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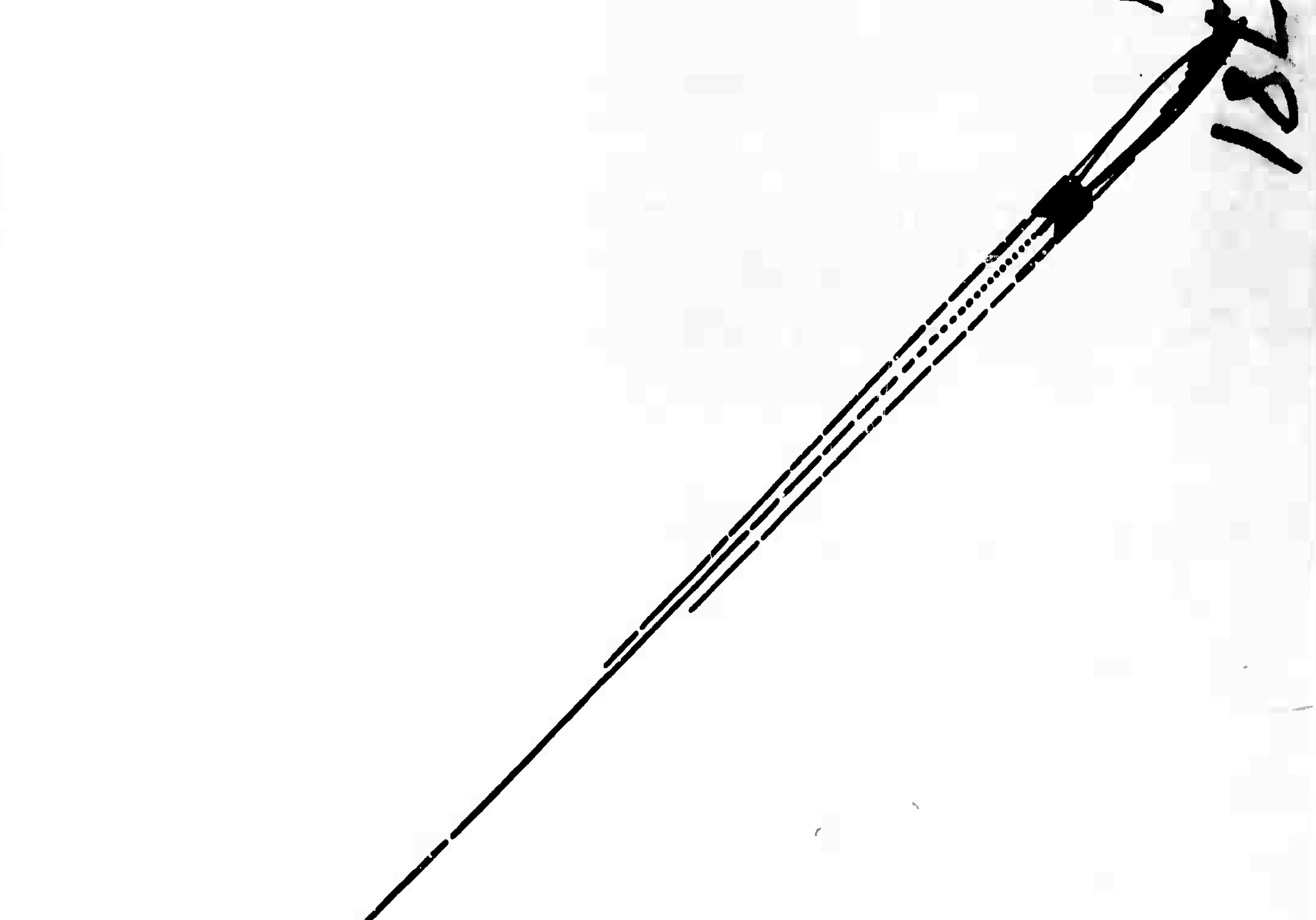
HEATING AND COOLING OF ROCKET MOTORS



NOTS 382

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27 APRIL 1951

U.S. NAVAL ORDNANCE TEST STATION

U. S. NAVAL ORDNANCE TEST STATION, INYOKERN
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HEATING AND COOLING OF ROCKET MOTORS

By

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NOTS 382

China Lake, California
27 April 1951

ACKNOWLEDGMENT

The discussion of the evaluation of the heat-transfer coefficient presented in Appendix A was contributed by M. L. Jackson. The authors further acknowledge the work of R. R. Helus, H. R. Welton, L. E. Schilberg, and A. E. Harris, from whose investigations the data for the evaluations presented in this report were obtained.

FOREWORD

This report summarizes several sets of data on the heating and cooling of rocket motors in ovens. The data were analyzed according to available theory, and similarities in performance were studied to obtain simple generalized correlations. As the data used were not originally obtained for this type of analysis, the need for more pertinent data is apparent. It is believed, however, that the generalized correlations presented here will aid in predicting the time-temperature behavior of loaded rocket motors to a degree of accuracy sufficient for most purposes.

The original work of accumulating the data began early in 1949 under Bureau of Ordnance Task Assignment NOTS-2-Re3e-520-1. Further work was done under Task Assignments NOTS-20-Re2d-429-2 and NOTS-19-Re3e-516-2. The analysis presented here was begun in July and finished in September 1950.

This report was reviewed for technical accuracy by W. P. Reid, R. H. Olds, and M. L. Jackson.

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ABSTRACT

Heating and cooling data for the 2.0-in., 2.75-in., 5.0-in. (HPAG), and 5.25-in. (Weapon A) motors with internal-burning grains have been analyzed. The relation of Y, the fractional approach of initial to final temperature, to θ, the time of heating or cooling, is given by the general equation $Y = c \cdot 10^{-a\theta}$. Values of the constants a and c have been evaluated and found to compare reasonably well with those derived from the Gurney-Lurie type of charts for infinite, homogeneous, solid cylinders.

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INTRODUCTION

As the burning characteristics of a propellant grain are dependent on its temperature, some means of predicting the temperature at any particular time is of primary importance. In the computation of rocket trajectories and ultimately in the setting of the sight, the temperature of the propellant is a critical factor. In the field of internal ballistics, temperature has a direct bearing on the reaction pressure and burning rate. Information concerning the *unsteady-state* heat transfer to a rocket is required by aircraft designers in the designing of rocket launchers with temperature control.

In previous studies at the Naval Ordnance Test Station,[1,2,3] the time-temperature relationships during heating and cooling of the 2.75-in. and Weapon A motors were presented by means of simple temperature-time curves or, in one case, in terms of the difference between the temperature at a certain point on the grain and that of the oven in which the grain was placed. No attempt was made to generalize the data so that they might be applied to predict the temperature-time relations of other rounds. It is the purpose of this report to analyze the experimental data and reduce them to simple equations so as to afford some basis of prediction for other rocket motors.

THEORETICAL BACKGROUND

Heating and cooling of rocket motors constitutes a case of thermal conduction in the unsteady state, in which the temperature at any point in the motor is dependent on both the elapsed time after removal of the motor from the oven and the location of the particular point in the motor. For an infinitely long, homogeneous, solid cylinder, the differential equation (taken from Ref. 1) expressing the variation of temperature t at any radius r with respect to time θ is,

$$\frac{\partial t}{\partial \theta} = \alpha\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\left(\frac{\partial t}{\partial r}\right)\right] \qquad (1)$$

where α is the thermal diffusivity.

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[1] Helus, R. R., and H. R. Welton. *Measurement of Temperature Gradients in Loaded, Solid Propellants in the 2.75-In. Round*, Oct. 28, 1949, Special Report AJO 41063-3045. CONFIDENTIAL.

[2] Schilberg, L. E. *Routine Temperature-Time Heating and Cooling Investigations of Charge 5303 in Motor NOTS Model 118 (Weapon A)*, 5 July 1949, XTT-2-520-1 (41019-3011) JOR. CONFIDENTIAL.

[3] Schilberg, L. E., and A. E. Harris. *Heating and Cooling Rate Studies on 2.75-In. Rocket Motor NOTS Model 101*, 21 August 1950, 4052 Re2d 429-2 JOR. CONFIDENTIAL.

For the case of a solid cylinder of radius r_o at a uniform temperature t_i placed in surroundings of temperature t_o, a solution has been presented (Ref. 1) in the form of a series for the boundary condition

$$\left(\frac{\partial t}{\partial r}\right)_{r=r_o} = -\frac{h}{k}(t_o - t) \tag{2}$$

where h is the heat-transfer coefficient and k is the thermal conductivity. The series converges rapidly, and the substitution of real values in the equation shows that for sufficiently large values of θ all of the terms after the first may be disregarded. The approximate solution so obtained has the form

$$\frac{t_o - t}{t_o - t_i} = \frac{2J_1(R_1)}{R_1[J_0^2(R_1) + J_1^2(R_1)]} J_0\left(R_1 \frac{r}{r_o}\right) 10^{-R_1^2 \alpha\theta/2.303 r_o^2} \tag{3}$$

where

$J_0(R_1)$ = Bessel function of the first kind and zero order of (R_1)
$J_1(R_1)$ = Bessel function of the first kind and first order of (R_1)
r_o = outside radius of cylinder, ft
(R_1) = the first positive root of $R\,J_1(R) = \frac{hr_o}{k} J_0(R)$

Equation 3 has the general form

$$Y = c \cdot 10^{-bX} \tag{4}$$

where the several terms have the meaning, by comparison with Eq. 3,

$$Y = \frac{t_o - t}{t_o - t_i} \tag{5}$$

$$X = \frac{\alpha\theta}{r_o^2} = \frac{k\theta}{C_p \rho r_o^2} \tag{6}$$

$$c = \frac{2J_1(R_1)}{R_1[J_0^2(R_1) + J_1^2(R_1)]} J_0\left(R_1 \frac{r}{r_o}\right) \tag{7}$$

$$b = \frac{R_1^2}{2.303} \tag{8}$$

Gurney and Lurie (Ref. 2) have presented graphs of Eq. 3 in the form of log Y plotted against X (X = the relative-time ratio). These graphs show families of lines for various positions within the cylinder n and surface resistivities m, where

$$m = \frac{k}{hr_o} \tag{9}$$

$$n = \frac{r}{r_o} \tag{10}$$

Examples of this kind of plot may also be found in Ref. 3. The families of lines are observed to be straight parallel lines for a given value of m for values of X greater than about 0.3.

This method of plotting will be used because it has a particular advantage for experimental data in that the straight line results over most of the region of interest. The Gurney-Lurie type of chart for an infinite cylinder may serve as a convenient comparison between theoretical and experimental data, although a rocket motor does not behave strictly as an infinite, homogeneous, solid cylinder. The original Gurney-Lurie charts, however, present only limited values of m and X. The extended charts, recently worked out by Heisler (Ref.4) provide sufficiently accurate comparison over the range desired.

EXPERIMENTAL DATA

The data available for this analysis were taken mostly from the routine temperature-cycling tests made in connection with the temperature conditioning of propellant grains prior to static firing. The extent of this analysis is therefore limited, since the experiments were not designed for the specific purpose of observing unsteady-state heat transfer.

An estimate of the average heat-transfer coefficient h was made from the data obtained; details of the calculation are given in Appendix A. At the present time, the available heating and cooling data are limited to the 2.0-in., 2.75-in., HPAG, and Weapon A motors.[1,2,3]

Physical constants of the H-9 powder charge, used in the 2.75-in. rocket motor, such as thermal conductivity, specific gravity, and specific heat, are as follows:

$$k = 4.5 \times 10^{-4} \frac{\text{cal}}{(\text{sec})(\text{cm})(^\circ\text{C})}$$

$$= 0.109 \frac{\text{Btu}}{(\text{hr})(\text{ft})(^\circ\text{F})}$$

$$C_p = 0.42 \frac{\text{Btu}}{(\text{lb mass})(^\circ\text{F})}$$

$$\rho = 101.2 \frac{\text{lb mass}}{\text{cu ft}}$$

The above constants were taken from the report cited in footnote 1. Values of h and α, which were calculated for specific conditions, are cited below:

$$h = 3.6 \frac{\text{Btu}}{(\text{hr})(\text{sq ft})(^\circ\text{F})}$$

$$\alpha = 0.00256 \frac{\text{sq ft}}{\text{hr}}$$

EXPERIMENTAL PROCEDURE

A brief description of the procedure used in testing one of the motors is given at this point to indicate the accuracy which may be expected from the results. (The report cited in footnote 3, page 1, gives this procedure in full.)

The motor used was the motor for the 2.75-in. folding-fin, air-to-air rocket. It consisted of an aluminum tube containing a 6-point star, internal-burning propellant grain of H-9 powder inhibited with 6 laps of cellulose acetate. Neither the head nor the fin assembly was attached to the motor. Instead, thin cardboard disks covered with layers of aluminum foil were attached to the tube ends to minimize heat transfer through the internal perforation. Thermocouples, made of 30-gage iron and constantan wire, were inserted in the motor or propellant grain parallel to the cylindrical axis and 2 in. from one end of the motor.

The thermocouple which measured the oven air temperature (Tc. 1 in all motors except the Weapon A, where it was designated Tc. 5) was attached to the motor, the junction being held 2 in. away from the tube and shielded from radiation with aluminum foil. Figure 1 shows the location of the thermocouples.

A calibrated Brown Electronik recording potentiometer was used to record the temperature at intervals of 6 sec between successive readings of the several thermocouple locations. The same conditioning ovens usually used in preparation for static firing were used in this work. The air moving normal to the rocket axis varied in speed between 200 and 300 fpm.

The rocket motor under test was brought to desired initial temperature by allowing it to stand in an oven overnight or until every point in the motor had reached approximately the predetermined initial temperature (t_i). It was then quickly transferred to a second oven maintained at the temperature t_o, and thermocouple readings were recorded until every point in the motor had approached to within 2°F of the ambient temperature (t_o).

METHOD OF PLOTTING DATA

The most simple and direct method of presenting the data is to plot temperature versus time on regular coordinate paper, as shown in Fig. 2. This method does not, however, yield much information on heating and cooling characteristics, nor does it provide a general correlation.

As discussed previously, the behavior of rockets of different sizes and materials may be correlated by plotting the dimensionless ratio log Y as ordinate against X as abscissa, using semilogarithmic paper. The resulting curve is a straight line over most of the range, whose general equation may be presented by Eq. 4,

$$Y = c \cdot 10^{-bX}$$

where c and b, both dimensionless, are intercept and slope, respectively, of the straight line.

Since the rockets covered in this analysis are made of similar materials, it is sufficient to plot log Y against θ on semilogarithmic paper, yielding straight lines whose general equation is now

$$Y = c \cdot 10^{-a\theta} \qquad (11)$$

with c having the same value as in Eq. 4, and a as the slope, but having the dimension of hr^{-1} or min^{-1}, depending on the units used in plotting the data.

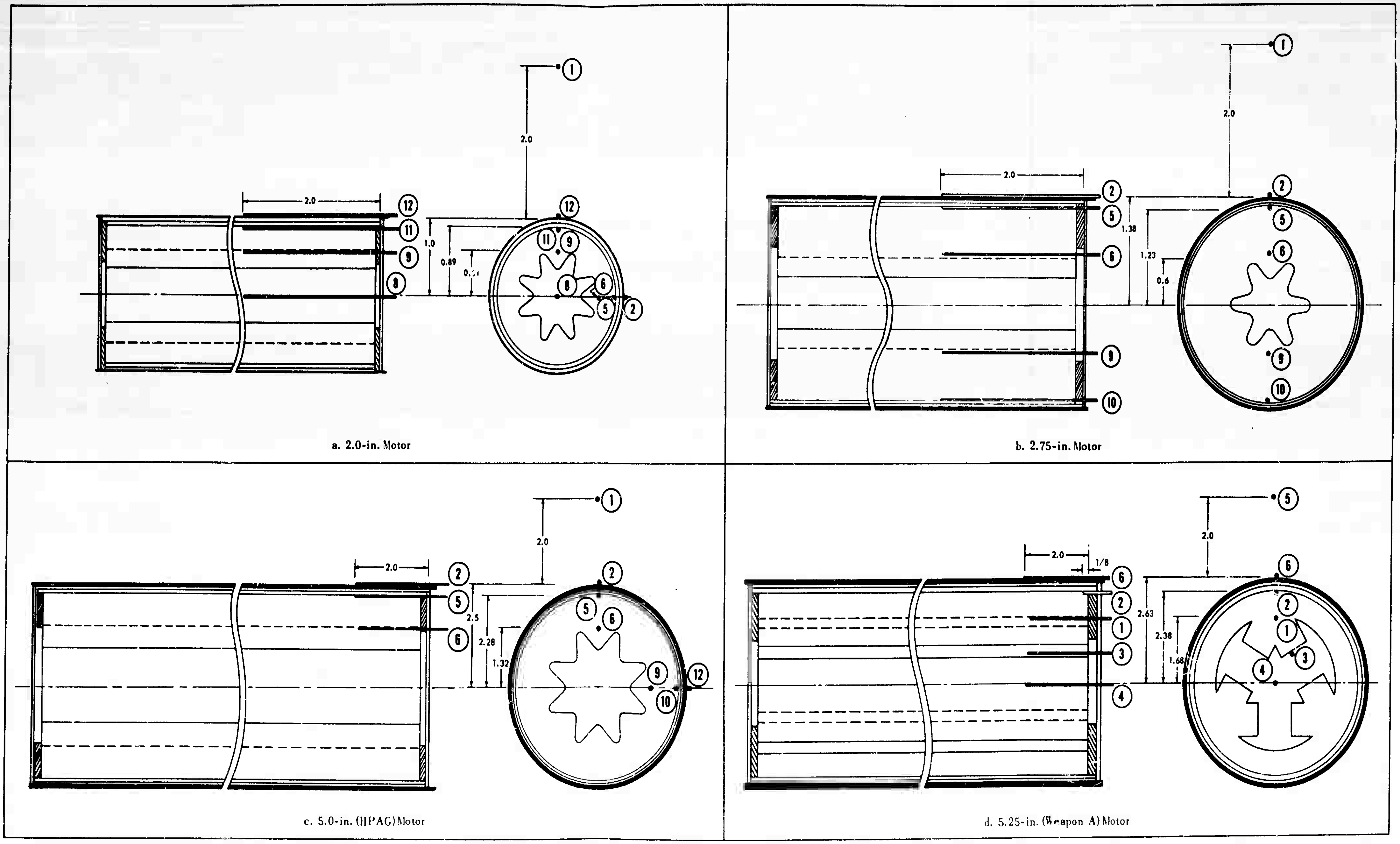

FIG 1. Diagram of Thermocouple Locations. Thin cardboard disks covered with aluminum foil are attached to the ends of the motor tubes. Numbers in circles indicate positions of thermocouples.

