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UNITED STATES AIR FORCE

School of AVIATION MEDICINE

DISCRIMINATORY ANALYSIS

II. FACTOR ANALYSIS AND DISCRIMINATION

PROJECT NUMBER 21-49-004
REPORT NUMBER 2
(Formerly Project Number 21-02-105)

PROJECT REPORT

DISCRIMINATORY ANALYSIS

II. FACTOR ANALYSIS AND DISCRIMINATION

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PROJECT NUMBER 21-49-004
REPORT NUMBER 2*
(Formerly Project Number 21-02-105)

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USAF SCHOOL OF AVIATION MEDICINE
RANDOLPH FIELD, TEXAS
OCTOBER 1950

Precis

OBJECT:

To survey the literature on factor analysis theory in order to establish the connection between factoring and discriminating.

SUMMARY:

The theory of factor analysis is cogently expressed in matrix notation. Some of the mathematical statistical vagaries of factor analysis are alluded to. Several hundred references on factor analysis are summarized.

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DISCRIMINATORY ANALYSIS - II - FACTOR ANALYSIS AND DISCRIMINATION

EVELYN FIX OCT'50 82PP

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AVIATION MEDICINE (19) BIOMETRICS
GENERAL (0) STATISTICAL ANALYSIS

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SECTION 1.

Introduction.

The inclusion of a section on factor analysis in a monograph designed to review the literature on discriminatory analysis may require explanation. It seems apparent that a technique as closely related to the central problem of classification as factor analysis would enter into the survey but the decision to devote a section to it was based mainly on two considerations. One is the very widespread interest in the subject which has reflected itself in the enormous amount of literature in which the various authors have been concerned with many different aspects of factor analysis and its relations to other statistical techniques. The second reason relates to the research phase of the project. We found that, in a great many cases, the populations in which we will be interested have been analyzed by the factor method. It seemed relevant then to make more than a cursory survey of the field.

However, because factor analysis is subsidiary to the main subject of this report and because the material is readily accessible and well-documented, a somewhat different plan has been adopted for this section. No attempt has been made to present a complete bibliography since this would mean a duplication of existing lists [Dodd (1928), Wolfle (1940) and Swineford and Holzinger (1936 on)]. The papers given in the bibliography have been classified according to their main point of interest and are listed with reference to this classification after the pertinent discussion. Also, since there are excellent texts and survey articles which summarize and compare the existing techniques, and since a complete discussion would be impractical, we were concerned with giving a picture of the basic problem and then briefly sketching in the developments. The stress placed upon the questions which are to be answered by factor analysis differs from that found in psychological discussions since the motivation is quite different. In this connection, it should be mentioned that the recently started periodical, the British Journal of Psychology--Statistical Section, has a definite policy of presenting expository papers designed for the non-expert in its field. There have been excellent articles on factor analysis and its relations to various topics.

I would like to thank Professor J. L. Hodges, Jr. and Professor E. L. Lehmann for their very great help in the preparation of this section.

Bibliographies: Dodd (1928), Swineford and Holzinger (1936 on), Wolfle (1940).

Texts: Brown and Thomson (1940), Burt (1940), Holzinger and Harman (1941), Kelley (1928, 1935), Spearman (1927), Thomson (1948a), Thorndike, E. L. (1913), Thurstone (1935, 1947).

Survey articles: Bartlett (1937), Burt (1949a, b), Conrad (1944), Cureton (1939), Darmois (1940), Hartog et al (1936), Holzinger (1930, 1942, 1944), Irwin (1943a, b), Lino (1933), Piaggio (1933a), Thomson (1938a, f, 1939, 1946), Wolfle (1940).

SECTION 2.

Origin of Factor Analysis.

The starting point of any stream of thought is always poorly defined but we have selected the two situations which seem to have given definite formulations to the motives and methods of factor analysis, one preceding the terminology now employed, the other, a catalytic agent. We have described them in some detail because they give an answer to the question often asked, "Why do people factor?" and are suggestive of the lines of thought which factor analysts have followed.

The first situation was one of practical importance. The Central Metric Office, New Scotland Yard, was faced with the problem of constructing a filing system which would permit ready classification and identification of criminals on the basis of their physical measurements. The system of measures tentatively adopted was that of Bertillon and consisted in twelve measures on each individual. The procedure followed was to divide each measure into three classes, small, medium and large, so that each class contained approximately equal numbers of individuals. This meant that each individual would fall into one of 3^{12} categories when all twelve measures were considered, and it was their hope that each of these categories would contain approximately equal numbers. Actually, of course, this did not happen and the Central Office then used the practical expedient of breaking up each of the three divisions small, medium, large of one measure, say knee height into three subdividions formed by such values of a second measure, say stature, so that there are approximately equal numbers in each of the subdivisions. This means that for stature the division points for small knee height are not necessarily the same as the division points for large knee heights. If this method is continued by breaking up each of these nine subdivisions into three parts formed by such values of a third measure, say cubit, so that there are approximately equal numbers, and so on until all twelve measures have been accounted for, one arrives at the goal of having approximately equal numbers in each of the 3^{12} categories. However, practical as the method may be it leaves a great deal to be desired theoretically. Galton, who was formulating his ideas on correlation at this time, became interested in the problem and pointed out that the measures were not independent and therefore that one could not hope for equal numbers in their first trial. He proposed that the observed measurements be "transmuted" into a set of independent measures and suggested the method of transformation which is now known as simple or unweighted summation in factor analysis terminology. We find, also, in Galton's papers the suggestion that correlations may be regarded as being partly due to common causes and that when there are several unobservable causes the resulting variation may be regarded as the sum of the separate contributions. Edgeworth and Karl Pearson became interested in the problem and the solution by means of "principal axes" better known now as the method of principal components, was given by Pearson in 1901.

It seems relevant to translate this problem into the terminology of factor analysis. One should remember that most of the work in factor analysis has been done in psychology and education and that, therefore, the language used is more directly applicable to these fields. Frequently a word has a different connotation in mathematics and statistics. For simplicity, let us consider only three of the Bertillon measures, say stature, cubit and knee height. In this example, the population in which we are interested is British criminals and the battery of tests is the set of three measurements which are to be made either on each individual of the population or on each individual in a sample of individuals selected from the population of criminals. Thus, test is used here as the generic name for any measurable quantity. Now suppose that we measure each criminal in Great Britain in the year 1880 with respect to stature, cubit and knee height. Then these measures are known as the test scores or the scores, that is the score of individual A on the test "stature" would in this case be the height of A. The scores are statistical variables and will ordinarily be correlated. In this instance, we would be surprised indeed if the length of a man's arm from the elbow to the end of the middle finger were not related to the length of his leg from the knee to the ground. It seems inevitable that one should look then for the reason that a taller man, say, should have both longer legs and longer arms. It also seems plausible that if we can find this reason or common cause, that it will have more stable properties than the scores themselves. But just as it is possible to take more than one measure of length on an individual, we must admit the possibility of more than one reason for the correlation we observe. The next step consists in visualizing these reasons as statistical variables, that is, we think of them as hypothetical tests for which each individual has a score but, unfortunately, a score which is not directly determinable. The simplest assumption that can be made, and one that does not seem too unreasonable, is that the observed score is simply the sum of the hypothetical scores. However, one feels intuitively that causes will have different influences on different tests and one is led, therefore, to consider a weighted rather than an unweighted sum. In factor analysis terminology, the hypothetical tests are the factors, the amount of any factor which an individual possesses is his factor score for that factor and, finally, the weights whibh we attach to the factors in the summation are the factor loadings.

To return to our example, we might postulate the existence of a single factor, say, a size factor. This would be termed a general factor since it has non-zero loadings for all tests. If our hypothesis were true, perfect correlation would exist since stature, cubit and knee length would all be proportional to the size factor. This, however, is not true but it may be that the departures from perfect proportionality are due to chance acceleration or retardation of the growth mechanism, that is, we could think of each of our scores as being composed of a "true factor score" plus an amount due to chance fluctuations. The latter is known as the error factor. If we think of the scores of each individual as representing a point in a three-dimensional space, this first hypothesis would mean that all of the points would lie exactly on a straight line in this space while the second would assume a scattering of points about the line, rather like a pencil of points. Again, we may discard this hypothesis and say that we need more than an error factor to explain the scatter. We may then postulate two factors with or without an error factor. If either of these assumptions are correct, the points would spread out on a plane or cluster about a plane in a wafer-like arrangement. If the second factor had non-zero loadings in at least two of the tests it would be called a common factor.

A common factor that has non-zero loadings in all tests is usually termed a general factor. However, if our second factor has only one non-zero loading it would be called a specific factor. The psychologist makes a distinction between the specific factor and the error factor on one hand and between the specific factor and the common factor on the other. Consider the case of a battery of three tests, an arithmetic test, a language test and a form-recognition test. The experimenter believes that there is a certain basic ability, which will operate no matter which test we consider. Therefore, he postulates a general factor, say an intelligence factor. But he believes that there is another ability connected with numbers and how much or how little of this ability an individual has will not affect his score on either the language or the form-recognition tests. Similarly, he feels that there is an ability which comes into use on the language test but not on the other two, likewise with respect to the form-recognition test. Also, he is aware of the fact that the scores he observes are subject to random errors, say from judgment in scoring, a momentary distraction of an individual while taking the test, etc. For his analysis, then, he would postulate that the arithmetic score is composed of an intelligence factor score (general factor) plus a number factor score (specific factor) plus an error factor score; the language score equals an intelligence factor score (general factor) plus a word fluency score (specific factor) plus an error score; and lastly the form-recognition score equals an intelligence factor score (general factor) plus a visualizing factor score (specific factor) plus an error factor score.

You will notice that there is an embarrassing amount of freedom--if we have the scores on each of the three tests for ten individuals, there will be 30 equations but we have postulated one general plus three specific plus 30 error factors or

34 variables. In the theory built upon the hypothesis that each test contains a specific factor, it has been found that one can isolate the general and common factors but that one cannot separate the specific factor from the error factor. The main concern of the psychologist has been with common factors since he is interested in finding new abilities or factors rather than in discovering all the abilities present in a given test. The last phase of factor analysis is concerned with giving psychological significance to the factors extracted. There has been much discussion as to whether factors are primarily mathematical artifacts or whether they represent the ultimate and basic mental abilities or whether they lie somewhere between the two extremes. Most psychologists belong to the middle of the road school and emphasize the exploratory nature of the technique.

