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SOME PROBLEMS OF HEAT TRANSFER IN ROCKETS

by

John Beck, Jr., J. Barkley Rosser, and Harry Siller

FINAL REPORT

Series B · Number 3

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ALLEGANY BALLISTICS LABORATORY

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SOME PROBLEMS OF HEAT TRANSFER IN ROCKETS

Final Report Series B Number 3

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Preface and Acknowledgments

The work described in this report is pertinent to the projects designated by the War Department as OD-26, "Jet Propulsion," and CWS-22, "Rocket Projection of Chemical Munitions," and to the project designated by the Navy Department as NO-33, "Rocket Propellants and Ballistics." It is referred to in the Allegany Ballistics Laboratory files as Project W-6.1

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The Allegany Ballistics Laboratory editorial staff edited the report. The staff of the Technical Reports Section, Office of the Chairman, National Defense Research Committee prepared this report for publication.

B

TABLE OF CONTENTS

List of Tables

c

List of Figures

Figure		Page

List of Symbols

$\underline{A}$	Function of $\underline{q}$ and T_o chosen to fit the boundary conditions.
$\underline{a}$	Thickness of the wall (cm).
$\underline{B}$	Function of $\underline{q}$ and T_o chosen to fit the boundary conditions.
$\underline{c}$	Thermal capacity per unit volume (cal/cm^3-^{o}C).
$\underline{D}$	Time differentiating operator, $\partial/\partial t$.
$F'(P)$	Derivative of $F(D)$ with respect to $\underline{D}$ evaluated at $\underline{P}$.
f_o, f_n	Coefficients in the Fourier series, $T(t_1) = T_o \sum_{n=o}^{\infty} f_n \cos n\pi x/a$.
f_o^o	Approximate value of f_o, that is, Q^o/ac or

$$f_o^o = \frac{\alpha}{\beta} t \sum_{n=o}^{\infty} \frac{(-1)^n (\alpha\sqrt{t}/\beta)^n}{\Gamma(\frac{1}{2}n + 2)}.$$

$\underline{h}$	$K(\rho v)^{0.8}$ (cal/cm^{2}sec ^{o}C).
J_o, J_1	Bessel functions of the first kind, zeroth and first order, respectively.
$\underline{K}$	Proportionality constant in heat transfer equation (cal/cm$^{0.4}$sec$^{0.2}$gm$^{0.8}$ ^{o}C).
$\underline{L}$	Heat of fusion of TNT (cal/gm).
$\underline{M}$	Melting point of TNT (^{o}C).
$\underline{P}$	Roots of the equation $F(D) = 0$.
$\underline{Q}$	Heat transferred per unit area (cal/cm^2).
q^2	$(c/k)D$.
$\underline{r}$	Radius in the cylindrical coordinate system with axis in the axis of the wire (cm).
r_1	Radius of the wire (cm).
$\underline{T}$	Temperature as a function of position and time (^{o}C).
$\overline{T}$	Average temperature at a given time t_1 (^{o}C).
T_o	Temperature of the gas (^{o}C).
T_1	Temperature of the surface, or temperature for $x = 0$ (^{o}C).
T_s	Temperature of the inside surface of the wall (^{o}C).
$\underline{t}$	Time (sec).

D

t_1	Burning time of the propellant (sec).
u	$\frac{1}{4}(\beta D)^{-1/2}$
v	Average velocity of the gas (cm/sec).
Z	Distance the melted surface of TNT recedes from the wall, $Z \propto t^{1/2}$ (cm).
α	ah/k or $r_1 h/k$
β	$a^2 c/k$ or $r_1^2\, c/k$ (sec).
$\Gamma(k)$	Gamma function of k.
γ	$Z/t^{1/2}$
∇^2	Laplace operator, $(\partial^2/\partial x^2) + (\partial^2/\partial y^2) + (\partial^2/\partial z^2)$.
k	Thermal conductivity (cal/cm-sec-°C).
ρ	Density of gas [Part II] or density of TNT [Part IV] (gm/cm^3).
θ	$\sqrt{-(a^2 c/k)D}$.
ϕ	$\sqrt{-(r_1^2\, c/k)D}$.

SOME PROBLEMS OF HEAT TRANSFER IN ROCKETS

Abstract

Some of the problems connected with the heating of rocket parts by the flowing gas are treated by the method outlined by Hirschfelder, Garten, and Hougen, but using the Heaviside operational calculus for solving the differential equations. These problems are related to the performance of rockets through the effect of heat transfer on the cooling of the propellant gas, on the strength of the chamber walls and trap, and on the stability of high explosive carried by the rockets. Application of the calculations to the 4-1/2 in. rocket indicates that the trap wires reach an average temperature of more than 1000°C in the hottest cross section; that the wall of the burster tube reaches a temperature of about 350°C a fraction of a second after the charge is fired, which temperature is high enough to cause TNT to detonate; and that a thin layer of insulating material on the outside of the burster tube is adequate to keep its temperature below a safe limit. A supplementary solution for use with small <u>t</u> is given.

1. INTRODUCTION

In several respects it is important to be able to estimate the transfer of heat from the propellant gases to the confining walls of rockets. Any complete system of internal ballistics must depend on such as estimate. The heating of trap wires imposes a restriction on design of traps. In the 4-1/2 in. rocket with burster tube, which is the principal device discussed in connection with the present theoretical treatment, particular interest attaches to the effect of the heating on the high explosive in the burster tube.

The treatment of the physical problem will follow the pattern outlined by Hirschfelder, Garten and Hougen. 1/ In solving the equations, however, the powerful operational calculus of Heaviside and the method of Laplace transforms will be used. The accuracy of the results thus obtained is limited by our assumptions concerning the physical properties of steel and TNT, but it is believed that enough information is obtained to help in certain problems of design, particularly of the burster tube in the 4-1/2 in. rockets.

It is assumed throughout that the density of gas flow and the temperature of the gas are constant during the burning. Actually, both the decrease in pressure and increase in the area of the channel as the gas flows through the latter tend to make the density of flow less toward the end of the burning interval. In view of the fact that the heat-transfer coefficient <u>h</u> will be chosen to fit an experimental result, the principal effect of this erroneous assumption concerning constancy of flow density and temperature is to distort the distribution of temperature, but probably not to introduce large errors in the calculated values of total heat transferred. Changes in the volume thermal capacity and thermal conductivity with temperature are neglected, despite the fact that the thermal conductivity of steel first increases above room temperature and then decreases

1/ J. O. Hirschfelder, W. Garten, Jr., and O. Hougen, <u>Heat conduction, gas flow, and heat transfer in guns</u>, NDRC Report A-87 (OSRD 868).

as the temperature is raised further. The time-rate of transfer of heat across unit area of interface between the gas and the metal is assumed to be given by the expression,[1/]

$$dQ/dt = K(\rho v)^{0.8} (T_o - T_1), \qquad \text{(I)} \; 2/$$

in which ρ is the density of the gas, v is its average velocity, and T_o and T_1 are the temperature of gas and surface, respectively. When ρ, v, and t are in cgs units, Q is in calories per square centimeter, and the temperature difference is in degrees centigrade, K has the numerical value 0.0023 for smooth surfaces, and is somewhat larger for rough surfaces. In terms of the heat-transfer coefficient h, Eq. (I) becomes $dQ/dt = h(T_o - T_1)$.

It should be stated at the outset that there is a priori a large uncertainty about the value of the heat-transfer coefficient h appropriate to the surfaces in question. However, since C. A. Boyd, of the Jet Propulsion Laboratory, Indian Head, Maryland, has measured the maximum temperature reached inside the wall of the burster tube of the 4½-in. rocket after the burning of a normal charge, the heat-transfer coefficient can be estimated, and the related quantities deduced in turn.

Three problems are considered: first, the distribution of temperature in walls bounded by parallel plane surfaces, these being taken to be satisfactory approximations to the walls with cylindrical surfaces actually encountered; second, the extent of melting of the high explosive, supposed here to be TNT, in the burster tube of the 4½-in. or similar rocket; and third, the temperature distribtuion in cylindrical trap wires.

An outline of the results may be set forth briefly. As previously pointed out, they depend on the value of 360°C found by Boyd for the maximum temperature reached inside the burster tube of the 4½-in. rocket. The corresponding heat-transfer coefficient h is 0.183 cal/cm²sec °C, which give

$$h = 0.0032\ (\rho v)^{0.8}\ \text{cal/cm}^2\text{-sec-}^\circ\text{C},$$

where ρ is the density of the gas in grams per cubic centimeter and v is its velocity in centimeters per second. The rise in temperature on the inside of the wall of the tube 0.2 sec after the start of burning is calculated to be 97 percent of the extreme rise. The temperature of the surface next to the gases rises to 730°C near the end of the burster tube during the burning of a normal charge of 7/8-in. stick powder. If use is made of a charge of powder with 1-in. web, weighing 6 lb, and burning 0.65 sec, the maximum temperature on the inner side is 540°C. If the inner surfaces of a burster tube filled with TNT is at 360°C, a layer of TNT less than 2 mm thick would be melted at the end of a flight of 5 sec. Insulating the surface next to the gas with a layer ½ mm thick of some substance similar to porcelain would reduce the maximum temperature reached inside the tube to about 180°C.

The surface of the trap wires of the 4½-in. rocket in the region where the flow of gas is greatest reaches about 1230°C, but the average temperature over the same cross section of the wire is only 1050°C. A method for determining the radial distribution of temperature in the wire is given in Part V.

1/ See footnote page 1.
2/ If an equation or its equivalent appears in the Appendix of this report, it is given the number of the equation in the appendix. Other equations in the text are numbered with Roman numerals.

II. HEAT TRANSFER TO THE WALLS OF THE BURSTER TUBE

We start with Eq. (I) expressed in the form,

$$dQ/dt = h(T_o - T_1), \tag{II}$$

in which the heat-transfer coefficient $\underline{h}$ has been written for $K(\rho v)^{0.8}$. It is assumed that $\underline{h}$ does not vary during the burning of the propellant. The differential equation of heat conduction is put in the form used by Heaviside,

$$\nabla^2 T = q^2 T. \tag{III}$$

Here ∇^2 is the Laplace operator and q^2 is written for cD/k (where $\underline{c}$ is the thermal capacity per unit volume, k is the thermal conductivity, and $\underline{D}$ is the time-differentiating operator, $\partial/\partial t$). Heaviside showed [3] that under certain conditions the problem may be solved by integrating Eq. (III) with $\underline{q}$ considered as constant, fitting the boundary conditions with the operational form of the solution thus obtained and then converting the operational to an algebraic form. We usually require that the problem be reducible to one dimension, and that the quantity governing the condition of the system, such as a temperature or rate of flow of heat, be zero up to the initial time. It should be explained at this point that the zero of the scale of temperature is arbitrarily chosen as that of the initial state of the system. In this report temperatures will be referred to 20°C.

The problem of transfer of heat to the burster tube in the $4\frac{1}{2}$-in. rocket is considered as one of transfer to a wall of infinite extent bounded by parallel plane surfaces. It is supposed that conduction of heat through the inside surface of the wall is negligible in the intervals of time of interest here, so that one boundary condition is that at the inside surface $\partial T/\partial x = 0$. The other boundary condition expresses the equality of the heat transferred to the surface next to the gas and the heat conducted away from the surface; the corresponding equation is

$$h(T_o - T_1) = -k(\partial T/\partial x)_1. \tag{5}$$

The subscript 1 attached to the differential coefficient indicates that it refers to the surface adjacent to the hot gas. The form of Eq. (III) appropirate for our one-dimensional system is

$$d^2T/dx^2 = q^2 T. \tag{1}$$

The solution in operational form is $T = Ae^{-qx} + Be^{qx}$. It is shown in the Appendix that when the boundary conditions are imposed this becomes

$$T = \frac{h \cosh q(a - x)}{h \cosh qa + k q \sinh qa} T_o, \tag{8}$$

and that $\underline{T}$ is given in algebraic form by the expression,

$$T = T_o - T_o \sum \frac{2\alpha e^{-\theta^2 t/\beta} \cos [\theta(1 - x/a)]}{[\alpha(\alpha + 1) + \theta^2] \cos \theta}, \tag{17}$$

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[3] O. Heaviside, Electromagnetic theory ("The Electrician" Printing and Publishing Co., London 1899). In further references this work will be designated as "Electromagnetic theory".

in which θ takes the values of all the positive roots of the equation, [4]

$$\alpha \cos \theta = \theta \sin \theta. \tag{16}$$

In Eqs. (8) and (17) x is taken as increasing from zero as it goes through the wall from the surface next to the gas, a is the thickness of the wall, α is defined as ah/k, and β is defined as a^2c/k. We take the density of steel to be 7.8 gm/cm^3, its specific heat to be 0.11 cal/gm $^{\circ}C$, and its thermal conductivity to be 0.11 cal/cm sec $^{\circ}C$. The thickness of the wall of the burster tube is approximately 0.3 cm. In the $4\frac{1}{2}$-in. rocket loaded with a normal charge, 1.8 kg of powder is in front of the end of the burster tube. This exhausts in 0.2 sec through a channel with a median area of cross section equal to 37 cm^2, giving an average density of flow of 160 gm/cm^2sec. We find that α probably lies in the interval from 0.35 to 0.55, and that β is equal to 0.70 sec. With these values of the parameters, and with t not too small, the series of Eq. (17) converges very rapidly. For t equal to 0.1 sec, two terms of the summation suffice for accuracy to 1 part in 100. For t less than 0.1, one can use the series of Eq. (57), Sec. 6 of the Appendix.

