^2}\left\{F_{34}\left[(1-\mu^2 s_{3R})\sin\lambda_3 + \mu(1+s_{3R})\cos\lambda_3\right] + F_{23F}\left[(1-\mu^2 s_{2F})\cos(\psi_2+\alpha_{P2}) - \mu(1+s_{2F})\sin(\psi_2+\alpha_{P2})\right] - Q_3\left[\sin\gamma_3 + \mu\cos\gamma_3\right]\right\} \quad \text{(H-129)}$$

The sum $\tilde{F}_{x3} + \tilde{F}_{y3}$ of equation (H-127) is now made up of equations (H-128) and (H-129) in the sense of equation (A-3b)

$$\tilde{F}_{x3} + \tilde{F}_{y3} = F_{34}A_{31} + F_{23F}A_{32} + Q_3A_{33} + F_{34}A_{34} + F_{23F}A_{35} + Q_3A_{36} \quad \text{(H-130)}$$

where

$$A_{31} = \left| \frac{(1 - \mu^2 s_{3R})\cos\lambda_3 - \mu(1 + s_{3R})\sin\lambda_3}{1 + \mu^2} \right| \qquad \text{(H-131)}$$

$$A_{32} = \left| \frac{-\mu(1 + s_{2F})\cos(\psi_2 + \alpha_{P2}) - (1 - \mu^2 s_{2F})\sin(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \qquad \text{(H-132)}$$

$$A_{33} = \left| \frac{\mu\sin\gamma_3 - \cos\gamma_3}{1 + \mu^2} \right| \qquad \text{(H-133)}$$

$$A_{34} = \left| \frac{(1 - \mu^2 s_{3R})\sin\lambda_3 + \mu(1 + s_{3R})\cos\lambda_3}{1 + \mu^2} \right| \qquad \text{(H-134)}$$

$$A_{35} = \left| \frac{(1 - \mu^2 s_{2F})\cos(\psi_2 + \alpha_{P2}) - \mu(1 + s_{2F})\sin(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \qquad \text{(H-135)}$$

$$A_{36} = \left| \frac{\sin\gamma_3 + \mu\cos\gamma_3}{1 + \mu^2} \right| \qquad \text{(H-136)}$$

Equation (H-130) is now substituted into equation (H-127) and the result is solved for F_{23F}

$$F_{23F} = \frac{-F_{34}C_{16} - Q_3C_{17}}{C_{18}} \qquad \text{(H-137)}$$

where

$$C_{16} = \mu\rho_3(A_{31} + A_{34}) + a_{G3}\left[\sin(\phi_3 + \delta_{G3} - \lambda_3) - \mu a_{3R}\cos(\phi_3 + \delta_{G3} - \lambda_3)\right] - \mu a_{3R}\rho_{G3}$$

$$C_{17} = \mu\rho_3(A_{33} + A_{36})$$

$$C_{18} = \mu\rho_3(A_{32} + A_{35}) - \varepsilon_2$$

II. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 2

Figure H-8 shows the free body diagram of the gear and pinion set no. 2.

The forces of pinion no. 3 on gear no. 2 are equal and opposite to those given by equations (H-121) and (H-122), i.e.,

$$\bar{F}_{32F} = F_{23F}\bar{n}_{NF2} \tag{H-138}$$

and

$$\bar{F}_{f32F} = -\mu s_{2F} F_{23F}\bar{n}_{F2} \tag{H-139}$$

The forces of gear no. 1 on pinion no. 2 are those of equations (H-84) and (H-85).

The pivot friction is accounted for in the usual manner and the centrifugal force $\bar{Q}_2$ is given by equation (H-45).

Force equilibrium is given by

$$F_{23F}\bar{n}_{NF2} - \mu s_{2F} F_{23F}\bar{n}_{F2} + F_{12F}\bar{n}_{NF1} + \mu s_{1F} F_{12F}\bar{n}_{F1} + \bar{Q}_2 + F_{x2}\bar{i} - \mu F_{y2}\bar{i} + F_{y2}\bar{j} + \mu F_{x2}\bar{j} = 0 \tag{H-140}$$

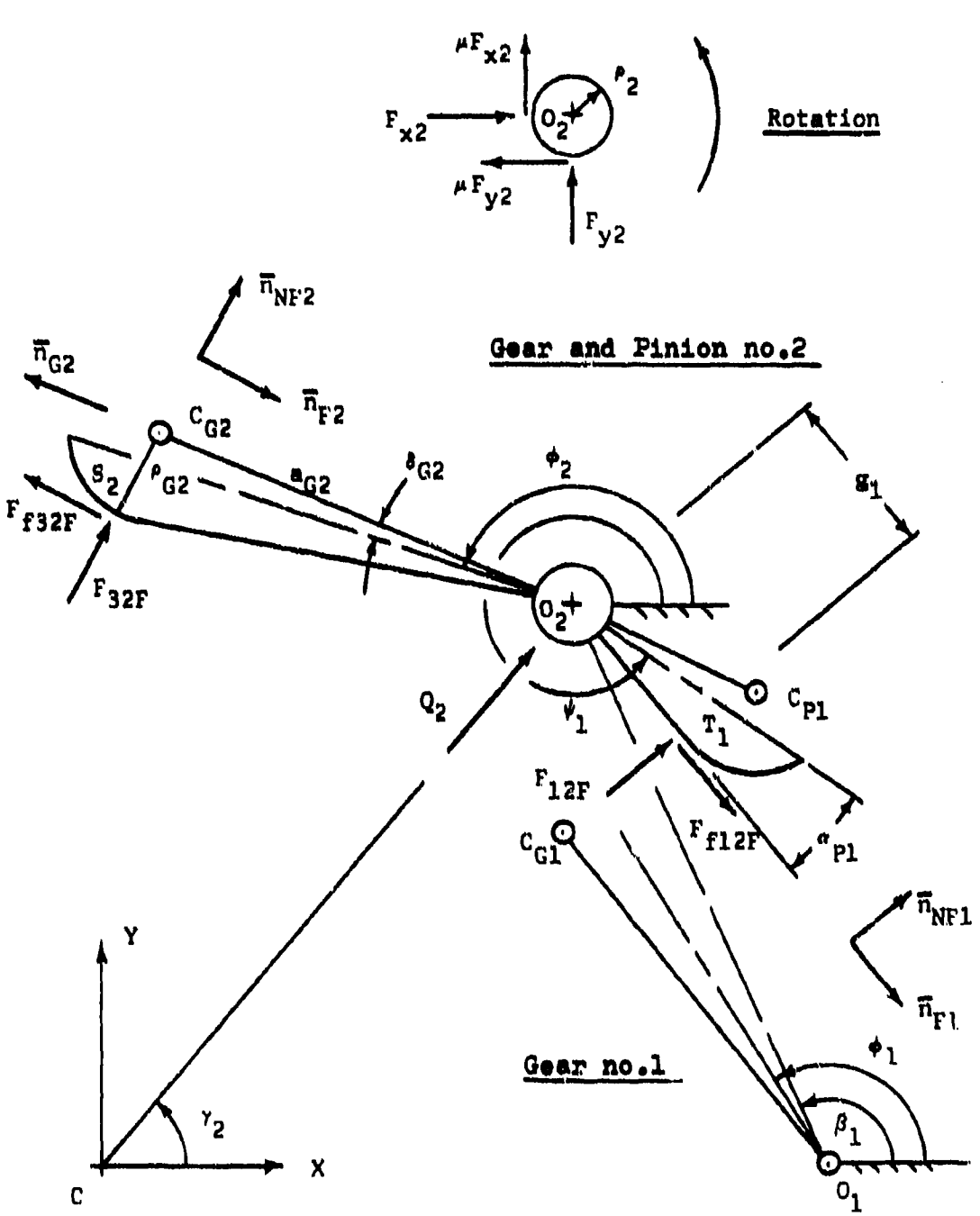

FIGURE H-8

FREE BODY DIAGRAM OF GEAR & PINION NO. 2

MESH NO. 2: ROUND ON FLAT

MESH NO. 1: ROUND ON FLAT

Moment equilibrium about point O_2 requires

$$-\mu\rho_2(\tilde{F}_{x2} + \tilde{F}_{y2})\bar{k} + [a_{G2}\bar{n}_{G2} - \rho_{G2}\bar{n}_{NF2}] \times [F_{23F}\bar{n}_{NF2} - \mu s_{2F}F_{23F}\bar{n}_{F2}]$$

$$+ g_1\bar{n}_{F1} \times F_{12F}\bar{n}_{NF1} = 0 \qquad \text{(H-141)}$$

Equation (H-140) gives the component equations

$$-F_{23F}\sin(\psi_2 + \alpha_{P2}) - \mu s_{2F}F_{23F}\cos(\psi_2 + \alpha_{P2}) + Q_2\cos\gamma_2 + F_{x2} - \mu F_{y2}$$

$$- F_{12F}\sin(\psi_1 - \alpha_{P1}) + \mu s_{1F}F_{12F}\cos(\psi_1 - \alpha_{P1}) = 0 \qquad \text{(H-142)}$$

and

$$F_{23F}\cos(\psi_2 + \alpha_{P2}) - \mu s_{2F}F_{23F}\sin(\psi_2 + \alpha_{P2}) + Q_2\sin\gamma_2 + F_{y2} + \mu F_{x2}$$

$$+ F_{12F}\cos(\psi_1 - \alpha_{P1}) + \mu s_{1F}F_{12F}\sin(\psi_1 - \alpha_{P1}) = 0 \qquad \text{(H-143)}$$

The scalar form of the moment equation (H-141) becomes

$$-\mu\rho_{2}(\tilde{F}_{x2} + \tilde{F}_{y2}) + a_{G2}F_{23F}\left[\cos(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2})\right.$$

$$\left. + \mu s_{2F}\sin(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2})\right] - \mu s_{2F}\rho_{G2}F_{23F} + \varepsilon_1 F_{12F} = 0$$

(H-144)

Simultaneous solution of the component equations (H-142) and (H-143) for F_{x2} and F_{y2} leads to

$$F_{x2} = \frac{1}{1 + \mu^2}\left\{F_{23F}\left[(1 + \mu^2 s_{2F})\sin(\psi_2 + \alpha_{P2}) + \mu(s_{2F} - 1)\cos(\psi_2 + \alpha_{P2})\right]\right.$$

$$+ F_{12F}\left[(1 - \mu^2 s_{1F})\sin(\psi_1 - \alpha_{P1}) - \mu(1 + s_{1F})\cos(\psi_1 - \alpha_{P1})\right]$$

$$\left. - Q_2\left[\mu\sin\gamma_2 + \cos\gamma_2\right]\right\} \qquad \text{(H-145)}$$

and

$$F_{y2} = \frac{1}{1 + \mu^2}\left\{F_{23F}\left[\mu(s_{2F} - 1)\sin(\psi_2 + \alpha_{P2}) - (1 + \mu^2 s_{2F})\cos(\psi_2 + \alpha_{P2})\right]\right.$$

$$- F_{12F}\left[\mu(1 + s_{1F})\sin(\psi_1 - \alpha_{P1}) + (1 - \mu^2 s_{1F})\cos(\psi_1 - \alpha_{P1})\right]$$

$$\left. + Q_2\left[\mu\cos\gamma_2 - \sin\gamma_2\right]\right\} \qquad \text{(H-146)}$$

The sum $\tilde{F}_{x2} + \tilde{F}_{y2}$ of equation (H-144) is now made up of equations (H-145) and (H-146) in the sense of equation (A-3b)

$$\tilde{F}_{x2} + \tilde{F}_{y2} = F_{23F}A_{37} + F_{12F}A_{38} + Q_2A_{39} + F_{23F}A_{40} + F_{12F}A_{41} + Q_2A_{42} \quad \text{(H-147)}$$

where

$$A_{37} = \left| \frac{(1 + \mu^2 s_{2F})\sin(\psi_2 + \alpha_{P2}) + \mu(s_{2F} - 1)\cos(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \quad \text{(H-148)}$$

$$A_{38} = \left| \frac{(1 - \mu^2 s_{1F})\sin(\psi_1 - \alpha_{P1}) - \mu(1 + s_{1F})\cos(\psi_1 - \alpha_{P1})}{1 + \mu^2} \right| \quad \text{(H-149)}$$

$$A_{39} = \left| \frac{\mu \sin\gamma_2 + \cos\gamma_2}{1 + \mu^2} \right| \quad \text{(H-150)}$$

$$A_{40} = \left| \frac{\mu(s_{2F} - 1)\sin(\psi_2 + \alpha_{P2}) - (1 + \mu^2 s_{2F})\cos(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \quad \text{(H-151)}$$

$$A_{41} = \left| \frac{\mu(1 + s_{1F})\sin(\psi_1 - \alpha_{P1}) + (1 - \mu^2 s_{1F})\cos(\psi_1 - \alpha_{P1})}{1 + \mu^2} \right| \quad \text{(H-152)}$$

$$A_{42} = \left| \frac{\mu \cos\gamma_2 - \sin\gamma_2}{1 + \mu^2} \right| \quad \text{(H-153)}$$

Equation (H-147) is now substituted into equation (H-144) and the result is solved for F_{12F}

$$F_{12F} = \frac{- F_{23F}C_{19} + Q_2C_{20}}{C_{21}} \qquad \text{(H-154)}$$

where

$$C_{19} = -\mu\rho_2(A_{37} + A_{40}) + s_{G2}\left[\cos(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2}) + \mu s_{2F}\sin(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2})\right] - \mu s_{2F}\rho_{G2}$$

$$C_{20} = \mu\rho_2(A_{39} + A_{42})$$

$$C_{21} = -\mu\rho_2(A_{38} + A_{41}) + \varepsilon_1$$

III. FORCE AND MOMENT EQUILIBRIA OF INPUT GEAR NO. 1

While the numerical values of the force F_{21F} and its associated friction force F_{f12F}, both acting on gear no. 1, are peculiar to the present combination of contact phases, its functional relationship to the input moment M_{1n} and the centrifugal force Q_1 is identical to that derived in section 1b-II of this appendix. (See also Figure H-6.)

According to equation (H-115), one obtains for F_{12F}

$$F_{12F} = \frac{M_{1n} - Q_1 c_{14}}{c_{15}} \qquad \text{(H-155)}$$

IV. MOMENT INPUT-OUTPUT RELATIONSHIP

Equations (H-154) and (H-155), both in F_{12F}, are set equal to each other and the result is solved for F_{23F}

$$F_{23F} = \frac{-C_{21}}{C_{15}C_{19}}(M_{in} - Q_1C_{14}) + Q_2\frac{C_{20}}{C_{19}} \qquad \text{(H-156)}$$

The above is now equated to equation (H-137) and the result is solved for F_{34}

$$F_{34} = \frac{C_{18}C_{21}}{C_{15}C_{16}C_{19}}(M_{in} - Q_1C_{14}) - Q_2\frac{C_{18}C_{20}}{C_{16}C_{19}} - Q_3\frac{C_{17}}{C_{16}} \qquad \text{(H-157)}$$

Finally, equation (H-157) is equated to equation (H-19), which corresponds to the round on round phase of mesh no. 3. The result is solved for the equilibrant moment M_{043} (for case 3: RFF)

$$M_{043} = M_{in}\frac{C_2C_{18}C_{21}}{C_{15}C_{16}C_{19}} - Q_1\frac{C_2C_{14}C_{18}C_{21}}{C_{15}C_{16}C_{19}} - Q_2\frac{C_2C_{18}C_{20}}{C_{16}C_{19}} - Q_3\frac{C_2C_{17}}{C_{16}}$$

$$- Q_4C_1 \qquad \text{(H-158)}$$

d. CASE NO. 4: RFR

For this contact combination force F_{34} may be taken from the results of case no. 1 [see equation (H-19)], since mesh no. 3 is in the round on round phase of motion. The force F_{23F} of case no. 3, i.e., equation (H-137), also is incorporated.

The input-output relationship of the gear and pinion combination no. 2 must be newly derived, i.e., the force F_{12} must be expressed in terms of the contact force F_{32F} and the centrifugal force Q_2. Finally, the results of the equilibrium equations for the input gear no. 1 of case no. 1 are used. For this case of contact, the force F_{12} is given by equation (H-78).

I. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 2

Figure H-9 shows the free body diagram of gear and pinion set no. 2 with the necessary portions of mesh no. 1.

The forces of pinion no. 3 on gear no. 2 were given by equations (H-138) and (H-139)

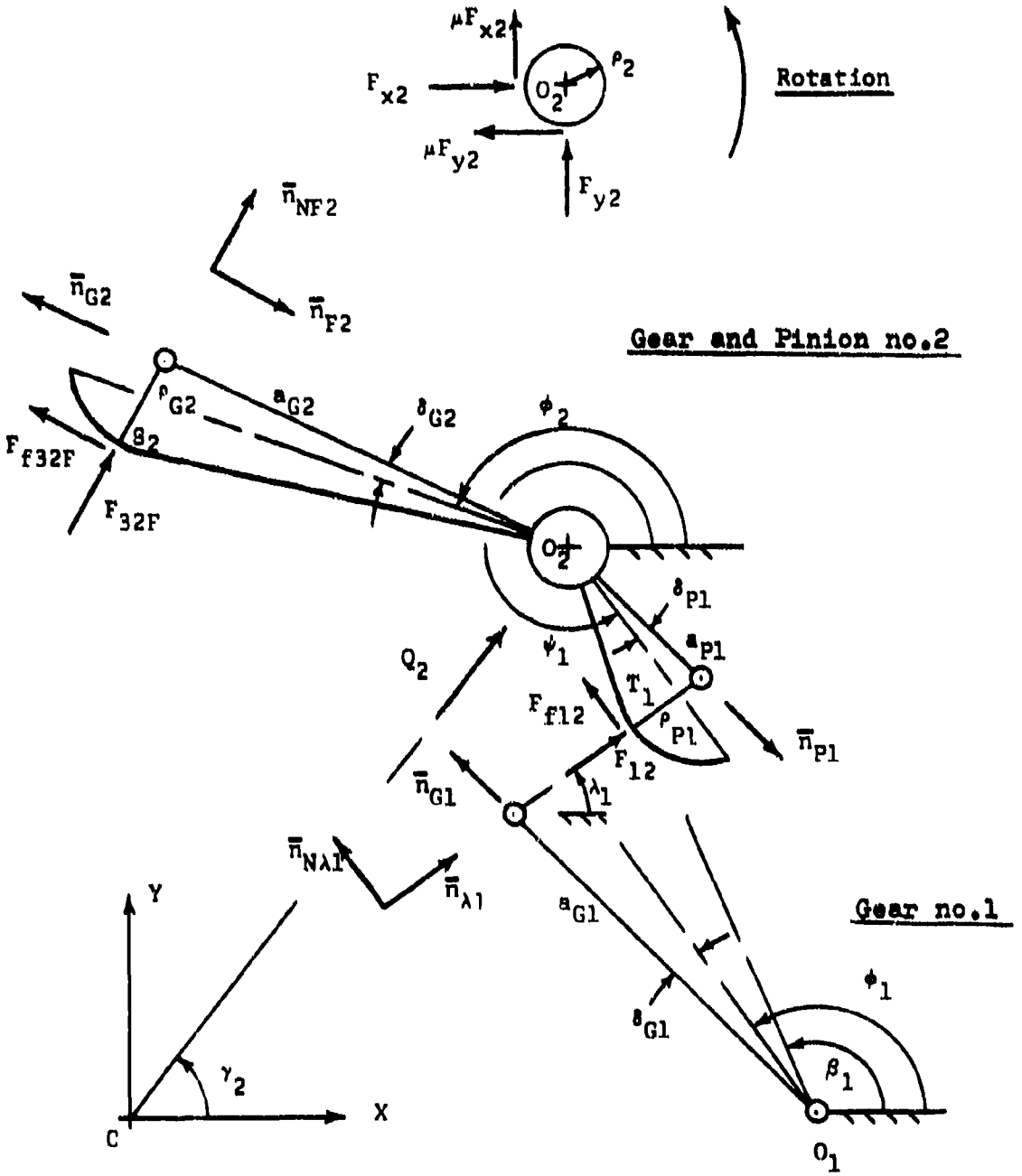

FIGURE H-9

FREE BODY DIAGRAM OF GEAR & PINION NO. 2

MESH NO. 2: ROUND ON FLAT

MESH NO. 1: ROUND ON ROUND

$$\bar{F}_{32F} = F_{23F}\bar{n}_{NF2} \tag{H-159}$$

and

$$\bar{F}_{f32F} = -\mu s_{2F}F_{23F}\bar{n}_{F2} \tag{H-160}$$

The forces of gear no. 1 on pinion no. 2 are given by equations (H-43) and (H-44)

$$\bar{F}_{12} = F_{12}\bar{n}_{\lambda 1} \tag{H-161}$$

and

$$\bar{F}_{f12} = \mu s_{1R}F_{12}\bar{n}_{N\lambda 1} \tag{H-162}$$

The centrifugal force $\bar{Q}_2$ is given by equation (H-45). The pivot reactions and friction forces are handled as before.

Force equilibrium is given by

$$F_{23F}\bar{n}_{NF2} - \mu s_{2F}F_{23F}\bar{n}_{F2} + F_{12}\bar{n}_{\lambda 1} + \mu s_{1R}F_{12}\bar{n}_{N\lambda 1} + \bar{Q}_2 + F_{x2}\bar{i} - \mu F_{y2}\bar{i} + F_{y2}\bar{j} + \mu F_{x2}\bar{j} = 0 \tag{H-163}$$

Moment equlibrium about point O_2 is given by

$$-\mu\rho_2(\tilde{F}_{x2} + \tilde{F}_{y2}) + [a_{G2}\bar{E}_{G2} - \rho_{G2}\bar{E}_{NF2}] \times [F_{23F}\bar{n}_{NF2} - \mu s_{2F}F_{23F}\bar{n}_{F2}]$$

$$+ [a_{P1}\bar{n}_{P1} - \rho_{P1}\bar{n}_{\lambda 1}] \times [F_{12}\bar{n}_{\lambda 1} + \mu s_{1R}F_{12}\bar{n}_{N\lambda 1}] = 0 \qquad \text{(H-164)}$$

Equation (H-163) gives the following component equations

$$-F_{23F}\sin(\psi_2 + \alpha_{P2}) - \mu s_{2F}F_{23F}\cos(\psi_2 + \alpha_{P2}) + F_{x2} - \mu F_{y2} + F_{12}\cos\lambda_1$$

$$- \mu s_{1R}F_{12}\sin\lambda_1 + Q_2\cos\gamma_2 = 0 \qquad \text{(H-165)}$$

and

$$F_{23F}\cos(\psi_2 + \alpha_{P2}) - \mu s_{2F}F_{23F}\sin(\psi_2 + \alpha_{P2}) + F_{y2} + \mu F_{x2} + F_{12}\sin\lambda_1$$

$$+ \mu s_{1R}F_{12}\cos\lambda_1 + Q_2\sin\gamma_2 = 0 \qquad \text{(H-166)}$$

The scalar form of the moment equation (H-164) becomes

$$- \rho_2(\tilde{F}_{x2} + \tilde{F}_{y2}) + a_{G2}F_{23F}[\cos(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2}) + \mu s_{2F}\sin(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2})] - \mu s_{2F}\rho_{G2}F_{23F} - \mu s_{1R}\rho_{P1}F_{12} + a_{P1}F_{12}[\mu s_{1R}\cos(\psi_1 + \delta_{P1} - \lambda_1) - \sin(\psi_1 + \delta_{P1} - \lambda_1)] = 0 \quad \text{(H-167)}$$

Simultaneous solution of the force component equations (H-165) and (H-166) for F_{x2} and F_{y2} results in

$$F_{x2} = \frac{1}{1 + \mu^2}\left\{F_{23F}[(1 + \mu^2 s_{2F})\sin(\psi_2 + \alpha_{P2}) - \mu(1 - s_{2F})\cos(\psi_2 + \alpha_{P2})] + F_{12}[\mu(s_{1R} - 1)\sin\lambda_1 - (1 + \mu^2 s_{1R})\cos\lambda_1] - Q_2[\mu\sin\gamma_2 + \cos\gamma_2]\right\} \quad \text{(H-168)}$$

and

$$F_{y2} = \frac{1}{1 + \mu^2}\left\{F_{23F}[\mu(s_{2F} - 1)\sin(\psi_2 + \alpha_{P2}) - (1 + \mu^2 s_{2F})\cos(\psi_2 + \alpha_{P2})] + F_{12}[-(1 + \mu^2 s_{1R})\sin\lambda_1 + \mu(1 - s_{1R})\cos\lambda_1] + Q_2[-\sin\gamma_2 + \mu\cos\gamma_2]\right\} \quad \text{(H-169)}$$

The sum $\tilde{F}_{x2} + \tilde{F}_{y2}$ of equation(H-167) is now made up of equations (H-168) and (H-169) in the usual manner

$$\tilde{F}_{x2} + \tilde{F}_{y2} = F_{23F}A_{43} + F_{12}A_{44} + Q_2A_{45} + F_{23F}A_{46} + F_{12}A_{47} + Q_2A_{48} \qquad \text{(H-170)}$$

where

$$A_{43} = \left| \frac{(1 + \mu^2 s_{2F})\sin(\psi_2 + \alpha_{P2}) - \mu(1 - s_{2F})\cos(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \qquad \text{(H-171)}$$

$$A_{44} = \left| \frac{\mu(s_{1R} - 1)\sin\lambda_1 - (1 + \mu^2 s_{1R})\cos\lambda_1}{1 + \mu^2} \right| \qquad \text{(H-172)}$$

$$A_{45} = \left| \frac{\mu\sin\gamma_2 + \cos\gamma_2}{1 + \mu^2} \right| \qquad \text{(H-173)}$$

$$A_{46} = \left| \frac{\mu(s_{2F} - 1)\sin(\psi_2 + \alpha_{P2}) - (1 + \mu^2 s_{2F})\cos(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \qquad \text{(H-174)}$$

$$A_{47} = \left| \frac{-(1 + \mu^2 s_{1R})\sin\lambda_1 + \mu(1 - s_{1R})\cos\lambda_1}{1 + \mu^2} \right| \qquad \text{(H-175)}$$

$$A_{48} = \left| \frac{-\sin\gamma_2 + \mu\cos\gamma_2}{1 + \mu^2} \right| \qquad \text{(H-176)}$$

Equation (H-170) is now substituted into the moment equation (H-167) and the result is solved for F_{12}

$$F_{12} = \frac{- F_{23F}C_{22} + Q_2C_{23}}{C_{24}} \qquad \text{(H-177)}$$

where

$$C_{22} = -\mu\rho_2(A_{43} + A_{46}) + a_{G2}[\cos(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2}) + \mu a_{2F}\sin(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2})] - \mu a_{2F}\rho_{G2}$$

$$C_{23} = \mu\rho_2(A_{45} + A_{48})$$

$$C_{24} = -\mu\rho_2(A_{44} + A_{47}) + a_{P1}[\mu a_{1R}\cos(\psi_1 + \delta_{P1} - \lambda_1) - \sin(\psi_1 + \delta_{P1} - \lambda_1)] - \mu a_{1R}\rho_{P1}$$

II. MOMENT INPUT-OUTPUT RELATIONSHIP

Equations (H-177) and (H-78) are now set equal to each other and the result is solved for F_{23F}

$$F_{23F} = \frac{-C_{24}}{C_{10}C_{22}}(M_{in} - Q_1C_9) + Q_2\frac{C_{23}}{C_{22}} \qquad \text{(H-178)}$$

The above is now equated to equation (H-137) to obtain F_{34}

$$F_{34} = \frac{C_{18}C_{24}}{C_{10}C_{16}C_{22}}(M_{in} - Q_1C_9) - Q_2\frac{C_{18}C_{23}}{C_{16}C_{22}} - Q_3\frac{C_{17}}{C_{16}} \qquad \text{(H-179)}$$

Finally equations (H-179) and (H-19) are set equal to each other and the equilibrant moment M_{o44} (For case 4: RFR) is determined

$$M_{o44} = M_{in}\frac{C_2C_{18}C_{24}}{C_{10}C_{16}C_{22}} - Q_1\frac{C_2C_9C_{18}C_{24}}{C_{10}C_{16}C_{22}} - Q_2\frac{C_2C_{18}C_{23}}{C_{16}C_{22}} - Q_3\frac{C_2C_{17}}{C_{16}} - Q_4C_1 \qquad \text{(H-180)}$$

e. CASE NO. 5: FFF

For this contact combination it is necessary to determine new expressions for the force F_{34F} of gear no. 3 on pinion no. 4, and for force F_{23F} of gear no. 2 on pinion no. 3.

Equation (H-156), derived for case 3, and which relates force F_{23F} to the input moment M_{in}, may be used for the determination of the final input-output relationship.

I. FORCE AND MOMENT EQUILIBRIA OF PINION NO. 4

Figure H-10 gives the free body diagram of pinion no. 4 in the round on flat phase of motion with gear no. 3.

The equilibrant moment M_{o4} acts in a clockwise direction and opposes the counter-clockwise rotation of the pinion.

The normal contact force $\bar{F}_{34F}$ is given by

$$\bar{F}_{34F} = F_{34F}\bar{n}_{NF3} \qquad \text{(H-181)}$$

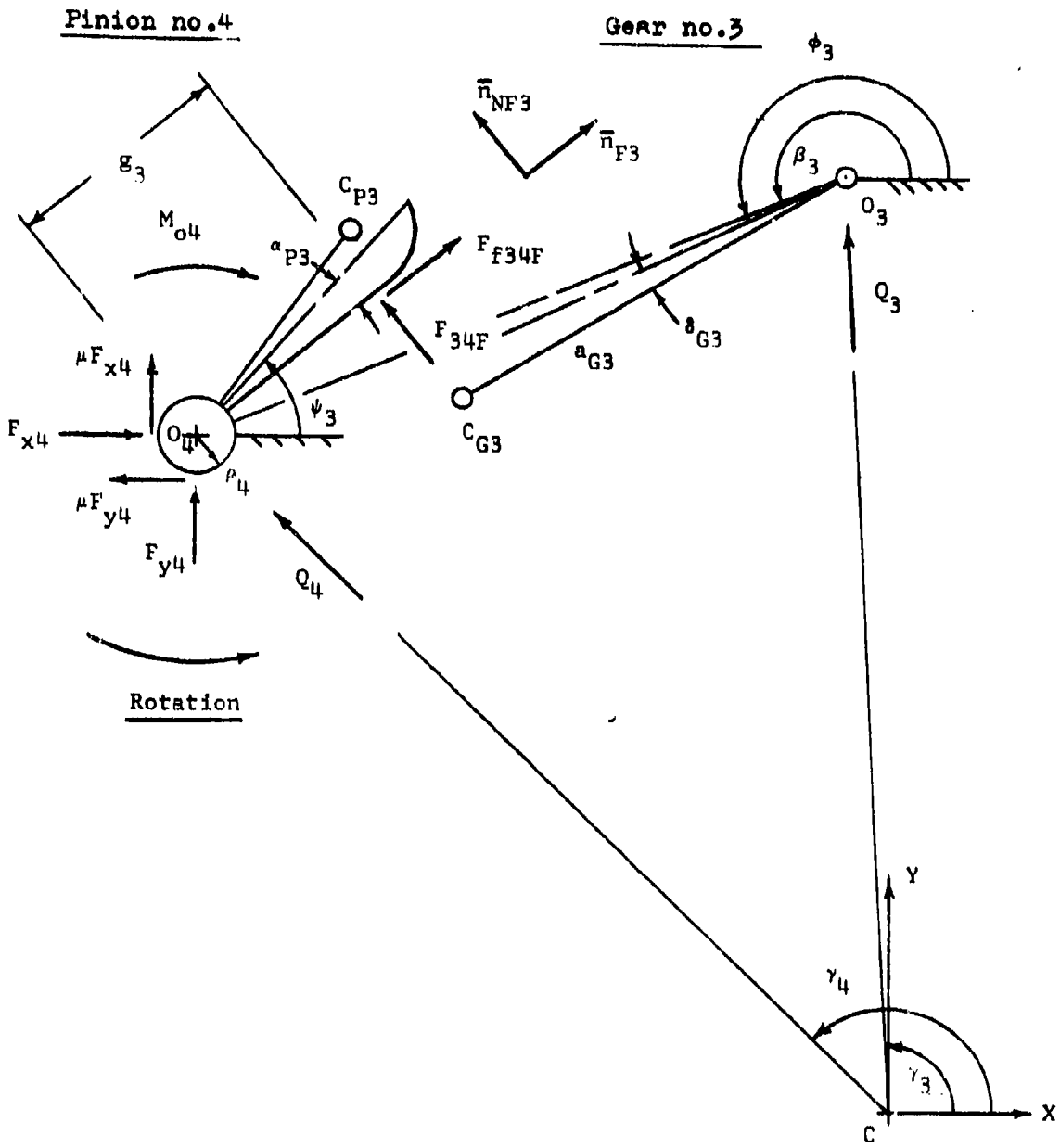

FIGURE H-10

FREE BODY DIAGRAM OF PINION NO. 4
MESH NO. 3: ROUND ON FLAT

The associated friction force becomes

$$\bar{F}_{f34F} = \mu s_{3F} F_{34F} \bar{n}_{F3} \qquad \text{(H-182)}$$

The centrifugal force $\bar{Q}_4$ is given by equation (H-5) and the pivot friction forces are chosen such that they oppose rotation.

Force equilibrium is given by

$$F_{34F}\bar{n}_{NF3} + \mu s_{3F} F_{34F} \bar{n}_{F3} + F_{x4}\bar{i} - \mu F_{y4}\bar{i} + F_{y4}\bar{j} + \mu F_{x4}\bar{j} + Q_4(\cos\gamma_4 \bar{i} + \sin\gamma_4 \bar{j}) = 0 \qquad \text{(H-183)}$$

Moment equilibrium about point O_4 requires that

$$-M_{O4}\bar{k} - \mu\rho_4(\tilde{F}_{x4} + \tilde{F}_{y4})\bar{k} + g_3 \bar{n}_{F3} \times F_{34F}\bar{n}_{NF3} = 0 \qquad \text{(H-184)}$$

Note that the friction force F_{f34F} does not exert a moment about point O_4, since its line of action passes through it.

Equation (H-183) furnishes the following component equations

$$-F_{34F}\sin(\psi_3 - \alpha_{p3}) + \mu s_{3F} F_{34F}\cos(\psi_3 - \alpha_{p3}) + F_{x4} - \mu F_{y4} + Q_4\cos\gamma_4 = 0 \qquad \text{(H-185)}$$

and

$$F_{34F}\cos(\psi_3 - \alpha_{p3}) + \mu s_{3F}F_{34F}\sin(\psi_3 - \alpha_{p3}) + F_{y4} + \mu F_{x4} + Q_4\sin\gamma_4 = 0 \qquad \text{(H-186)}$$

The scalar form of the moment equation (H-184) becomes

$$-M_{o4} - \mu\rho_4(\tilde{F}_{x4} + \tilde{F}_{y4}) + s_3F_{34F} = 0 \qquad \text{(H-187)}$$

Simultaneous solution of equations (H-185) and (H-186) for F_{x4} and F_{y4} results in

$$F_{x4} = \frac{1}{1+\mu^2}\left\{F_{34F}\left[(1 - \mu^2 s_{3F})\sin(\psi_3 - \alpha_{p3}) - \mu(1 + s_{3F})\cos(\psi_3 - \alpha_{p3})\right] + Q_4\left[-\mu\sin\gamma_4 - \cos\gamma_4\right]\right\} \qquad \text{(H-188)}$$

and

$$F_{y4} = \frac{1}{1+\mu^2}\left\{F_{34F}\left[-\mu(1 + s_{3F})\sin(\psi_3 - \alpha_{p3}) - (1 - \mu^2 s_{3F})\cos(\psi_3 - \alpha_{p3})\right] + Q_4\left[-\sin\gamma_4 + \mu\cos\gamma_4\right]\right\} \qquad \text{(H-189)}$$

The sum $\tilde{F}_{x4} + \tilde{F}_{y4}$ of equation (H-187) is now made up of equations (H-188) and (H-189) in the sense of equation (A-3b)

$$\tilde{F}_{x4} + \tilde{F}_{y4} = F_{34F}A_{49} + Q_4A_{50} + F_{34F}A_{51} + Q_4A_{52} \qquad \text{(H-190)}$$

where

$$A_{49} = \left| \frac{(1 - \mu^2 s_{3F})\sin(\psi_3 - \alpha_{P3}) - \mu(1 + s_{3F})\cos(\psi_3 - \alpha_{P3})}{1 + \mu^2} \right| \qquad \text{(H-191)}$$

$$A_{50} = \left| \frac{-\mu\sin\gamma_4 - \cos\gamma_4}{1 + \mu^2} \right| \qquad \text{(H-192)}$$

$$A_{51} = \left| \frac{-\mu(1 + s_{3F})\sin(\psi_3 - \alpha_{P3}) - (1 - \mu^2 s_{3F})\cos(\psi_3 - \alpha_{P3})}{1 + \mu^2} \right| \qquad \text{(H-193)}$$

$$A_{52} = \left| \frac{-\sin\gamma_4 + \mu\cos\gamma_4}{1 + \mu^2} \right| \qquad \text{(H-194)}$$

Equation (H-190) is now substituted into equation (H-187) and the result is solved for F_{34F}

$$F_{34F} = \frac{M_{04} + Q_4C_{25}}{C_{26}} \qquad \text{(H-195)}$$

where

$$C_{25} = \mu\rho_4(A_{50} + A_{52})$$

$$C_{26} = \beta_3 - \mu\rho_4(A_{49} + A_{51})$$

II. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 3

Figure H-11 gives the free body diagram of gear and pinion combination no. 3. Both mesh 3 and mesh 2 are in their round on flat phase of contact.

