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Abstract

The constant volume (CV) limit of pulsed propulsion was explored theoretically, where the combustion chamber was approximated as being time-varying but spatially uniform, while the nozzle flow was approximated as being one dimensional but quasi-steady. The CV limit was explored for the isentropic blow down of a constant γ ideal gas for fixed expansion ratios and for variable expansion ratios which were adjusted to match the pressure ratio at all times. The CV limit calculations were notable in that all the fixed expansion ratio results could be expressed as analytical solutions. The CV limit was then compared with two relevant constant pressure (CP) cycles. Among the several conclusions were that using a fixed expansion ratio nozzle during a CV blow down did not exert more than a 3% performance penalty over using a variable expansion ratio nozzle as long as the fixed expansion ratio nozzle was optimized for the blow down. Another major conclusion was that, while the impulse produced by a CV device could significantly exceed that of a CP device operating at the fill pressure of the CV device at elevated ambient pressures (*e.g.*, 1 atm), the impulse produced by a CP device could actually exceed, albeit only slightly, the impulse produced by a CV device when the ambient pressure was near zero, such as would occur in the near vacuum conditions in space.

Nomenclature

A - area
c - speed of sound
c_F - thrust coefficient
c_p - specific heat at constant pressure
F - thrust
$g(\gamma)$ - eq. (4)
I - impulse
$\dot{m}$ - rate of mass flow
M - Mach number
v - velocity
P - pressure
r - ρ / ρ_0
R - gas constant
t - time
T - temperature
V - volume

Greek

ε - expansion ratio
ϕ - P / P_∞
γ - ratio of specific heats
ι - $I / \rho_0 c_0 V$; dimensionless impulse
ρ - density
Ω, Φ - eq. (23)
τ - $t / c_0 A^* V$; dimensionless time

Subscripts

cp - pertaining to a constant pressure cycle
e - nozzle exit
f - fill condition
lc - pertaining to limit cycle operation
0 - initial condition before start of blow down
∞ - ambient condition

Superscripts

$*$ - throat condition

1. Introduction

Recent interest in pulsed detonation propulsion has spawned a number of attempts to model the system performance, the results from which have so far varied widely[1]. The constant volume (CV) limit of pulsed propulsion, as used here, refers to a limiting case for pulsed propulsion cycles which is approached when blow down times become much longer than characteristic wave transit times in the combustion chamber. Compared with how a practical pulsed propulsion device is likely to operate, the CV limit probably underpredicts the thrust (unrealistically small nozzles), but the specific impulse is probably close or equal to an upper bound. Thus the CV limit is a useful reference case. Some analytical solutions for the CV limit are explored here for the isentropic blow down of a constant γ ideal gas at fixed expansion ratios. The results are compared with the case where the expansion ratio is continuously varied to match the pressure ratio at all times during the blow down.

2. Formulation

2.1 General

The impulse produced by the unsteady blow down of the combustion gases in the CV limit is the integral of the thrust $F = \dot{m}v_e + (P_e - P_\infty)A_e$ over time, where $\dot{m}$ is the mass flow rate, P_∞ is the ambient pressure, and v_e, P_e, and A_e are the velocity, pressure, and area, respectively, at the exit plane. Under the transformation $dt = (dt/d\rho)d\rho = -(V/\dot{m})d\rho$ and the further transformation $r \equiv \rho/\rho_0$, where V is the combustion chamber volume and ρ_0 is the initial chamber density after combustion but before blow down at time $t = 0$, the total impulse I and blow down time t may be expressed as

$$I(t) = \int_0^t F(t)dt = \rho_0 V \int_r^1 (v_e + (P_e - P_\infty)A_e/\dot{m})dr, \quad (1)$$

$$t = \int_0^t dt = \rho_0 V \int_r^1 dr/\dot{m} \quad (2)$$

2.2 Fixed Nozzles

For isentropic flow, the instantaneous exit velocity is $v_e = \sqrt{2c_p(T - T_e)}$, where T and T_e are the instantaneous chamber and exit temperatures, respectively. Pressures and densities will be related in a simple fashion to temperatures according to $T/T_0 = (\rho/\rho_0)^{\gamma-1} = (P/P_0)^{(\gamma-1)/\gamma}$, where γ is the ratio of specific heats and where the subscript "0" denotes conditions in the chamber at time $t = 0$. With $c_0 = \sqrt{\gamma R T_0}$ and $c_p = \gamma R/(\gamma - 1)$, the exit velocity can be expressed as

$$v_e = c_0 r^{(\gamma-1)/2}\left(\frac{2}{\gamma-1}\right)^{1/2}\sqrt{1 - r_e^{\gamma-1}}, \quad (3)$$

where $r_e \equiv \rho_e/\rho$ is the ratio of the density at the exit plane to the density in the chamber. For isentropic nozzle flow, the exit density ratio is related to the expansion ratio $\varepsilon \equiv A_e/A^*$, where A^* is the throat area, by (*see* eq. 5.3 in ref. 2):

$$\varepsilon = \frac{g(\gamma)}{r_e\sqrt{1 - r_e^{\gamma-1}}};$$

$$g(\gamma) \equiv \left(\frac{\gamma-1}{2}\right)^{1/2}\left(\frac{2}{\gamma+1}\right)^{(1/2)(\gamma+1)/(\gamma-1)} \quad (4)$$