To return to the history of factor analysis, after Galton, the next significant development in the thought pattern was due to Spearman (1904). Spearman was the first to notice that when one deals with the intercorrelations between many variables and arranges them in the familiar matrix pattern, then the coefficients in any two columns tend to have a constant ratio throughout the column. Spearman called this tendency "hierarchical order" and in explanation he put forth the hypothesis that all the correlations were due to a single factor which is present in every test but in varying degrees. In addition, each test has a second factor which is peculiar to the test itself. This hypothesis is now known as the theory of two factors, although, in fact, the number of factors is one plus the number of tests. It is interesting in view of the present quest for meaningful factors that Spearman designated the general factor by the letter "g" because he was reluctant to attach psychological significance to it.

Articles of historical interest: Edgeworth (1892a, b, 1896), Galton (1888), Garnett (1920), Pearson (1896a, b, 1901, 1902, 1911), Spearman (1904), Yule (1897, 1907).

SECTION 3.

Statement of the Problem of Factor Analysis.

The problem of factor analysis is one of analysing a set of variates, called factors, consistent with certain facts and assumptions. Let Y_i, $i = 1, 2, \cdots, p$ be p random variables which have been normalized so that

$$E(Y_i) = 0, \; E(Y_i^2) = 1, \quad i = 1, 2, \cdots, p,$$

and denote the coefficient of correlation between Y_i and Y_j by ρ_{ij}

$$\rho_{ij} = E(Y_i Y_j), \quad i, j = 1, 2, \cdots, p.$$

It is postulated that

$$(1) \qquad Y_i = \sum_{k=1}^{m} c_{ik} X_k + a_i, \quad i = 1, 2, \cdots, p, \quad m \leq p,$$

where X_k, $k = 1, 2, \cdots, m$ and a_i, $i = 1, 2, \cdots, p$ are variables and the c_{ik} are constant coefficients. It is further postulated that

$$E(X_k) = E(a_i) = 0, \quad k = 1, 2, \cdots, m, \; i = 1, 2, \cdots, p,$$

and that a_i is uncorrelated with all other variables so that

(2) $E(a_i a_j) = E(a_i X_k) = 0$ for all values of i, j and k, $i \neq j$.

Then, for all i, j, $i \neq j$

(3) $$E(Y_i^2) = \sum_{k=1}^{m} c_{ik}^2 \ \phi_{ii} + \sigma_i^2 = 1$$

and

(4) $$\rho_{ij} = E(Y_i Y_j) = \sum_{k=1}^{m} \sum_{h=1}^{m} c_{ik} c_{jh} \ \phi_{ij} \ , \quad i \neq j,$$

where

$$\phi_{ii} = E(X_i)^2$$

$$\phi_{ij} = E(X_i X_j)$$

$$\sigma_i^2 = E(a_i^2).$$

It will be convenient to use the terminology of factor analysis as defined in the preceding section. Thus, the values of Y_i are the scores on the i-th test; the values of X_k are the factor scores on the k-th common factor; the values of a_i are the combined specific factor score plus the error factor score on the i-th test; and, finally, c_{ik} is the factor loading of the k-th factor in the i-th test. The sum of the specific and error factor is sometime called the uniqueness factor.

Now consider a random sample of n individuals. We are now introducing a second source of randomness. Any statement that we make regarding the composition of Y_i is subject to

Substituting the sample estimates in (7) and designating the matrices on the right hand side by R_1 and R_2 respectively, we have an expression for R which can be identified with (7), giving

$$R_1 = FPP'F' , \quad \text{and} \quad R_2 = E^2.$$

We notice that R_1 differs from R only in the diagonal elements. The matrix R_1 is called the reduced correlation matrix and its rank is equal to the number of common factors m. In practice, due to the errors of sampling the rank of R_1 will be p and one faces the question of whether or not the observed matrix R_1 can be sufficiently well approximated by a matrix of lower rank m.

The quantity $\sum_{k=1}^{m} c_{ik}^{2} \phi_{ii}^{2} = h_i^2$, say, which appears in the diagonal of the reduced correlation matrix is called the communality and, for each i, h_i^2 represents that part of the total variance (unity) which is due to the common factors.

This may be clearer if we consider again the three measures on criminals. Suppose that we postulate one general factor and error factors. Using H for stature, A for cubit and K for knee height, we have

$$H = c_h X + a_1,$$

$$A = c_a X + a_2,$$

$$K = c_k X + a_3.$$

Suppose further that we have the normalized scores of 100 criminals on these tests (measures) and that we compute the sample correlation coefficients, say,

$$r_{ha} = .5,$$

$$r_{hk} = .7,$$

$$r_{ak} = .4.$$

Then,

$$R = \begin{Vmatrix} 1 & .5 & .7 \\ .5 & 1 & .4 \\ .7 & .4 & 1 \end{Vmatrix}.$$

Now it is possible that we know something about our errors. Say that we know

$$\sigma_i^2 = .2, \quad i = 1, 2, 3.$$

From this it follows that the communalities would be

$$h_i^2 = 1 - .2 = .8, \quad i = 1, 2, 3,$$

and the reduced correlation matrix would be

$$R_1 = \begin{Vmatrix} .8 & .5 & .7 \\ .5 & .8 & .4 \\ .7 & .4 & .8 \end{Vmatrix}.$$

If our hypothesis is true, R_1 is of rank 1 which would mean that every 2-rowed determinant is zero which would seem very unlikely in this case. If we now make the hypothesis that

a_i includes a specific factor for each test, we cannot even hope to estimate the σ_i^2 from other sources. However, the reduced matrix is still of rank 1 if our hypothesis is true and from R_1 we can select the proper 2-rowed determinants to estimate h_i^2,

$$.4h_1^2 = .5 \times .7,$$

$$.7h_2^2 = .4 \times .5,$$

and

$$.5h_3^2 = .4 \times .7.$$

Then having determined the values of the h's from these equations we can substitute them in the matrix R_1 and make a decision to accept or reject our hypothesis. If we decide that the matrix is of rank 1 and that the non-zero values of the principal 2-rowed determinants are due to sampling errors, we will then go on to find the loading matrix F. The methods of factoring either the correlation matrix or the reduced correlation matrix are discussed in the next section.

We may summarize the situation in the following terms. Geometrically, the scores $y_{i,j}$ $(i = 1, 2, \cdots, p)$ of each individual can be regarded as the coordinates of a point in a p-dimensional space. The mathematical problem of factor analysis consists in selecting a new set of axes of reference by which to describe the complex of points. In particular, the question of dimensionality is important. We would like to find the space of minimum dimensions in which the points, except for the errors of sampling, may be said to lie.

SECTION 4.

Methods of Factoring.

All the methods of factoring try to transform either the correlation matrix or the reduced correlation matrix into a matrix of factor loadings which satisfactorily reproduces the data in some sense. In most methods, the aim is to reproduce the correlations but there is some variation in aims. The reproduction can be accomplished in many ways and the various methods arise from disagreement as to which way is best.

a) The method of principal axes or principal components. In 1901, Karl Pearson proposed essentially the method that Hotelling favored in his well-known paper of 1933 on principal components. Hotelling's article gives a precise statistical treatment with direct reference to factor analysis and the sampling theory involved.

In this method, it is assumed that the X's are independently and normally distributed. The second assumption is made that there are as many factors (Hotelling's components) as tests. However, as Hotelling's procedure is an iterative one, it can be stopped after the extraction of any number of components if one deems that the residual variance is small enough to be due to random errors alone. The assumption made with respect to the a's will determine whether R or R_1 is to be factored. Further, since it is assumed that the common factors are independent, we have either R or R_1 equal to the product of a matrix by its transpose.

The criterion is that each factor (component) shall be so selected that it will account for as large a part of the residual variance as possible. Geometrically the solution corresponds to rotating the rectangular axes $Y_1, \cdots, Y_p$ so that the new coordinate axes lie along the principal axes of the ellipsoids of uniform density. Computationally, the solution involves finding the roots of the determinantal equation $|R - \lambda I| = 0$, where I is the identity matrix. It is interesting that the application of other criteria will give the same solution. Girshick (1936) showed that if all the tests have equal variances of errors of measurement, then the principal component is (i) that linear function of the Y's which has the least variance due to errors of measurement, (ii) that linear function of the Y's such that the sum of squares of the correlation between the function and each variate is a maximum, (iii) equivalent to the maximum likelihood estimates of the factor loadings.

The method of principal components has often been attacked by psychologists on the grounds that there is no decrease in the number of dimensions, that there is no provision for specific factors and that the components are without psychological significance. The first two objections were answered by both Hotelling and Girshick in that one need not extract all p factors and that one can factor R_I by this method as well as R. However, the fact remains that the components do not have psychological meaning in the sense that the loadings are not invariant if other tests be added to the battery. It should also be noted that although theoretically the method is very satisfactory the labor involved in the computations may be excessive.

Articles on the method of principal components: Davis (1947b), Girshick (1936), Green (1950), Holzinger (1946a), Hotelling (1933), Roos (1937), Thomson (1934).

b) Spearman's theory of two factors.

Spearman's hypotheses of one general factor "g" and p specific factors is expressed by setting $m = 1$ and $a_{ij} \neq 0$. Then it follows that the reduced correlation matrix R_1 must be of rank one and, from this, that

$$\rho_{ij}\,\rho_{rs} - \rho_{is}\,\rho_{jr} = 0,$$

which is the well-known tetrad-difference equation. The value h_i^2 of the communality can then be determined from

$$\rho_{rs}\,h_i^2 - \rho_{ri}\,\rho_{si} = 0.$$

There has been disagreement among the psychologists with regard to the existence of "g". Thomson has shown that the "hierarchy" of Spearman can be formed without the existence of a general factor. In connection with the application of this method in practice, the experimenter discards tests from his battery until he arrives at a battery which satisfies the tetrad-difference criteria. This would seem a rather circular process.

Articles on the two factor theory: Alexander (1935), Bartlett (1935), Brown (1934, 1935), Brown and Stephenson (1933), Burt (1909, 1949d), Darmois (1936), Garnett (1933), Garrett and Anastasi (1932), Girault (1948), Guilford and Michael (1948), Holzinger (1930, 1944), Irwin (1933, 1935), Line and Hedman (1933, 1935), Piaggio (1933b), Spearman (1904, 1933), Stephenson (1935), Thomson (1916, 1919, 1934, 1935), Weinberg (1945, 1946), Wilson, E. B. (1928, 1934), Wilson, J. H. (1935), Wishart (1933).

c) Holzinger's bi-factor theory.