The forces of pinion 4 on gear 3 are equal to, but opposite in direction to, those given by equations (H-181) and (H-182)

$$\bar{F}_{43F} = - F_{34F}\bar{n}_{NF3} \tag{H-196}$$

and

$$\bar{F}_{f43F} = - \mu s_{3F} F_{34F}\bar{n}_{F3} \tag{H-197}$$

The forces of gear 2 on pinion 3 are given by

$$\bar{F}_{23F} = - F_{23F}\bar{n}_{NF2} \tag{H-198}$$

and

$$\bar{F}_{f23F} = \mu s_{2F} F_{23F}\bar{n}_{F2} \tag{H-199}$$

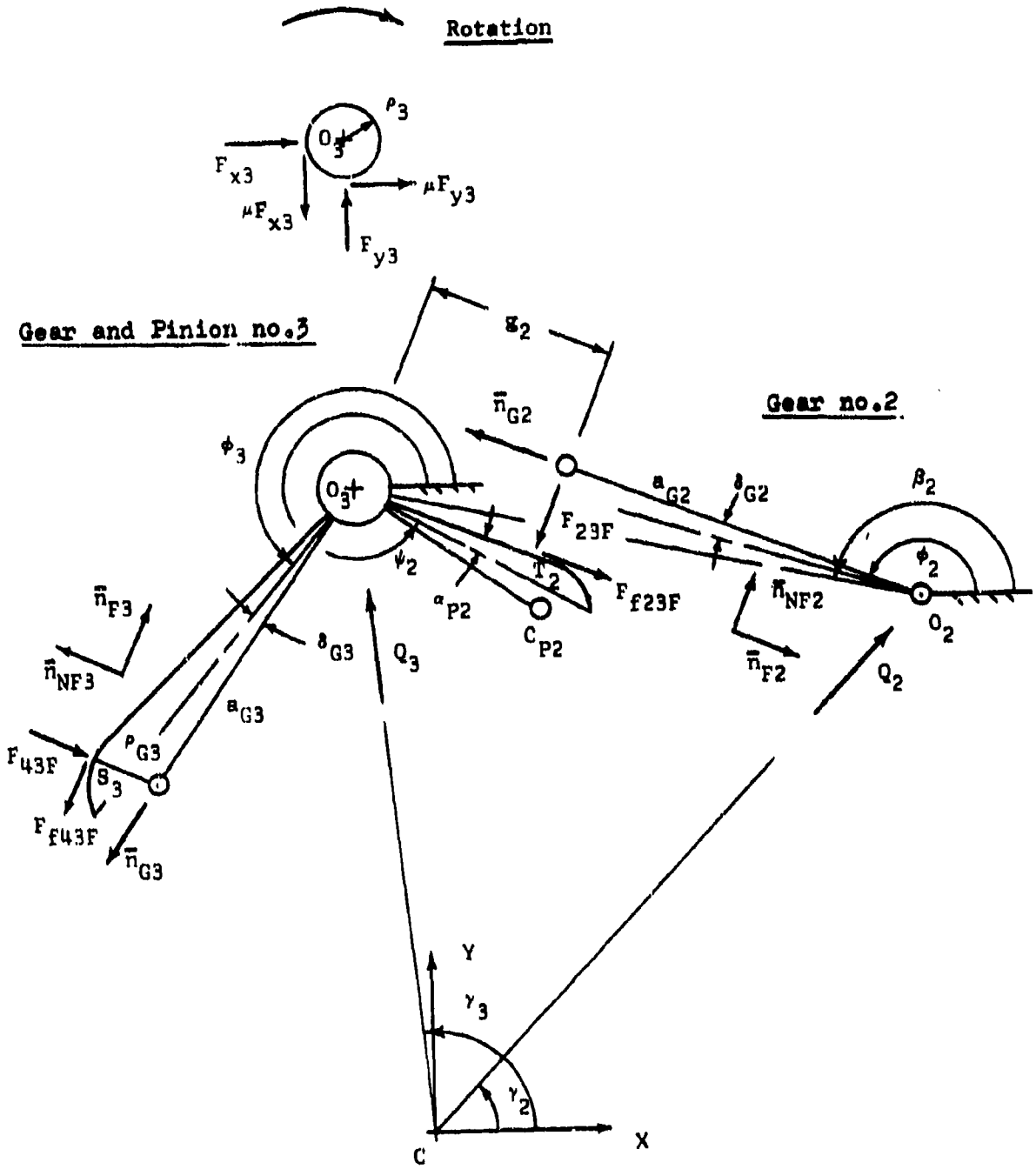

FIGURE H-11

FREE BODY DIAGRAM OF GEAR & PINION NO. 3

MESH NO. 3: ROUND ON FLAT

MESH NO. 2: ROUND ON FLAT

The pivot forces and moments are chosen in the usual manner and the centrifugal force $\bar{Q}_3$ is defined by equation (H-24).

Force equilibrium is given by

$$-F_{34F}\bar{n}_{NF3} - \mu s_{3F}F_{34F}\bar{n}_{F3} - F_{23F}\bar{n}_{NF2} + \mu s_{2F}F_{23F}\bar{n}_{F2} + \bar{Q}_3 + F_{x3}\bar{i} + \mu F_{y3}\bar{i} + F_{y3}\bar{j} - \mu F_{x3}\bar{j} = 0 \qquad \text{(H-200)}$$

Moment equilibrium about point O_3 requires that

$$\mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3})\bar{k} + [s_{G3}\bar{n}_{G3} + \rho_{G3}\bar{n}_{NF3}] \times [-F_{34F}\bar{n}_{NF3} - \mu s_{3F}F_{34F}\bar{n}_{F3}] + s_2\bar{n}_{F2} \times (-)F_{23F}\bar{n}_{NF2} = 0 \qquad \text{(H-201)}$$

Note that the friction force F_{f23F} does not exert a moment about point O_3.

Equation (H-200) furnishes, after all necessary substitutions, the following component equations:

$$F_{34F}\sin(\psi_3 - \alpha_{P3}) - \mu s_{3F}F_{34F}\cos(\psi_3 - \alpha_{P3}) + Q_3\cos\gamma_3 + F_{x3} + \mu F_{y3} + F_{23F}\sin(\psi_2 + \alpha_{P2}) + \mu s_{2F}F_{23F}\cos(\psi_2 + \alpha_{P2}) = 0 \qquad \text{(H-202)}$$

and

$$-F_{34F}\cos(\psi_3 - \alpha_{P3}) - \mu s_{3F}F_{34F}\sin(\psi_3 - \alpha_{P3}) + Q_3\sin\gamma_3 + F_{y3} - \mu F_{x3}$$

$$- F_{23F}\cos(\psi_2 + \alpha_{P2}) + \mu s_{2F}F_{23F}\sin(\psi_2 + \alpha_{P2}) = 0 \qquad \text{(H-203)}$$

The scalar form of the moment equation (H-201) becomes

$$\mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3}) + s_{G3}F_{34F}\left[-\cos(\phi_3 + \delta_{G3} - \psi_3 + \alpha_{P3})\right.$$

$$\left. + \mu s_{3F}\sin(\phi_3 + \delta_{G3} - \psi_3 + \alpha_{P3})\right] + \mu s_{3F}\rho_{G3}F_{34F} - s_2F_{23F} = 0$$

(H-204)

Simultaneous solution of the component equations (H-202) and (H-203) for F_{x3} and F_{y3} gives

$$F_{x3} = \frac{1}{1 + \mu^2}\left\{F_{34F}\left[-(1 + \mu^2 s_{3F})\sin(\psi_3 - \alpha_{P3}) + \mu(s_{3F} - 1)\cos(\psi_3 - \alpha_{P3})\right]\right.$$

$$+ F_{23F}\left[(\mu^2 s_{2F} - 1)\sin(\psi_2 + \alpha_{P2}) - \mu(1 + s_{2F})\cos(\psi_2 + \alpha_{P2})\right]$$

$$\left. + Q_3\left[\mu\sin\gamma_3 - \cos\gamma_3\right]\right\} \qquad \text{(H-205)}$$

and

$$F_{y3} = \frac{1}{1+\mu^2}\Big\{F_{34F}\big[\mu(s_{3F}-1)\sin(\psi_3-\alpha_{P3}) + (1+\mu^2 s_{3F})\cos(\psi_3-\alpha_{P3})\big] + F_{23F}\big[-\mu(1+s_{2F})\sin(\psi_2+\alpha_{P2}) + (1-\mu^2 s_{2F})\cos(\psi_2+\alpha_{P2})\big] + Q_3\big[-\sin\gamma_3 - \mu\cos\gamma_3\big]\Big\} \quad \text{(H-206)}$$

The sum $\tilde{F}_{x3} + \tilde{F}_{y3}$ is now made up from equations (H-205) and (H-206)

$$\tilde{F}_{x3} + \tilde{F}_{y3} = F_{34F}A_{53} + F_{23F}A_{54} + Q_3A_{55} + F_{34F}A_{56} + F_{23F}A_{57} + Q_3A_{58} \quad \text{(H-207)}$$

where

$$A_{53} = \left|\frac{-(1+\mu^2 s_{3F})\sin(\psi_3-\alpha_{P3}) + \mu(s_{3F}-1)\cos(\psi_3-\alpha_{P3})}{1+\mu^2}\right| \quad \text{(H-208)}$$

$$A_{54} = \left|\frac{(\mu^2 s_{2F}-1)\sin(\psi_2+\alpha_{P2}) - \mu(1+s_{2F})\cos(\psi_2+\alpha_{P2})}{1+\mu^2}\right| \quad \text{(H-209)}$$

$$A_{55} = \left|\frac{\mu\sin\gamma_3 - \cos\gamma_3}{1+\mu^2}\right| \quad \text{(H-210)}$$

$$A_{56} = \left| \frac{\mu(s_{3F} - 1)\sin(\psi_3 - \alpha_{P3}) + (1 + \mu^2 s_{3F})\cos(\psi_3 - \alpha_{P3})}{1 + \mu^2} \right| \qquad \text{(H-211)}$$

$$A_{57} = \left| \frac{-\mu(1 + s_{2F})\sin(\psi_2 + \alpha_{P2}) + (1 - \mu^2 s_{2F})\cos(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \qquad \text{(H-212)}$$

$$A_{58} = \left| \frac{\sin\gamma_3 + \mu\cos\gamma_3}{1 + \mu^2} \right| \qquad \text{(H-213)}$$

Equation (H-207) is now substituted into equation (H-204).

The result is solved for F_{23F}

$$F_{23F} = \frac{- F_{34F}C_{27} - Q_3 C_{28}}{C_{29}} \qquad \text{(H-214)}$$

where

$$C_{27} = \mu\rho_3(A_{53} + A_{56}) + s_{G3}\left[-\cos(\phi_3 + \delta_{G3} - \psi_3 + \alpha_{P3}) + \mu s_{3F}\sin(\phi_3 + \delta_{G3} - \psi_3 + \alpha_{P3})\right] + \mu s_{3F}\rho_{G3}$$

$$C_{28} = \mu\rho_3(A_{55} + A_{58})$$

$$C_{29} = \mu\rho_3(A_{54} + A_{57}) - s_2$$

III. MOMENT INPUT-OUTPUT RELATIONSHIP

Equation (H-156) is an expression for F_{23F} as a function of the moment M_{in} and the centrifugal forces Q_1 and Q_2, when both meshes no. 1 and no. 2 are in the round on flat phase of motion. This expression for F_{23F} is now equated to equation (H-214). The result is solved for F_{34F}

$$F_{34F} = \frac{C_{21}C_{29}}{C_{15}C_{19}C_{27}}(M_{in} - Q_1C_{14}) - Q_2\frac{C_{20}C_{29}}{C_{19}C_{27}} - Q_3\frac{C_{28}}{C_{27}} \qquad \text{(H-215)}$$

The above is now equated to equation (H-195) and the resulting expression is used to determine the equilibrant moment M_{045} (for Case 5: FFF)

$$M_{045} = M_{in}\frac{C_{21}C_{26}C_{29}}{C_{15}C_{19}C_{27}} - Q_1\frac{C_{14}C_{21}C_{26}C_{29}}{C_{15}C_{19}C_{27}} - Q_2\frac{C_{20}C_{26}C_{29}}{C_{19}C_{27}}$$

$$- Q_3\frac{C_{26}C_{28}}{C_{27}} - Q_4C_{25} \qquad \text{(H-216)}$$

f. CASE NO. 6: FFR

MOMENT INPUT-OUTPUT RELATIONSHIP

The moment input-output relationship for this contact combination can be assembled entirely from previously derived component relationships. As for case no. 4, mesh no. 1 is in the round on round phase while mesh no. 2 is in the round on flat phase. Therefore, equation (H-178), which relates the force F_{23F} to the input moment M_{in}, may be used. The input-output relationship of the gear and pinion set no. 3, i.e. the relationship between the forces F_{34F} and F_{23F}, is given by equation (H-214) of case no. 5. The force F_{34F} may be obtained from equation (H-195). This expression was also derived for a round on flat contact in case no. 5.

Thus, equation (H-178) is first set equal to equation (H-214) and the result is solved for the force F_{34F}

$$F_{34F} = \frac{C_{24}C_{29}}{C_{10}C_{22}C_{27}}(M_{in} - Q_1C_9) - Q_2\frac{C_{23}C_{29}}{C_{22}C_{27}} - Q_3\frac{C_{28}}{C_{27}} \qquad \text{(H-217)}$$

The above expression is now set equal to equation (H-195). This then allows the determination of the equilibrant moment M_{046} for the present contact combination.

$$M_{046} = M_{in}\frac{C_{24}C_{26}C_{29}}{C_{10}C_{22}C_{27}} - Q_1\frac{C_9C_{24}C_{26}C_{29}}{C_{10}C_{22}C_{27}} - Q_2\frac{C_{23}C_{26}C_{29}}{C_{22}C_{27}}$$

$$- Q_3\frac{C_{26}C_{28}}{C_{27}} - Q_4C_{25} \qquad \text{(H-218)}$$

g. CASE NO. 7: FRR

For the present contact combination the expression for force F_{34F} may be taken over from equation (H-195) of case no. 5. The input-output relationship of gear and pinion set no. 3, which relates the forces F_{34F} and F_{23}, must be newly derived. The relationship between force F_{23} and the input moment M_{in} is taken from case no. 1 in the form of equation (H-79).

I. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 3

Figure H-12 gives the free body diagram of gear and pinion set no. 3. Mesh no. 3 is in the round on flat phase of contact, while mesh no. 2 is in the round on round one.

The forces of pinion no. 4 on gear no. 3 are equal to, but opposite in direction to those given by equations (H-181) and (H-182)

$$\bar{F}_{43F} = - F_{34F}\bar{n}_{NF3} \qquad \text{(H-219)}$$

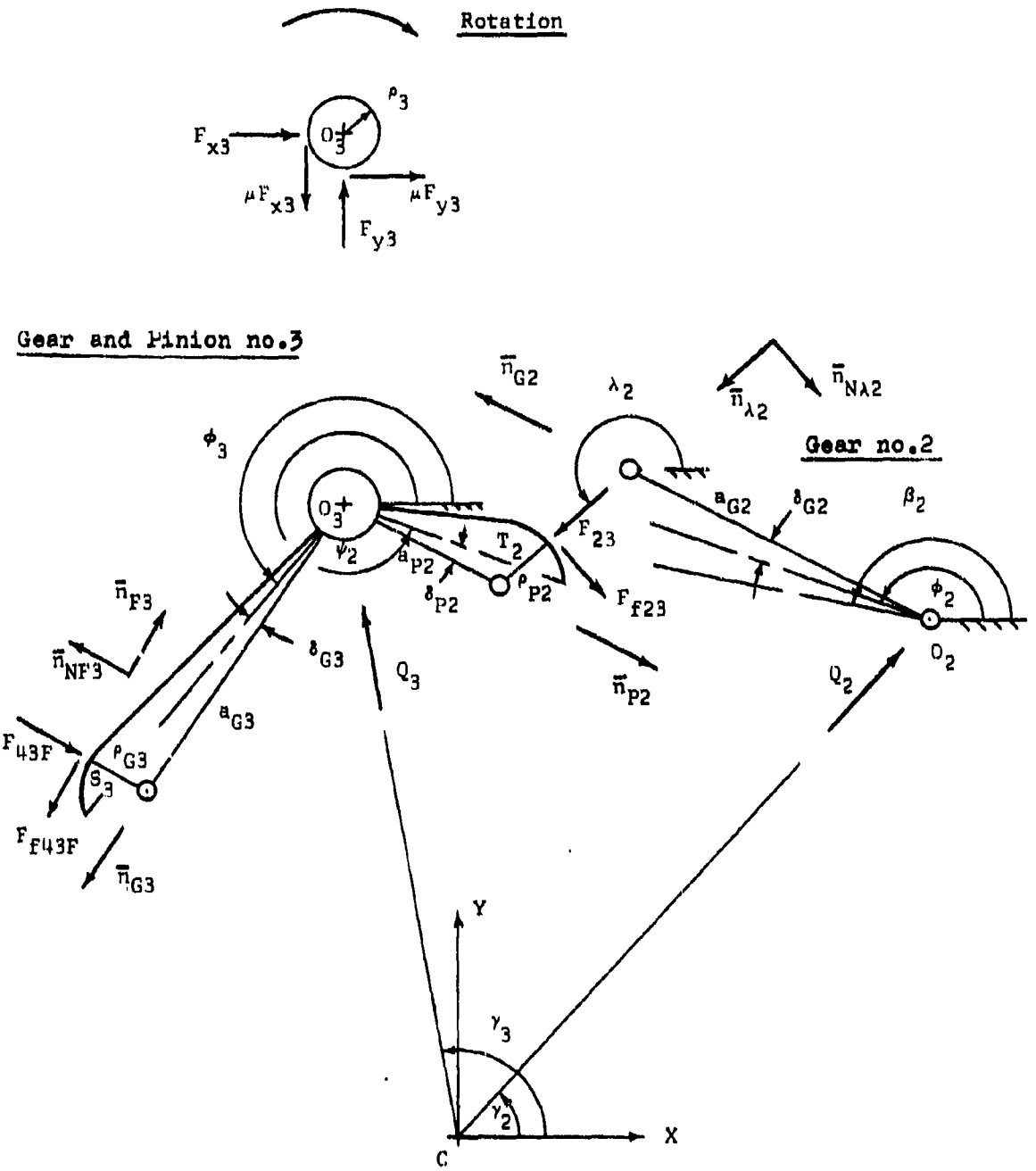

FIGURE H-12

FREE BODY DIAGRAM OF GEAR & PINION NO. 3
MESH NO. 3: ROUND ON FLAT
MESH NO. 2: ROUND ON ROUND

and

$$\bar{F}_{f43F} = -\mu s_{3F} F_{34F} \bar{n}_{F3} \qquad \text{(H-220)}$$

The forces of gear no. 2 on pinion no. 3 are given by

$$\bar{F}_{23} = F_{23} \bar{n}_{\lambda 2} \qquad \text{(H-221)}$$

and

$$\bar{F}_{f23} = \mu s_{2R} F_{23} \bar{n}_{N\lambda 2} \qquad \text{(H-222)}$$

The pivot reactions are chosen in the same manner as before.

The centrifugal force $\bar{Q}_3$ was defined by equation (H-24).

Force equilibrium of the gear set requires

$$-F_{34F} \bar{n}_{NF3} - \mu s_{3F} F_{34F} \bar{n}_{F3} + F_{23} \bar{n}_{\lambda 2} + \mu s_{2R} F_{23} \bar{n}_{N\lambda 2} + \bar{Q}_3 + F_{x3} \bar{I}$$

$$+ \mu F_{y3} \bar{i} + F_{y3} \bar{j} - \mu F_{x3} \bar{j} = 0 \qquad \text{(H-223)}$$

Moment equilibrium about point O_3 is given by

$$\mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3})\bar{k} + [a_{G3}\bar{n}_{G3} + \rho_{G3}\bar{n}_{NF3}] \times [-F_{34F}\bar{n}_{NF3} - \mu s_{3F}F_{34F}\bar{n}_{F3}]$$

$$+ [a_{P2}\bar{n}_{P2} - \rho_{P2}\bar{n}_{\lambda 2}] \times [F_{23}\bar{n}_{\lambda 2} + \mu s_{2R}F_{23}\bar{n}_{N\lambda 2}] = 0 \qquad \text{(H-224)}$$

Equation (H-223) gives the following component expressions

$$F_{34F}\sin(\psi_3 - \alpha_{P3}) - \mu s_{3F}F_{34F}\cos(\psi_3 - \alpha_{P3}) + Q_3\cos\gamma_3 + F_{x3} + \mu F_{y3}$$

$$+ F_{23}\cos\lambda_2 - \mu s_{2R}F_{23}\sin\lambda_2 = 0 \qquad \text{(H-225)}$$

and

$$-F_{34F}\cos(\psi_3 - \alpha_{P3}) - \mu s_{3F}F_{34F}\sin(\psi_3 - \alpha_{P3}) + Q_3\sin\gamma_3 + F_{y3} - \mu F_{x3}$$

$$+ F_{23}\sin\lambda_2 + \mu s_{2R}F_{23}\cos\lambda_2 = 0 \qquad \text{(H-226)}$$

The scalar form of the moment equation (H-224) becomes

$$\mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3}) + a_{G3}F_{34F}[-\cos(\phi_3 + \delta_{G3} - \psi_3 + \alpha_{P3})$$

$$+ \mu a_{3F}\sin(\phi_3 + \delta_{G3} - \psi_3 + \alpha_{P3})] + \mu a_{3F}\rho_{G3}F_{34F}$$

$$+ a_{P2}F_{23}[-\sin(\psi_2 - \delta_{P2} - \lambda_2) + \mu a_{2R}\cos(\psi_2 - \delta_{P2} - \lambda_2)]$$

$$- \mu a_{2R}\rho_{P2}F_{23} = 0 \qquad \text{(H-227)}$$

Simultaneous solution of the component equations (H-225) and (H-226) for F_{x3} and F_{y3} results in:

$$F_{x3} = \frac{1}{1 + \mu^2}\left\{F_{34F}[-(1 + \mu^2 a_{3F})\sin(\psi_3 - \alpha_{P3}) + \mu(a_{3F} - 1)\cos(\psi_3 - \alpha_{P3})]\right.$$

$$+ F_{23}[\mu(1 + a_{2R})\sin\lambda_2 + (\mu^2 a_{2R} - 1)\cos\lambda_2]$$

$$\left. + Q_3[\mu\sin\gamma_3 - \cos\gamma_3]\right\} \qquad \text{(H-228)}$$

and

$$F_{y3} = \frac{1}{1+\mu^2}\Big\{F_{34F}\big[\mu(s_{3F}-1)\sin(\psi_3-\alpha_{P3}) + (1+\mu^2 s_{3F})\cos(\psi_3-\alpha_{P3})\big] + F_{23}\big[(\mu^2 s_{2R}-1)\sin\lambda_2 - \mu(1+s_{2R})\cos\lambda_2\big] + Q_3\big[-\sin\gamma_3 - \mu\cos\gamma_3\big]\Big\} \quad \text{(H-229)}$$

The sum $\tilde{F}_{x3} + \tilde{F}_{y3}$ in equation (H-227) is now made up from equations (H-228) and (H-229) in the sense of equation (A-3b)

$$\tilde{F}_{x3} + \tilde{F}_{y3} = F_{34F}A_{59} + F_{23}A_{60} + Q_3A_{61} + F_{34F}A_{62} + F_{23}A_{63} + Q_3A_{64} \quad \text{(H-230)}$$

where

$$A_{59} = \left|\frac{-(1+\mu^2 s_{3F})\sin(\psi_3-\alpha_{P3}) + \mu(s_{3F}-1)\cos(\psi_3-\alpha_{P3})}{1+\mu^2}\right| \quad \text{(H-231)}$$

$$A_{60} = \left|\frac{\mu(1+s_{2R})\sin\lambda_2 + (\mu^2 s_{2R}-1)\cos\lambda_2}{1+\mu^2}\right| \quad \text{(H-232)}$$

$$A_{61} = \left|\frac{\mu\sin\gamma_3 - \cos\gamma_3}{1+\mu^2}\right| \quad \text{(H-233)}$$

$$A_{62} = \left| \frac{\mu(s_{3F} - 1)\sin(\psi_3 - \alpha_{P3}) + (1 + \mu^2 s_{3F})\cos(\psi_3 - \alpha_{P3})}{1 + \mu^2} \right| \qquad \text{(H-234)}$$

$$A_{63} = \left| \frac{(\mu^2 s_{2R} - 1)\sin\lambda_2 - \mu(1 + s_{2R})\cos\lambda_2}{1 + \mu^2} \right| \qquad \text{(H-235)}$$

$$A_{64} = \left| \frac{-\sin\gamma_3 - \mu\cos\gamma_3}{1 + \mu^2} \right| \qquad \text{(H-236)}$$

Equation (H-230) is now substituted into the moment equation (H-227) and the resulting expression is solved for F_{23}

$$F_{23} = \frac{-F_{34F}C_{30} - Q_3 C_{31}}{C_{32}} \qquad \text{(H-237)}$$

where

$$C_{30} = \mu\rho_3(A_{59} + A_{62}) - s_{G3}\left[\cos(\phi_3 + \delta_{G3} - \psi_3 + \alpha_{P3}) - \mu s_{3F}\sin(\phi_3 + \delta_{G3} - \psi_3 + \alpha_{P3})\right] + \mu s_{3F}\rho_{G3}$$

$$C_{31} = \mu\rho_3(A_{61} + A_{64})$$

$$C_{32} = \mu\rho_3(A_{60} + A_{63}) - a_{P2}\left[\sin(\psi_2 - \delta_{P2} - \lambda_2) - \mu s_{2R}\cos(\psi_2 - \delta_{P2} - \lambda_2)\right]$$

$$- \mu s_{2R}\rho_{P2}$$

II. MOMENT INPUT-OUTPUT RELATIONSHIP

Equation (H-79), which gives the force F_{23} in terms of M_{in}, Q_1 and Q_2 for the appropriate contact combinations, is now set equal to equation (H-237). Subsequently, one finds the following formulation for F_{34F}:

$$F_{34F} = \frac{C_8C_{32}}{C_6C_{10}C_{30}}(M_{in} - Q_1C_9) - Q_2\frac{C_7C_{32}}{C_6C_{30}} - Q_3\frac{C_{31}}{C_{30}} \qquad \text{(H-238)}$$

The above expression is now set equal to equation (H-195) which gives F_{34F} in terms of M_{04} and Q_4. The determination of the equilibrant moment M_{047} (for case 7: FRR) is now possible. Thus,

$$M_{047} = M_{1n}\frac{C_8C_{26}C_{32}}{C_6C_{10}C_{30}} - Q_1\frac{C_8C_9C_{26}C_{32}}{C_6C_{10}C_{30}} - Q_2\frac{C_7C_{26}C_{32}}{C_6C_{30}}$$

$$- Q_3\frac{C_{26}C_{31}}{C_{30}} - Q_4C_{25} \qquad \text{(H-239)}$$

h. CASE NO. 8: FRF

MOMENT INPUT-OUTPUT RELATIONSHIP

The input-output relationship for this contact combination can also be assembled entirely from existing expressions. With mesh no. 2 in the round on round phase of contact and mesh no. 1 in the round on flat one, the relationship between force F_{23} and the moment M_{in} is that of case no. 2. Equation (H-116) is applicable. The input-output relationship of gear and pinion set no. 3, which relates F_{34F} to F_{23}, was derived for case no. 7 and is given by equation (H-237). Finally, with mesh no. 3 in the round on flat phase, one uses equation (H-195) for the relationship between force F_{34F} and moment M_{04}.
Thus, equation (H-116) is first set equal to equation (H-237) to obtain an expression for force F_{34F}

$$F_{34F} = \frac{C_{13}C_{32}}{C_{11}C_{15}C_{30}}(M_{in} - Q_1C_{14}) - Q_2\frac{C_{12}C_{32}}{C_{11}C_{30}} - Q_3\frac{C_{31}}{C_{30}} \qquad \text{(H-240)}$$

The above expression is now set equal to equation (H-195). This allows the determination of M_{048} (for case 8: FRF)

$$M_{048} = M_{in}\frac{C_{13}C_{26}C_{32}}{C_{11}C_{15}C_{30}} - Q_1\frac{C_{13}C_{14}C_{26}C_{32}}{C_{11}C_{15}C_{30}} - Q_2\frac{C_{12}C_{26}C_{32}}{C_{11}C_{30}}$$

$$- Q_3\frac{C_{26}C_{31}}{C_{30}} - Q_4C_{25} \qquad \text{(H-241)}$$

2. INPUT-OUTPUT ANALYSIS OF TWO STEP-UP GEAR TRAIN

The following gives derivations for the moment input-output relationships associated with the four possible contact combinations of a two step-up gear train. (See Table H-2).

a. CASE NO. 1: RR

For this contact combination, the relationship between the equilibrant moment M_{03} and force F_{23}, both acting on pinion no. 3, must first be newly obtained. The relationships between forces F_{23} and F_{12} of gear and pinion set no. 2, as well as between force F_{12} and input moment M_{in} of gear no. 1, may be taken from case no. 1 of the three step-up gear train analysis. Equations (H-61) and (H-78), respectively, are applicable.

I. FORCE AND MOMENT EQUILIBRIA OF PINION NO. 3

Figure H-13 shows a schematic free body diagram of pinion no. 3 in the round on round phase of contact. The equilibrant

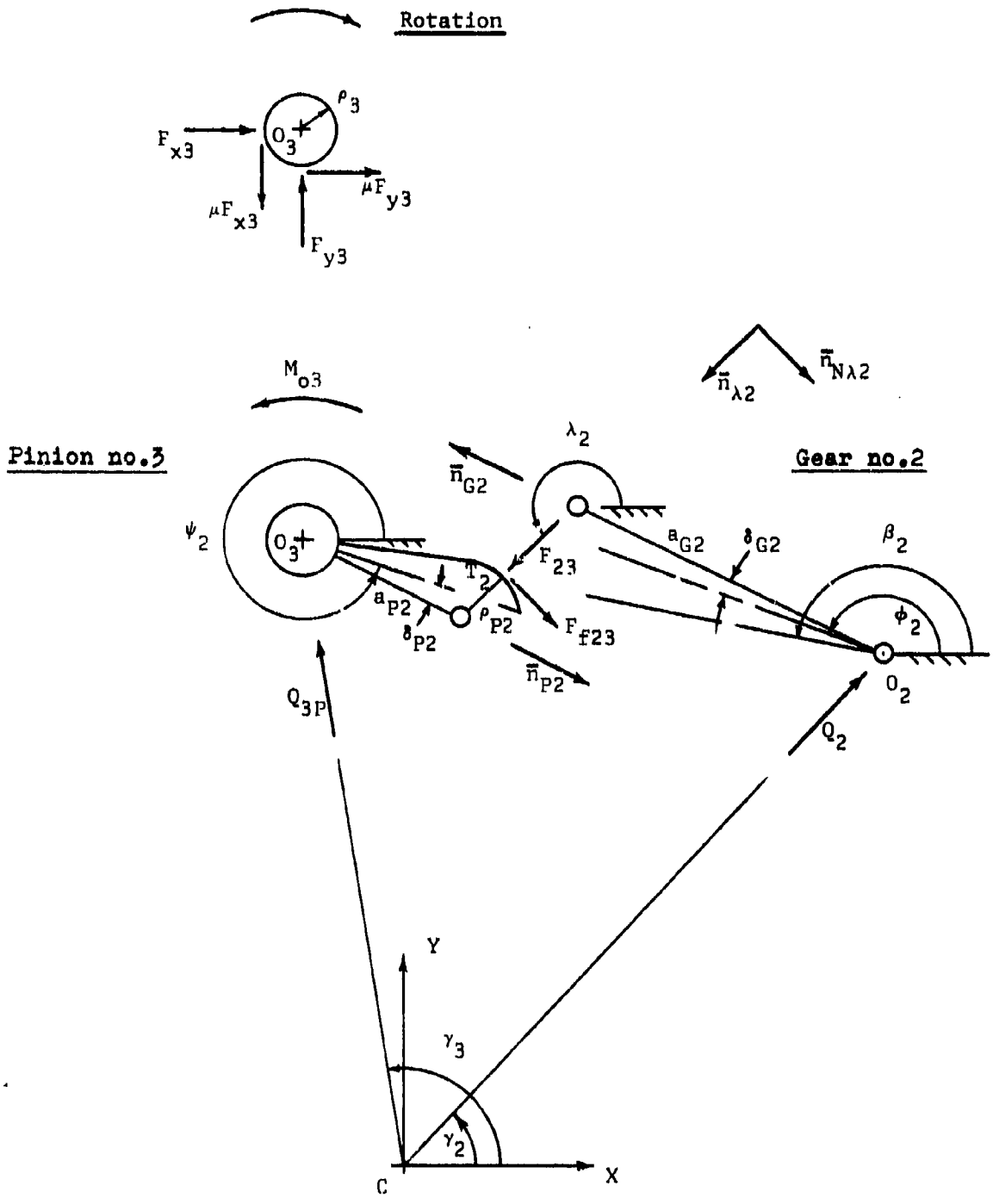

FIGURE H-13

FREE BODY DIAGRAM OF PINION NO. 3
MESH NO. 2: ROUND ON ROUND

moment M_{o3} acts in the direction opposite to the clockwise rotation of the pinion. The normal contact force of gear no. 2 on pinion no. 3 is given by

$$\bar{F}_{23} = F_{23}\bar{n}_{\lambda 2} \tag{H-242}$$

The associated friction force becomes

$$\bar{F}_{f23} = \mu_{s_{2R}} F_{23}\bar{n}_{N\lambda 2} \tag{H-243}$$

The pivot reactions are chosen in the usual manner.