Equation (4) has two solutions for r_e corresponding to subsonic and supersonic flow at the exit plane. These solutions will depend only on ε and γ. For supersonic flow, the density ratio r_e will therefore remain constant during blow down as long as compression waves do not enter the nozzle, and eq. (3) will be analytically integrable when inserted into eq. (1). Compression/expansion waves will remain external to the nozzle for all r such that

$$r > \frac{1}{r_e}\left(\frac{\phi_e'}{\phi_0}\right)^{1/\gamma}, \quad (5)$$

where $\phi_0 \equiv P_0/P_\infty$ and $\phi_e \equiv P_e/P_\infty$ are the ratios of the initial and exit pressures, respectively, to the ambient pressure, and where ϕ_e' is the critical value of ϕ_e below which a compression wave enters the nozzle. The critical value of ϕ_e is given by (*see* eqs. 2.48a and 5.2 of ref. 2)

$$\phi_e' = \left[1 + \frac{2\gamma}{\gamma+1}(M_e^2 - 1)\right]^{-1}, \quad (6)$$

where

$$M_e^2 = \frac{2}{\gamma-1}\left[\left(\frac{1}{r_e}\right)^{\gamma-1} - 1\right]. \quad (7)$$

While eq. (5) holds, the flow will also be choked, and the throat conditions will be functions only of γ. For a throat density and velocity given by (*see* eq. 2.35a of ref. 2) $\rho^* = r\rho_0[2/(\gamma+1)]^{1/(\gamma-1)}$ and $v^* = \sqrt{\gamma R T^*} = c_0(2/(\gamma+1))^{1/2} r^{\gamma-1}$, and with $P_e = P_0 r_e^\gamma r^\gamma$, the instantaneous mass flow and thrust become

$$\dot{m}(r) = \rho^* A^* v^* = \rho_0 c_0 A^* g(\gamma)\left(\frac{2}{\gamma-1}\right)^{1/2} r^{(\gamma+1)/2} \quad (8)$$

$$F(r)/P_0 A^* g(\gamma) = r^\gamma \gamma \frac{2}{\gamma-1}\sqrt{1 - r_e^{\gamma-1}} + \quad (9)$$
$$+ \frac{1}{r_e\sqrt{1 - r_e^{\gamma-1}}}\left(r_e^\gamma r^\gamma - \frac{P_\infty}{P_0}\right)$$

where eq. (4) has also been used. Thus all terms in eq. (1) will integrable analytically.

Define $\iota \equiv I/\rho_0 c_0 V$ and $\tau \equiv t c_0 A^*/V$ to be a dimensionless impulse and a dimensionless time, respectively. Then integrating eqs. (1) and (2) gives

$$\iota(r) = \sqrt{1-r_e^{\gamma-1}}\left(\frac{2}{\gamma-1}\right)^{1/2}\left(\frac{2}{\gamma+1}\right)a(r) + \qquad (10)$$

$$+\frac{1}{\gamma r_e\sqrt{1-r_e^{\gamma-1}}}\left(\frac{\gamma-1}{2}\right)^{1/2}\left[r_e^{\gamma}\frac{2}{\gamma+1}a(r)-\frac{1}{\phi_0}\left(\frac{2}{\gamma-1}\right)b(r)\right]$$

$$\tau(r) = \frac{1}{g(\gamma)}\left(\frac{2}{\gamma-1}\right)^{1/2} b(r), \qquad (11)$$

where

$$a(r) \equiv 1 - r^{(\gamma+1)/2} \qquad b(r) \equiv (1/r)^{(\gamma-1)/2} - 1. \qquad (12)$$

The quantity ι/τ will be related to the average thrust $\bar{F} = I/t$. Noting that $c_0^2 = \gamma RT_0$ and $P_0 = \rho_0 RT_0$, then combining constants after dividing leads to

$$\gamma\iota/\tau = \bar{F}/P_0A^* \equiv \bar{c}_F, \qquad (13)$$

where $\bar{c}_F$ is the average thrust coefficient for the blow down.

2.3 Variable Nozzles

Fixed expansion ratio nozzles will be optimized for at most only a single value of r during the blow down; losses will occur at all other values. To assess what the performance would be if these losses were not present, a variable nozzle can be envisioned where the expansion ratio is continuously adjusted to match the pressure ratio at all times. The variable nozzle limit produces the maximum possible impulse, and is therefore also useful as a reference case.

With the exit pressure always matched to the ambient pressure, only the exit velocity term $v_e = \sqrt{2c_p(T-T_e)}$ in eq. (1) will be of concern. For fixed nozzles, it was found that the term $T_e/T = r_e^{\gamma-1}$ was a constant, but here the exit temperature is fixed by the exit conditions $P_e = P_\infty$, $\rho_e = \rho_0(1/\phi_0)^{1/\gamma}$, and $T_e = T_0(1/\phi_0)^{(\gamma-1)/\gamma}$. Therefore T_e/T will not be constant in this case. The dimensionless impulse in this case becomes

$$\iota(r) = \left(\frac{2}{\gamma-1}\right)^{1/2}\int_r^1 \sqrt{r^{\gamma-1} - (1/\phi_0)^{(\gamma-1)/\gamma}}\,dr. \qquad (14)$$

Eq. (14) cannot be integrated analytically but may easily be integrated numerically.

The blow down time for the variable nozzle limit will depend on the mass flow, and calculation of the mass flow will depend on whether the exit area or the throat area is varied to match the pressure ratio. The mass flow is best calculated using the area which is being held constant. If the exit area is assumed to be varied, as might approximately be the case with an automatically compensating nozzle such as a plug nozzle, the mass flow and blow down time based on a constant throat area are still given by eqs. (8) and (11), respectively. If the throat area is varied, then the mass flow and blow down time for a fixed exit area are

$$\dot{m} = \rho_e A_e v_e = \rho_0 c_0 A_e\left(\frac{2}{\gamma-1}\right)^{1/2}\left(\frac{1}{\phi_0}\right)^{1/\gamma}\sqrt{r^{\gamma-1} - (1/\phi_0)^{(\gamma-1)/\gamma}} \qquad (15)$$

$$\tau_e(r) = \phi_0^{1/\gamma}\left(\frac{\gamma-1}{2}\right)^{1/2}\int_r^1 dr\Big/\sqrt{r^{\gamma-1} - (1/\phi_0)^{(\gamma-1)/\gamma}} \qquad (16)$$

where $\tau_e = tc_0A_e/V$ is defined this time in terms of the exit area. Eq. (16) also cannot be integrated analytically but may easily be integrated numerically. Inasmuch as shocks and expansion waves will not occur when the expansion ratio is always matched to the instantaneous pressure ratio, eqs. (14)-(16) above for the variable nozzle case will apply until the expansion ratio reduces to unity and the flow becomes unchoked, *i.e.*, for all r such that

$$r > \left(\frac{\gamma+1}{2}\right)^{1/(\gamma-1)}\left(\frac{1}{\phi_0}\right)^{1/\gamma} \qquad (17)$$