By integrating over the thickness of the wall, one can determine the average temperature at a given time t_1, for example, the burning time of the propellant. It is

$$\overline{T} = T_o - T_o \sum \frac{2\alpha^2 e^{-\theta^2 t_1/\beta}}{\theta^2[\alpha(\alpha + 1) + \theta^2]}, \tag{IV}$$

where the values of θ are given again by Eq. (16). The derivations of this and following equations are given in the Appendix. The value of $\overline{T}$ at the end of the burning time is a good approximation to the maximum temperature reached inside the burster tube, as radiation and conduction could not be expected to change the heat content of the wall rapidly. By calculating $\overline{T}$ for a series of values of α, using 3200°K for the temperature of the gas, it was found that the experimental figure, a rise of about 340°C, is given with α equal to 0.5. This figure corresponds to a value of

$$0.0032 \text{ cal/gm}^{0.4}\text{sec}^{0.2}\text{gm}^{0.8}\ {}^{\circ}C$$

for the coefficient K in the expression for the heat-transfer coefficient h, which means that the surface in question is not effectively smooth.

The distribution of temperature after the burning of the propellant was determined by using the usual Fourier analysis. It was assumed that no heat was passing through either side of the wall. If we write

$$\frac{T}{T_o} = \sum_{m=0}^{\infty} f_m \exp\left[\frac{-m^2\pi^2}{\beta}(t - t_1)\right] \cos\frac{m\pi x}{a},$$

---

[4] The practical solution of this equation is treated in Sec. 5 of the Appendix.

[see Eq. (21) in the Appendix] we get

$$f_0 = \bar{T}/T_0 \tag{V}$$

and

$$f_m = \sum \frac{4\alpha^2 e^{-\theta^2 t_1/\beta}}{(m^2\pi^2 - \theta^2)[\alpha(\alpha+1) + \theta^2]}, \quad (m>0) . \tag{20}$$

In the last three equations t_1 represents the burning time of the propellant. Figures 1 to 3 show the temperature as a function of position and time, as given by Eqs. (17), (18), (V), and (20), with $\alpha = 0.5$, $\beta = 0.70$ sec, $t_1 = 0.20$ sec, and $T_0 = 3200°K$. It is significant that a temperature high enough to start decomposition of TNT is reached a small fraction of a second after the charge starts to burn.

It would be expected that if the time is short in comparison with β, the quantity of heat transferred to a wall of finite thickness would be slightly less than the value for a semi-infinite solid, as the reflected wave would have little effect on the temperature of the hot surface. The value for the semi-infinite solid provides a convenient approximation and permits accurate evaluation of the correct figure for the wall of finite thickness by means of deviation curves. For the semi-infinite solid we have the boundary condition of Eq. (5) and the condition that T approaches zero when x increases without limit. The operational solution [see Eq. (30) in this Appendix] is

$$Q = \frac{T_0 h\sqrt{\delta k D}}{h + \sqrt{\delta k D}}\, t .$$

Since f_0^0, the approximate value of f_0, is Q/acT_0, we get from Eq. (32) the algebraic form,

$$f_0^0 = \frac{\alpha}{\beta} t \sum_{n=0}^{\infty} \frac{(-1)^n(\alpha\sqrt{t/\beta})^n}{\Gamma\left(\frac{n+4}{2}\right)}.$$

Figure 4 and Table I give $\beta f_0^0/\alpha t$ as a function of $\alpha\sqrt{t/\beta}$. Figure 5 gives the deviation curves,

$$\frac{\beta}{\alpha t}(f_0^0 - f_0),$$

as a function of α for a series of values of t/β. Figure 6 provides a means of estimating f_0^0 accurately. It is the deviation curve showing the difference between

$$\beta f_0^0/\alpha t \text{ and } \frac{1}{1 + (3/4)\,\alpha\sqrt{t/\beta}}.$$

Table I. <u>Heat transferred to a semi-infinite solid in terms of the heat transfer coefficient and the properties of the solid.</u>

$\alpha\sqrt{t/\beta}$	$\beta f_o^o/\alpha t$	$\alpha\sqrt{t/\beta}$	$\beta f_o^o/\alpha t$	$\alpha\sqrt{t/\beta}$	$\beta f_o^o/\alpha t$
0.0	1.000	0.5	0.720	1.0	0.556
.1	0.929	.6	.680	1.1	.531
.2	.867	.7	.644	1.2	.509
.3	.812	.8	.612	1.3	.488
.4	.763	.9	.583	1.4	.469

The use of Figs. 5 and 6 may be illustrated by calculating the temperature reached inside the burster tube of a standard $4\frac{1}{2}$-in. projectile when the propellant charge consists of a single grain of powder, weighing 6 lb, burning in 0.65 sec, and giving a gas temperature of 2800°K. It is supposed that the total length of the charge is 11 in., with 2 in. of this length extending behind the burster tube. The median port area is found to be 46 cm^2 and, since only 9 in. of powder contribute to the flow, the median density of flow is 75 gm/cm^2 sec and <u>h</u> is 0.101 cal/cm^2 sec °C. The corresponding values of $\underline{\alpha}$ and t/β are 0.275 and 0.93, and the other quantities that determine $\underline{\alpha}$ and β are as previously given. From Fig. 6 we find that for $\alpha\sqrt{t/\beta}$ equal to 0.264,

$$\beta f_o^o/\alpha t = \frac{1}{1 + (3/4)\ (0.264)} - 0.003 = 0.832.$$

In Fig. 5 we find that when α is 0.275 and t/β is 0.93,

$$\beta f_o/\alpha t = \beta f_o^o/\alpha t - 0.011 = 0.821,$$

for which we get $f_o = 0.209$. The rise in temperature is found from Eq. (V) to be 522°C, making the maximum temperature on the inward side of the wall of the burster tube about 540°C.

An estimate of the error introduced by neglecting the cylindrical form of the burster tube was made by comparing the heat transferred to unit surface of a semi-infinite solid with a plane wall, with that transferred to unit surface of a solid cylinder of the same diameter as the outside of the tube, under the conditions previously given for the usual charge. The method of making the calculation for the cylinder appears in the Appendix. The error turns out to be only 0.45 percent, which is negligible in comparison with the errors associated with the assumption that the properties of the steel do not change with temperature.

III. EFFECT OF HEATING ON TNT IN THE BURSTER TUBE

In a test carried out at the request of Division 3, H. Henkin of Division 8 observed that TNT in a sealed tube detonated after 25 sec at 360°C. Clearly it would not be safe to use an uninsulated burster tube in the $4\frac{1}{2}$-in. projectile of the present design, unless the absorption of heat by the melting of the TNT gives a substantial cooling of the wall. Neither the date on liquid TNT nor the solution of the mathematical problem required for a determination of the cooling is available. The magnitude of the effect may be estimated by solving the problem on the assumptions that the thermal conductivity and thermal capacity per unit volume are the same for liquid as for solid TNT, and that the inward side of the wall is a plane surface kept at a constant temperature. In a solution attributed to Franz Neumann,[5] it is found that the melting surface recedes from the heated surface in such a way that its distance Z is proportional to $t^{1/2}$, and that the total amount of heat transmitted from unit area of the wall in the time t is

$$Q = 2\sqrt{ckt/\pi}\,(T_2 - M) \Big/ \sqrt{2/\pi}\int_0^{\gamma\sqrt{c/2k}} \exp\left(-\tfrac{1}{2}x^2\right) dx, \qquad \text{(VI)}$$

where the proportionality factor,

$$\gamma = Z/t^{\frac{1}{2}}, \qquad \text{(VII)}$$

is given as a root of the equation,

$$\frac{(T_2 - M)(1/\sqrt{2\pi})\exp(-\gamma^2 c/4k)}{M\,\sqrt{2/\pi}\int_0^{\gamma\sqrt{c/2k}}\exp\left(-\tfrac{1}{2}x^2\right)dx} - \frac{(1/\sqrt{2\pi})\exp(-\gamma^2 c/4k)}{1-\sqrt{2/\pi}\int_0^{\gamma\sqrt{c/2k}}\exp\left(-\tfrac{1}{2}x^2\right)dx} = \frac{\sqrt{2}L\rho\gamma}{4M\sqrt{ck}}. \qquad \text{(VIII)}$$

In these equations T_2 is the constant temperature of the wall, c and k refer to TNT, M is the melting point of the TNT, L is its heat of fusion, and ρ is its density. E. Hutchinson[6] has given the thermal conductivity of TNT at 30°C as 51.1×10^{-4} watt/cm-°C, (or 1.22×10^{-3} cal/cm-sec-°C), the density as 1.67 gm/cm³, and the diffusivity k/c as 1.94×10^{-3} cm²/sec. The value 93.5 ± 3.5 joule/gm or 22.3cal/gm, is given by the International Critical Tables for the heat of fusion at 80°C, the melting point. The proportionality factor γ and the coefficient of $t^{\frac{1}{2}}$ in Eq. (VI) were calculated for two values of the temperature of the wall, 340° and 360°C, both with initial temperatures of 20°C. The results and some related figures are given in Table II.

Table II. Rate of melting of, and heat absorbed by, TNT.

Temperature of wall (°C)	340	360
γ (cm/sec$^{\frac{1}{2}}$)	0.067	0.070
$Q/t^{1/2}$ (cal/cm²sec$^{\frac{1}{2}}$)	10.9	11.9
Q for t = 0.2 sec (cal/cm²)	4.9	5.3
Z for t = 0.2 sec (cm)	0.030	0.031
Z for t = 5 sec (cm)	0.15	0.16

[5] Weber-Riemann, Partielle Differential-Gleichungen, vol. II, p. 117. Apparently the only restriction on this solution is that the densities of solid and liquid be the same.

[6] "The thermal conductivity of explosive materials," Advisory Council on Scientific Research and Technical Development Report A. C. 2861, Oct. 15, 1942.

A decrease of 5 cal/cm^2 in the heat content of a wall of steel 0.3 cm thick would correspond to a drop in temperature of about 19°C. Thus, cooling by the TNT could hardly keep the maximum temperature of the inward side of the wall below 340°C. On the other hand, the thickness of the layer of TNT melted is so small for moderately short times of flight that it would not be expected to interfere with the detonation of the whole mass.

IV. EFFECT OF A LAYER OF INSULATION ON THE ABSORPTION OF HEAT BY THE BURSTER TUBE

If a layer of insulating material is interposed between the gas and the wall of the burster tube, much less heat is transferred to the wall because the surface of the insulating material reaches a much higher temperature than would a steel surface. The exact calculation of the transfer of heat through an insulating layer is unduly complicated, but an upper limit for the heat transferred may be calculated easily by assuming that the surface of the insulator adjacent to the wall is kept at the initial temperature during the burning of the charge. Under this assumption we find that $\underline{Q}$, the heat passing through unit area of the outer surface of the insulating layer in the time $\underline{t}$, is given by the expression,

$$Q = hT_0 \left\{ \frac{t}{\alpha + 1} + 2\alpha\beta \sum \frac{1 - e^{-\theta^2 t/\beta}}{\theta^2[\alpha(\alpha + 1) + \theta^2]} \right\} , \qquad (26)$$

in which the values of $\underline{\theta}$ are the positive roots of the equation

$$\alpha \sin \theta + \theta \cos \theta = 0. \qquad (27)$$

The calculation was carried out for an insulating layer 0.5 mm thick having the thermal conductivity $\underline{k}$ and thermal capacity per unit volume $\underline{c}$ of porcelain. We take $k = 4 \times 10^{-3}$ cal/cm sec °C, $c = 0.6$ cal/cm³, $T_0 = 2900$°C, $t = 0.2$ sec, and $h = 0.183$ cal/cm²sec °C. Then $\alpha = 2.29$ and $\beta = 0.375$ sec, and we find $Q = 46$ cal/cm². The thermal capacity per square centimeter of the wall with the insulating layer is $(0.3)(0.86) + (0.05)(0.6) = 0.29$ cal/cm² °C. Thus the upper limit for the rise in temperature is $46/0.29 = 159$°C, and hence the temperature will not rise above 180°C. Since the material chosen as an example is not an expecially good insulator, and TNT does not decompose rapidly below 240°C, it is clear that the problem of providing adequate insulation for the burster tube when the standard charge is used may be reduced to that of getting a thin layer of refractory material to adhere to the wall during the burning of the charge. If we make a similar calculation for the charge described in Part II, weighing 6 lb and burning for 0.65 sec, we find a limit of about 300°C for the maximum temperature; so we see that with such a charge the problem of insulation is more critical.