This is an extension of the two-factor theory to allow for the presence of group factors. Holzinger divides his battery of tests into, say, r groups, $G_1, G_2, \cdots, G_r$, and postulates that for any test i belonging to G_h, $h = 1, 2, \cdots, r$,

$$y_{ij} = c_{i1}x_{1j} + c_{ih}x_{hj} + a_{ij}, \quad \begin{array}{l} i = 1, 2, \cdots, p, \\ j = 1, 2, \cdots, n. \end{array}$$

For the purpose of selecting the G_i, he has defined the B-coefficient, the coefficient of belonging, which is the ratio (expressed as a per cent) of the average of the intercorrelations of a subset or group of variables to their average correlation with all remaining variables. A decisive change in the B-coefficient indicates the two tests belong to different groups.

We see that $m = r + 1$, $a_{ij} \neq 0$, and the further assumption is made that the X's are uncorrelated. Since the matrix R_1 is then of rank $(r + 1)$ all $(r + 2)$-rowed determinants must be zero and the solution is analogous to the two-factor one. In using this technique also, the experimenter selects the tests in his battery to agree with the criterion.

Articles on the bi-factor theory: Holzinger (1946b), Holzinger and Swineford (1939, 1946), Swineford (1947), Swineford and Holzinger (1942), Thomson (1938b).

d) The multiple-factor method.

This method postulates both common and specific factors but with no particular insistence on either a general factor or the way in which the factors are distributed within the tests. It is assumed that $0 < m < n$, $a_{ij} \neq 0$ and that the X's are uncorrelated. Since a specific factor is postulated, the reduced correlation matrix, $R_1 = FPP'F' = FF'$, since $PP' = I$, is factored. The criterion is to select the communalities h_i^2 so that the rank of R_1 namely m, will be a minimum.

The procedure which is most widely used is Thurstone's centroid method. The first centroid is essentially an average of all tests included in the average. This method is also the one of the multiple factoring methods which seems most susceptible to statistical treatment, since the factors found can be interpreted either as first approximations to Hotelling's principal components or as correlations between each variable and the sum of variables.

Computationally, the centroid method has a great deal to recommend it. It is much easier and faster than the method of principal components.

Articles on multiple factor theory: Alexander (1935), Burt (1949c, 1950a), Clark (1944), Cox (1939), Holley (1947), Holzinger (1944, 1946a), Jeffries (1948), Kellogg (1948), MacCrone and Starfield (1949), Roff (1937), Thomson (1938d), Thurstone (1933, 1938, 1949a, b).

Articles on communalities: Medland (1947), Mosier (1939), Roff (1936), Rosner (1948).

e) Oblique factors and simple structure.

An objection to the factors obtained by the previous methods is that they have no psychological significance nor are any such claims made for them. Thurstone believed that, if the factor loadings would remain invariant for any given test when analyzed in different batteries then the factors would be meaningful.

Thurstone arrived at simple structure by considering the factor space. In this space each test can be represented by a point with coordinates equal to the loadings of the factors. He felt that if one would rotate the axes until the resulting figure was in some simple form, the new axes would have meaning. In a space of three factors, imagine the set of points representing the p tests and join these points to the origin. Now extend the vectors through the points and cut these extensions by a plane perpendicular to the first centroid or first factor extracted. If simple structure exists, the pattern of dots made by the intersections of the extended vectors with the plane will form a triangle. The factors that Thurstone has named primary will be the edges of this pyramid. The criterion for determining the position of the rotated axes is to maximize the number of zero loadings. Not all batteries can be transformed into simple structure but if it can be attained then Thurstone believes that the factors will have psychological significance and certain invariance properties. It is clear that Thurstone does not insist on orthogonality after rotation; that is, he permits the factors to be correlated. The other point of difference between simple structure and other methods of factoring is that there can be no general factor. If there is a general factor it will appear in the correlations between the factors found by simple structure.

Articles on simple structure and oblique factors: Cattell (1946, 1947b, 1949a), Cattell and Tiner (1949), Harris and Knoel (1948), Holzinger (1947), Reyburn and Raath (1949), Saunders (1949), Thomson (1936a, 1949), Thurstone (1946c), Yela (1949).

f) Maximum likelihood solution.

The maximum likelihood solution was given first by Girshick and later by Lawley (1940). Both Girshick and Lawley assumed that the X's and a's were independent and normally distributed. In a second paper, Lawley gives the solution under the assumptions that the a's but not necessarily the X's are independent random variables, normally distributed with common variance. This is theoretically perhaps the most satisfying solution of all.

Articles on the maximum likelihood solution: Emmett (1949b), Girshick (1936), Lawley (1940, 1941, 1943a, 1950), Young (1941).

g) Approximation to a matrix by one of lower rank.

There have been some interesting papers on this subject which have treated the mathematical aspects of matrix approximation with an indication of how the solutions apply to the problem of factor analysis.

Articles on the approximation of a matrix: Eckart and Young (1936), Guttman (1944), Householder and Young (1938), Young (1937).

h) Inverted factor techniques.

Stephenson who was interested in the isolation of personality traits suggested the use of existing techniques of factor analysis to study the correlations between individuals rather than tests. Thus instead of the usual correlation table involving the correlations between tests, we would have a correlation table involving correlations between persons. The factors of such a correlation matrix would be types of individuals rather than traits of personality. Stephenson attached the label of inverted factor technique to this process. Burt advocates the principle that factors should be chosen so that the same results are given whether we analyse tests or persons with the factors and loadings of the one analysis being the loadings and factors of the other.

Articles on the inverted factor technique: Burt (1937, 1938), Cattell and Rhymer (1947), Peel (1946), Sandler (1949), Stafford and Hsu (1947), Stephenson (1936a).

SECTION 5

Computational Procedures

Since any discussion of procedures would be long and technical, pertinent papers are listed without comment.

Articles:

a) On matrix theory: Aitken (1937a, b), Albert (1944a, b), Dwyer (1941), Etherington (1932), Fruchter (1949), Hotelling (1943a, b, 1949), Ledermann (1936, 1937a, 1937b, 1938b, 1939a), Slater (1947), Waugh and Dwyer (1945), Young and Householder (1938).

b) On iterative and simplified procedures: Adcock (1946), Berry (1945), Flood (1940), Guttman (1944), Holzinger (1949), Horst (1937), Hotelling (1933, 1936b), Kelley and Salisbury (1926), Luborsky and Hornaday (1948), Richardson (1950), Spearman (1934), Thomson (1936b), Ullman (1944), Van Boven (1947), Waugh (1945), Wherry (1949), Zimmerman (1946).

c) On the estimation of mental factors: Bartlett(1938), Burt (1944a), Guttman (1940), Guttman and Cohen (1943), Herman (1938), Ledermann (1938c, 1939b), Thomson (1937, 1938a).

SECTION 6

Related Methods of Analysis

There are other methods of analysis of correlations whose objectives are closely allied to factor analyses. They include the cluster analysis of Tryon, the confluence analysis of Frisch and the path coefficients of Wright. Tryon has a very simple profile analysis to detect similarities between groups and he also has a more precise but a more laborious method which gives however more stable results. Frisch became interested in the clustering of tests through his concern with multiple regression equations. He wanted to be reasonably certain that, for example, the three variables he was using to predict a fourth were not on the same straight line and that he would not get a fictitious plane. Delaporte has given a graphic method for the location of group factors. His method depends upon the overlapping of confidence intervals determined for the ratios of the correlation coefficients. There would seem to be some danger that if the number of variables is very large, the probability of non-overlap due purely to errors of chance would not be neglible.

Articles on related methods of analysis: Cattell (1949b), Delaporte (1939a, b, 1944, 1946a, b, 1947), Frisch (1934), Geary (1948), Gengerelli (1948), Koopmans (1937), Tryon (1949), Wright (1934), Young (1937).

SECTION 7

Distribution Theory and Tests of Significance

The variables to be analysed are subject to sampling errors of two quite different kinds. In the first place, we have a sample of n individuals from a finite or an infinite population of individuals and, in the second place, a sample of tests from a finite or infinite population of tests. While it is quite likely that the sample of individuals has been randomly selected, it is quite possible that the tests have been selected with a particular purpose in view and do not, therefore, form a random sample of tests. The limiting distributions of many of the statistics used in factor analysis are known. Thus, theory has been developed with respect to the roots of the determinantal equation in the method of principal components, to the tetrad differences, the canonical correlation coefficients and so on. Various tests of significance have been devised for the rank of a matrix, the tetrad differences and the residual variances. Riersøl and Koopmans have done very interesting work on the identifiability of the variates.

Articles:

a) On distribution theory: Anderson (1948), Bartlett (1947a, b), Doob (1935), Hsu, P. L. (1939, 1941a, 1941b, c, 1949), Spearman (1937).

b) On tests of significance: Emmett (1935), Hoel (1937, 1939), Hotelling (1933), McNemar (1941, 1942), Mosier (1939), Saunders (1948), Wishart (1928).

c) On consistency of the factors: Fiske (1948), Harsh (1940), Sisk (1939), Young and Householder (1940).

d) On the effect of selection: Aitken (1935, 1936), Cureton and Dunlap (1930), Degan (1948), Lawley (1943b), Ledermann (1938a, 1939c), Thomson (1937, 1938c), Thomson and Ledermann (1939), Thurstone (1945a).

SECTION 8.

Relation of Factor Analysis to Other Statistical Techniques.

a) Internal and external factor analysis.

The phrase, internal and external factor analysis, due to Bartlett, expresses the contrast of the transformation of a single set of variables into components (internal or factor analysis) to the transformation of one group of variables into components with reference to an external criterion, namely the other group of variables (external or Hotelling's most predictable criterion). Bartlett has written a good expository article on this subject. Thomson has been interested in the application of the most predictable criterion with reference to battery reliability.

Articles on internal and external factor analysis: Baten (1941), Bartlett (1941, 1948), Hotelling (1935, 1936a) Peel (1948), Thomson (1940b, 1947, 1948b).

b) Regression theory and factor analysis.

The essential difference between multiple regression and factor analysis is in the fact that multiple regression is concerned with a dependent variable while there are no dependent variables in factor analysis. In both analyses, one works with the same matrix of correlations and with the roots of the same determinantal equation, so that techniques of one can often be used in the other.

Articles on regression theory and factor analysis: Dwyer (1939, 1940), Guttman and Cohen (1943), Rao and Slater (1949).

c) Analysis of variance and factor analysis:

Burt, in an expository article, has pointed out the similarities of analysis of variance and factor-analysis. Bartlett has given the warning that the analogy should not be pushed too far since factor analysis treats the different tests as different variables, while analysis of variance would attempt to treat them all in terms on one variable.

Articles on analysis of variance and factor analysis: Bartlett (1947a, 1948), Burt (1947a).

SECTION 9.

The Applications of Factor Analysis.

a) The military personnel.