2.4 Limit Cycle Operation

In repetitive operation, intake valves would be opened at some point during the blow down in order to introduce fresh propellants for the next cycle. Any remaining combustion products not yet expelled would be pushed out, or purged, by the incoming fresh propellants. If the purge of the remaining combustion products can be approximated as a constant pressure process, and the value of r is large enough to prevent shocks from entering the nozzle, then the above equations can be used to calculate the time to purge the remaining mass and the additional impulse produced. Specifically, the time required to purge the remaining mass is $t_{cp}(r) = \rho V/\dot{m} = \rho_0 Vr/\dot{m}(r)$, and the additional impulse produced is $I_{cp}(r) = F(r)t_{cp}(r)$, where $F(r)$ and $\dot{m}(r)$ are given by eqs. (8) and (9) at a constant value of r during the purge. The additional dimensionless impulse and blow down times can still be expressed in the forms of eqs. (10) and (11), but with the functions $a(r)$ and $b(r)$ given instead by

$$a_{cp}(r) = \frac{\gamma+1}{2}r^{(\gamma+1)/2} \qquad b_{cp}(r) = \frac{\gamma-1}{2}r^{(\gamma-1)/2} \qquad (18)$$

where the subscript "*cp*" stands for the these constant pressure additions. The total impulse for the entire cycle can then be expressed, again in the forms of eqs. (10) and (11), but this time with the functions $a(r)$ and $b(r)$ given by

$$a_{lc}(r) = a(r) + a_{cp}(r) = 1 + \frac{\gamma-1}{2}r^{(\gamma+1)/2} \qquad (19)$$

$$b_{lc}(r) = b(r) + b_{cp}(r) = \frac{\gamma+1}{2}\left(\frac{1}{r}\right)^{(\gamma-1)/2} - 1$$

where the subscript "*lc*" stands for "limit cycle." For limit cycle operation with variable nozzles, the total cycle impulse is given instead by the sum of the constant pressure impulse with the impulse calculated by eq. (14), and the total cycle blow down time is the sum of the constant pressure time plus the blow down time calculated by eq. (11) or eq. (16). The expansion ratio always matches the pressure ratio in the variable nozzle case, so the exit density ratio must be set to $r_e = (1/r)(1/\phi_0)^{1/\gamma}$.

The density ratio r cannot be specified arbitrarily if limit cycle operation is to be achieved. Limit cycle operation requires the blow down to proceed at least until the chamber pressure reaches the fill pressure at which fresh propellants are introduced. This gives $r \le (P_f / P_0)^{1/\gamma}$, where P_f is the fill pressure. The inequality sign indicates that the blow down can proceed to a pressure lower than the fill pressure if the fresh reactants undergo compression during the fill. Given that $\rho_0 = \rho_f$ for CV heat addition, the ratio P_f / P_0 could be on the order of 1000 times smaller for propellants in an initial liquid state compared to an initial gaseous state.

For a given density ratio r, the blow down of the remaining combustion products in a CP mode produces a greater impulse than if the blow down had been permitted to proceed to a lower density ratio. Therefore, given a choice, the density ratio r should be selected to be the maximum value possible consistent with limit cycle operation and available injection pressure, namely $r = (P_f / P_0)^{1/\gamma}$.

2.5 Comparisons with Constant Pressure Devices

Comparisons of the dimensionless impulse or the thrust coefficient with corresponding quantities for constant pressure (CP) processes can be performed if the normalization constants are the same in both cases. Here the normalization constants are chosen to be those of the CV process, leading to the correction factors Ω and Φ in the resulting expressions below. Assuming the expansion ratio for the CP process is chosen to match the pressure ratio, the resulting dimensionless impulse, blow down time, and average thrust coefficient become

$$\iota_{cp} = \frac{1}{\Omega}\left(\frac{2}{\gamma-1}\right)^{1/2} \sqrt{1-(\Phi/\phi_0)^{(\gamma-1)/\gamma}} \qquad (20)$$

$$\tau_{cp} = \frac{\Phi}{\Omega}\frac{1}{g(\gamma)}\left(\frac{\gamma-1}{2}\right)^{1/2} \qquad (21)$$

$$\bar{c}_{F,cp} = \gamma g(\gamma)\frac{2}{\gamma-1}\frac{1}{\Phi}\sqrt{1-(\Phi/\phi_0)^{(\gamma-1)/\gamma}}\,, \qquad (22)$$

where

$$\Omega \equiv c_0 / c_{cp} \qquad \Phi \equiv P_0 / P_{cp} \qquad (23)$$

and where $\iota_{cp} \equiv I_{cp} / \rho_0 c_0 V$, $\tau_{cp} \equiv t_{cp} c_0 A^* / V$, $\bar{c}_{F,cp} \equiv \bar{F} / P_0 A^*$, and the values of ρ_0, c_0, P_0, and V are those of the constant volume process. The quantity τ_{cp} in this context can be interpreted to be the dimensionless time required by the constant pressure process to expel the same mass as the initial mass of the constant volume process.