V. HEATING OF TRAP WIRES

The trap wires in the Budd model of the 4½-in. rocket are 0.162 in. in diameter; that is, the radius is 0.206 cm, so that it is certainly necessary to take account of the cylindrical form of the wires in calculating the distribution of temperature in the wires, or even the average temperature at the end of the burning time. The appropriate solution of the differential equation of heat flow is

$$T = J_0(iqr)A, \tag{IX}$$

in which $\underline{A}$ is a function of the time chosen to satisfy the boundary condition, J_0 represents the Bessel function of the first kind and zeroth order, and $\underline{r}$ is the radius in the cylindrical coordinate system with axis in the axis of the wire. The Bessel function of the second kind is not included because we require the temperature to be finite at the axis. The operational solution satisfying the boundary condition on the transfer of heat is

$$T = \frac{hJ_0(iqr)}{hJ_0(iqr_1) + ikqJ_0'(iqr_1)}\, T_0, \tag{35}$$

where r_1 is written for the radius of the wire. The corresponding algebraic form is

$$T = T_0 - T_0 \sum \frac{2\alpha e^{-\beta^2 t/\beta} J_0(r\beta/r_1)}{(\alpha^2 + \beta^2)\, J_0(\beta)}, \tag{37}$$

with $\alpha \equiv r_1 h/K$, $k \equiv r_1^2 c/K$, and the values of $\underline{\beta}$ are given by the positive roots of the equation,

$$\alpha J_0(\beta) = \beta J_1(\beta). \tag{38}$$

In computing the surface temperature for small values of $\underline{t}$ it is convenient to use an asymptotic expansion of Eq. (35), in the form

$$\begin{aligned} T_1 = 2\alpha T_0\sqrt{t/\pi\beta}\, \Big\{ 1 &+ \tfrac{1}{2}\sqrt{\pi}(\tfrac{1}{2} - \alpha)\,(t/\beta)^{1/2} + (2/3)(3/8 - \alpha + \alpha^2)(t/\beta) \\ &+ \tfrac{1}{2}\sqrt{\pi}\,[3/8 - \alpha + (3/2)\alpha^2 - \alpha^3](t/\beta)^{3/2} \\ &+ (4/15)\,[63/128 - (9/8)\alpha + (15/8)\alpha^2 - 2\alpha^3 - \alpha^4]\,(t/\beta)^2 + \cdots \Big\}. \end{aligned} \tag{X}$$

The proper value to use for the median density of flow past the trap wires has not been determined with any certainty. It seems reasonable to suppose that it would be somewhat larger than that on the outside of the grains, since the ratio of median burning surface to median port area is much larger. However, some of the gas probably escapes to the outer side between the grains, which are arranged three on a wire. If we take 15 percent as the part of the gas from the second grain that goes through the perforation of the grain farthest back, it can be shown that we get about 190 gm/cm² sec for the median density of flow. We then find $h = 0.21$ cal/cm² sec °C, and $\alpha = 0.39$. Figure 7 shows the distribution of temperature in a wire for the conditions $\alpha = 0.4$, $\beta = 0.33$, $T_0 = 2900$°C (corresponding to about 3200°K), and $t = 0.2$ sec. The calculated surface temperature is well below the melting point of the steel. It has been observed, however, that after use in one or two rounds the wires have a polished appearance, such as would be expected if there were superficial melting. It may well be that the value selected for the density of flow is too small; it is also possible that the neglected changes in the physical properties of the steel with temperature would lead to a considerable error in defect in the calculated temperature.

Just as in the case of the wall with plane surfaces, it is convenient to represent the total heat transferred from the gas in terms of the deviation from the value for the semi-infinite solid. Figure 8 shows the deviations of Q/htT_o from Q^o/htT_o, the value for the semi-infinite solid. The last quantity is the same as the $\beta f_o^o/\alpha t$ used in connection with the plane wall and given in Figs. 4 and 6 and Table I. Comparison of Figs. 5 and 8 makes apparent the short time required to produce in cylinders significant departures from the conditions holding for the semi-infinite solid.

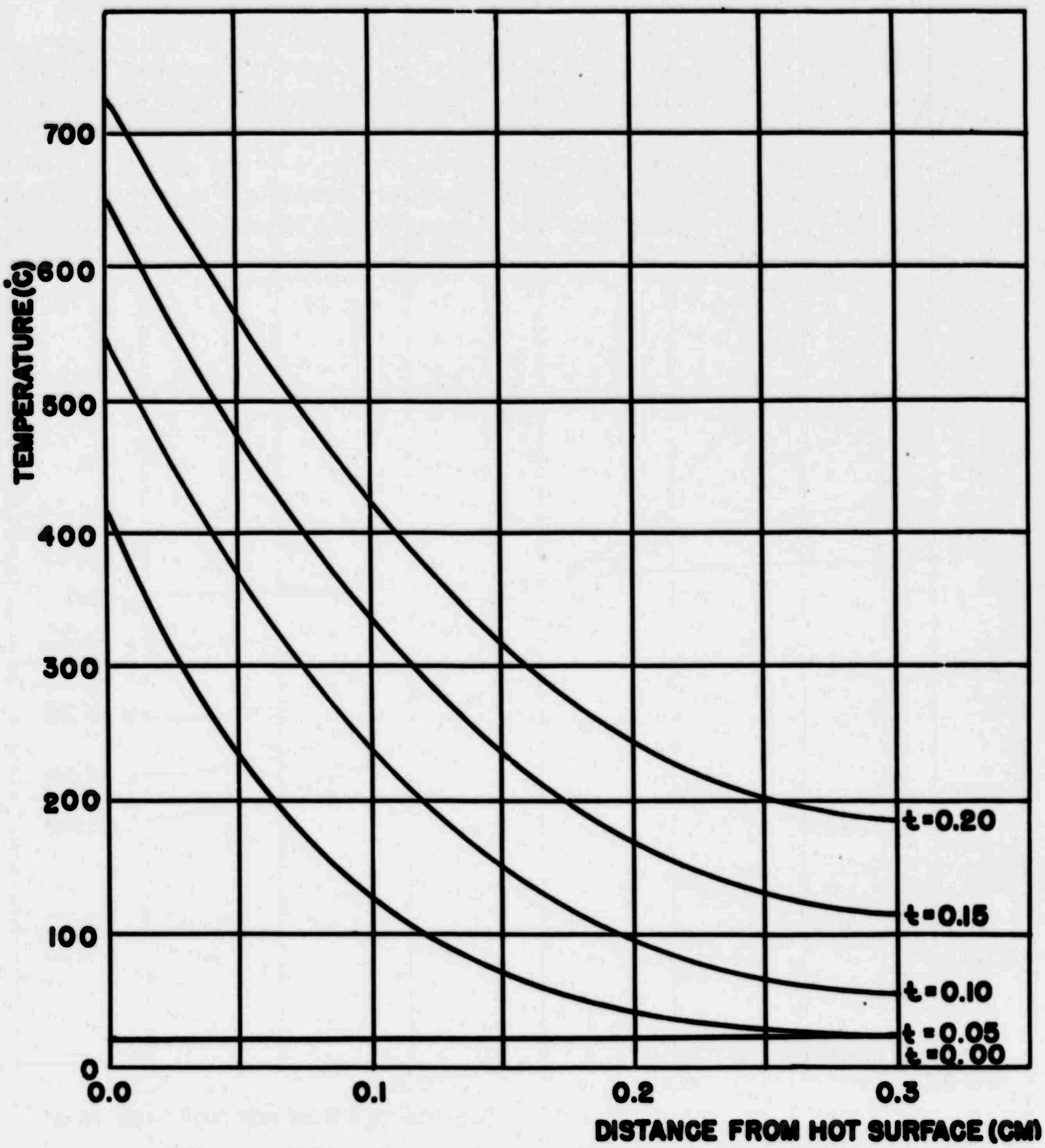

FIGURE I. DISTRIBUTION OF TEMPERATURE IN WALL 0.3 CM THICK, DURING BURNING OF THE CHARGE.

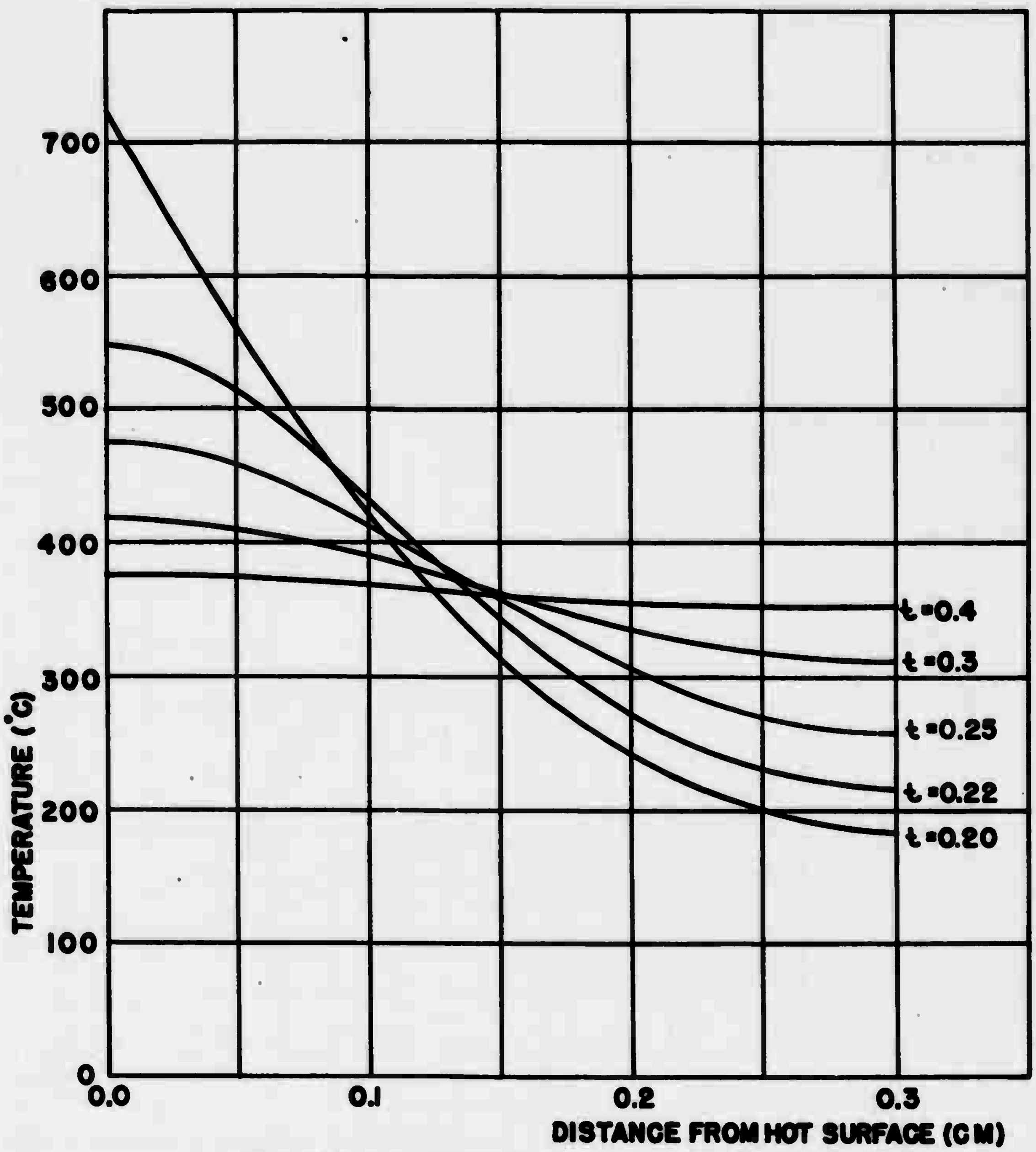

FIGURE 2. DISTRIBUTION OF TEMPERATURE IN WALL 0.3CM THICK, AFTER BURNING IS COMPLETED.

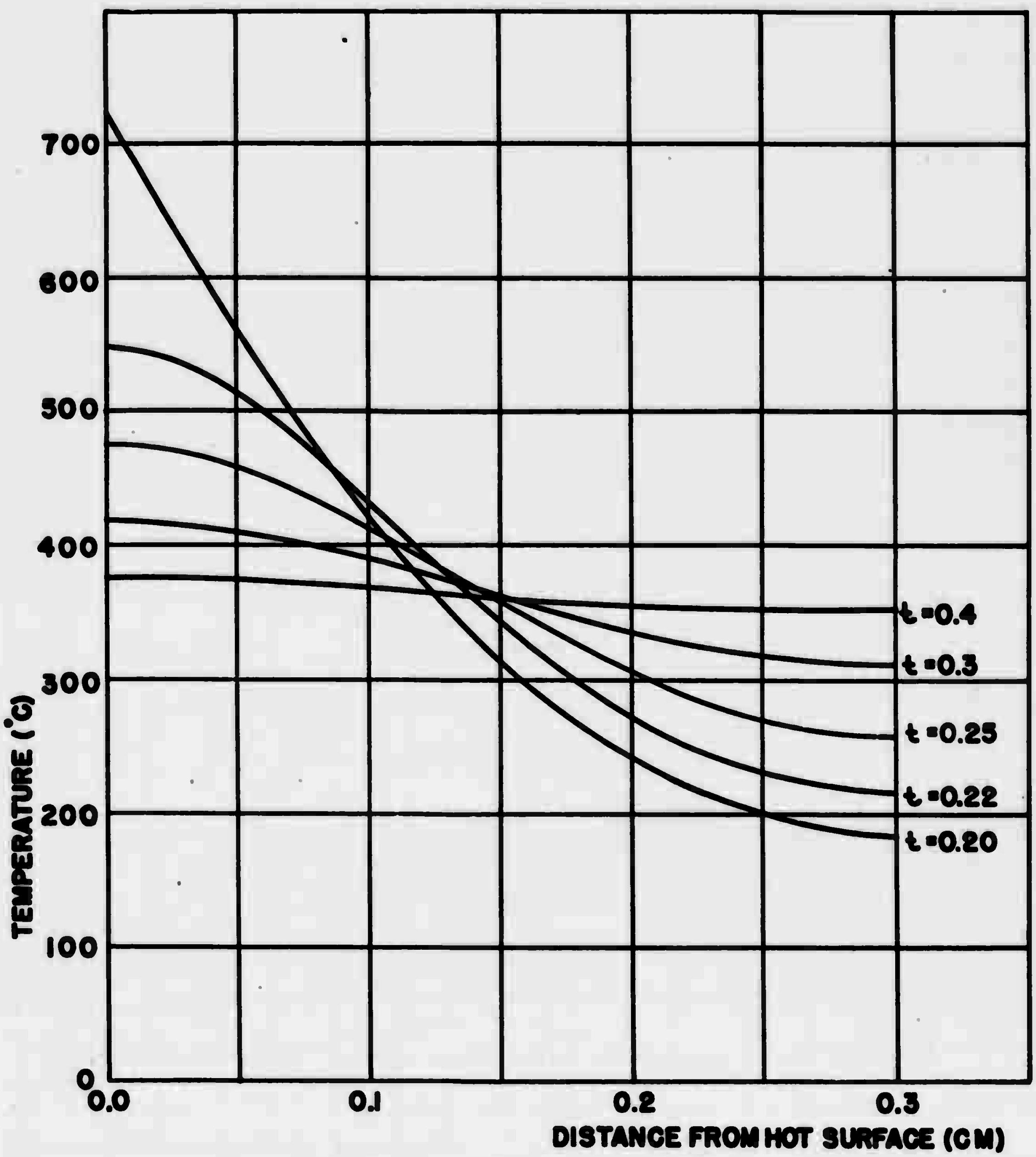

FIGURE 2. DISTRIBUTION OF TEMPERATURE IN WALL 0.3CM THICK, AFTER BURNING IS COMPLETED.