The centrifugal force $\bar{Q}_{3P}$ is now due to the mass m_{3P} of the pinion alone, i.e.,

$$\bar{Q}_{3P} = Q_{3P}(\cos\gamma_3\bar{I} + \sin\gamma_3\bar{J}) \tag{H-244}$$

where

$$Q_{3P} = m_{3P}R_3\omega^2 \tag{H-245}$$

Force equilibrium of the pinion is assured by

$$F_{23}\bar{n}_{\lambda 2} + \mu s_{2R}F_{23}\bar{n}_{N\lambda 2} + F_{x3}\bar{i} + \mu F_{y3}\bar{i} + F_{y3}\bar{j} - \mu F_{x3}\bar{j} + \bar{Q}_{3P} = 0 \qquad \text{(H-246)}$$

Moment equilibrium about point O_3 is given by

$$\mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3})\bar{k} + M_{o3}\bar{k} + \left[a_{P2}\bar{n}_{P2} - \rho_{P2}\bar{n}_{\lambda 2}\right] \times \left[F_{23}\bar{n}_{\lambda 2} + \mu s_{2R}F_{23}\bar{n}_{N\lambda 2}\right] = 0 \qquad \text{(H-247)}$$

Equation (H-246) gives the following component expressions

$$F_{23}\cos\lambda_2 - \mu s_{2R}F_{23}\sin\lambda_2 + F_{x3} + \mu F_{y3} + Q_{3P}\cos\gamma_3 = 0 \qquad \text{(H-248)}$$

and

$$F_{23}\sin\lambda_2 + \mu s_{2R}F_{23}\cos\lambda_2 + F_{y3} - \mu F_{x3} + Q_{3P}\sin\gamma_3 = 0 \qquad \text{(H-249)}$$

The scalar form of the moment equation (H-247) becomes

$$\mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3}) + M_{o3} - \mu s_{2R}\rho_{P2}F_{23} + a_{P2}F_{23}\left[-\sin(\psi_2 - \delta_{P2} - \lambda_2)\right.$$

$$\left. + \mu s_{2R}\cos(\psi_2 - \delta_{P2} - \lambda_2)\right] = 0 \qquad \text{(H-250)}$$

Simultaneous solution of equations (H-248) and (H-249) for the forces F_{x3} and F_{y3} gives

$$F_{x3} = \frac{1}{1 + \mu^2}\left\{F_{23}\left[\mu(1 + s_{2R})\sin\lambda_2 + (\mu^2 s_{2R} - 1)\cos\lambda_2\right]\right.$$

$$\left. + Q_{3P}\left[\mu\sin\gamma_3 - \cos\gamma_3\right]\right\} \qquad \text{(H-251)}$$

and

$$F_{y3} = \frac{1}{1 + \mu^2}\left\{F_{23}\left[(\mu^2 s_{2R} - 1)\sin\lambda_2 - \mu(1 + s_{2R})\cos\lambda_2\right]\right.$$

$$\left. + Q_{3P}\left[-\sin\gamma_3 - \mu\cos\gamma_3\right]\right\} \qquad \text{(H-252)}$$

The sum $\tilde{F}_{x3} + \tilde{F}_{y3}$ of equation (H-250) is now made up from equations (H-251) and (H-252) in the sense of equation (A-3b)

$$\tilde{F}_{x3} + \tilde{F}_{y3} = F_{23}A_{65} + Q_{3P}A_{66} + F_{23}A_{67} + Q_{3P}A_{68} \qquad \text{(H-253)}$$

where

$$A_{65} = \left| \frac{\mu(1 + s_{2R})\sin\lambda_2 + (\mu^2 s_{2R} - 1)\cos\lambda_2}{1 + \mu^2} \right| \qquad \text{(H-254)}$$

$$A_{66} = \left| \frac{\mu\sin\gamma_3 - \cos\gamma_3}{1 + \mu^2} \right| \qquad \text{(H-255)}$$

$$A_{67} = \left| \frac{(\mu^2 s_{2R} - 1)\sin\lambda_2 - \mu(1 + s_{2R})\cos\lambda_2}{1 + \mu^2} \right| \qquad \text{(H-256)}$$

$$A_{68} = \left| \frac{-\sin\gamma_3 - \mu\cos\gamma_3}{1 + \mu^2} \right| \qquad \text{(H-257)}$$

Equation (H-253) is now substituted into the moment equation (H-250) and the result is solved for F_{23}

$$F_{23} = \frac{-M_{o3} - Q_{3P}C_{33}}{C_{34}} \qquad \text{(H-258)}$$

where

$$C_{33} = \mu\rho_3(A_{66} + A_{68})$$

$$C_{34} = \mu\rho_3(A_{65} + A_{67}) - \mu s_{2R}\rho_{P2}$$

$$+ s_{P2}\left[\mu s_{2R}\cos(\psi_2 - \delta_{P2} - \lambda_2) - \sin(\psi_2 - \delta_{P2} - \lambda_2)\right]$$

II. MOMENT INPUT-OUTPUT RELATIONSHIP

In the derivation for case 1 of the three step-up gear train it was shown that if one sets equations (H-61) and (H-78) equal to each other, one obtains the following relationship [i.e., equation (H-79)] between force F_{23} and the input moment M_{in}:

$$F_{23} = \frac{-C_8}{C_6C_{10}} (M_{in} - Q_1C_9) + Q_2 \frac{C_7}{C_6} \qquad \text{(H-259)}$$

The above expression is now set equal to equation (H-258), and the result is solved for the equilibrant moment M_{o31} (for case 1: RR)

$$M_{o31} = M_{in} \frac{C_8C_{34}}{C_6C_{10}} - Q_1 \frac{C_8C_9C_{34}}{C_6C_{10}} - Q_2 \frac{C_7C_{34}}{C_6} - Q_{3p}C_{33} \qquad \text{(H-260)}$$

b. CASE NO. 2: RF

MOMENT INPUT-OUTPUT RELATIONSHIP

The moment equation for the present case, in which mesh no. 1 is in the round on flat phase and mesh no. 2 in the round on round one, may be derived entirely from existing relationships. Equation (H-116) gives an expression for the force F_{23} in terms of the input moment M_{in} and the centrifugal forces Q_1 and Q_2 for the present combination of contacts in both meshes. When this expression, from case no. 2 of the three step-up gear train, is set equal to equation (H-259) of case no. 1 of the two step-up gear train, one obtains for M_{o32} (case no. 2: RF)

$$M_{o32} = M_{in}\frac{C_{13}C_{34}}{C_{11}C_{15}} - Q_1\frac{C_{13}C_{14}C_{34}}{C_{11}C_{15}} - Q_2\frac{C_{12}C_{34}}{C_{11}} - Q_{3p}C_{33} \qquad \text{(H-261)}$$

c. CASE NO. 3: FF

For this contact combination, where both meshes no. 1 and no.2 are in the round on flat phase, the relationship between the equilibrant moment M_{03} and the normal contact force F_{23F}, both acting on pinion no. 3, must first be determined. The resulting expression in F_{23F} can then be set equal to the relationship between F_{23F} and the input moment M_{in}, which is given by equation (H-156), and which was derived in conjunction with case no. 3 of the three step-up gear train.

I. FORCE AND MOMENT EQUILIBRIA OF PINION NO. 3

Figure H-14 shows the free body diagram of pinion no. 3 in the round on flat phase of contact. Again, the equilibrant moment M_{03} acts in a counter-clockwise direction. The normal force F_{23F} of gear no. 2 on pinion no. 3 is given by

$$\bar{F}_{23F} = - F_{23F}\bar{n}_{NF2} \qquad \text{(H-262)}$$

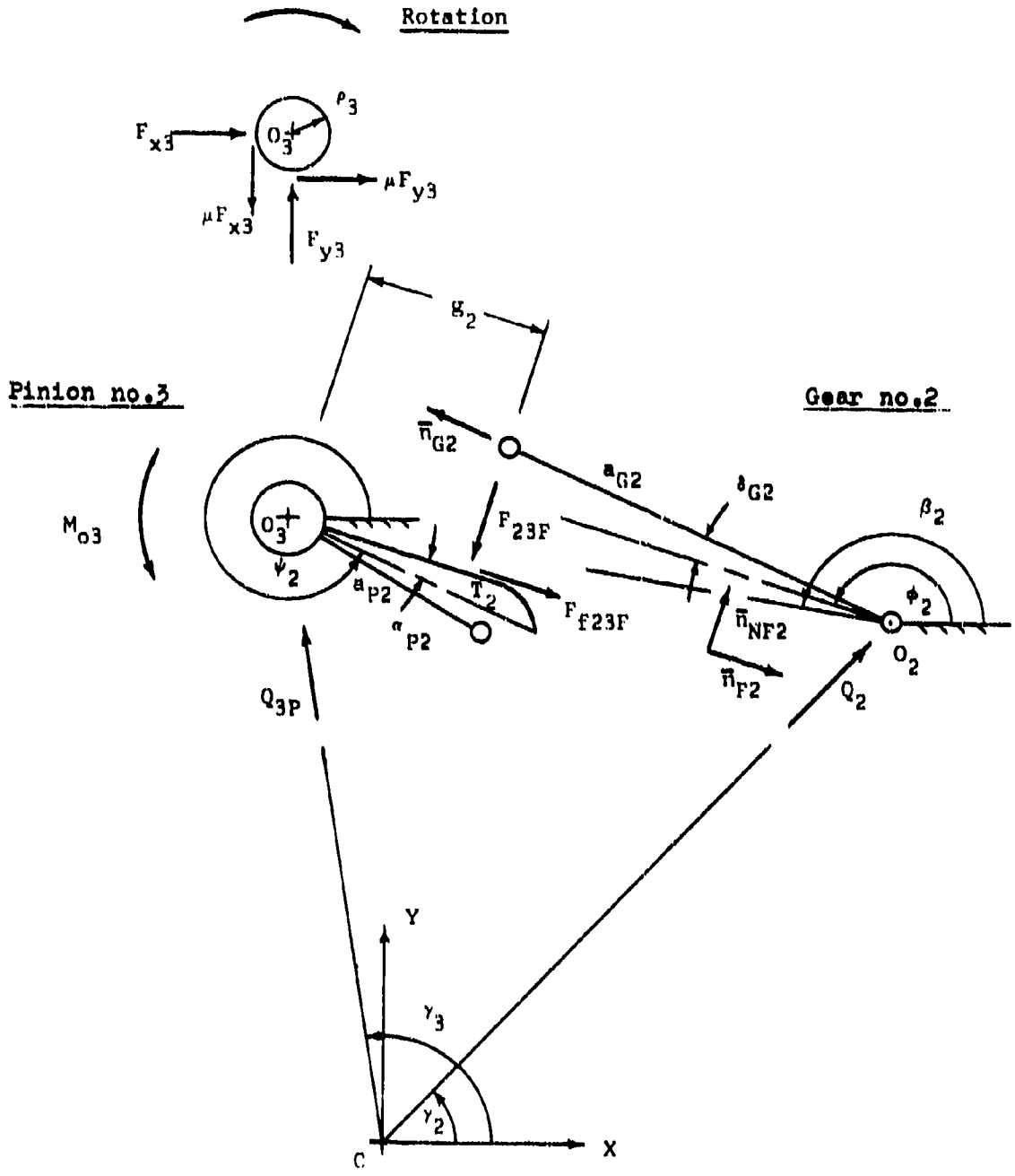

FIGURE H-14

FREE BODY DIAGRAM OF PINION NO. 3
MESH NO. 2: ROUND ON FLAT

The associated friction force of gear no. 2 on pinion no. 3 becomes

$$\bar{F}_{f23F} = \mu_{s_{2F}} F_{23F} \bar{n}_{F2} \qquad \text{(H-263)}$$

The pivot reactions and pivot friction forces are chosen in the usual manner. The centrifugal force $\bar{Q}_{3P}$ was given previously by equation (H-244).

The force equilibrium expression becomes

$$-F_{23F}\bar{n}_{NF2} + \mu_{s_{2F}} F_{23F}\bar{n}_{F2} + F_{x3}\bar{I} + \mu F_{y3}\bar{I} + F_{y3}\bar{J} - \mu F_{x3}\bar{J} + \bar{Q}_{3P}$$

$$= 0 \qquad \text{(H-264)}$$

Moment equilibrium about point O_3 is assured by

$$\mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3})\bar{k} + M_{o3}\bar{k} + \varepsilon_2\bar{n}_{F2} \times (-)F_{23F}\bar{n}_{NF2} = 0 \qquad \text{(H-265)}$$

As always before, the friction force F_{f23F} exerts no moment about point O_3.

Equation (H-264) gives the following component expressions

$$F_{23F}\sin(\psi_2 + \alpha_{P2}) + \mu s_{2F}F_{23F}\cos(\psi_2 + \alpha_{P2}) + F_{x3} + \mu F_{y3} + Q_{3P}\cos\gamma_3 = 0 \qquad \text{(H-266)}$$

and

$$-F_{23F}\cos(\psi_2 + \alpha_{P2}) + \mu s_{2F}F_{23F}\sin(\psi_2 + \alpha_{P2}) + F_{y3} - \mu F_{x3} + Q_{3P}\sin\gamma_3 = 0 \qquad \text{(H-267)}$$

The scalar form of the moment equation (H-265) is given by

$$\mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3}) + M_{o3} - s_2F_{23F} = 0 \qquad \text{(H-268)}$$

Simultaneous solution of equations (H-266) and (H-267) for F_{x3} and F_{y3} leads to

$$F_{x3} = \frac{1}{1 + \mu^2}\left\{F_{23F}\left[(\mu^2 s_{2F} - 1)\sin(\psi_2 + \alpha_{P2}) - \mu(1 + s_{2F})\cos(\psi_2 + \alpha_{P2})\right] + Q_{3P}\left[\mu\sin\gamma_3 - \cos\gamma_3\right]\right\} \qquad \text{(H-269)}$$

$$F_{y3} = \frac{1}{1+\mu^2}\left\{F_{23F}\left[-\mu(1+s_{2F})\sin(\psi_2+\alpha_{P2}) + (1-\mu^2 s_{2F})\cos(\psi_2+\alpha_{P2})\right]\right.$$
$$\left. + Q_{3P}\left[-\sin\gamma_3 - \mu\cos\gamma_3\right]\right\} \qquad \text{(H-270)}$$

The sum $\tilde{F}_{x3} + \tilde{F}_{y3}$ of equation (H-268) is now made up of equations (H-269) and (H-270) in the sense of equation (A-3b)

$$\tilde{F}_{x3} + \tilde{F}_{y3} = F_{23F}A_{69} + Q_{3P}A_{70} + F_{23F}A_{71} + Q_{3P}A_{72} \qquad \text{(H-271)}$$

where

$$A_{69} = \left|\frac{(\mu^2 s_{2F} - 1)\sin(\psi_2+\alpha_{P2}) - \mu(1+s_{2F})\cos(\psi_2+\alpha_{P2})}{1+\mu^2}\right| \qquad \text{(H-272)}$$

$$A_{70} = \left|\frac{\mu\sin\gamma_3 - \cos\gamma_3}{1+\mu^2}\right| \qquad \text{(H-273)}$$

$$A_{71} = \left|\frac{-\mu(1+s_{2F})\sin(\psi_2+\alpha_{P2}) + (1-\mu^2 s_{2F})\cos(\psi_2+\alpha_{P2})}{1+\mu^2}\right| \qquad \text{(H-274)}$$

$$A_{72} = \left|\frac{-\sin\gamma_3 - \mu\cos\gamma_3}{1+\mu^2}\right| \qquad \text{(H-275)}$$

Equation (H-271) is now substituted into the moment equation (H-268), and the result is solved for F_{23F}

$$F_{23F} = \frac{- M_{03} - Q_{3P}C_{35}}{C_{36}} \qquad \text{(H-276)}$$

where

$$C_{35} = \mu\rho_3(A_{70} + A_{72})$$

$$C_{36} = \mu\rho_3(A_{69} + A_{71}) - \delta_2$$

II. MOMENT INPUT-OUTPUT RELATIONSHIP

The moment input-output relationship for the present case is obtained, as stated earlier, by setting equation (H-276) equal to equation (H-156) and solving the resulting expression for the equilibrant moment M_{033} (case no. 3: FF)

$$M_{033} = M_{1n}\frac{C_{21}C_{36}}{C_{15}C_{19}} - Q_1\frac{C_{14}C_{21}C_{36}}{C_{15}C_{19}} - Q_2\frac{C_{20}C_{36}}{C_{19}} - Q_{3P}C_{35} \qquad \text{(H-277)}$$

d. CASE NO. 4: FR

MOMENT INPUT-OUTPUT RELATIONSHIP

The moment input-output relationship for this case, where mesh no. 1 is in round on round contact while mesh no. 2 is in the round on flat phase, may also be derived entirely by assembling existing relationships.

Equation (H-276), of the previous section, gives force F_{23F} in terms of the equilibrant moment M_{o3} when mesh no. 2 is in the round on flat phase. Equation (H-178), derived for case no. 4 (RFR) of the three step-up gear train, relates this force F_{23F} to the input moment M_{in}.

Thus, one first sets equations (H-178) and (H-276) equal to each other and then solves the result for M_{o34} (case 4: FR)

$$M_{o34} = M_{in}\frac{C_{24}C_{36}}{C_{10}C_{22}} - Q_1\frac{C_9C_{24}C_{36}}{C_{10}C_{22}} - Q_2\frac{C_{23}C_{36}}{C_{22}} - Q_{3P}C_{35} \qquad \text{(H-278)}$$

APPENDIX I

COMPUTER MODELS FOR THREE AND TWO STEP-UP GEAR TRAINS WITH CLOCK TEETH OPERATING IN A SPIN ENVIRONMENT

The following appendix contains descriptions, listings and sample outputs of the computer models relating to step-up gear trains containing clock (ogival) gear teeth:

1. Program CLOCK 3: Point and cycle efficiencies for three pass clock (ogival) step-up gear train in spin environment.

2. Program CLOCK 4: Point and cycle efficiencies for two pass clock (ogival) step-up gear train in spin environment.

The relevant background, the input parameters, the manner of the computations and the form of the output of each program are discussed in detail. The program proper forms the last part of each section.

1. Program CLOCK 3: Point and Cycle Efficiencies for Three Pass Clock (Ogival) Step-Up Gear Train in Spin Environment

The kinematics of program CLOCK 3 is based on the work in Appendix G, while the moment input-output relationships are derived in section 1 of Appendix H. The nomenclature of the program is chosen to coincide as much as possible with the above appendices. It is to be noted that, even though the fuze related geometry produces different expressions for the various meshes, the kinematic computations of the individual meshes are very similar to those shown in CLOCK 1 in Appendix F for the single mesh in the standard position. It is also assumed that all three meshes will have been tested by program CLOCK 1 for their geometric suitability, i.e. whether there is enough room for tip radii.

a. Input Parameters (see Program CLOCK 3, below)

The following parameters represent the input data for the program (for explanations and nomenclature, see sections 1 and 2 of Appendix F, as well as section 3 of Appendix C):

MU, coefficient of friction, as before

RPM, spin velocity

CAPRP1, CAPRP2, CAPRP3, RP2, RP3, RP4, pitch radii of gears and pinions with nomenclature of Fig. G-1

RHOG1, RHOG2, RHOG3, RHOP1, RHOP2, RHOP3, radii of curvature of circular arc portion of gear and pinion teeth

ACG1, ACG2, ACG3* = a_{CG_i}, distance from the center of rotation of the gear of the i^{th} mesh to the center of curvature of the circular arc portion of the gear tooth. (Unless otherwise noted, this and all following numbering schemes refer to those associated with the mesh mechanics as given in the text of Appendices G and H.)

ACP1, ACP2, ACP3 = a_{CP_i}, distance from the center of rotation of the pinion of the i^{th} mesh to the center of curvature of the circular arc portion of the pinion tooth

R1, R2, R3, R4 = $\mathcal{R}_i$ (nomenclature of Fig. G-1)

TG1, TG2, TG3, TP1, TP2, TP3, maximum thickness of gear and pinion teeth (mesh nomenclature)

NG1, NG2, NG3, NP2, NP3, NP4, numbers of teeth in various gears and pinions (nomenclature of Fig. G-1)

RHO1, RHO2, RHO3, RHO4, gear and/or pinion pivot radii (nomenclature of Fig. G-1)

M1, M2, M3, M4, masses of gear and/or pinion combinations

MD, see program INVOL 3

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*Since many parts of the computer program were written before the nomenclature for these distances was changed in the report from a_{CG_i} and a_{CP_i} to a_{G_i} and a_{P_i}, there is a certain discrepancy between the program and the report.

K, range divisor

PHDOT1 = -1, all velocity computations are based on this value. The input motion in the fuze gearing model is negative (see Fig. (G-1)).

b. Computations (see also COMMENT cards in program)

I. Computation of Gear Tooth Parameters

The tooth parameters of the gears and pinions of all three meshes are first computed. These computations are essentially the same as those shown in program CLOCK 1 for a single mesh. Certain parameters are omitted because they have been checked separately by using CLOCK 1 and are not required for the kinematics of CLOCK 3.

In addition, the pivot to pivot distances B1, B2 and B3 are obtained.

II. Computation of MIN, GAMMAS and BETAS

To begin with, the program computes the input moment

$$MIN = M_{in} = md^2\omega^2 \qquad (I\text{-}1)$$

Subsequently, the angles γ_2, γ_3, γ_4 and β_1, β_2, β_3 are established according to the expressions of section 6b of Appendix A.

III. Computation of Other Parameters

The angles $\Delta\varphi_1$ and $\Delta\psi_1$ between the centerlines of adjacent gear and pinion teeth, respectively, are determined in this

section of the computations. In addition, the lengths L_1 are found (see eqs. (G-7), (G-53) and (G-88)). Finally, the centrifugal forces Q_1, Q_2, Q_3 and Q_4 are computed according to eqs. (H-65), (H-46), (H-25) and (H-6), respectively.

IV. Preliminary Computations for Mesh 1

A. Determination of Transition Angle

The primary consideration for determining the transition angles in the fuze related clock gear meshes is identical with that used in Appendix F. The transition angle ψ_T is established as that angle for which, depending upon whether the input angle φ has counterclockwise or clockwise motion, a small increase or decrease in φ, respectively, will cause the associated value of g to become smaller than its transition value f_p. Since the gear of mesh 1 turns in a clockwise direction, the above increment of φ will be negative.

The program uses this criterion in the following manner:

(a) Transition angles ψ_{1T1} and ψ_{1T2} are computed according to eq. (G-39).

(b) The subroutine TRANS1 (which is valid for meshes in which the input gear has clockwise rotation, as is the case also for mesh 3) is called, and the angle φ_{1T1} (PHIT), which is associated with ψ_{1T1}, is computed with the help of eqs. (G-40) and (G-41).

(c) The angle φ is made slightly smaller than φ_{1T1} to produce the angle PHINEXT, and eq. (G-29) is used to find the associated angle PSINEX. Since there are two such angles, the

subroutine selects the one which is closest in value to the transition angle ψ_{1T1}. Subsequently, the associated value of g_{11} is computed according to eq. (G-27).

(d) Steps (b) and (c) are then repeated identically for the second transition angle ψ_{1T2}. This results in the determination of g_{12}.

(e) Control returns to the main program, and that value of ψ_{1T} is chosen for which the associated value of g_1 is smaller than f_{p1}.

For checking, a subsidiary test, which is similar to the one shown in Appendix F, is added to the program. It is based on the idea that, for the correct transition angle ψ_{1T}, the line representing the flat portion of the pinion will make a smaller angle with the centerline O_1O_2 than will be the case for the incorrect ones. TEST11 and TEST12 find these angles with the help of the expressions shown below. These expressions hold for all values of β_1 and make use of a new variable ψ_{test}, which had to be introduced since the tests require that the transition angles be expressed in a range between -180° and $+180^\circ$. Thus,

For $0^\circ < \psi_{test} < 180^\circ$

$$TEST11 = |\pi - \beta_1 + \psi_{test} - \alpha_{P1}| \qquad (I\text{-}2)$$

For $-180^\circ < \psi_{test} < 0^\circ$

$$TEST12 = |\pi + \beta_1 - (\psi_{test} + 2\pi - \alpha_{P1})| \qquad (I\text{-}3)$$

To determine the angle ψ_{test}, let

$$\psi_{test} = \psi_{1T} \quad \text{if} \quad -180^{\circ} < \psi_{1T} < 180^{\circ} \tag{I-4}$$

$$\psi_{test} = \psi_{1T} + 2\pi \quad \text{if} \quad \psi_{1T} < -180^{\circ} \tag{I-5}$$

$$\psi_{test} = \psi_{1T} - 2\pi \quad \text{if} \quad \psi_{1T} > 180^{\circ} \tag{I-6}$$

B. Determination of Correct Sign for Round on Flat Regime

The sign preceding the square root in eq. (G-29), for the round on flat regime, is determined with the help of Φ_{1T}. The condition yielding that angle ψ_{1F} which is closest to the angle ψ_{1T} governs. The variable SIGN1F is used for the sign in question.

C. Computation of Final and Initial Values of Φ_1 and ψ_1

The final and initial values of the gear and pinion angles Φ_1 and ψ_1, respectively, are found by continuously evaluating the round on flat regime eq. (G-29), using the previously determined value of SIGN1F, and simultaneously checking the contact condition for the subsequent set of teeth as given by eq. (G-46). This loop is initiated at the transition angle Φ_{1T}, and it is terminated when the condition of eq. (G-46) is met. This allows the determination of the angles PHI1F and PSI1FF, at which the first set of teeth loses contact, as well as of the angles PHI1I and PSI1I at which the second set of teeth simultaneously comes into engagement. The initial engagement angles PHI1I and PSI1I are obtained by adding φ_1 to the "loss of contact" angle PHI1F

and by subtracting $\Delta\psi_1$ from the "loss of contact" angle PSI1FF.

D. Determination of Correct Sign for Round on Round Regime

Eq. (G-12) is used to determine the angle ψ_1, while the gear and pinion are in the round on round regime. The correct sign for this expression is obtained by comparing the value ψ_1, as computed with PHI1I, with the value for PSI1I. SIGN1R is the variable used for the desired sign.

V. Preliminary Computations for Mesh 2

A. Determination of Transition Angle

The primary criterion for determining the transition angle is again similar to that used in Appendix F and described earlier for mesh 1.

(a) Transition angles ψ_{2T1} and ψ_{2T2} are computed according to eq. (G-79).

(b) The subroutine TRANS2, which is valid for meshes in which the input gear has counterclockwise rotation, is called, and the angle φ_{2T1}, which is associated with ψ_{2T1} is computed, with the help of eqs. (G-80) and (G-81).

(c) The angle φ_2 is made slightly larger than φ_{2T1} to produce the angle PHINEXT, and eq. (G-71) is used to find the associated output angle PSINEX. Since there are two such angles, the subroutine selects the one which is closest to the transition value ψ_{2T1}. Subsequently, the associated value of g_{21} is computed according to eq. (G-69).

(d) Steps (b) and (c) are then repeated identically for the second transition angle ψ_{2T2}. This results in the determination of g_{22}.

(e) Control returns to the main program, and that value of ψ_{2T} is chosen for which the associated value of g_2 is smaller than f_{p2}.

The procedure for the associated subsidiary test for the transition angle is similar to that for mesh 1 and is given by

For $0^\circ < \psi_{test} < 180^\circ$

$$TEST21 = |\pi - \beta_2 + \psi_{test} + \alpha_{p2}| \qquad (I\text{-}7)$$

For $-180^\circ < \psi_{test} < 0^\circ$

$$TEST22 = |\beta_2 + \pi - (\psi_{test} + 2\pi + \alpha_{p2})| \qquad (I\text{-}8)$$

To determine the angle ψ_{test}, let

$$\psi_{test} = \psi_{2T} \quad \text{if} \quad -180^\circ < \psi_{2T} < 180^\circ \qquad (I\text{-}9)$$

$$\psi_{test} = \psi_{2T} + 2\pi \quad \text{if} \quad \psi_{2T} < -180^\circ \qquad (I\text{-}10)$$

$$\psi_{test} = \psi_{2T} - 2\pi \quad \text{if} \quad \psi_{2T} > 180^\circ \qquad (I\text{-}11)$$

B. Determination of Correct Sign for Round on Flat Regime

The sign preceding the square root in eq. (G-71), for the round on flat regime, is determined with the help of ψ_{2T}. The condition yielding that angle ψ_{2F} which is closest to the angle ψ_{2T} governs. The variable SIGN2F is used for the sign in question.

C. Computation of Final and Initial Values of Φ_2 and ψ_2

The final and initial values of the gear and pinion angles Φ_2 and ψ_2, respectively, are found by continuously evaluating the round on flat eq. (G-71), using the previously determined value of SIGN2F, and simultaneously checking the contact condition for the subsequent set of teeth, as given by eq. (G-86). This loop is initiated at the transition angle Φ_{2T} and is terminated when the condition of eq. (G-86) is met. (Recall that in meshes 1 and 3 the driving gear turns clockwise, while in mesh 2 it turns in a counterclockwise direction.) This allows the determination of the two angles PHI2F and PSI2FF at which the first set of teeth loses contact as well as of the angles PHI2I and PSI2I at which the second set of teeth simultaneously comes into contact. The initial engagement angles PHI2I and PSI2I are obtained by subtracting $\Delta\Phi_2$ from the "loss of contact" angle PHI2F and by adding $\Delta\psi_2$ to the "loss of contact" angle PSI2FF.

D. Determination of Correct Sign for Round on Round Regime

Eq. (G-58) is used to determine the angle ψ_2 while the gear and pinion are in the round on round phase of motion. The correct sign for this expression is obtained by comparing the value of ψ_2, as computed with PHI2I, with the previously obtained value for PSI2I. SIGN2R is the variable used for the desired sign.

VI. Preliminary Computations for Mesh 3

A. Determination of Transition Angle

The determination of the transition angles for mesh 3 runs along parallel lines to the one shown for mesh 1 since the driving gear also rotates in a clockwise direction. In all cases, the parameters of section 3 of Appendix G are used.

(a) Transition angles ψ_{3T1} and ψ_{3T2} are computed with the help of eq. (G-99).

(b) The subroutine TRANS1 determines the angle φ_{3T1}, associated with ψ_{3T1}, according to eqs. (G-100) and (G-101).

(c) PHINEXT, which is now obtained by a decrease of the angle φ_3 from φ_{3T1}, serves as the input variable of eq. (G-94), and is used to determine PSINEX. Appropriate controls, as described before, determine the angle ψ_{3T1}. In addition, the associated value of g_{31} is computed with the help of eq. (G-95).

(d) Steps (b) and (c) are again repeated for the second transition angle ψ_{3T2} and g_{22} is determined.

(e) After control is returned to the main program, that value of ψ_{3T} is chosen for which the associated value of g_3 is smaller than f_{p3}.

The subsidiary test for the transition angles runs parallel to that described for mesh 1, i.e.,

For $0^\circ < \psi_{test} < 180^\circ$

$$TEST31 = |\pi - \beta_3 + \psi_{test} - \alpha_{p3}| \qquad (I\text{-}12)$$

For $-180^{\circ} < \psi_{test} < 0^{\circ}$

$$TEST32 = |\pi + \beta_3 - (\psi_{test} + 2\pi - \alpha_{P3})| \qquad (I\text{-}13)$$

To determine the angle ψ_{test}, let

$$\psi_{test} = \psi_{3T} \quad \text{if} \quad -180^{\circ} < \psi_{3T} < 180^{\circ} \qquad (I\text{-}14)$$

$$\psi_{test} = \psi_{3T} + 2\pi \quad \text{if} \quad \psi_{3T} < -180^{\circ} \qquad (I\text{-}15)$$

$$\psi_{test} = \psi_{3T} - 2\pi \quad \text{if} \quad \psi_{3T} > 180^{\circ} \qquad (I\text{-}16)$$

B. Determination of Correct Sign for Round on Flat Regime

The sign preceding the square root in eq. (G-94), for the round on flat regime, is determined with the help of the angle φ_{3T}. The condition yielding that angle ψ_{3F} which is closest to the angle ψ_{3T} will govern. The variable SIGN3F is used for the sign in question.

C. Computation of Final and Initial Values of φ_3 and ψ_3

The final and initial values of the gear and pinion angles φ_3 and ψ_3, respectively, are found by continuously evaluating the round on flat regime eq. (G-94), using the previously determined value of SIGN3F, and simultaneously checking the contact condition for the subsequent set of teeth, as given by eq. (G-102). This loop is initiated at the transition angle φ_{3T}, and it is terminated when the condition of eq. (G-102) is met. This allows the determination of the two angles PHI3F and PSI3FF at which the first set of teeth loses contact as well

as the angles PHI3I and PSI3I at which the second set of teeth simultaneously comes into contact. The initial engagement angles PHI3I and PSI3I are obtained by adding $\Delta\varphi_3$ to the "loss of contact" angle PHI3F and by subtracting $\Delta\psi_3$ from the "loss of contact" angle PSI3FF.

D. Determination of Correct Sign for Round on Round Regime

Eq. (G-87) is used to determine the angle ψ_3 while the gear and pinion are in the round on round phase of motion. The correct sign for this expression is obtained by comparing the value of ψ_3, as computed with PHI3I, with the previously obtained value for PSI3I. SIGN3R is the variable used for the desired sign.

VII. Gear Train Motion Model: Kinematics, Point and Cycle Efficiency

The simulation of the gear train model, which is necessary for the computation of both POINTEF and CYCLEFF, is found in a loop, starting with statement label no. 29 (card no. 458) and ending with card no. 812. The motions of the individual driving gears are initialized at their respective angles PHI1I, PHI2I and PHI3I. (This again is arbitrary.) The meshes will be in round on round contact until they reach their respective transition angles PHI1T, PHI2T and PHI3T. Once the transition angles are passed, the meshes will be in round on flat contact. These regimes continue until the final angles PHI1F, PHI2F and PHI3F are reached.

The increment DDPHI1 of the angle PHI1 of the input gear 1

is obtained from an adaptation of eqs. (A-211) and (A-213), in which tooth numbers, rather than base circle radii, are used. The increment DDPHI2 of gear 2 is related to the increment of the pinion angle PSI1. Similarly, the increment DDPHI3 is obtained with the help of the pinion angle PSI2.

While the motion of gear 1 is terminated when the angle PHI1 reaches the angle PHI1F (or rather PHI1F + DDPHI1 for moment summation purposes), both gears 2 and 3 must be reset to their respective starting angles whenever their final angles PHI2F and PHI3F are reached.

The appropriate choice of moment equation depends upon which of the eight possible combinations of contact conditions, as indicated by Table H-1, is applicable.

The following discusses the kinematics of the individual meshes as well as the determination of the point and cycle efficiencies in greater detail.

A. Kinematics

(1) Mesh 1

Depending on whether PHI1 is larger or smaller than PHI1T, the parameters of the round on round or the round on flat regime are computed. (Recall that gear 1 turns in a clockwise direction.)

For the round on round phase, the following calculations are made:

ψ_1, according to eq. (G-11), and with the help of the previously determined SIGN1R

λ_1, according to eqs. (G-13) and (G-14)

$\dot{\phi}_1$, according to eq. (G-15)

$V_{S1/T1_R}$, according to eq. (G-20)

s_{1R}, according to eq. (H-1) as adapted to mesh 1

For the round on flat phase, the following calculations are made:

ψ_1, according to eq. (G-29), and with the help of the previously determined SIGN1F

g_1, according to eq. (G-27)

$\dot{\phi}_1$, according to eq. (G-30)

$V_{S1/T1_F}$, according to eq. (G-33)

s_{1F}, according to eq. (H-2) as adapted to mesh 1

(2) Mesh 2

The increment DDPHI2 for each round of computation is obtained with the help of the change in the angle ψ_1 between the present and the previous computation, i.e., as shown at statement label no. 31

$$\text{DDPHI2} = \text{PSI1} - \text{PSI1P} \qquad \text{(I-17)}$$

For the first round of computations, the "previous" ψ_1, i.e., PSI1P, is equal to PSI1I.

It must be recalled that gear 2 rotates in a positive direction, and therefore, the angle ϕ_2 increases with continued

motion. The angle PHI2 is re-indexed to PHI2I once it becomes larger than PHI2F.

As for mesh 1, comparison with the transition angle decides whether the mesh is in the round on round or in the round on flat regime.

The following round on round parameters are calculated:

ψ_2, according to eq. (G-58), and with the help of the previously determined SIGN2R

λ_2, according to eqs. (G-59) and (G-60)

Note that the "input angular velocity" for mesh 2, i.e., $\dot{\varphi}_2$, equals the momentary value of $\dot{\psi}_1$.

$\dot{\psi}_2$, according to eq. (G-61)

$V_{S2/T2_R}$, according to eq. (G-63)

s_{2R}, according to eq. (H-1) as adapted to mesh 2

For the round on flat phase, the following calculations are made:

ψ_2, according to eq. (G-71), and with the help of the previously determined SIGN2F

ε_2, according to eq. (G-69)

Again, $\dot{\varphi}_2$ equals the momentary value of $\dot{\psi}_1$

$\dot{\psi}_2$, according to eq. (G-72)

$V_{S2/T2_F}$, according to eq. (G-74)

s_{2F}, according to eq. (H-2) as adapted to mesh 2

(3) Mesh 3

The increment DDPHI3, for each round of computation, is obtained with the help of the change in the angle ψ_2 between the present and the previous computation, i.e., as shown at statement label no. 33:

$$\text{DDPHI3} = \text{PSI2} - \text{PSI2P} \qquad \text{(I-18)}$$

For the first round of computations, the "previous" ψ_2, i.e., PSI2P, is equal to PSI2I.

Gear 3 rotates in a negative (clockwise) direction, and therefore, the angle φ_3 decreases with continued rotation. The angle PHI3, which represents this angle, is re-indexed to PHI3I once it becomes smaller than PHI3F.

As for meshes 1 and 2, comparison with the applicable transition angle decides whether the mesh is in the round on round or in the round on flat regime.

The following round on round parameters are calculated:

ψ_3, according to eq. (G-87), and with the help of the previously determined SIGN3R

λ_3, according to eqs. (G-89) and (G-90)

Note that the "input angular velocity" for mesh 3, i.e., $\dot{\varphi}_3$, equals the momentary value of $\dot{\psi}_2$.

$\dot{\psi}_3$, according to eq. (G-91)

$V_{S3/T3_R}$, according to eq. (G-92)

s_{3R}, according to eq. (H-1) as adapted to mesh 3

For the round on flat phase, the following calculations are made:

ψ_3, according to eq. (G-94), and with the help of the previously determined SIGN3F

s_3, according to eq. (G-95)

Again, $\dot{\varphi}_3$ is equal to the momentary value of $\dot{\psi}_2$.

$\dot{\psi}_3$, according to eq. (G-96)

$V_{S3/T3F}$, according to eq. (G-97)

s_{3F}, according to eq. (H-2) as adapted to mesh 3

B. Moment Computations, Point and Cycle Efficiencies

Regardless of the combination of contact conditions, the point efficiency is computed according to eq. (3), i.e.,

$$\epsilon_p = \text{POINTEF} = K_{ratio} \frac{M_{o4i}}{M_{in}} \qquad \text{(I-19)}$$

where, with $\dot{\varphi}_1 = -1$,

$$K_{ratio} = |\dot{\psi}_3| \qquad \text{(I-20)}$$

The cycle efficiency determination is based on eq. (C-10) in Appendix C, which represents an adaptation of eq. (4):

$$\epsilon_c = \frac{\Delta\varphi_1 \, \Sigma \epsilon_p}{\varphi_{1FIN} - \varphi_{1IN}} \qquad \text{(I-21)}$$

(See page C-18.) The associated expression in the program, at statement label no. 45 becomes

CYCLEFF = -MTOT*DDPHI1/(PHI1F-PHI1I) (I-22)

where

MTOT = MTOT + POINTEF (I-23)

The moment computations begin with the statement label no. 35 , and initially consist of the determination of the variables A1 to A64 and C1 to C32 of section 1 of Appendix H. The governing contact combination (see also Table H-1) is determined with the help of the 8 moment control statements, which start with card no. 737 . Once the appropriate combination is established, the program is directed to one of the 8 associated moment expressions. These expressions for M_{o4i} coincide in nomenclature with those given by eqs. (H-81), (H-118), (H-158), (H-180), (H-216), (H-218), (H-239) and (H-241). They are listed in the above order, beginning with statement label no. 36 and ending with statement label no. 43 .