3. Results and Discussion

3.1 Effect of Thermochemistry

The dimensionless impulse and blow down time, eqs. (10)-(12), depend explicitly on r and implicitly only on γ, r_e (or ε), and ϕ_0. Of these, only γ depends on the thermochemistry, but this dependency is weak. The major influence of thermochemistry comes through the initial speed of sound c_0 used to normalize the impulse. Thus the specific impulse $I_s = \iota c_0$ is maximized when c_0 is maximized. This in turn implies that the optimum thermochemistry is that which maximizes the initial combustion temperature T_0 and minimizes the molecular weight. The same general guidance is known to also apply to constant pressure devices. *Thus constant volume devices should optimize at approximately the same mixture ratios as constant pressure devices.* Also, like constant pressure devices, the thrust is maximized by maximizing the initial chamber pressure and the throat area, as implied by eq. (9).

3.2 Fixed Nozzles

Much can be understood about the blow down of fixed nozzles by examining conventional steady state thrust coefficient curves such as can be found in standard texts.[3] The thrust coefficient is defined in eq. (13), where P_0 in the steady state case is interpreted to be the steady chamber pressure of a constant pressure device. A set of these curves for the steady state case is given in Fig. 1 for $\gamma = 1.2$. These curves were generated by setting $r = 1$ in eq. (18), substituting into eqs. (10) and (11), and then calculating $\bar{c}_F$ per eq. (13). The curves reproduce those given in ref. 3. Dimensionless isobars (curves of constant ϕ_0) initially increase with ε, reach a maximum, then decrease to a minimum where a shock enters the nozzle and the above formulation is no longer valid. The curve for $\phi_0 = \infty$ reaches a maximum only at $\varepsilon = \infty$. A curve connecting the maxima indicates the expansion ratio producing the maximum thrust for a given isobar.

The blow down of a fixed nozzle CV device proceeds along a vertical path of constant ε between two isobars. The thrust produced will be some average between the two isobars. Picking any two isobars in Fig. 1, say between $\phi_0 = 50$ and $\phi_0 = 20$, and following the vertical line between them for various ε, it can be envisioned that the average thrust will reach a maximum neither at large ε nor at $\varepsilon = 1$, but at some optimum ε in between. However, the average thrust cannot be calculated directly from Fig. 1, because the thrust coefficient is proportional to the thrust divided by the chamber pressure, not the thrust alone. A blow down from $\phi_0 = 50$ to $\phi_0 = 20$ is replotted in Fig. 2 where the curve for $\phi_0 = 20$ is normalized by the same pressure as for $\phi_0 = 50$, making all curves proportional to the thrust. This is accomplished by multiplying the thrust coefficient of Fig 1. by 0.4 for $\phi_0 = 20$. The curve labeled "b.d." gives the average thrust coefficient for the blow down and is seen to be an average of the curves for $\phi_0 = 50$ and $\phi_0 = 20$, as expected. The variable nozzle case shown in Fig. 2 will be discussed later.

Average thrust coefficients are plotted as functions of ϕ_0 and ε for blow downs to $r = 0.75$, 0.5, and 0.25 in Figs. 3, 4, and 5, respectively. Dimensionless blow down times are plotted as a function of r for two values of γ in Fig. 6. As can be seen from eq. (11) and Fig. 6, blow down times are independent of ϕ_0, and depend only weakly on γ. Because τ is fixed and independent of ϕ_0 for a fixed r, the curves in Figs. 3, 4, and 5 will also be proportional to the dimensionless impulse ι, as can be shown from eq. (13). However, the constant of proportionality will be different for each r, increasing as r decreases, because τ increases as r decreases. The constant of proportionality is given in the figures. The overall trend in these figures is that the average thrust coefficient decreases as r decreases. This is due to τ increasing more rapidly than ι as r decreases, because ι also increases as r decreases.

The expansion ratio which produces the maximum impulse can in principle be found by taking the first derivative of eq. (10) with respect to r_e and setting it equal to zero. The resulting expression cannot be solved analytically, however, so the optimum expansion ratio is computed here by finding the maximum impulse using a numerical search. The optimum expansion ratio is plotted as a function of ϕ_0 and r in Fig. 7, and the dimensionless impulse produced at the optimum expansion ratio is plotted in Fig. 8. Because r is variable, τ is also, so the dimensionless impulse and the average thrust coefficient curves are no longer directly proportional as they were in earlier figures, where r was fixed. The optimum thrust coefficient is plotted in Fig. 9.

3.3 Variable Nozzles

Because the expansion ratio is always optimized for variable nozzles, the blow down of variable nozzles proceeds along the curve of maximum thrust in Fig. 1. The dimensionless impulse produced can be computed as a function of ϕ_0 and r by integrating eq. (14), and compared with the performance of fixed nozzles operating at the optimum expansion ratio. The results are not easily viewed in the form of Fig. 8, however, so they are plotted instead here in Fig. 10 as the impulse penalty in using optimized fixed nozzles compared to the variable nozzle case, based on the percentage of the variable nozzle impulse. The penalty is shown not to exceed about 3% for all cases calculated. The reason the penalty is not more severe can be traced to the flatness of the curves in Fig. 1 near the points of maximum thrust. This is illustrated more clearly in Fig. 2, where the difference between the variable and optimized fixed nozzle blow down paths is shown. As can be seen, the maximum instantaneous thrust coefficient is not that much higher for the variable nozzle case than for optimized fixed nozzles, leading to calculated average thrust coefficients which are very close.

3.4 Comparison with Constant Pressure Devices

By normalizing with the same constants as used for the CV device, the correction factors Ω and Φ as defined in eq. (23) and used in eqs. (20)-(22) allow the normalized quantities for CP devices to be directly compared with CV devices at any arbitrary CP chamber pressure and speed of sound (temperature). However, only certain cases will be of interest here. It is presumed that the two cycles are to be compared for the same propellant combination and at the optimum mixture ratio for each cycle, the latter of which as discussed above should be approximately the same in both cases. If T_f is the initial fill temperature before combustion, and q is the heat of reaction per unit mass for a given propellant combination and mixture ratio, then the initial CP temperature after combustion will be approximately $T_{cp} = T_f + q/c_p$, while the initial CV temperature after combustion will be approximately $T_0 = T_f + \gamma q/c_p$. Thus Ω will vary only over a very narrow range near unity, namely from $\Omega = 1$ for $q = 0$ to $\Omega = \gamma^{1/2}$ for $q/c_p >> T_f$. For $\gamma = 1.2$, this corresponds to $1 \le \Omega \le 1.095$.