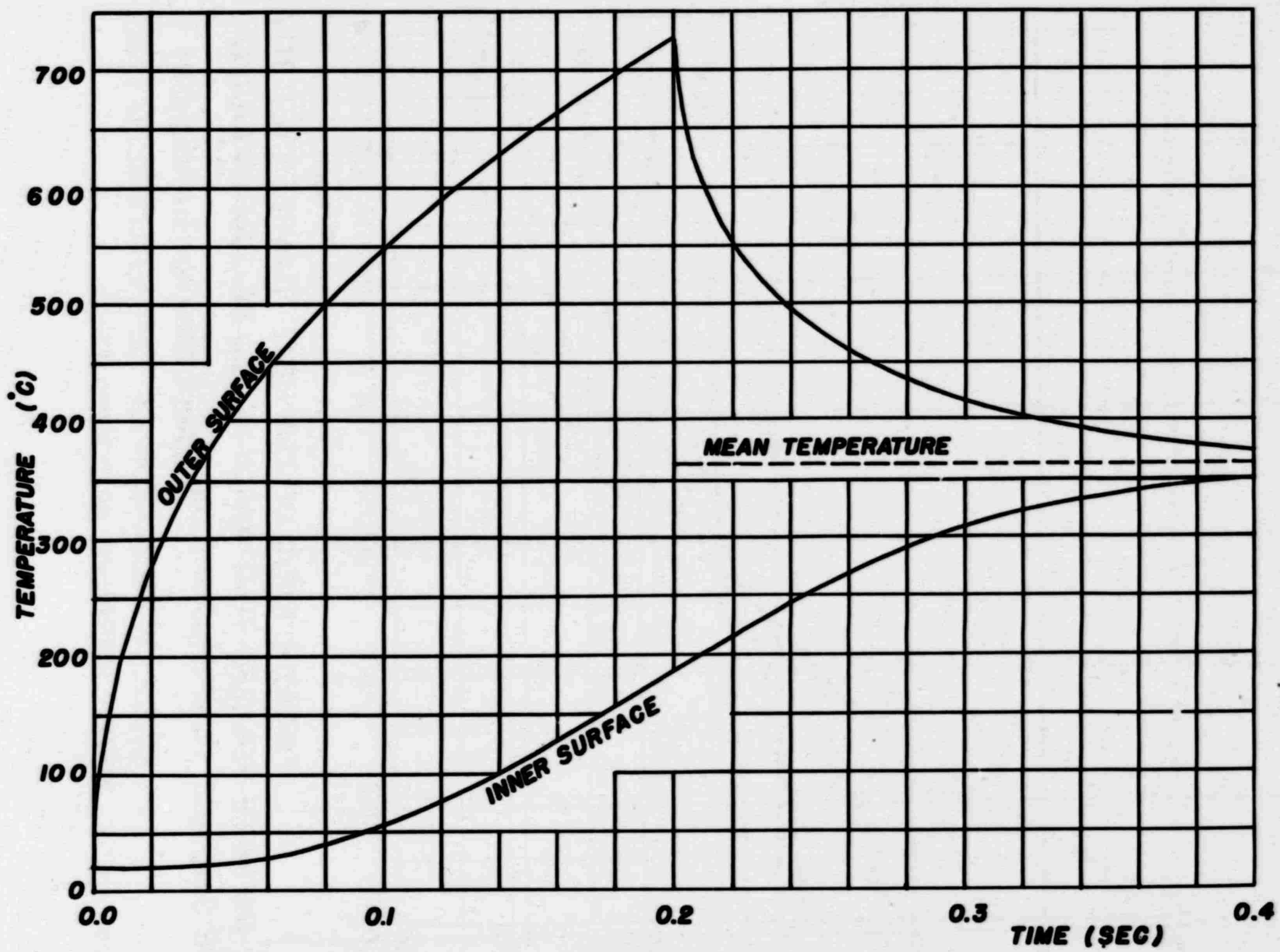

FIGURE 3. SURFACE TEMPERATURES AS A FUNCTION OF TIME, DURING AND AFTER BURNING.

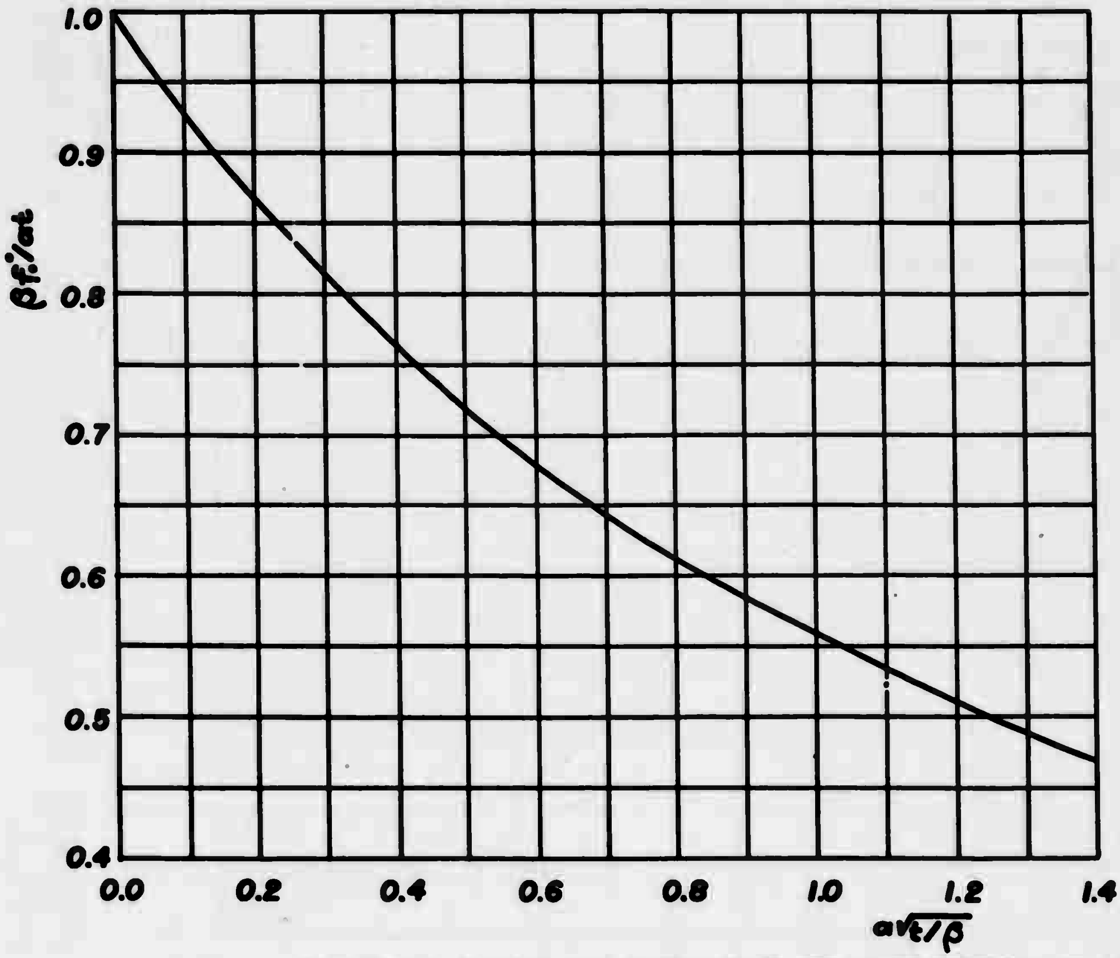

FIGURE 4. RATIO OF THE HEAT TRANSFERRED TO A SEMI-INFINITE SOLID, TO THAT WHICH WOULD BE TRANSFERRED IF THE SURFACE TEMPERATURE DID NOT RISE.

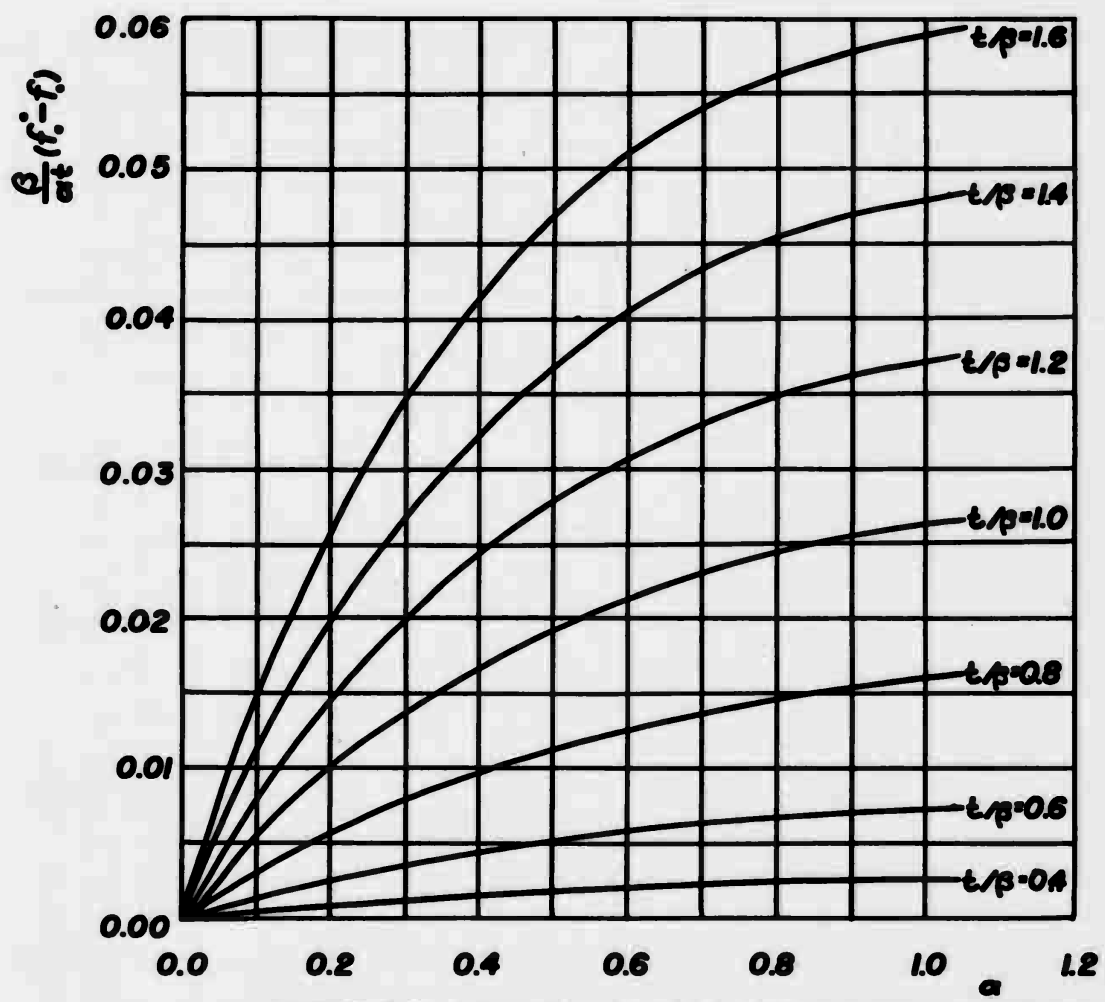

FIGURE 5. DEVIATION CURVES SHOWING THE RELATION BETWEEN THE HEAT TRANSFERRED TO A SEMI-INFINITE SOLID AND TO FINITE WALLS.