In devising the control statements, the manner of rotation of the individual mesh input gears had to be taken into account. Thus:

For mesh 3:

Round on round (R) corresponds to

PHI3I > PHI3 > PHI3T

Round on flat (F) corresponds to

PHI3T > PHI3 > PHI3F

For mesh 2:

Round on round (R) corresponds to

PHI2I < PHI2 < PHI2T

Round on flat (F) corresponds to

PHI2T < PHI2 < PHI2F

For mesh 1:

Round on round (R) corresponds to

PHI1I > PHI1 > PHI1T

Round on flat (F) corresponds to

PHI1T > PHI1 > PHI1F

c. Output (see Program CLOCK 3, below)

The output of the program is best explained with the help of the sample problem at the end of the program.

I. Input Parameters

Mesh 1

CAPRP1 = R_{P1} = .47725 in. (1.212 cm)

RP2 = r_{P2} = .09085 in. (0.231 cm)

ACG1 = a_{G1} = .47725 in. (1.212 cm)

ACP1 = a_{P1} = .09085 in. (0.231 cm)

RHOG1 = ρ_{G1} = .03870 in. (0.098 cm)

RHOP1 = ρ_{P1} = .01740 in. (0.044 cm)

TG1 = t_{G1} = .03480 in. (0.088 cm)

TP1 = t_{P1} = .02800 in. (0.071 cm)

NG1 = n_{G1} = 42

NP2 = n_{P2} = 8

Mesh 2

CAPRP2 = R_{P2} = .20670 in. (0.525 cm)

RP3 = r_{P3} = .06890 in. (0.175 cm)

ACG2 = a_{G2} = .20670 in. (0.525 cm)

ACP2 = a_{P2} = .06890 in. (0.175 cm)

RHOG2 = ρ_{G2} = .02070 in. (0.053 cm)

RHOP2 = ρ_{P2} = .01040 in. (0.026 cm)

TG2 = t_{G2} = .02520 in. (0.064 cm)

TP2 = t_{P2} = .02080 in. (0.053 cm)

NG2 = n_{G2} = 27

NP3 = n_{P3} = 9

Mesh 3

CAPRP3 = R_{P3} = .17560 in. (0.446 cm)

RP4 = r_{P4} = .05905 in. (0.150 cm)

ACG3 = a_{G3} = .17560 in. (0.446 cm)

ACP3 = a_{P3} = .05905 in. (0.150 cm)

RHOG3 = ρ_{G3} = .01910 in. (0.049 cm)

RHOP3 = ρ_{P3} = .00875 in. (0.022 cm)

TG3 = t_{G3} = .02170 in. (0.055 cm)

TP3 = t_{P3} = .01750 in. (0.044 cm)

NG3 = n_{G3} = 27

NP4 = n_{P4} = 9

In addition

MU = μ = .2

RPM = 1000

M1 = m_1 = $.69515 \times 10^{-4}$ lb-sec^2/in. (12.171 g)

M2 = m_2 = .97028 x 10^{-5} lb-sec^2/in. (1.699 g)

M3 = m_3 = .70027 x 10^{-5} lb-sec^2/in. (1.226 g)

M4 = m_4 = .79188 x 10^{-6} lb-sec^2/in. (0.139 g)

R1 = $\mathcal{R}_1$ = .750 in. (1.905 cm)

R2 = $\mathcal{R}_2$ = .750 in. (1.905 cm)

R3 = $\mathcal{R}_3$ = .750 in. (1.905 cm)

R4 = $\mathcal{R}_4$ = .750 in. (1.905 cm)

RHO1 = ρ_1 = .060 in.(0.152 cm)

RHO2 = ρ_2 = .030 in.(0.076 cm)

RHO3 = ρ_3 = .025 in.(0.064 cm)

RHO4 = ρ_4 = .020 in.(0.051 cm)

MD = md^2 = .15 x 10^{-4} lb-sec^2 in. (16.944 g - cm^2)

K = 25

II. Computed Values

At the beginning of the output, one finds MIN = M_{in}. Subsequently, the following are listed for each mesh:

f_{Pi}, the length of the pinion flats

β_i, the fuze body pivot to pivot line angles

ψ_{Ti} and φ_{Ti}, the transition angles as well as the associated subsidiary tests

φ_{INi} and ψ_{INi}, the initial angles

φ_{FINi} and ψ_{FINi}, the final angles

Finally, for the full range of the input angle φ_1, the point efficiency POINTEF is listed, in addition to other parameters which are useful for checking purposes. Note that DPSI1, DPSI2

and DPSI3 represent $\dot{\psi}_1$, $\dot{\psi}_2$ and $\dot{\psi}_3$, respectively. The cycle efficiency CYCLEFF is found at the end of the output.