The next question is which of the various pressures encountered over the CV blow down, or not encountered, should be used for a CP comparison. Only two possibilities will be considered here. The first is the case where the pressure of the CP device is the same as the fill pressure of the CV device, namely $\Phi = P_0 / P_f$. This would allow a comparison of two devices having equivalent feed systems. Assuming the purge pressure is the same as the feed pressure, limit cycle operation gives $r = (P_f / P_0)^{1/\gamma}$. Given further that $P_0 = \rho_0 R T_0$ and that $\rho_0 = \rho_f$ for a CV device, where ρ_f is the density after propellant fill but before combustion, then $P_0 / P_f = 10$ would be representative of propellants in an initial gaseous state, while $P_0 / P_f = 1000$ would approach propellants in an initial liquid state. A CP device operating at the same pressure as the fill pressure of the CV device will be referred to here as a "CP$_f$" device for brevity.

The second possibility will be to compare the performance of combustion chambers experiencing the same material stresses. A very rough approximation of this would be a CP device operating at the same pressure as the peak pressure of a CV device, namely $\Phi = 1$. A CP device operating at the same pressure as the peak pressure of the CV device will be referred to here as a "CP$_p$" device, again for brevity.

The ratio of the dimensionless impulse produced by a CP$_f$ device to the dimensionless impulse of a CV device is presented in Fig. 11, and the ratio of the dimensionless impulse produced by the CP$_p$ device to the dimensionless impulse of a CV device is presented in Fig. 12. Because the same quantities are used to nondimensionalize the impulse in all cases, these plots also give the ratio of the total impulses and the ratio of the specific impulses. The expansion ratios used for the CP and CV devices are those which produce the maximum possible impulse in each case, namely a fixed expansion ratio which matches the fixed pressure ratio for the CP device, and a variable expansion ratio to continuously match the variable pressure ratio for the CV device. The CV device is operated in limit cycle mode where the blow down is performed to the maximum value of r consistent with limit cycle operation, $r = (P_f / P_0)^{1/\gamma}$. It is assumed that $q / c_p >> T_f$ in all cases, giving $\Omega = \gamma^{1/2}$.

Curves are drawn in Figs. 11 and 12 for different ratios of the peak pressure to the fill pressure, P_0 / P_f. Increases in this ratio indicate either more energetic chemistry (greater temperature/pressure rise), or the presence of condensed phases in the initial fill. Increases over two orders of magnitude as shown in the figures would be largely the result of the presence of condensed phases. Increases in P_0 / P_f always increase the difference between the CP and CV performance. The right hand extent of all the curves is dictated by the shock limit, beyond which a shock enters the nozzle and the present formulation no longer applies. The left hand extent of the curves as drawn in the figures is limited solely for reasons of graphical clarity; the curves in both figures actually all extend to $1/\phi_0 = 0$. The limiting values of the curves at $1/\phi_0 = 0$ are given in the figures. These limiting values are exactly the same for both the CP$_f$ and CP$_p$ cases. The reason for this is that eq. (20) becomes independent of the chamber pressure when $1/\phi_0 = 0$.

Away from $1/\phi_0 = 0$, the impulse of a CP$_f$ device is shown in Fig. 11 to be inferior to that of the CV device, and the impulse of a CP$_p$ device is shown in Fig. 12 to be superior to that of the CV device. The relative differences diminish as $1/\phi_0$ approaches zero. At large enough $1/\phi_0$, the relative differences are significantly larger than the 3% maximum performance penalty shown in Fig. 10 attributed to using an optimized fixed nozzle instead of a variable nozzle. This implies that, at large enough $1/\phi_0$, the performance penalty associated with the practical simplicity of using an optimized fixed nozzle might be tolerable compared with the complications of developing a variable nozzle. To assess whether a practically attainable device could fall within a regime where such losses might be tolerable, a reference case consisting of the approximate operating point of a stoichiometric kerosine/air (jet fuel/air) system operating at 1 atm ambient pressure and a 3 atm fill pressure is plotted as a black dot in Fig. 11. The operating point is shown to be well within a regime where such performance losses might be tolerable. Whether such losses would in fact be tolerable in a practical system would of course depend on all the other system losses.

The case $1/\phi_0 = 0$ is an important limit in that it represents operation where the ambient pressure is a vacuum, *i.e.*, in space. As $1/\phi_0$ approaches zero, the difference between a CP$_f$ device and a CP$_p$ device vanishes, and the impulse of the CP device can become either slightly higher or slightly lower than the impulse of the CV device, depending on the magnitude of P_0 / P_f. The reason for this may be found in deriving the following analytical expression for ι / ι_{cp}, which is possible when $1/\phi_0 = 0$. For limit cycle operation, integrate eq. (14) from $r = (P_f / P_0)^{1/\gamma}$ to $r = 1$, which now can be done analytically, and add the impulse from the constant pressure part of the cycle by using the functions from eq. (18) in eq. (10). Note that $r_e = 0$ ($\varepsilon = \infty$) when $1/\phi_0 = 0$. Divide the result into eq. (20). The result is

$$\frac{\iota_{cp}}{\iota} = \frac{1}{\Omega}\left(\frac{\gamma+1}{2}\right)\left[1+\frac{\gamma-1}{2}\left(\frac{P_f}{P_0}\right)^{(\gamma+1)/2\gamma}\right]^{-1}. \quad (24)$$

The maximum value of the expression occurs as $P_0/P_f \to \infty$, which for $\Omega = \gamma^{1/2}$ and $\gamma = 1.2$ is $\iota/\iota_{cp} = 1.004$. As P_0/P_f becomes smaller, the magnitude of the impulse ratio also becomes smaller, until eventually it is reduced to a value less than unity. For $P_0/P_f = 10$, this value becomes $\iota/\iota_{cp} = 0.992$. Limit values of ι/ι_{cp} at $1/\phi_0 = 0$ shown in Figs 11 and 12 were calculated using eq. (24).

