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PARALLEL "T" STABILIZING NETWORKS FOR AC SERVOS

REPORT
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RADIATION LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE - MASSACHUSETTS

NDRC
Div. 14
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Radiation Laboratory

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PARALLEL "T" STABILIZING NETWORKS FOR AC SERVOS

Abstract

In Part I a method of analysis of linear carrier frequency systems is indicated, and applied in particular to parallel "T" servos using the Diehl two-phase control motor. Improvements are described by the Nyquist Bode procedure for analysis of a feedback amplifier or servo-mechanism.

Part II contains detailed design procedures and tables of values for various forms of parallel "T" networks. The several forms arise from the fact that there are five time constants in the parallel "T", four of which are independent. The remaining degrees of freedom may be used to obtain the most suitable input and output impedances for the source and load impedances with which the parallel "T" is to be used.

Part III contains design formulae and tables of values for Bridge "T" and Wien Bridge proportional-derivative networks.

In Part IV various methods of obtaining the required 90° phase difference of the 60-cycle voltages on the fixed and control windings of the Diehl motor are discussed and compared. In order to obtain a large phase shift it is necessary to add either a series input or a load impedance to the parallel "T", or to use a phase-shifting network preceding or following the parallel "T". Formulae and tables of values for phase lag networks are given.

In Part V tolerance requirements on the components of parallel "T", bridge "T", and phase lag networks are derived.

Finally in Part VI the method is indicated by means of which the design formulae of Part II were obtained.

A. Sobczyk

Approved by: N.B. Nichols
Leader, Group 83
I. A. Getting
Head, Division 8

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PARALLEL "T" STABILIZING NETWORKS FOR AC SERVOS

Part I - Introduction

CARRIER FREQUENCY SYSTEMS

The reader is assumed to be familiar with elementary Laplace transform theory, the modern equivalent of the Heaviside operational calculus. This subject is excellently presented in the recent book: Gardner-Barnes, Transients in Linear Systems, Vol. I.

1. Condition for Equivalence to a d.c. System; Quadrature Voltage

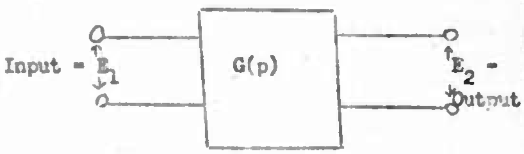


Fig. 1

The steady-state output of a linear system for a sinusoidal input may be calculated conveniently as follows. Let the transfer characteristic of the linear system be G(p), as in Fig. 1. In complex representation, for an input $E_1 = \exp(j\omega t)$, the output is $E_2 = G(j\omega) \exp(j\omega t)$. For an input $E_1 = \exp(-j\omega t)$, the output is $E_2 = \overline{G(j\omega)} \exp(-j\omega t)$. (A bar over a complex number indicates the conjugate complex number; thus if $N = A + jB$, then $\overline{N} = A - jB$.) To calculate the output for a real input, say $\cos \omega t$, apply the principle of superposition: $E_1 = \cos \omega t = \frac{1}{2} [\exp (j\omega t) + \exp (-j\omega t)]$, and the (real) output is $E_2 = \frac{1}{2} [G(j\omega)\exp(j\omega t) + \overline{G(j\omega)} \exp (-j\omega t)]$.

Let m denote a modulating angular frequency, ω_o the carrier angular frequency. Suppose a modulated carrier $E_1 = \cos mt \cos \omega_o t = \frac{1}{2} \{\exp[j(\omega_o + m)t] + \exp [-j(\omega_o + m)t] + \exp [j(\omega_o - m)t] + \exp [-j(\omega_o - m)t]\}$ is impressed on the linear system G(p). Then the output is $E_2 = \frac{1}{2}\mathcal{R} \{G[j(\omega_o + m)] \exp [j(\omega_o + m)t] + G[j(\omega_o - m)] \exp [j(\omega_o - m)t]\} = \frac{1}{2} \{|G[j(\omega_o + m)]| \cos [(\omega_o + m)t + \phi] + |G[j(\omega_o - m)]| \cos [(\omega_o - m)t + \psi]$ where $\phi = \angle G[j(\omega_o + m)], \psi = \angle G[j(\omega_o - m)]$. This may be expanded in the form $E_2 = \frac{1}{2}\left|G[j(\omega_o + m)]\right| \{\cos(mt + \phi_1)\cos(\omega_o t + \phi_2) - \sin(mt + \phi_1)\sin(\omega_o t + \phi_2)\} + \frac{1}{2}\left|G[j(\omega_o - m)]\right| \{\cos(mt + \psi_1)\cos(\omega_o t + \psi_2) + \sin(mt + \psi_1)\sin(\omega_o t + \psi_2)\}$ where $\phi_1, \phi_2, \psi_1, \psi_2$ are arbitrary angles such that $\phi_1 + \phi_2 = \phi$, $\psi_2 - \psi_1 = \psi$. We take $\psi_1 = \phi_1$, $\psi_2 = \phi_2$ to obtain the form $E_2 = \frac{1}{2}\{\left|G[j(\omega_o - m)]\right| + \left|G[j(\omega_o + m)]\right|\}\cos(mt + \phi_1)\cos(\omega_o t + \phi_2) + \frac{1}{2}\{\left|G[j(\omega_o - m)]\right| - \left|G[j(\omega_o + m)]\right|\}\sin(mt + \phi_1)\sin(\omega_o t + \phi_2)$, $\phi_1 = \frac{1}{2}(\phi - \psi)$, $\phi_2 = \frac{1}{2}(\phi + \psi)$, which is a resolution into a main component and a quadrature component having the minimum possible amplitude.

The amplitude of the main component is the average of the amplitudes $|G^+|$ and $|G^-|$ of the upper and lower sidebands; the phase of the carrier is the average of the phases of the respective sidebands, while the modulation phase is one-half the difference. The amplitude of the quadrature component is one-half the difference of the amplitudes of the lower and upper sidebands; if $|G^-| > |G^+|$ the quadrature component lags the main component, and the carrier and modulation phases of the quadrature component lag those of the main component by 90°. The relationships between the sidebands, main component (m.c.), quadrature component (q.c.), and their phase angles, are indicated vectorially in Fig. 1a.