The items in the following list either refer to the methods that were actually used in the placement of men during the last war or more directly to the use of factor analysis on data from the armed forces. Factor analysis has been used quite extensively both in this country and in Great Britain.

Books: Deemer (1947), DuBois (1947), Eysenck (1947), U.S., OSS (1948), Vernon and Parry (1949).

Articles on application of factor analysis to military personnel:

Banks (1948a, 1949), Burt (1944b, 1947b), Burt and Banks (1947), Carter and Dudek (1947), Composite factor analysis summary (1945), Comrey (1949), Cottle (1950), Davis (1947a), Deemer and Rafferty (1949), Dudek (1948), Findley and Andregg (1949), Guilford (1944, 1947), Guilford and Zimmerman (1947), Michael (1949), Rafferty and Deemer (1949), Roff (1950), Vernon (1946, 1947a).

b) Other applications.

In this list, ther are, in addition to the direct applications of factor analysis, listings which give general methods used in practice as counseling techniques.

Books: Burt (1942), Horst, Wallin, Guttman (1935), Kelly (1914), Thorndike, E. L. (1934), Thorndike, R. L. (1949).

Articles on applications: Andrews (1943), Balinski (1941), Brogden (1944), Brodgen and Thomas (1943), Burt (1916, 1948a, 1950b), Banks (1948b), Cattell (1947a, 1948b), Cattell and Wispe (1948), Chapman (1948), Clarke (1940), Coombs and Salter (1949), Davidson and Carroll (1945), Davis (1944), Emmett (1949a), Eysenck and Crown (1949), Flanagan (1935), Fruchter (1948), Gage (1947), Goodman (1944), Gosnell and Schmidt (1936), Guilford (1948), Halstead (1945), Harris (1948a, b), Hellfritzsch (1945), Herdan (1943), Heston (1943, 1947), Howie (1945), Hsu, E. H. (1946, 1947, 1948), Hsu, E. H. and Sherman (1946), Jones, F. N. (1948), Jones, L. V. (1949), Kleir (1949), Kleemeier and Dudek (1950), Lovell (1945), McCraw (1949), Marzolf and Larsen (1945), North (1949), Peel (1949), Richards (1941), Rimoldi (1948a, b), Roff (1949), Salisbury (1924), Sastry (1941), Sen (1950), Smalzried and Remmers (1943), Snedecor (1936), Staff of D.O.A. (1945), Stone (1947), Swineford (1949), Taylor (1947), Thomson (1940a, 1941), Thurstone (1946a, b), Thurstone and Thurstone (1941), Tschechtelin (1944), Vernon (1947b, 1949, 1950a, b), Wilson and Worcester (1934), Wittenborn (1945, 1949), Wittenborn and Larsen (1944), Woodrow (1939, 1945).

BIBLIOGRAPHY

ADCOCK, C. J., 1946. Simplified factor analysis. Occup. Psych., 20, 183-198.

AITKEN, A. C., 1935. Note on selection from a multivariate normal population. Proc. Edin. Math. Soc., Ser. 2, 4, 106-110.

——— 1936. A further note on multivariate selection. Proc. Edin. Math. Soc., Ser. 2, 5, 37-40.

——— 1937a. The evaluation of a certain triple-product matrix. Proc. Roy. Soc. Edin., 57, 172-181.

——— 1937b. The evaluation of the latent roots and vectors of a matrix. Proc. Roy. Soc. Edin., 57, 269-304.

ALBERT, A. A., 1944a. The matrices of factor analysis. Proc. Nat. Acad. Sci., 30, 90-95.

——— 1944b. The minimum rank of a correlation matrix. Proc. Nat. Acad. Sci., 30, 144-148.

ALEXANDER, W. P., 1935. Intelligence, concrete and abstract--a study in differential traits. Cambridge University Press, 177 pp.

ANASTASI, A. (see Garrett, H. E.).

ANDERSON, T. W., 1948. The asymptotic distributions of the roots of certain determinantal equations. J.R.S.S., B, 10, 132-139.

ANDREGG, N. B. (see Findley, W. G.).

ANDREWS, T. G., 1943. A factorial analysis of responses to the comic as a study in personality. J. Gen. Psych., 28, 209-224.

BALINSKI, B., 1941. An analysis of the mental factors of various age groups from nine to sixty. Genet. Psych. Mon., 23, 191-234.

BANKS, C., 1948a. Flying ability and body-build. B. J. Psych., Stat., 1, 107-113.

1948b. Primary personality factors in women: a re-analysis. B. J. Psych., Stat., 1, 204-218.

1949. Factor analysis of assessments for army recruits. B. J. Psych., Stat., 2, 76-89.

(see Burt, C.).

BARTLETT, M. S., 1935. The statistical estimation of G. B. J. Psych., 26, 199-206.

1937a. The development of correlations among genetic components. Ann. Eugen., 7, 299-302.

1937b. The statistical conception of mental factors. B. J. Psych., 28, 97-104.

1938. Methods of estimating mental factors. Nature, 141, 609-610.

1941. The statistical significance of canonical correlations. Biom., 32, 29-37.

1947a. The general canonical correlation distribution. Ann. Math. Stat., 18, 1-17.

BARTLETT, M. S., 1947b. Multivariate analysis. Supp. J.R.S.S., 9, 176-197.

1948. Internal and external factor analysis. B. J. Psych., Stat., 1, 73-81.

BATEN, W. D., 1941. How to determine which of two variables is better for predicting a third variable. J. Amer. Soc. Agron., 33, 695-699.

BERRY, C. E., 1945. A criterion of convergence for the classical iterative method of solving linear simultaneous equations. Ann. Math. Stat., 16, 398-400.

BRODGEN, H. E., 1944. A multiple-factor analysis of the character trait intercorrelations published by Sister Mary McDonough. J. Ed. Psych., 35, 397-410.

and Thomas, W. F., 1943. The primary traits in personality items purporting to measure sociability. J. Psych., 16, 85-97.

BROWN, W., 1934. The theory of two factors versus the sampling theory of mental ability. Nature, 133, 724-725.

1935. A note on the theory of two factors versus the sampling theory of mental ability. B. J. Psych., 25, 395-398.

and Stephenson, W., 1933. A test of the theory of two factors. B. J. Psych., 23, 352-370.

and Thomson, G. H., 1940. The essentials of mental measurement. Cambridge University Press, 256 pp.

BURT, C., 1909. Experimental tests of general intelligence. B. J. Psych., 3, 94-176.

BURT, C., 1916. The distribution and relations of educational abilities. London County Council, 93 pp.

1937. Correlation between persons. B. J. Psych., 28, 59-96.

1938. The analysis of temperament. B. J. Med. Psych., 17, 158-188.

1940. The factors of the mind; an introduction to factor analysis in psychology. University of London Press, 509 pp.

1944a. Mental abilities and mental factors. B. J. Ed. Psych., 14, 85-94.

1944b. Statistical problems in the evaluation of army tests. Psych., 9, 219-235.

1947a. A comparison of factor analysis and analysis of variance. B. J. Psych., Stat., 1, 3-26.

1947b. Factor analysis and physical types. Psych., 12, 171-188

1948a. Factor analysis and canonical correlations. B. J. Psych., Stat., 1, 95-106.

1948b. The factorial study of temperamental traits. B. J. Psych., Stat., 1, 178-203.

1949a. Alternative methods of factor analysis and their relations to Pearson's method of "principal axes." B. J. Psych., Stat., 2, 98-121.

1949b. Structure of the mind: a review of the results of factor analysis. B. J. Ed. Psych., 19, 176-199.

1949c. Subdivided factors. B. J. Psych., Stat., 2, 41-63.

BURT, C., 1949d. The two-factor theory. B. J. Psych., Stat., 2, 151-179.

1950a. Group factor analysis. B. J. Psych., Stat., 3, 40-75.

1950b. Symposium on the selection of pupils for different types of secondary schools: IX--conclusion. B. J. Ed. Psych., 20, 1-10.

and Banks, C., 1947. A factor analysis of body measurements for British adult males. Ann. Eugen., 13, 238-256.

(see Hartog, P.).

BURTT, H. E., 1942. Principles of employment psychology. Rev. ed. Harper and Bros., 568 pp.

CARROLL, J. B. (see Davidson, W. M.).

CARTER, L. F., and Dudek, F. J., 1947. The use of psychological techniques in measuring and critically analyzing navigators' flight performance. Psych., 12, 31-42.

CATTELL, A. K. S. (see Cattell, R. B.).

CATTELL, R. B., 1946. Simple structure in relation to some alternative factorizations of the personality sphere. J. Gen. Psych., 35, 225-238.

1947a. Confirmation and clarification of primary personality factors. Psych., 12, 197-220.

1947b. Oblique, second-order, and co-operative factors in personality. J. Gen. Psych., 36, 3-22.

1948a. The integration of factor analysis with psychology.

J. Ed. Psych., 39, 227-236.

CATTELL, R. B., 1948b. The primary personality factors in women compared with those in men. B. J. Psych., Stat., 1, 114-130.

1949a. A note on factor invariance and the identification of factors. B. J. Psych., Stat., 2, 134-139.

1949b. r_p and other coefficients of pattern similarity. Psych., 14, 279-298.

Cattell, A. K. S., and Rhymer, R. M., 1947. P-technique demonstrated in determining psycho-physiological source traits in a normal individual. Psych., 12, 267-288.

and Tiner, L. G., 1949. Varieties of structural rigidity. J. Pers., 17, 321-341.

and Wispe, L. G., 1948. Dimensions of syntality in small groups. J. Soc. Psych., 28, 57-78.

CHAPMAN, R. L., 1948. The Mac Quarrie test for mechanical ability. Psych., 13, 175-179.

CLARK, M. P., 1944. Changes in primary mental abilities with age. Arch. Psych., No. 291, 1-30.

CLARKE, E. R., 1940. Predictable accuracy in examinations. B. J. Psych., Mon. Supp., No. 24, 48 pp.

COHEN, J. (see Guttman, L.).

Composite factor analysis summary, 1945. AAF Sch. Av. Med., Department of Records and Analyses, 1045th AAF Base Unit, September, 1945.

COMREY, A. L., 1949. Factorial study of achievement in West Point courses. Ed. Psych. Meas., 9, 2, 193-209.

CONRAD, H. S., 1944. Statistical methods related to test construction and evaluation. Rev. Educ. Res., 14, 110-126.

COOMBS, C. H., and Satter, G. A., 1949. A factorial approach to job families. Psych., 14, 33-42.

COTTLE, W. C., 1950. A factorial study of the Multiphasic, Strong, Kuder, and Bell inventories using a population of adult males. Psych., 15, 25-47.