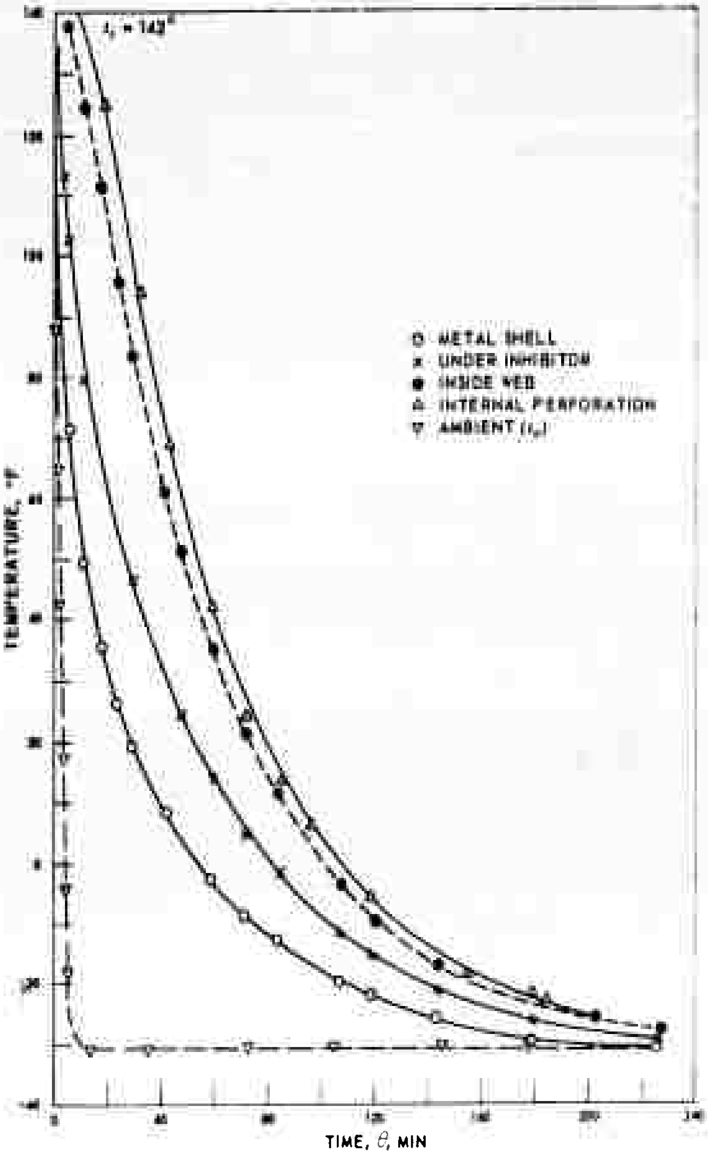

FIG. 2. Time-Temperature Curve, Cooling of 2.75-in. Motor (142 to -30°F).

Figures 3–5 show graphically the results for the 2.75-in. motor at different thermocouple locations as plotted by this method. Similar graphs for the other motors appear in Appendix B. All of the graphs represent the data after they have been smoothed out.

METHOD OF CALCULATION

The original temperature readings (contained in the reports cited in footnotes 1–3) were taken from the recorder charts at convenient time intervals. The arithmetical mean temperatures of corresponding points located 90 or 180 deg apart were used in the event that readings of each pair of thermocouples differed. The equilibrium temperature, measured by Tc. 1 (see Fig. 1 for location of this thermocouple)[4] in the first oven used was taken as the initial temperature t_i; the

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[4] In the case of the Weapon A, this thermocouple is Tc. 5.

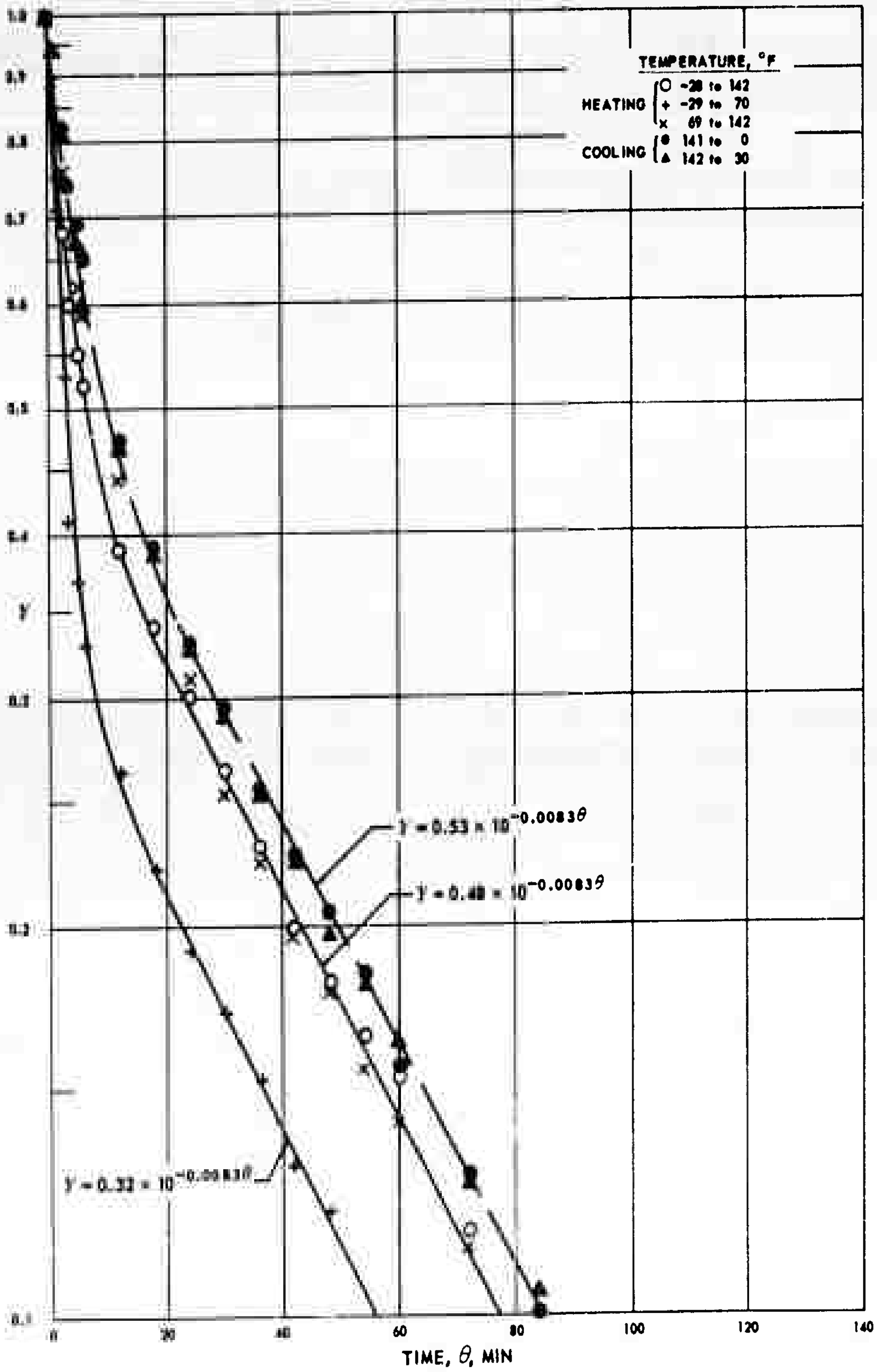

FIG. 3. Temperature History of 2.75-in. Motor During Heating and Cooling; Thermocouple Position: Metal Shell.

instantaneous temperature, measured by Tc. 1 in the second oven, was considered to be the temperatures of the surroundings, t_o. Knowing these temperatures, the ratio $Y = (t_o - t)/(t_o - t_i)$ was calculated for every temperature t. The values of Y were then plotted against the corresponding values of θ.

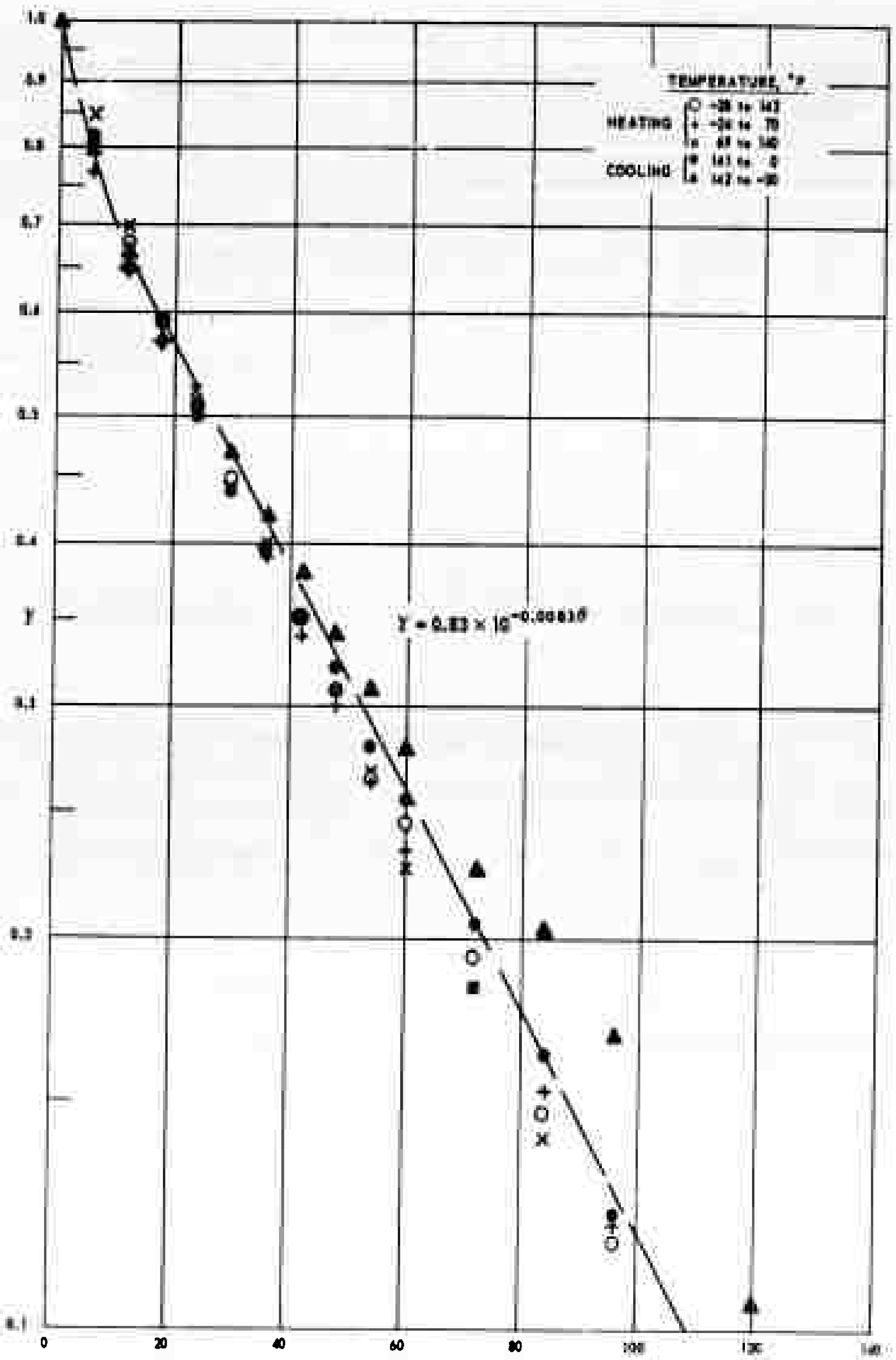

FIG. 4. Temperature History of 2.75-in. Motor During Heating and Cooling; Thermocouple Position: Under Inhibitor.

The constants, dimensionless intercept c and slope a for Eq. 11, are determined from the straight-line portions of Fig. 3–5 and are shown in Table 1. Values of a and b for all of the motors are summarized in Table 2, and values for c are summarized in Table 3.

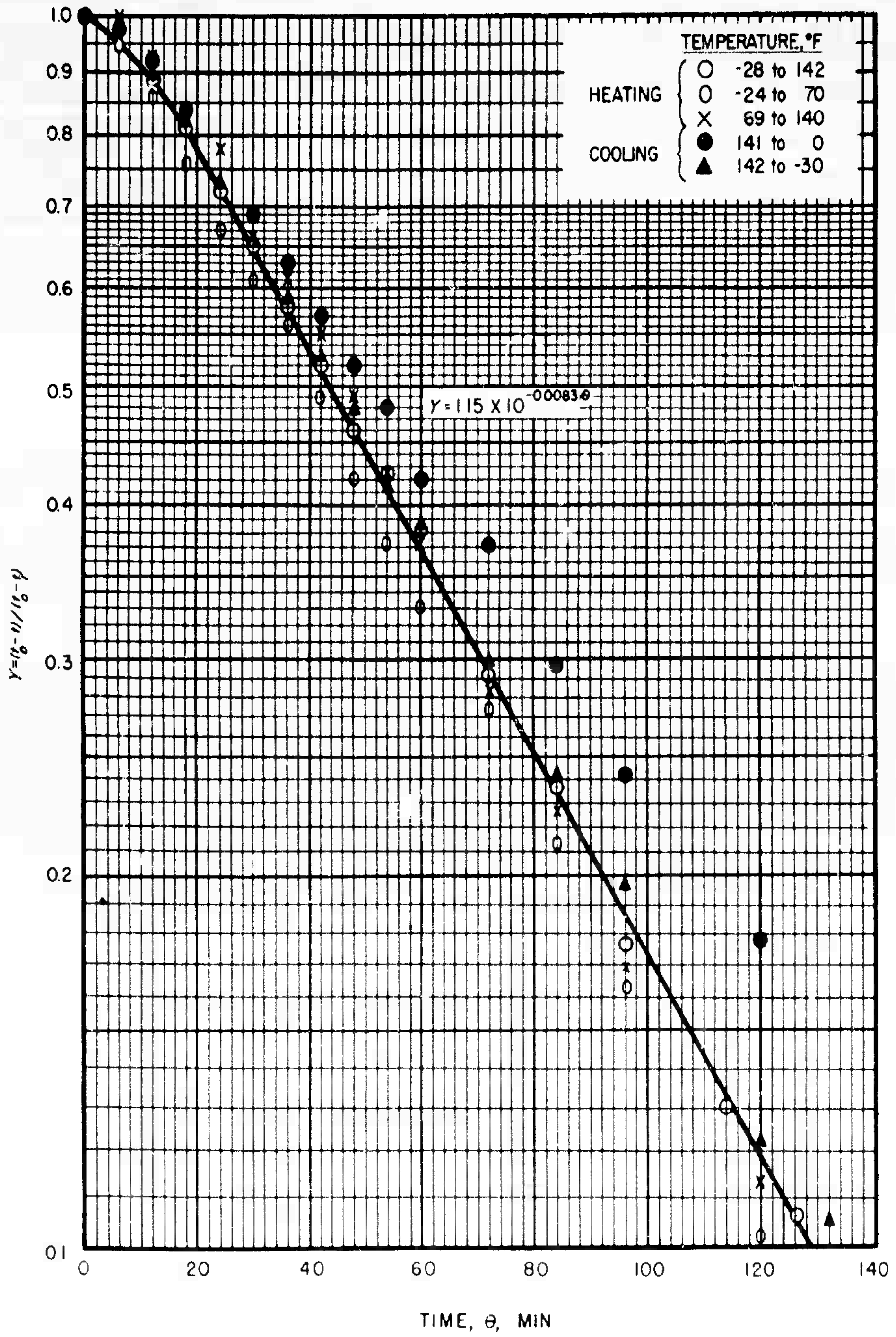

FIG. 5. Temperature History of 2.75-in. Motor During Heating and Cooling: Thermocouple Position: Inside Web.