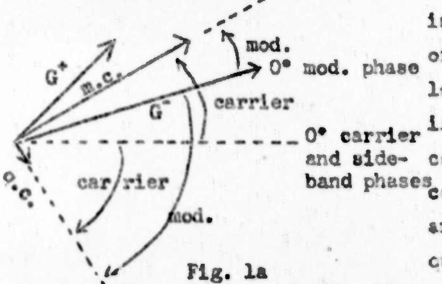
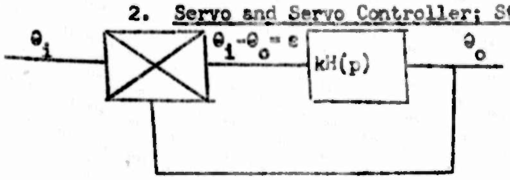


Fig. 1a

Evidently if the amplitude characteristic of the linear system G (p) is symmetrical about the carrier frequency, $|G[j(\omega_o - m)]| = |G[j(\omega_o + m)]|$, the quadrature component disappears, and the effect of G(p) is similar to the effect of a d.c. system on the modulation, with a phase shift ϕ_2 of the carrier. In this form, the phase shift ϕ_2 may vary with the modulating frequency; if desired a different resolution may be made with a constant carrier phase shift angle in the main component.

2. Servo and Servo Controller; Stability

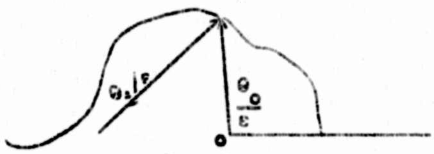


$\theta_o = kH(p)\varepsilon, \quad \frac{\theta_o}{\theta_i} = \frac{kH(p)}{1 + kH(p)}$

Fig. 2

Fig. 2 is the familiar servo diagram. The output θ_o is related to the error, or difference of the input θ_i and output, through the transfer characteristic H(p), multiplied by a gain factor k.

The portion of the servo marked k H(p) is called the servo controller, and k H(p) is the transfer characteristic for the servo controller.



$\varepsilon = \theta_i - \theta_o$

$\frac{\theta_o}{\varepsilon} = \frac{\theta_i}{\varepsilon} - 1$

Fig. 4

The plot of $\theta_o/\varepsilon = k\,H(j\omega)$ in the complex plane, as ω ranges from 0 to ∞, is the well-known Nyquist diagram for the servo. If the origin is shifted to the point -1, the same plot is also the graph of θ_i/ε, since $\varepsilon = \theta_i - \theta_o$, or $1 = \theta_i/\varepsilon - \theta_o/\varepsilon$. (See Fig. 4)

The degree of stability of the servo is expressed by the peak height of the frequency characteristic, that is the value of $M = \max \left| \theta_o/\theta_i \right|$. The greater the value of the ratio M, the less stable is the servo. Also, for any particular application, of course the frequency ω for which the maximum value M is attained, is important. Stability corresponding to values of M between 1.0 and 2.0 usually is satisfactory.

In the Nyquist plane, the system of curves of equal values of the ratio $M = \left| \theta_o/\theta_i \right|$ are circles of diameter $(M + 1)^{-1} + (M - 1)^{-1}$, with centers at $-1 - \frac{1}{2}[(M - 1)^{-1} - (M + 1)^{-1}]$.

Given the Nyquist graph for a stable servo, the peak height M is the value of M for that circle of the system which is just tangent to the graph.

For easy determination of appropriate values of the gain factor k, and also for other reasons, the following procedure (Bode and Nichols) is advantageous. In rectangular coordinates, plot the ratio $|\theta_o/k\varepsilon| = |H(j\omega)|$ in decibels, as a function of the logarithm of the angular frequency ω. Also plot the phase margin = $[\sphericalangle H(j\omega) - \pi]$ as a function of log ω. From the resulting chart, decibels corresponding to phase margin may be easily determined; or also the latter may be determined directly from $|H(j\omega)|$ and $\sphericalangle H(j\omega)$. Plot decibels versus phase margin to obtain a transformed Nyquist graph. Let this graph be referred to as A. Figure 5 (Nichols) gives the curves which correspond, under the same transformation, to the system of circles of equal values of M. Figure 5 is then superposed on A and translated up or down until the curve corresponding to the desired peak $|\theta_o/\theta_i| = M$ just touches the graph in A. The position of 0 db. in Fig. 5 on A then is read, to determine the value of the gain factor k in decibels. If 0 db. in Fig. 5 corresponds to -x db., then 20 log k = x. (For this purpose it would be convenient to have the curves of Fig. 5 on a sheet of transparent material.) The frequency at which the peak value M will be attained then may be determined by referring the decibel or phase margin coordinate of the point of contact to the earlier chart.

The ratio of error to input, $E = |\varepsilon/\theta_i|$, for any setting of k_v and any frequency of the input, may be read by comparing the plot A of the servo controller with the dashed-line curves in Fig. 5. (The E-curves are reflections in the 0 db. axis of the corresponding M curves.) Similarly, the phase