COX, G. M., 1939. The multiple factor theory in terms of common elements. Psych., 4, 59-68.

CROWN, S. (see Eysenck, H. J.).

CURETON, E. E., 1939. The principal compulsions of factor analysis. Harv. Ed. Rev., 9, 287-295.

and Dunlap, J. W., 1930. Some effects of heterogeneity on the theory of factors. Amer. J. Psych., 42, 608-620.

DANGER, J. (see Weinberg, D.).

DARMOIS, G., 1936. Sur "l'indetermination" du facteur general dans la theorie de Spearman. Mathematica, 12, 211-216.

—— 1940. Les mathematiques de la psychologie. Mem. Sci. Math. Paris, Fasc. 98, 49 pp.

DAVENPORT, K. S. and Remmers, H. H., 1950. Factors in state characteristics related to average A-12 V-12 test scores. J. Ed. Psych., 41, 110-115.

DAVIDSON, W. M., and Carroll, J. B., 1945. Speed and level components in time-limit scores: a factor analysis. Ed. Psych. Meas., 5, 411-427.

DAVIS, F. B., 1944. Fundamental factors of comprehension in reading. Psych., 9, 185-197.

DAVIS, F. B., 1946. A brief comment on Thurstone's note on a reanalysis of Davis' reading tests. Psych., 11, 249-255.

(Ed.), 1947a. The AAF qualifying examination. AAF Av. Psych. Prog. Res. Rep., No. 6, 266 pp.

1947b. The interpretation of principal-axis factors. J. Ed. Psych., 38, 471-481.

DEEMER, W. L., Jr., (Ed.), 1947. Records, analysis, and test procedures. AAF Av. Psych. Prog. Res. Rep., No. 18, 621 pp.

and Rafferty, J. A., 1949. Experimental evaluation of psychiatric interview for prediction of success in pilot training. J. Av. Med., 20, 238-250.

(see Rafferty, J. A.).

DEGAN, J. W., 1948. A note on the effects of selection in factor analysis. Psych., 13, 87-89.

DELAPORTE, P., 1939a. Une méthode d'analyse des corrélations. C. R. Acad. Sci. Paris, 208, 1960-1963.

1939b. Une méthode d'analyse des corrélations et son application. C. R. Acad. Sci. Paris, 209, 142-145.

1945. Vérification de l'efficacité d'une méthode d'analyse factorielle. C. R. Acad. Sci. Paris, 220, 212-214.

1946a. Prolongement de la méthode d'analyse factorielle de Spearman en utilisant la statistique mathématique. Biotyp., 8, No.'s 3,4.

DELAPORTE, P., 1946b. Sur l'estimation des corrélations des caractères avec le facteur général et les facteurs de groupe et sur l'écart-type de cette estimation, en analyse factorielle. C. R. Acad. Sci. Paris, 222, 525-526.

1947. Une nouvelle methode d'analyse factorielle (mimeograph). Inst. Int. Stat., 16 pp.

DODD, S. C., 1928. The theory of factors. Psych. Rev., 35, 211-234, 261-279.

DOOB, J. L., 1935. The limiting distributions of certain statistics. Ann. Math. Stat., 6, 160-169.

DU BOIS, P. (Ed.), 1947. The classification program. AAF Av. Psych. Prog. Res. Rep., No. 2, 394 pp.

DUDEK, F. J., 1948. The dependence of factorial composition of aptitude tests upon population differences among pilot trainees: I. The isolation of factors. Ed. Psych. Meas., 8, 613-633.

(see Carter, L. F.).

(see Kleemeier, R. W.).

DUNLAP, J. W. (see Cureton, E. E.).

DUREA, M. A., and Nonman, R. D., 1948. The significance of weighted and unweighted items in differentiating between groups. J. Gen. Psych., 38, 217-227.

DWYER, P. S., 1939. The contribution of an orthogonal multiple factor solution to multiple correlation. Psych., 4, 163-171.

1940. The evaluation of multiple and partial correlation coefficients from the factorial matrix. Psych., 5, 211-232.

DWYER, P. S., 1941. The evaluation of linear forms. Psych., 6, 355-365.

(see Waugh, F. V.).

ECKART, C., and Young, G., 1936. The approximation of one matrix by another of lower rank. Psych., 1, 211-218.

EDGEWORTH, F. Y., 1892a. Correlated averages. Phil. Mag., 34, Ser. 5, 190-204.

1892b. The law of error and correlated averages. Phil. Mag., 34, Ser. 5, 518-526.

1896. Supplementary notes on statistics. J. R. S. S., 59, 529-539.

EMMETT, W. G., 1936. Sampling error and the two-factor theory. B. J. Psych., 26, 362-387.

1949a. Evidence of a space factor at 11+ and earlier. B. J. Psych., Stat., 2, 3-16.

1949b. Factor analysis by Lawley's method of maximum likelihood. B. J. Psych., Stat., 2, 90-97.

ETHERINGTON, I. M. H., 1932. On errors in determinants. Proc. Edin. Math. Soc., Ser. 2, β, 107-117.

EYSENCK, H. J., 1947. Dimensions of personality. Kogan Paul, Trench, Trubner and Co., Ltd., 308 pp.

FINDLEY, W. G., and Andregg, N. B., 1949. A statistical critique of the U.S.A.F.I. tests of general educational development. Psych., 14, 47-60.

FISKE, D. W., 1948. Consistency of the factorial structure of personality from different sources (abstract). Amer. Psych., 3, 360.

FLANAGAN, J. C., 1935. Factor analysis in the study of personality. Stanford University Press, 103 pp.

FLOOD, M. M., 1940. A computational procedure for the method of principal components. Psych., 5, 169-172.

FRISCH, R., 1934. Statistical confluence analysis by means of complete regression systems. Univ. Øk. Inst., No. 5, 192 pp.

FRUCHTER, B., 1948. The nature of verbal fluency. Ed. Psych. Meas., 8, 33-47.

1949. Note on the computation of the inverse of a triangular matrix. Psych., 14, 89-93.

GAGE, N. L., 1947. Scaling and factorial design in opinion poll analysis. Purdue Univ. Div. Ed. Ref. No. 61, 87 pp.

GALTON, F., 1888. Co-relations and their measurement, chiefly from anthropometric data. Proc. Roy. Soc., 45, 135-140.

GARNETT, J. C. M., 1920. On certain independent factors in mental measurements. Proc. Roy. Soc., A, 96, 91-111.

1933. Linear transformations of hierarchical systems. Nature, 132, 676-677.

GARRETT, H. E., and Anastasi, A., 1932. The tetrad-difference criterion and the measurement of mental traits. Ann. N. Y. Acad. Sci., 33, 233-282.

GAYLORD, R. H. (see Wherry, R. J.).

GEARY, R. C., 1948. Studies in relations between economic series. J.R.S.S., B, 10, 140-158.

GENGERELLI, J. A., 1948. A binomial method for analyzing psychological functions. Psych., 13, 69-77.

GIRAULT, M., 1948. Sur la notion de facteur commun en analyse factorielle generale. C. R. Acad. Sci. Paris, 227, 499-500.

GIRSHICK, M. A., 1936. Principal components. J.A.S.A., 31, 519-528.

GOODMAN, C. H., 1944. Prediction of college success by means of Thurstone's primary abilities tests. Ed. Psych. Meas., 4, 125-140.

GUILFORD, J. P. (Ed.), 1944. Annual report of Psychological Research Unit No. 3. Santa Ana Army Air Base.

(Ed.), 1947. Printed classification tests. A.A.F. Av. Psych. Prog. Res. Rep., No. 5, 919 pp.

1948. Factor analysis in a test-development program. Psych. Rev., 55, 79-94.

and Michael, W. B., 1948. Approaches to univocal factor scores. Psych., 13, 1-22.

and Zimmerman, W. S., 1947. Some A.A.F. findings concerning aptitude factors. Occup., 26, 154-159.

GUTTMAN, L., 1940. Multiple rectilinear prediction and the resolution into components. Psych., 5, 75-99.

1944. General theory and methods for matric factoring. Psych., 9, 1-16.

GUTTMAN, L., and Cohen, J., 1943. Multiple rectilinear prediction and the resolution into components: II. Psych., 8, 169-183.
(see Horst, P.)

HALSTEAD, W. C., 1945. A power factor (P) in general intelligence: the effect of brain injuries. J. Psych., 20, 57-64.

HARMAN, H. H., 1938. Systems of regression equations for the estimation of factors. J. Ed. Psych., 29, 431-441.
(see Holzinger, K. J.).

HARRIS, C. W., 1948a. An exploration of language skill patterns. J. Ed. Psych., 39, 321-336.

1948b. A factor analysis of selected Senate roll calls, 80th Congress. Ed. Psych Meas., 8, 583-591.

1949. Projections of three types of factor pattern. J. Exp. Ed., 17, 335-346.

and Knoell, D. M., 1948. The oblique solution in factor analysis. J. Ed. Psych., 39, 385-403.

HARSH, C. M., 1940. Constancy and variation in patterns of factor loadings. J. Ed. Psych., 31, 335-359.

HARTOG, P., and Rhodes, E. C., with a memorandum by Burt, C., 1936. The marks of examiners--being a comparison of marks allotted to examination scripts by independent examiners and boards of examiners, together with a section on a viva voce examination. MacMillan and Co., Ltd., 344 pp.

HEDMAN, H. B. (see Line, W.).

HELLFRITZSCH, A. G., 1945. A factor analysis of teacher abilities. J. Exp. Ed., 14, 166-199.

HERDAN, G., 1943. The logical and analytical relationship between the theory of accidents and factor analysis. J.R.S.S., 106, 125-142.

HESTON, J. C., 1943. A factor analysis of some clinical performance tests. J. Appl. Psych., 27, 135-149.

1947. The graduate record examination vs. other measures of aptitude and achievement. Ed. Psych. Meas., 7, 618-630.

HOEL, P. G., 1937. A significance test for component analysis. Ann. Math. Stat., 8, 149-158.

1939. A significance test for minimum rank in factor analysis. Psych., 4, 245-253.

HOLLEY, J. W., 1947. A note on the reflection of signs in the extraction of centroid factors. Psych., 12, 263-265.

HOLZINGER, K. J., 1930. Statistical resume of the Spearman two-factor theory. University of Chicago Press, 1930, 43 pp.