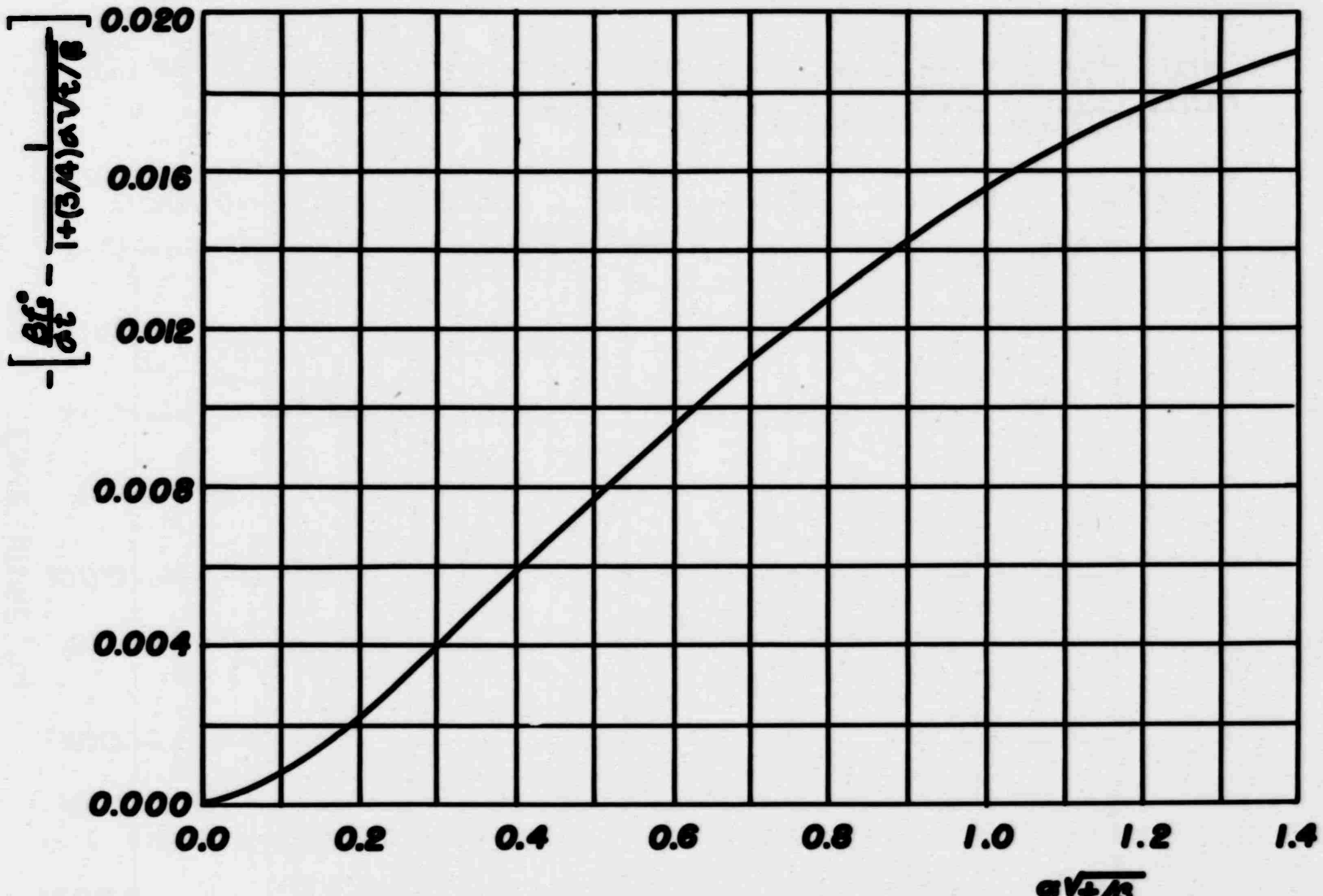

FIGURE 6. DEVIATION CURVE SHOWING THE ERROR IN AN APPROXIMATE FORMULA FOR THE HEAT TRANSFERRED TO A SEMI-INFINITE SOLID.

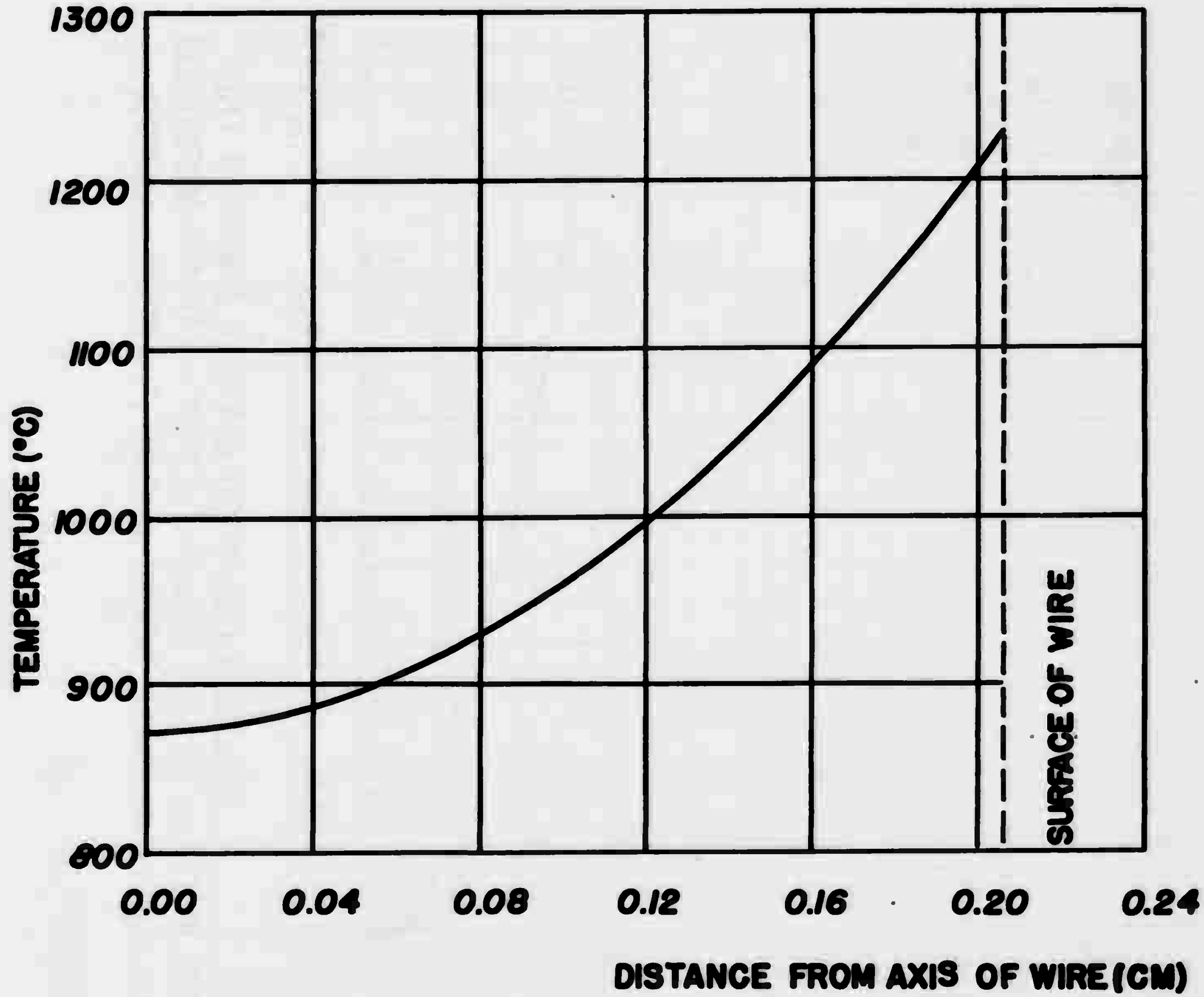

FIGURE 7. DISTRIBUTION OF TEMPERATURE IN A TRAP WIRE AT THE END OF BURNING.

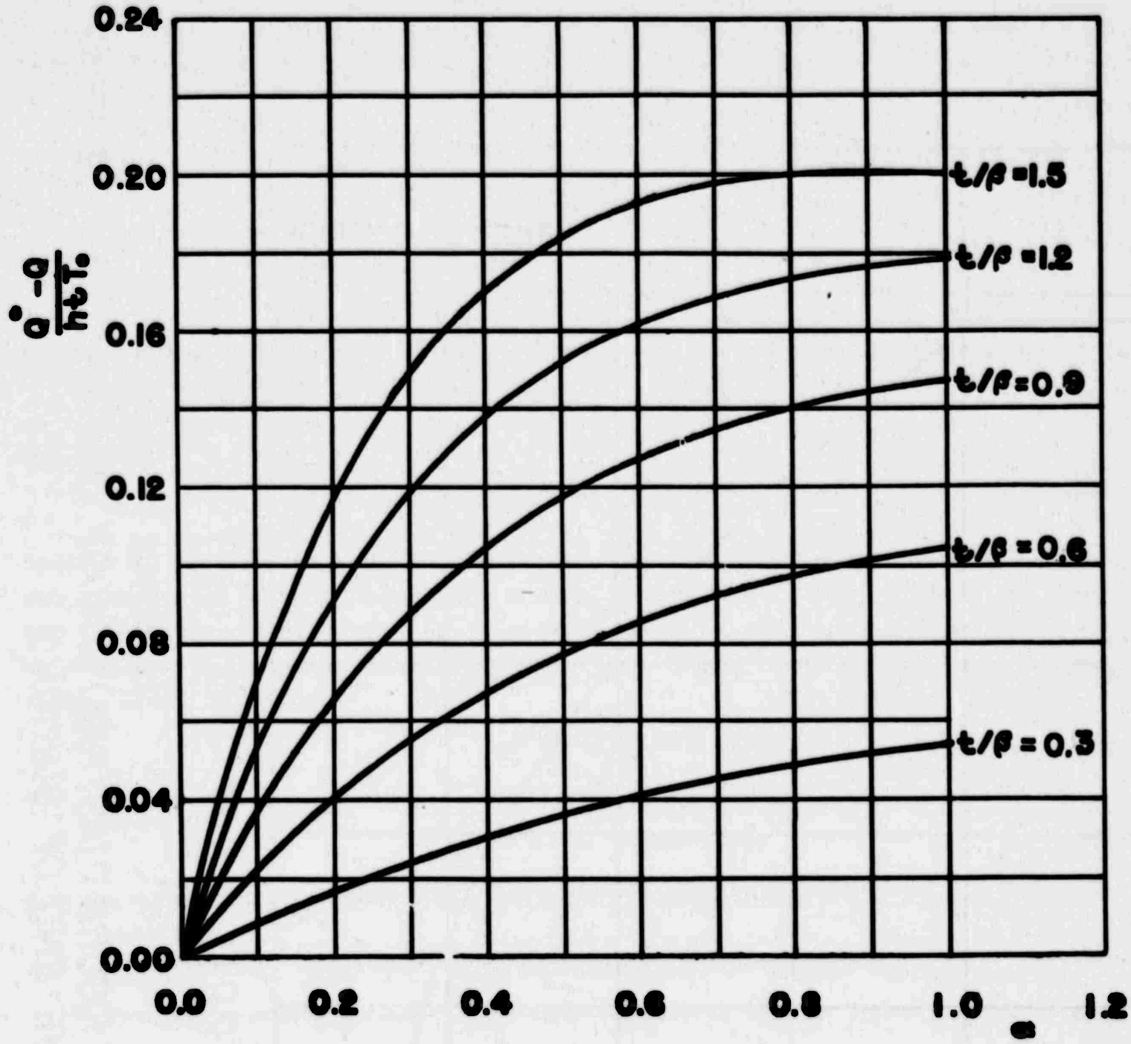

FIGURE 8. DEVIATION CURVES SHOWING THE RELATION BETWEEN QUANTITIES OF HEAT TRANSFERRED TO A SEMI-INFINITE SOLID AND TO FINITE CYLINDERS.

APPENDIX

Derivation of Equations

1. Solution by Heaviside method

The differential equation for the rate of transfer of heat across unit area of interface between the mediums is

$$d^2T/dx^2 = q^2T, \tag{1}$$

in which

$$q^2 = \frac{c}{k}\frac{\partial}{\partial t}. \tag{2}$$

The complete solution of Eq. (1), considered as a total differential equation in T and x, is

$$T = e^{qx}A + e^{-qx}B; \tag{3}$$

A and B are functions of q and T_o, chosen to fit the boundary conditions, which are that

$$(\partial T/\partial x)_{x=a} = 0 \;, \tag{4}$$

and

$$h(T_o - T_1) = -k(\partial T/\partial x)_{x=o} \,. \tag{5}$$

Equation (4) expresses the condition that no heat flows through the inward side of the wall (the side away from the hot gas), and Eq. (5), in which T_1 is written for $T(0)$, expresses the condition that the flow of heat through the outer side, that is, at the interface, is determined by the difference in temperature across the boundary layer of gas. When we insert T from Eq. (3) in Eqs. (4) and (5), we get

$$qe^{qa}A - qe^{-qa}B = 0 \tag{6}$$

and

$$h(T_o - A - B) = -k(qA - qB). \tag{7}$$

With the values of A and B given by Eqs. (6) and (7), Eq. (3) becomes[7]

$$T = \frac{h \cosh q\,(a - x)}{h \cosh qa + kq \sinh qa}T_o. \tag{8}$$

To simplify the notation and make the time-differentiating operator appear explicitly, we put

$$\alpha \equiv ah/k, \tag{9}$$

$$\beta \equiv a^2c/k, \tag{10}$$

$$D \equiv \partial/\partial t. \tag{11}$$

---

[7] It is understood that all operands contain as a factor a function of t that is zero for $t < 0$ and unity for $t > 0$.