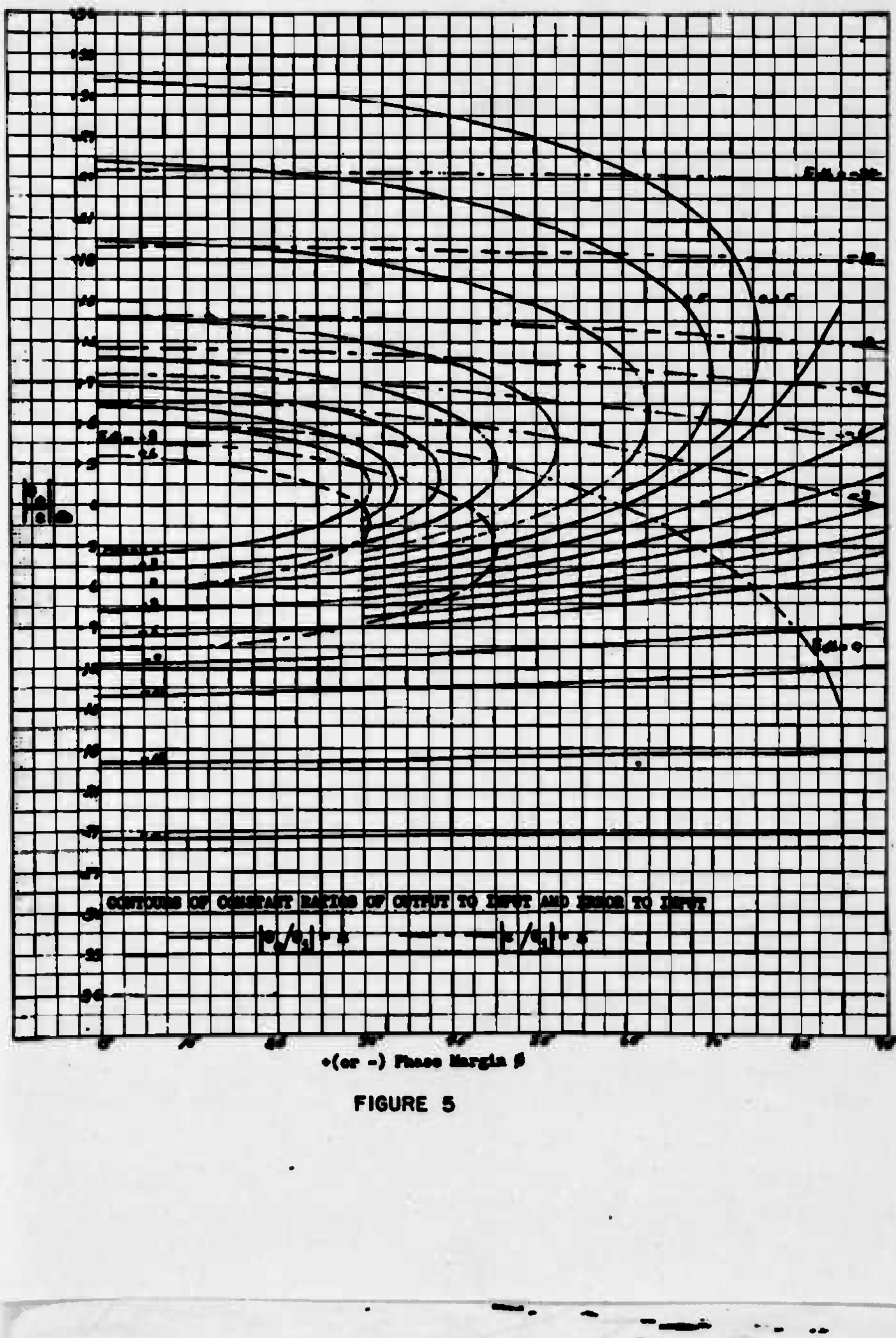

CONTOURS OF CONSTANT RATIOS OF OUTPUT TO INPUT AND ERROR TO INPUT
+(or -) Phase Margin ϕ

FIGURE 5

shift ψ of output with respect to input, $\psi = \angle(\theta_o/\theta_i)$, may be determined by superposing A on the dashed-line curves of Fig. 5a.

If the Nyquist plane is the complex z-plane, then the above transformed plane is essentially the w-plane for the transformation $w = \log z$. For if $z = re^{j\theta}$, $\log z = \log r + j\theta$. Since log z is analytic, if rectangular coordinate scales in the w-plane were chosen such that 1 radian and 1 neper = 8.686db were equal units of distance, then angles would be preserved by the transformation. That is, an orthogonal intersection of curves in the Nyquist z-plane would be transformed into an orthogonal intersection.

W. Hurewicz suggests a procedure for graphical construction of the above decibel-phase graph which may be very convenient for servo controller characteristics H(p) of the form

$$H(p) = \frac{\theta_o}{k_v \varepsilon} = \frac{(T_1 p + 1)\ (T_2 p + 1)\ \ldots}{(U_1 p + 1)\ (U_2 p + 1)\ \ldots}$$

Let the z-plane be transformed into the decibel-phase plane of Fig. 5b. The curves in Fig. 5b are images of the lines $z = \pm 1, \pm 2, \pm 4, \pm 8, \ldots, z = \pm 0.5, \pm 0.25, \pm .125, \ldots,$ $Z = \pm 1j, \pm 2j, \ldots, Z = \pm 0.5j, \pm 0.25j, \ldots$ in the z-plane. By means of the curves, for each frequency ω the transformed vectors $(T_1 j\omega + 1), \ldots, (-U_1 j\omega - 1), \ldots$ are located in the decibel-phase plane; and $H(j\omega) = \theta_o/k_v\varepsilon$ is then simply the sum of these vectors.

A plot indicating $|\theta_o/\theta_i|_{db}$ and $\angle(\theta_o/\theta_i)$, with frequency ω as parameter, may be directly constructed on a grid system similar to Fig. 5b, if the db. scale is sufficiently extended, and if there are curves corresponding to $z = \pm k$ and $z = \pm jh$ for a set of values of k, h sufficient for the required accuracy. (A unit change of z corresponds to a very small distance at the top of Fig. 5b, and to a very large distance at the bottom.)