TABLE 1. CONSTANTS IN EQ. 11 FOR 2.75-IN. ROCKET MOTOR

Thermocouple			Heating		Cooling	
Number	Location	r, in.	c	a, min^{-1}	c	a, min^{-1}
2	outside metal	1.38	0.48*	0.0083	0.53	0.0083
5 & 10	under inhibitor	1.23	0.83	0.0083	0.83	0.0083
6 & 9	inside web	0.60	1.15	0.0083	1.15	0.0083

*Average value for -28 to 140°F and 69 to 140°F. Data for -29 to 70°F gave $c = 0.32$.

TABLE 2. VALUES OF a AND b AS FUNCTIONS OF m

Rocket motor	h	m	a, min^{-1}	b	$1/b$
2.0-in.	4.08	0.320	0.017	2.77	0.361
2.75-in.	3.60	0.264	0.0083	2.67	0.393
HPAG	2.83	0.185	0.0037	3.77	0.265
Weapon A	2.77	0.180	0.0041	4.61	0.217

TABLE 3. VALUES OF c AS A FUNCTION OF n

Rocket motor	r_o, ft	m	In metal			Under inhibitor			Inside web		
			c		n	c		n	c		n
			Heat	Cool		Heat	Cool		Heat	Cool	
2.0-in.	0.0835	0.320	0.34	0.54	1.0	0.84	1.13	0.894	1.06	1.39	0.40
2.75-in.	0.115	0.264	0.48	0.53	1.0	0.83	0.83	0.891	1.15	1.15	0.43
HPAG	0.208	0.185	0.52	0.62	1.0	0.80	0.81	0.921	1.20	1.13	0.52
Weapon A	0.219	0.180	0.31	0.45	1.0	0.50	0.61	0.867	1.20	1.27	0.64

COMPARISON OF ACTUAL RESULTS WITH THEORETICAL CURVES

Since both the theoretical and the experimental results can be represented by straight lines over most of the range, only the slopes and the intercepts of these straight-line portions need to be compared. As discussed previously under "Methods of Plotting Data," Eq. 4 and 11 are identical except for the dimensions of the slopes. It can be shown that for a having the dimensions of min^{-1},

$$b = \frac{60 r_o^2 a}{\alpha} \tag{12}$$

Table 2 contains values of b as converted from a.

The theoretical equation predicts that for a given surface resistivity m, the slope b is independent of the position ratio n. The experimental results of Fig. 3–5 and of Table 1 confirm this behavior.

From the Heisler chart, an empirical equation relating the slopes b and surface resistivities m may be obtained graphically as

$$b = \frac{\Delta \log Y}{\Delta X} = \frac{1}{1.1m + 0.4} \tag{13}$$

The reciprocal of the slope is plotted versus m in Fig. 6 so that a straight line may be obtained. The experimental values of the slope are also shown on this figure.

Equation 7 indicates that the intercept for any particular value of m is dependent only on the radial distance from the center of the cylinder, or more explicitly, c is a function of the position ratio n. Figure 7 compares the experimental intercepts c for the 2.75-in. motor with the c of the corresponding theoretical solid cylinder by plotting these as a function of n. Similar curves, for the 2.0-in., HPAG, and Weapon A motors, are found in Appendix C.

Examination of Fig. 6 reveals that for a given m the reciprocal of the slope ($1/b$) in the theoretical relation is greater than the actual value by an approximately constant value, as

$$\frac{1}{b_T} - \frac{1}{b_A} \simeq 0.38 \tag{14}$$

where

b_T = the theoretical slope for a given m

b_A = the actual slope for the same m

Since $1/b$ is the measure of time required for the temperature to change a given fraction of the initial temperature differential, Eq. 14 indicates that it takes a longer time to heat or cool a solid cylinder of propellant to a given temperature than it does a rocket motor. The reason for the discrepancy appears to be attributable to the original assumption that the average thermal conductivity, density, and specific heat of the loaded motor are equal to those of the H-9 powder.

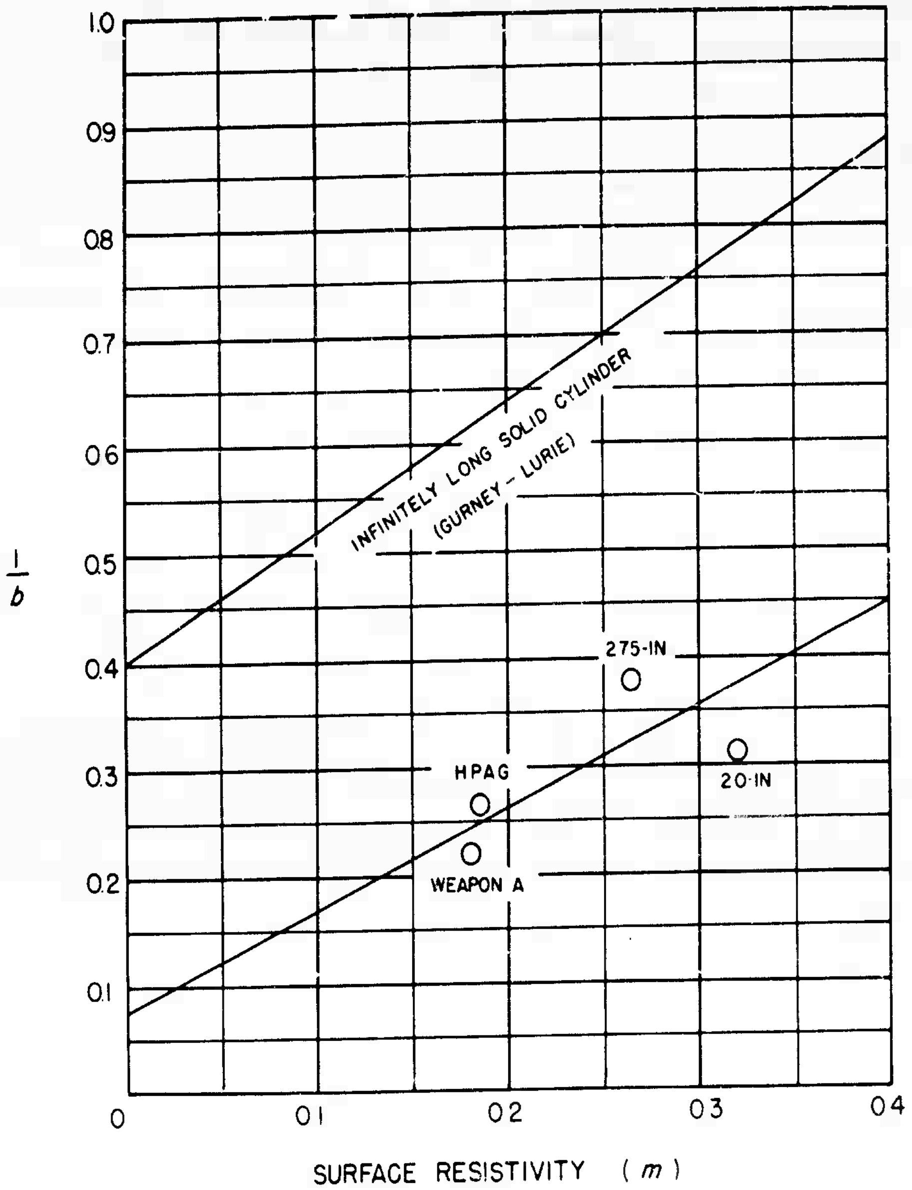

FIG.6. Comparison of Actual With Theoretical Slope.

It is noted in the section on "Experimental Procedure" and in Fig. 1 that the thermocouples are inserted 2 in. or less from one end of the grain. Therefore, the actual experiment did not correspond to the conditions for an infinite cylinder. Some heat transferred longitudinally from the end may have affected the temperature distribution at this point.

Depending on the use to which the heat-transfer data will be put, the comparison of the rocket motors with an infinite, homogeneous, solid cylinder may or may not be seriously in error. If only the time to reach a uniform temperature is required, the use of the charts for solid cylinders will be on the safe side because the time calculated is longer than necessary (see Eq. 14). If the actual temperature distribution in the rocket motor at the time of firing is desired, appreciable error may result.

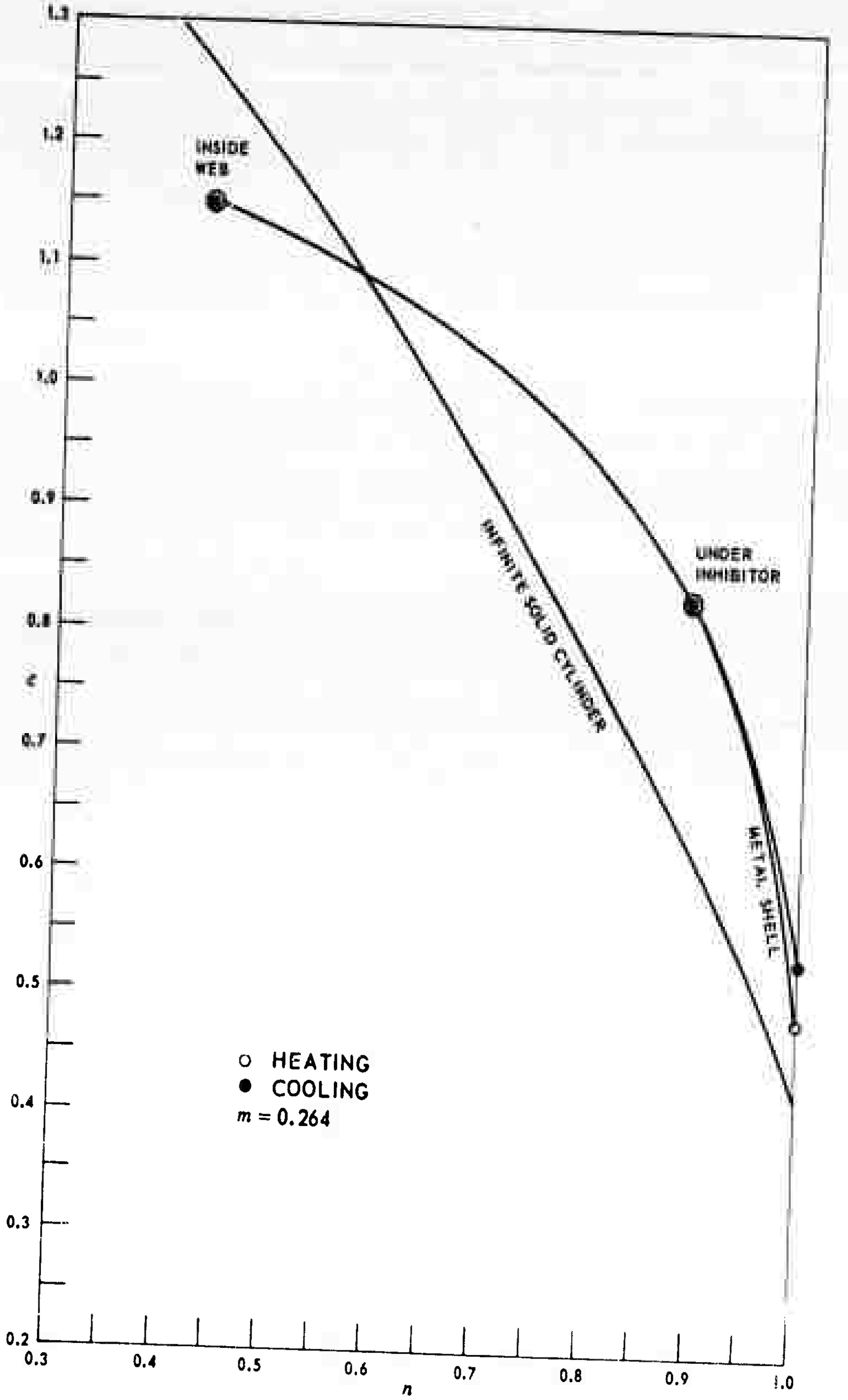

FIG. 7. Comparison of Actual With Theoretical Intercepts for 2.75-in. Rocket Motor.

APPLICATIONS

ILLUSTRATIVE PROBLEMS

Consideration of several illustrative problems will aid the reader in applying the graphs and equations.

Problem 1. A 2.75-in. rocket is removed suddenly from a conditioning oven after having been brought to an equilibrium temperature of −32°F and is transported to the range for firing. The outside temperature is 98°F, and the time required between the removal of the rocket and the closing of the firing switch is 1 hr 20 min. Assuming the heat-transfer rates to be about those of the ovens studied, what is the temperature of a point 3/4 in. from the outside surface at the time of firing?

Solution. A point 3/4 in. from the outside surface would be inside the web, hence Fig. 5 could be used for this case as a good approximation.

$$Y = \frac{t_o - t}{t_o - t_i} = \frac{98 - t}{98 - (-32)} = \frac{98 - t}{130}$$

$$\theta = 80 \text{ min}$$

From Fig. 5 the point on the Y-axis corresponding to 80 min is 0.25. Hence

$$Y = 0.25 = \frac{98 - t}{130}$$

$$t = 65.3\,^\circ\text{F}$$

The temperature may also be obtained by applying Eq. 11 as follows: From Tables 1 and 3 under 2.75-in. rocket motor may be found

$$a = 0.0083$$

$$c = 1.15$$

Substituting these constants in Eq. 11

$$Y = 1.15 \times 10^{-0.0083 \times 80}$$

$$= \frac{1.15}{10^{0.664}} = \frac{1.15}{4.61}$$

$$= 0.25$$

$$\frac{98 - t}{130} = 0.25$$

$$t = 65.5\,^\circ\text{F}$$

Problem 2. A new 6.5-in. rocket is to be tested on the range. It is desired to know the temperature-time relationship inside the rocket for fire control, but time does not permit an extensive experimental evaluation of the constants for the rocket. A simple experiment, in which only one thermocouple is tacked onto the motor wall, is performed, and an equation is obtained for the temperature history of the metal

$$Y = 1.71 \times 10^{-0.00126\theta} \tag{15}$$

The grain, made of H-9 powder, has an outside diameter of 6.13 in. The problem is to find the constant c for the point under the inhibitor and for a point 2.0 in. from the axis of the grain.

Solution. Applicable physical constants of H-9 powder are

$$k = 0.109 \frac{\text{Btu}}{(\text{hr})(\text{ft})(^\circ\text{F})}$$

$$C_p = 0.4 \frac{\text{Btu}}{(\text{lb})(^\circ\text{F})}$$

$$\rho = 101.2 \frac{\text{lb}}{\text{cu ft}}$$

$$r_o = \frac{6.5}{12 \times 2} = 0.27 \text{ ft}$$

Since $a = 0.00126 \text{ min}^{-1}$, from Eq. 15, and since $r_o = 0.27$ ft, then, using the H-9 constants,

$$\begin{aligned} b &= 60\rho C_p r^2 a/k \\ &= 60 \times 101.2 \times 0.4 \times (0.27)^2(0.00126/0.109) \\ &= 2.07 \end{aligned}$$

or

$$1/b = 0.482$$

From Fig. 6, using the Gurney-Lurie line, the value of m corresponding to this value of $1/b$ is 0.0685.

To find the values of c at various positions (n), Fig. 8 was constructed by using Eq. 7 and the values of R_1 from Ref. 1. From this figure at $m = 0.0685$, the following n and c values are obtained:

n	c
1.0	0.117
0.8	0.528
0.6	0.935
0.4	1.275
0.2	1.507

Figure 9 is plotted from the values above, and the c-value at any radial distance in the rocket may then be determined. At the point under the inhibitor, $r = 6.13/2$ (outside radius of the grain), $n = 0.943$, and $c = 0.232$. In the same manner, c for the point 2.0 in. from the axis ($n = 0.615$) is found to be 0.905. The value of a for all points is 0.00126, as determined experimentally in Eq. 15. The above values suffice to establish the temperature-time relationship.