4.0 Conclusions

Analytical solutions of the constant volume limit of pulsed propulsion for fixed expansion ratios have been explored and compared with the case where a variable nozzle is used to match the pressure ratio at all times during the blow down. The solutions apply for all supersonic flows where compression waves remain exterior to the nozzle. It was found that:

1. CV devices should optimize at approximately the same mixture ratio as CP devices, namely the mixture ratio which maximizes the initial temperature and minimizes the molecular weight.
2. The thrust of CV devices is maximized in the same way as for CP devices, namely by maximizing the initial pressure and the throat area.
3. The blow down time depends on $r = \rho/\rho_0$, only weakly on γ, and is independent of the initial pressure ratio $\phi_0 = P_0/P_\infty$.
4. In general an optimum expansion ratio exists which maximizes the impulse produced by the blow down of a CV device at a fixed expansion ratio.
5. Using an optimized fixed expansion ratio nozzle results in an impulse penalty that does not exceed 3% (for the cases considered) of the impulse that would be produced using the more complicated variable expansion ratio nozzle.
6. A comparison between a CP device operating at its optimum expansion ratio and a CV device in limit cycle operation with a variable expansion ratio leads to the following additional conclusions:
   a. Except near $1/\phi_0 = P_\infty/P_0 = 0$, the impulse produced by a CV device is superior to that of a CP device operating at the same pressure as the fill pressure of the CV device, but inferior to the impulse produced by a CP device operating at the same pressure as the peak pressure of the CV device.
   b. The magnitude of the difference between a CV device and the two CP devices considered increases as $1/\phi_0$ increases.
   c. At large enough $1/\phi_0$, the relative difference between a CV device and the two CP devices considered is significantly larger than the impulse penalty associated with using an optimized fixed expansion ratio instead of a variable nozzle on the CV device.
   d. The regime where the relative difference between a CV device and a CP device is significantly larger than the impulse penalty associated with using an optimized fixed expansion ratio on the CV device appears to be practically achievable for at least the one reference case considered, which involved kerosine/air operation at 1 atm ambient pressure.
   e. Near $1/\phi_0 = P_\infty/P_0 = 0$, which is an important limit in that it represents operation at the near zero ambient pressure of space, the difference between the two CP devices considered vanishes, and the impulse of the CP device can become either slightly higher or slightly lower than the impulse of the CV device, depending on the magnitude of P_0/P_f.

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References

1. Kailasanath, K., Patnaik, G., and Li, C., "Computational Studies of Pulse Detonation Engines: A Status Report," 35th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, paper AIAA 99-2634, 20-24 June 1999, Los Angeles, CA.
2. Leipman, H.W., and Roshko, A., *Elements of Gasdynamics*, John Wylie & Sons, 1957.
3. Sutton, G.P., *Rocket Propulsion Elements*, 6th ed., John Wiley & Sons, 1992.

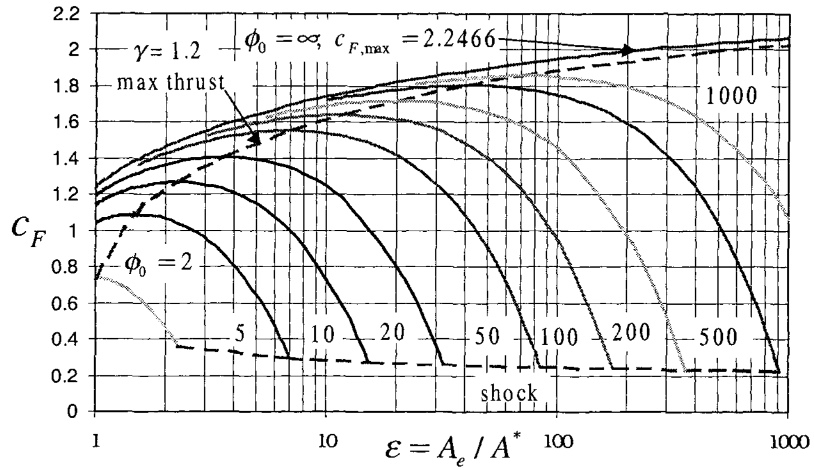

Figure 1. Steady state thust coefficients

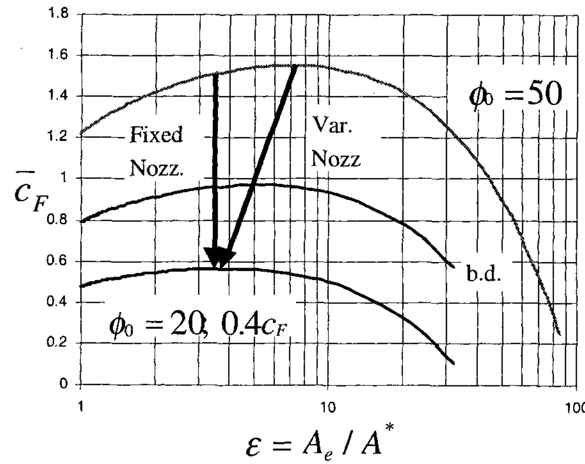

Figure 2. Blow down from $\phi_0 = 50$ to $\phi_0 = 20$, where "b.d." curve is for the blow down. Arrows indicate paths taken for fixed and variable nozzles.