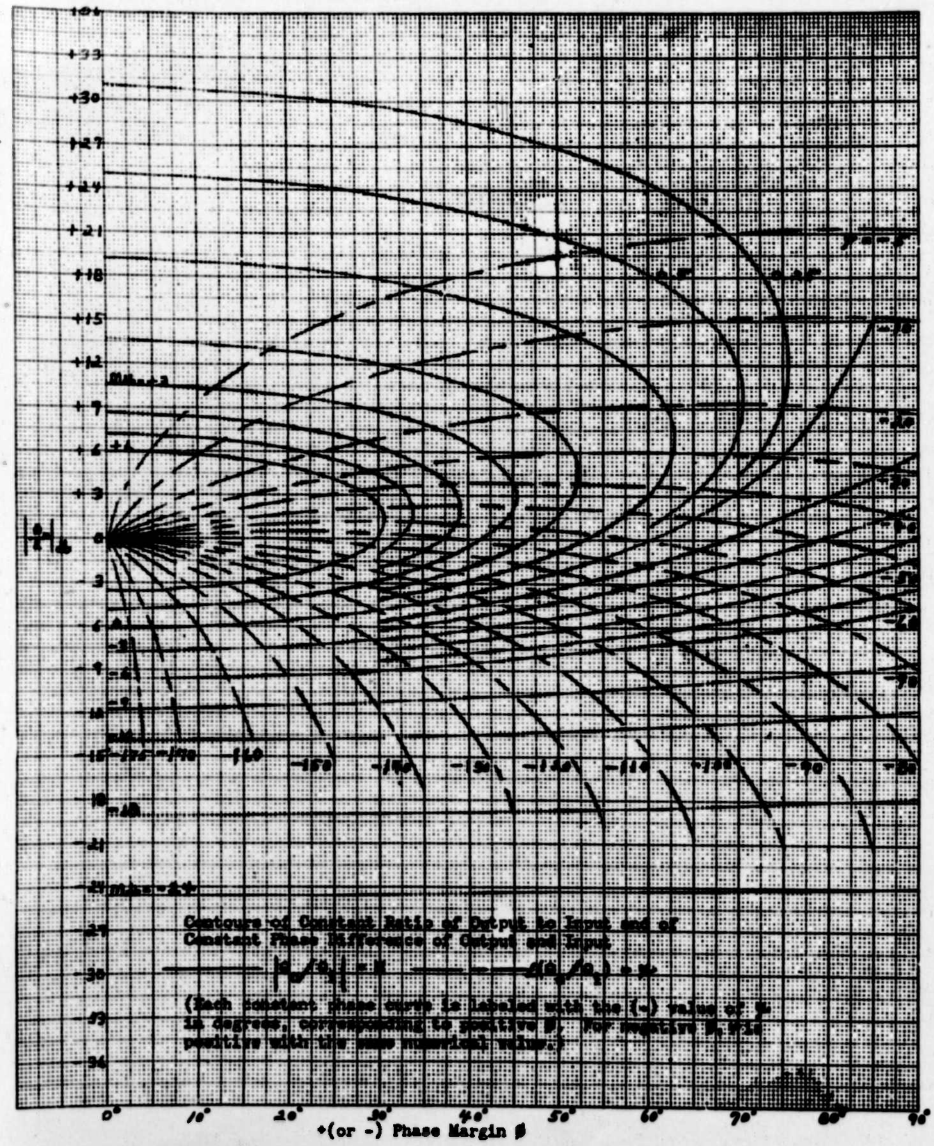

Contours of Constant Ratio of Output to Input and of
Constant Phase Difference of Output and Input
+(or -) Phase Margin ø
0° 10° 20° 30° 40° 50° 60° 70° 80° 90°
+33 +30 +27 +24 +21 +18 +15 +12 +9 +6 +3 0
-150 -140 -130 -120 -110 -100 -90 -80