PREDICTION FOR OTHER CONDITIONS

It is possible to give a value for the constant c for various other conditions than those in Problem 2, such as various air velocities, heating times, grain compositions, and rocket sizes. In Problem 2, the constant c for the desired

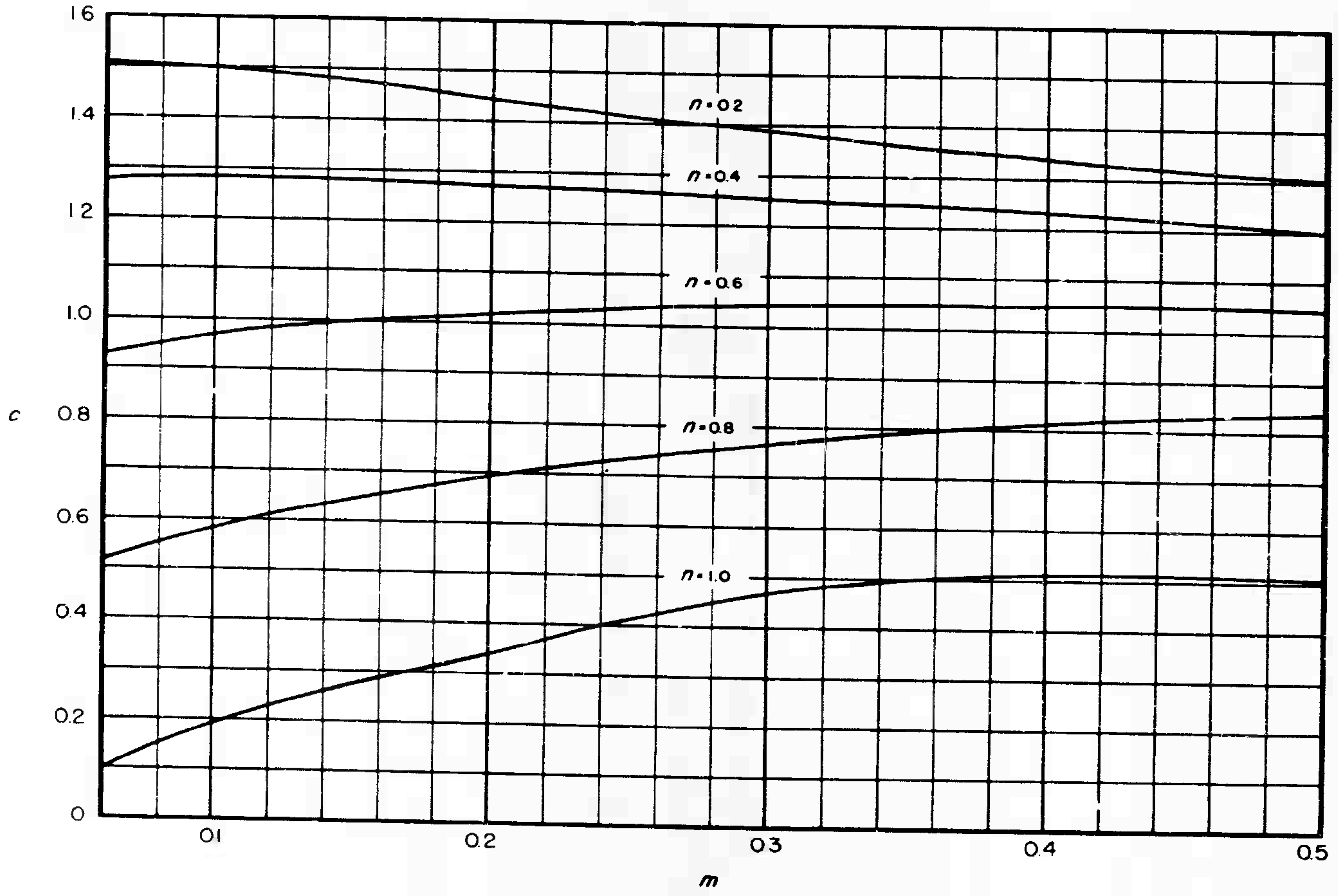

FIG. 8. Chart for Determining Intercepts With Respect to Position Ratios.

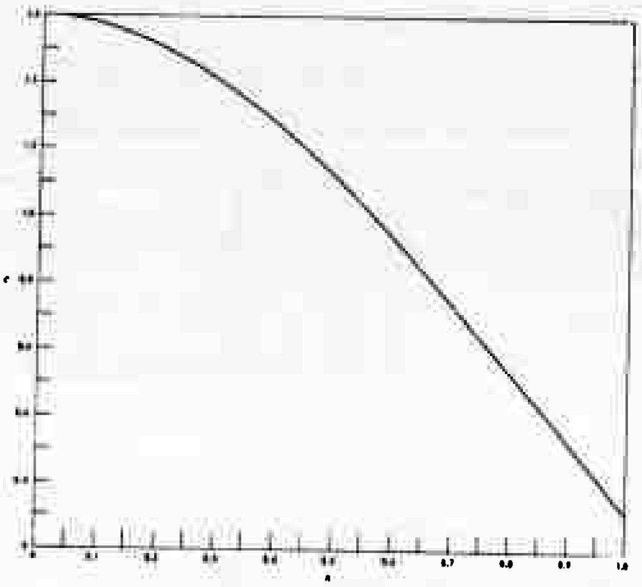

FIG. 9. Intercept vs. Position Ratio for a Hypothetical Rocket (Problem 2). Outside diameter 6.5-in.; m = 0.0684.

points was obtained from Fig. 6 and 9; this infers that if m (or k/hr_o) is known, the time-temperature relation at any point may be determined. At the present time, insufficient experimental data are available for complete evaluation of c as a function of n; however, the theoretical relation derived from Eq. 7, or Gurney-Lurie or Heisler charts, will give a rough approximation.

IMPORTANCE OF FILM COEFFICIENT

The m used in Table 2 was based on the surface heat-transfer coefficient (or film coefficient) h derived in Appendix A. Examination of the data in Table 4 (Appendix A) shows that the experimental h varies from 1.03 to 6.44 Btu/(hr)(sq ft)(°F), whereas the other two properties, k and r_o, required to determine m were quite accurately determined.

In Appendix A, Eq. 18 and the discussion clearly show that h determined under one set of oven conditions is by no means a constant for all uses of that oven. The air velocity, the orientation of the rocket in the oven, and alterations in the air duct affect the magnitude of h. It is, therefore, imperative that h be determined separately for each type of use.

LIMITATIONS OF THE EQUATIONS

Equations 4 and 11, when used with the appropriate constants from Tables 1 and 2, may be applied to motors of similar size, composition, and grain configuration. For rocket motors which depart too far in these respects from the models mentioned, a new set of constants will have to be evaluated from experimental data. It must also be borne in mind that in the derivation of these equations certain simplifying assumptions were made. The conditions assumed necessarily impose certain limitations on the application of the constants to Eq. 4 and 11. For example, the equations cannot be used when the rocket is exposed to several different temperatures in a short time; nor do they apply to short periods of heating and cooling.

Although no rigid calculation was made as to probable error, a 30 percent error in temperature or time is possible.

SUMMARY

The heating and cooling data of the 2.0-in., 2.75-in., HPAG, and Weapon A rocket motors have been analyzed and are shown to correlate well with the equation

$$Y = c \cdot 10^{-a\theta}$$

Constants of the equation have been evaluated for all of the motors. They are of the same order of magnitude as those derived from the Gurney-Lurie type of charts. The accurate prediction of temperature-time behavior in rocket motors, however, requires definite knowledge of the surface film coefficient.

RECOMMENDATIONS

The importance of the surface film coefficient h and the factors that influence its value have been discussed, and it has been mentioned in Appendix A that the inconsistency of the values of h as calculated from the data may be due to a lack of uniformity in oven conditions. These observations suggest the necessity for future work to include the following.

1. Separate determination of the film coefficient under controlled and known oven conditions for several available ovens.

2. Design and construction of a test cell for the accurate determination of the heat-transfer coefficient of rocket motors. This could be used to determine values from which the heating and cooling characteristics of rockets can be determined.

3. Construction of one or more special test ovens in which conditions may be precisely controlled. These could produce specific heat-transfer conditions so that studies of the ballistic behavior of rockets in the unsteady state can be accomplished.

It is further recommended that theoretical studies be made on heat transfer to concentric hollow cylinders so as to afford closer prediction of the temperature distributions in the propellant grain than those predicted from a theoretical solid cylinder.

Appendix A

EVALUATION OF HEAT-TRANSFER COEFFICIENT

The heat-transfer coefficient used in this report was based on data contained in the report cited in footnote 1. The following is a résumé of the method by which the coefficient was evaluated.

The heat transferred by convection from the surrounding air to the exterior of the rocket motor is given by the equation

$$dQ/d\theta = q = hA\Delta t \tag{16}$$

Values of Q for the 2.75-in. rocket motor were obtained by graphical integration (footnote 1) over several time intervals of the equation:

$$\text{heat content} = 2\pi l \rho C_p \int_{r_1}^{r_2} [t(r) r] dr \tag{17}$$

Values of h for the heat transferred across the air space from the motor tube to the grain were then calculated. Since the Δt used was a fairly small difference between two temperatures, each subject to errors of observation, the values obtained were inexact. Further, a large number of tests would be required to study the variation of this coefficient due to changes from round to round. For practical use,[5] the external film coefficient, which is obtained from the Δt, is preferable. Since Y in Fig. 3 was obtained for the metal tube, $(Y)\Delta t_i$ will give the desired Δt for the external heat-transfer coefficient. Table 4 summarizes the individual values of $q'/Y\Delta t_i$ or $q'/\Delta t$. The arithmetical average [0.067 (Btu)/(min) (°F)] of the six tests is taken. If this average is divided by the exterior area of the tube, 1.11 sq ft, and multiplied by 60 to convert the time into hours, an average heat-transfer coefficient of 3.6 Btu/(hr)(sq ft)(°F) is obtained.

It should be remembered that the above value of h is calculated only for the 2.75-in. rocket motor under a certain set of oven conditions. The value obtained may be compared to that given in the literature for forced convection by an equation derived from a graph in Ref. 3

$$hD_o/k_f = 0.24(D_o V \rho_a/\mu)^{0.6} \tag{18}$$

which applies for $(D_o V \rho_a/\mu)$ from 1,000 to 50,000.

It is seen from the above equation that h is a function of the diameter of the cylinder as well as the velocity of the fluid (air), the physical properties of the fluid, and the temperature. Since all the rocket motors were tested under similar oven conditions and similar temperature ranges, V, ρ_a, k_f, and μ are the same for

---

[5] Since the heat retained by the metal shell is small compared to the total heat transferred to the grain, the error introduced by using the q values of the report cited in footnote 1 is not appreciable.

TABLE 4. SUMMARY OF DATA FOR HEAT TRANSFER

Temperature range, °F	Time interval selected, min in oven	Midpoint of time interval, min	q', Btu/min	Y*	$q'/Y(t_o - t_i)$
−30 to 70	0–10	5	3.7	0.45	0.082
	10–20	15	2.41	0.37	0.065
	20–40	30	2.35	0.28	0.084
	40–60	50	1.6	0.195	0.082
	60–80	70	1.0	0.13	0.077
Average	....	....	....	....	*0.078*
70 to −30	0–10	5	2.7	0.57	0.047
	10–20	15	1.8	0.49	0.037
	20–30	25	1.8	0.42	0.043
	30–50	40	2.0	0.34	0.059
	50–80	65	1.3	0.23	0.057
Average	....	......	......	....	*0.049*
70 to 130	0–10	5	1.5	0.64	0.039
	10–22	16	1.17	0.50	0.039
	22–30	26	1.25	0.40	0.052
	30–40	35	1.3	0.33	0.065
	40–60	50	1.15	0.24	0.080
Average	....	....	....	....	*0.055*
130 to 70	0–10	5	2.5	0.35	0.119
	10–20	15	1.3	0.32	0.068
	20–30	25	1.7	0.29	0.098
	30–40	35	0.3	0.26	0.019
	40–60	50	0.7	0.23	0.051
Average	....	....	....	....	*0.071*
−30 to 130	0–10	5	3.5	0.53	0.041
	10–20	10	3.51	0.47	0.047
	20–40	30	4.1	0.33	0.078
	40–70	55	2.35	0.20	0.073
Average	....	....	....	....	*0.060*
130 to −30	0–10	5	4.5	0.25	0.112
	10–20	15	3.5	0.23	0.095
	20–40	30	2.45	0.19	0.0665
	40–70	55	2.0	0.15	0.0833
Average	....	....	....	....	*0.0892*
GRAND AV.	....	....	....	....	*0.067*

*Taken from best line through actual data and not from generalized lines.

all cases. The variation of h for each rocket is dependent then on the diameter alone. This makes possible the estimation of the h-values for other motors by multiplying the h-value for the 2.75-in. motor by the conversion factor of $(2.75/12\ D_o)^{0.4}$.

DISCUSSION

The method of obtaining h set forth in the report cited in footnote 1, as modified here, does not give precise values. The film coefficient was evaluated by the graph from which Eq. 18 was obtained (limits of greatest possible variation in the heating oven were taken as 140 to −30°F and 200 to 300 fpm for air velocities) with the following results:

Assumed film temperature, t, °F	*Air velocity, V, fpm*	*Film coefficient, h, Btu/(hr)(sq ft)(°F)*
140	200	2.31
	300	2.96
80	200	2.40
	300	2.96
30	200	2.43
	300	3.07
−30	200	2.52
	300	3.17

The probable value of h within ±20 percent should be 2.80 from this calculation unless undue turbulence exists in the air stream, in which case h would be up to 50 percent greater.

The values calculated from the data in footnote 1 vary from about 1 to 6.5, with an average for one series given as 3.6. The value of 3.6 is probably reasonable, but the variation of h encountered in the evaluation is unexpected and is probably due to the changing of the ovens; 8 ovens were used, all of which had slightly different conditions.

A great deal of confidence cannot therefore be placed in the comparison of the observed with the predicted results. However, when using the data for rockets under similar conditions of heating or cooling, the results can probably be used with reasonable success.

Appendix B

TEMPERATURE HISTORY (Y VERSUS θ) DURING HEATING AND COOLING

(FIG. 10–24)

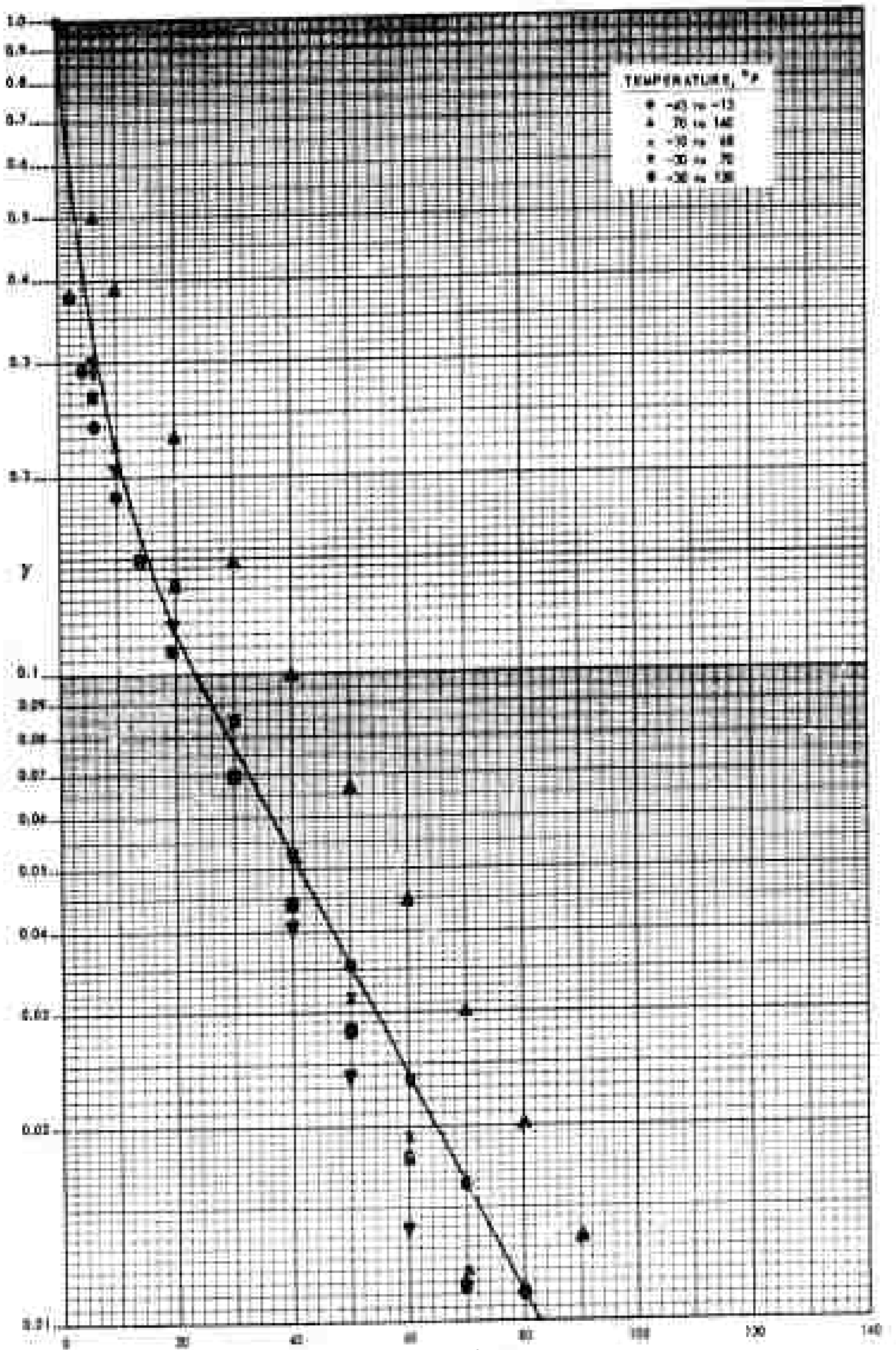

FIG. 10. 2.0-in. Motor During Heating; Thermocouple Position: Metal Shell.