Then we have

$$T = \frac{\alpha \cosh \sqrt{\beta D}\,(1 - x/a)}{\alpha \cosh \sqrt{\beta D} + \sqrt{\beta D}\sinh \sqrt{\beta D}}\,T_o. \tag{12}$$

Heaviside has shown[8] that operational solutions of this sort may be expressed in algebraic form as a series, developed in the following way. If we have a solution,

$$V = \frac{1}{F(D)}\,G(D), \tag{14}$$

it may be expressed algebraically as

$$V = \frac{G(0)}{F(0)} + \sum \frac{e^{Pt}G(P)}{P\,F'(P)}. \tag{15}$$

The summation is over the values of P, the roots of the equation $F(P) = 0$. The symbol $F'(P)$ is used to denote the derivative of $F(D)$ with respect to D, evaluated at P. To apply this theorem to Eq. (12), we take

$$F(D) = \alpha \cosh \sqrt{\beta D} + \sqrt{\beta D}\sinh \sqrt{\beta D}\ , \tag{13}$$

and

$$G(D) = \alpha T_o \cosh \sqrt{\beta D}\,(1 - x/a). \tag{13'}$$

Then we require the roots in P of the equation $F(P) = 0$. It may be shown[9] that all the roots of the equation

$$\cosh z + z \sinh z = 0$$

are pure imaginary numbers, so that all the roots P are negative real numbers. Thus it is convenient to substitute $i\theta$ for $\sqrt{\beta D}$, and use the roots of the equivalent equation

$$H(\theta) = \alpha \cos \theta - \theta \sin \theta = 0. \tag{16}$$

We now have

$$\begin{aligned} PF'(P) &= \tfrac{1}{2}\theta_n H'(\theta_n) \\ &= \tfrac{1}{2}\theta_n\,[-\alpha \sin \theta_n - \sin\theta_n - \theta_n \cos \theta_n], \end{aligned}$$

where the θ_n are roots of Eq. (16) with the index n indicating the greatest multiple of π not exceeding θ_n. Then, by Eq. (16),

$$PF'(P) = -\tfrac{1}{2}\,[\alpha(\alpha + 1) + \theta_n^2]\cos \theta_n.$$

Since $P = -\theta_n^2/\beta$, the resulting expression for the temperature is

$$T = T_o - T_o \sum_{n=0}^{\infty} \frac{2\alpha e^{-\theta_n^2 t/\beta}\cos\,[\theta_n\,(1 - x/a)]}{[\alpha(\alpha + 1) + \theta_n^2]\cos \theta_n}. \tag{17}$$

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[8] Electromagnetic theory, vol. 2, p. 127.

[9] Churchill, Modern operational mathematics in engineering, p. 258.

At the end of the burning time, say at the time t_1, the distribution of temperature may be represented by a Fourier series in the form

$$T(t_1) = T_o(f_o + \sum_{m=1}^{\infty} f_m \cos m\pi x/a). \tag{18}$$

This form is chosen to make the distribution symmetrical about $x = 0$ and $x = a$, to conform with the assumption that no heat flows through the surfaces at any time after t_1. The coefficients f_m may be evaluated in the usual way, giving the results

$$f_o = 1 - \sum_{n=o}^{\infty} \frac{2\alpha^2 e^{-\theta_n^2 t_1/\beta}}{\theta_n^2[\alpha(\alpha+1) + \theta_n^2]}, \tag{19}$$

$$f_m = \sum_{n=o}^{\infty} \frac{4\alpha^2 e^{-\theta_n^2 t_1/\beta}}{(m^2\pi^2 - \theta_n^2)[\alpha(\alpha+1) + \theta_n^2]}, \quad (m > 0). \tag{20}$$

Then the distribution of temperature after burning is given by the expression,

$$T = T_o\left\{f_o + \sum_{m=1}^{\infty} f_m \exp\left[-\frac{m^2\pi^2 (t - t_1)}{\beta}\right] \cos m\pi x/a\right\}. \tag{21}$$

2. <u>Heat transfer to insulating layer</u>

In order to get an upper bound for the heat transferred to the wall covered with an insulating layer, we assume that the interface between the insulator and the steel is maintained at the initial temperature. If we apply the same symbols for the layer of insulation that we used for the wall proper, we have the condition that $T = 0$ when $x = a$, in addition to the condition of Eq. (5). We use the solution in the form of Eq. (3), and get Eq. (7) and the equation

$$e^{qa}A + e^{-qa}B = 0 \tag{22}$$

for the evaluation of $\underline{A}$ and $\underline{B}$. The resulting operational solution for T_1 is

$$T_1 = \frac{h}{k} \frac{\sinh qa}{(h/k)\sinh qa + q\cosh qa} T_o. \tag{23}$$

The heat transferred to unit area from the gas in the time $\underline{t}$ is

$$Q = \int_0^t h(T_o - T_1)dt,$$

or

$$Q = hD^{-1} \frac{q\cosh qa}{(h/k)\sinh qa + q\cosh qa} T_o. \tag{24}$$

Now we substitute from Eqs. (9, 10, and 11) and obtain

$$Q = hD^{-1} \frac{\sqrt{\beta D}\cosh\sqrt{\beta D}}{\alpha\sinh\sqrt{\beta D} + \sqrt{\beta D}\cosh\sqrt{\beta D}} T_o. \tag{25}$$

The expansion theorem may be applied here to give

$$Q = hT_o D^{-1}\left[\frac{1}{\alpha + 1} + 2\alpha \sum \frac{1 - e^{-\theta^2 t/\beta}}{\alpha(\alpha + 1) + \theta^2}\right], \tag{26}$$

where the values of θ are the positive roots of the equation

$$\alpha \sin\theta + \theta\cos\theta = 0. \tag{27}$$

Integration of Eq. (26) gives the result

$$Q = hT_o\left\{\frac{t}{\alpha + 1} + 2\alpha\beta \sum \frac{1 - e^{-\theta^2 t/\beta}}{\theta^2[\alpha(\alpha + 1) + \theta^2]}\right\}. \tag{28}$$

3. Transfer of heat to a semi-infinite solid

In this case the boundary conditions are (i) that the temperature approaches zero as x increases without limit, and (ii) the condition of Eq. (5). The operational form of the solution for the temperature of the surface is

$$T_1 = \frac{h}{h + kq} T_o. \tag{29}$$

This solution may be expanded in either ascending or descending powers of D, giving asymptotic and convergent series, respectively, for T_1. The convergent series is convenient for the intervals of time of interest in this report. The quantity actually wanted is Q, or $hD^{-1}(T_o - T_1)$, which is found to be

$$\begin{aligned} Q &= \frac{hD^{-1}}{1 + (h/k)q^{-1}} T_o \\ &= hT_o \frac{1}{1 + \frac{h}{\sqrt{ck}} D^{-1/2}} t. \end{aligned} \tag{30}$$

We may expand the fraction in Eq. (30) by the binomial theorem and use the resulting operators according to the generalized definition,

$$D^n t^m = \frac{\Gamma(m + 1)}{\Gamma(m - n + 1)} t^{m-n}, \tag{31}$$

applicable only for power series in t. The result is

$$Q = hT_o t \sum_{n=0}^{\infty} \frac{(-1)^n (h\sqrt{t/ck})^n}{\Gamma(\frac{1}{2}n + 2)}. \tag{32}$$

4. Distribution of temperature in cylinders

The general solution of the equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = q^2 T \quad ,$$

may be written as

$$T = J_o(iqr)\,A + Y_o(iqr)\,B,$$

where J_o and Y_o are the Bessel functions of the first and second kinds and zeroth order. Since we require the temperature to be finite when $r = 0$, we put $B = 0$ and so can immediately write

$$T = \frac{J_o(iqr)}{J_o(iqr_1)}\,T_1\ , \qquad (33)$$

in which the subscripts 1 refer to the surface of the cylinder. The remaining boundary condition is

$$h(T_o - T_1) = k\left(\frac{dT}{dr}\right)_{r=r_1}. \qquad (34)$$

From Eqs. (33) and (34) we find that

$$T = \frac{hJ_o(iqr)}{hJ_o(iqr_1) + ikq\,J_o'(iqr_1)}\,T_o. \qquad (35)$$

This equation is exactly analogous to Eq. (8) for the plane wall.

Now put $\alpha = r_1h/k$ and $\beta = r_1^2\,c/k$. Then

$$T = \frac{\alpha J_o(\sqrt{-\beta D}\,r/r_1)}{\alpha J_o(\sqrt{-\beta D}) + \sqrt{-\beta D}\,J_o'(\sqrt{-\beta D})}\,T_o. \qquad (36)$$

In order to get into the denominator a function that has real roots, we make the substitution $\phi = \sqrt{-\beta D}$. The denominator of the general term in the solution expanded according to Heaviside's expansion theorem is

$$\frac{1}{2}\phi_n\frac{\partial}{\partial\phi_n}[\alpha J_o(\phi_n) + \phi_n J_o'(\phi_n)] = \frac{1}{2}[\alpha\phi_n J_o'(\phi_n) + \phi_n J_o'(\phi_n) + \phi_n^2 J_o''(\phi_n)]$$

$$= \frac{1}{2}\left\{\alpha\phi_n J_o'(\phi_n) + \phi_n J_o'(\phi_n) + \phi_n^2\left[-J_o(\phi_n) - \frac{1}{\phi_n}J_o'(\phi_n)\right]\right\},$$

or

$$\frac{1}{2}\phi_n\frac{\partial}{\partial\phi_n}[\alpha J_o(\phi_n) + \phi_n J_o'(\phi_n)] = -\frac{1}{2}[\alpha^2 J_o(\phi_n) + \phi_n^2 J_o(\phi_n)]$$

$$= -\frac{1}{2}(\alpha^2 + \phi_n^2)\,J_o(\phi_n).$$

Thus the algebraic form of $\underline{T}$ is

$$T = T_o - T_o\sum_{n=o}^{\infty}\frac{2\alpha e^{-\phi_n^2 t/\beta}\,J_o(\phi_n r/r_1)}{(\alpha^2 + \phi_n^2)\,J_o(\phi_n)}, \qquad (37)$$

with the ϕ_n given as the positive roots of the equation

$$\alpha J_o(\phi) = \phi J_1(\phi). \tag{38}$$

The last equation depends on the relation $J_1(\phi) = -J_o'(\phi)$.

In the case of the cylinder we may take advantage of the possibility of expanding the Bessel functions in asymptotic series to get an asymptotic expansion for T_1, useful for calculating either the surface temperature or the heat transferred at small values of the time. We take the solution in the form of Eq. (36) and use the asymptotic expansion[10]

$$J_o(ix) = \frac{e^x}{\sqrt{2\pi x}} \sum_{n=o}^{\infty} \frac{[\Gamma(n+\frac{1}{2})]^2}{n!\,(2x)^n}. \tag{39}$$

Since $Q = D^{-1}h(T_o - T_1)$, we get from Eq. (36)

$$Q = hD^{-1} \frac{\sqrt{-\beta D}\, J_o'(\sqrt{-\beta D})}{\alpha J_o(\sqrt{-\beta D}) + \sqrt{-\beta D}\, J_o'(\sqrt{-\beta D})} T_o. \tag{40}$$

On carrying out the division of the series for J_o and J_o', we get the expression

$$Q = hD^{-1} \frac{1}{1 + \frac{\alpha}{\sqrt{\beta D}}[1 + 2u + 6u^2 + 24u^3 + 126u^4 + \dots]} T_o, \tag{41}$$

in which $\underline{u}$ is written for $\frac{1}{2}(\beta D)^{-1/2}$. We may now invert the series in the denominator to get

$$\begin{aligned} Q = hD^{-1} [&1 - 4\alpha u + (16\alpha^2 - 8\alpha)u^2 - (64\alpha^3 - 64\alpha^2 + 24\alpha)u^3 \\ &+ (256\alpha^4 - 384\alpha^3 + 256\alpha^2 - 96\alpha)u^4 \\ &- (1024\alpha^5 - 2048\alpha^4 + 1920\alpha^3 - 1152\alpha^2 + 504\alpha)u^5 + \dots]T_o \end{aligned}$$

or

$$\begin{aligned} Q = hD^{-1} [&1 - \alpha(\beta D)^{-1/2} + \alpha(\alpha - \tfrac{1}{2})(\beta D)^{-1} - \alpha(\alpha^2 - \alpha + \tfrac{3}{8})(\beta D)^{-3/2} + \\ &\alpha(\alpha^3 - \tfrac{3}{2}\alpha^2 + \alpha - \tfrac{3}{8})(\beta D)^{-2} - \alpha(\alpha^4 - 2\alpha^3 + \tfrac{15}{8}\alpha^2 - \tfrac{9}{8}\alpha + \tfrac{63}{128})(\beta D)^{-5/2} + \dots]T_o \end{aligned} \tag{42}$$

This equation may be integrated to give an asymptotic series for $\underline{Q}$ in the form

$$\begin{aligned} Q = htT_o\Bigg[&1 - \frac{\alpha}{\Gamma(\frac{5}{2})}\left(\frac{t}{\beta}\right)^{1/2} + \frac{\alpha(\alpha - \frac{1}{2})}{\Gamma(3)}\left(\frac{t}{\beta}\right) - \frac{\alpha(\alpha^2 - \alpha + \frac{3}{8})}{\Gamma(\frac{7}{2})}\left(\frac{t}{\beta}\right)^{3/2} \\ &+ \frac{\alpha(\alpha^3 - \frac{3}{2}\alpha^2 + \alpha - \frac{3}{8})}{\Gamma(4)}\left(\frac{t}{\beta}\right)^2 - \frac{\alpha(\alpha^4 - 2\alpha^3 + \frac{15}{8}\alpha^2 - \frac{9}{8}\alpha + \frac{63}{128})}{\Gamma(\frac{9}{2})}\left(\frac{t}{\beta}\right)^{5/2} + \dots\Bigg]. \end{aligned} \tag{43}$$

Equations (32) and (43) may be compared to bring out the effect of the cylindrical form on the heating. The terms in Eq. (43) corresponding to Eq. (32) are those multiplied by the highest

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[10] _Electromagnetic theory_, vol. 2, p. 240.

powers of $\underline{\alpha}$. We might rewrite Eq. (32) in the form

$$Q = htT_0\left[1 - \frac{\alpha}{\Gamma\left(\frac{5}{2}\right)}\left(\frac{t}{\beta}\right)^{1/2} + \frac{\alpha^2}{\Gamma(3)}\left(\frac{t}{\beta}\right) - \frac{\alpha^3}{\Gamma\left(\frac{7}{2}\right)}\left(\frac{t}{\beta}\right)^{3/2} + \dots\right],$$

to exhibit the correspondence between the two series.