FIGURE 5A

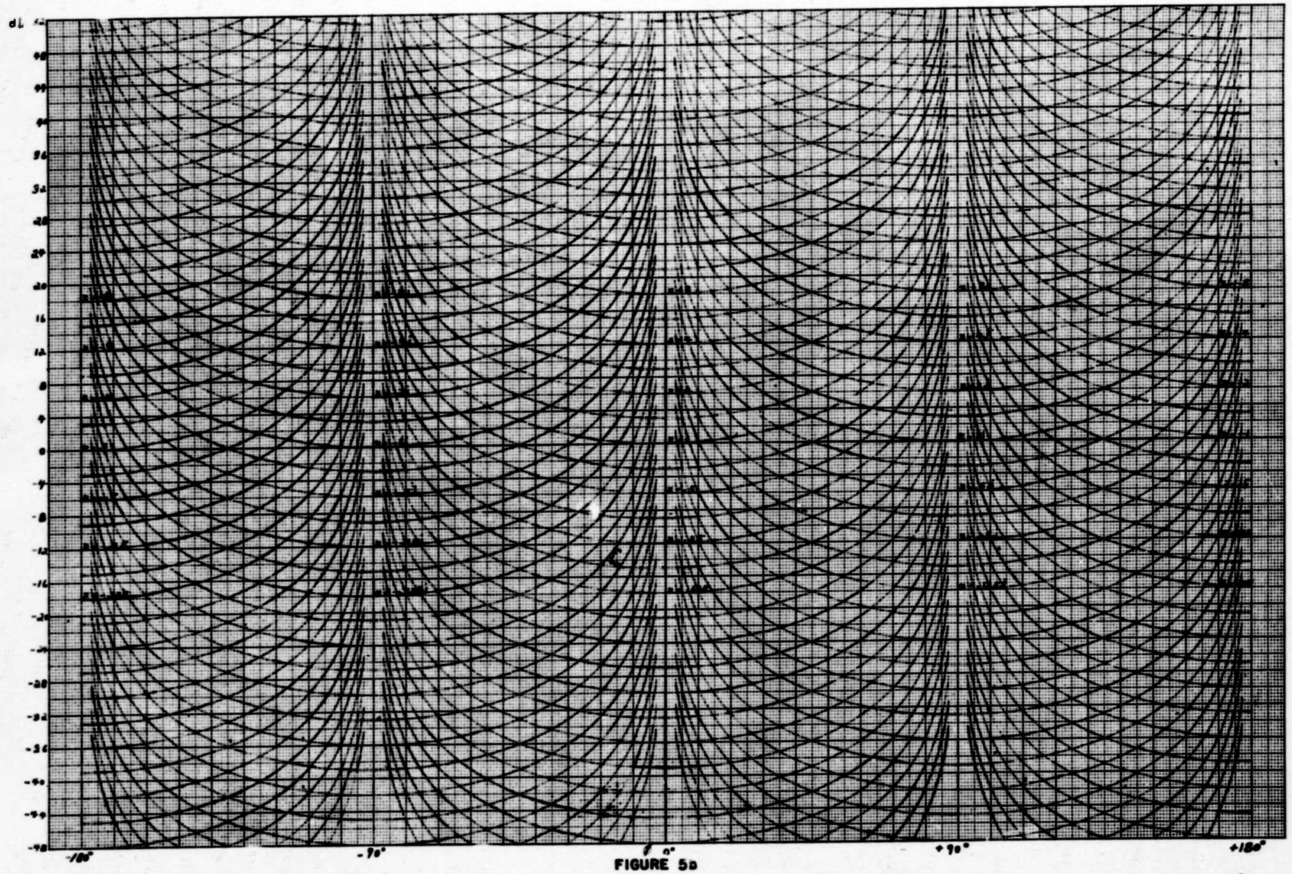


FIGURE 5D

For since $\varepsilon = \theta_i - \theta_c$, $\theta_o/\theta_i = 1/(\varepsilon/\theta_o + 1)$, $\log(\theta_o/\theta_i) = -\log(\varepsilon/\theta_o + 1)$. Thus one would plot on Fig. 5b the curve of $|k_v\varepsilon/\theta_o|_{db} = -|\theta_o/k_v\varepsilon|_{db}$ and $\angle(k_v\varepsilon/\theta_o) = \angle(\theta_o/k_v\varepsilon) \pm 180°$. The ε/θ_o curve would be obtained from the $k_v\varepsilon/\theta_o$ curve by a translation corresponding to subtraction of $(k_v)_{db}$; and points on the curve corresponding to $\varepsilon/\theta_o + 1$ are obtained by proceding along the curves $z = \mathcal{L}(\varepsilon/\theta_o)$ from $z = \mathcal{R}(\varepsilon\theta_o)$ to $z = \mathcal{R}(\varepsilon/\theta_o) + 1$. Changing the sign of the db and phase scales, this curve is then the plot of the amplitude and phase of θ_o/θ_i.

An alternative method of construction of the θ_o/θ_i curve would be based on the relationships $\theta_i/\varepsilon = 1 + \theta_o/\varepsilon$, $\log(\theta_o/\theta_i) = \log(\theta_o/\varepsilon) - \log(1 + \theta_o/\varepsilon)$. The $(1 + \theta_o/\varepsilon)$ curve could easily be constructed from the θ_o/ε curve, by means of the $z = \pm k$ and $z = \pm jh$ curves; and the θ_o/θ_i curve then would be obtained by laying out from the origin the vector differences of corresponding points on the θ_o/ε and $(1 + \theta_o/\varepsilon)$ curves.

3. Equation for Two-Phase Motor

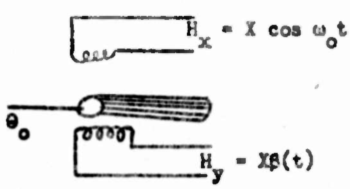


Fig. 6

Let $H_x = X \cos \omega_o t$, X = constant, be the voltage applied to the fixed winding; and let $H_y = X\beta(t)$ be the voltage applied to the control winding. (Fig. 6.) The differential equation relating the output θ_o of the motor and the input voltage $\beta(t)$ has been derived by Nichols; it is

$$T\ddot{\theta}_o + [1 + \cos 2\omega_o t + 2\beta^2(t)]\dot{\theta}_o = 2\,\omega_o\beta(t)\sin\omega_o t + 2\dot{\beta}(t)\cos\omega_o t \quad (1)$$

where T is a time constant associated with the motor and the load on the output. (In the derivation of this equation, the effect of circulating rotor currents on the stator field was neglected.)

For small amplitudes of the input $\beta(t)$, if the high-frequency term $\cos 2\omega_o t$ is neglected, the above equation simplifies to the linear equation

$$T\ddot{\theta}_o + \dot{\theta}_o = 2\,\omega_o\beta(t)\sin\omega_o t + 2\dot{\beta}(t)\cos\omega_o t \quad (2)$$

If $\beta(t) = \beta_s \sin \omega_o t$ ($\beta(t)$ then is 90° out of phase with the fixed winding excitation), the motor will run at constant speed $2\omega_o \beta_s$. If $\beta(t) = \beta_c \cos \omega_o t$, the right hand side of the equation is zero, and the motor will not run.

If $\beta(t) = \cos mt \sin \omega_o t$, the right hand side is $2\omega_o \cos mt - m \sin mt \sin 2\omega_o t$. If $\beta(t) = \cos mt \cos \omega_o t$, the right hand side is $- m \sin mt\ (1 + \cos 2\omega_o t)$. Thus if $\beta(t) = \beta_1 \cos mt \sin (\omega_o t + \phi)$ and the high frequency terms are neglected, the right hand side is $\beta_1 [2\omega_o \cos \phi \cos mt - m \sin \phi \sin mt] = \beta_1 (4\omega_o^2 \cos^2 \phi + m^2 \sin^2 \phi)^{1/2} \cos (mt + \gamma) = 2\beta_1 \omega_o \cos \phi \sec \gamma \cos (mt + \gamma)$, where $\tan \gamma = (m/2\omega_o) \tan \phi$.

Any carrier frequency servo using the above motor now may be analyzed by the method of section 2. The characteristic of the servo controller for the a.c. servo is the result of the combination of a characteristic $G(p)$ as in section 1 with the motor characteristic. The output of $G(p)$ is applied to the control winding of the motor. As in section 1, if the input to $G(p)$ is $\varepsilon_1 \cos mt \cos \omega_o t$, that is $\varepsilon = \varepsilon_1 \cos mt$, the output is

$$E_2 = \varepsilon_1 [A(m) \cos(mt + \phi_1) \cos(\omega_o t + \phi_o + \phi) + B(m) \sin (mt + \phi_1) \sin (\omega_o t + \phi_o + \phi)]$$

where $A(m) = \frac{1}{2}[|G^-| + |G^+|] = \frac{1}{2}[|G[j(\omega_o - m)]| + |G[j(\omega_o + m)]|]$

$B(m) = \frac{1}{2}[|G^-| - |G^+|]$, $\phi^+ = \angle G[j(\omega_o + m)]$, $\phi^- = \angle G[j(\omega_o - m)]$,