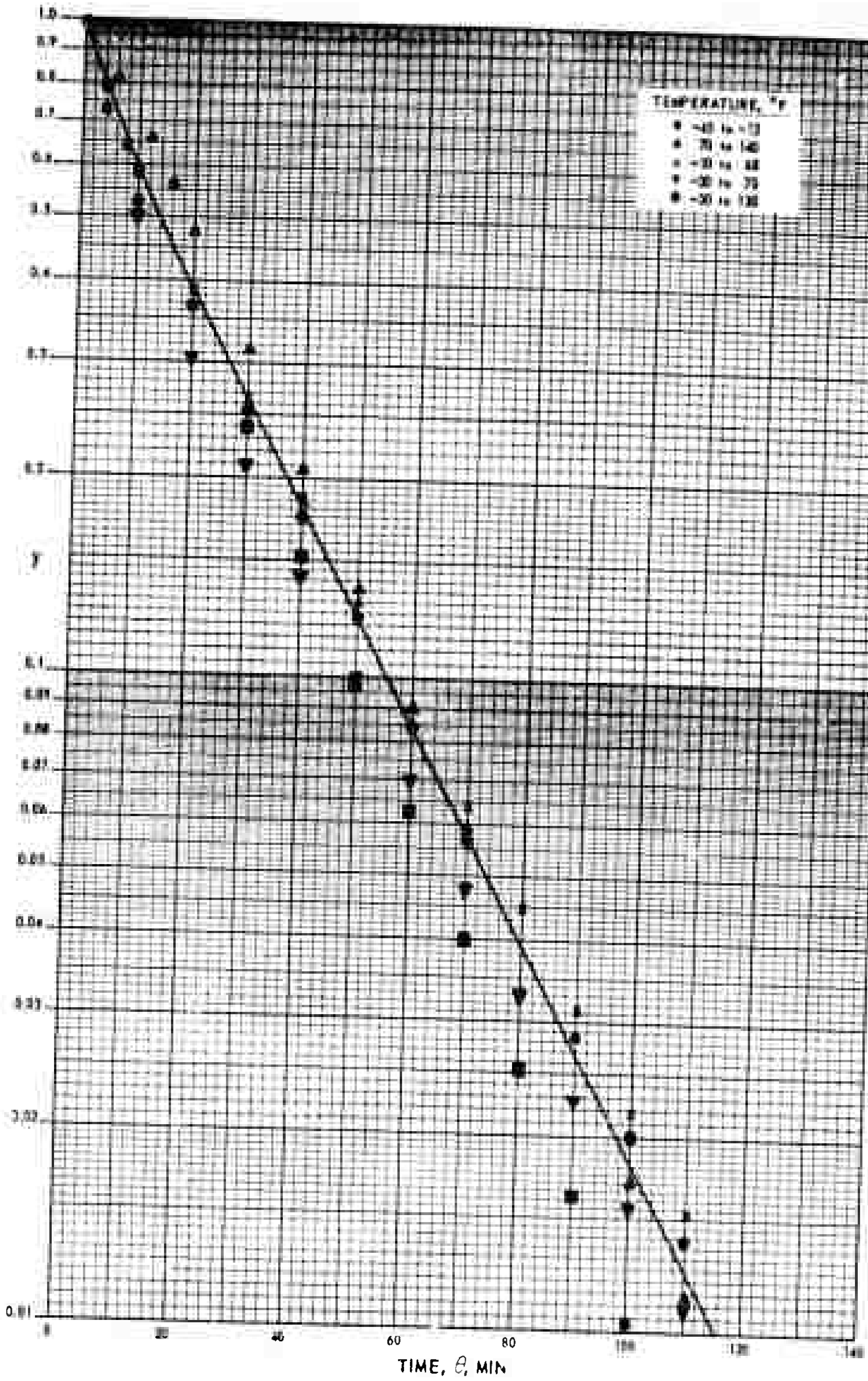

FIG. 11. 2.0-in. Motor During Heating; Thermocouple Position: Under Inhibitor.

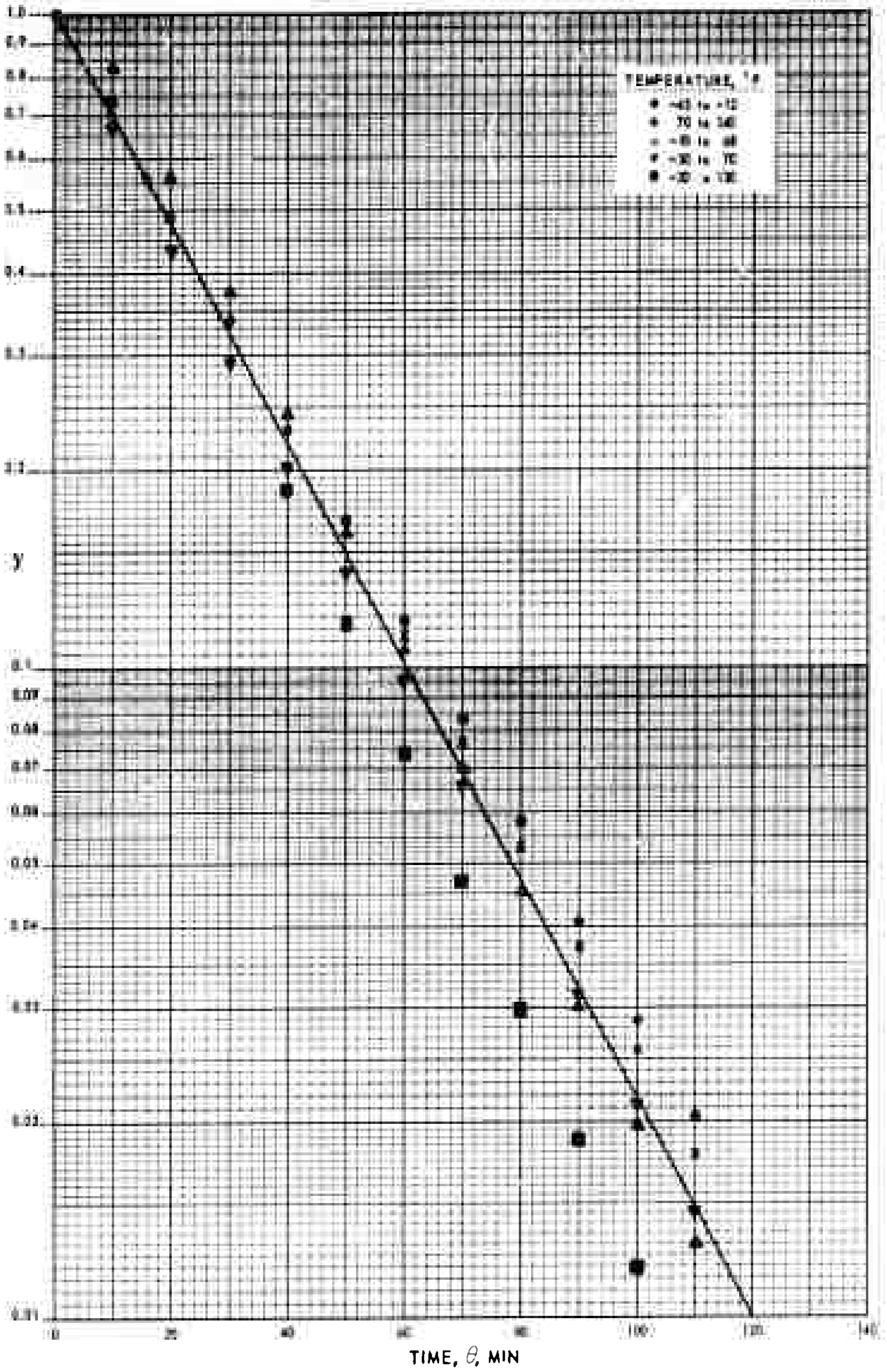

FIG. 12. 2.0-in. Motor During Heating; Thermocouple Position: Inside Web.

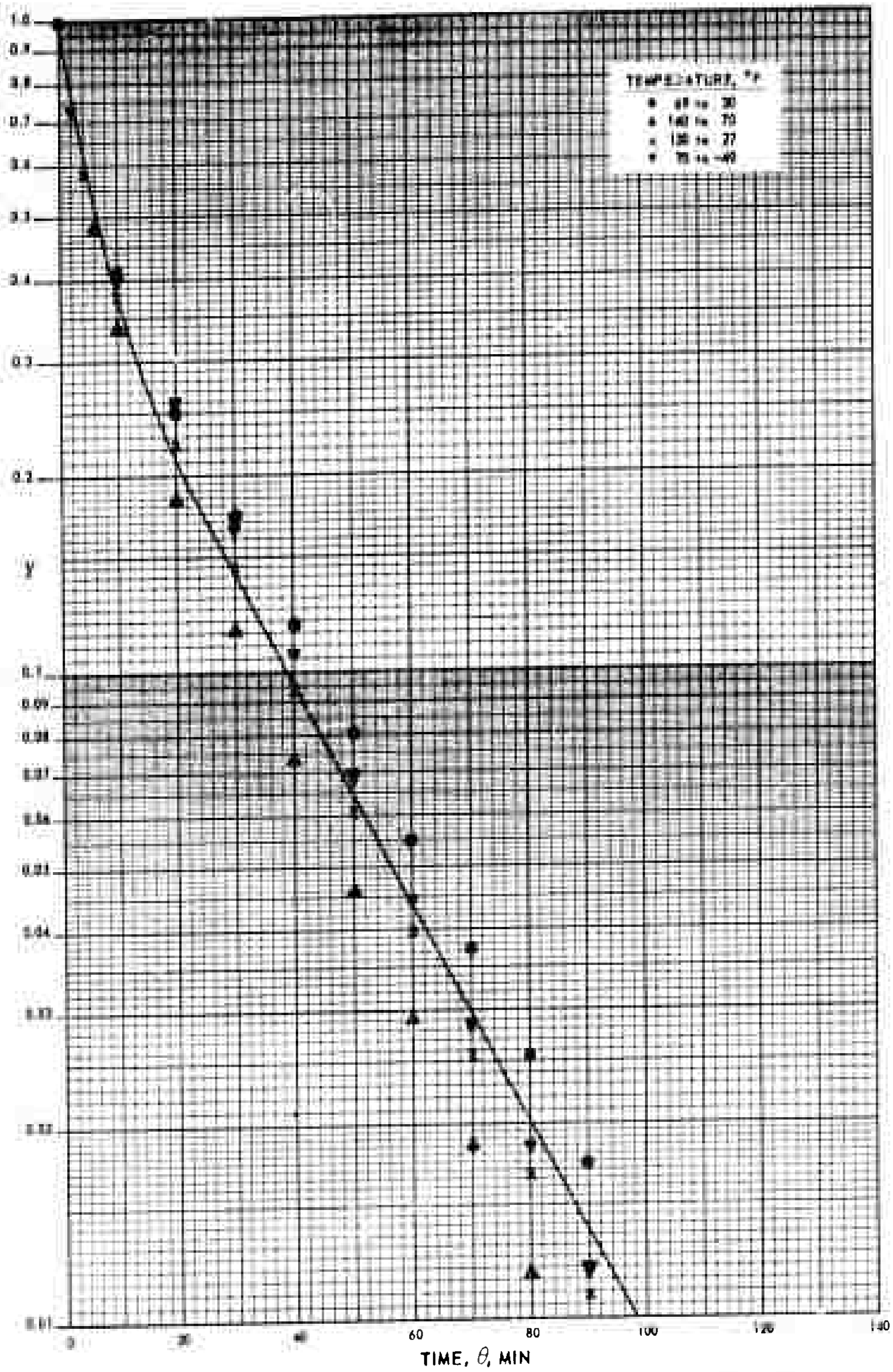

FIG. 13. 2.0-in. Motor During Cooling; Thermocouple Position: Metal Shell.

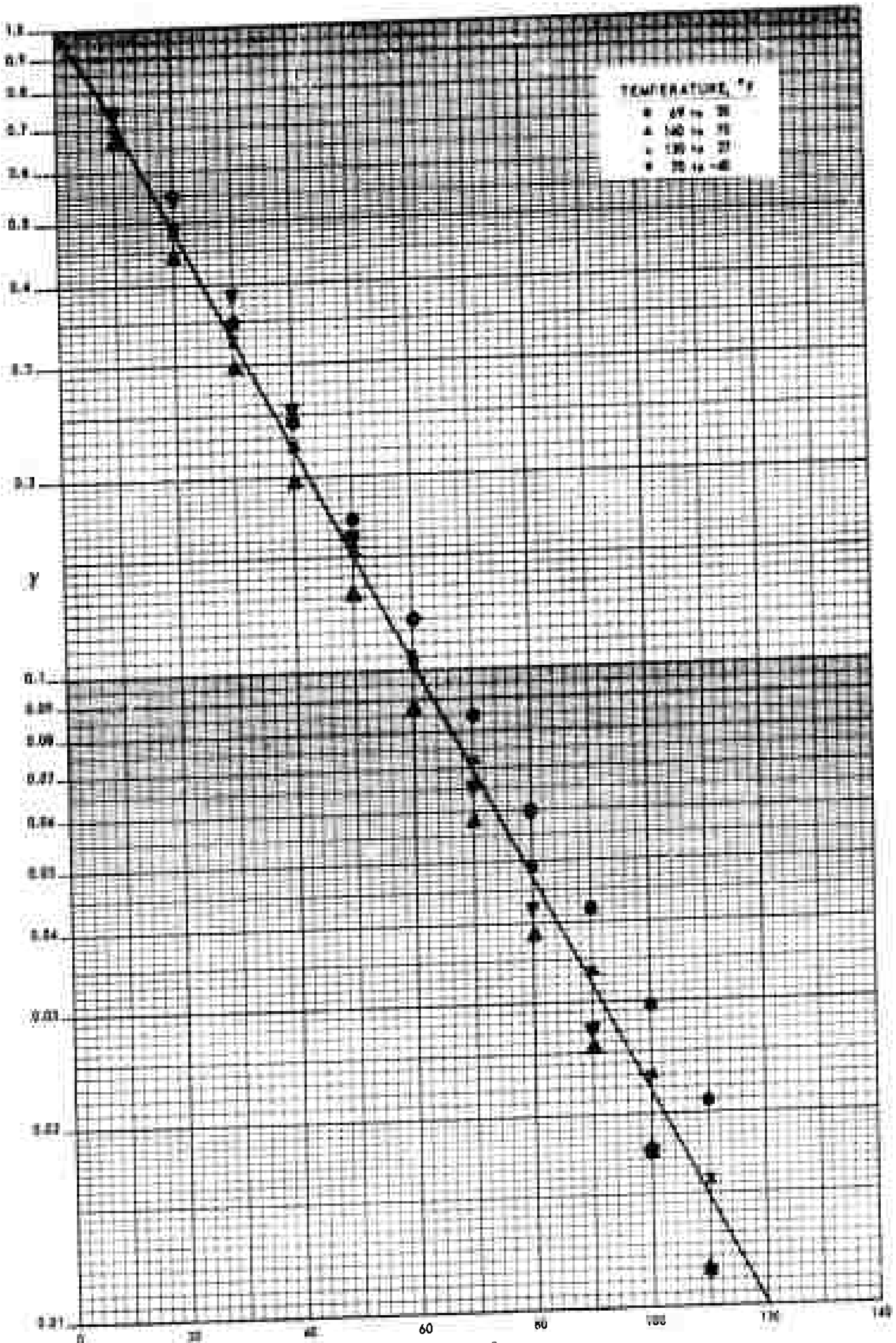

FIG.14. 2.0-in. Motor During Cooling; Thermocouple Position: Under Inhibitor.

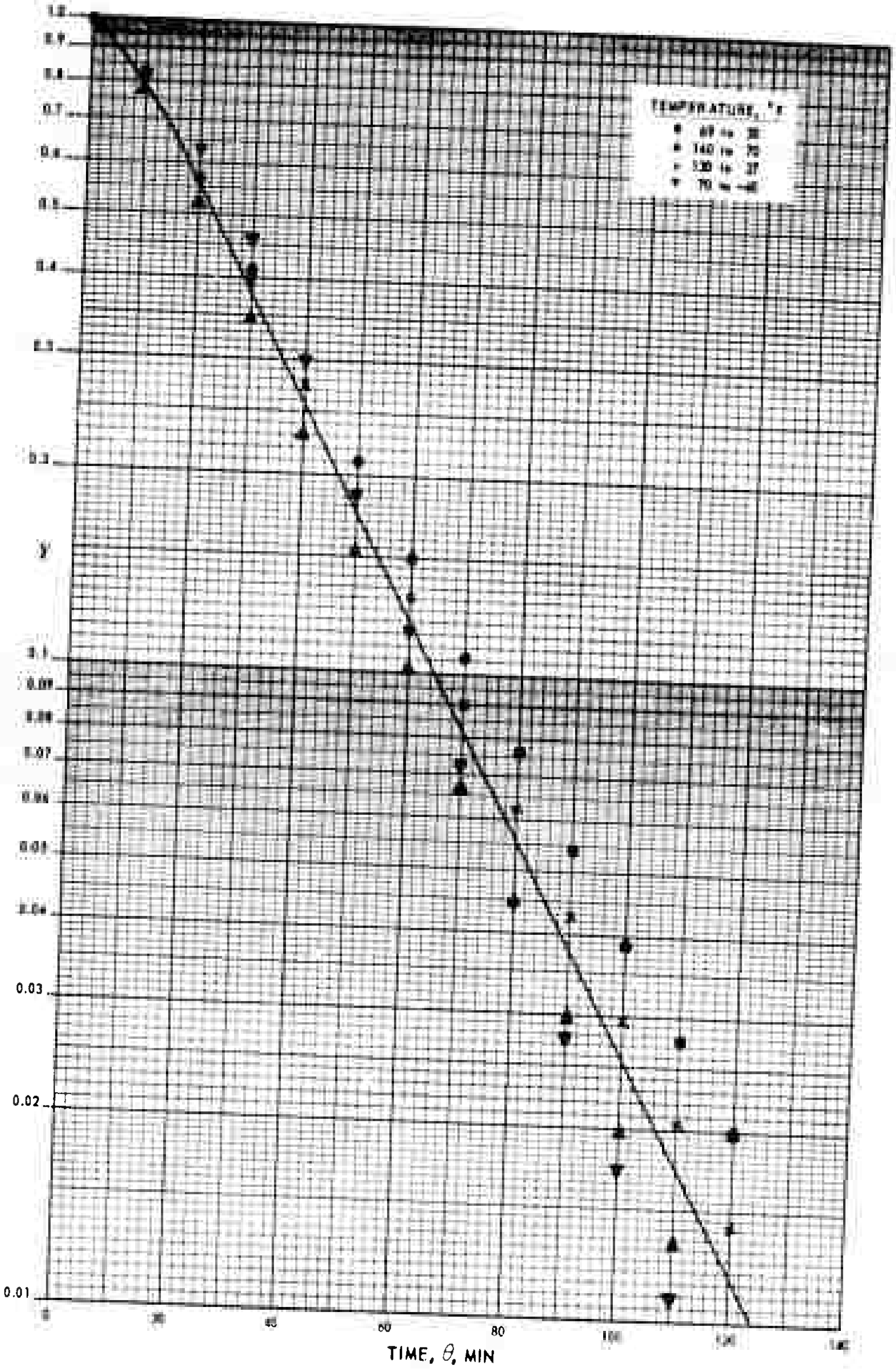

FIG. 15. 2.0-in. Motor During Cooling; Thermocouple Position: Inside Web.