5. Solution of $\alpha \cos \theta = \theta \sin \theta$

We give at the end of this section a table (A-I) of solutions of the equation $\underline{\alpha} \cos \theta = \theta \sin \theta$. Results for θ_1, θ_2 and θ_3 (good to within six, two, and one in the fourth decimal place respectively for all values of $\underline{\alpha}$ between 0 and 5) may be obtained by linear interpolation in Table A-I.

For small values of $\underline{\alpha}$ we can proceed as shown in the following illustration:

Example 1: Given $\alpha = 0.14$; find θ_0

From the given table we find $0 < \theta_0 < 0.48$. A few rapid trials with a table of trigonometric functions and a slide rule or other calculating device narrowed $\underline{\theta}$ down to the interval between 0.36 and 0.37; and in fact we compute:

$$0.36 \tan(0.36) = 0.13550$$
$$0.32 \tan(0.37) = 0.14351$$

The usual interpolation procedure gives

$$\theta_0 = 0.36 + \frac{0.14000 - 0.13550}{0.14351 - 0.13550} \times 0.01 = 0.36562.$$

We have used a table in which the trigonometric tangent is given to five decimal places and the argument in radians to two. Yet we have obtained accuracy within 4 units in the fourth decimal place for θ_0 as shown by comparison with the correct result $\theta_0 = 0.36565577$ which was got by using a more extensive table of tangents. The table actually used is that given in the "Handbook of Chemistry and Physics."

We now consider a case where the given value of $\underline{\alpha}$ is greater than 5 and hence lies outside of the range of the table.

Example 2: Given $\alpha = 6.5$; find θ_1.

We know that

$$\pi < \theta_1 < 2\pi.$$

If we write $\theta_1 = \pi + \phi$, we have to solve the equation

$$(\pi + \phi)\tan(\pi + \phi) = 6.5,$$

or, what comes to the same thing,

$$(\pi + \phi) \tan \phi = 6.5, \qquad 0 < \phi < \pi.$$

We get some assistance from the given table by noting that $\theta_1 = 4.033690$ for $\underline{\alpha} = 5.0$ and that the difference in θ_1 for each interval of 0.25 in $\underline{\alpha}$ is roughly 0.02. Hence we might try as an initial value of ϕ,

$$\phi = 4.033690 + 6(0.02) - 3.141593 \simeq 1.01.$$

This turns out to be a good guess, and we compute:

$$(3.141593 + 1.00) \tan (1.00) = 6.4501$$

$$(3.141593 + 1.01) \tan (1.01) = 6.6102.$$

By interpolation

$$\phi = 1.00 + \frac{6.5000 - 6.4501}{6.6102 - 6.4501} \times 0.01 = 1.00312^{11/},$$

so that

$$\theta_1 = \pi + \phi = 3.14159 + 1.00312 = 4.14471.$$

Finally, we show how to find the value of θ_n if $n > 3$.

<u>Example 3</u>: Given $\alpha = 3.0$; find θ_7

Since $7\pi < \theta_7 < 8\pi$,

let

$$\theta_7 = 7\pi + \phi = 21.991151 + \phi, \qquad 0 < \phi < \pi,$$

so that $\theta_7 \tan \theta_7 = 3.0$ is equivalent to

$$(\phi + 7\pi) \tan \phi = 3.0.$$

We know that the greater the value of $\underline{n}$, the closer does θ_n lie to $n\pi$. Hence ϕ is rather small and we seek its value in that part of the table where the argument is small. By trial we find

$$(21.991151 + 0.13) \tan (0.13) = 2.8921$$
$$(21.991151 + 0.14) \tan (0.14) = 3.1187.$$

By interpolation,

$$\phi = 0.13 + \frac{3.0000 - 2.8921}{3.1187 - 2.8921} \times 0.01 = 0.13476^{12/},$$

hence

$$\theta_7 = 21.99115 + 0.13476 = 22.12591.$$

11/ This may be compared with the correct value, $\phi_1 = 1.00315011$.

12/ The correct value of ϕ is 0.13476578.

Some fairly accurate formulas for Θ_n as a function of $\underline{\alpha}$ have been devised. For instance,

$$\Theta_0 \simeq \sqrt{\frac{(45\alpha + 105) - \sqrt{(45\alpha + 105)^2 - (420\alpha^2 + 4200\alpha)}}{2\alpha + 20}}, \tag{44}$$

is good to within one unit in the fourth decimal place for $0 \leq \alpha \leq 5$.

Also

$$\Theta_n \simeq \frac{n\pi(2\alpha + 3) + \sqrt{9\pi^2 n^2 + 36\alpha + 12\alpha^2}}{2\alpha + 6}, \tag{45}$$

is quite good for large $\underline{n}$. Even for small $\underline{n}$ it is fair, except $n = 0$. For $n = 1$, it is good to within one unit in the second decimal place for $0 \leq \alpha \leq 5$. For $n = 2$, it is good to within two units in the third decimal place for $0 \leq \alpha \leq 5$. For $n = 3$, it is good to within five units in the fourth decimal place for $0 \leq \alpha \leq 5$.

We give the above formulas purely as mathematical curiosities, since experience has shown that for the purpose of computing Θ_n for a specific $\underline{n}$ and $\underline{\alpha}$, the interpolation method given above is far handier than use of either (44) or (45).

6. Solution by the Laplace Transformation

We now obtain a solution to the problem treated in Sec. 1 suitable for computation with small values of the time $\underline{t}$.

It is required to obtain solutions $T(x,t)$ for the "heat equation"

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}, \tag{46}$$

under the boundary conditions

$$\left(\frac{\partial T}{\partial x}\right)_{x=0} = -\alpha\left[1 - T(0,t)\right], \tag{47}$$

$$\left(\frac{\partial T}{\partial x}\right)_{x=1} = 0, \tag{48}$$

and the initial condition

$$T(x,0) = 0, \tag{49}$$

where $\alpha \geq 0$.

If we denote the Laplace transform of $\underline{T}$ with respect to $\underline{t}$ by T_L then we have by definition

$$T_L = T_L(x,s) = \int_0^\infty e^{-st}\, T(x,t)\, dt. \tag{50}$$

From (46), (47) and (48) respectively we get

$$\frac{\partial^2 T_L}{\partial x^2} = sT_L, \tag{51}$$

$$\left(\frac{\partial T_L}{\partial x}\right)_{x=0} = -\alpha\left[\frac{1}{s} - T_L\,(o,s)\right], \tag{52}$$

$$\left(\frac{\partial T_L}{\partial x}\right)_{x=1} = 0 \quad . \tag{53}$$

The solution of (51) as an ordinary differential equation in T_L and $\underline{x}$ is

$$T_L = A \cosh x\sqrt{s} + B \sinh x\sqrt{s} \quad ,$$

where $\underline{A}$ and $\underline{B}$ are functions of $\underline{s}$ alone, to be determined from the conditions (52) and (53). They are easily found to be

$$A = \frac{\alpha}{s(\sqrt{s}\tanh\sqrt{s} + \alpha)} \quad \text{and } B = -\frac{\alpha\tanh\sqrt{s}}{s(\sqrt{s}\tanh\sqrt{s} + \alpha)} \quad .$$

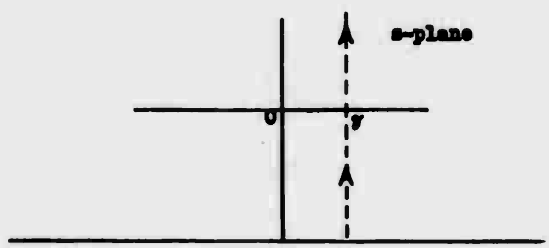
Hence, finally,

$$T_L\,(x,s) = \frac{\alpha\cosh\left[\sqrt{s}\,(1-x)\right]}{s(\sqrt{s}\sinh\sqrt{s} + \alpha\cosh\sqrt{s})} \quad . \tag{54}$$

Now in order to find T (x,t) we have to find the inverse of T_L (x,s) with respect to the Laplace transformation. Hence[13]

$$T\,(x,t) = \frac{\alpha}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{e^{st}\cosh\left[\sqrt{s}\,(1-x)\right]\,ds}{s(\sqrt{s}\sinh\sqrt{s} + \alpha\cosh\sqrt{s})} \quad , \tag{55}$$

where $\underline{\gamma}$ is any real number such that all the poles of the integrand have a real part less than $\underline{\gamma}$. The origin is clearly a pole and it can be shown[14] that all the other poles lie on the negative half of the real axis. Hence it is sufficient to take $\gamma > 0$ (Sketch 1).



SKETCH 1

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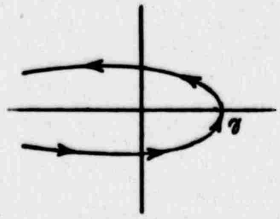
[13] Churchill: "Modern Operational Mathematics in Engineering," p. 159 Theorem 5.
[14] Churchill: "Modern Operational Mathematics in Engineering," Chap. IX particularly p. 258.

In order to evaluate the integral (55) we made a change of path and change of variable. First we deform the path shown in Sketch 1 into a parabola in the s-plane whose equation is

$$R(s) = \gamma - \frac{[I(s)]^2}{4\gamma}$$

This path is shown in Sketch 2.



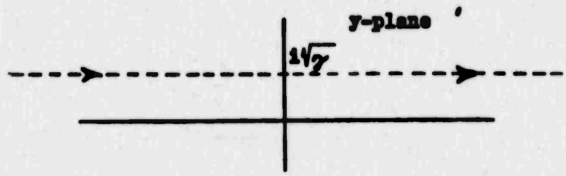
SKETCH 2

It will be noted that this deformation can be effected without crossing any of the poles of the integrand. It may also be verified that because of the exponential factor in the integrand the portion of the path joining the line to the parabola contributes nothing, in the limit, to the value of the integral. Hence the value of the integral is not altered.

Now we change the variable of integration by means of the transformation

$$s = -y^2$$

The path of Sketch 2 becomes, after reversing the sense of integration, a straight line in the y-plane parallel to the real axis and at a distance $\sqrt{\gamma}$ above it (Sketch 3).



SKETCH 3

With this change of variable, the integral (55) becomes

$$T(x,t) = -\frac{\alpha}{\pi i}\int_L \frac{e^{-ty^2}\cos[y(1-x)]\,dy}{y(\sin y - h\cos y)} \tag{56}$$

where we denote by L the path of integration shown in Sketch 3. Now we replace the trigonometric functions appearing in the integrand by their values in terms of exponentials,

$$\sin y = \frac{e^{iy} - e^{-iy}}{2i}; \qquad \cos y = \frac{e^{iy} + e^{-iy}}{2i}$$

and then expand the integrands in powers of e^{iy}.

Thus we get

$$T\,(x,t) = -\frac{\alpha}{\pi}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-ty^2}}{y(y+i\alpha)}\left(\frac{y-i\alpha}{y+i\alpha}\right)^n e^{iy(2n+x)}dy$$

$$-\frac{\alpha}{\pi}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-ty^2}}{y(y+i\alpha)}\left(\frac{y-i\alpha}{y+i\alpha}\right)^n e^{iy(2n+2-x)}dy\,.$$

Upon making the change of variable $u = ty^2$ this becomes

$$T(x,t) = -\frac{\alpha t^{3/2}}{\pi}\sum_{n=0}^{\infty} e^{\frac{-(x+2n)^2}{4t}}\int_{-\infty}^{\infty} e^{-u^2}\frac{\left[\sqrt{t}u + i\left(n+\frac{x}{2}-t\alpha\right)\right]^n du}{\left[\sqrt{t}u + i\left(n+\frac{x}{2}\right)\right]\left[\sqrt{t}u + i\left(n+\frac{x}{2}+t\alpha\right)\right]^{n+1}}$$

$$-\frac{\alpha t^{3/2}}{\pi}\sum_{n=0}^{\infty} e^{\frac{-(2n+2-x)^2}{4t}}\int_{-\infty}^{\infty} e^{-u^2}\frac{[\sqrt{t}u + i\,(2n+2-x-t\alpha)]^n\,du}{\left[\sqrt{t}u+\frac{1}{2}\,(2n+2-x)\right]\left[\sqrt{t}u + i\,(2n+2-x+t\alpha)\right]^{n+1}}\,. \tag{57}$$

We now proceed to the evaluation of the integrals appearing in (57). As a matter of convenience we note that these integrals are essentially of the form

$$I_n = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{e^{-w^2}(w+ic)^n\,dw}{(w+ia)(w+ib)^{n+1}}\,, \tag{58}$$

where $\underline{n}$ is a non-negative integer. We shall make special use of the cases $n = 0$ and $n = 1$.