1942. Why do people factor? Psych., 7, 147-156.

1944. The relationship between the centroid and Spearman's methods. J. Ed. Psych., 35. 347-351.

1946. A comparison of the principal-axis and centroid factors. J. Ed. Psych., 37, 449-472.

1947. Factoring factors. J. Ed. Psych., 38, 321-328.

1949. Applications of the simple method of factor analysis. J. Ed. Psych., 40, 129-142.

HOLZINGER, K. J., and Harman, H. H., 1941. Factor analysis--
a synthesis of factorial methods. University of
Chicago Press, 417 pp.

and Swineford, F., 1939. A study in factor analysis:
the stability of a bi-factor solution. Supp. Ed.
Mon., No. 48, 91 pp.

and Swineford, F., 1946. The relation of two bi-factors
to achievement in geometry and other subjects. J.
Ed. Psych., 37, 257-265.

(see Swineford, F. J.).

HORNADAY, J. (see Lubonsky, B.).

HORST, P., 1937. A method of factor analysis by means of which
all coordinates of the factor matrix are given simul-
taneously. Psych., 2, 225-236.

Wallin, P., Guttman, L., et al, 1941. The prediction of
personal adjustment. Soc. Sci. Res. Coun., 455 pp.

HOTELLING, H., 1933. Analysis of a complex of statistical
variables into principal components. J. Ed. Psych.,
24, 417-441, 498-520.

1935. The most predictable criterion. J. Ed. Psych.,
26, 139-142.

1936a. Relations between two sets of variates. Biom.,
28, 321-377.

1936b. Simplified calculation of principal components.
Psych., 1, 27-35.

1943a. Some new methods in matrix calculation. Ann. Math.
Stat., 14, 1-34.

HOTELLING, H., 1943b. Further points on matrix calculation and simultaneous equations. Ann. Math. Stat., 14, 440-441.

1949. Practical problems of matrix calculation. Proc. Berk. Symp. Math. Stat. Prob., University of California Press, 275-293.

HOUSEHOLDER, A. S., and Young, G., 1938. Matrix approximation and latent roots. Amer. Math. Month., 45, 165-171.

(see Young, G.).

HOWIE, D., 1945. Aspects of personality in the classroom: a study of ratings on personal qualities for a group of schoolboys. B. J. Psych., 36, 15-28.

HSÜ, E. H., 1946. A factorial analysis of olfaction. Psych., 11, 31-42.

1947. The Rorschach responses and factor analysis. J. Gen. Psych., 38, 129-138.

1948. An experimental demonstration of factor analysis. J. Gen. Psych., 38, 235-241.

and Sherman, M., 1946. The factorial analysis of the electro-encephalogram. J. Psych., 21, 189-196.

(see Stafford, J. W.).

HSU, P. L., 1939. On the distribution of roots of certain determinantal equations. Ann. Eugen., 9, 250-258.

1941a. Canonical reduction of the general regression problem. Ann. Eugen., 11, 42-46.

1941b. On the limiting distribution of roots of a determinantal equation. J. Lond. Math. Soc., 16, 183-194.

HSU, P. L., 1941c. On the limiting distribution of the canonical correlations. Biom., 32, 38-45.

1949. The limiting distribution of functions of sample means and application to testing hypotheses. Proc. Berk. Symp. Math. Stat. Prob., University of California Press, 359-402.

IRWIN, J. O., 1933. A critical discussion of the single-factor theory. B. J. Psych., 23, 371-381.

1934a. Correlation methods in psychology. B. J. Psych., 25, 86-91.

1934b. Statistical methods in psychology--the present position of the theory of two factors. Proc. Inst. Int. Stat., Sess. 22, 15 pp.

1935. On the indeterminacy in the estimate of g. B. J. Psych., 25, 393-394.

JEFFRIES, L. A., 1948. The nature of "primary abilities." Amer. J. Psych., 61, 107-111.

JONES, F. N., 1948. A factor analysis of visibility data. Amer. J. Psych., 61, 361-369.

JONES, L. V., 1949. A factor analysis of the Stanford-Binet at four age levels. Psych., 14, 299-331.

KELLEY, T. L., 1914. Educational guidance--an experimental study in the analysis and prediction of ability of high school pupils. Teachers College, Columbia University, 116 pp.

KELLEY, T. L., 1928. Crossroads in the mind of man. Stanford University Press, 238 pp.

1935. Essential traits of mental life--the purposes and principles underlying the selection and measurement of independent mental factors, together with computation tables. Harvard University Press, 145 pp.

and Salisbury, F. S., 1926. An iteration method for determining multiple correlation constants. J.A.S.A., 21, 282-292.

KELLOGG, C. E., 1948. Note: A correction for Thurstone's multiple factor analysis. Can. J. Psych., 2, 137-139.

KEIR, G., 1949. The progressive matrices as applied to school children. B. J. Psych., Stat., 2, 140-150.

KLEEMEIER, R. W., and Dudek, F. J., 1950. Factorial investigation of flexibility. Ed. Psych. Meas., 10, 107-118.

KNOELL, D. M. (see Harris, C. W.).

KOOPMANS, T., 1937. Linear regression analysis of economic time series. Neder. Econ. Inst., No. 20, 150 pp.

LARSEN, A. H. (see Marzolf, S. S.).

LARSEN, R. P. (see Wittenborn, J. R.).

LAWLEY, D. N., 1940. The estimation of factor loadings by the method of maximum likelihood. Proc. Roy. Soc. Edin., 60, 64-82.

1941. Further investigations in factor estimation. Proc. Roy. Soc. Edin., A, 61, 176-185.

1943a. The application of the maximum likelihood method to factor analysis. B. J. Psych., 33, 172-175.

LAWLEY, D. N., 1943b. A note on Karl Pearson's selection formulae. Proc. Roy. Soc. Edin., 62, 28-30.

1950. Factor analysis by maximum likelihood: a correction. B. J. Psych., Stat., 3, 76.

LEDERMANN, W., 1936. Some mathematical remarks concerning boundary conditions in the factorial analysis of ability. Psych., 1, 165-174.

1937a. On an upper limit for the latent roots of a certain class of matrices. J. Lond. Math. Soc., 12, 12-18.

1937b. On the rank of the reduced correlational matrix in multiple-factor analysis. Psych., 2, 85-93.

1938a. Note on Professor Godfrey H. Thomson's article "The influence of univariate selection on factorial analysis of ability." B. J. Psych., 29, 69-73.

1938b. The orthogonal transformations of a factorial matrix into itself. Psych., 3, 181-187.

1938c. A shortened method of estimation of mental factors by regression. Nature, 141, 650.

1939a. On a problem concerning matrices with variable diagonal elements. Proc. Roy. Soc. Edin., 60, 1-17.

1939b. On a shortened method of estimation of mental factors by regression. Psych., 4, 109-116.

1939c. Sampling distribution and selection in a normal population. Biom., 30, 295-304.

(see Thomson, G. H.).

LINE, W., 1933. Factorial analysis and its relationship to psychological method. B. J. Psych., 24, 187-198.

and Hedman, H. B., 1933. A simplified statement of the two-factor theory. J. Ed. Psych., 24, 195-220.

and Hedman, H. B., 1935. The reversibility of proof. J. Exp. Ed., 3, 216-224.

LOEVINGER, J., 1947. A systematic approach to the construction and evaluation of tests of ability. Psych. Mon., 61, No. 4, 49 pp.

1948. The technic of homogeneous tests compared with some aspects of "scale analysis" and factor analysis. Psych. Bull., 45, 507-529.

LOVELL, C., 1945. A study of the factor structure of thirteen personality variables. Ed. Psych. Meas., 5, 335-350.

LUBORSKY, L. B., and Hornaday, J., 1948. A mechanical factor-rotator for demonstration. Amer. J. Psych., 61, 104-106.

MAC CRONE, I. D., and Starfield, A., 1949. A comparative study in multiple-factor analysis of "neurotic tendency." Psych., 14, 1-20.

MCCRAW, L. W., 1949. Factor analysis of motor learning: a condensation of a dissertation. Res. Quart., 20, 316-335.

MCNEMAR, Q., 1941. On the sampling errors of factor loadings. Psych., 6, 141-152.

1942. On the number of factors. Psych., 7, 9-18.

1945. The mode of operation of suppresant variables.

Amer. J. Psych., 58. 554-555.

MARZOLF, S. S., and Larsen, A. H., 1945. Statistical interpretation of symptoms illustrated with a factor analysis of problem check list items. Ed. Psych. Meas., 5, 285-294.

MEDLAND, F. F., 1947. An empirical comparison of methods of communality estimation. Psych., 12, 101-109.

MEEHL, P. E., 1945. A simple algebraic development of Horst's suppressor variables. Amer. J. Psych., 58, 550-554.

MICHAEL, W. B., 1949. Factor analyses of tests and criteria: a comparative study of two AAF pilot populations. Psych. Mon., 63, No. 3, 55 pp.

(see Guilford, J. P.).

MORRIS, C. M., 1939. A critical analysis of certain performance tests. J. Gen. Psych., 54, 85-105.

MOSIER, C. I., 1939. Influence of chance error on simple structure; an empirical investigation of the effect of chance error and estimated communalities on simple structure in factorial analysis. Psych., 4, 33-44.

NORMAN, R. D. (see Durea, M. A.).

NORTH, R. D., Jr., 1949. Analysis of the personality dimensions of introversion-extroversion. J. Pers., 17, 352-367.

PARRY, J. B. (see Vernon, P. E.).

PEARSON, K., 1896a. Contributions to the mathematical theory of evolution. Note on reproductive selection. J.R.S.S., 59, 398-402.

1896b. Mathematical contributions to the theory of evolution--III. Regression, heredity, and panmixia. Phil. Trans., A, 187, 253-318.

PEARSON, K., 1901. On lines and planes of closest fit to systems of points in space. Phil. Mag., Ser. 6, 2, 559-572.

1902. On the influence of natural selection on the variability and correlation of organs. Phil. Trans., A, 200, 1-16.

1912. On the general theory of the influence of selection on correlation and variation. Biom., 8, 437-443.

PEEL, E. A., 1946. A new method for analyzing aesthetic preferences: some theoretical considerations. Psych., 11, 129-137.

1948. Prediction of a complex criterion and battery reliability. B. J. Psych., Stat., 1, 84-94.

1949. Symposium on selection of pupils for different types of secondary schools: evidence of a practical factor at the age of eleven. B. J. Ed. Psych., 19, 1-15.

PIAGGIO, H. T. H., 1933a. Mathematics and psychology. Math. Gaz., 17, 36-42.

1933b. Three sets of conditions necessary for the existence of a g that is real and unique except in sign. B. J. Psych., 24, 88-105.

RAATH, M. J. (see Reyburn, H. A.).

RAFFERTY, J. A., and Deemer, W. L., Jr., 1949. Statistical

evaluation of the experimental arma of World War II--
factor analysis of items. U.S.A.F. Sch. Av. Med.
Prog. Rep., 7 pp.