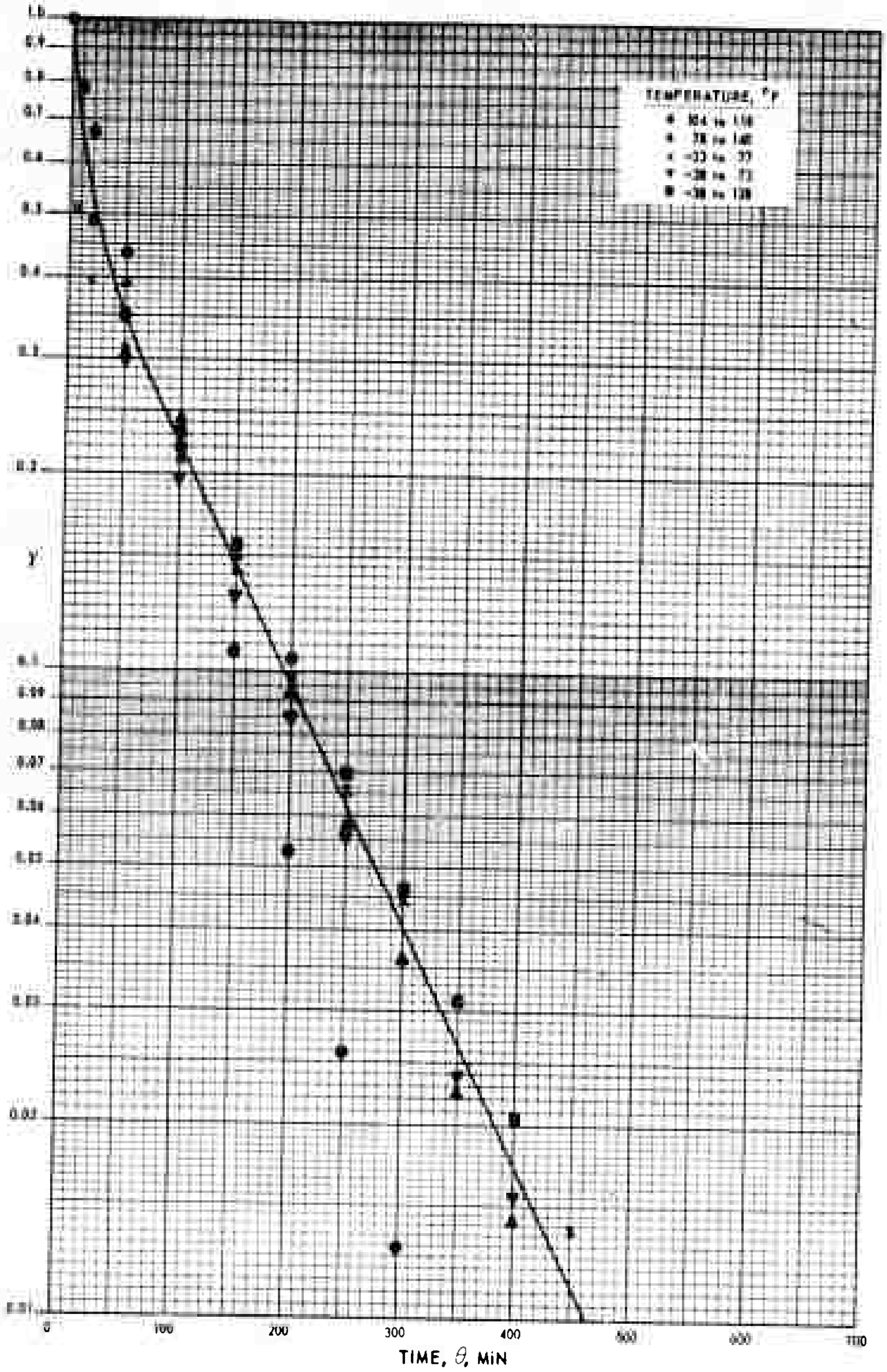

FIG. 16. HPAG Motor During Heating; Thermocouple Position: Metal Shell.

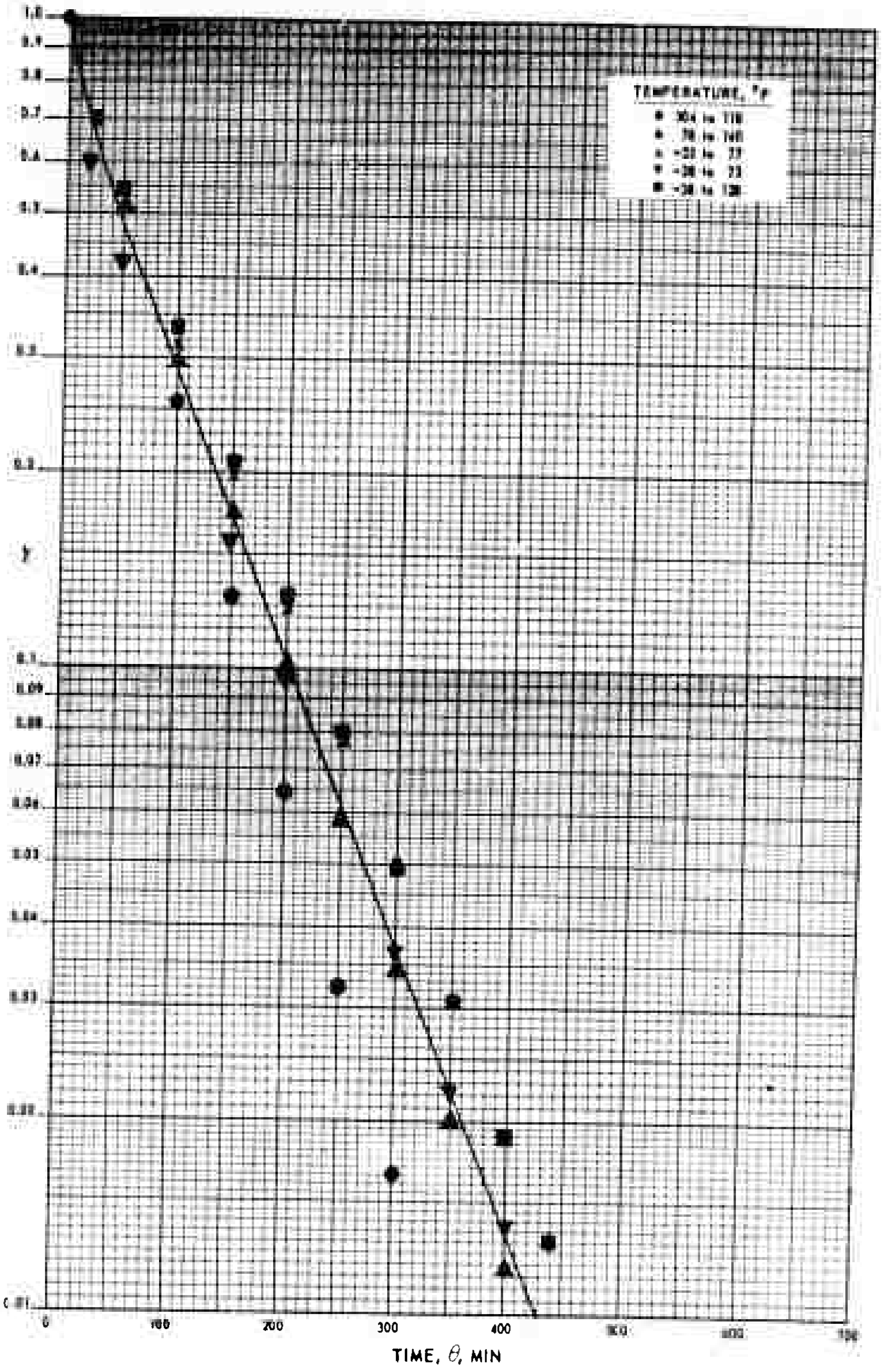

FIG. 17. HPAG Motor During Heating; Thermocouple Position: Under Inhibitor.

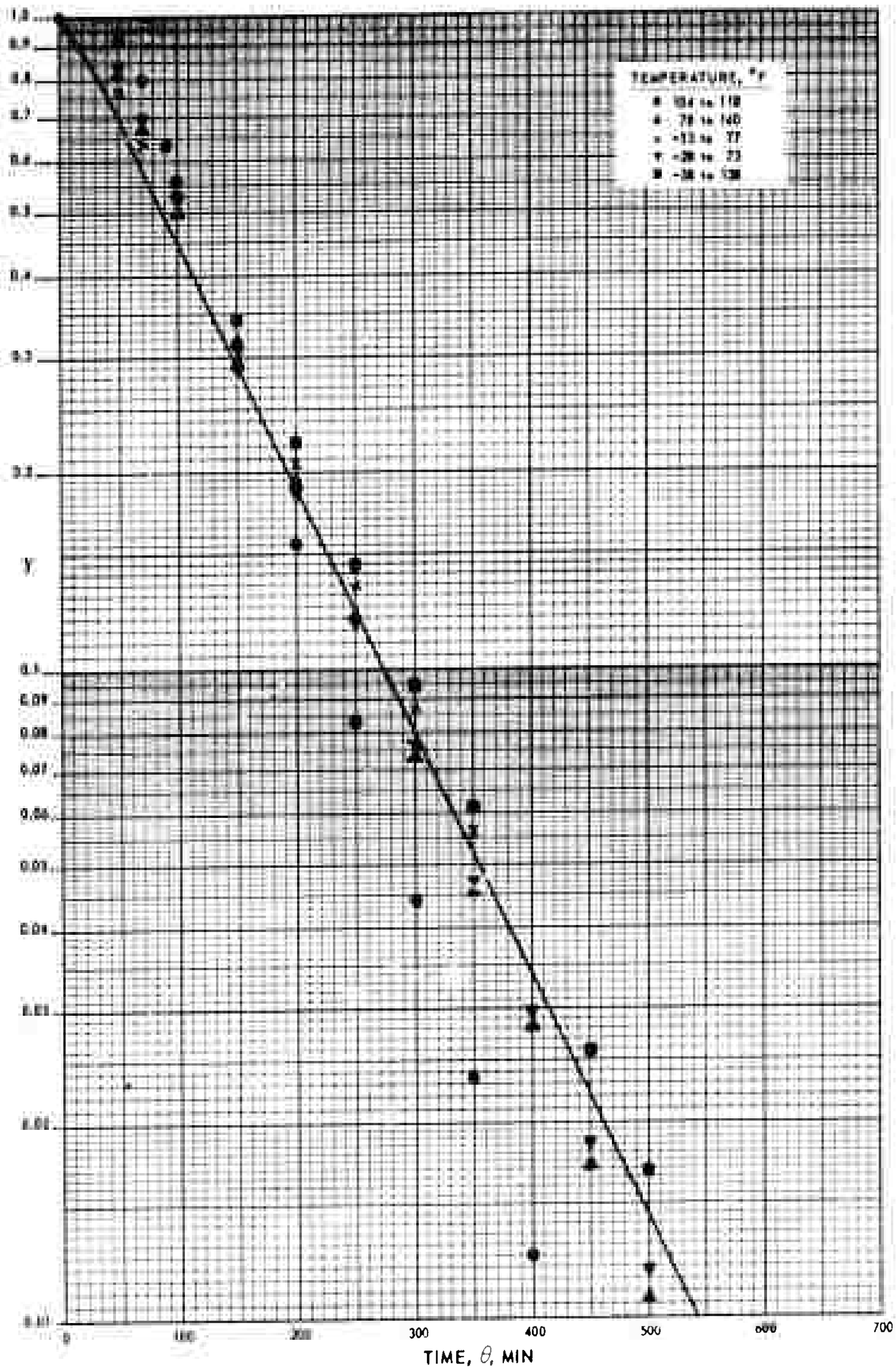

FIG. 18. HPAG Motor During Heating; Thermocouple Position: Inside Web.

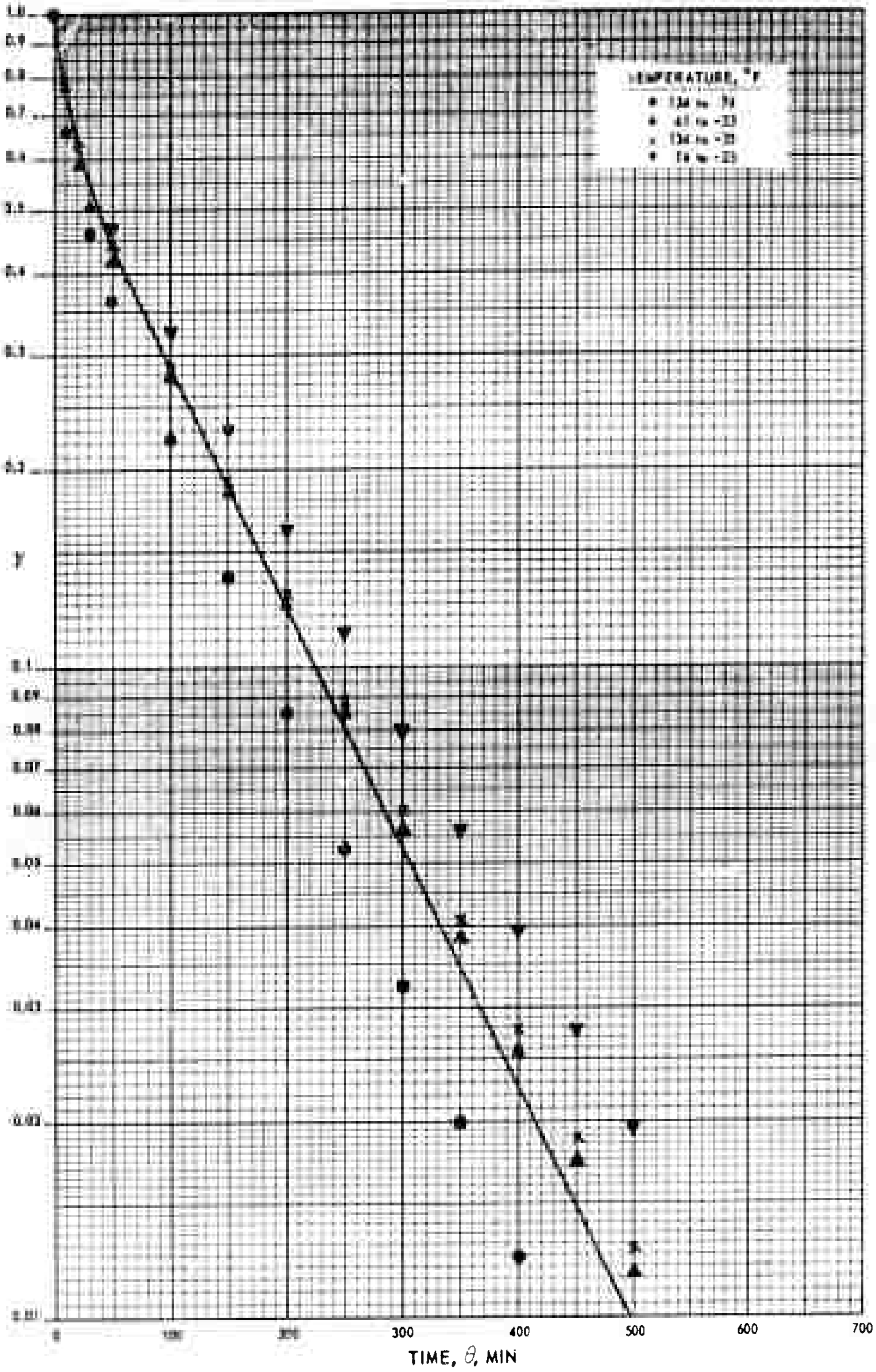

FIG. 19. HPAG Motor During Cooling; Thermocouple Position: Metal Shell.