$\phi_1 = \frac{1}{2}(\phi^+ - \phi^-)$, $\phi = \frac{1}{2}(\phi^+ + \phi^-) - \phi_o$, $\phi_o = \angle G(j\omega_o)$. Suppose that $\cos (\omega_o t + \phi_o)$ is the proper carrier phase for the control winding of the motor. Then when $\varepsilon = \varepsilon_1 \cos mt$, by the preceding paragraph, the differential equation of the motor is

$$T\ddot{\theta}_o + \dot{\theta}_o = 2\varepsilon_1 [A(m)\omega_o \cos \phi \sec \gamma \cos (mt + \phi_1 + \gamma) + B(m)\omega_o \sin \phi \sec \gamma' \cos (mt + \phi_1 + \gamma' - \tfrac{\pi}{2})] \qquad (3)$$

$$= [2A(m)\omega_o - B(m)m]\cos\phi \cos(mt + \phi_1) + [2B(m)\omega_o - A(m)m]\sin\phi \sin(mt + \phi_1)$$

where $\tan\gamma = m \tan\phi/2\omega_o$, $\tan\gamma' = -m \cot\phi/2\omega_o$. From this Θ_o/ε may be easily calculated, and the Nyquist or equivalent decibel-phase margin diagram constructed, following the procedure outlined in the next section.

4. Construction of Gain-Phase Margin Diagram for Any G(p) With Motor of 6 3

From the differential equation (3) for $\varepsilon = \cos mt$, the angle of $\Theta_o/k_v\varepsilon$ evidently is:

$$\angle\frac{\Theta_o}{k_v\varepsilon} = \phi_1 + \alpha - 90° - \text{arc tan } mT \qquad (4)$$

where

$$\tan\alpha = -\frac{[2\omega_o B(m) - m\,A(m)]\sin\phi}{[2\omega_o A(m) - m\,B(m)]\cos\phi} \qquad (5)$$

Thus we have

$$\text{Phase Margin} = 90° + \phi_1 + \alpha - \text{arc tan } mT \qquad (6)$$

With quantities as in the previous section, we have also

$$\left|\frac{\Theta_o}{k_v\varepsilon}\right| = \frac{\{[2\omega_o A(m) - m\,B(m)]^2\cos^2\phi + [2\omega_o B(m) - m\,A(m)]^2\sin^2\phi\}^{1/2}}{2\omega_o|G(j\omega_o)|m\,(1 + m^2T^2)^{1/2}}$$

$$= \frac{[2\omega_o A(m) - m\,B(m)]\cos\phi\cos(\text{arc tan } mT)}{2m\omega_o|G(j\omega_o)|\cos\alpha} \qquad (7)$$

where k_v = (output rate corresponding to error)/error is the "velocity error constant" of the servo. In general, it is convenient to read arc tan x and cos (arc tan x) $= 1/(1 + x^2)^{1/2}$ simultaneously from a table of natural trigonometric functions.

A tabular procedure is recommended for obtaining the values of phase margin and corresponding ratios $|\Theta_o/k_v\varepsilon|$. For successive values of modulating frequency m, find and enter in the table:
ϕ^+, ϕ^-, ϕ, $\cos\phi$, $|G^-|$, $|G^+|$, A(m), B(m), $[2\omega_o A(m) - m\,B(m)]$, $\tan\alpha$, $\cos\alpha$ from G(p), mT, cos (arc tan mT) from the motor time constant T, then finally phase margin, ratio $|k_v\varepsilon/\Theta_o|$, and $|\Theta_o/k_v\varepsilon|$ in db = $-|k_v\varepsilon/\Theta_o|$ in db.

For a carrier frequency of 60 cycles, that is $\omega_o = 2\pi 60 = 377$, the following table of values, used in the construction of the decibel-phase margin diagrams in the next section, will be useful to the reader in constructing diagrams for any additional cases of $G(p)$ in which he may be interested:

arc tan $m\tau$	$m\tau$	$(1 + m^2\tau^2)^{-1/2}$ = cos (arc tan $m\tau$)	m for τ = 0.2 sec	$\frac{m}{2\pi}$ in cps.	$\frac{2\omega_o - m}{2(\omega_o - m)}$	$\frac{2\omega_o + m}{2(\omega_o + m)}$	$\frac{11\omega_o - m}{2(\omega_o - m)}$	$\frac{11\omega_o + m}{2(\omega_o + m)}$	$\frac{13\omega_o - m}{2(\omega_o - m)}$	$\frac{13\omega_o + m}{2(\omega_o + m)}$	$\frac{\omega_o - m}{\omega_o}$	$\frac{\omega_o + m}{\omega_o}$	arc tan $\frac{15}{\omega_o} m$	cos (arc tan $\frac{15}{\omega_o} m$)
50°	1.192	.643	5.96	0.95	1.008	.992	.924	.910	1.093	1.074	.985	1.015	13.3°	.973
60°	1.732	.500	8.66	1.38	1.012	.989	.925	.909	1.097	1.070	.978	1.022	19.0°	.9455
70°	2.75	.342	13.75	2.19	1.019	.982	.933	.902	1.105	1.063	.963	1.036	28.65°	.8775
80°	5.67	.1736	28.35	4.52	1.041	.965	.951	.886	1.131	1.043	.925	1.075	48.4°	.664
85.	12.56	.0794	62.8	10.0	1.100	.929	1.00	.857	1.200	1.000	.835	1.167	68.2°	.3714
87°	19.08	.0523	95.4	15.2	1.169	.899	1.06	.832	1.281	.966	.741	1.252	75.2°	.258
88°	28.64	.0349	148.3	22.8	1.324	.859	1.16	.806	1.462	.919	.607	1.392	80.4°	.167
89°	57.29	.0175	286.5	45.6	2.583	.784	2.23	.737	2.930	.831	.240	1.76	85°	.0872

5. Proportional- Derivative Frequency-Phase Characteristics

The ideal proportional-derivative characteristic for a carrier frequency servo is $G(j\omega) = 1 + jT_d(\omega - \omega_o)$. It may be shown that if a modulated carrier voltage $m(t) \cos \omega_o t$ is impressed on this characteristic, the output is $[m(t) + T_d \dot{m}(t)]\cos \omega_o t$. The ratio T_d of the coefficients of the differentiated and proportional components is called the "derivative time constant".