7. Evaluation of I_n

Let us define the function

$$F(a) = e^{a^2}\int_a^{\infty} e^{-x^2}dx = e^{a^2}\left[\frac{\sqrt{\pi}}{2} - \int_0^a e^{-x^2}dx\right]. \tag{59}$$

Then

$$F'(a) = 2a\,F(a) - 1\,. \tag{60}$$

From a well-known definite integral,

$$1 = \frac{2x}{\sqrt{\pi}}\int_0^{\infty} e^{-x^2y^2}dy;$$

whence

$$e^{-x^2} = \frac{2x}{\sqrt{\pi}}\int_0^{\infty} e^{-x^2(1+y^2)}dy.$$

Integration of both sides with respect to $\underline{x}$ from $\underline{a}$ to $\underline{\infty}$ gives

$$\int_a^\infty e^{-x^2}\,dx = \frac{1}{\sqrt{\pi}}\int_0^\infty \frac{e^{-a^2(1+y^2)}}{1+y^2}\,dy,$$

so that

$$F(a) = \frac{1}{\sqrt{\pi}}\int_0^\infty \frac{e^{-a^2y^2}}{1+y^2}\,dy. \tag{61}$$

Setting $y = \frac{w}{a}$, we get

$$F(a) = \frac{a}{\sqrt{\pi}}\int_0^\infty \frac{e^{-w^2}}{w^2+a^2}\,dw. \tag{62}$$

Using the integral expression for $\underline{F}$ given in (61) we can write

$$2xe^{-s^2x^2}F(x) = \frac{2x}{\sqrt{\pi}}\int_0^\infty \frac{e^{-(s^2+y^2)x^2}}{1+y^2}\,dy.$$

Integrating with respect to $\underline{x}$ from $\underline{a}$ to $\underline{\infty}$ gives

$$\int_a^\infty 2xe^{-s^2x^2}F(x)\,dx = \frac{1}{\sqrt{\pi}}\int_0^\infty \frac{e^{-(s^2+y^2)a^2}}{(1+y^2)(s^2+y^2)}\,dy. \tag{63}$$

We note the following relation, which can be easily verified by differentiating both sides with respect to $\underline{a}$:

$$\int_a^\infty 2xe^{-s^2x^2}F(x)dx = \frac{e^{-s^2a^2}}{s^2-1}\left[F(a) - \frac{1}{s}F(sa)\right].$$

Equating the right-hand side of this identity with that of (63) and multiplying both sides by $e^{s^2a^2}$ we get

$$\frac{1}{s^2-1}\left[F(a) - \frac{1}{s}F(sa)\right] = \frac{1}{\sqrt{\pi}}\int_0^\infty \frac{e^{-y^2a^2}}{(1+y^2)(s^2+y^2)}\,dy.$$

Upon making the substitutions

$$y = \frac{w}{a}, \quad s = \frac{b}{a};$$

this becomes

$$\frac{1}{b^2-a^2}\left[bF(a) - aF(b)\right] = \frac{ab}{\sqrt{\pi}}\int_0^\infty \frac{e^{-w^2}dw}{(w^2+a^2)(w^2+b^2)}. \tag{64}$$

From (62),

$$\frac{F(a)}{a} = \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{e^{-w^2}(w^2 + b^2)\, dw}{(w^2 + a^2)(w^2 + b^2)} ,$$

$$= \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{w^2 e^{-w^2}\, dw}{(w^2 + a^2)(w^2 + b^2)} + \frac{b^2}{\sqrt{\pi}} \int_0^\infty \frac{e^{-w^2}\, dw}{(w^2 + a^2)(w^2 + b^2)} .$$

If for the right most integral we substitute its value as given by (64), we get

$$\frac{1}{\sqrt{\pi}} \int_0^\infty \frac{w^2 e^{-w^2}\, dw}{(w^2 + a^2)(w^2 + b^2)} = \frac{1}{(b^2 - a^2)} \Big[bF(b) - aF(a) \Big] . \tag{65}$$

We have

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^\infty \frac{w e^{-w^2}\, dw}{(w^2 + a^2)(w^2 + b^2)} = 0 ,$$

since the integrand is an odd function of the variable of integration.

By use of this and (64) and (65),

$$I_0 = \frac{1}{\sqrt{\pi}} \int_{-\infty}^\infty \frac{e^{-w^2}\, dw}{(w + ia)(w + ib)} ,$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^\infty \frac{w^2 e^{-w^2}\, dw}{(w^2 + a^2)(w^2 + b^2)} - \frac{i\,(b + a)}{\sqrt{\pi}} \int_{-\infty}^\infty \frac{w e^{-w^2}\, dw}{(w^2 + a^2)(w^2 + b^2)}$$

$$- \frac{ab}{\sqrt{\pi}} \int_{-\infty}^\infty \frac{e^{-w^2}\, dw}{(w^2 + a^2)(w^2 + b^2)} ,$$

$$= \frac{2}{b - a} \Big[F(b) - F(a) \Big] . \tag{66}$$

Differentiation of both sides of (66) with respect to <u>b</u> gives

$$- \frac{i}{\sqrt{\pi}} \int_{-\infty}^\infty \frac{e^{-w^2}\, dw}{(w + ia)(w + ib)^2} = \frac{2}{b - a} \Big[2bF(b) - 1 \Big] - \frac{2}{(b - a)^2} \Big[F(b) - F(a) \Big] . \tag{67}$$

Now

$$\frac{\partial^m}{\partial b^m} I_0 = \frac{(-1)^m m!}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-w^2}\,dw}{(w + ia)[w + ib]^{m+1}} ,$$

$$I_n = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-w^2}\,[(w + ib) + i(c - b)]^n}{(w + ia)[w + ib]^{n+1}}\,dw ,$$

$$= \frac{1}{\sqrt{\pi}} \sum_{m=0}^{n} \frac{n!}{m!\,(n - m)!} \int_{-\infty}^{\infty} \frac{e^{-w^2}(w + ib)^{n-m}\,[i(c - b)]^m}{(w + ia)[w + ib]^{n+1}}\,dw ,$$

$$= \sum_{m=0}^{n} \frac{n!\,i^m(c - b)^m}{m!(n - m)!} \cdot \frac{1}{(-1)^m m!} \frac{\partial^m}{\partial b^m} I_0 ,$$

$$= \sum_{m=0}^{n} \frac{n!\,(b - c)^m}{(m!)^2(n - m)!} \frac{\partial^m}{\partial b^m} I_0 .$$

Since $\frac{\partial^m}{\partial b^m} I_0$ can be computed by differentiating both sides of (66) $\underline{m}$ times, we have a means of computing I_n. In particular, if we set $n = 1$ we get

$$I_1 = I_0 + (b-c)\frac{\partial I_0}{\partial b} = \frac{2}{b-a}\Big[F(b)-F(a)\Big] + 2(b-c)\left[\frac{F'(b)}{b-a} - \frac{F(b) - F(a)}{(b - a)^2}\right],$$

$$= \frac{2}{b - a}\Big[F(b) - F(a)\Big] + 2(b - c)\left[\frac{2bF(b) - 1}{b - a} - \frac{F(b) - F(a)}{(b - a)^2}\right]. \qquad (68)$$

When $\underline{a}$ differs but little from $\underline{b}$, one can get useful approximate formulas for I_0 and I_1. By Taylor's Series,

$$F(b) = F(a) + (b - a)\,F'(a) + \frac{(b - a)^2}{2!} F''(a) + \dots .$$

Hence by (66)

$$I_0 = 2\,[F'(a) + \frac{b - a}{2!} F''(a) + \frac{(b - a)^2}{3!} F'''(a) + \dots] .$$

Then

$$I_1 = I_0 + (b - c)\frac{\partial}{\partial b} I_0$$

$$= 2\,[F'(a) + \frac{b - a}{2!} F''(a) + \frac{(b - a)^2}{3!} F'''(a) + \dots]$$

$$+ 2\,(b - c)\left[\frac{F''(a)}{2} + (b - a)\,\frac{F'''(a)}{3} + \dots\right],$$

$$= 2\left[F'(a) + \frac{2b - a - c}{2!} F''(a) + \dots\right] .$$

By (60)

$$F'(a) = 2aF(a) - 1 ,$$

$$F''(a) = (4a^2 + a) F(a) - 2a ,$$

so that

$$I_1 \cong 2 [k F(a) + \ell] ,$$

where

$$k = 2b - c + a + 2a^2(2b - c - a),$$

and

$$\ell = a(2b - c - a) - 1.$$

A table (A-II) of $F(x)$ for x between 0 and 6 is provided. For $x \geq 1$, linear interpolation is good to four decimal places, and for $0 \leq x \leq 1$, linear interpolation is good to three decimal places.

Table A-I. Positive roots of $\theta \sin \theta = \alpha \cos \theta$

α	θ_0	θ_1	θ_2	θ_3
0.00	0.000000	3.141593	6.283185	9.424778
.25	.480094	3.219099	6.322705	9.451223
.50	.653271	3.292309	6.361620	9.477486
.75	.771359	3.361135	6.399844	9.503533
1.00	.860334	3.425618	6.437298	9.529334
1.25	.930756	3.485897	6.473921	9.554863
1.50	.988241	3.542166	6.509659	9.580092
1.75	1.036197	3.594652	6.544473	9.604998
2.00	1.076874	3.643597	6.578334	9.629560
2.25	1.111839	3.689246	6.611220	9.653760
2.50	1.142227	3.731838	6.643121	9.677580
2.75	1.168884	3.771604	6.674032	9.701007
3.00	1.192459	3.808765	6.703956	9.724027
3.25	1.213455	3.843514	6.732901	9.746632
3.50	1.232272	3.876050	6.760880	9.768813
3.75	1.249230	3.906545	6.787910	9.790563
4.00	1.264592	3.935157	6.814014	9.811878
4.25	1.278569	3.962036	6.839204	9.832755
4.50	1.291341	3.987314	6.863515	9.853193
4.75	1.303056	4.011116	6.886970	9.873192
5.00	1.313837	4.033570	6.909595	9.892753

Table A-II.* Table of $F(x) = e^{x^2} \int_x^{\infty} e^{-y^2} dy$

x	F(x)	x	F(x)	x	F(x)	x	F(x)
0.00	0.8862	1.25	0.3260	2.50	0.1868	3.75	0.1290
0.05	.8384	1.30	.3170	2.55	.1836	3.80	.1274
0.10	.7945	1.35	.3084	2.60	.1804	3.85	
0.15	.7541	1.40	.3002	2.65	.1774	3.90	.1244
0.20	.7170	1.45	.2924	2.70	.1745	4.00	.1214
0.25	.6827	1.50	.2850	2.75	.1716	4.1	.1186
0.30	.6510	1.55	2779	2.80	.1689	4.2	.1159
0.35	.6217	1.60	.2711	2.85	.1662	4.3	.1134
0.40	.5945	1.65	.2647	2.90	.1636	4.4	.1109
0.45	.5692	1.70	.2585	2.95	.1611	4.5	.1085
0.50	.5456	1.75	.2526	3.00	.1586	4.6	.1063
0.55	.5237	1.80	.2469	3.05	.1563	4.7	.1041
0.60	.5032	1.85	.2414	3.10	.1540	4.8	.1020
0.65	.4840	1.90	.2362	3.15	.1517	4.9	.1000
0.70	.4661	1.95	.2312	3.20	.1495	5.0	.0981
0.75	.4493	2.00	.2263	3.25	.1474	5.1	.0963
0.80	.4335	2.05	.2217	3.30	.1453	5.2	.0945
0.85	.4186	2.10	.2172	3.35	.1433	5.3	.0927
0.90	.4046	2.15	.2129	3.40	.1414	5.4	.0911
0.95	.3914	2.20	.2088	3.45	.1395	5.5	.0895
1.00	.3789	2.25	.2048	3.50	.1376	5.6	.0879
1.05	.3672	2.30	.2009	3.55	.1358	5.7	.0864
1.10	.3560	2.35	.1972	3.60	.1341	5.8	.0850
1.15	.3455	2.40	.1936	3.65	.1323	5.9	.0836
1.20	.3355	2.45	.1902	3.70	.1307	6.0	.0822

*This table is abridged from Table 2, Column 2, given in ABL Final Report B-2.1 (CSRD 5861), "Part I: Methods of Computation," pp 208, 209, and 210.

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Problems are discussed on performance of rockets through cooling effect of heat transfer of the propellant gases, strength of chamber walls and trap, and high explosive stability. Application of calculations to 4-1/2-in. rocket indicates trap wires reach temperatures over 1000°C and wall of burster tube reaches temperature of 350°C. Thin layer of insulating material on outside of burster tube is adequate to keep temperature below safe limit.

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