RAFFERTY, J. A. (see Deemer, W. L., Jr.).

RAO, C. R., and Slater, P., 1949. Multivariate analysis applied to differences between neurotic groups. B. J. Psych., Stat., 2, 17-29.

REMMERS, H. H. (see Davenport, K. S.; Smalzried, H. T.).

REYBURN, H. A., and Raath, M. J., 1949. Simple structure: a critical examination. B. J. Psych., Stat., 2, 125-133.

RHODES, E. C. (see Hartog, P.).

RHYMER, R. M. (see Cattell, R. B.).

RICHARDS, T. W., 1941. Genetic emergence of factor specificity. Psych., 6, 37-42.

RICHARDSON, L. F., 1950. A method for computing principal axes. B. J. Psych., Stat., 3, 16-20.

RIMOLDI, H. J. A., 1948a. A note on Raven's progressive matrices test. Ed. Psych. Meas., 8, 347-352.

1948b. Study of some factors related to intelligence. Psych., 13, 27-46.

ROFF, M. F., 1936. Some properties of the communality in multiple factor theory. Psych., 1, 2, 1-6.

1937. The relation between results obtainable with raw and corrected correlation coefficients in multiple factor analysis. Psych., 2, 35-39.

1949. Factorial study of the Fels parent behavior scales.

Child Dev., 20, 29-45.

ROFF, M. F., 1950. Personnel selection and classification procedures: a factorial analysis. U.S.A.F. Sch. Av. Med. Proj. Rep., 23 pp.

ROOS, C. F., 1937. A general invariant fit for lines and planes where all the variates are subject to error. Metron, 13, 1-20.

ROSNER, B., 1948. An algebraic solution for the communalities. Psych., 13, 181-184.

SALISBURY, F. S., 1924. A quantitative evaluation of some of the factors determining certain administrative procedures in the public schools of California(thesis). Stanford University.

(see Kelley, T. L.).

SANDLER, J., 1949. The reciprocity principle as an aid to factor analysis. B. J. Psych., Stat., 2, 180-187.

SASTRY, N. S. N., 1941. Can there be a factor-analysis of aesthetic judgment? Sankhya, 5, 313-316.

SATTER, G. A. (see Coombs, C. H.).

SAUNDERS, D. R., 1948. Factor analysis I: Some effects of chance error. Psych., 13, 251-257.

1949. Factor analysis: A note concerning rotation of axes to simple structure. Ed. Psych. Meas., 9, 4, 753-756.

SCHMIDT, M. J. (see Gosnell, H. F.).

SEN, A., 1950. A statistical study of the Rorschach test. B. J. Psych., Stat., 3, 21-39.

coefficients. Proc. Roy. Soc., A, 95, 400-408.

THOMSON, G. H., 1934. Hotelling's method modified to give Spearman's g. J. Ed. Psych., 25, 366-374.

1935. On complete families of correlation coefficients, and their tendency to zero tetrad-differences; including a statement of the sampling theory of abilities. B. J. Psych., 26, 63-92.

1936a. Boundary conditions in the common factor space, in the factorial analysis of ability. Psych., 1, 155-163.

1936b. Some points of mathematical technique in the factorial analysis of ability. J. Ed. Psych., 27, 37-54.

1937. Selection and mental factors. Nature, 140, 934.

1938a. A discussion on certain problems in psychology. Proc. Roy. Soc., B, 125, 418-419.

1938b. The estimation of specific and bi-factors. J. Ed. Psych., 29, 355-362.

1938c. The influence of univariate selection on the factorial analysis of ability. B. J. Psych., 28, 451-459.

1938d. Maximising the specific factors in the analysis of ability. B. J. Ed. Psych., 8, 255-264.

1938e. Methods of estimating mental factors. Nature, 141, 246.

1938f. Recent development of statistical method in psychology-II. Occup. Psych., 12, 319-325.

THOMSON, G. H., 1939. The factorial analysis of ability: I. The present position and the problems confronting us. Agreement and disagreement in factor analysis: a summing up. B. J. Psych., 30, 71-77, 105-108.

1940a. An analysis of performance test scores of a representative group of Scottish children. University of London Press, 58 pp.

1940b. Weighting for battery reliability and prediction. B. J. Psych., 30, 357-366.

1941. The speed factor in performance tests. B. J. Psych., 32, 131-135.

1946. Some recent work in factorial analysis, and a retrospect. Presidential address to British Pyschological Society, University of London Press, 16 pp.

1947. The maximum correlation of two weighted batteries. B. J. Psych., Stat., 1, 27-34.

1948a. The factorial analysis of human ability. 3rd ed. University of London Press, 392 pp.

1948b. Note on the relations of two weighted batteries. B. J. Psych., Stat., 1, 82-83.

1949. On estimating oblique factors. B. J. Psych., Stat., 2, 1-2.

and Ledermann, W., 1939. The influence of multivariate selection on the factorial analysis of ability. B. J. Psych., 29, 288-306.

(see Brown, W.).

THORNDIKE, E. L., 1913. An introduction to the theory of mental and social measurements. 2nd ed., Teachers College, Columbia University, 277 pp.

THORNDIKE, E. L., 1934. Prediction of vocational success. The Commonwealth Fund, 284 pp.

1949. Personnel selection--test and measurement techniques. John Wiley and Sons, 358 pp.

THURSTONE, L. L., 1933. The theory of multiple factors. Edwards Bros., 65 pp.

1935. The vectors of the mind; multiple-factor analysis for the isolation of primary traits. University of Chicago Press, 266 pp.

1938. Primary mental abilities. Psychometric Mon. 1, No. 1, 121 pp.

1945a. The effects of selection in factor analysis. Psych., 10, 165-198.

1945b. The prediction of choice. Psych., 10, 237-253.

1946a. Factor analysis and body types. Psych., 11, 15-21.

1946b. Note on a reanalysis of Davis' reading tests. Psych., 11, 185-188.

1946c. A single plane method of rotation. Psych., 11, 71-79.

1947. Multiple-factor analysis--a development and expansion of "The Vectors of Mind." University of Chicago Press, 535 pp.

1949a. Note about the multiple group method. Psych., 14, 43-45.

THURSTONE, L. L., 1949b. Primary abilities. Occup., 27 527-529.

and Thurstone, T. Q., 1941. Factorial studies of intelligence. Psychometric Mon., No. 2, 94 pp.

THURSTONE, T. Q. (see Thurstone, L. L.).

TRYON, R. C., 1949. Cluster analysis, Revised ed. (hectographed lecture notes). University of California.

TSCHECHTELIN, S. M.A, 1944. Factor analysis of children's personality rating scale. J. Psych., 18, 197-200.

ULLMAN, J., 1944. The probability of convergence of an iterative process of inverting a matrix. Ann. Math. Stat., 15, 205-213.

U. S. Office of Strategic Services, 1948. Assessment of men. Rinehart and Co., 541 pp.

VAN BOVEN, A., 1947. A modified Aitken pivotal condensation method for partial regression and multiple correlation. Psych., 12, 127-133.

VERNON, P. E., 1946. Statistical methods in the selection of navy and army personnel. Supp J.R.S.S., 8, 139-153.

1947a. Psychological tests in the Royal Navy, Army and A.T.S. Occup. Psych., 21, 53-74.

1947b. The variations of intelligence with occupation, age, and locality. B. J. Psych., Stat., 1, 52-63.

1949. The structure of practical abilities. Occup. Psych., 23, 81-96.

1950a. An application of factorial analysis to the study of test items. B. J. Psych., Stat., 3, 1-15.

VERNON, P. E., 1950b. Psychological studies of the mental quality of the population. B. J. Ed. Psych., 20, 35-42.

and Parry, J. B., 1949. Personnel selection in the British forces. University of London Press, 324 pp.

WALLIN, P. (see Horst, P.).

WAUGH, F. V., 1945. A note concerning Hotelling's method of inserting a partitioned matrix. Ann. Math. Stat., 16, 216-217.

and Dwyer, P. S., 1945. Compact computation of the inverse of a matrix. Ann. Math. Stat., 16, 259-271.

WEINBERG, D., 1945. Une experience de controle des methodes d'analyse factorielle. C. R. Acad. Sci. Paris, 220, 214-216.

and Danger, J., 1946. Controle experimental des methodes d'analyse factorielle. Biotyp., 8, 56-74.

WHERRY, R. J., 1946. Test selection and suppressor variables. Psych., 11, 239-247.

1949. A new iterative method for correcting erroneous communality estimates in factor analysis. Psych., 14, 231-241.

and Gaylord, R. H., 1943. The concept of test and item reliability in relation to factor pattern. Psych., 8, 247-269.

WILSON, E. B., 1928. On hierarchical correlation systems. Proc. Nat. Acad. Sci., 14, 283-291.

WILSON, E. B., 1934. On resolution into generals and spe-

cifics. Proc. Nat. Acad. Sci., 20, 193-196.
and Worcester, J., 1934. The resolution of four tests. Proc. Nat. Acad. Sci., 20, 189-192.
WILSON, J. H., 1935. The exactness of "g" as determined by certain intelligence tests. B. J. Psych., 26, 93-98.
WISHART, J., 1928. Sampling errors in the theory of two factors. B. J. Psych., 19, 181-187.
1933. The two-factor theory of intelligence. Nature, 132, 677.
WISPE, L. G. (see Cattell, R. B.).
WITTENBORN, J. R., 1945. Mechanical ability, its nature and measurement: I. An analysis of the variables employed in the preliminary Minnesota experiment. Ed. Psych. Meas., 5, 241-260.
1949. Factor analysis of discrete responses to the Rorschach ink blots. J. Cons. Psych., 13, 335-340.
and Larsen, R. P., 1944. A factorial study of achievement in college German. J. Ed. Psych., 35, 39-48.
WOLFLE, D., 1940. Factor analysis to 1940. Psychometric Mon., No. 3, 69 pp., University of Chicago Press.
WOODROW, H., 1939. The common factors in fifty-two mental tests. Psych., 4, 99-108.
1945. Intelligence and improvement in school subjects. J. Ed. Psych., 36, 155-166.
WORCESTER, J. (see Wilson, E. B.).

WRIGHT, S., 1934. The method of path coefficients. Ann. Math. Stat., 5, 161-215.

YELA, M., 1949. Application of the concept of simple structure to Alexander's data. Psych., 14, 121-135.

YOUNG, G., 1937. Matrix approximations and subspace fitting. Psych., 2, 21-25.