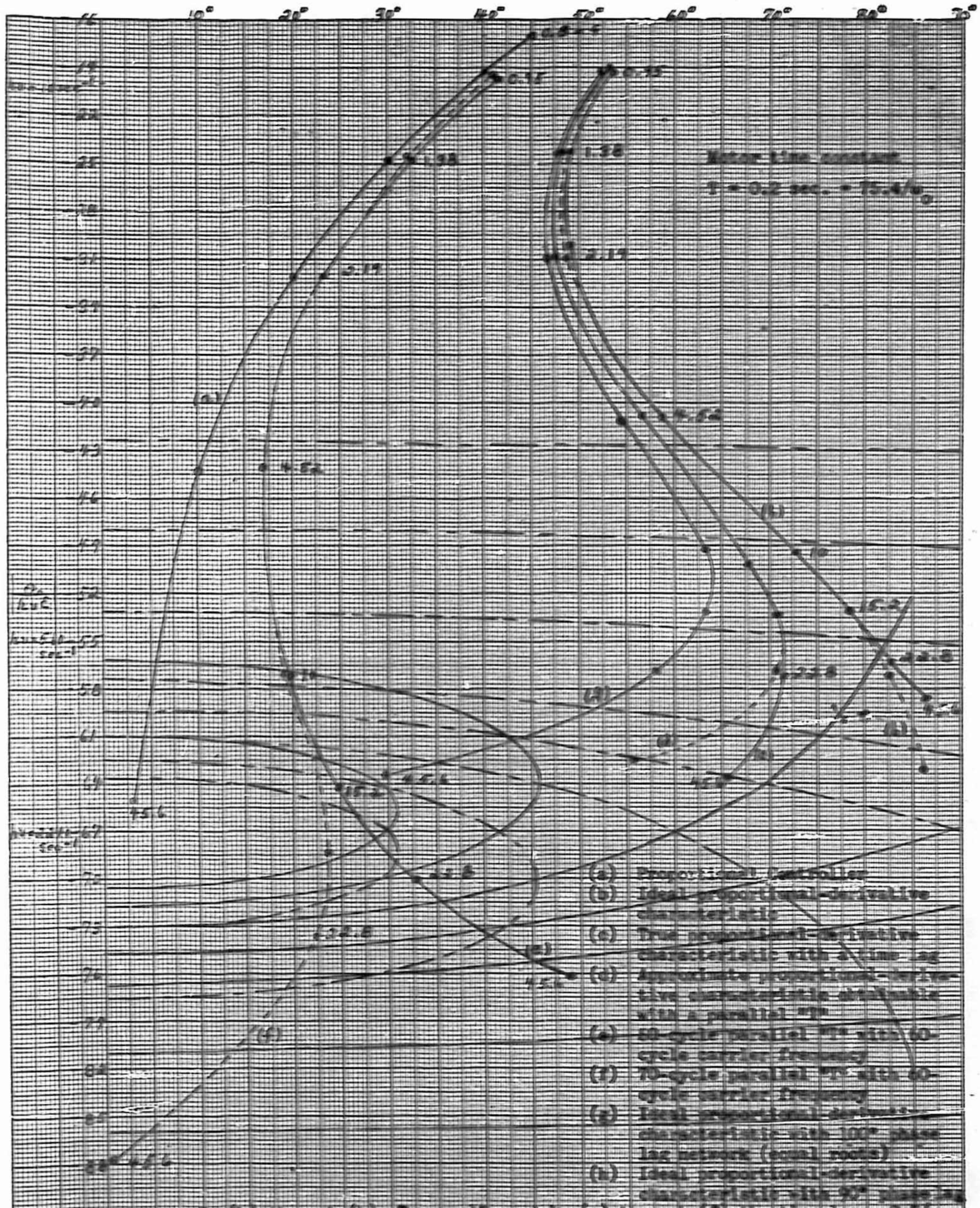
The approximation to the ideal proportional derivative characteristic which it is possible to obtain with the parallel "T" network is

$$G(j\omega) = \frac{\ell}{T_d} \frac{1 + jT_d \frac{\omega+\omega_o}{2\omega}(\omega-\omega_o)}{1 + j\ell \frac{\omega + \omega_o}{2\omega} (\omega - \omega_o)} \qquad (8)$$

where the time lag ℓ cannot be larger than $1/\omega_o$.

It has been found experimentally that different servo setups require different values of T_d for best stability; however T_d is not at all critical. Values of $T_d\omega_o$ in the range of 5 to 50 have been found useful.

Curve (a) in Fig. 7 is the decibel-phase margin plot for a simple proportional controller, that is, G(p) is identically 1 or a constant, so that the controller characteristic H(p) is the motor characteristic $1/(Tp + 1)p$. Comparing with Fig. 5, evidently even with low gains (k_v in the vicinity of 10 sec^{-1}), there are peaks of large amplitude at low frequencies. In an actual servo system, where there are time lags not included in curve (a) (as for example a small time lag in the coupling circuits between stages of a vacuum tube amplifier), as the gain is increased, the servo output quickly will begin to oscillate violently and continuously, with no input; that is, the servo will be unstable.



In (b) through (h), $T_d\omega_o = 15$, and in (c) through (f) the time lag = $0.3/\omega_o$

FIGURE 7 GAIN MARGIN DIAGRAMS

Curve (b) in Fig. 7 is the corresponding plot for the servo controller consisting of the ideal proportional-derivative characteristic with $T_d \omega_o = 15$, and the simplified motor of §3, with $T = 0.2$ second. This value of T is approximately the value for the Diehl FPE-49-2 motor. Comparing with Fig. 5, evidently the servo now is very stable at all values of k_v. That is, if $k_v = 560 \sec^{-1} \sim 55$db, the frequency response $|\theta_o/\theta_i|$ is nearly flat from 0 to 20 cps or higher, and the output θ_o of the servo is a very faithful copy of the input θ_i, even though θ_i may be of such a nature as to have components of rather high frequency. In particular the output for a step displacement of the input θ_i will also be very nearly a step, and will have no overshoot. This is unlike the situation for the proportional servo (with the same velocity error constant), where for a step input the output has many overshoots and only gradually settles down to the displacement corresponding to the input displacement.