Program CLOCK 3

```
      1                PROGRAM CLOCK3(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)                  A   1
                  C                                                                        A   2
                  C     POINT AND CYCLE EFFICIENCIES FOR THREE PASS CLOCK (OGIVAL) STEP UP  A   3
                  C     GEAR TRAIN IN SPIN ENVIRONMENT                                     A   4
      5           C                                                                        A   5
                       REAL MU,LAMDA1,LAMDA2,LAMDA3,MO4,MO41,MO42,MO43,MO44,MO45,MO46,MO4  A   6
                      17,MO48,LX1,LY1,L1,LX2,LY2,L2,LX3,LY3,L3,NP2,NP3,NP4,NG1,NG2,NG3,MI  A   7
                      2N,M1,M2,M3,M4,MTOT,MD,LL1,LL2,LL3,K                                A   8
                     1 READ (5,61) MU,RPM,CAPRP1,CAPRP2,CAPRP3,RP2,RP3,RP4,ACG1,ACG2,ACG3  A   9
      10              1,ACP1,ACP2,ACP3,ISTOP                                               A  10
                       READ (5,82) R1,R2,R3,R4                                             A  11
                       READ (5,62) RHOG1,RHOG2,RHOG3,RHOP1,RHOP2,RHOP3                     A  12
                       READ (5,63) TG1,TG2,TG3,TP1,TP2,TP3                                 A  13
                       READ (5,64) NG1,NG2,NG3,NP2,NP3,NP4                                 A  14
      15               READ (5,85) M1,M2,M3,M4                                             A  15
                       READ (5,65) RHO1,RHO2,RHO3,RHO4,MD,K                                A  16
                       PI=3.14159                                                          A  17
                       Z=PI/180.                                                           A  18
                       OMEGA=RPM*2.*PI/60.                                                 A  19
      20               OM2=OMEGA*OMEGA                                                     A  20
                       PHDOT1=-1.                                                          A  21
                  C                                                                        A  22
                  C     COMPUTATION OF GEAR TOOTH PARAMETERS                               A  23
                  C                                                                        A  24
      25               CXG1=RHOG1-TG1/2.                                                   A  25
                       DELG1=ASIN(CXG1/CAPRP1)                                             A  26
                       CXP1=RHOP1-TP1/2.                                                   A  27
                       DELP1=ASIN(CXP1/RP2)                                                A  28
                       GAMMP1=ASIN(RHOP1/RP2)                                              A  29
      30               ALPHP1=GAMMP1-DELP1                                                 A  30
                       FP1=ACP1*COS(GAMMP1)                                                A  31
                       B1=CAPRP1+RP2                                                       A  32
                       CXG2=RHOG2-TG2/2.                                                   A  33
                       DELG2=ASIN(CXG2/CAPRP2)                                             A  34
      35               CXP2=RHOP2-TP2/2.                                                   A  35
                       DELP2=ASIN(CXP2/RP3)                                                A  36
                       GAMMP2=ASIN(RHOP2/RP3)                                              A  37
                       ALPHP2=GAMMP2-DELP2                                                 A  38
                       FP2=ACP2*COS(GAMMP2)                                                A  39
      40               B2=CAPRP2+RP3                                                       A  40
                       CXG3=RHOG3-TG3/2.                                                   A  41
                       DELG3=ASIN(CXG3/CAPRP3)                                             A  42
                       CXP3=RHOP3-TP3/2.                                                   A  43
                       DELP3=ASIN(CXP3/RP4)                                                A  44
      45               GAMMP3=ASIN(RHOP3/RP4)                                              A  45
                       ALPHP3=GAMMP3-DELP3                                                 A  46
                       FP3=ACP3*COS(GAMMP3)                                                A  47
                       B3=CAPRP3+RP4                                                       A  48
                  C                                                                        A  49
      50          C     COMPUTATION OF MIN, GAMMAS AND BETAS                               A  50
                  C                                                                        A  51
                       MIN=MD*OM2                                                          A  52
                       DELTA2=ACOS(((CAPRP1+RP2)*(CAPRP1+RP2)+R1*R1-R2*R2)/(2.*R1*(CAPRP1  A  53
```

```
                1+RP2)))                                                          A  54
     55          DELTA3=ACOS(((CAPRP2+RP3)*(CAPRP2+RP3)+R2*R2-R3*R3)/(2.*R2*(CAPRP2  A  55
                1+RP3)))                                                          A  56
                 DELTA4=ACOS(((CAPRP3+RP4)*(CAPRP3+RP4)+R3*R3-R4*R4)/(2.*R3*(CAPRP3  A  57
                1+RP4)))                                                          A  58
                 GAMMA2=ACOS((R1*R1+R2*R2-(CAPRP1+RP2)*(CAPRP1+RP2))/(2.*R1*R2))  A  59
     60          GAMMA3P=ACOS((R2*R2+R3*R3-(CAPRP2+RP3)*(CAPRP2+RP3))/(2.*R2*R3))  A  60
                 GAMMA3=GAMMA2+GAMMA3P                                            A  61
                 GAMMA4P=ACOS((R3*R3+R4*R4-(CAPRP3+RP4)*(CAPRP3+RP4))/(2.*R3*R4))  A  62
                 GAMMA4=GAMMA3+GAMMA4P                                            A  63
                 BETA1=PI-DELTA2                                                  A  64
     65          BETA2=GAMMA2+PI-DELTA3                                           A  65
                 BETA3=GAMMA3+PI-DELTA4                                           A  66
                 BETA1D=BETA1/Z                                                   A  67
                 BETA2D=BETA2/Z                                                   A  68
                 BETA3D=BETA3/Z                                                   A  69
     70          WRITE (6,66) MIN,MU,RPM,CAPRP1,CAPRP2,CAPRP3,RP2,RP3,RP4,ACG1,ACG2  A  70
                1,ACG3,ACP1,ACP2,ACP3                                             A  71
                 WRITE (6,83) R1,R2,R3,R4                                         A  72
                 WRITE (6,67) RHOG1,RHOG2,RHOG3,RHOP1,RHOP2,RHOP3                 A  73
                 WRITE (6,68) TG1,TG2,TG3,TP1,TP2,TP3                             A  74
     75          WRITE (6,69) NG1,NG2,NG3,NP2,NP3,NP4                             A  75
                 WRITE (6,86) M1,M2,M3,M4                                         A  76
                 WRITE (6,70) RHO1,RHO2,RHO3,RHO4,MD,K,PHDOT1                     A  77
                 WRITE (6,84) FP1,FP2,FP3                                         A  78
                 WRITE (6,71) BETA1D,BETA2D,BETA3D                                A  79
     80        C                                                                  A  80
               C     COMPUTATION OF OTHER PARAMETERS                              A  81
               C                                                                  A  82
                 DPHI1=360./NG1*Z                                                 A  83
                 DPSI1=360./NP2*Z                                                 A  84
     85          DPHI2=360./NG2*Z                                                 A  85
                 DPSI2=360./NP3*Z                                                 A  86
                 DPHI3=360./NG3*Z                                                 A  87
                 DPSI3=360./NP4*Z                                                 A  88
                 L1=RHOG1+RHOP1                                                   A  89
     90          L2=RHOG2+RHOP2                                                   A  90
                 L3=RHOG3+RHOP3                                                   A  91
                 Q1=M1*R1*OM2                                                     A  92
                 Q2=M2*R2*OM2                                                     A  93
                 Q3=M3*R3*OM2                                                     A  94
     95          Q4=M4*R4*OM2                                                     A  95
               C                                                                  A  96
               C     PRELIMINARY COMPUTATIONS FOR MESH 1                          A  97
               C                                                                  A  98
               C     DETERMINATION OF TRANSITION ANGLE OF MESH 1                  A  99
    100        C                                                                  A 100
                 A1T=RHOG1*COS(BETA1+ALPHP1)+FP1*SIN(BETA1+ALPHP1)                A 101
                 B1T=-RHOG1*SIN(BETA1+ALPHP1)+FP1*COS(BETA1+ALPHP1)               A 102
                 C1T=(ACG1*ACG1-RHOG1*RHOG1-B1*B1-FP1*FP1)/(2.*B1)                A 103
                 ROOT1T=A1T*A1T+B1T*B1T-C1T*C1T                                   A 104
    105          Y1T1=A1T+SQRT(ROOT1T)                                            A 105
                 Y1T2=A1T-SQRT(ROOT1T)                                            A 106
```

```
                X1T=B1T+C1T                                                    A 107
                PSI1T1=2.*ATAN2(Y1T1,X1T)                                      A 108
                PSI1T2=2.*ATAN2(Y1T2,X1T)                                      A 109
  110           PSITE11=PSI1T1                                                 A 110
                PSITE12=PSI1T2                                                 A 111
                IF (PSI1T1.GT.PI) PSITE11=PSI1T1-2.*PI                         A 112
                IF (PSI1T1.LT.PI) PSITE11=PSI1T1+2.*PI                         A 113
                IF (PSI1T2.GT.PI) PSITE12=PSI1T2-2.*PI                         A 114
  115           IF (PSI1T2.LT.PI) PSITE12=PSI1T2+2.*PI                         A 115
                IF (PSITE11.GE.3.) TEST11=ABS(PI-BETA1+PSITE11-ALPHP1)/Z       A 116
                IF (PSITE12.GE.0.) TEST12=ABS(PI-BETA1+PSITE12-ALPHP1)/Z       A 117
                IF (PSITE11.LT.0.) TEST11=ABS(PI+BETA1-(PSITE11+2.*PI-ALPHP1))/Z A 118
                IF (PSITE12.LT.0.) TEST12=ABS(PI+BETA1-(PSITE12+2.*PI-ALPHP1))/Z A 119
  120           IF (PSI1T1.LT.0.) PSI1T1=PSI1T1+2.*PI                          A 120
                IF (PSI1T2.LT.0.) PSI1T2=PSI1T2+2.*PI                          A 121
                PSI1T1D=PSI1T1/Z                                               A 122
                PSI1T2D=PSI1T2/Z                                               A 123
                WRITE (6,46) PSI1T1D,TEST11                                    A 124
  125           WRITE (6,47) PSI1T2D,TEST12                                    A 125
                CALL TRANS1 (RHOG1,ALPHP1,BETA1,FP1,ACG1,B1,DELG1,Z,PSI1T1,PHI1T1, A 126
               1G11)                                                           A 127
                IF (G11.GT.FP1) GO TO 2                                        A 128
                PHI1T=PHI1T1                                                   A 129
  130           PSI1T=PSI1T1                                                   A 130
                GO TO 4                                                        A 131
              2 CALL TRANS1 (RHOG1,ALPHP1,BETA1,FP1,ACG1,B1,DELG1,Z,PSI1T2,PHI1T2, A 132
               1G12)                                                           A 133
                IF (G12.LT.FP1) GO TO 3                                        A 134
  135           WRITE (6,72)                                                   A 135
                STOP                                                           A 136
              3 PHI1T=PHI1T2                                                   A 137
                PSI1T=PSI1T2                                                   A 138
              4 IF (PHI1T.LT.0.) PHI1T=PHI1T+2.*PI                             A 139
  140           IF (PSI1T.LT.0.) PSI1T=PSI1T+2.*PI                             A 140
                PHI1TD=PHI1T/Z                                                 A 141
                PSI1TD=PSI1T/Z                                                 A 142
                WRITE (6,73) PHI1TD,PSI1TD                                     A 143
          C                                                                    A 144
  145     C     DETERMINATION OF CORRECT SIGN FOR ROUND ON FLAT REGIME OF MESH 1 A 145
          C                                                                    A 146
                A1F=ACG1*COS(PHI1T+DELG1+ALPHP1)-B1*COS(BETA1+ALPHP1)          A 147
                B1F=-ACG1*SIN(PHI1T+DELG1+ALPHP1)+B1*SIN(BETA1+ALPHP1)         A 148
                C1F=RHOG1                                                      A 149
  150           ROOT1F=A1F*A1F+B1F*B1F-C1F*C1F                                 A 150
                Y1F1=A1F+SQRT(ROOT1F)                                          A 151
                Y1F2=A1F-SQRT(ROOT1F)                                          A 152
                X1F=B1F+C1F                                                    A 153
                PSI1F1=2.*ATAN2(Y1F1,X1F)                                      A 154
  155           PSI1F2=2.*ATAN2(Y1F2,X1F)                                      A 155
                IF (PSI1F1.LT.0.) PSI1F1=PSI1F1+2.*PI                          A 156
                IF (PSI1F2.LT.0.) PSI1F2=PSI1F2+2.*PI                          A 157
                IF (ABS(PSI1F1-PSI1T).LT.ABS(PSI1F2-PSI1T)) GO TO 5            A 158
                SIGN1F=-1.                                                     A 159
```

```
                  GO TO 6                                                          A 160
                5 SIGN1F=1.                                                        A 161
C                                                                                  A 162
C     COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI FOR MESH 1            A 163
C                                                                                  A 164
                6 DO 7 I=1,2000                                                    A 165
                  PHID1=PHI1TD-(I-1.)/100.                                         A 166
                  PHI1=PHID1*Z                                                     A 167
                  A1F=ACG1*COS(PHI1+DELG1+ALPHP1)-B1*COS(BETA1+ALPHP1)             A 168
                  B1F=-ACG1*SIN(PHI1+DELG1+ALPHP1)+B1*SIN(BETA1+ALPHP1)            A 169
                  C1F=RHOG1                                                        A 170
                  ROOT1F=A1F*A1F+B1F*B1F-C1F*C1F                                   A 171
                  Y1F=A1F+SIGN1F*SQRT(ROOT1F)                                      A 172
                  X1F=B1F+C1F                                                      A 173
                  PSI1F=2.*ATAN2(Y1F,X1F)                                          A 174
                  IF (PSI1F.LT.0.) PSI1F=PSI1F+2.*PI                               A 175
                  LX1=B1*COS(BETA1)+ACP1*COS(PSI1F-DPSI1+DELP1)-ACG1*COS(PHI1+DPHI1+ A 176
                 1DELG1)                                                           A 177
                  LY1=B1*SIN(BETA1)+ACP1*SIN(PSI1F-DPSI1+DELP1)-ACG1*SIN(PHI1+DPHI1+ A 178
                 1DELG1)                                                           A 179
                  LL1=SQRT(LX1*LX1+LY1*LY1)                                        A 180
                  DELEL1=LL1-L1                                                    A 181
                  IF (DELEL1.LE.0.) GO TO 8                                        A 182
                7 CONTINUE                                                         A 183
                8 PHI1F=PHI1                                                       A 184
                  PSI1FF=PSI1F                                                     A 185
                  PHI1I=PHI1F+DPHI1                                                A 186
                  PSI1I=PSI1FF-DPSI1                                               A 187
                  IF (PSI1I.LT.0.) PSI1I=PSI1I+2.*PI                               A 188
                  PHI1ID=PHI1I/Z                                                   A 189
                  PSI1ID=PSI1I/Z                                                   A 190
                  PHI1FD=PHI1F/Z                                                   A 191
                  PSI1FD=PSI1FF/Z                                                  A 192
                  WRITE (6,74) PHI1ID,PSI1ID,PHI1FD,PSI1FD                         A 193
C                                                                                  A 194
C     DETERMINATION OF CORRECT SIGN FOR ROUND ON ROUND REGIME OF MESH 1            A 195
C                                                                                  A 196
                  A1R=ACG1*SIN(PHI1I+DELG1-DELP1)-B1*SIN(BETA1-DELP1)              A 197
                  B1R=ACG1*COS(PHI1I+DELG1-DELP1)-B1*COS(BETA1-DELP1)              A 198
                  C1R=(ACP1*ACP1+ACG1*ACG1+B1*B1-L1*L1-2.*ACG1*B1*COS(PHI1I+DELG1-BE A 199
                 1TA1))/(2.*ACP1)                                                  A 200
                  ROOT1R=A1R*A1R+B1R*B1R-C1R*C1R                                   A 201
                  Y1R1=A1R+SQRT(ROOT1R)                                            A 202
                  Y1R2=A1R-SQRT(ROOT1R)                                            A 203
                  X1R=B1R+C1R                                                      A 204
                  PSI1R1=2.*ATAN2(Y1R1,X1R)                                        A 205
                  PSI1R2=2.*ATAN2(Y1R2,X1R)                                        A 205
                  IF (PSI1R1.LT.0.) PSI1R1=PSI1R1+2.*PI                            A 207
                  IF (PSI1R2.LT.0.) PSI1R2=PSI1R2+2.*PI                            A 208
                  IF (ABS(PSI1I-PSI1R1).LT.ABS(PSI1I-PSI1R2)) GO TO 9              A 209
                  SIGN1R=-1.                                                       A 210
                  GO TO 10                                                         A 211
                9 SIGN1R=1.                                                        A 212
```

```
      C                                                                 A 213
      C     PRELIMINARY COMPUTATIONS FOR MESH 2                         A 214
      C                                                                 A 215
      C     DETERMINATION OF TRANSITION ANGLE OF MESH 2                 A 216
      C                                                                 A 217
   10 A2T=-RHOG2*COS(BETA2-ALPHP2)+FP2*SIN(BETA2-ALPHP2)                A 218
      B2T=RHOG2*SIN(BETA2-ALPHP2)+FP2*COS(BETA2-ALPHP2)                 A 219
      C2T=(ACG2*ACG2-RHOG2*RHOG2-B2*B2-FP2*FP2)/(2.*B2)                 A 220
      ROOT2T=A2T*A2T+B2T*B2T-C2T*C2T                                    A 221
      Y2T1=A2T+SQRT(ROOT2T)                                             A 222
      Y2T2=A2T-SQRT(ROOT2T)                                             A 223
      X2T=B2T+C2T                                                       A 224
      PSI2T1=2.*ATAN2(Y2T1,X2T)                                         A 225
      PSI2T2=2.*ATAN2(Y2T2,X2T)                                         A 226
      PSITE21=PSI2T1                                                    A 227
      PSITE22=PSI2T2                                                    A 228
      IF (PSI2T1.GT.PI) PSITE21=PSI2T1-2.*PI                            A 229
      IF (PSI2T1.LT.PI) PSITE21=PSI2T1+2.*PI                            A 230
      IF (PSI2T2.GT.PI) PSITE22=PSI2T2-2.*PI                            A 231
      IF (PSI2T2.LT.PI) PSITE22=PSI2T2+2.*PI                            A 232
      IF (PSITE21.GE.0.) TEST21=ABS(PI-BETA2+PSITE21+ALPHP2)/Z          A 233
      IF (PSITE21.LT.0.) TEST21=ABS(BETA2+PI-(PSITE21+2.*PI+ALPHP2))/Z  A 234
      IF (PSITE22.GE.0.) TEST22=ABS(PI-BETA2+PSITE22+ALPHP2)/Z          A 235
      IF (PSITE22.LT.0.) TEST22=ABS(BETA2+PI-(PSITE22+2.*PI+ALPHP2))/Z  A 236
      IF (PSI2T1.LT.0.) PSI2T1=PSI2T1+2.*PI                             A 237
      IF (PSI2T2.LT.0.) PSI2T2=PSI2T2+2.*PI                             A 238
      PSI2T1D=PSI2T1/Z                                                  A 239
      PSI2T2D=PSI2T2/Z                                                  A 240
      WRITE (6,48) PSI2T1D,TEST21                                       A 241
      WRITE (6,49) PSI2T2D,TEST22                                       A 242
      CALL TRANS2 (RHOG2,ALPHP2,BETA2,FP2,ACG2,B2,DELG2,Z,PSI2T1,PHI2T1,A 243
     1G21)                                                              A 244
      IF (G21.GT.FP2) GO TO 11                                          A 245
      PHI2T=PHI2T1                                                      A 246
      PSI2T=PSI2T1                                                      A 247
      GO TO 13                                                          A 248
   11 CALL TRANS2 (RHOG2,ALPHP2,BETA2,FP2,ACG2,B2,DELG2,Z,PSI2T2,PHI2T2,A 249
     1G22)                                                              A 250
      IF (G22.LT.FP2) GO TO 12                                          A 251
      WRITE (6,75)                                                      A 252
      STOP                                                              A 253
   12 PHI2T=PHI2T2                                                      A 254
      PSI2T=PSI2T2                                                      A 255
   13 IF (PHI2T.LT.0.) PHI2T=PHI2T+2.*PI                                A 256
      IF (PSI2T.LT.0.) PSI2T=PSI2T+2.*PI                                A 257
      PHI2TD=PHI2T/Z                                                    A 258
      PSI2TD=PSI2T/Z                                                    A 259
      WRITE (6,76) PHI2TD,PSI2TD                                        A 260
      C                                                                 A 261
      C     DETERMINATION OF CORRECT SIGN FOR ROUND ON FLAT REGIME OF MESH 2 A 262
      C                                                                 A 263
      A2F=ACG2*COS(PHI2T-DELG2-ALPHP2)-B2*COS(BETA2-ALPHP2)             A 264
      B2F=-ACG2*SIN(PHI2T-DELG2-ALPHP2)+B2*SIN(BETA2-ALPHP2)            A 265
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                C2F=-RHOG2                                                        A 266
                ROOT2F=A2F*A2F+B2F*B2F-C2F*C2F                                    A 267
                Y2F1=A2F+SQRT(ROOT2F)                                             A 268
                Y2F2=A2F-SQRT(ROOT2F)                                             A 269
  270           X2F=B2F+C2F                                                       A 270
                PSI2F1=2.*ATAN2(Y2F1,X2F)                                         A 271
                PSI2F2=2.*ATAN2(Y2F2,X2F)                                         A 272
                IF (PSI2F1.LT.0.) PSI2F1=PSI2F1+2.*PI                             A 273
                IF (PSI2F2.LT.0.) PSI2F2=PSI2F2+2.*PI                             A 274
  275           IF (ABS(PSI2F1-PSI2T).LT.ABS(PSI2F2-PSI2T)) GO TO 14              A 275
                PSI2F1D=PSI2F1/Z                                                  A 276
                PSI2F2D=PSI2F2/Z                                                  A 277
                SIGN2F=-1.                                                        A 278
                GO TO 15                                                          A 279
  280         14 SIGN2F=1.                                                        A 280
              C                                                                   A 281
              C     COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI FOR MESH 2  A 282
              C                                                                   A 283
             15 DO 16 I=1,1000                                                    A 284
  285           PHID2=PHI2TD+(I-1.)/100.                                          A 285
                PHI2=PHID2*Z                                                      A 286
                A2F=ACG2*COS(PHI2-DELG2-ALPHP2)-B2*COS(BETA2-ALPHP2)              A 287
                B2F=-ACG2*SIN(PHI2-DELG2-ALPHP2)+B2*SIN(BETA2-ALPHP2)             A 288
                C2F=-RHOG2                                                        A 289
  290           ROOT2F=A2F*A2F+B2F*B2F-C2F*C2F                                    A 290
                Y2F=A2F+SIGN2F*SQRT(ROOT2F)                                       A 291
                X2F=B2F+C2F                                                       A 292
                PSI2F=2.*ATAN2(Y2F,X2F)                                           A 293
                IF (PSI2F.LT.0.) PSI2F=PSI2F+2.*PI                                A 294
  295           LX2=B2*COS(BETA2)+ACP2*COS(PSI2F+DPSI2-DELP2)-ACG2*COS(PHI2-DPHI2-  A 295
               1DELG2)                                                            A 296
                LY2=B2*SIN(BETA2)+ACP2*SIN(PSI2F+DPSI2-DELP2)-ACG2*SIN(PHI2-DPHI2-  A 297
               1DELG2)                                                            A 298
                LL2=SQRT(LX2*LX2+LY2*LY2)                                         A 299
  300           DELEL2=LL2-L2                                                     A 300
                IF (DELEL2.LE.0.) GO TO 17                                        A 301
             16 CONTINUE                                                          A 302
             17 PHI2F=PHI2                                                        A 303
                PSI2FF=PSI2F                                                      A 304
  305           PHI2I=PHI2F-DPHI2                                                 A 305
                PSI2I=PSI2FF+DPSI2                                                A 306
                IF (PSI2I.GT.2.*PI) PSI2I=PSI2I-2.*PI                             A 307
                PHI2ID=PHI2I/Z                                                    A 308
                PSI2ID=PSI2I/Z                                                    A 309
  310           PHI2FD=PHI2F/Z                                                    A 310
                PSI2FD=PSI2FF/Z                                                   A 311
                WRITE (6,77) PHI2ID,PSI2ID,PHI2FD,PSI2FD                          A 312
              C                                                                   A 313
              C     DETERMINATION OF CORRECT SIGN FOR ROUND ON ROUND REGIME OF MESH 2  A 314
  315         C                                                                   A 315
                A2R=B2*SIN(BETA2+DELP2)-ACG2*SIN(PHI2I-DELG2+DELP2)               A 316
                B2R=B2*COS(BETA2+DELP2)-ACG2*COS(PHI2I-DELG2+DELP2)               A 317
                C2R=(L2*L2-B2*B2-ACG2*ACG2-ACP2*ACP2+2.*ACG2*B2*COS(PHI2I-DELG2-BE  A 318
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     1TA2))/(2.*ACP2)                                                    A 319
      ROOT2R=A2R*A2R+B2R*B2R-C2R*C2R                                     A 320
      Y2R1=A2R+SQRT(ROOT2R)                                              A 321
      Y2R2=A2R-SQRT(ROOT2R)                                              A 322
      X2R=B2R+C2R                                                        A 323
      PSI2R1=2.*ATAN2(Y2R1,X2R)                                          A 324
      PSI2R2=2.*ATAN2(Y2R2,X2R)                                          A 325
      IF (PSI2R1.LT.0.) PSI2R1=PSI2R1+2.*PI                              A 326
      IF (PSI2R2.LT.0.) PSI2R2=PSI2R2+2.*PI                              A 327
      IF (ABS(PSI2I-PSI2R1).LT.ABS(PSI2I-PSI2R2)) GO TO 18               A 328
      SIGN2R=-1.                                                         A 329
      GO TO 19                                                           A 330
   18 SIGN2R=1.                                                          A 331
C                                                                        A 332
C     PRELIMINARY COMPUTATIONS FOR MESH 3                                A 333
C                                                                        A 334
C     DETERMINATION OF TRANSITION ANGLE OF MESH 3                        A 335
C                                                                        A 336
   19 A3T=RHOG3*COS(BETA3+ALPHP3)+FP3*SIN(BETA3+ALPHP3)                  A 337
      B3T=-RHOG3*SIN(BETA3+ALPHP3)+FP3*COS(BETA3+ALPHP3)                 A 338
      C3T=(ACG3*ACG3-RHOG3*RHOG3-B3*B3-FP3*FP3)/(2.*B3)                  A 339
      ROOT3T=A3T*A3T+B3T*B3T-C3T*C3T                                     A 340
      Y3T1=A3T+SQRT(ROOT3T)                                              A 341
      Y3T2=A3T-SQRT(ROOT3T)                                              A 342
      X3T=B3T+C3T                                                        A 343
      PSI3T1=2.*ATAN2(Y3T1,X3T)                                          A 344
      PSI3T2=2.*ATAN2(Y3T2,X3T)                                          A 345
      PSITE31=PSI3T1                                                     A 346
      PSITE32=PSI3T2                                                     A 347
      IF (PSI3T1.GT.PI) PSITE31=PSI3T1-2.*PI                             A 348
      IF (PSI3T1.LT.PI) PSITE31=PSI3T1+2.*PI                             A 349
      IF (PSI3T2.GT.PI) PSITE32=PSI3T2-2.*PI                             A 350
      IF (PSI3T2.LT.PI) PSITE32=PSI3T2+2.*PI                             A 351
      IF (PSITE31.GE.0.) TEST31=ABS(PI-BETA3+PSITE31-ALPHP3)/Z           A 352
      IF (PSITE31.LT.0.) TEST31=ABS(PI+BETA3-(PSITE31+2.*PI-ALPHP3))/Z   A 353
      IF (PSITE32.GE.0.) TEST32=ABS(PI-BETA3+PSITE32-ALPHP3)/Z           A 354
      IF (PSITE32.LT.0.) TEST32=ABS(PI+BETA3-(PSITE32+2.*PI-ALPHP3))/Z   A 355
      IF (PSI3T1.LT.0.) PSI3T1=PSI3T1+2.*PI                              A 356
      IF (PSI3T2.LT.0.) PSI3T2=PSI3T2+2.*PI                              A 357
      PSI3T1D=PSI3T1/Z                                                   A 358
      PSI3T2D=PSI3T2/Z                                                   A 359
      WRITE (6,50) PSI3T1D,TEST31                                        A 360
      WRITE (6,51) PSI3T2D,TEST32                                        A 361
      CALL TRANS1 (RHOG3,ALPHP3,BETA3,FP3,ACG3,B3,DELG3,Z,PSI3T1,PHI3T1,  A 362
     1G31)                                                               A 363
      IF (G31.GT.FP3) GO TO 20                                           A 364
      PHI3T=PHI3T1                                                       A 365
      PSI3T=PSI3T1                                                       A 366
      GO TO 22                                                           A 367
   20 CALL TRANS1 (RHOG3,ALPHP3,BETA3,FP3,ACG3,B3,DELG3,Z,PSI3T2,PHI3T2,  A 368
     1G32)                                                               A 369
      IF (G32.LT.FP3) GO TO 21                                           A 370
      WRITE (5,78)                                                       A 371
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                      STOP                                                      A 372
                   21 PHI3T=PHI3T2                                              A 373
                      PSI3T=PSI3T2                                              A 374
                   22 IF (PHI3T.LT.0.) PHI3T=PHI3T+2.*PI                        A 375
                      IF (PSI3T.LT.0.) PSI3T=PSI3T+2.*PI                        A 376
                      PHI3TD=PHI3T/Z                                            A 377
                      PSI3TD=PSI3T/Z                                            A 378
                      WRITE (6,79) PHI3TD,PSI3TD                                A 379
                C                                                               A 380
                C     DETERMINATION OF CORRECT SIGN FOR ROUND ON FLAT REGIME OF MESH 3  A 381
                C                                                               A 382
                C                                                               A 383
                      A3F=ACG3*COS(PHI3T+DELG3+ALPHP3)-B3*COS(BETA3+ALPHP3)     A 384
                      B3F=-ACG3*SIN(PHI3T+DELG3+ALPHP3)+B3*SIN(BETA3+ALPHP3)    A 385
                      C3F=RHOG3                                                 A 386
                      ROOT3F=A3F*A3F+B3F*B3F-C3F*C3F                            A 387
                      Y3F1=A3F+SQRT(ROOT3F)                                     A 388
                      Y3F2=A3F-SQRT(ROOT3F)                                     A 389
                      X3F=B3F+C3F                                               A 390
                      PSI3F1=2.*ATAN2(Y3F1,X3F)                                 A 391
                      PSI3F2=2.*ATAN2(Y3F2,X3F)                                 A 392
                      IF (PSI3F1.LT.0.) PSI3F1=PSI3F1+2.*PI                     A 393
                      IF (PSI3F2.LT.0.) PSI3F2=PSI3F2+2.*PI                     A 394
                      IF (ABS(PSI3F1-PSI3T).LT.ABS(PSI3F2-PSI3T)) GO TO 23      A 395
                      SIGN3F=-1.                                                A 396
                      GO TO 24                                                  A 397
                   23 SIGN3F=1.                                                 A 398
                C                                                               A 399
                C     COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI FOR MESH 3  A 400
                C                                                               A 401
                   24 DO 25 I=1,2000                                            A 402
                      PHID3=PHI3TD-(I-1.)/100.                                  A 403
                      PHI3=PHID3*Z                                              A 404
                      A3F=ACG3*COS(PHI3+DELG3+ALPHP3)-B3*COS(BETA3+ALPHP3)      A 405
                      B3F=-ACG3*SIN(PHI3+DELG3+ALPHP3)+B3*SIN(BETA3+ALPHP3)     A 406
                      C3F=RHOG3                                                 A 407
                      ROOT3F=A3F*A3F+B3F*B3F-C3F*C3F                            A 408
                      Y3F=A3F+SIGN3F*SQRT(ROOT3F)                               A 409
                      X3F=B3F+C3F                                               A 410
                      PSI3F=2.*ATAN2(Y3F,X3F)                                   A 411
                      IF (PSI3F.LT.0.) PSI3F=PSI3F+2.*PI                        A 412
                      LX3=B3*COS(BETA3)+ACP3*COS(PSI3F-DPSI3+DELP3)-ACG3*COS(PHI3+DPHI3+  A 413
                     1DELG3)                                                    A 414
                      LY3=B3*SIN(BETA3)+ACP3*SIN(PSI3F-DPSI3+DELP3)-ACG3*SIN(PHI3+DPHI3+  A 415
                     1DELG3)                                                    A 416
                      LL3=SQRT(LX3*LX3+LY3*LY3)                                 A 417
                      DELEL3=LL3-L3                                             A 418
                      IF (DELEL3.LE.0.) GO TO 26                                A 419
                   25 CONTINUE                                                  A 420
                   26 PHI3F=PHI3                                                A 421
                      PSI3FF=PSI3F                                              A 422
                      PHI3I=PHI3F+DPHI3                                         A 423
                      PSI3I=PSI3FF-DPSI3                                        A 424
                      IF (PSI3I.LT.0.) PSI3I=PSI3I+2.*PI
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      IF (PSI3I.LT.0.) PSI3I=PSI3I+2.*PI                                A 425
      PHI3ID=PHI3I/Z                                                    A 426
      PSI3ID=PSI3I/Z                                                    A 427
      PHI3FD=PHI3F/Z                                                    A 428
      PSI3FD=PSI3FF/Z                                                   A 429
      WRITE (6,80) PHI3ID,PSI3ID,PHI3FD,PSI3FD                          A 430
C                                                                       A 431
C     DETERMINATION OF CORRECT SIGN OF ROUND ON ROUND REGIME FOR MESH 3 A 432
C                                                                       A 433
      A3R=ACG3*SIN(PHI3I+DELG3-DELP3)-B3*SIN(BETA3-DELP3)               A 434
      B3R=ACG3*COS(PHI3I+DELG3-DELP3)-B3*COS(BETA3-DELP3)               A 435
      C3R=(ACP3*ACP3+ACG3*ACG3+B3*B3-L3*L3-2.*ACG3*B3*COS(PHI3I+DELG3-BE A 436
     1TA3))/(2.*ACP3)                                                   A 437
      ROOT3R=A3R*A3R+B3R*B3R-C3R*C3R                                    A 438
      Y3R1=A3R+SQRT(ROOT3R)                                             A 439
      Y3R2=A3R-SQRT(ROOT3R)                                             A 440
      X3R=B3R+C3R                                                       A 441
      PSI3R1=2.*ATAN2(Y3R1,X3R)                                         A 442
      PSI3R2=2.*ATAN2(Y3R2,X3R)                                         A 443
      IF (PSI3R1.LT.0.) PSI3R1=PSI3R1+2.*PI                             A 444
      IF (PSI3R2.LT.0.) PSI3R2=PSI3R2+2.*PI                             A 445
      IF (ABS(PSI3I-PSI3R1).LT.ABS(PSI3I-PSI3R2)) GO TO 27              A 446
      SIGN3R=-1.                                                        A 447
      GO TO 28                                                          A 448
   27 SIGN3R=1.                                                         A 449
C                                                                       A 450
C     GEAR TRAIN MOTION MODEL, KINEMATICS                               A 451
C                                                                       A 452
   28 DOPHI1=NP2*NP3*(PHI3I-PHI3F)/(K*NG1*NG2)                          A 453
      PHI1=PHI1I+DOPHI1                                                 A 454
      WRITE (6,52)                                                      A 455
   29 PHI1=PHI1-DOPHI1                                                  A 456
      PHI1D=PHI1/Z                                                      A 457
      IF (PHI1.LE.PHI1F+DOPHI1) GO TO 45                                A 458
C                                                                       A 459
C     MESH 1                                                            A 460
C                                                                       A 461
      IF (PHI1.LE.PHI1T) GO TO 30                                       A 462
      A1R=ACG1*SIN(PHI1+DELG1-DELP1)-B1*SIN(BETA1-DELP1)                A 463
      B1R=ACG1*COS(PHI1+DELG1-DELP1)-B1*COS(BETA1-DELP1)                A 464
      C1R=(ACP1*ACP1+ACG1*ACG1+B1*B1-L1*L1-2.*ACG1*B1*COS(PHI1+DELG1-BET A 465
     1A1))/(2.*ACP1)                                                    A 466
      ROOT1R=A1R*A1R+B1R*B1R-C1R*C1R                                    A 467
      Y1R=A1R+SIGN1R*SQRT(ROOT1R)                                       A 468
      X1R=B1R+C1R                                                       A 469
      PSI1=2.*ATAN2(Y1R,X1R)                                            A 470
      IF (PSI1.LT.0.) PSI1=PSI1+2.*PI                                   A 471
      PSI1D=PSI1/Z                                                      A 472
      IF (ABS(PHI1-PHI1I).LT.0.0001) PSI1I=PSI1                         A 473
      IF(ABS(PHI1-PHI1I).LT..0001)PSI1P=PSI1I                           A 474
      SLAM1=(B1*SIN(BETA1)+ACP1*SIN(PSI1+DELP1)-ACG1*SIN(PHI1+DELG1))/L1 A 475
      CLAM1=(B1*COS(BETA1)+ACP1*COS(PSI1+DELP1)-ACG1*COS(PHI1+DELG1))/L1 A 476
      LAMDA1=ATAN2(SLAM1,CLAM1)                                         A 477
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      IF (LAMDA1.LT.0.) LAMDA1=LAMDA1+2.*PI                              A 478
      PSDOT1=PHDOT1*ACG1*(B1/ACP1*SIN(PHI1+DELG1-BETA1)+SIN(PHI1-PSI1+DE A 479
     1LG1-DELP1))/(A1R*COS(PSI1)-B1R*SIN(PSI1))                          A 480
      VST1R=PHDOT1*(ACG1*COS(PHI1+DELG1-LAMDA1)+RHOG1)-PSDOT1*(ACP1*COS( A 481
     1PSI1+DELP1-LAMDA1)-RHOP1)                                          A 482
      S1R=VST1R/ABS(VST1R)                                               A 483
      GO TO 31                                                           A 484
   30 A1F=ACG1*COS(PHI1+DELG1+ALPHP1)-B1*COS(BETA1+ALPHP1)               A 485
      B1F=-ACG1*SIN(PHI1+DELG1+ALPHP1)+B1*SIN(BETA1+ALPHP1)              A 486
      C1F=RHOG1                                                          A 487
      ROOT1F=A1F*A1F+B1F*B1F-C1F*C1F                                     A 488
      Y1F=A1F+SIGN1F*SQRT(ROOT1F)                                        A 489
      X1F=B1F+C1F                                                        A 490
      PSI1=2.*ATAN2(Y1F,X1F)                                             A 491
      IF (PSI1.LT.0.) PSI1=PSI1+2.*PI                                    A 492
      PSI1D=PSI1/Z                                                       A 493
      G1=(ACG1*SIN(PHI1+DELG1)+RHOG1*COS(PSI1-ALPHP1)-B1*SIN(BETA1))/SIN A 494
     1(PSI1-ALPHP1)                                                      A 495
      PSDOT1=PHDOT1*(ACG1*COS(PHI1-PSI1+DELG1+ALPHP1))/(A1F*COS(PSI1)-B1 A 496
     1F*SIN(PSI1))                                                       A 497
      VST1F=PHDOT1*(ACG1*SIN(PSI1-ALPHP1-PHI1-DELG1)-RHOG1)              A 498
      S1F=VST1F/ABS(VST1F)                                               A 499
C                                                                        A 500
C     MESH 2                                                             A 501
C                                                                        A 502
   31 DDPHI2=PSI1-PSI1P                                                  A 503
      IF (ABS(PHI1-PHI1I).LT.0.0001) PHI2=PHI2I                          A 504
      PHI2=PHI2+DDPHI2                                                   A 505
      PSI1P=PSI1                                                         A 506
      IF (PHI2.GT.PHI2F) PHI2=PHI2I                                      A 507
      PHI2D=PHI2/Z                                                       A 508
      IF (PHI2.GE.PHI2T) GO TO 32                                        A 509
      A2R=B2*SIN(BETA2+DELP2)-ACG2*SIN(PHI2-DELG2+DELP2)                 A 510
      B2R=B2*COS(BETA2+DELP2)-ACG2*COS(PHI2-DELG2+DELP2)                 A 511
      C2R=(L2*L2-B2*B2-ACG2*ACG2-ACP2*ACP2+2.*ACG2*B2*COS(PHI2-DELG2-BET A 512
     1A2))/(2.*ACP2)                                                     A 513
      ROOT2R=A2R*A2R+B2R*B2R-C2R*C2R                                     A 514
      Y2R=A2R+SIGN2R*SQRT(ROOT2R)                                        A 515
      X2R=B2R+C2R                                                        A 516
      PSI2=2.*ATAN2(Y2R,X2R)                                             A 517
      IF (PSI2.LT.0.) PSI2=PSI2+2.*PI                                    A 518
      PSI2D=PSI2/Z                                                       A 519
      IF (ABS(PHI2-PHI2I).LT.0.0001) PSI2I=PSI2                          A 520
      IF(ABS(PHI2-PHI2I).LT.0.0001)PSI2P=PSI2I                           A 521
      SLAM2=(B2*SIN(BETA2)+ACP2*SIN(PSI2-DELP2)-ACG2*SIN(PHI2-DELG2))/L2 A 522
      CLAM2=(B2*COS(BETA2)+ACP2*COS(PSI2-DELP2)-ACG2*COS(PHI2-DELG2))/L2 A 523
      LAMDA2=ATAN2(SLAM2,CLAM2)                                          A 524
      IF (LAMDA2.LT.0.) LAMDA2=LAMDA2+2.*PI                              A 525
      PHDOT2=PSDOT1                                                      A 526
      PSDOT2=PHDOT2*ACG2*(-SIN(PHI2-PSI2-DELG2+DELP2)-B2/ACP2*SIN(PHI2-D A 527
     1ELG2-BETA2))/(A2R*COS(PSI2)-B2R*SIN(PSI2))                         A 528
      VST2R=PHDOT2*(ACG2*COS(PHI2-DELG2-LAMDA2)+RHOG2)-PSDOT2*(ACP2*COS( A 529
     1PSI2-DELP2-LAMDA2)-RHOP2)                                          A 530
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                  S2R=VST2R/ABS(VST2R)                                                A 531
                  GO TO 33                                                            A 532
               32 A2F=-ACG2*COS(PHI2-DELG2-ALPHP2)-B2*COS(BETA2-ALPHP2)               A 533
                  B2F=-ACG2*SIN(PHI2-DELG2-ALPHP2)+B2*SIN(BETA2-ALPHP2)               A 534
   535            C2F=-RHOG2                                                          A 535
                  ROOT2F=A2F*A2F+B2F*B2F-C2F*C2F                                      A 536
                  Y2F=A2F+SIGN2F*SQRT(ROOT2F)                                         A 537
                  X2F=B2F+C2F                                                         A 538
                  PSI2=2.*ATAN2(Y2F,X2F)                                              A 539
   540            IF (PSI2.LT.0.) PSI2=PSI2+2.*PI                                     A 540
                  PSI2O=PSI2/Z                                                        A 541
                  G2=(ACG2*SIN(PHI2-DELG2)-RHOG2*COS(PSI2+ALPHP2)-B2*SIN(BETA2))/SIN  A 542
                 1(PSI2+ALPHP2)                                                       A 543
                  PHDOT2=PSDOT1                                                       A 544
   545            PSDOT2=(PHDOT2*ACG2*COS(PHI2-DELG2-ALPHP2-PSI2))/(A2F*COS(PSI2)-B2  A 545
                 1F*SIN(PSI2))                                                        A 546
                  VST2F=PHDOT2*(ACG2*SIN(PSI2+ALPHP2-PHI2+DELG2)+RHOG2)               A 547
                  S2F=VST2F/ABS(VST2F)                                                A 548
               C                                                                      A 549
   550         C     MESH 3                                                           A 550
               C                                                                      A 551
               33 DDPHI3=PSI2-PSI2P                                                   A 552
                  IF (ABS(PHI1-PHI1I).LT.0.0001) PHI3=PHI3I                           A 553
                  PHI3=PHI3+DDPHI3                                                    A 554
   555            PSI2P=PSI2                                                          A 555
                  IF (PHI3.LT.PHI3F) PHI3=PHI3I                                       A 556
                  PHI3O=PHI3/Z                                                        A 557
                  IF (PHI3.LE.PHI3T) GO TO 34                                         A 558
                  A3R=ACG3*SIN(PHI3+DELG3-DELP3)-B3*SIN(BETA3-DELP3)                  A 559
   560            B3R=ACG3*COS(PHI3+DELG3-DELP3)-B3*COS(BETA3-DELP3)                  A 560
                  C3R=(ACP3*ACP3+ACG3*ACG3+B3*B3-L3*L3-2.*ACG3*B3*COS(PHI3+DELG3-BET  A 561
                 1A3))/(2.*ACP3)                                                      A 562
                  ROOT3R=A3R*A3R+B3R*B3R-C3R*C3R                                      A 563
                  Y3R=A3R+SIGN3R*SQRT(ROOT3R)                                         A 564
   565            X3R=B3R+C3R                                                         A 565
                  PSI3=2.*ATAN2(Y3R,X3R)                                              A 566
                  IF (PSI3.LT.0.) PSI3=PSI3+2.*PI                                     A 567
                  IF (ABS(PHI3-PHI3I).LT.0.0001) PSI3=PSI3I                           A 568
                  PSI3O=PSI3/Z                                                        A 569
   570            SLAM3=(B3*SIN(BETA3)+ACP3*SIN(PSI3+DELP3)-ACG3*SIN(PHI3+DELG3))/L3  A 570
                  CLAM3=(B3*COS(BETA3)+ACP3*COS(PSI3+DELP3)-ACG3*COS(PHI3+DELG3))/L3  A 571
                  LAMDA3=ATAN2(SLAM3,CLAM3)                                           A 572
                  IF (LAMDA3.LT.0.0) LAMDA3=LAMDA3+2.*PI                              A 573
                  PHDOT3=PSDOT2                                                       A 574
   575            PSDOT3=PHDOT3*ACG3*(B3/ACP3*SIN(PHI3+DELG3-BETA3)+SIN(PHI3-PSI3+DE  A 575
                 1LG3-DELP3))/(A3R*COS(PSI3)-B3R*SIN(PSI3))                           A 576
                  VST3R=PHSDT3*(ACG3*COS(PHI3+DELG3-LAMDA3)+RHOG3)-PSDOT3*(ACP3*COS(  A 577
                 1PSI3+DELP3-LAMDA3)-RHOP3)                                           A 578
                  S3R=VST3R/ABS(VST3R)                                                A 579
   580            GO TO 35                                                            A 580
               34 A3F=ACG3*COS(PHI3+DELG3+ALPHP3)-B3*COS(BETA3+ALPHP3)                A 581
                  B3F=-ACG3*SIN(PHI3+DELG3+ALPHP3)+B3*SIN(BETA3+ALPHP3)               A 582
                  C3F=RHOG3                                                           A 583
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                ROOT3F=A3F*A3F+B3F*B3F-C3F*C3F                                    A 584
585             Y3F=A3F*SIGN3F*SQRT(ROOT3F)                                       A 585
                X3F=B3F*C3F                                                       A 586
                PSI3=2.*ATAN2(Y3F,X3F)                                            A 587
                IF (PSI3.LT.0.) PSI3=PSI3+2.*PI                                   A 588
                PSI3D=PSI3/Z                                                      A 589
590             G3=(ACG3*SIN(PHI3+DELG3)+RHOG3*COS(PSI3-ALPHP3)-B3*SIN(BETA3))/SIN A 590
               1(PSI3-ALPHP3)                                                     A 591
                PHDOT3=PSDOT2                                                     A 592
                PSDOT3=PHDOT3*ACG3*COS(PHI3-PSI3+DELG3+ALPHP3)/(A3F*COS(PSI3)-B3F* A 593
               1SIN(PSI3))                                                        A 594
595             VST3F=PHDOT3*(ACG3*SIN(PSI3-ALPHP3-PHI3-DELG3)-RHOG3)             A 595
                S3F=VST3F/ABS(VST3F)                                              A 596
              C                                                                   A 597
              C     MOMENT COMPUTATIONS                                           A 598
              C                                                                   A 599
600           35 DN=1.+MU*MU                                                      A 600
                A1=ABS((MU*SIN(GAMMA4)+COS(GAMMA4))/DN)                           A 601
                A2=ABS((MU*(S3R-1.)*SIN(LAMDA3)-(1.+MU*MU*S3R)*COS(LAMDA3))/DN)   A 602
                A3=ABS((SIN(GAMMA4)-MU*COS(GAMMA4))/DN)                           A 603
                A4=ABS(((1.+MU*MU*S3R)*SIN(LAMDA3)+MU*(S3R-1.)*COS(LAMDA3))/DN)   A 604
605             A5=ABS(((1.-MU*MU*S3R)*COS(LAMDA3)-MU*(1.+S3R)*SIN(LAMDA3))/DN)   A 605
                A6=ABS((MU*(1.+S2R)*SIN(LAMDA2)-(1.-MU*MU*S2R)*COS(LAMDA2))/DN)   A 606
                A7=ABS((MU*SIN(GAMMA3)-COS(GAMMA3))/DN)                           A 607
                A8=ABS(((1.-MU*MU*S3R)*SIN(LAMDA3)+MU*(1.+S3R)*COS(LAMDA3))/DN)   A 608
                A9=ABS(((MU*MU*S2R-1.)*SIN(LAMDA2)-MU*(1.+S2R)*COS(LAMDA2))/DN)   A 609
610             A10=ABS((SIN(GAMMA3)+MU*COS(GAMMA3))/DN)                          A 610
                A11=ABS(((1.+MU*MU*S2R)*COS(LAMDA2)-MU*(S2R-1.)*SIN(LAMDA2))/DN)  A 611
                A12=ABS((MU*(S1R-1.)*SIN(LAMDA1)-(1.+MU*MU*S1R)*COS(LAMDA1))/DN)  A 612
                A13=ABS((-COS(GAMMA2)-MU*SIN(GAMMA2))/DN)                         A 613
                A14=ABS(((1.+MU*MU*S2R)*SIN(LAMDA2)-MU*(1.-S2R)*COS(LAMDA2))/DN)  A 614
615             A15=ABS((MU*(1.-S1R)*COS(LAMDA1)-(1.+MU*MU*S1R)*SIN(LAMDA1))/DN)  A 615
                A16=ABS((MU*COS(GAMMA2)-SIN(GAMMA2))/DN)                          A 616
                A17=ABS(((1.-MU*MU*S1R)*COS(LAMDA1)-MU*(1.+S1R)*SIN(LAMDA1))/DN)  A 617
                A18=ABS(1./DN)                                                    A 618
                A19=ABS(((1.-MU*MU*S1R)*SIN(LAMDA1)+MU*(1.+S1R)*COS(LAMDA1))/DN)  A 619
620             A20=ABS(MU/DN)                                                    A 620
                A21=ABS((MU*(1.-S2R)*SIN(LAMDA2)+(1.+MU*MU*S2R)*COS(LAMDA2))/DN)  A 621
                A22=ABS(((1.-MU*MU*S1F)*SIN(PSI1-ALPHP1)-MU*(1.+S1F)*COS(PSI1-ALPH A 622
               1P1))/DN)                                                          A 623
                A23=ABS((MU*SIN(GAMMA2)+COS(GAMMA2))/DN)                          A 624
625             A24=ABS(((1.+MU*MU*S2R)*SIN(LAMDA2)-MU*(1.-S2R)*COS(LAMDA2))/DN)  A 625
                A25=ABS((-MU*(1.+S1F)*SIN(PSI1-ALPHP1)+(MU*MU*S1F-1.)*COS(PSI1-ALP A 626
               1HP1))/DN)                                                         A 627
                A26=ABS((-SIN(GAMMA2)+MU*COS(GAMMA2))/DN)                         A 628
                A27=ABS((-(1.+MU*MU*S1F)*SIN(PSI1-ALPHP1)+MU*(S1F-1.)*COS(PSI1-ALP A 629
630            1HP1))/DN)                                                         A 630
                A28=ABS(1./DN)                                                    A 631
                A29=ABS((MU*(S1F-1.)*SIN(PSI1-ALPHP1)+(1.+MU*MU*S1F)*COS(PSI1-ALPH A 632
               1P1))/DN)                                                          A 633
                A30=ABS(MU/DN)                                                    A 634
635             A31=ABS(((1.-MU*MU*S3R)*COS(LAMDA3)-MU*(1.+S3R)*SIN(LAMDA3))/DN)  A 635
                A32=ABS((-MU*(1.+S2F)*COS(PSI2+ALPHP2)-(1.-MU*MU*S2F)*SIN(PSI2+ALP A 636
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     1HP2))/DN)                                                                  A 637
      A33=ABS((MU*SIN(GAMMA3)-COS(GAMMA3))/DN)                                   A 638
      A34=ABS((MU*(1.+S3R)*COS(LAMDA3)+(1.-MU*MU*S3R)*SIN(LAMDA3))/DN)           A 639
640   A35=ABS(((1.-MU*MU*S2F)*COS(PSI2+ALPHP2)-MU*(1.+S2F)*SIN(PSI2+ALPH        A 640
     1P2))/DN)                                                                   A 641
      A36=ABS((SIN(GAMMA3)+MU*COS(GAMMA3))/DN)                                   A 642
      A37=ABS((1.+MU*MU*S2F)*SIN(PSI2+ALPHP2)+MU*(S2F-1.)*COS(PSI2+ALPH          A 643
     1P2))/DN)                                                                   A 644
645   A38=ABS(((1.-MU*MU*S1F)*SIN(PSI1-ALPHP1)-MU*(1.+S1F)*COS(PSI1-ALPH        A 645
     1P1))/DN)                                                                   A 646
      A39=ABS((MU*SIN(GAMMA2)+COS(GAMMA2))/DN)                                   A 647
      A40=ABS((MU*(S2F-1.)*SIN(PSI2+ALPHP2)-(1.+MU*MU*S2F)*COS(PSI2+ALPH         A 648
     1P2))/DN)                                                                   A 649
650   A41=ABS((-MU*(1.+S1F)*SIN(PSI1-ALPHP1)+(MU*MU*S1F-1.)*COS(PSI1-ALP        A 650
     1HP1))/DN)                                                                  A 651
      A42=ABS((MU*COS(GAMMA2)-SIN(GAMMA2))/DN)                                   A 652
      A43=ABS(((1.+MU*MU*S2F)*SIN(PSI2+ALPHP2)-MU*(1.-S2F)*COS(PSI2+ALPH         A 653
     1P2))/DN)                                                                   A 654
655   A44=ABS((MU*(S1R-1.)*SIN(LAMDA1)-(1.+MU*MU*S1R)*COS(LAMDA1))/DN)           A 655
      A45=ABS((MU*SIN(GAMMA2)+COS(GAMMA2))/DN)                                   A 656
      A46=ABS((MU*(S2F-1.)*SIN(PSI2+ALPHP2)-(1.+MU*MU*S2F)*COS(PSI2+ALPH         A 657
     1P2))/DN)                                                                   A 658
      A47=ABS((-(1.+MU*MU*S1R)*SIN(LAMDA1)+MU*(1.-S1R)*COS(LAMDA1))/DN)          A 659
660   A48=ABS((-SIN(GAMMA2)+MU*COS(GAMMA2))/DN)                                  A 660
      A49=ABS(((1.-MU*MU*S3F)*SIN(PSI3-ALPHP3)-MU*(1.+S3F)*COS(PSI3-ALPH         A 661
     1P3))/DN)                                                                   A 662
      A50=ABS((-MU*SIN(GAMMA4)-COS(GAMMA4))/DN)                                  A 663
      A51=ABS((-MU*(1.+S3F)*SIN(PSI3-ALPHP3)-(1.-MU*MU*S3F)*COS(PSI3-ALP         A 664
665  1HP3))/DN)                                                                   A 665
      A52=ABS((-SIN(GAMMA4)+MU*COS(GAMMA4))/DN)                                  A 666
      A53=ABS((-(1.+MU*MU*S3F)*SIN(PSI3-ALPHP3)+MU*(S3F-1.)*COS(PSI3-ALP         A 667
     1HP3))/DN)                                                                  A 668
      A54=ABS(((MU*MU*S2F-1.)*SIN(PSI2+ALPHP2)-MU*(1.+S2F)*COS(PSI2+ALPH         A 669
670  1P2))/DN)                                                                   A 670
      A55=ABS((MU*SIN(GAMMA3)-COS(GAMMA3))/DN)                                   A 671
      A56=ABS((MU*(S3F-1.)*SIN(PSI3-ALPHP3)+(1.+MU*MU*S3F)*COS(PSI3-ALPH         A 672
     1P3))/DN)                                                                   A 673
      A57=ABS((-MU*(1.+S2F)*SIN(PSI2+ALPHP2)+(1.-MU*MU*S2F)*COS(PSI2+ALP         A 674
675  1HP2))/DN)                                                                   A 675
      A58=ABS((-SIN(GAMMA3)-MU*COS(GAMMA3))/DN)                                  A 676
      A59=ABS((-(1.+MU*MU*S3F)*SIN(PSI3-ALPHP3)+MU*(S3F-1.)*COS(PSI3-ALP         A 677
     1HP3))/DN)                                                                  A 678
      A60=ABS((MU*(1.+S2R)*SIN(LAMDA2)+(MU*MU*S2R-1.)*COS(LAMDA2))/DN)           A 679
680   A61=ABS((MU*SIN(GAMMA3)-COS(GAMMA3))/DN)                                   A 680
      A62=ABS((MU*(S3F-1.)*SIN(PSI3-ALPHP3)+(1.+MU*MU*S3F)*COS(PSI3-ALPH         A 681
     1P3))/DN)                                                                   A 682
      A63=ABS((MU*MU*S2R-1.)*SIN(LAMDA2)-MU*(1.+S2R)*COS(LAMDA2))/DN)            A 683
      A64=ABS((-SIN(GAMMA3)-MU*COS(GAMMA3))/DN)                                  A 684
685   C1=MU*RHO4*(A1+A3)                                                         A 685
      C2=ACP3*(MU*S3R*COS(PSI3+DELP3-LAMDA3)-SIN(PSI3+DELP3-LAMDA3))-MU*         A 686
     1(RHOP3*S3R+RHO4*(A2+A4))                                                   A 687
      C3=MU*RHO3*(A5+A8)-MU*S3R*RHOG3+ACG3*(SIN(PHI3+DELG3-LAMDA3)-MU*S3         A 688
     1R*COS(PHI3+DELG3-LAMDA3))                                                  A 689
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      C4=MU*RHO3*(A7+A10)                                                A 690
      C5=MU*RHO3*(A6+A9)-MU*S2R*RHOP2+ACP2*(MU*S2R*COS(PSI2-DELP2-LAMDA2 A 691
     1)-SIN(PSI2-DELP2-LAMDA2))                                          A 692
      C6=ACG2*(SIN(PHI2-DELG2-LAMDA2)-MU*S2R*COS(PHI2-DELG2-LAMDA2))-MU* A 693
     1RHO2*(A11+A14)-MU*RHOG2*S2R                                        A 694
      C7=MU*RHO2*(A13+A16)                                               A 695
      C8=-(ACP1*(SIN(PSI1+DELP1-LAMDA1)-MU*S1R*COS(PSI1+DELP1-LAMDA1))+M A 696
     1U*RHO2*(A12+A15)+MU*S1R*RHOP1)                                     A 697
      C9=MU*RHO1*(A18+A20)                                               A 698
      C10=MU*RHO1*(A17+A19)+ACG1*(SIN(PHI1+DELG1-LAMDA1)-MU*S1R*COS(PHI1 A 699
     1+DELG1-LAMDA1))-MU*S1R*RHOG1                                       A 700
      C11=ACG2*(SIN(PHI2-DELG2-LAMDA2)-MU*S2R*COS(PHI2-DELG2-LAMDA2))-MU A 701
     1*RHO2*(A21+A24)-MU*RHOG2*S2R                                       A 702
      C12=MU*RHO2*(A23+A26)                                              A 703
      C13=G1-MU*RHO2*(A22+A25)                                           A 704
      C14=MU*RHO1*(A28+A30)                                              A 705
      C15=MU*RHO1*(A27+A29)+MU*S1F*RHOG1+ACG1*(MU*S1F*SIN(PHI1+DELG1-PSI A 706
     11+ALPHP1)-COS(PHI1+DELG1-PSI1+ALPHP1))                             A 707
      C16=MU*RHO3*(A31+A34)+ACG3*(SIN(PHI3+DELG3-LAMDA3)-MU*S3R*COS(PHI3 A 708
     1+DELG3-LAMDA3))-RHOG3*MU*S3R                                       A 709
      C17=MU*RHO3*(A33+A36)                                              A 710
      C18=MU*RHO3*(A32+A35)-G2                                           A 711
      C19=-MU*RHO2*(A37+A40)+ACG2*(COS(PHI2-DELG2-PSI2-ALPHP2)+MU*S2F*SI A 712
     1N(PHI2-DELG2-PSI2-ALPHP2))-MU*S2F*RHOG2                            A 713
      C20=MU*RHO2*(A39+A42)                                              A 714
      C21=-MU*RHO2*(A38+A41)+G1                                          A 715
      C22=-MU*RHO2*(A43+A46)+ACG2*(COS(PHI2-DELG2-PSI2-ALPHP2)+MU*S2F*SI A 716
     1N(PHI2-DELG2-PSI2-ALPHP2))-MU*S2F*RHOG2                            A 717
      C23=MU*RHO2*(A45+A48)                                              A 718
      C24=-MU*RHO2*(A44+A47)+ACP1*(MU*S1R*COS(PSI1+DELP1-LAMDA1)-SIN(PSI A 719
     11+DELP1-LAMDA1))-MU*S1R*RHOP1                                      A 720
      C25=MU*RHO4*(A50+A52)                                              A 721
      C26=G3-MU*RHO4*(A49+A51)                                           A 722
      C27=MU*RHO3*(A53+A56)+ACG3*(-COS(PHI3+DELG3-PSI3+ALPHP3)+MU*S3F*SI A 723
     1N(PHI3+DELG3-PSI3+ALPHP3))+MU*S3F*RHOG3                            A 724
      C28=MU*RHO3*(A55+A58)                                              A 725
      C29=MU*RHO3*(A54+A57)-G2                                           A 726
      C30=MU*RHO3*(A59+A62)-ACG3*(COS(PHI3+DELG3-PSI3+ALPHP3)-MU*S3F*SIN A 727
     1(PHI3+DELG3-PSI3+ALPHP3))+MU*S3F*RHOG3                             A 728
      C31=MU*RHO3*(A61+A64)                                              A 729
      C32=MU*RHO3*(A60+A63)-ACP2*(SIN(PSI2-DELP2-LAMDA2)-MU*S2R*COS(PSI2 A 730
     1-DELP2-LAMDA2))-MU*S2R*RHOP2                                       A 731
C                                                                        A 732
C     MOMENT CONTROL                                                     A 733
C                                                                        A 734
      IF ((PHI1.GE.PHI1T).AND.(PHI2.LE.PHI2T).AND.(PHI3.GE.PHI2T)) GO TO A 735
     1 36                                                                A 736
      IF ((PHI1.LE.PHI1T).AND.(PHI2.LE.PHI2T).AND.(PHI3.GE.PHI3T)) GO TO A 737
     1 37                                                                A 738
      IF ((PHI1.LE.PHI1T).AND.(PHI2.GE.PHI2T).AND.(PHI3.GE.PHI3T)) GO TO A 739
     1 38                                                                A 740
      IF ((PHI1.GE.PHI1T).AND.(PHI2.GE.PHI2T).AND.(PHI3.GE.PHI3T)) GO TO A 741
     1 39                                                                A 742
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      IF ((PHI1.LE.PHI1T).AND.(PHI2.GE.PHI2T).AND.(PHI3.LE.PHI3T)) GO TO  A 743
     1 40                                                                 A 744
  745 IF ((PHI1.GE.PHI1T).AND.(PHI2.GE.PHI2T).AND.(PHI3.LE.PHI3T)) GO TO  A 745
     1 41                                                                 A 746
      IF ((PHI1.GE.PHI1T).AND.(PHI2.LE.PHI2T).AND.(PHI3.LE.PHI3T)) GO TO  A 747
     1 42                                                                 A 748
      IF ((PHI1.LE.PHI1T).AND.(PHI2.LE.PHI2T).AND.(PHI3.LE.PHI3T)) GO TO  A 749
  750 1 43                                                                A 750
C                                                                         A 751
C     MOMENT EXPRESSIONS                                                  A 752
C                                                                         A 753
   36 MO41=MIN*C2*C5*C8/(C3*C6*C10)-Q4*C1-Q2*C2*C5*C7/(C3*C6)-Q3*C2*C4/C  A 754
  755 13-Q1*C2*C5*C8*C9/(C3*C6*C10)                                       A 755
      MO4=MO41                                                            A 756
      POINTEF=ABS(PSDOT3)*MO4/MIN                                         A 757
      WRITE (6,53) PHI1D,PHI2D,PHI3D,PSI1D,PSI2D,PSI3D,PSDOT1,PSDOT2,PSD  A 758
     1OT3,S1R,S2R,S3R,POINTEF                                             A 759
  760 GO TO 44                                                            A 760
   37 MO42=MIN*C2*C5*C13/(C3*C11*C15)-Q1*C2*C5*C13*C14/(C3*C11*C15)-Q2*C  A 761
     12*C5*C12/(C3*C11)-Q3*C2*C4/C3-Q4*C1                                 A 762
      MO4=MO42                                                            A 763
      POINTEF=ABS(PSDOT3)*MO4/MIN                                         A 764
  765 WRITE (6,54) PHI1D,PHI2D,PHI3D,PSI1D,PSI2D,PSI3D,PSDOT1,PSDOT2,PSD  A 765
     1OT3,S2R,S3R,S1F,G1,POINTEF                                          A 766
      GO TO 44                                                            A 767
   38 MO43=MIN*C2*C18*C21/(C15*C16*C19)-Q1*C2*C14*C18*C21/(C15*C16*C19)-  A 768
     1Q2*C2*C18*C20/(C16*C19)-Q3*C2*C17/C16-Q4*C1                         A 769
  770 MO4=MO43                                                            A 770
      POINTEF=ABS(PSDOT3)*MO4/MIN                                         A 771
      WRITE (6,55) PHI1D,PHI2D,PHI3D,PSI1D,PSI2D,PSI3D,PSDOT1,PSDOT2,PSD  A 772
     1OT3,S3R,S1F,G1,S2F,G2,POINTEF                                       A 773
      GO TO 44                                                            A 774
  775 39 MO44=MIN*C2*C18*C24/(C16*C16*C22)-Q1*C2*C9*C18*C24/(C16*C16*C22)-Q  A 775
     12*C2*C18*C23/(C16*C22)-Q3*C2*C17/C16-Q4*C1                          A 776
      MO4=MO44                                                            A 777
      POINTEF=ABS(PSDOT3)*MO4/MIN                                         A 778
      WRITE (6,56) PHI1D,PHI2D,PHI3D,PSI1D,PSI2D,PSI3D,PSDOT1,PSDOT2,PSD  A 779
  780 1OT3,S1R,S3R,S2F,G2,POINTEF                                         A 780
      GO TO 44                                                            A 781
   40 MO45=MIN*C21*C26*C29/(C15*C19*C27)-Q1*C14*C21*C26*C29/(C15*C19*C27  A 782
     11)-Q2*C20*C26*C29/(C19*C27)-Q3*C26*C28/C27-Q4*C25                   A 783
      MO4=MO45                                                            A 784
  785 POINTEF=ABS(PSDOT3)*MO4/MIN                                         A 785
      WRITE (6,57) PHI1D,PHI2D,PHI3D,PSI1D,PSI2D,PSI3D,PSDOT1,PSDOT2,PSD  A 786
     1OT3,S1F,G1,S2F,G2,S3F,G3,POINTEF                                    A 787
      GO TO 44                                                            A 788
   41 MO46=MIN*C24*C26*C29/(C18*C22*C27)-Q1*C9*C24*C26*C29/(C18*C22*C27)  A 789
  790 1-Q2*C23*C26*C29/(C22*C27)-Q3*C26*C28/C27-Q4*C25                    A 790
      MO4=MO46                                                            A 791
      POINTEF=ABS(PSDOT3)*MO4/MIN                                         A 792
      WRITE (6,58) PHI1D,PHI2D,PHI3D,PSI1D,PSI2D,PSI3D,PSDOT1,PSDOT2,PSD  A 793
     1OT3,S1R,S2F,G2,S3F,G3,POINTEF                                       A 794
  795 GO TO 44                                                            A 795
```

```
     42 MD47=MIN*C8*C26*C32/(C6*C10*C30)-Q1*C8*C9*C26*C32/(C6*C10*C30)-Q2*   A 796
       1C7*C26*C32/(C6*C30)-Q3*C26*C31/C30-Q4*C25                            A 797
        MD4=MD47                                                             A 798
        POINTEF=ABS(PSDOT3)*MD4/MIN                                          A 799
  800   WRITE (6,59) PHI1D,PHI2D,PHI3D,PSI1D,PSI2D,PSI3D,PSDOT1,PSDOT2,PSD   A 800
       1OT3,S1R,S2R,S3F,G3,POINTEF                                           A 801
        GO TO 44                                                             A 802
     43 MD48=MIN*C13*C26*C32/(C11*C15*C30)-Q1*C13*C14*C26*C32/(C11*C15*C30   A 803
       1)-Q2*C12*C26*C32/(C11*C30)-Q3*C26*C31/C30-Q4*C25                     A 804
  805   MD4=MD48                                                             A 805
        POINTEF=ABS(PSDOT3)*MD4/MIN                                          A 806
        WRITE (6,60) PHI1D,PHI2D,PHI3D,PSI1D,PSI2D,PSI3D,PSDOT1,PSDOT2,PSD   A 807
       1OT3,S2R,S1F,G1,S3F,G3,POINTEF                                        A 808
     44 MTOT=MTOT+POINTEF                                                    A 809
  810   GO TO 29                                                             A 810
     45 CYCLEFF=-MTOT*DDPHI1/(PHI1F-PHI1I)                                   A 811
        WRITE (6,61) CYCLEFF                                                 A 812
        MTOT=0.                                                              A 813
        IF (ISTOP.NE.0) GO TO 1                                              A 814
  815   STOP                                                                 A 815
C                                                                            A 816
C                                                                            A 817
     46 FORMAT (6X,9HPSI1T1D =,F9.4,3X,8HTEST11 =,F9.4)                      A 818
     47 FORMAT (6X,9HPSI1T2D =,F9.4,3X,8HTEST12 =,F9.4//)                    A 819
  820 48 FORMAT (6X,9HPSI2T1D =,F9.4,3X,8HTEST21 =,F9.4)                     A 820
     49 FORMAT (6X,9HPSI2T2D =,F9.4,3X,8HTEST22 =,F9.4//)                    A 821
     50 FORMAT (6X,9HPSI3T1D =,F9.4,3X,8HTEST31 =,F9.4)                      A 822
     51 FORMAT (6X,9HPSI3T2D =,F9.4,3X,8HTEST32 =,F9.4//)                    A 823
     52 FORMAT (120H0      PHI1  PHI2  PHI3  PSI1  PSI2  PSI3  DPSI1  DPSI2  A 824
  825   1  DPSI3  S1R  S2R  S3R  S1F  G1    S2F  G2    S3F  G3    POINTEF   A 825
       2/)                                                                   A 826
     53 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),3(F3.0,2X),36X,F5.3)                A 827
     54 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),5X,3(F3.0,2X),F5.3,26X,F5.3)        A 828
     55 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),10X,F3.0,2X,F3.0,2X,F5.3,2X,F3.0,   A 829
  830  12X,F5.3,14X,F5.3)                                                    A 830
     56 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),F3.0,7X,F3.0,14X,F3.0,2X,F5.3,14X   A 831
       1,F5.3)                                                               A 832
     57 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),15X,3(F3.0,2X,F5.3,2X),F5.3)        A 833
     58 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),F3.0,24X,F3.0,2X,F5.3,2X,F3.0,2X,   A 834
  835  1F5.3,2X,F5.3)                                                        A 835
     59 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),2(F3.0,2X),29X,F3.0,2X,F5.3,2X,F5   A 836
       1.3)                                                                  A 837
     60 FORMAT (6X,6(F4.0,2X),3(F5.0,2X),5X,F3.0,7X,F3.0,2X,F5.3,14X,F3.0,   A 838
       12X,F5.3,2X,F5.3)                                                     A 839
  840 61 FORMAT (F10.3,F10.0/6F10.5/6F10.5/I1)                               A 840
     62 FORMAT (6F10.4)                                                      A 841
     63 FORMAT (6F10.4)                                                      A 842
     64 FORMAT (6F10.0)                                                      A 843
     65 FORMAT (4F10.4/F10.4/F10.6)                                          A 844
  845 66 FORMAT (1H1,5X,5HMIN =,F8.4,3X,4HMU =,F6.3,3X,5HRPM =,F6.0//6X,8HC   A 845
       1APR1 =,F8.5,3X,8HCAPRF2 =,F8.5,3X,8HCAPGP3 =,F8.5//6X,5HRP2 =,F8.    A 846
       25,3X,5HRP3 =,F8.5,3X,5HRP4 =,F8.5//6X,6HACG1 =,F8.5,3X,6HACG2 =,F8   A 847
       3.5,3X,6HACG3 =,F8.5//6X,6HACP1 =,F8.5,3X,6HACP2 =,F8.5,3X,6HACP3 =   A 848
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      4,F8.5/)                                                          A 849
850   67 FORMAT (6X,7HRHOG1 =,F8.5,3X,7HRHOG2 =,F8.5,3X,7HRHOG3 =,F8.5,3X,7 A 850
        1HRHOP1 =,F8.5,3X,7HRHOP2 =,F8.5,3X,7HRHOP3 =,F8.5/)            A 851
      68 FORMAT (6X,5HTG1 =,F8.5,3X,5HTG2 =,F8.5,3X,5HTG3 =,F8.5,3X,5HTP1 = A 852
        1,F8.5,3X,5HTP2 =,F8.5,3X,5HTP3 =,F8.5/)                        A 853
      69 FORMAT (6X,5HNG1 =,F5.0,3X,5HNG2 =,F5.0,3X,5HNG3 =,F5.0    5HNP2 = A 854
855     1,F5.0,3X,5HNP3 =,F5.0,3X,5HNP4 =,F5.0/)                        A 855
      70 FORMAT (6X,6HRHO1 =,F6.3,3X,6HRHO2 =,F6.3,3X,6HRHO3 =,F6.3,3X,6HRH A 856
        1O4 =,F6.3//6X,4HMO =,E12.4//6X,3HK =,F6.1//6X,8HPHOGT1 =,F5.1/) A 857
      71 FORMAT (6X,8HBETA1D =,F8.4,3X,8HBETA2D =,F8.4,3X,8HBETA3D =,F8.4// A 858
        1)                                                              A 859
850   72 FORMAT (6X,30HSOMETHING IS WRONG WITH MESH 1)                  A 860
      73 FORMAT (6X,6HPHI1TD =,F8.4,3X,8HPSI1TD =,F8.4)                 A 861
      74 FORMAT (6X,8HPHI1ID =,F8.4,3X,8HPSI1ID =,F8.4,3X,8HPHI1FD =,F8.4,3 A 862
        1X,8HPSI1FD =,F8.4//)                                           A 863
      75 FORMAT (6X,30HSOMETHING IS WRONG WITH MESH 2)                  A 864
865   76 FORMAT (6X 8HPHI2TD =,F8.4,3X,8HPSI2TD =,F8.4)                 A 865
      77 FORMAT (6X,8HPHI2ID =,F8.4,3X,8HPSI2ID =,F8.4,3X,8HPHI2FD =,F8.4,3 A 866
        1X,8HPSI2FD =,F8.4//)                                           A 867
      78 FORMAT (6X,30HSOMETHING IS WRONG WITH MESH 3)                  A 868
      79 FORMAT (6X,8HPHI3TD =,F8.4,3X,8HPSI3TD =,F8.4)                 A 869
870   80 FORMAT (6X,8HPHI3ID =,F8.4,3X,8HPSI3ID =,F8.4,3X,8HPHI3FD =,F8.4,3 A 870
        1X,8HPSI3FD =,F8.4//)                                           A 871
      81 FORMAT (1H0,5X,18HCYCLE EFFICIENCY =,F5.3)                     A 872
      82 FORMAT (4F10.5)                                                A 873
      83 FORMAT (6X,4HR1 =,F8.5,3X,4HR2 =,F8.5,3X,4HR3 =,F8.5,3X,4HR4 =,F8. A 874
875     1S/)                                                            A 875
      84 FORMAT (6X,5HFP1 =,F8.5,3X,5HFP2 =,F8.5,3X,5HFP3 =,F8.5/)      A 876
      85 FORMAT (4E15.5)                                                A 877
      86 FORMAT (6X,4HM1 =,E15.5,3X,4HM2 =,E15.5,3X,4HM3 =,E15.5,3X,4HM4 =, A 878
        1E15.5/)                                                        A 879
880     END                                                             A 880-
```

```
      SUBROUTINE TRANS1 (RHOG,ALPHP,BETA,FP,ACG,B,DELG,Z,PSIT,PHIT,G)
      PI=3.14159
      ST=(-RHOG*COS(PSIT-ALPHP)+B*SIN(BETA)+FP*SIN(PSIT-ALPHP))/ACG
      CT=(RHOG*SIN(PSIT-ALPHP)+B*COS(BETA)+FP*COS(PSIT-ALPHP))/ACG
      PHIT=ATAN2(ST,CT)-DELG
      PHINEXT=PHIT-.1*Z
      AF=ACG*COS(PHINEXT+DELG+ALPHP)-B*COS(BETA+ALPHP)
      BF=-ACG*SIN(PHINEXT+DELG+ALPHP)+B*SIN(BETA+ALPHP)
      CF=RHOG
      ROOTF=AF*AF+BF*BF-CF*CF
      YF1=AF+SQRT(ROOTF)
      YF2=AF-SQRT(ROOTF)
      XF=BF+CF
      PSINEX1=2.*ATAN2(YF1,XF)
      PSINEX2=2.*ATAN2(YF2,XF)
      IF (PSINEX1.LT.0.) PSINEX1=PSINEX1+2.*PI
      IF (PSINEX2.LT.0.) PSINEX2=PSINEX2+2.*PI
      IF (ABS(PSINEX1-PSIT).LT.ABS(PSINEX2-PSIT)) GO TO 1
      PSINEXT=PSINEX2
      GO TO 2
    1 PSINEXT=PSINEX1
    2 G=(ACG*SIN(PHINEXT+DELG)+RHOG*COS(PSINEXT-ALPHP)-B*SIN(BETA))/SIN(
     1PSINEXT-ALPHP)
      RETURN
      END
```

```
      SUBROUTINE TRANS2 (RHOG,ALPHP,BETA,FP,ACG,B,DELG,Z,PSIT,PHIT,G)      C   1
      PI=3.14159                                                           C   2
      ST=(RHOG*COS(PSIT+ALPHP)+B*SIN(BETA)+FP*SIN(PSIT+ALPHP))/ACG         C   3
      CT=(-RHOG*SIN(PSIT+ALPHP)+B*COS(BETA)+FP*COS(PSIT+ALPHP))/ACG        C   4
      PHIT=ATAN2(ST,CT)+DELG                                               C   5
      PHINEXT=PHIT+.1*Z                                                    C   6
      AF=ACG*COS(PHINEXT-DELG-ALPHP)-B*COS(BETA-ALPHP)                     C   7
      BF=-ACG*SIN(PHINEXT-DELG-ALPHP)+B*SIN(BETA-ALPHP)                    C   8
      CF=-RHOG                                                             C   9
      ROOTF=AF*AF+BF*BF-CF*CF                                              C  10
      YF1=AF+SQRT(ROOTF)                                                   C  11
      YF2=AF-SQRT(ROOTF)                                                   C  12
      XF=BF+CF                                                             C  13
      PSINEX1=2.*ATAN2(YF1,XF)                                             C  14
      PSINEX2=2.*ATAN2(YF2,XF)                                             C  15
      IF (PSINEX1.LT.0.) PSINEX1=PSINEX1+2.*PI                             C  16
      IF (PSINEX2.LT.0.) PSINEX2=PSINEX2+2.*PI                             C  17
      IF (ABS(PSINEX1-PSIT).LT.ABS(PSINEX2-PSIT)) GO TO 1                  C  18
      PSINEXT=PSINEX2                                                      C  19
      GO TO 2                                                              C  20
    1 PSINEXT=PSINEX1                                                      C  21
    2 G=(ACG*SIN(PHINEXT-DELG)-RHOG*COS(PSINEXT+ALPHP)-B*SIN(BETA))/SIN(   C  22
     1PSINEXT+ALPHP)                                                       C  23
      RETURN                                                               C  24
      END                                                                  C  25
```

```
MIN =   .1645   MU =  .200   RPM = 1000.

CAPRP1 =  .47725   CAPRP2 =  .20670   CAPRP3 =  .17560

RP2 =  .09085   RP3 =  .06890   RP4 =  .05905

ACG1 =  .47725   ACG2 =  .20670   ACG3 =  .17560

ACP1 =  .09085   ACP2 =  .06890   ACP3 =  .05905

R1 =  .75000   R2 =  .75000   R3 =  .75000   R4 =  .75000

RHOG1 =  .03870   RHOG2 =  .02070   RHOG3 =  .01910   RHOP1 =  .01740   RHOP2 =  .01C40   RHOP3 =  .00875

TG1 =  .03480   TG2 =  .02520   TG3 =  .02170   TP1 =  .02800   TP2 =  .02080   TP3 =  .01750

NG1 =  42.   NG2 =  27.   NG3 =  27.   NP2 =  8.   NP3 =  9.   NP4 =  9.

M1 =     .69515E-04   M2 =     .97028E-05   M3 =     .70027E-05   M4 =     .79188E-06

RHO1 =  .060   RHO2 =  .030   RHO3 =  .025   RHO4 =  .020

MD =   .1500E-04

K =  25.0

PHDOT1 = -1.0

FP1 =  .08917   FP2 =  .06811   FP3 =  .05840

BETA1D =112.2552   BETA2D =145.0978   BETA3D =164.6850


PSI1T1D = 305.6017   TEST11 =   4.4495
PSI1T2D = 343.6259   TEST12 =  42.4736


PHI1TD =113.5016   PSI1TD =305.6017
PHI1ID =115.9930   PSI1ID =292.4283   PHI1FD =107.4216   PSI1FD =337.4283


PSI2T1D = 286.9442   TEST21 =  29.4719
PSI2T2D = 312.0785   TEST22 =   4.3377


PHI2TD =142.0461   PSI2TD =312.0785
PHI2ID =136.7828   PSI2ID =330.9521   PHI2FD =150.1181   PSI2FD =290.9521


PSI3T1D = 357.5003   TEST31 =   4.2938
PSI3T2D =  25.1348   TEST32 =  31.9282


PHI3TD =166.7857   PSI3TD =357.5003
PHI3ID =173.6791   PSI3ID =336.8610   PHI3FD =160.3457   PSI3FD = 16.8610


PHI1  PHI2  PHI3  PSI1  PSI2  PSI3  DPSI1  DPSI2  DPSI3  S1R  S2R  S3R  S1F   G1    S2F   G2   S3F   G3   POINTEF

116.  137.  174.  292.  331.  337.     5.   -16.    59.   1.  -1.   1.                                      .468
```