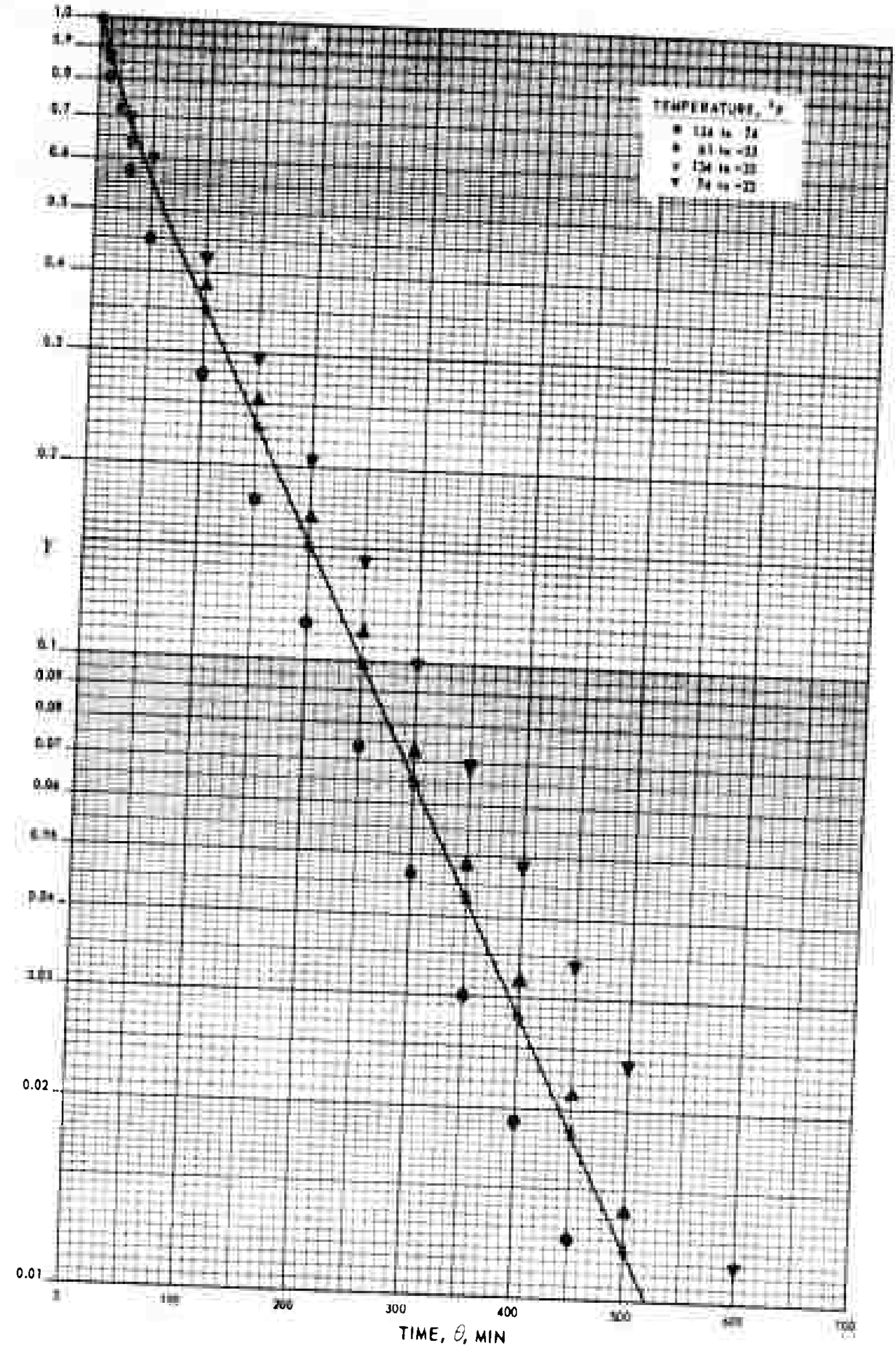

FIG. 20. HPAG Motor During Cooling; Thermocouple Position: Under Inhibitor.

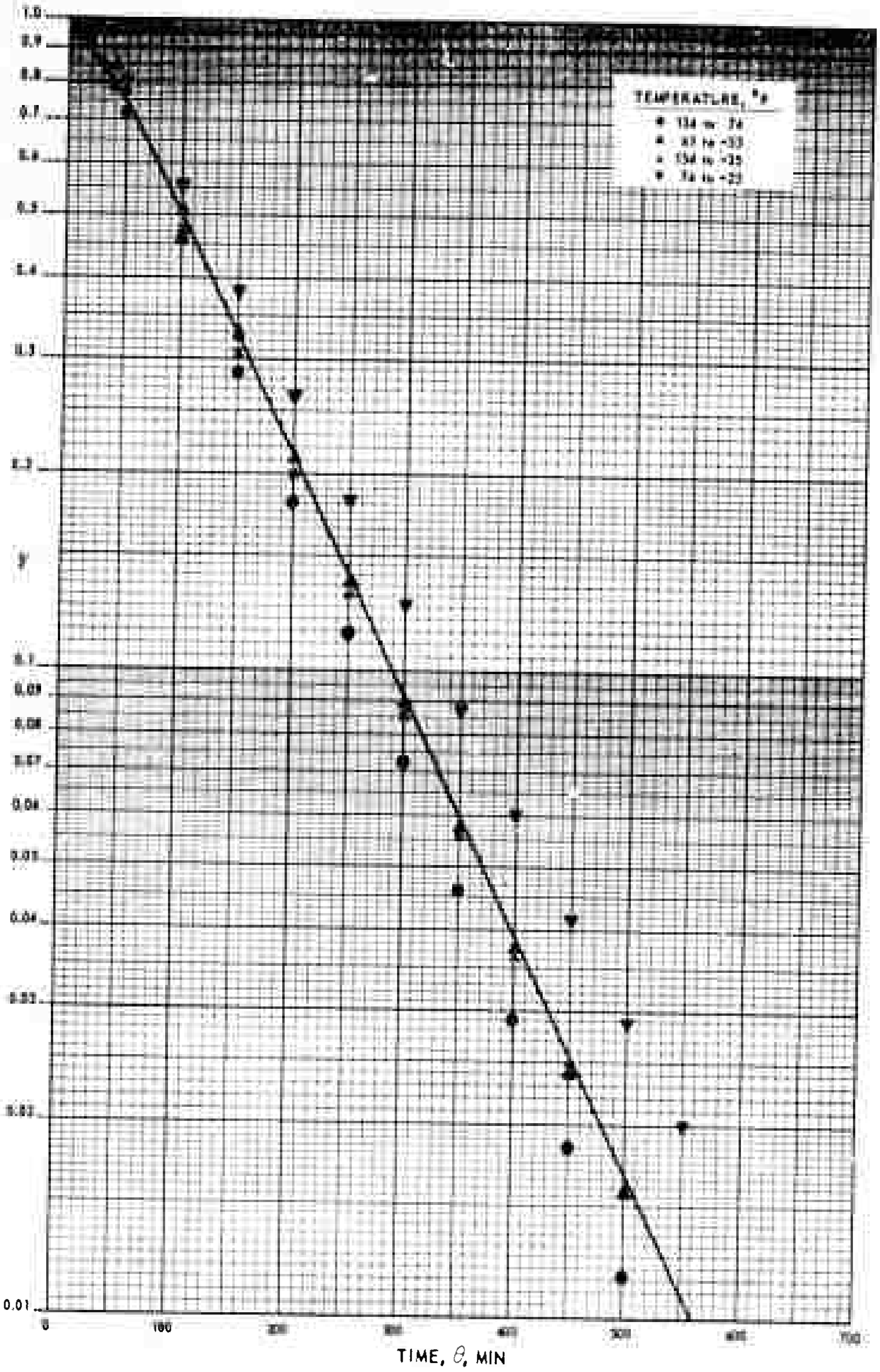

FIG. 21. HPAG Motor During Cooling; Thermocouple Position: Inside Web.

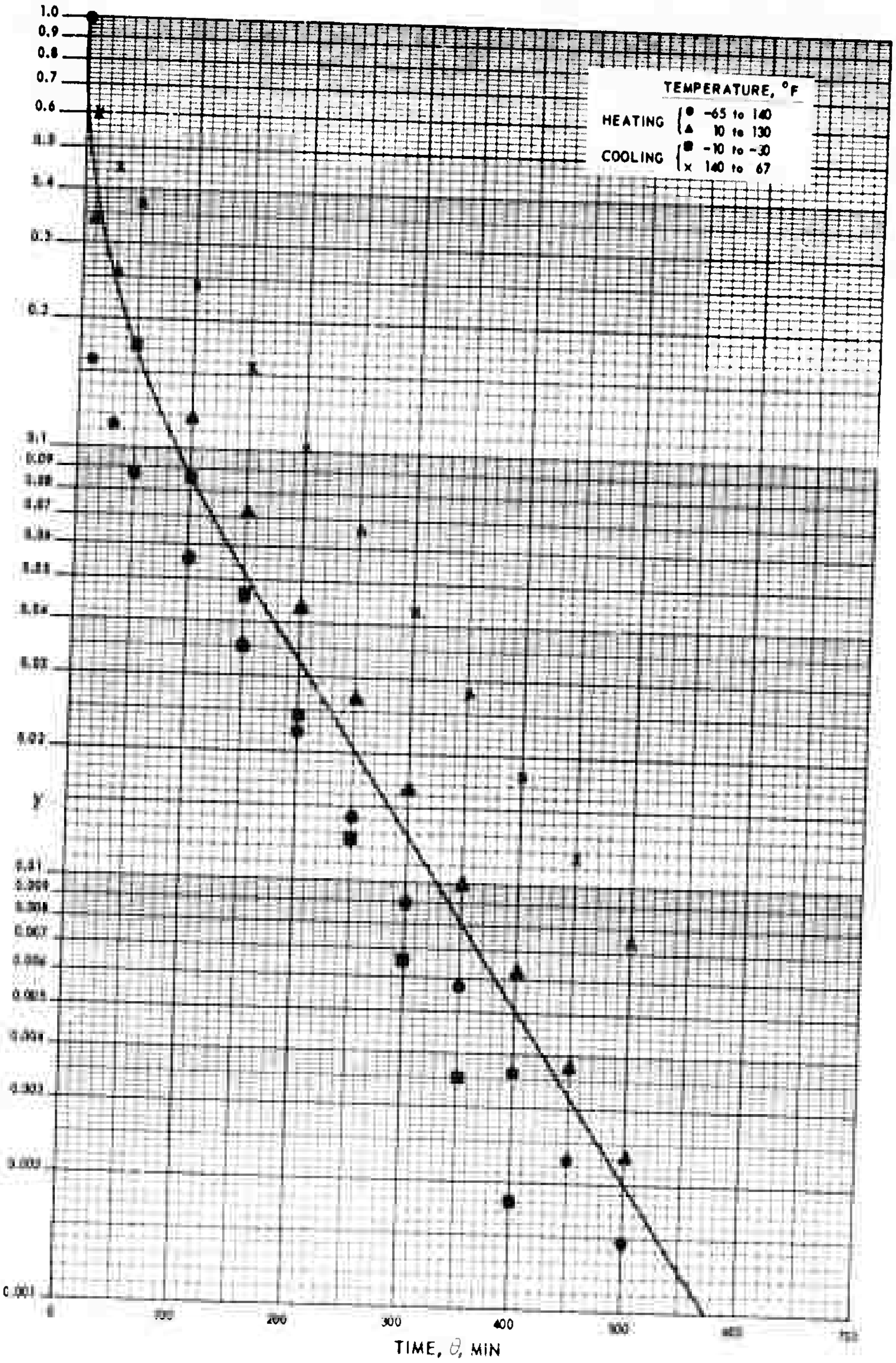

FIG. 22. Weapon A Motor During Heating and Cooling; Thermocouple Position: Metal Shell.

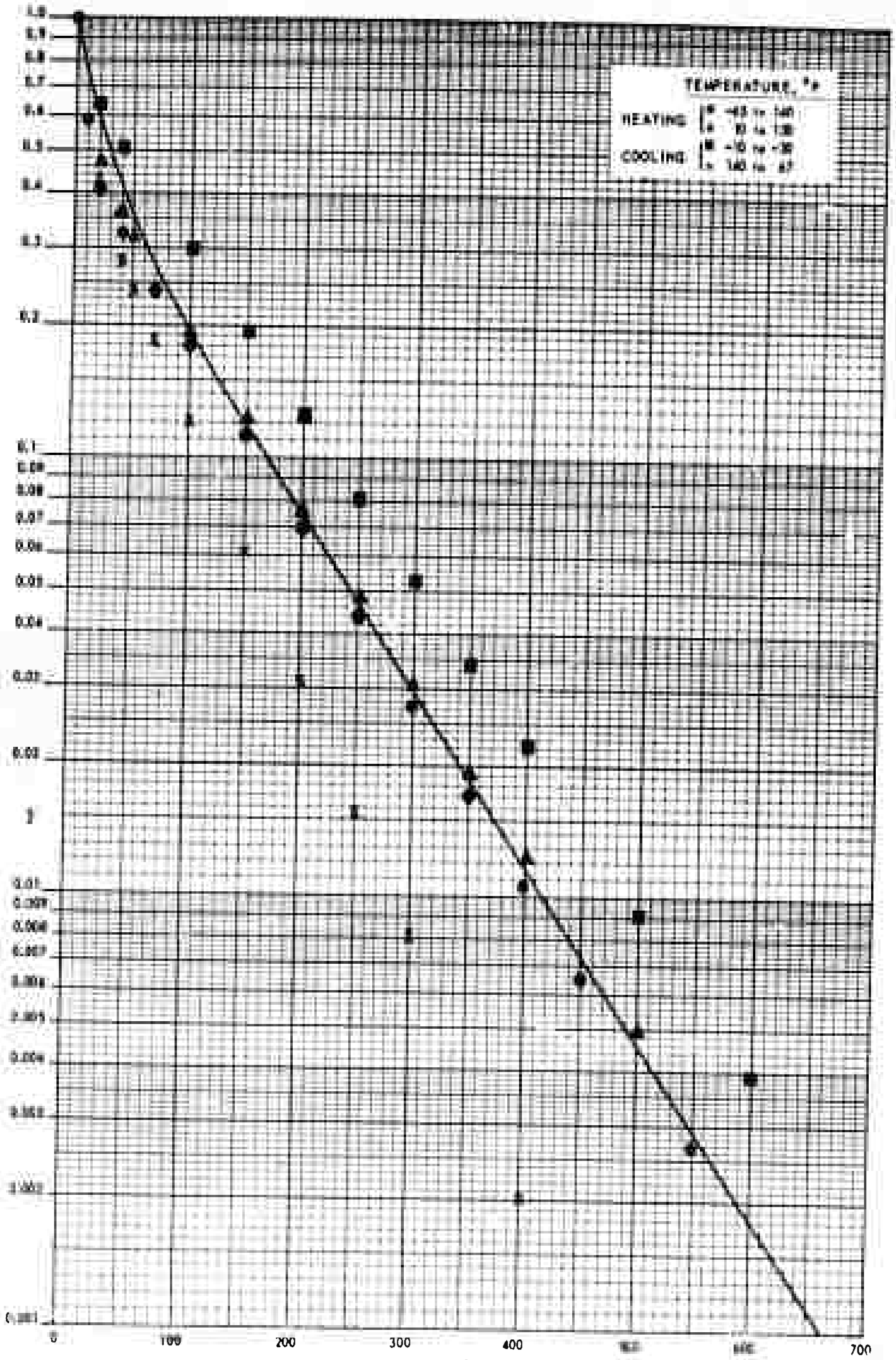

FIG. 23. Weapon A Motor During Heating and Cooling; Thermocouple Position: Under Inhibitor.