1939. Factor analysis and the index of clustering. Psych., 4, 201-208.

1941. Maximum likelihood estimation and factor analysis. Psych., 6, 49-53.

and Householder, A. S., 1938. Discussion of a set of points in terms of their mutual distances. Psych., 3, 19-22, 126.

and Householder, A. S., 1940. Factorial invariance and significance. Psych., 5, 47-56.

(see Eckart, C.).

(see Householder, A. S.).

YULE, G. U., 1897. On the theory of correlation. J.R.S.S., 60, 812-824.

1907. On the theory of correlation for any number of variables. Proc. Roy. Soc., A, 79, 182-193.

ZIMMERMAN, W. S., 1946. A simple graphical method for orthogonal rotation of axes. Psych., 11, 51-55.

(see Guilford, J. P.).

In the following alphabetical list of periodicals, the abbreviations are those used in the three bibliographies. The Library of Congress reference number is given wherever this information was available.

Abbreviation	Title
Abh. d. Deut. Seef.	Abhandlungen des Deutschen Seefischereivereins.
Amer. Ed. Res. Ass., Off. Rep.	American Educational Research Association Official Report. L13.A425.
Amer. J. Phys. Anthrop.	American Journal of Physical Anthropology. GN1.A55.
Amer. J. Psych.	American Journal of Psychology. BF1.A5.
Amer. J. Publ. H.	American Journal of Public Health. RA421.A41.
Amer. Math. Month.	American Mathematical Monthly. QA1.A515.
Amer. Nat.	American Naturalist. QH1.A5.
Amer. Psych.	American Psychologist. BF1.A55.
Ann. Eugen.	Annals of Eugenics. HQ750.A1A5.
Ann. Math. Stat.	Annals of Mathematical Statistics. HA1.A8.
Ann. N. Y. Acad. Sci.	Annals of the New York Academy of Sciences. Q11.N5.

Abbreviation	Title
Arch. Naturgesch.	Archiv für Naturgeschichte (Zeitschrift für systematische Zoologie). QL1.A57.
Arch. Psych.	Archives of Psychology. BF21.A7.
AAF Av. Psych. Prog. Res. Rep.	Army Air Forces Aviation Psychology Program Research Reports.
---	Biometrics. QH301.B48.
Biom.	Biometrika. QH301.B5.
Biotyp.	Biotypologie (Bulletin de la Societe de Biotypologie).
B. J. Ed. Psych.	British Journal of Educational Psychology. LB1051.A2B7.
B. J. Med. Psych.	British Journal of Medical Psychology. RC321.B83.
B. J. Psych.	British Journal of Psychology, General Section. BF1.B7.
B. J. Psych., Mon. Supp.	British Journal of Psychology, Monograph Supplements.
B. J. Psych., Stat.	British Journal of Psychology, Statistical Section.
Bull. Int. Acad. Sci. Crac., A	Bulletin International de l'Academie Polonaise des Sciences et des Lettres, Classe des Sciences Mathematiques et Naturelles, Series A, Cracovie. AS142.K84.

Abbreviation	Title
Bull. Amer. Math. Soc.	Bulletin of the American Mathematical Society. QA1.A52.
Bull. Calc. Math. Soc.	Bulletin of the Calcutta Mathematical Society. QA1.C25.
Bull. Calc. Stat. Ass.	Bulletin of the Calcutta Statistical Association.
Bull. Can. Psych. Ass.	Bulletin of the Canadian Psychological Association.
Can. J. Psych.	Canadian Journal of Psychology. BF1.C3.
Child Dev.	Child Development. HQ750.A1C45.
C. R. Acad. Sci. Paris	Comptes Rendus Hebdomodaires des Seances de l'Academie des Sciences, Paris. Q46.A14.
Cur. Sci.	Current Science. Q1.C78.
Econ.	Econometrica. H31.E2.
Ed. Psych. Meas.	Educational and Psychological Measurements. BF1.E3.
---	Experientia. Q1.A1E9.
Genet. Psych. Mon.	Genetic Psychology Monographs.
---	Genetics.
Harv. Ed. Rev.	Harvard Educational Review. L11.H3.
Hum. Biol.	Human Biology. GN1.H8.
---	India Rubber World. TS1870.I4.
Indus. Eng. Chem., Anal. Ed.	Industrial and Engineering Chemistry, Analytical Edition. TP1.I615.

Abbreviation	Title
I. F. R. B.	Industrial Fatigue Research Board, Medical Research Council, London.
I. H. R. B.	Industrial Health Research Board, Medical Research Council, London.
Inst, Int. Stat.	Institut International de Statistique.
Int. Rev. d. ges. Hydr. u. Hydr.	Internationale Revue der gesamten Hydrobiologie und Hydrographie. 25321.I5.
Iowa. St. Col. J. Sci.	Iowa State College Journal of Science Q1.I6.
J. As. Soc. Beng.	Journal and Proceedings of the Asiatic Society of Bengal, New Series. A5472.B33.
J. Appl. Psych.	Journal of Applied Psychology. BF1.J55.
J. Av. Med.	Journal of Aviation Medicine. TL555.A1A4.
J. Cons. Psych.	Journal of Consulting Psychology. BF1.J575.
J. Ed. Psych.	Journal of Educational Psychology. LB1051.A2J6.
J. Exp. Ed.	Journal of Experimental Education. L11.J77.
J. For.	Journal of Forestry. SD1.S63.
J. Gen. Psych.	Journal of General Psychology. BF1.J64.
J. Gen.	Journal of Genetics.
J. Pers.	Journal of Personality. BF1.J66.
J. Psych.	Journal of Psychology. BF1.J67.

Abbreviation	Title
J. Soc. Psych.	Journal of Social Psychology. HM251.A1J6.
J. Amer. Soc. Agron.	Journal of the American Society of Agronomy. S22.A7.
J. A. S. A.	Journal of the American Statistical Association. HA1.A6.
J. Lond. Math. Soc.	Journal of the London Mathematical Society. QA1.L53.
J. Roy. Anthrop. Inst.	Journal of the Royal Anthropological Institute of Great Britain and Ireland.
J. R. S. S.	Journal of the Royal Statistical Society. HA1.R8.
J. R. S. S., B	Journal of the Royal Statistical Society, Series B (Methodological).
Phil. Mag.	London, Edinburgh and Dublin Philosophical Magazine and Journal of Science. Q1.P5.
---	Man in India. GN1.M3.
---	Mathematica, Cluj.
Math. Gaz.	Mathematical Gazette.
Mem. Sci. Math. Paris	Memorial des Sciences Mathematiques, Paris. QA1.M6.
Mem. Ist. Ital. Idrobiol.	Memorie dell'Istituto Italiano di Idrobiologia, Pallanza.

Abbreviation	Title
---	Metron. HA1.M4.
Mich. Acad. Sci.	Michigan Academy of Science, Arts and Letters. Q11.M56.
---	Nature, London. Q1.N2.
Neder. Econ. Inst.	Nederlandsch Economisch Instituut.
Occup. Psych.	Occupational Psychology. T58.A2N35.
Occup.	Occupations. HF5381.045.
Phil. Mag.	(See under "London.")
Phil. Trans., A	Philosophical Transactions of the Royal Society of London, Series A. Q41.L8.
Proc. Amer. Soc. An. Prod.	Proceedings of the American Society of Animal Production. SF95.A6.
Proc. Berk. Symp. Math. Stat. Prob.	Proceedings of the Berkeley Symposium on Mathematical Statistics and Probability. QA276.B4.
Proc. Camb. Phil. Soc.	Proceedings of the Cambridge Philosophical Society. Q41.C17.
Proc. Edin. Math. Soc.	Proceedings of the Edinburgh Mathematical Society. QA1.E2.
Proc. Inst. Int. Stat.	Proceedings of the Institut International de Statistique.
Proc. Nat. Acad. Sci.	Proceedings of the National Academy of Sciences of the U.S.A. Q11.N26.
Proc. Nat. Inst. Sci. Ind.	Proceedings of the National Institute of Sciences of India. Q73.N3.

Abbreviation	Title
Proc. Roy. Soc.	Proceedings of the Royal Society of London. Q41.L7.
Proc. Roy. Soc., A	Proceedings of the Royal Society of London, Series A.
Proc. Roy. Soc., B	Proceedings of the Royal Society of London, Series B.
Proc. Roy. Soc. Edin.	Proceedings of the Royal Society of Edinburgh. Q41.E21.
Proc. Roy. Soc. Edin., A	Proceedings of the Royal Society of Edinburgh, Section A.
Proc. Sec. Int. Biom. Conf.	Proceedings of the Second International Biometrics Conference.
Psych. Bull.	Psychological Bulletin. BF1.P75.
Psych. Mon.	Psychological Monographs. BF1.P8.
Psych. Rev.	Psychological Review. BF1.P7.
Psych.	Psychometrika. BF1.P86.
Psychometric Mon.	Psychometric Monographs.
Publ. H. Rep.	Public Health Reports. RA11.B17.
Purdue Univ. Div. Ed. Ref.	Purdue University Division of Educational Reference. LB2305.P79.
Rec. Ind. Mus.	Records of the Indian Museum, Calcutta.
Res. Quart.	Research Quarterly of the American Association for Health, Physical Education, and Recreation.
Rev. Ed. Res.	Review of Educational Research. L11.R35.
---	Sankhya. HA1.S3.

Abbreviation	Title
Sch. Rev.	School Review. L11.S551.
Sci. Cult.	Science and Culture. QH1.S35.
Stat. Res. Mem.	Statistical Research Memoirs. HA1.L6.
Stud. Cons. Inst. Fin.	Studies in Consumer Instalment Financing, National Bureau of Economic Research. HF5568.D87.
Supp. Ed. Mon.	Supplementary Educational Monographs.
Supp. J. R. S. S.	Supplement to the Journal of the Royal Statistical Society. HA1.R8.
Trans. Amer. Math. Soc.	Transactions of the American Mathematical Society. QA1.A522.
USAF Sch. Av. Med. Proj. Rep.	United States Air Force School of Aviation Medicine Project Reports.
USDA Tech. Bull.	United States Department of Agriculture Technical Bulletin. S21.A72.
Univ. Calif. Publ. Zool.	University of California Publications in Zoology. QL1.C15.
Univ. Øk. Inst.	Universitetets Økonomiske Institutt. HA33.F92.
Verh. K. Akad. Wetens. Amst., Afd. Nat.	Verhandelingen der Koninklijke Akademie van Wetenschappen te Amsterdam, Afdeeling Natuurkunde.
Zeits. für Konst.	Zeitschrift für Konstitutionslehre.
---	Zoologica, New York. QL1.N6.

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DISCRIMINATORY ANALYSIS - II - FACTOR ANALYSIS AND DISCRIMINATION

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