116.	137.	173.	293.	330.	339.	5.	-16.	49.	1.	-1.	1.						.476
116.	137.	173.	293.	330.	340.	5.	-16.	48.	1.	-1.	1.						.484
116.	137.	172.	293.	329.	342.	5.	-16.	48.	1.	-1.	1.						.492
116.	137.	172.	293.	329.	343.	5.	-16.	47.	1.	-1.	1.						.499
116.	138.	171.	293.	328.	345.	5.	-16.	47.	1.	-1.	1.						.506
116.	138.	170.	294.	328.	347.	5.	-16.	47.	1.	-1.	1.						.512
116.	138.	170.	294.	327.	348.	5.	-16.	47.	1.	-1.	1.						.519
116.	138.	169.	294.	327.	350.	5.	-16.	47.	1.	-1.	1.						.526
116.	138.	169.	294.	326.	351.	5.	-16.	47.	1.	-1.	1.						.532
116.	139.	168.	294.	326.	353.	5.	-16.	47.	1.	-1.	1.						.539
116.	139.	168.	294.	325.	355.	5.	-16.	47.	1.	-1.	-1.						.531
116.	139.	167.	295.	324.	356.	5.	-16.	47.	1.	-1.	-1.						.529
116.	139.	167.	295.	324.	358.	5.	-16.	47.	1.	-1.						1. .058	.527
116.	139.	166.	295.	323.	359.	5.	-16.	48.	1.	-1.						1. .058	.525
115.	139.	166.	295.	323.	1.	5.	-16.	48.	1.	-1.						1. .057	.524
115.	140.	165.	295.	322.	3.	5.	-16.	48.	1.	-1.						1. .057	.523
115.	140.	165.	295.	322.	4.	5.	-16.	48.	1.	-1.						1. .057	.521
115.	140.	164.	296.	321.	6.	5.	-16.	48.	1.	-1.						1. .056	.520
115.	140.	164.	296.	321.	8.	5.	-16.	48.	1.	-1.						1. .056	.519
115.	140.	163.	296.	320.	9.	5.	-16.	48.	1.	-1.						1. .056	.517
115.	141.	162.	296.	320.	11.	5.	-16.	47.	1.	-1.						1. .056	.515
115.	141.	162.	296.	319.	12.	5.	-16.	47.	1.	-1.						1. .056	.514
115.	141.	161.	297.	319.	14.	5.	-16.	46.	1.	-1.						1. .055	.512
115.	141.	161.	297.	318.	15.	5.	-16.	45.	1.	-1.						1. .056	.510
115.	141.	174.	297.	318.	337.	5.	-16.	49.	1.	-1.	1.						.521
115.	141.	173.	297.	317.	339.	5.	-16.	48.	1.	-1.	1.						.529
115.	142.	173.	297.	316.	340.	5.	-16.	48.	1.	-1.	1.						.537
115.	142.	172.	297.	316.	342.	5.	-16.	48.	1.	1.	1.						.551
115.	142.	172.	298.	315.	343.	5.	-16.	47.	1.	1.	1.						.554
115.	142.	171.	298.	315.	345.	5.	-16.	47.	1.	1.	1.						.558
115.	142.	170.	298.	314.	347.	5.	-16.	47.	1.	1.	1.						.562
115.	142.	170.	298.	314.	348.	5.	-16.	47.	1.	1.	1.						.565
115.	143.	169.	298.	313.	350.	5.	-16.	47.	1.	1.	1.						.569
115.	143.	169.	298.	313.	351.	5.	-16.	48.	1.	1.	1.						.572
115.	143.	168.	299.	312.	353.	5.	-16.	48.	1.	1.	1.						.575
115.	143.	168.	299.	312.	355.	5.	-16.	48.	1.		-1.	1.	.068				.563
115.	143.	167.	299.	311.	356.	5.	-16.	48.	1.		-1.	1.	.068				.557
115.	144.	167.	299.	311.	358.	5.	-16.	49.	1.			1.	.068	1.	.058		.551
115.	144.	166.	299.	310.	360.	5.	-16.	49.	1.			1.	.067	1.	.053		.545
115.	144.	166.	300.	309.	1.	5.	-16.	50.	1.			1.	.067	1.	.057		.540
115.	144.	165.	300.	309.	3.	5.	-16.	50.	1.			1.	.067	1.	.057		.535
115.	144.	164.	300.	308.	5.	5.	-16.	50.	1.			1.	.067	1.	.057		.530
115.	144.	164.	300.	308.	6.	5.	-16.	50.	1.			1.	.067	1.	.056		.525
115.	145.	163.	300.	307.	8.	5.	-16.	50.	1.			1.	.067	1.	.056		.520
114.	145.	163.	300.	307.	10.	5.	-16.	49.	1.			1.	.067	1.	.056		.515
114.	145.	162.	301.	306.	11.	5.	-16.	49.	1.			1.	.066	1.	.056		.510
114.	145.	162.	301.	306.	13.	5.	-16.	48.	1.			1.	.066	1.	.056		.505
114.	145.	161.	301.	305.	15.	5.	-16.	47.	1.			1.	.066	1.	.056		.500
114.	146.	161.	301.	304.	16.	5.	-16.	47.	-1.			1.	.066	1.	.056		.509
114.	146.	174.	301.	304.	337.	5.	-16.	50.	-1.		1.	1.	.066				.518
114.	146.	173.	302.	303.	339.	5.	-16.	50.	-1.		1.	1.	.066				.522
114.	146.	173.	302.	303.	340.	5.	-16.	49.	-1.		1.	1.	.066				.525
114.	146.	172.	302.	302.	342.	5.	-16.	49.	-1.		1.	1.	.066				.528
114.	146.	171.	302.	302.	344.	5.	-16.	48.	-1.		1.	1.	.066				.531
114.	147.	171.	302.	301.	345.	5.	-16.	48.	-1.		1.	1.	.066				.534
114.	147.	170.	302.	301.	347.	5.	-16.	48.	-1.		1.	1.	.066				.537
114.	147.	170.	303.	300.	348.	5.	-16.	48.	-1.		1.	1.	.066				.539
114.	147.	169.	303.	300.	350.	5.	-16.	48.	-1.		1.	1.	.066				.541
114.	147.	169.	303.	299.	352.	5.	-16.	48.	-1.		1.	1.	.066				.544
114.	147.	168.	303.	298.	353.	5.	-16.	47.	-1.		-1.	1.	.066				.539
114.	148.	168.	303.	298.	355.	5.	-16.	47.	-1.		-1.	1.	.066				.532

114.	148.	167.	304.	297.	356.	5.	-16.	47.	-1.		-1.			1.	.066				.526
114.	148.	167.	304.	297.	358.	5.	-16.	47.	-1.					1.	.066	1.	.058		.520
114.	148.	166.	304.	296.	360.	5.	-16.	48.	-1.					1.	.066	1.	.058		.515
114.	148.	166.	304.	296.	1.	5.	-16.	48.	-1.					1.	.066	1.	.057		.509
114.	149.	165.	304.	295.	3.	5.	-15.	47.	-1.					1.	.066	1.	.057		.504
114.	149.	164.	304.	295.	4.	5.	-15.	47.	-1.					1.	.066	1.	.057		.499
114.	149.	64.	305.	294.	6.	5.	-15.	47.	-1.					1.	.066	1.	.056		.494
114.	149.	163.	305.	294.	8.	5.	-15.	46.	-1.					1.	.066	1.	.056		.490
114.	149.	163.	305.	293.	9.	5.	-15.	46.	-1.					1.	.066	1.	.056		.485
114.	149.	162.	305.	293.	11.	5.	-15.	45.	-1.					1.	.066	1.	.056		.480
114.	150.	162.	305.	292.	12.	5.	-15.	44.	-1.					1.	.066	1.	.056		.475
114.	150.	161.	305.	292.	14.	5.	-15.	43.	-1.					1.	.067	1.	.056		.470
113.	150.	161.	306.	291.	15.	5.	-15.	42.				1.	.089	1.	.067	1.	.056		.465
113.	137.	161.	306.	331.	15.	5.	-16.	48.		-1.		1.	.089			1.	.056		.481
113.	137.	160.	306.	330.	17.	5.	-16.	47.		-1.		1.	.089			1.	.056		.479
113.	137.	174.	306.	330.	337.	5.	-16.	51.		-1.	1.	1.	.089						.492
113.	137.	173.	306.	329.	339.	5.	-16.	50.		-1.	1.	1.	.089						.499
113.	138.	173.	307.	329.	340.	5.	-16.	49.		-1.	1.	1.	.089						.506
113.	138.	172.	307.	328.	342.	5.	-16.	49.		-1.	1.	1.	.088						.513
113.	138.	171.	307.	328.	344.	5.	-16.	49.		-1.	1.	1.	.088						.519
113.	138.	171.	307.	327.	345.	5.	-16.	48.		-1.	1.	1.	.088						.525
113.	138.	170.	307.	327.	347.	5.	-16.	48.		-1.	1.	1.	.088						.531
113.	138.	170.	308.	326.	348.	5.	-16.	48.		-1.	1.	1.	.088						.537
113.	139.	169.	308.	325.	350.	5.	-16.	48.		-1.	1.	1.	.088						.543
113.	139.	169.	308.	325.	352.	5.	-16.	48.		-1.	1.	1.	.088						.548
113.	139.	168.	308.	324.	353.	5.	-16.	49.		-1.	-1.	1.	.088						.546
113.	139.	168.	308.	324.	355.	5.	-16.	49.		-1.	-1.	1.	.088						.542
113.	139.	167.	308.	323.	357.	5.	-16.	49.		-1.	-1.	1.	.087						.539
113.	140.	166.	309.	323.	358.	5.	-16.	49.		-1.		1.	.087			1.	.058		.536
113.	140.	166.	309.	322.	0.	5.	-16.	50.		-1.		1.	.087			1.	.058		.533
113.	140.	165.	309.	322.	2.	5.	-16.	50.		-1.		1.	.087			1.	.057		.530
113.	140.	165.	309.	321.	3.	5.	-16.	50.		-1.		1.	.087			1.	.057		.528
113.	140.	164.	309.	320.	5.	5.	-16.	50.		-1.		1.	.087			1.	.056		.525
113.	140.	164.	310.	320.	7.	5.	-16.	50.		-1.		1.	.087			1.	.056		.523
113.	141.	163.	310.	319.	9.	5.	-16.	50.		-1.		1.	.087			1.	.056		.520
113.	141.	163.	310.	319.	10.	5.	-16.	50.		-1.		1.	.087			1.	.056		.517
113.	141.	162.	310.	318.	12.	5.	-16.	49.		-1.		1.	.087			1.	.056		.514
113.	141.	162.	310.	318.	14.	5.	-16.	48.		-1.		1.	.086			1.	.056		.511
113.	141.	161.	310.	317.	15.	5.	-17.	48.		-1.		1.	.086			1.	.056		.508
113.	142.	160.	311.	317.	17.	5.	-17.	47.		-1.		1.	.086			1.	.056		.505
113.	142.	174.	311.	316.	337.	5.	-17.	51.		1.	1.	1.	.086						.525
113.	142.	173.	311.	315.	339.	6.	-17.	51.		1.	1.	1.	.086						.529
112.	142.	173.	311.	315.	340.	6.	-17.	50.		1.	1.	1.	.086						.532
112.	142.	172.	311.	314.	342.	6.	-17.	50.		1.	1.	1.	.086						.536
112.	143.	171.	312.	314.	344.	6.	-17.	50.		1.	1.	1.	.086						.538
112.	143.	171.	312.	313.	345.	6.	-17.	50.		1.	1.	1.	.086						.541
112.	143.	170.	312.	313.	347.	6.	-17.	50.		1.	1.	1.	.086						.543
112.	143.	170.	312.	312.	349.	6.	-17.	50.			1.	1.	.086	1.	.068				.546
112.	143.	169.	312.	311.	350.	6.	-17.	50.			1.	1.	.085	1.	.068				.548
112.	143.	169.	313.	311.	352.	6.	-17.	50.			1.	1.	.085	1.	.068				.549
112.	144.	168.	313.	310.	354.	6.	-17.	50.			-1.	1.	.085	1.	.068				.541
112.	144.	167.	313.	310.	356.	6.	-17.	51.			-1.	1.	.085	1.	.067				.534
112.	144.	167.	313.	309.	357.	6.	-17.	51.			-1.	1.	.085	1.	.067				.527
112.	144.	166.	313.	309.	359.	6.	-17.	52.				1.	.085	1.	.067	1.	.058		.520
112.	144.	166.	313.	308.	1.	6.	-17.	52.				1.	.085	1.	.067	1.	.057		.514
112.	145.	165.	314.	307.	2.	6.	-17.	52.				1.	.085	1.	.067	1.	.057		.507
112.	145.	165.	314.	307.	4.	6.	-17.	52.				1.	.085	1.	.067	1.	.057		.501
112.	145.	164.	314.	306.	6.	6.	-17.	52.				1.	.085	1.	.066	1.	.056		.496
112.	145.	163.	314.	306.	8.	6.	-17.	52.				1.	.085	1.	.066	1.	.056		.490
112.	145.	163.	314.	305.	10.	6.	-17.	52.				1.	.085	1.	.066	1.	.056		.484
112.	145.	162.	315.	305.	11.	6.	-17.	51.				1.	.085	1.	.066	1.	.056		.478

112.	146.	162.	315.	304.	13.	6.	-17.	50.			1.	.084	1.	.066	1.	.056	.472
112.	146.	161.	315.	303.	15.	6.	-17.	49.			1.	.084	1.	.066	1.	.056	.467
112.	146.	161.	315.	303.	16.	6.	-17.	48.			1.	.084	1.	.066	1.	.056	.461
112.	146.	174.	315.	302.	337.	6.	-17.	52.		1.	1.	.084	1.	.066			.469
112.	146.	173.	316.	302.	339.	6.	-17.	52.		1.	1.	.084	1.	.066			.473
112.	147.	173.	316.	301.	340.	6.	-17.	51.		1.	1.	.084	1.	.066			.476
112.	147.	172.	316.	301.	342.	6.	-17.	50.		1.	1.	.084	1.	.066			.479
112.	147.	171.	316.	300.	344.	6.	-17.	50.		1.	1.	.084	1.	.066			.482
112.	147.	171.	316.	299.	345.	6.	-17.	49.		1.	1.	.084	1.	.066			.484
112.	147.	170.	316.	299.	347.	6.	-17.	49.		1.	1.	.084	1.	.066			.487
111.	148.	170.	317.	298.	349.	6.	-17.	49.		1.	1.	.084	1.	.066			.489
111.	148.	169.	317.	298.	350.	6.	-16.	49.		1.	1.	.084	1.	.066			.491
111.	148.	169.	317.	297.	352.	6.	-16.	49.		1.	1.	.084	1.	.066			.493
111.	148.	168.	317.	297.	354.	6.	-16.	49.		-1.	1.	.084	1.	.066			.488
111.	148.	167.	317.	296.	355.	6.	-16.	48.		-1.	1.	.084	1.	.066			.480
111.	148.	167.	318.	296.	357.	6.	-16.	48.		-1.	1.	.084	1.	.066			.475
111.	149.	166.	318.	295.	359.	6.	-16.	48.			1.	.083	1.	.066	1.	.058	.469
111.	149.	166.	318.	294.	0.	6.	-16.	48.			1.	.083	1.	.066	1.	.058	.464
111.	149.	165.	318.	294.	2.	6.	-16.	48.			1.	.083	1.	.066	1.	.057	.459
111.	149.	165.	318.	293.	4.	6.	-16.	48.			1.	.083	1.	.066	1.	.057	.455
111.	149.	164.	319.	293.	5.	6.	-15.	48.			1.	.083	1.	.066	1.	.056	.450
111.	150.	164.	319.	292.	7.	6.	-15.	47.			1.	.083	1.	.066	1.	.056	.446
111.	150.	163.	319.	292.	8.	6.	-15.	46.			1.	.083	1.	.067	1.	.056	.442
111.	150.	163.	319.	291.	10.	6.	-15.	46.			1.	.083	1.	.067	1.	.056	.437
111.	137.	163.	319.	331.	10.	6.	-17.	51.	-1.		1.	.083			1.	.056	.451
111.	137.	162.	319.	330.	12.	5.	-17.	50.	-1.		1.	.083			1.	.056	.449
111.	137.	162.	320.	330.	13.	5.	-17.	49.	-1.		1.	.083			1.	.056	.447
111.	137.	161.	320.	329.	15.	5.	-17.	48.	-1.		1.	.083			1.	.056	.445
111.	138.	160.	320.	329.	17.	5.	-17.	47.	-1.		1.	.083			1.	.056	.444
111.	138.	174.	320.	328.	337.	5.	-17.	51.	-1.	1.	1.	.083					.456
111.	138.	173.	320.	328.	339.	5.	-17.	51.	-1.	1.	1.	.083					.462
111.	138.	173.	321.	327.	340.	5.	-16.	50.	-1.	1.	1.	.083					.459
111.	138.	172.	321.	326.	342.	5.	-16.	49.	-1.	1.	1.	.083					.475
111.	138.	171.	321.	326.	344.	5.	-16.	49.	-1.	1.	1.	.083					.481
111.	139.	171.	321.	325.	345.	5.	-15.	49.	-1.	1.	1.	.083					.487
111.	139.	170.	321.	325.	347.	5.	-16.	49.	-1.	1.	1.	.093					.492
111.	139.	170.	321.	324.	349.	5.	-16.	48.	-1.	1.	1.	.083					.498
111.	139.	169.	322.	324.	350.	5.	-16.	48.	-1.	1.	1.	.083					.503
111.	139.	169.	322.	323.	352.	5.	-16.	48.	-1.	1.	1.	.083					.508
111.	140.	168.	322.	323.	353.	5.	-16.	48.	-1.	-1.	1.	.083					.505
110.	140.	168.	322.	322.	355.	5.	-16.	49.	-1.	-1.	1.	.082					.502
110.	140.	167.	322.	321.	357.	5.	-16.	49.	-1.	-1.	1.	.082					.499
110.	140.	166.	323.	321.	358.	5.	-16.	49.	-1.		1.	.082			1.	.058	.496
110.	140.	166.	323.	320.	0.	5.	-16.	49.	-1.		1.	.082			1.	.058	.494
110.	140.	165.	323.	320.	2.	5.	-16.	50.	-1.		1.	.082			1.	.057	.491
110.	141.	165.	323.	319.	3.	5.	-16.	50.	-1.		1.	.082			1.	.057	.489
110.	141.	164.	323.	319.	5.	5.	-16.	50.	-1.		1.	.082			1.	.056	.487
110.	141.	164.	323.	318.	7.	5.	-16.	49.	-1.		1.	.082			1.	.056	.485
110.	141.	163.	324.	318.	8.	5.	-16.	49.	-1.		1.	.082			1.	.056	.482
110.	141.	163.	324.	317.	10.	5.	-16.	49.	-1.		1.	.082			1.	.056	.480
110.	142.	162.	324.	317.	12.	5.	-16.	48.	-1.		1.	.082			1.	.056	.477
110.	142.	162.	324.	316.	13.	5.	-16.	47.	1.		1.	.082			1.	.056	.480
110.	142.	161.	324.	315.	15.	5.	-16.	47.	1.		1.	.082			1.	.056	.475
110.	142.	160.	325.	315.	17.	5.	-16.	46.	1.		1.	.082			1.	.056	.470
110.	142.	174.	325.	314.	337.	5.	-16.	50.	1.	1.	1.	.082					.479
110.	142.	173.	325.	314.	339.	5.	-16.	49.	1.	1.	1.	.082					.482
110.	143.	173.	325.	313.	340.	5.	-16.	49.	1.	1.	1.	.082					.486
110.	143.	172.	325.	313.	342.	5.	-16.	48.	1.	1.	1.	.082					.488
110.	143.	171.	325.	312.	343.	5.	-16.	48.	1.	1.	1.	.082					.491
110.	143.	171.	326.	312.	345.	5.	-16.	48.		1.	1.	.082	1.	.068			.493
110.	143.	170.	325.	311.	347.	5.	-16.	48.		1.	1.	.082	1.	.068			.495