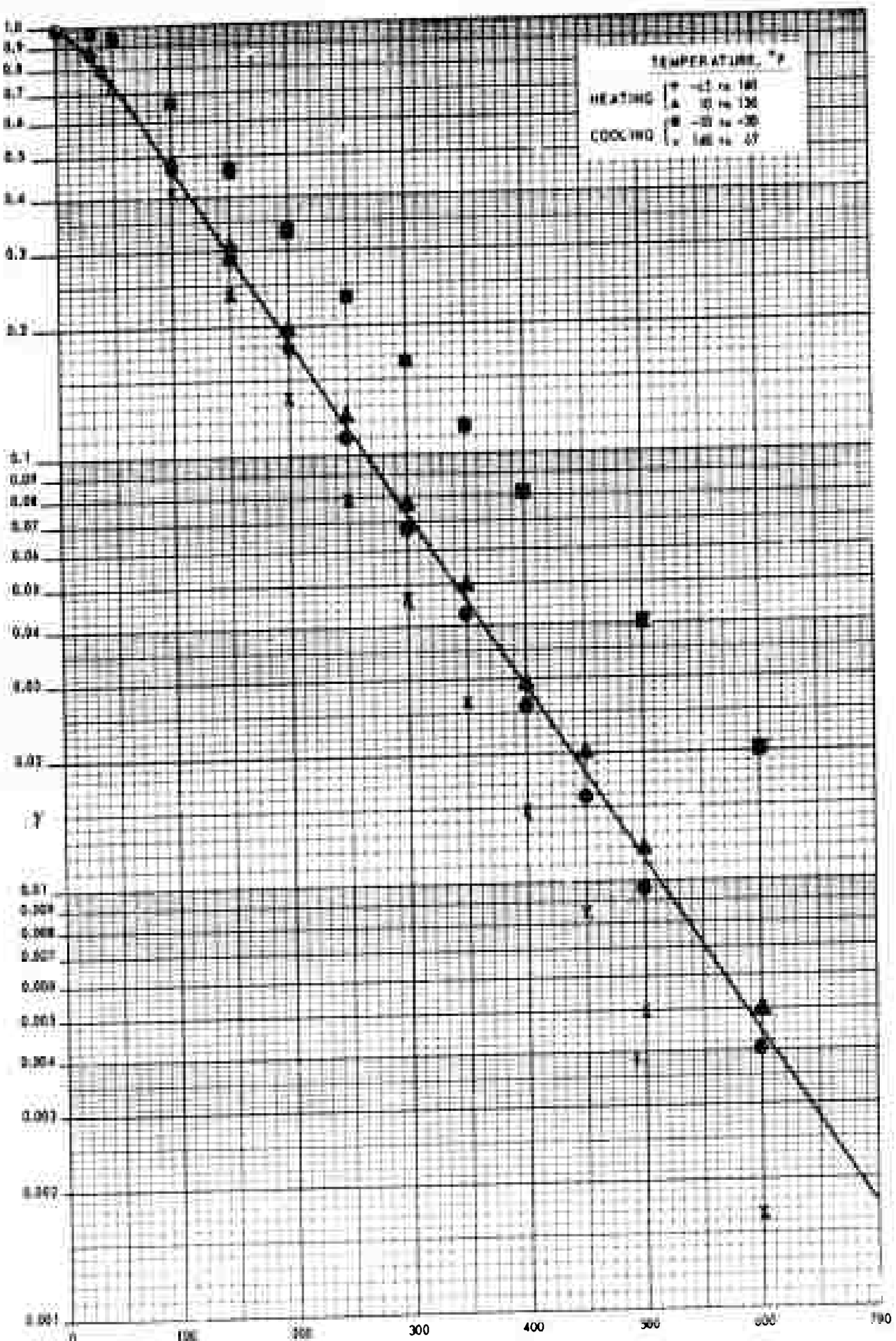

FIG. 24. Weapon A Motor During Heating and Cooling; Thermocouple Position: Inside Web.

Appendix C

INTERCEPT VERSUS POSITION DATA

(FIG. 25–27)

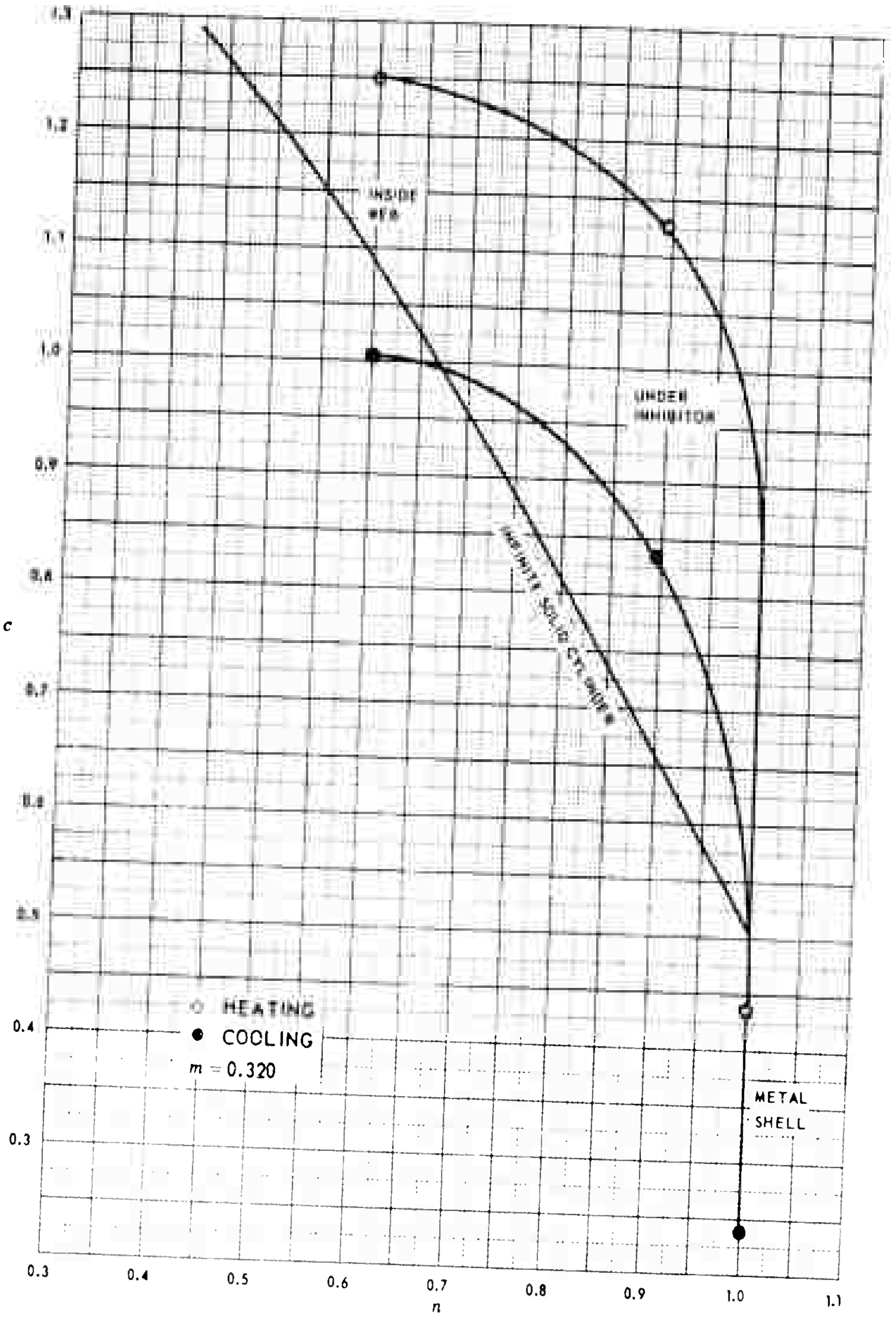

FIG. 25. Comparison of Actual Intercepts With Theoretical for the 2.0-in. Rocket Motor.

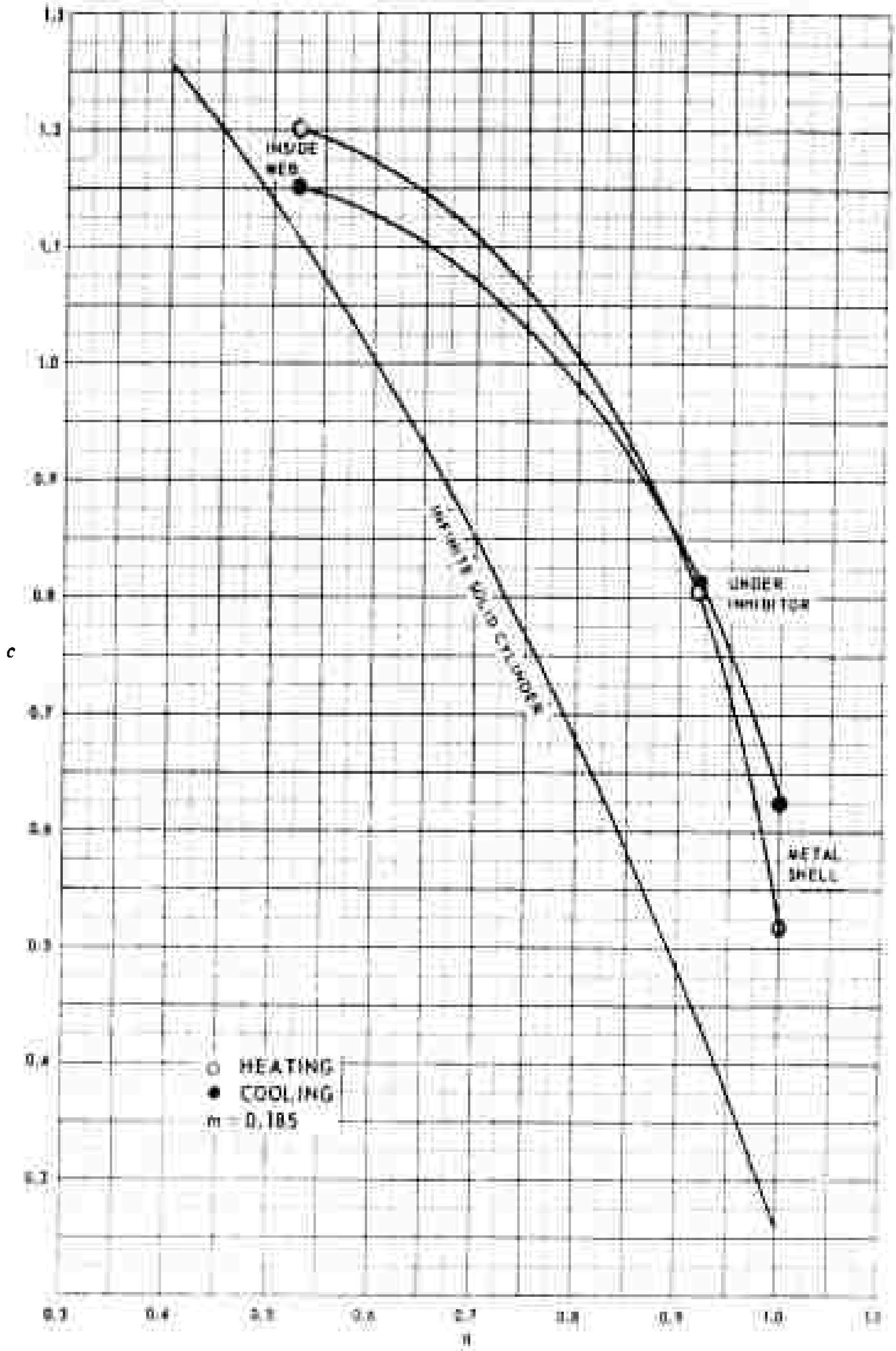

FIG. 26. Comparison of Actual Intercepts With Theoretical for the HPAG Rocket Motor.

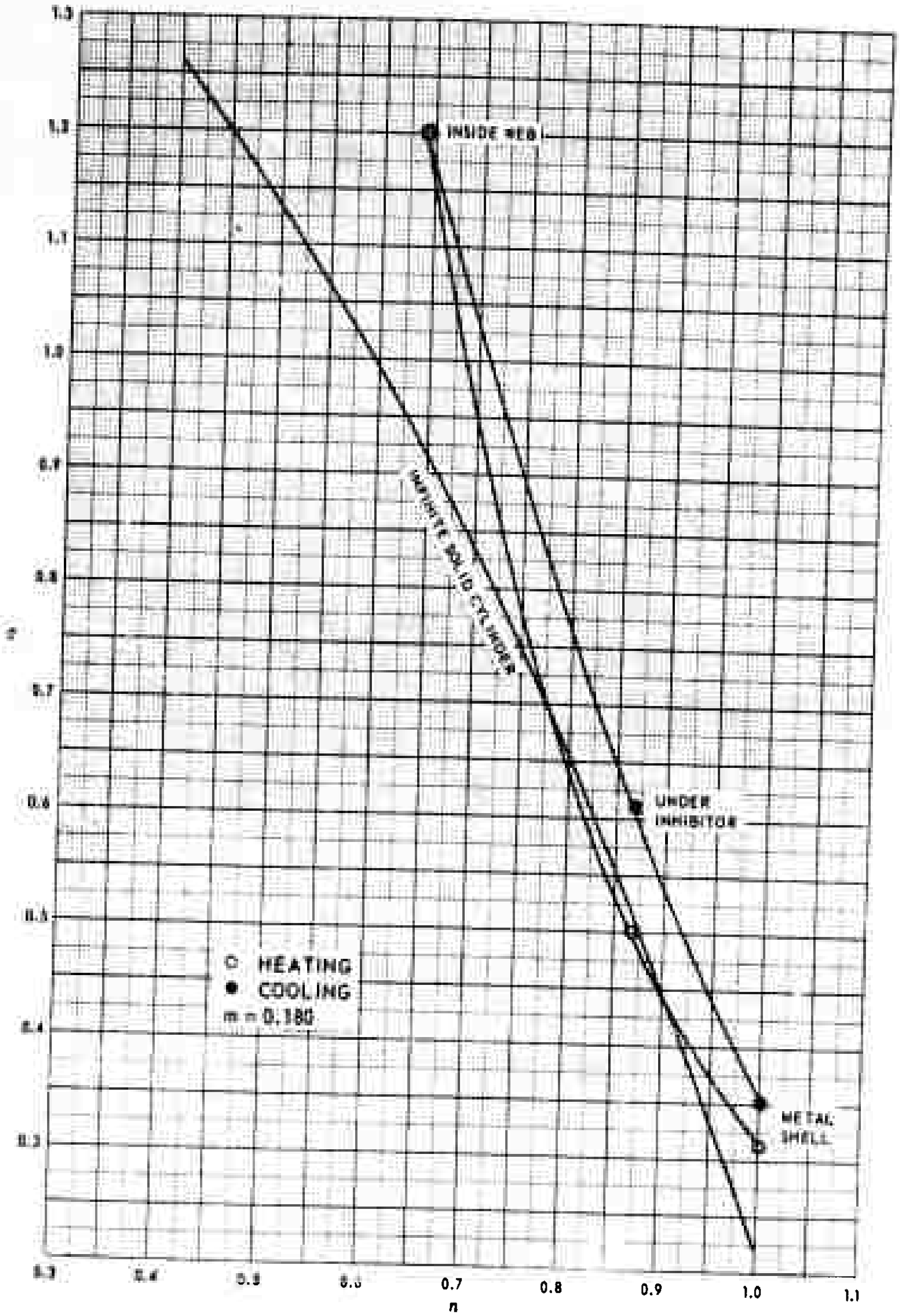

FIG. 27. Comparison of Actual Intercepts With Theoretical for the Weapon A Motor.

NOMENCLATURE

A	Peripheral area of the cylinder, sq ft
a	Slope of line defined by Eq. 11, hr^{-1} or min^{-1}
b	Slope of line defined by Eq. 4, dimensionless
C_p	Specific heat, Btu/(lb mass)(°F)
c	Intercept of lines defined by Eq. 4 and 11, dimensionless
D_o	Outside diameter of the cylinder, ft
h	Surface heat-transfer coefficient or film coefficient, Btu/(hr)(sq ft)(°F)
J	Notation of Bessel function: $J_0(x)$, Bessel function of first kind and zero order of x $J_1(x)$, Bessel function of first kind and first order of x
k	Thermal conductivity of the solid body, Btu/(hr)(sq ft)(°F per ft)
k_f	Thermal conductivity of air film, Btu/(hr)(sq ft)(°F per ft)
L	Distance through which heat is conducted, ft
l	Length of the rocket motor, ft
m	k/hr_o, surface resistivity, dimensionless
n	r/r_o, position ratio, dimensionless
Q	Quantity of heat transferred, Btu
q	Rate of heat transfer, Btu/hr
R	A positive root of $RJ_1(R) = \frac{hr_o}{k} J_0(R)$
r	Redial distance of point considered, measured from the axis of the cylinder, ft
r_o	Outside radius of the cylinder, ft
t	Temperature at time and position considered, °F
t_o	Temperature of the surroundings, °F
t_i	Temperature of the solid at zero time, °F
Δt	Temperature difference between two points considered
Δt_i	Temperature difference between surroundings and initial temperature of the body $(t_o - t_i)$, °F
V	Velocity of air passing the rocket, ft/hr
X	$\frac{k\theta}{C_p \rho r_o^2}$, dimensionless ratio
Y	Ratio of temperature difference, $(t_o - t)/(t_o - t_i)$, dimensionless

α Thermal diffusivity $k/C_p\rho$, dimensionless

ρ Density of propellant, lb mass/cu ft

ρ_a Density of air, lb mass/cu ft

μ Absolute viscosity of fluid (air), lb mass/(hr)(ft)

θ Time, hr or min

REFERENCES

1. Jakob, Max. Heat Transfer. New York, Wiley, 1941. Vol. I.
2. Gurney, H. P., and J. Lurie. "Chart for Estimating Temperature Distributions in Heating or Cooling Solid Shapes," IND ENG CHEM, Vol. 15(November 1923), pp. 170–72.
3. McAdams, W. H. Heat Transmission, 2nd ed. New York, McGraw-Hill, 1942.
4. Heisler, M. P. "Temperature Charts for Induction and Constant Temperature Heating," AM SOC MECH ENGR, TRANS, Vol. 69 (April 1947), p.227.

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