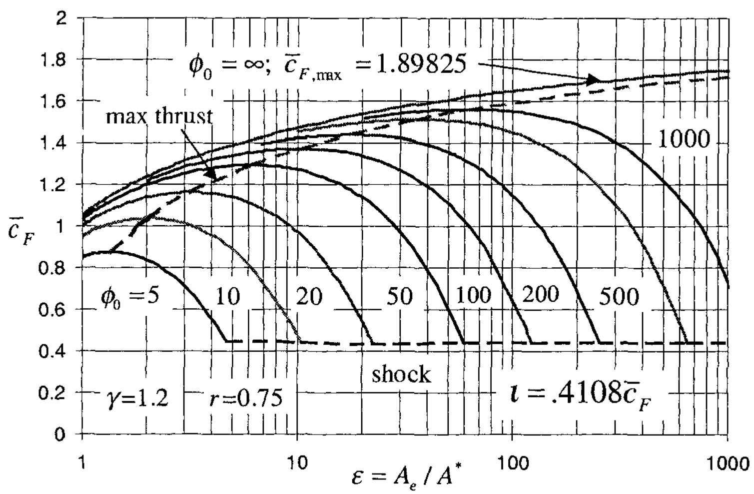

Figure 3. Blow down to $r = 0.75$

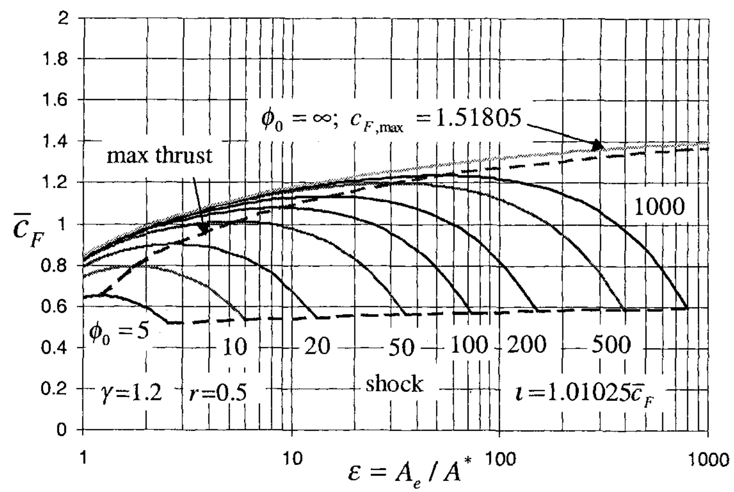

Figure 4. Blow down to $r = 0.5$

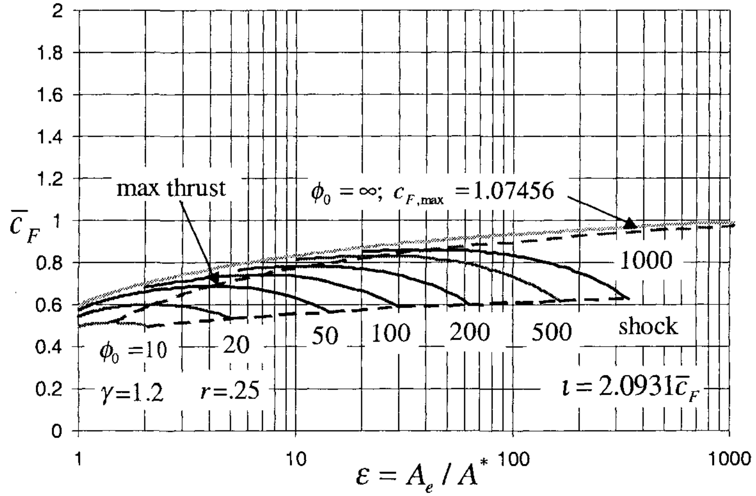

Figure 5. Blow down to $r = 0.25$

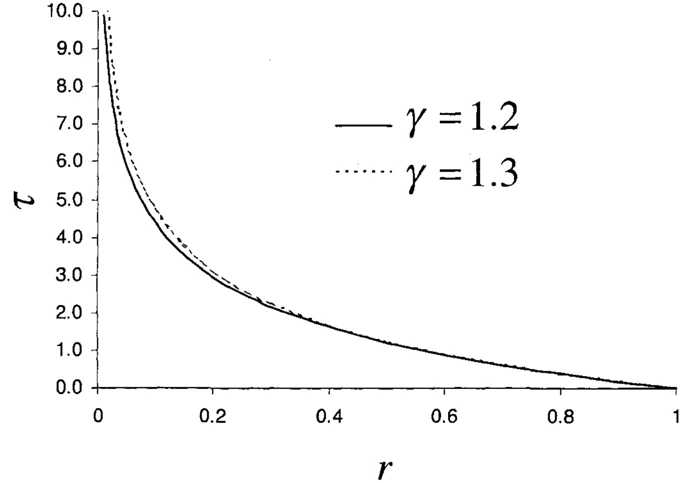

Figure 6. Blow down times

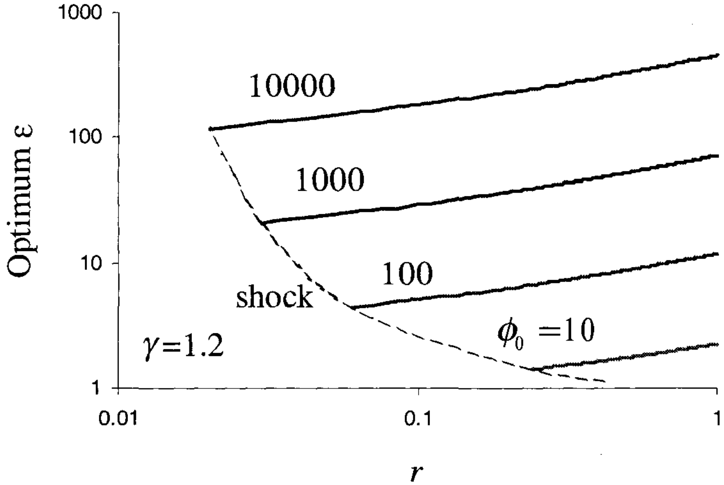

Figure 7. Optimum expansion ratios

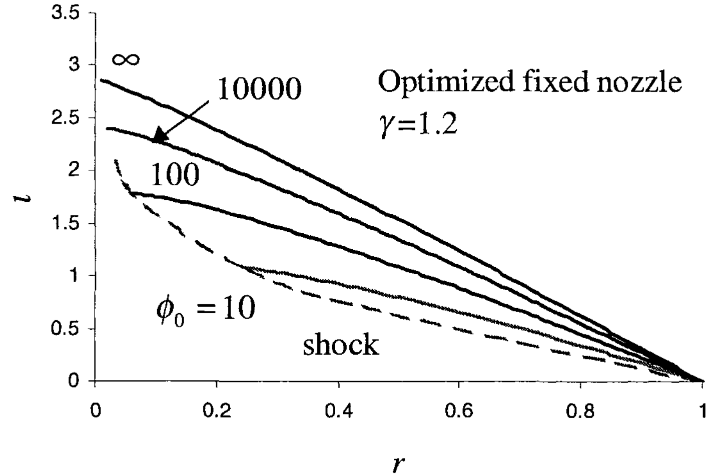

Figure 8. Dimensionless impulse at the optimimum expansion ratio