110.	144.	170.	326.	311.	348.	5.	-16.	48.			1.	1.	.082	1.	.068			.497
110.	144.	169.	326.	310.	350.	5.	-16.	48.			1.	1.	.082	1.	.067			.499
110.	144.	169.	326.	309.	352.	5.	-16.	48.			1.	1.	.082	1.	.067			.501
110.	144.	168.	327.	309.	353.	5.	-16.	48.			1.	1.	.082	1.	.067			.503
110.	144.	168.	327.	308.	355.	5.	-16.	48.			-1.	1.	.082	1.	.067			.490
110.	144.	167.	327.	308.	356.	5.	-16.	48.			-1.	1.	.082	1.	.067			.484
110.	145.	167.	327.	307.	358.	5.	-16.	48.				1.	.082	1.	.067	1.	.058	.478
110.	145.	166.	327.	307.	360.	5.	-16.	49.				1.	.082	1.	.067	1.	.058	.473
109.	145.	166.	327.	306.	1.	5.	-16.	49.				1.	.082	1.	.066	1.	.057	.468
109.	145.	165.	328.	306.	3.	5.	-16.	49.				1.	.082	1.	.066	1.	.057	.463
109.	145.	164.	329.	305.	5.	5.	-16.	49.				1.	.082	1.	.066	1.	.057	.458
109.	145.	164.	328.	305.	6.	5.	-16.	49.				1.	.082	1.	.066	1.	.056	.453
109.	146.	163.	328.	304.	8.	5.	-16.	48.				1.	.082	1.	.066	1.	.056	.448
109.	146.	163.	328.	303.	10.	5.	-16.	48.				1.	.082	1.	.066	1.	.056	.443
109.	146.	162.	328.	303.	11.	5.	-16.	47.				1.	.082	1.	.066	1.	.056	.438
109.	146.	162.	329.	302.	13.	5.	-16.	46.				1.	.082	1.	.066	1.	.056	.434
109.	146.	161.	329.	302.	14.	5.	-16.	45.				1.	.082	1.	.066	1.	.056	.429
109.	147.	161.	329.	301.	16.	5.	-16.	44.				1.	.082	1.	.066	1.	.056	.424
109.	147.	174.	329.	301.	337.	5.	-15.	48.			1.	1.	.082	1.	.066			.431
109.	147.	173.	329.	300.	338.	5.	-15.	47.			1.	1.	.082	1.	.066			.435
109.	147.	173.	330.	300.	340.	5.	-15.	46.			1.	1.	.082	1.	.066			.438
109.	147.	172.	330.	299.	342.	5.	-15.	46.			1.	1.	.082	1.	.066			.440
109.	147.	172.	330.	299.	343.	5.	-15.	45.			1.	1.	.082	1.	.066			.443
109.	148.	171.	330.	298.	345.	5.	-15.	45.			1.	1.	.082	1.	.066			.445
109.	148.	171.	330.	298.	346.	5.	-15.	44.			1.	1.	.082	1.	.066			.447
109.	148.	170.	330.	297.	348.	5.	-15.	44.			1.	1.	.083	1.	.066			.449
109.	148.	170.	331.	297.	349.	5.	-15.	44.			1.	1.	.083	1.	.066			.451
109.	148.	169.	331.	296.	351.	5.	-15.	43.			1.	1.	.083	1.	.066			.453
109.	148.	169.	331.	296.	352.	5.	-14.	43.			1.	1.	.083	1.	.066			.455
109.	149.	168.	331.	295.	354.	5.	-14.	43.			-1.	1.	.083	1.	.066			.449
109.	149.	168.	331.	295.	355.	5.	-14.	43.			-1.	1.	.083	1.	.066			.444
109.	149.	167.	331.	294.	356.	5.	-14.	42.			-1.	1.	.083	1.	.066			.440
109.	149.	167.	332.	294.	358.	5.	-14.	42.				1.	.083	1.	.066	1.	.058	.435
109.	149.	166.	332.	293.	359.	5.	-14.	42.				1.	.083	1.	.066	1.	.058	.431
109.	149.	166.	332.	293.	1.	5.	-14.	42.				1.	.083	1.	.066	1.	.057	.427
109.	150.	165.	332.	292.	2.	5.	-14.	42.				1.	.083	1.	.066	1.	.057	.424
109.	150.	165.	332.	292.	4.	5.	-14.	41.				1.	.083	1.	.066	1.	.057	.420
109.	150.	164.	332.	292.	5.	5.	-13.	41.				1.	.083	1.	.067	1.	.056	.416
108.	150.	164.	333.	291.	6.	5.	-13.	41.				1.	.083	1.	.067	1.	.056	.413
108.	137.	164.	333.	331.	6.	5.	-15.	45.	-1.			1.	.083			1.	.056	.427
108.	137.	163.	333.	330.	8.	5.	-15.	45.	-1.			1.	.083			1.	.056	.426
108.	137.	163.	333.	330.	9.	5.	-15.	44.	-1.			1.	.083			1.	.056	.425
108.	137.	162.	333.	329.	11.	5.	-15.	44.	-1.			1.	.083			1.	.056	.423
108.	137.	162.	333.	329.	12.	5.	-15.	43.	-1.			1.	.083			1.	.056	.422
108.	138.	161.	334.	326.	14.	5.	-14.	42.	-1.			1.	.083			1.	.056	.420
108.	138.	161.	334.	328.	15.	5.	-14.	41.	-1.			1.	.083			1.	.058	.419
108.	138.	160.	334.	327.	17.	5.	-14.	41.	-1.			1.	.083			1.	.058	.417
108.	138.	174.	334.	327.	337.	5.	-14.	44.	-1.	1.		1.	.083					.428
108.	138.	173.	334.	327.	338.	5.	-14.	43.	-1.	1.		1.	.083					.434
108.	138.	173.	334.	326.	340.	5.	-14.	43.	-1.	1.		1.	.083					.439
108.	139.	172.	334.	326.	341.	5.	-14.	42.	-1.	1.		1.	.084					.444
108.	139.	172.	335.	325.	343.	5.	-14.	42.	-1.	1.		1.	.084					.449
108.	139.	171.	335.	325.	344.	5.	-14.	42.	-1.	1.		1.	.084					.453
108.	139.	171.	335.	324.	345.	5.	-14.	41.	-1.	1.		1.	.084					.458
108.	139.	170.	335.	324.	347.	5.	-14.	41.	-1.	1.		1.	.084					.462
108.	139.	170.	335.	323.	348.	5.	-14.	41.	-1.	1.		1.	.084					.467
108.	139.	169.	335.	323.	350.	5.	-14.	41.	-1.	1.		1.	.084					.471
108.	140.	169.	336.	322.	351.	5.	-14.	41.	-1.	1.		1.	.084					.475
108.	140.	168.	336.	322.	352.	5.	-14.	41.	-1.	1.		1.	.084					.479
108.	140.	168.	336.	321.	354.	5.	-14.	41.	-1.	-1.		1.	.084					.474
108.	140.	168.	336.	321.	355.	5.	-14.	41.	-1.	-1.		1.	.084					.472

108.	140.	167.	336.	320.	357.	5.	-14.	41.	-1.	-1.	1.	.084				.470
108.	140.	167.	336.	320.	358.	5.	-14.	41.	-1.		1.	.084		1.	.058	.467
108.	141.	166.	337.	320.	359.	4.	-13.	41.	-1.		1.	.084		1.	.058	.465
108.	141.	166.	337.	319.	1.	4.	-13.	41.	-1.		1.	.084		1.	.057	.464
108.	141.	165.	337.	319.	2.	4.	-13.	41.	-1.		1.	.084		1.	.057	.462
108.	141.	165.	337.	318.	3.	4.	-13.	41.	-1.		1.	.085		1.	.057	.460
107.	141.	164.	337.	318.	5.	4.	-13.	41.	-1.		1.	.085		1.	.057	.458
107.	141.	164.	337.	317.	6.	4.	-13.	41.	-1.		1.	.085		1.	.056	.456

CYCLE EFFICIENCY = .491

2. Program CLOCK 4: Point and Cycle Efficiencies for Two Pass Clock (Ogival) Step-Up Gear Train in Spin Environment

The kinematics of program CLOCK 4 is again based on the work in Appendix G. The moment input-output relationships are derived in section 2 of Appendix H. This program is in many ways very similar to CLOCK 3 with the exception that only two meshes are involved, and therefore, wherever possible, reference will be made to CLOCK 3. Again, it is assumed that the two meshes will have been tested by program CLOCK 1 for their geometric suitability. The format of the following is identical to that used in section 1 of this appendix. For the sake of clarity, it will be helpful to refer to these parallel descriptions.

a. Input Parameters (see Program CLOCK 4, below)

The following parameters represent the input data for the program (for explanation, refer to section 1a of this appendix):

MU

RPM

CAPRP1, CAPRP2, RP2, RP3

RHOG1, RHOG2, RHOP1, RHOP2

ACG1, ACG2, ACP1, ACP2

R1, R2, R3

TG1, TG2, TP1, TP2

NG1, NG2, NP2, NP3

RHO1, RHO2, RHO3

M1, M2, M3

MD

K

PHDOT1 = -1

b. Computations (see also COMMENT cards in program)

I. Computation of Gear Tooth Parameters

The required computations are identical to those in CLOCK 3.

II. Computation of MIN, GAMMAS and BETAS

The input moment is computed in the manner of eq. (I-1). In addition, the angles γ_2, γ_3, β_1 and β_2 are found according to the expressions given in section 6b of Appendix A.

III. Computation of Other Parameters

The computation of the angles $\Delta\varphi_1$ and $\Delta\psi_1$, the length L_1 as well as the centrifugal forces Q_1, Q_2 and Q_3 (called Q_{3P} by eq. (H-245)) are identical to those described in the parallel section dealing with CLOCK 3.

IV. Preliminary Computations for Mesh 1

The preliminary computations for mesh 1 are identical to those given in section 1-IV of this appendix.

V. Preliminary Computations for Mesh 2

The preliminary computations for mesh 2 are identical to those given in section 1-V of this appendix.

VI. Gear Train Motion Model: Kinematics, Point and Cycle Efficiencies

The simulation of the gear train model, which is necessary for the determination of both POINTEF and CYCLEFF, is found in a loop starting with statement label no. 20 and ending with card no. 531. The motions of the individual driving gears are initialized at their respective angles PHI1I and PHI2I. The meshes will be in round on round contact until they reach their respective transition angles PHI1T and PHI2T. After the transition angles are passed, the meshes will be in round on flat contact. These regimes continue until the final angles PHI1F and PHI2F are reached.

The increment DDPHI1 of the input gear 1 is obtained from an adaptation of eqs. (A-207) and (A-208), in which tooth numbers, rather than base circle radii are used. The increment DDPHI2 of gear 2 is related to the increment of the pinion angle PSI1.

While the motion of gear 1 is terminated when the angle PHI1 reaches the magnitude PHI1F (or rather PHI1F + DDPHI1 for moment summation purposes), gear 2 must be reset to its starting angle PHI2I whenever its final angle PHI2F has been reached.

The appropriate choice of moment equation depends upon which of the four possible combinations of contact conditions, as indicated by Table H-2, is applicable.

The following discusses the kinematics of the individual meshes as well as the determination of the point and cycle efficiencies where they differ from the description in section 1 of this appendix.

A. Kinematics

The program only utilizes the kinematics of meshes 1 and 2. These are identical with those for program CLOCK 3, as given in section 1.

B. Moment Computations, Point and Cycle Efficiencies

Regardless of the combination of contact conditions, the point efficiency is computed according to eq. (3), i.e.,

$$\eta_p = \text{POINTEF} = K_{ratio} \frac{M_{o31}}{M_{in}} \qquad \text{(I-24)}$$

where, with $\dot{\phi}_1 = -1$

$$K_{ratio} = |\dot{\psi}_2| \qquad \text{(I-25)}$$

The cycle efficiency determination is based on eqs. (I-21) to (I-23).

The moment computations begin with the statement label no.24 , and initially consist of the determination of selected variables between A11 and A72 and selected variables between C6 and C36, as applicable to the analyses of section 2 of Appendix H. The governing contact combination (see also Table H-2) is determined with the help of the four moment control statements, which start with card no.498 . Once the appropriate combination is established, the program is directed to one of the four associated moment expressions. These expressions for M_{o31} coincide with those given by eqs. (H-260), (H-261), (H-277) and (H-278). They are listed in the above

order beginning with statement label no. 25 and ending with statement label no. 28 .

The rationale of the control statements for meshes 1 and 2 is identical to that given for program CLOCK 3 (see section 1-VIIB of this appendix).

c. Output (see Program CLOCK 4, below)

The output of the program is best explained with the help of the sample problem at the end of the program.

I. Input Parameters

Mesh 1

CAPRP1 = R_{P1} = .47725 in. (1.212 cm)

RP2 = r_{P2} = .09085 in. (0.231 cm)

ACG1 = a_{G1} = .47725 in. (1.212 cm)

ACP1 = a_{P1} = .09085 in. (0.231 cm)

RHOG1 = ρ_{G1} = .03870 in. (0.098 cm)

RHOP1 = ρ_{P1} = .01740 in. (0.044 cm)

TG1 = t_{G1} = .03480 in. (0.088 cm)

TP1 = t_{P1} = .02800 in. (0.71 cm)

NG1 = n_{G1} = 42

NP2 = n_{P2} = 8

Mesh 2

CAPRP2 = R_{P2} = .20670 in. (0.525 cm)

RP3 = r_{P3} = .06890 in. (0.175 cm)

ACG2 = a_{G2} = .20670 in. (0.525 cm)

ACP2 = a_{P2} = .06890 in. (0.175 cm)

RHOG2 = ρ_{G2} = .02070 in. (0.053 cm)

RHOP2 = ρ_{P2} = .01040 in. (0.026 cm)

TG2 = t_{G2} = .02520 in. (0.064 cm)

TP2 = t_{P2} = .02080 in. (0.053 cm)

NG2 = n_{G2} = 27

NP3 = n_{P3} = 9

In addition

MU = .2

RPM = 1000

M1 = m_1 = .69515 x 10^{-4} lb-sec^2/in. (12.171 g)

M2 = m_2 = .97028 x 10^{-5} lb-sec^2/in. (1.699 g)

M3 = m_3 = .10780 x 10^{-5} lb-sec^2/in. (0.189 g)

R1 = $\mathcal{R}_1$ = .750 in. (1.905 cm)

R2 = $\mathcal{R}_2$ = .750 in. (1.905 cm)

R3 = $\mathcal{R}_3$ = .750 in. (1.905 cm)

RHO1 = ρ_1 = .060 in. (0.152 cm)

RHO2 = ρ_2 = .030 in. (0.076 cm)

RHO3 = ρ_3 = .025 in. (0.051 cm)

MD = md^2 = .15 x 10^{-4} lb-sec^2 in. (16.944 g - cm^2)

K = 25

II. Computed Values

At the beginning of the output, one finds MIN = M_{in}. Subsequently, the following are listed for each mesh:

f_{Pi}, the length of the pinion flats

$\beta_{\perp}$, the fuze body pivot to pivot line angles

ψ_{T1} and φ_{T1}, the transition angles as well as the associated subsidiary tests

φ_{IN1} and ψ_{IN1}, the initial angles

φ_{FIN1} and ψ_{FIN1}, the final angles

Finally, for the full range of the input angle φ_1, the point efficiency POINTEF is listed, in addition to other parameters which are useful for checking purposes. Note that DPSI1 and DPSI2 represent $\dot{\psi}_1$ and $\dot{\psi}_2$, respectively. The cycle efficiency CYCLEFF is found at the end of the output.