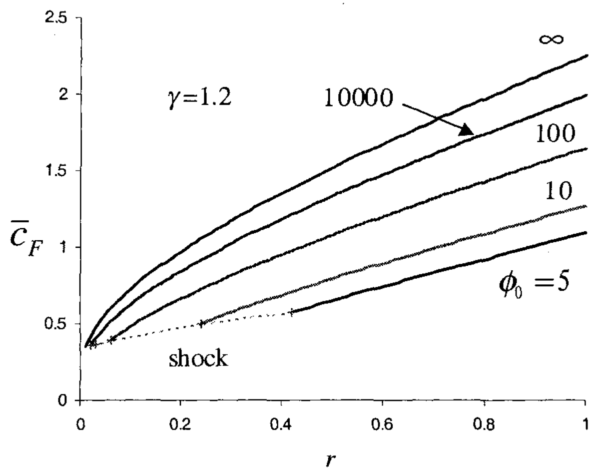

Figure 9. Thrust coefficient at the optimum expansion ratio

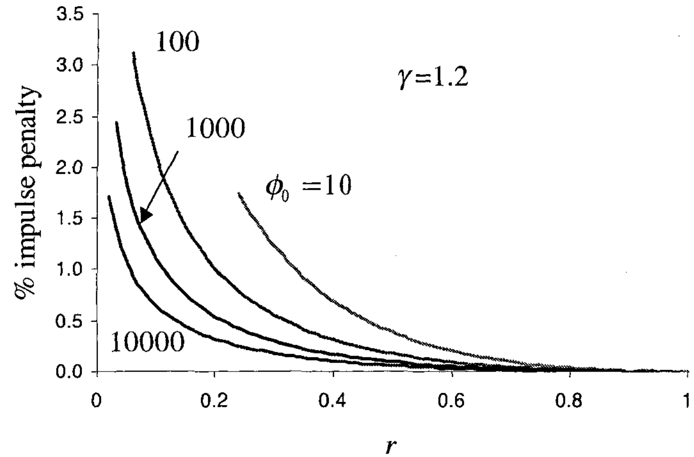

Figure 10. Impulse penalty of an optimized fixed nozzle Compared with a variable nozzle

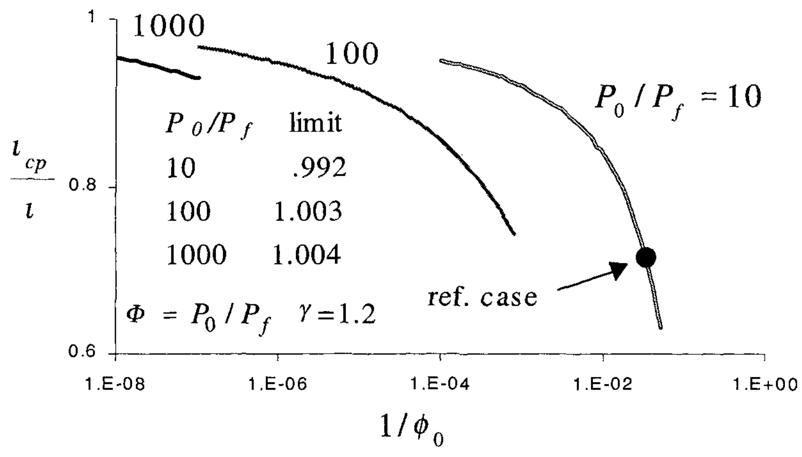

Figure 11. Comparison of CP_f and CV impulse. The reference case is stoichiometric kerosine/air at 1 atm ambient pressure and 3 atm fill pressure.

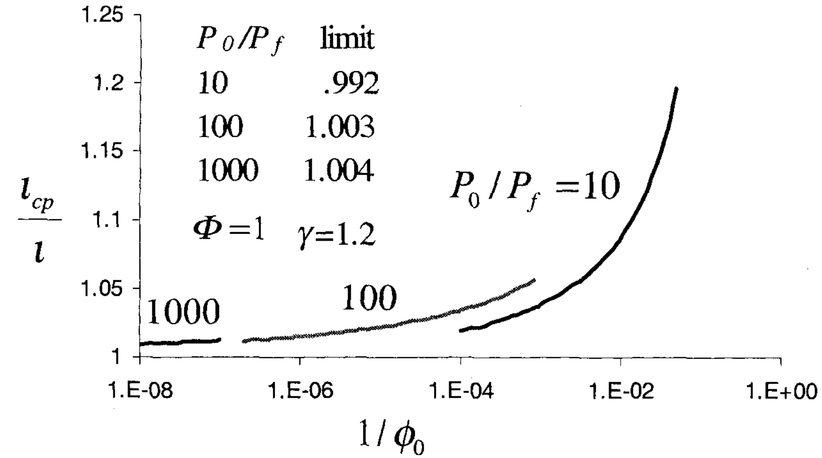

Figure 12. Comparison of CP_p and CV impulse.