Program CLOCK 4

```
      PROGRAM CLOCK4(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)                    A  1
C                                                                              A  2
C     POINT AND CYCLE EFFICIENCIES FOR TWO PASS CLOCK (OGIVAL) STEP-UP         A  3
C     GEAR TRAIN IN SPIN ENVIRONMENT                                           A  4
C                                                                              A  5
      REAL MU,LAMDA1,LAMDA2,LX1,LY1,L1,LX2,LY2,L2,NP2,NP3,NG1,NG2,MIN,M1       A  6
     1,M2,M3,MO3,MO31,MO32,MO33,MO34,MTOT,MD,LL1,LL2,K                         A  7
     1 READ (5,40) MU,RPM,CAPRP1,CAPRP2,RP2,RP3,ACG1,ACG2,ACP1,ACP2,ISTOP      A  8
      READ (5,58) R1,R2,R3                                                     A  9
      READ (5,41) RHOG1,RHOG2,RHOP1,RHOP2                                      A 10
      READ (5,42) TG1,TG2,TP1,TP2                                              A 11
      READ (5,43) NG1,NG2,NP2,NP3                                              A 12
      READ (5,61) M1,M2,M3                                                     A 13
      READ (5,44) RHO1,RHO2,RHO3,MD,K,J1,J2                                    A 14
      PI=3.14159                                                               A 15
      Z=PI/180.                                                                A 16
      OMEGA=RPM*2.*PI/60.                                                      A 17
      OM2=OMEGA*OMEGA                                                          A 18
      PHOOT1=-1.                                                               A 19
C                                                                              A 20
C     COMPUTATION OF GEAR TOOTH PARAMETERS FOR BOTH MESHES                     A 21
C                                                                              A 22
      CXG1=RHOG1-TG1/2.                                                        A 23
      DELG1=ASIN(CXG1/CAPRP1)                                                  A 24
      CXP1=RHOP1-TP1/2.                                                        A 25
      DELP1=ASIN(CXP1/RP2)                                                     A 26
      GAMMP1=ASIN(RHOP1/RP2)                                                   A 27
      ALPHP1=GAMMP1-DELP1                                                      A 28
      FP1=ACP1*COS(GAMMP1)                                                     A 29
      B1=CAPRP1+RP2                                                            A 30
      CXG2=RHOG2-TG2/2.                                                        A 31
      DELG2=ASIN(CXG2/CAPRP2)                                                  A 32
      CXP2=RHOP2-TP2/2.                                                        A 33
      DELP2=ASIN(CXP2/RP3)                                                     A 34
      GAMMP2=ASIN(RHOP2/RP3)                                                   A 35
      ALPHP2=GAMMP2-DELP2                                                      A 36
      FP2=ACP2*COS(GAMMP2)                                                     A 37
      B2=CAPRP2+RP3                                                            A 38
C                                                                              A 39
C     COMPUTATION OF MIN, GAMMAS AND BETAS                                     A 40
C                                                                              A 41
      MIN=MD*OM2                                                               A 42
      DELTA2=ACOS(((CAPRP1+RP2)*(CAPRP1+RP2)+R1*R1-R2*R2)/(2.*R1*(CAPRP1       A 43
     1+RP2)))                                                                  A 44
      DELTA3=ACOS(((CAPRP2+RP3)*(CAPRP2+RP3)+R2*R2-R3*R3)/(2.*R2*(CAPRP2       A 45
     1+RP3)))                                                                  A 46
      GAMMA2=ACOS((R1*R1+R2*R2-(CAPRP1+RP2)*(CAPRP1+RP2))/(2.*R1*R2))          A 47
      GAMM3P=ACOS((R2*R2+R3*R3-(CAPRP2+RP3)*(CAPRP2+RP3))/(2.*R2*R3))          A 48
      GAMMA3=GAMMA2+GAMM3P                                                     A 49
      BETA1=PI-DELTA2                                                          A 50
      BETA2=GAMMA2+PI-DELTA3                                                   A 51
      BETA1D=BETA1/Z                                                           A 52
      BETA2D=BETA2/Z                                                           A 53
```

```
      WRITE (6,45) MIN,MU,RPM,CAPRP1,CAPRP2,RP2,RP3,ACG1,ACG2,ACP1,ACP2   A  54
   55 WRITE (6,59) R1,R2,R3                                               A  55
      WRITE (6,46) RHOG1,RHOG2,RHOP1,RHOP2                                A  56
      WRITE (6,47) TG1,TG2,TP1,TP2                                        A  57
      WRITE (6,48) NG1,NG2,NP2,NP3                                        A  58
      WRITE (6,62) M1,M2,M3                                               A  59
   60 WRITE (6,49) RHO1,RHO2,RHO3,MD,K,PHDOT1                             A  60
      WRITE (6,60) FP1,FP2                                                A  61
      WRITE (6,50) BETA1D,BETA2D                                          A  62
C                                                                         A  63
C     COMPUTATION OF OTHER PARAMETERS                                     A  64
   65 C                                                                   A  65
      DPHI1=360./NG1*Z                                                    A  66
      DPSI1=360./NP2*Z                                                    A  67
      DPHI2=360./NG2*Z                                                    A  68
      DPSI2=360./NP3*Z                                                    A  69
   70 L1=RHOG1+RHOP1                                                      A  70
      L2=RHOG2+RHOP2                                                      A  71
      Q1=M1*R1*OM2                                                        A  72
      Q2=M2*R2*OM2                                                        A  73
      Q3=M3*R3*OM2                                                        A  74
   75 C                                                                   A  75
C     PRELIMINARY COMPUTATIONS FOR MESH 1                                 A  76
C                                                                         A  77
C     DETERMINATION OF TRANSITION ANGLE OF MESH 1                         A  78
C                                                                         A  79
   80 A1T=RHOG1*COS(BETA1+ALPHP1)+FP1*SIN(BETA1+ALPHP1)                   A  80
      B1T=-RHOG1*SIN(BETA1+ALPHP1)+FP1*COS(BETA1+ALPHP1)                  A  81
      C1T=(ACG1*ACG1-RHOG1*RHOG1-B1*B1-FP1*FP1)/(2.*B1)                   A  82
      ROOT1T=A1T*A1T+B1T*B1T-C1T*C1T                                      A  83
      Y1T1=A1T+SQRT(ROOT1T)                                               A  84
   85 Y1T2=A1T-SQRT(ROOT1T)                                               A  85
      X1T=B1T+C1T                                                         A  86
      PSI1T1=2.*ATAN2(Y1T1,X1T)                                           A  87
      PSI1T2=2.*ATAN2(Y1T2,X1T)                                           A  88
      PSITE11=PSI1T1                                                      A  89
   90 PSITE12=PSI1T2                                                      A  90
      IF (PSI1T1.GT.PI) PSITE11=PSITE11-2.*PI                             A  91
      IF (PSI1T1.LT.PI) PSITE11=PSITE11+2.*PI                             A  92
      IF (PSI1T2.GT.PI) PSITE12=PSITE12-2.*PI                             A  93
      IF (PSI1T2.LT.PI) PSITE12=PSITE12+2.*PI                             A  94
   95 IF (PSITE11.GE.0.) TEST11=ABS(PI-BETA1+PSITE11-ALPHP1)/Z            A  95
      IF (PSITE11.LT.0.) TEST11=ABS(PI+BETA1-(PSITE11+2.*PI-ALPHP1))/Z    A  96
      IF (PSITE12.GE.0.) TEST12=ABS(PI-BETA1+PSITE12-ALPHP1)/Z            A  97
      IF (PSITE12.LT.0.) TEST12=ABS(PI+BETA1-(PSITE12+2.*PI-ALPHP1))/Z    A  98
      IF (PSI1T1.LT.0.) PSI1T1=PSI1T1+2.*PI                               A  99
  100 IF (PSI1T2.LT.0.) PSI1T2=PSI1T2+2.*PI                               A 100
      PSI1T1D=PSI1T1/Z                                                    A 101
      PSI1T2D=PSI1T2/Z                                                    A 102
      WRITE (6,31) PSI1T1D,TEST11                                         A 103
      WRITE (6,32) PSI1T2D,TEST12                                         A 104
  105 CALL TRANS1 (RHOG1,ALPHP1,BETA1,FP1,ACG1,B1,DELG1,Z,PSI1T1,PHI1T1,  A 105
     1G11)                                                                A 106
```

```
                      IF (G11.GT.FP1) GO TO 2                                   A 107
                      PHI1T=PHI1T1                                              A 108
                      PSI1T=PSI1T1                                              A 109
      110             GO TO 4                                                   A 110
                    2 CALL TRANS1 (RHOG1,ALPHP1,BETA1,FP1,ACG1,B1,DELG1,Z,PSI1T2,PHI1T2,  A 111
                     1G12)                                                      A 112
                      IF (G12.LT.FP1) GO TO 3                                   A 113
                      WRITE (6,51)                                              A 114
      115             STOP                                                      A 115
                    3 PHI1T=PHI1T2                                              A 116
                      PSI1T=PSI1T2                                              A 117
                    4 IF (PHI1T.LT.0.) PHI1T=PHI1T+2.*PI                        A 118
                      IF (PSI1T.LT.0.) PSI1T=PSI1T+2.*PI                        A 119
      120             PHI1TD=PHI1T/Z                                            A 120
                      PSI1TD=PSI1T/Z                                            A 121
                      WRITE (6,52) PHI1TD,PSI1TD                                A 122
                C                                                               A 123
                C     DETERMINATION OF CORRECT SIGN FOR ROUND ON FLAT REGIME OF MESH 1  A 124
      125       C                                                               A 125
                      A1F=ACG1*COS(PHI1T+DELG1+ALPHP1)-B1*COS(BETA1+ALPHP1)     A 126
                      B1F=-ACG1*SIN(PHI1T+DELG1+ALPHP1)+B1*SIN(BETA1+ALPHP1)    A 127
                      C1F=RHOG1                                                 A 128
                      ROOT1F=A1F*A1F+B1F*B1F-C1F*C1F                            A 129
      130             Y1F1=A1F+SQRT(ROOT1F)                                     A 130
                      Y1F2=A1F-SQRT(ROOT1F)                                     A 131
                      X1F=B1F+C1F                                               A 132
                      PSI1F1=2.*ATAN2(Y1F1,X1F)                                 A 133
                      PSI1F2=2.*ATAN2(Y1F2,X1F)                                 A 134
      135             IF (PSI1F1.LT.0.) PSI1F1=PSI1F1+2.*PI                     A 135
                      IF (PSI1F2.LT.0.) PSI1F2=PSI1F2+2.*PI                     A 136
                      IF (ABS(PSI1F1-PSI1T).LT.ABS(PSI1F2-PSI1T)) GO TO 5       A 137
                      SIGN1F=-1.                                                A 138
                      GO TO 6                                                   A 139
      140           5 SIGN1F=1.                                                 A 140
                C                                                               A 141
                C     COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI FOR MESH 1  A 142
                C                                                               A 143
                    6 DO 7 I=1,2000                                             A 144
      145             PHID1=PHI1TD-(I-1.)/100.                                  A 145
                      PHI1=PHID1*Z                                              A 146
                      A1F=ACG1*COS(PHI1+DELG1+ALPHP1)-B1*COS(BETA1+ALPHP1)      A 147
                      B1F=-ACG1*SIN(PHI1+DELG1+ALPHP1)+B1*SIN(BETA1+ALPHP1)     A 148
                      C1F=RHOG1                                                 A 1-9
      150             ROOT1F=A1F*A1F+B1F*B1F-C1F*C1F                            A
                      Y1F=A1F+SIGN1F*SQRT(ROOT1F)
                      X1F=B1F+C1F
                      PSI1F=2.*ATAN2(Y1F,X1F)                                     .53
                      IF (PSI1F.LT.0.) PSI1F=PSI1F+2.*PI                        A 154
      155             LX1=B1*COS(BETA1)+ACP1*COS(PSI1F-DPSI1+DELP1)-ACG1*COS(PHI1+DPHI1+  A 155
                     1DELG1)                                                    A 156
                      LY1=B1*SIN(BETA1)+ACP1*SIN(PSI1F-DPSI1+DELP1)-ACG1*SIN(PHI1+DPHI1+  A 157
                     1DELG1)                                                    A 158
                      LL1=SQRT(LX1*LX1+LY1*LY1)                                 A 159
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```
                  DELEL1=LL1-L1                                                    A 160
                  IF (DELEL1.LE.0.) GO TO 8                                        A 161
                7 CONTINUE                                                         A 162
                8 PHI1F=PHI1                                                       A 163
                  PSI1FF=PSI1F                                                     A 164
                  PHI1I=PHI1F+DPHI1                                                A 165
                  PSI1I=PSI1FF-DPSI1                                               A 166
                  IF (PSI1I.LT.0.) PSI1I=PSI1I+2.*PI                               A 167
                  PHI1ID=PHI1I/Z                                                   A 168
                  PSI1ID=PSI1I/Z                                                   A 169
                  PHI1FD=PHI1F/Z                                                   A 170
                  PSI1FD=PSI1FF/Z                                                  A 171
                  WRITE (6,53) PHI1ID,PSI1ID,PHI1FD,PSI1FD                         A 172
            C                                                                      A 173
            C     DETERMINATION OF CORRECT SIGN FOR ROUND ON ROUND REGIME OF MESH 1 A 174
            C                                                                      A 175
                  A1R=ACG1*SIN(PHI1I+DELG1-DELP1)-B1*SIN(BETA1-DELP1)              A 176
                  B1R=ACG1*COS(PHI1I+DELG1-DELP1)-B1*COS(BETA1-DELP1)              A 177
                  C1R=(ACP1*ACP1+ACG1*ACG1+B1*B1-L1*L1-2.*ACG1*B1*COS(PHI1I+DELG1-BE A 178
                 1TA1))/(2.*ACP1)                                                  A 179
                  ROOT1R=A1R*A1R+B1R*B1R-C1R*C1R                                   A 180
                  Y1R1=A1R+SQRT(ROOT1R)                                            A 181
                  Y1R2=A1R-SQRT(ROOT1R)                                            A 182
                  X1R=B1R+C1R                                                      A 183
                  PSI1R1=2.*ATAN2(Y1R1,X1R)                                        A 184
                  PSI1R2=2.*ATAN2(Y1R2,X1R)                                        A 185
                  IF (PSI1R1.LT.0.) PSI1R1=PSI1R1+2.*PI                            A 186
                  IF (PSI1R2.LT.0.) PSI1R2=PSI1R2+2.*PI                            A 187
                  IF (ABS(PSI1I-PSI1R1).LT.ABS(PSI1I-PSI1R2)) GO TO 9              A 188
                  SIG1R=-1.                                                        A 189
                  GO TO 10                                                         A 190
                9 SIG1R=1.                                                         A 191
            C                                                                      A 192
            C     PRELIMINARY COMPUTATIONS FOR MESH 2                              A 193
            C                                                                      A 194
            C     DETERMINATION OF TRANSITION ANGLE OF MESH 2                      A 195
            C                                                                      A 196
               10 A2T=-RHOG2*COS(BETA2-ALPHP2)+FP2*SIN(BETA2-ALPHP2)               A 197
                  B2T=RHOG2*SIN(BETA2-ALPHP2)+FP2*COS(BETA2-ALPHP2)                A 198
                  C2T=(ACG2*ACG2-RHOG2*RHOG2-B2*B2-FP2*FP2)/(2.*B2)                A 199
                  ROOT2T=A2T*A2T+B2T*B2T-C2T*C2T                                   A 200
                  Y2T1=A2T+SQRT(ROOT2T)                                            A 201
                  Y2T2=A2T-SQRT(ROOT2T)                                            A 202
                  X2T=B2T+C2T                                                      A 203
                  PSI2T1=2.*ATAN2(Y2T1,X2T)                                        A 204
                  PSI2T2=2.*ATAN2(Y2T2,X2T)                                        A 205
                  PSITE21=PSI2T1                                                   A 206
                  PSITE22=PSI2T2                                                   A 207
                  IF (PSI2T1.GT.PI) PSITE21=PSITE21-2.*PI                          A 208
                  IF (PSI2T1.LT.PI) PSITE21=PSITE21+2.*PI                          A 209
                  IF (PSI2T2.GT.PI) PSITE22=PSITE22-2.*PI                          A 210
                  IF (PSI2T2.LT.PI) PSITE22=PSITE22+2.*PI                          A 211
                  IF (PSITE21.GE.0.) TEST21=ABS(PI-BETA2+PSITE21+ALPHP2)/Z         A 212
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```
      IF (PSITE21.LT.0.) TEST21=ABS(PI+BETA2-(PSITE21+2.*PI+ALPHP2))/Z      A 213
      IF (PSITE22.GE.0.) TEST22=ABS(PI-BETA2+PSITE22+ALPHP2)/Z              A 214
215   IF (PSITE22.LT.0.) TEST22=ABS(PI+BETA2-(PSITE22+2.*PI+ALPHP2))/Z      A 215
      IF (PSI2T1.LT.0.) PSI2T1=PSI2T1+2.*PI                                 A 216
      IF (PSI2T2.LT.0.) PSI2T2=PSI2T2+2.*PI                                 A 217
      PSI2T1D=PSI2T1/Z                                                      A 218
      PSI2T2D=PSI2T2/Z                                                      A 219
220   WRITE (6,33) PSI2T1D,TEST21                                           A 220
      WRITE (6,34) PSI2T2D,TEST22                                           A 221
      CALL TRANS2 (RHOG2,ALPHP2,BETA2,FP2,ACG2,B2,DELG2,Z,PSI2T1,PHI2T1,    A 222
     1G21)                                                                  A 223
      IF (G21.GT.FP2) GO TO 11                                              A 224
225   PHI2T=PHI2T1                                                          A 225
      PSI2T=PSI2T1                                                          A 226
      GO TO 13                                                              A 227
   11 CALL TRANS2 (RHOG2,ALPHP2,BETA2,FP2,ACG2,B2,DELG2,Z,PSI2T2,PHI2T2,    A 228
     1G22)                                                                  A 229
230   IF (G22.LT.FP2) GO TO 12                                              A 230
      WRITE (6,54)                                                          A 231
      STOP                                                                  A 232
   12 PHI2T=PHI2T2                                                          A 233
      PSI2T=PSI2T2                                                          A 234
235   13 IF (PHI2T.LT.0.) PHI2T=PHI2T+2.*PI                                 A 235
      IF (PSI2T.LT.0.) PSI2T=PSI2T+2.*PI                                    A 236
      PHI2TD=PHI2T/Z                                                        A 237
      PSI2TD=PSI2T/Z                                                        A 238
      WRITE (6,55) PHI2TD,PSI2TD                                            A 239
240   C                                                                     A 240
      C     DETERMINATION OF CORRECT SIGN FOR ROUND ON FLAT REGIME OF MESH 2 A 241
      C                                                                     A 242
      A2F=ACG2*COS(PHI2T-DELG2-ALPHP2)-B2*COS(BETA2-ALPHP2)                 A 243
      B2F=-ACG2*SIN(PHI2T-DELG2-ALPHP2)+B2*SIN(BETA2-ALPHP2)                A 244
245   C2F=-RHOG2                                                            A 245
      ROOT2F=A2F*A2F+B2F*B2F-C2F*C2F                                        A 246
      Y2F1=A2F+SQRT(ROOT2F)                                                 A 247
      Y2F2=A2F-SQRT(ROOT2F)                                                 A 248
      X2F=B2F+C2F                                                           A 249
250   PSI2F1=2.*ATAN2(Y2F1,X2F)                                             A 250
      PSI2F2=2.*ATAN2(Y2F2,X2F)                                             A 251
      IF (PSI2F1.LT.0.) PSI2F1=PSI2F1+2.*PI                                 A 252
      IF (PSI2F2.LT.0.) PSI2F2=PSI2F2+2.*PI                                 A 253
      IF (ABS(PSI2F1-PSI2T).LT.ABS(PSI2F2-PSI2T)) GO TO 14                  A 254
255   PSI2F1D=PSI2F1/Z                                                      A 255
      PSI2F2D=PSI2F2/Z                                                      A 256
      SIGN2F=-1.                                                            A 257
      GO TO 15                                                              A 258
   14 SIGN2F=1.                                                             A 259
260   C                                                                     A 260
      C     COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI FOR MESH 2 A 261
      C                                                                     A 262
   15 DO 16 I=1,1000                                                        A 263
      PHID2=PHI2TD+(I-1.)/100.                                              A 264
265   PHI2=PHID2*Z                                                          A 265
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      A2F=ACG2*COS(PHI2-DELG2-ALPHP2)-B2*COS(BETA2-ALPHP2)                A 266
      B2F=-ACG2*SIN(PHI2-DELG2-ALPHP2)+B2*SIN(BETA2-ALPHP2)               A 267
      C2F=-RHOG2                                                          A 268
      ROOT2F=A2F*A2F+B2F*B2F-C2F*C2F                                      A 269
  270 Y2F=A2F+SIGN2F*SQRT(ROOT2F)                                         A 270
      X2F=B2F+C2F                                                         A 271
      PSI2F=2.*ATAN2(Y2F,X2F)                                             A 272
      IF (PSI2F.LT.0.) PSI2F=PSI2F+2.*PI                                  A 273
      LX2=B2*COS(BETA2)+ACP2*COS(PSI2F+DPSI2-DELP2)-ACG2*COS(PHI2-DPHI2-  A 274
     1DELG2)                                                              A 275
  275 LY2=B2*SIN(BETA2)+ACP2*SIN(PSI2F+DPSI2-DELP2)-ACG2*SIN(PHI2-DPHI2-  A 276
     1DELG2)                                                              A 277
      LL2=SQRT(LX2*LX2+LY2*LY2)                                           A 278
      DELEL2=LL2-L2                                                       A 279
  280 IF (DELEL2.LE.0.) GO TO 17                                          A 280
   16 CONTINUE                                                            A 281
   17 PHI2F=PHI2                                                          A 282
      PSI2FF=PSI2F                                                        A 283
      PHI2I=PHI2F-DPHI2                                                   A 284
  285 PSI2I=PSI2FF+DPSI2                                                  A 285
      IF (PSI2I.GT.2.*PI) PSI2I=PSI2I-2.*PI                               A 286
      PHI2ID=PHI2I/Z                                                      A 287
      PSI2ID=PSI2I/Z                                                      A 288
      PHI2FD=PHI2F/Z                                                      A 289
  290 PSI2FD=PSI2FF/Z                                                     A 290
      WRITE (6,56) PHI2ID,PSI2ID,PHI2FD,PSI2FD                            A 291
C                                                                         A 292
C     DETERMINATION OF CORRECT SIGN FOR ROUND ON ROUND REGIME OF MESH 2   A 293
C                                                                         A 294
  295 A2R=B2*SIN(BETA2+DELP2)-ACG2*SIN(PHI2I-DELG2+DELP2)                 A 295
      B2R=B2*COS(BETA2+DELP2)-ACG2*COS(PHI2I-DELG2+DELP2)                 A 296
      C2R=(L2*L2-B2*B2-ACG2*ACG2-ACP2*ACP2+2.*ACG2*B2*COS(PHI2I-DELG2-BE  A 297
     1TA2))/(2.*ACP2)                                                     A 298
      ROOT2R=A2R*A2R+B2R*B2R-C2R*C2R                                      A 299
  300 Y2R1=A2R+SQRT(ROOT2R)                                               A 300
      Y2R2=A2R-SQRT(ROOT2R)                                               A 301
      X2R=B2R+C2R                                                         A 302
      PSI2R1=2.*ATAN2(Y2R1,X2R)                                           A 303
      PSI2R2=2.*ATAN2(Y2R2,X2R)                                           A 304
  305 IF (PSI2R1.LT.0.) PSI2R1=PSI2R1+2.*PI                               A 305
      IF (PSI2R2.LT.0.) PSI2R2=PSI2R2+2.*PI                               A 306
      IF (ABS(PSI2I-PSI2R1).LT.ABS(PSI2I-PSI2R2)) GO TO 18                A 307
      SIGN2R=-1.                                                          A 308
      GO TO 19                                                            A 309
  310 18 SIGN2R=1.                                                        A 310
C                                                                         A 311
C     GEAR TRAIN MOTION MODEL, KINEMATICS                                 A 312
C                                                                         A 313
   19 DDPHI1=NP2*(PHI2F-PHI2I)/(K*NG1)                                    A 314
  315 PHI1=PHI1I+DDPHI1                                                   A 315
      WRITE (6,35)                                                        A 316
   20 PHI1=PHI1-DDPHI1                                                    A 317
      PHI1D=PHI1/Z                                                        A 318
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      IF (PHI1.LE.PHI1F+DDPHI1) GO TO 30                                A 319
                                                                        A 320
320   C                                                                 A 321
      C     MESH 1                                                      A 322
      C                                                                 A 323
      IF (PHI1.LE.PHI1T) GO TO 21                                       A 324
      A1R=ACG1*SIN(PHI1+DELG1-DELP1)-B1*SIN(BETA1-DELP1)                A 325
325   B1R=ACG1*COS(PHI1+DELG1-DELP1)-B1*COS(BETA1-DELP1)                A 326
      C1R=(ACP1*ACP1+ACG1*ACG1+B1*B1-L1*L1-2.*ACG1*B1*COS(PHI1+DELG1-BET A 327
     1A1))/(2.*ACP1)                                                    A 328
      ROOT1R=A1R*A1R+B1R*B1R-C1R*C1R                                    A 329
      Y1R=A1R+SIGN1R*SQRT(ROOT1R)                                       A 330
330   X1R=B1R+C1R                                                       A 331
      PSI1=2.*ATAN2(Y1R,X1R)                                            A 332
      IF (PSI1.LT.0.) PSI1=PSI1+2.*PI                                   A 333
      IF (ABS(PHI1-PHI1I).LT.0.0001)PSI1I=PSI1                          A 334
      IF (ABS(PHI1-PHI1I).LT.0.0001) PSI1P=PSI1I                        A 335
335   PSI1D=PSI1/Z                                                      A 336
      SLAM1=(B1*SIN(BETA1)+ACP1*SIN(PSI1+DELP1)-ACG1*SIN(PHI1+DELG1))/L1 A 337
      CLAM1=(B1*COS(BETA1)+ACP1*COS(PSI1+DELP1)-ACG1*COS(PHI1+DELG1))/L1 A 338
      LAMDA1=ATAN2(SLAM1,CLAM1)                                         A 339
      IF (LAMDA1.LT.0.) LAMDA1=LAMDA1+2.*PI                             A 340
340   PSDOT1=PHDOT1*ACG1*(B1/ACP1*SIN(PHI1+DELG1-BETA1)+SIN(PHI1-PSI1+DE A 341
     1LG1-DELP1))/(A1R*COS(PSI1)-B1R*SIN(PSI1))                         A 342
      VST1R=PHDOT1*(ACG1*COS(PHI1+DELG1-LAMDA1)+RHOG1)-PSDOT1*(ACP1*COS( A 343
     1PSI1+DELP1-LAMDA1)-RHOP1)                                         A 344
      S1R=VST1R/ABS(VST1R)                                              A 345
345   GO TO 22                                                          A 346
   21 A1F=ACG1*COS(PHI1+DELG1+ALPHP1)-B1*COS(BETA1+ALPHP1)              A 347
      B1F=-ACG1*SIN(PHI1+DELG1+ALPHP1)+B1*SIN(BETA1+ALPHP1)             A 348
      C1F=RHOG1                                                         A 349
      ROOT1F=A1F*A1F+B1F*B1F-C1F*C1F                                    A 350
350   Y1F=A1F+SIGN1F*SQRT(ROOT1F)                                       A 351
      X1F=B1F+C1F                                                       A 352
      PSI1=2.*ATAN2(Y1F,X1F)                                            A 353
      IF (PSI1.LT.0.) PSI1=PSI1+2.*PI                                   A 354
      PSI1D=PSI1/Z                                                      A 355
355   G1=(ACG1*SIN(PHI1+DELG1)+RHOG1*COS(PSI1-ALPHP1)-B1*SIN(BETA1))/SIN A 356
     1(PSI1-ALPHP1)                                                     A 357
      PSDOT1=PHDOT1*(ACG1*COS(PHI1-PSI1+DELG1+ALPHP1))/(A1F*COS(PSI1)-B1 A 358
     1F*SIN(PSI1))                                                      A 359
      VST1F=PHDOT1*(ACG1*SIN(PSI1-ALPHP1-PHI1-DELG1)-RHOG1)             A 360
360   S1F=VST1F/ABS(VST1F)                                              A 361
      C                                                                 A 362
      C     MESH 2                                                      A 363
      C                                                                 A 364
   22 DDPHI2=PSI1-PSI1P                                                 A 365
      IF (ABS(PHI1-PHI1I).LT.0.0001) PHI2=PHI2I                         A 366
365   PHI2=PHI2+DDPHI2                                                  A 367
      PSI1P=PSI1                                                        A 368
      IF (PHI2.GT.PHI2F) PHI2=PHI2I                                     A 369
      PHI2D=PHI2/Z                                                      A 370
370   IF (PHI2.GE.PHI2T) GO TO 23                                       A 371
      A2R=B2*SIN(BETA2+DELP2)-ACG2*SIN(PHI2-DELG2+DELP2)
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                B2R=B2*COS(BETA2+DELP2)-ACG2*COS(PHI2-DELG2+DELP2)                 A 372
                C2R=(L2*L2-B2*B2-ACG2*ACG2-ACP2*ACP2+2.*ACG2*B2*COS(PHI2-DELG2-BET A 373
               1A2))/(2.*ACP2)                                                    A 374
  375           ROOT2R=A2R*A2R+B2R*B2R-C2R*C2R                                     A 375
                Y2R=A2R+SIGN2R*SQRT(ROOT2R)                                       A 376
                X2R=B2R+C2R                                                       A 377
                PSI2=2.*ATAN2(Y2R,X2R)                                            A 378
                IF (PSI2.LT.0.) PSI2=PSI2+2.*PI                                   A 379
  380           IF (ABS(PHI2-PHI2I).LT.0.0001) PSI2I=PSI2                         A 380
                IF (ABS(PHI2-PHI2I).LT.0.0001) PSI2P=PSI2I                        A 381
                PSI2D=PSI2/Z                                                      A 382
                SLAM2=(B2*SIN(BETA2)+ACP2*SIN(PSI2-DELP2)-ACG2*SIN(PHI2-DELG2))/L2 A 383
                CLAM2=(B2*COS(BETA2)+ACP2*COS(PSI2-DELP2)-ACG2*COS(PHI2-DELG2))/L2 A 384
  385           LAMDA2=ATAN2(SLAM2,CLAM2)                                         A 385
                IF (LAMDA2.LT.0.) LAMDA2=LAMDA2+2.*PI                             A 386
                PHDOT2=PSDOT1                                                     A 387
                PSDOT2=PHDOT2*ACG2*(-SIN(PHI2-PSI2-DELG2+DELP2)-B2/ACP2*SIN(PHI2-D A 388
               1ELG2-BETA2))/(A2R*COS(PSI2)-B2R*SIN(PSI2))                        A 389
  390           VST2R=PHDOT2*(ACG2*COS(PHI2-DELG2-LAMDA2)+RHOG2)-PSDOT2*(ACP2*COS( A 390
               1PSI2-DELP2-LAMDA2)-RHOP2)                                         A 391
                S2R=VST2R/ABS(VST2R)                                              A 392
                GO TO 24                                                          A 393
             23 A2F=ACG2*COS(PHI2-DELG2-ALPHP2)-B2*COS(BETA2-ALPHP2)              A 394
  395           B2F=-ACG2*SIN(PHI2-DELG2-ALPHP2)+B2*SIN(BETA2-ALPHP2)             A 395
                C2F=-RHOG2                                                        A 396
                ROOT2F=A2F*A2F+B2F*B2F-C2F*C2F                                    A 397
                Y2F=A2F+SIGN2F*SQRT(ROOT2F)                                       A 398
                X2F=B2F+C2F                                                       A 399
  400           PSI2=2.*ATAN2(Y2F,X2F)                                            A 400
                IF (PSI2.LT.0.) PSI2=PSI2+2.*PI                                   A 401
                PSI2D=PSI2/Z                                                      A 402
                G2=(ACG2*SIN(PHI2-DELG2)-RHOG2*COS(PSI2+ALPHP2)-B2*SIN(BETA2))/SIN A 403
               1(PSI2+ALPHP2)                                                     A 404
  405           PHDOT2=PSDOT1                                                     A 405
                PSDOT2=(PHDOT2*ACG2*COS(PHI2-DELG2-ALPHP2-PSI2))/(A2F*COS(PSI2)-B2 A 406
               1F*SIN(PSI2))                                                      A 407
                VST2F=PHDOT2*(ACG2*SIN(PSI2+ALPHP2-PHI2+DELG2)+RHOG2)             A 408
                S2F=VST2F/ABS(VST2F)                                              A 409
  410       C                                                                     A 410
            C     MOMENT COMPUTATIONS                                             A 411
            C                                                                     A 412
             24 DN=1.+MU*MU                                                       A 413
                A12=ABS((MU*(S1R-1.)*SIN(LAMDA1)-(1.+MU*MU*S1R)*COS(LAMDA1))/DN)  A 415
  415           A11=ABS(((1.+MU*MU*S2R)*COS(LAMDA2)-MU*(S2R-1.)*SIN(LAMDA2))/DN)  A 414
                A13=ABS((-COS(GAMMA2)-MU*SIN(GAMMA2))/DN)                         A 416
                A14=ABS(((1.+MU*MU*S2R)*SIN(LAMDA2)-MU*(1.-S2R)*COS(LAMDA2))/DN)  A 417
                A15=ABS((MU*(1.-S1R)*COS(LAMDA1)-(1.+MU*MU*S1R)*SIN(LAMDA1))/DN)  A 418
                A16=ABS((MU*COS(GAMMA2)-SIN(GAMMA2))/DN)                          A 419
  420           A17=ABS(((1.-MU+MU*S1R)*COS(LAMDA1)-MU*(1.+S1R)*SIN(LAMDA1))/DN)  A 420
                A18=ABS(1./DN)                                                    A 421
                A19=ABS(((1.-MU+MU*S1R)*SIN(LAMDA1)+MU*(1.+S1R)*COS(LAMDA1))/DN)  A 422
                A20=ABS(MU/DN)                                                    A 423
                A21=ABS((MU*(1.-S2R)*SIN(LAMDA2)+(1.+MU*MU*S2R)*COS(LAMDA2))/DN)  A 424
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      A22=ABS(((1.-MU*MU*S1F)*SIN(PSI1-ALPHP1)-MU*(1.+S1F)*COS(PSI1-ALPH      A 425
     1P1))/DN)                                                                A 426
      A23=ABS((MU*SIN(GAMMA2)+COS(GAMMA2))/DN)                                 A 427
      A24=ABS(((1.+MU*MU*S2R)*SIN(LAMDA2)-MU*(1.-S2R)*COS(LAMDA2))/DN)         A 428
      A25=ABS((-MU*(1.+S1F)*SIN(PSI1-ALPHP1)+(MU*MU*S1F-1.)*COS(PSI1-ALP       A 429
     1HP1))/DN)                                                               A 430
      A26=ABS((-SIN(GAMMA2)+MU*COS(GAMMA2))/DN)                                A 431
      A27=ABS((-(1.+MU*MU*S1F)*SIN(PSI1-ALPHP1)+MU*(S1F-1.)*COS(PSI1-ALP       A 432
     1HP1))/DN)                                                               A 433
      A28=ABS(1./DN)                                                           A 434
      A29=ABS((MU*(S1F-1.)*SIN(PSI1-ALPHP1)+(1.+MU*MU*S1F)*COS(PSI1-ALPH       A 435
     1P1))/DN)                                                                A 436
      A30=ABS(MU/DN)                                                           A 437
      A37=ABS((1.+MU*MU*S2F)*SIN(PSI2+ALPHP2)+MU*(S2F-1.)*COS(PSI2+ALPH        A 438
     1P2))/DN)                                                                A 439
      A38=ABS(((1.-MU*MU*S1F)*SIN(PSI1-ALPHP1)-MU*(1.+S1F)*COS(PSI1-ALPH       A 440
     1P1))/DN)                                                                A 441
      A39=ABS((MU*SIN(GAMMA2)+COS(GAMMA2))/DN)                                 A 442
      A40=ABS((MU*(S2F-1.)*SIN(PSI2+ALPHP2)-(1.+MU*MU*S2F)*COS(PSI2+ALPH       A 443
     1P2))/DN)                                                                A 444
      A41=ABS((-MU*(1.+S1F)*SIN(PSI1-ALPHP1)+(MU*MU*S1F-1.)*COS(PSI1-ALP       A 445
     1HP1))/DN)                                                               A 446
      A42=ABS((MU*COS(GAMMA2)-SIN(GAMMA2))/DN)                                 A 447
      A43=ABS(((1.+MU*MU*S2F)*SIN(PSI2+ALPHP2)-MU*(1.-S2F)*COS(PSI2+ALPH       A 448
     1P2))/DN)                                                                A 449
      A44=ABS((MU*(S1R-1.)*SIN(LAMDA1)-(1.+MU*MU*S1R)*COS(LAMDA1))/DN)         A 450
      A45=ABS((MU*SIN(GAMMA2)+COS(GAMMA2))/DN)                                 A 451
      A46=ABS((MU*(S2F-1.)*SIN(PSI2+ALPHP2)-(1.+MU*MU*S2F)*COS(PSI2+ALPH       A 452
     1P2))/DN)                                                                A 453
      A47=ABS((-(1.+MU*MU*S1R)*SIN(LAMDA1)+MU*(1.-S1R)*COS(LAMDA1))/DN)        A 454
      A48=ABS((-SIN(GAMMA2)+MU*COS(GAMMA2))/DN)                                A 455
      A65=ABS((MU*(1.+S2R)*SIN(LAMDA2)+(MU*MU*S2R-1.)*COS(LAMDA2))/DN)         A 456
      A66=ABS((MU*SIN(GAMMA3)-COS(GAMMA3))/DN)                                 A 457
      A67=ABS(((MU*MU*S2R-1.)*SIN(LAMDA2)-MU*(1.+S2R)*COS(LAMDA2))/DN)         A 458
      A68=ABS((-SIN(GAMMA3)-MU*COS(GAMMA3))/DN)                                A 459
      A69=ABS(((MU*MU*S2F-1.)*SIN(PSI2+ALPHP2)-MU*(1.+S2F)*COS(PSI2+ALPH       A 460
     1P2))/DN)                                                                A 461
      A70=ABS((MU*SIN(GAMMA3)-COS(GAMMA3))/DN)                                 A 462
      A71=ABS((-MU*(1.+S2F)*SIN(PSI2+ALPHP2)+(1.-MU*MU*S2F)*COS(PSI2+ALP       A 463
     1HP2))/DN)                                                               A 464
      A72=ABS((-SIN(GAMMA3)-MU*COS(GAMMA3))/DN)                                A 465
      C6=ACG2*(SIN(PHI2-DELG2-LAMDA2)-MU*S2R*COS(PHI2-DELG2-LAMDA2))-MU*       A 466
     1RHO2*(A11+A14)-MU*RHOG2*S2R                                             A 467
      C7=MU*RHO2*(A13+A16)                                                     A 468
      C8=-(ACP1*(SIN(PSI1+DELP1-LAMDA1)-MU*S1R*COS(PSI1+DELP1-LAMDA1))+M       A 469
     1U*RHO2*(A12+A15)+MU*S1R*RHOP1)                                          A 470
      C9=MU*RHO1*(A18+A20)                                                     A 471
      C10=MU*RHO1*(A17+A19)+ACG1*(SIN(PHI1+DELG1-LAMDA1)-MU*S1R*COS(PHI1       A 472
     1+DELG1-LAMDA1))-MU*S1R*RHOG1                                            A 473
      C11=ACG2*(SIN(PHI2-DELG2-LAMDA2)-MU*S2R*COS(PHI2-DELG2-LAMDA2))-MU       A 474
     1*RHO2*(A21+A24)-MU*RHOG2*S2R                                            A 475
      C12=MU*RHO2*(A23+A26)                                                    A 476
      C13=G1-MU*RHO2*(A22+A25)                                                 A 477
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      C14=MU*RHO1*(A28+A30)                                                  A 478
      C15=MU*RHO1*(A27+A29)+MU*S1F*RHOG1+ACG1*(MU*S1F*SIN(PHI1+DELG1-PSI     A 479
     1I1+ALPHP1)-COS(PHI1+DELG1-PSI1+ALPHP1))                                A 480
      C19=-MU*RHO2*(A37+A40)+ACG2*(COS(PHI2-DELG2-PSI2-ALPHP2)+MU*S2F*SI     A 481
     1N(PHI2-DELG2-PSI2-ALPHP2))-MU*S2F*RHOG2                                A 482
      C20=MU*RHO2*(A39+A42)                                                  A 483
      C21=-MU*RHO2*(A38+A41)+G1                                              A 484
      C22=-MU*RHO2*(A43+A46)+ACG2*(COS(PHI2-DELG2-PSI2-ALPHP2)+MU*S2F*SI     A 485
     1N(PHI2-DELG2-PSI2-ALPHP2))-MU*S2F*RHOG2                                A 486
      C23=MU*RHO2*(A45+A48)                                                  A 487
      C24=-MU*RHO2*(A44+A47)+ACP1*(MU*S1R*COS(PSI1+DELP1-LAMDA1)-SIN(PSI     A 488
     1I1+DELP1-LAMDA1))-MU*S1R*RHOP1                                         A 489
      C33=MU*RHO3*(A66+A68)                                                  A 490
      C34=MU*RHO3*(A65+A67)-MU*S2R*RHOP2+ACP2*(MU*S2R*COS(PSI2-DELP2-LAM     A 491
     1DA2)-SIN(PSI2-DELP2-LAMDA2))                                           A 492
      C35=MU*RHO3*(A70+A72)                                                  A 493
      C36=MU*RHO3*(A69+A71)-G2                                               A 494
C                                                                            A 495
C     MOMENT CONTROL                                                         A 496
C                                                                            A 497
      IF ((PHI1.GE.PHI1T).AND.(PHI2.LE.PHI2T)) GO TO 25                      A 498
      IF ((PHI1.LE.PHI1T).AND.(PHI2.LE.PHI2T)) GO TO 26                      A 499
      IF ((PHI1.LE.PHI1T).AND.(PHI2.GE.PHI2T)) GO TO 27                      A 500
      IF ((PHI1.GE.PHI1T).AND.(PHI2.GE.PHI2T)) GO TO 28                      A 501
C                                                                            A 502
C     MOMENT EXPRESSIONS                                                     A 503
C                                                                            A 504
   25 MO31=MIN*C8*C34/(C6*C10)-Q1*C8*C9*C34/(C6*C10)-Q2*C7*C34/C6-Q3*C33     A 505
      MO3=MO31                                                               A 506
      POINTEF=ABS(PSDOT2)*MO3/MIN                                            A 507
      WRITE (6,36) PHI1D,PHI2D,PSI1D,PSI2D,PSDOT1,PSDOT2,S1R,S2R,POINTEF     A 508
      GO TO 29                                                               A 509
   26 MO32=MIN*C13*C34/(C11*C15)-Q1*C13*C14*C34/(C11*C15)-Q2*C12*C34/C11     A 510
     1-Q3*C33                                                                A 511
      MO3=MO32                                                               A 512
      POINTEF=ABS(PSDOT2)*MO3/MIN                                            A 513
      WRITE (6,37) PHI1D,PHI2D,PSI1D,PSI2D,PSDOT1,PSDOT2,S2R,S1F,G1,POIN     A 514
     1TEF                                                                    A 515
      GO TO 29                                                               A 516
   27 MO33=MIN*C21*C36/(C15*C19)-Q1*C14*C21*C36/(C15*C19)-Q2*C20*C36/C19     A 517
     1-Q3*C35                                                                A 518
      MO3=MO33                                                               A 519
      POINTEF=ABS(PSDOT2)*MO3/MIN                                            A 520
      WRITE (6,38) PHI1D,PHI2D,PSI1D,PSI2D,PSDOT1,PSDOT2,S1F,G1,S2F,G2,P     A 521
     1OINTEF                                                                 A 522
      GO TO 29                                                               A 523
   28 MO34=MIN*C24*C36/(C10*C22)-Q1*C9*C24*C36/(C10*C22)-Q2*C23*C35/C22-     A 524
     1Q3*C35                                                                 A 525
      MO3=MO34                                                               A 526
      POINTEF=ABS(PSDOT2)*MO3/MIN                                            A 527
      WRITE (6,39) PHI1D,PHI2D,PSI1D,PSI2D,PSDOT1,PSDOT2,S1R,S2F,G2,POIN     A 528
     1TEF                                                                    A 529
   29 MTOT=MTOT+POINTEF                                                      A 530
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                GO TO 20                                                        A 531
             30 CYCLEFF=-#TOT*DDPHI1/(PHI1F-PHI1I)                              A 532
                WRITE (6,57) CYCLEFF                                            A 533
                #TOT=0.                                                         A 534
    535         IF (ISTOP.NE.0) GO TO 1                                         A 535
                STOP                                                            A 536
          C                                                                     A 537
          C                                                                     A 538
             31 FORMAT (6X,9HPSI1T1D =,F9.4,3X,8HTEST11 =,F9.4)                 A 539
    540      32 FORMAT (6X,9HPSI1T2D =,F9.4,3X,8HTEST12 =,F9.4//)               A 540
             33 FORMAT (6X,9HPSI2T2D =,F9.4,3X,8HTEST21 =,F9.4)                 A 541
             34 FORMAT (6X,9HPSI2T2D =,F9.4,3X,8HTEST22 =,F9.4//)               A 542
             35 FORMAT (85H      PHI1  PHI2  PSI1  PSI2  DPSI1  DPSI2  S1R  S2R  S A 543
               11F  G1    S2F    G2   POINTEF/)                                A 544
    545      36 FORMAT (6X,4(F4.0,2X),2(F5.0,2X),2(F3.0,2X),24X,F5.3)           A 545
             37 FORMAT (6X,4(F4.0,2X),2(F5.0,2X),5X,F3.0,2X,F3.0,2X,F5.3,14X,F5.3) A 546
             38 FORMAT (6X,4(F4.0,2X),2(F5.0,2X),10X,2(F3.0,2X,F5.3,2X),F5.3)   A 547
             39 FORMAT (6X,4(F4.0,2X),2(F5.0,2X),F3.0,19X,F3.0,2X,F5.3,2X,F5.3) A 548
             40 FORMAT (F10.3,F10.0/4F10.5/4F10.5/I1)                           A 549
    550      41 FORMAT (4F10.4)                                                 A 550
             42 FORMAT (4F10.4)                                                 A 551
             43 FORMAT (4F10.0)                                                 A 552
             44 FORMAT (3F10.4/F10.6/F10.4)                                     A 553
             45 FORMAT (1H1,5X,5HMIN =,F8.4,3X,4HMU =,F6.3,3X,5HRPM =,F6.0//6X,8HC A 554
               1APRP1 =,F8.5,3X,8HCAPRP2 =,F8.5//6X,5HRP2 =,F8.5,3X,5HRP3 =,F8.5// A 555
    555        26X,6HACG1 =,F8.5,3X,6HACG2 =,F8.5,3X,6HACP1 =,F8.5,3X,6HACP2 =,F8. A 556
               35/)                                                             A 557
             46 FORMAT (6X,7HRHOG1 =,F8.5,3X,7HRHOG2 =,F8.5,3X,7HRHOP1 =,F8.5,3X,7 A 558
               1HRHOP2 =,F8.5/)                                                 A 559
    560      47 FORMAT (6X,5HTG1 =,F8.5,3X,5HTG2 =,F8.5,3X,5HTP1 =,F8.5,3X,5HTP2 = A 560
               1,F8.5/)                                                         A 561
             48 FORMAT (6X,5HMG1 =,F5.0,3X,5HMG2 =,F5.0,3X,5HMP2 =,F5.0,3X,5HMP3 = A 562
               1,F5.0/)                                                         A 563
             49 FORMAT (6X,6HRHO1 =,F6.3,3X,6HRHO2 =,F6.3,3X,6HRHO3 =,F6.3//6X,4HM A 564
    565        1D =,F10.6//6X,3HK =,F6.1//6X,8HPHOOT1 =,F5.1/)                  A 565
             50 FORMAT (6X,8HBETA1D =,F8.4,3X,8HBETA2D =,F8.4/)                 A 566
             51 FORMAT (6X,30HSOMETHING IS WRONG WITH MESH 1)                   A 567
             52 FORMAT (6X,8HPHI1TD =,F8.4,3X,8HPSI1TD =,F8.4)                  A 568
             53 FORMAT (6X,8HPHI1ID =,F8.4,3X,8HPSI1ID =,F8.4,3X,8HPHI1FD =,F8.4,3 A 569
    570        1X,8HPSI1FD =,F8.4//)                                            A 570
             54 FORMAT (6X,30HSOMETHING IS WRONG WITH MESH 2)                   A 571
             55 FORMAT (6X,8HPHI2TD =,F8.4,3X,8HPSI2TD =,F8.4)                  A 572
             56 FORMAT (6X,8HPHI2ID =,F8.4,3X,8HPSI2ID =,F8.4,3X,8HPHI2FD =,F8.4,3 A 573
               1X,8HPSI2FD =,F8.4//)                                            A 574
    575      57 FORMAT (1H0,5X,18HCYCLE EFFICIENCY =,F5.3)                      A 575
             58 FORMAT (3F10.5)                                                 A 576
             59 FORMAT (6X,4HR1 =,F8.5,3X,4HR2 =,F8.5,3X,4HR3 =,F8.5/)          A 577
             60 FORMAT (6X,5HFP1 =,F8.5,3X,5HFP2 =,F8.5/)                       A 578
             61 FORMAT (3E15.5)                                                 A 579
    580      62 FORMAT (6X,4HM1 =,E15.5,3X,4HM2 =,E15.5,3X,4HM3 =,E15.5/)       A 580
                END                                                             A 581-
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```
      SUBROUTINE TRANS1 (RHOG,ALPHP,BETA,FP,ACG,B,DELG,Z,PSIT,PHIT,G)      B   1
      PI=3.14159                                                           B   2
      ST=(-RHOG*COS(PSIT-ALPHP)+B*SIN(BETA)+FP*SIN(PSIT-ALPHP))/ACG        B   3
      CT=(RHOG*SIN(PSIT-ALPHP)+B*COS(BETA)+FP*COS(PSIT-ALPHP))/ACG         B   4
      PHIT=ATAN2(ST,CT)-DELG                                               B   5
      PHINEXT=PHIT-.1*Z                                                    B   6
      AF=ACG*COS(PHINEXT+DELG+ALPHP)-B*COS(BETA+ALPHP)                     B   7
      BF=-ACG*SIN(PHINEXT+DELG+ALPHP)+B*SIN(BETA+ALPHP)                    B   8
      CF=RHOG                                                              B   9
      ROOTF=AF*AF+BF*BF-CF*CF                                              B  10
      YF1=AF+SQRT(ROOTF)                                                   B  11
      YF2=AF-SQRT(ROOTF)                                                   B  12
      XF=BF+CF                                                             B  13
      PSINEX1=2.*ATAN2(YF1,XF)                                             B  14
      PSINEX2=2.*ATAN2(YF2,XF)                                             B  15
      IF (PSINEX1.LT.0.) PSINEX1=PSINEX1+2.*PI                             B  16
      IF (PSINEX2.LT.0.) PSINEX2=PSINEX2+2.*PI                             B  17
      IF (ABS(PSINEX1-PSIT).LT.ABS(PSINEX2-PSIT)) GO TO 1                  B  18
      PSINEXT=PSINEX2                                                      B  19
      GO TO 2                                                              B  20
    1 PSINEXT=PSINEX1                                                      B  21
    2 G=(ACG*SIN(PHINEXT+DELG)+RHOG*COS(PSINEXT-ALPHP)-B*SIN(BETA))/SIN(   B  22
     1PSINEXT-ALPHP)                                                       B  23
      RETURN                                                               B  24
      END                                                                  B  25
```

```
      SUBROUTINE TRANS2 (RHOG,ALPHP,BETA,FP,ACG,B,DELG,Z,PSIT,PHIT,G)                C    1
      PI=3.14159                                                                     C    2
      ST=(RHOG*COS(PSIT+ALPHP)+B*SIN(BETA)+FP*SIN(PSIT+ALPHP))/ACG                   C    3
      CT=(-RHOG*SIN(PSIT+ALPHP)+B*COS(BETA)+FP*COS(PSIT+ALPHP))/ACG                  C    4
      PHIT=ATAN2(ST,CT)+DELG                                                         C    5
      PHINEXT=PHIT+.1*Z                                                              C    6
      AF=ACG*COS(PHINEXT-DELG-ALPHP)-B*COS(BETA-ALPHP)                               C    7
      BF=-ACG*SIN(PHINEXT-DELG-ALPHP)+B*SIN(BETA-ALPHP)                              C    8
      CF=-RHOG                                                                       C    9
      ROOTF=AF*AF+BF*BF-CF*CF                                                        C   10
      YF1=AF+SQRT(ROOTF)                                                             C   11
      YF2=AF-SQRT(ROOTF)                                                             C   12
      XF=BF+CF                                                                       C   13
      PSINEX1=2.*ATAN2(YF1,XF)                                                       C   14
      PSINEX2=2.*ATAN2(YF2,XF)                                                       C   15
      IF (PSINEX1.LT.0.) PSINEX1=PSINEX1+2.*PI                                       C   16
      IF (PSINEX2.LT.0.) PSINEX2=PSINEX2+2.*PI                                       C   17
      IF (ABS(PSINEX1-PSIT).LT.ABS(PSINEX2-PSIT)) GO TO 1                            C   18
      PSINEXT=PSINEX2                                                                C   19
      GO TO 2                                                                        C   20
    1 PSINEXT=PSINEX1                                                                C   21
    2 G=(ACG*SIN(PHINEXT-DELG)-RHOG*COS(PSINEXT+ALPHP)-B*SIN(BETA))/SIN(            C   22
     1PSINEXT+ALPHP)
      RETURN                                                                         C   23
      END                                                                            C   24
                                                                                     C   25
```

MIN = .1645 MU = .200 RPM = 1000.

CAPRP1 = .47725 CAPRP2 = .20670

RP2 = .09085 RP3 = .06890

ACG1 = .47725 ACG2 = .20670 ACP1 = .09085 ACP2 = .06890

R1 = .75000 R2 = .75000 R3 = .75000

RHOG1 = .03870 RHOG2 = .02070 RHOP1 = .01740 RHOP2 = .01040

TG1 = .03480 TG2 = .02520 TP1 = .02800 TP2 = .02060

NG1 = 42. NG2 = 27. NP2 = 8. NP3 = 9.

M1 = .69514E-04 M2 = .97028E-05 M3 = .10780E-05

RHO1 = .060 RHO2 = .030 RHO3 = .025

MD = .000015

K = 25.0

PHDOT1 = -1.0

FP1 = .08917 FP2 = .06811

BETA1D =112.2552 BETA2D =145.0978

PSI1T1D = 305.6017 TEST11 = 4.4495
PSI1T2D = 343.6259 TEST12 = 42.4736

PHI1TD =113.5016 PSI1TD =305.6017
PHI1ID =115.9930 PSI1ID =292.4283 PHI1FD =107.4216 PSI1FD =337.4283

PSI2T2D = 286.9442 TEST21 = 29.4719
PSI2T2D = 312.0785 TEST22 = 4.3377

PHI2TD =143.0461 PSI2TD =312.0785
PHI2ID =136.7828 PSI2ID =330.9521 PHI2FD =150.1161 PSI2FD =290.9521

PHI1	PHI2	PSI1	PSI2	DPSI1	DPSI2	S1R	S2R	S1F	G1	S2F	G2	POINTEF
116.	137.	292.	331.	5.	-16.	1.	-1.					.617
116.	137.	293.	329.	5.	-16.	1.	-1.					.626
116.	138.	294.	328.	5.	-16.	1.	-1.					.634
116.	138.	294.	326.	5.	-16.	1.	-1.					.642
116.	139.	295.	324.	5.	-16.	1.	-1.					.650
115.	139.	295.	323.	5.	-16.	1.	-1.					.657
115.	140.	296.	321.	5.	-16.	1.	-1.					.665
115.	141.	296.	320.	5.	-16.	1.	-1.					.672
115.	141.	297.	318.	5.	-16.	1.	-1.					.679
115.	142.	297.	316.	5.	-16.	1.	-1.					.686
115.	142.	298.	315.	5.	-16.	1.	1.					.693
115.	143.	298.	313.	5.	-16.	1.	1.					.689
115.	143.	299.	312.	5.	-16.	1.				1.	.068	.685

115.	144.	299.	310.	5.	-16.	1.					1.	.067	.680
115.	144.	300.	308.	5.	-16.	1.					1.	.067	.675
114.	145.	300.	307.	5.	-16.	1.					1.	.067	.670
114.	145.	301.	305.	5.	-16.	1.					1.	.066	.665
114.	146.	302.	303.	5.	-16.	-1.					1.	.066	.675
114.	146.	302.	302.	5.	-16.	-1.					1.	.066	.668
114.	147.	303.	300.	5.	-16.	-1.					1.	.066	.660
114.	147.	303.	298.	5.	-16.	-1.					1.	.066	.652
114.	148.	304.	297.	5.	-16.	-1.					1.	.066	.645
114.	149.	304.	295.	5.	-15.	-1.					1.	.066	.636
114.	149.	305.	294.	5.	-15.	-1.					1.	.065	.631
114.	150.	305.	292.	5.	-15.	-1.					1.	.066	.624
113.	137.	306.	331.	5.	-16.		-1.	1.	.089				.643
113.	137.	306.	329.	5.	-16.		-1.	1.	.089				.649
113.	138.	307.	328.	5.	-16.		-1.	1.	.088				.653
113.	138.	309.	326.	5.	-16.		-1.	1.	.088				.658
113.	139.	308.	324.	5.	-16.		-1.	1.	.088				.662
113.	140.	309.	323.	5.	-16.		-1.	1.	.087				.666
113.	140.	309.	321.	5.	-16.		-1.	1.	.087				.669
113.	141.	310.	319.	5.	-16.		-1.	1.	.087				.673
113.	141.	310.	318.	5.	-16.		-1.	1.	.086				.676
113.	142.	311.	316.	5.	-17.		1.	1.	.086				.688
112.	142.	311.	314.	6.	-17.		1.	1.	.086				.679
112.	143.	312.	313.	6.	-17.		1.	1.	.086				.671
112.	143.	313.	311.	6.	-17.			1.	.085	1.	.068		.662
112.	144.	313.	309.	6.	-17.			1.	.085	1.	.067		.653
112.	145.	314.	307.	6.	-17.			1.	.085	1.	.067		.645
112.	145.	314.	306.	6.	-17.			1.	.085	1.	.066		.636
112.	146.	315.	304.	6.	-17.			1.	.084	1.	.066		.628
112.	146.	315.	302.	6.	-17.			1.	.084	1.	.066		.620
112.	147.	316.	301.	6.	-17.			1.	.084	1.	.066		.612
112.	147.	316.	299.	6.	-17.			1.	.084	1.	.066		.605
111.	148.	317.	297.	6.	-16.			1.	.084	1.	.066		.597
111.	148.	318.	296.	6.	-16.			1.	.084	1.	.066		.590
111.	149.	318.	294.	6.	-16.			1.	.083	1.	.066		.583
111.	150.	319.	292.	6.	-15.			1.	.083	1.	.066		.576
111.	137.	319.	331.	6.	-17.		-1.	1.	.083				.594
111.	137.	320.	329.	5.	-17.		-1.	1.	.083				.599
111.	138.	320.	328.	5.	-17.		-1.	1.	.083				.604
111.	138.	321.	326.	5.	-16.		-1.	1.	.083				.608
111.	139.	321.	324.	5.	-16.		-1.	1.	.083				.612
111.	140.	322.	323.	5.	-16.		-1.	1.	.083				.616
110.	140.	323.	321.	5.	-16.		-1.	1.	.082				.619
110.	141.	323.	319.	5.	-16.		-1.	1.	.082				.623
110.	141.	324.	318.	5.	-16.		-1.	1.	.082				.626
110.	142.	324.	316.	5.	-16.		1.	1.	.082				.637
110.	142.	325.	314.	5.	-16.		1.	1.	.082				.629
110.	143.	325.	313.	5.	-16.		1.	1.	.082				.622
110.	143.	326.	311.	5.	-16.			1.	.082	1.	.068		.615
110.	144.	326.	309.	5.	-16.			1.	.082	1.	.067		.607
110.	144.	327.	308.	5.	-16.			1.	.082	1.	.067		.600
109.	145.	327.	306.	5.	-16.			1.	.082	1.	.066		.592
109.	145.	328.	305.	5.	-16.			1.	.082	1.	.066		.585
109.	146.	328.	303.	5.	-16.			1.	.082	1.	.066		.578
109.	147.	329.	301.	5.	-16.			1.	.082	1.	.066		.572
109.	147.	330.	300.	5.	-15.			1.	.082	1.	.066		.565
109.	148.	330.	298.	5.	-15.			1.	.082	1.	.066		.559
109.	148.	331.	297.	5.	-15.			1.	.083	1.	.066		.553
109.	149.	331.	295.	5.	-14.			1.	.083	1.	.066		.547
109.	149.	332.	294.	5.	-14.			1.	.083	1.	.066		.542
109.	150.	332.	292.	5.	-14.			1.	.083	1.	.066		.536

108.	150.	333.	291.	5.	-13.		1.	.083	1.	.067	.531
108.	137.	333.	331.	5.	-15.	-1.	1.	.083			.551
108.	137.	334.	329.	5.	-15.	-1.	1.	.083			.555
108.	138.	334.	328.	5.	-14.	-1.	1.	.083			.559
108.	138.	334.	327.	5.	-14.	-1.	1.	.084			.563
108.	139.	335.	325.	5.	-14.	-1.	1.	.084			.566
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108.	141.	337.	320.	4.	-13.	-1.	1.	.084			.578

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