Curve (c) is the decibel-phase margin plot for the proportional-derivative characteristic $G(j\omega) = \dfrac{1 + jT_d(\omega - \omega_o)}{1 + j\ell(\omega - \omega_o)}$ with time lag $\ell = 0.5/\omega_o$, and $T_d\, \omega_o = 15$. Curve (d) is the plot for the approximate proportional-derivative characteristic (8), which may be obtained with the parallel "T" network, for the same values of ℓ and T_d.

In all curves of Fig. 7, the carrier phase is assumed to have the proper value for the control winding of the motor. To examine the effect of incorrect carrier phase, let us compare the right hand side of the equation in section 3,

$$2\,\omega_o \cos \phi \cos mt - m \sin \phi \sin mt,$$

with the output of the (d.c.) proportional-derivative characteristic $(1 + Ujm)$ when cos mt is applied,

$$\cos mt - m\, U \sin mt$$

Thus if the carrier phase is incorrect, then effectively another proportional-derivative characteristic $(1 \pm Ujm)$ is in cascade with the stabilizing characteristic $(1 + T_d jm)$, with $U = \tan \phi/2\omega_o$. The gain is reduced by the factor $\cos \phi$, although the actual voltage on the motor (and its heating effect) is not reduced. Also the maximum available torque of the motor, that is, the torque at saturation, is reduced by the factor $\cos \phi$. If gain and maximum torque reduction by a factor of 0.5 is tolerable, then as much as 60° phase shift may be allowed, since $\cos 60° = 0.5$. For errors ϕ in carrier phase up to 45° or 60°, $U\omega_o$ is less than 0.5 or 0.87, so for $T_d\omega_o > 5$, the effect on stability of $(1 \pm Ujm)$ is comparatively quite insignificant.

The high values of k_v attainable with the parallel "T", and the flat frequency characteristic, have been verified experimentally in a servo setup using 60 synchros and the Diehl FPE-49-2 motor. (We had $k_v = 5000$ sec^{-1} with satisfactory freedom from chatter; the only limitation on k_v seems to be backlash in the gear train.)

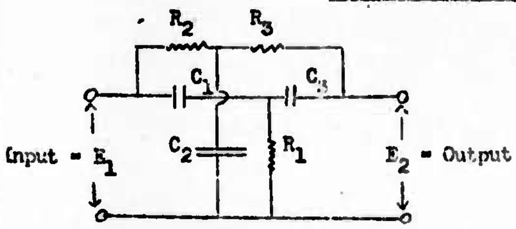
Curves (c) and (f) in Fig. 7 are decibel-phase margin plots for the servo controller consisting of the motor of section 3, and the parallel "T" approximate proportional-derivative characteristic, the characteristic however being tuned to frequencies $\omega_1 = 2\pi 70$ and $\omega_1 = 2\pi 50$, instead of to the carrier frequency $\omega_o = 2\pi 60$. There is then respectively a phase lag of 64.5°, or a phase lead of 62.1°, at carrier frequency ω_c. Including this phase shift, the phase of the carrier at the control winding of the motor is assumed to be correct (if not, the previous remark on the small effect of incorrect carrier phase still applies, and for phase errors up to 65° the stability is not appreciably different from that indicated by curves (e) and (f)). It is evident from the diagrams that when the parallel "T" network is tuned to a frequency as much as 10 cycles <u>lower</u> than the 60 cycle carrier, the stability still is very satisfactory at high values of k_v. However, when the parallel "T" network is tuned to a frequency 10 cycles higher than the 60 cycle carrier, even with the optimal setting of k_v (about 3000 sec^{-1}), there will be a peak in the region of 15-25 cps. of height greater than 2. (For deviations of ± 8 cycles, these conclusions also have been verified experimentally.)

6. Effect of Phase Lag Networks on Stability

Experimentally, it has been found satisfactory to shift the phase of the carrier to obtain the proper phase at the control winding, by inserting into the servo controller a one or two-section low pass R-C filter preceding the proportional-derivative parallel "T". (See Part IV, section 3.) When this is done, the proportional-derivative characteristic is multiplied by $1/(Up + 1)$ or by $1/(U_1p + 1)(U_2p + 1)$. The effect on stability to be expected is indicated by curves (g) and (h) in Fig. 7. Curve (h) will be approached as U becomes very large, or as one of U_1, U_2 approaches infinity and the other approaches zero. Curves (g) and (h) indicate, for a two-section filter to obtain say 100° phase lag, that better results, as regards keeping the amplification of noise or jitter and backlash chatter small, will be obtained by having the roots U_1^{-1} and U_2^{-1} as unequal as possible, than by having them nearly equal. Also, the high rate of decrease of phase margin with respect to error frequency in the range 15 - 45 cps., in combination with other time lags in the system not included in curve (g), may explain an observed tendency of the servos to oscillation at 30 cps.

Part II

Design Formulae for Parallel "T" Networks



$T_1 = R_1C_1$
$T_2 = R_2C_2$
$T_3 = R_3C_3$
$S_1 = R_1C_3$
$S_2 = R_2C_3$

Parallel "T" Network and Determining Time Constants

Fig. 8

The parallel "T" network is shown in Fig. 8. Let us define time constants $T_1 = R_1C_1$, $T_2 = R_2C_2$, $T_3 = R_3C_3$, $S_1 = R_1C_3$, $S_2 = R_2C_3$. Except for the impedance level, which always may be changed by simple scale transformation of the R's and C's, the parallel "T" is determined by the five constants T_1, T_2, T_3, S_1, S_2. For if these constants are known, and if any one of the components, say for example C_3, is arbitrarily specified, then the other components are determined:

$$R_1 = S_1/C_3,\ R_2 = S_2/C_3,\ R_3 = T_3/C_3,\ C_1 = T_1/R_1,\ C_2 = T_2/R_2.$$

1. Transfer Characteristic; Input and Output Impedances

The transfer characteristic of the parallel "T" is

$$\frac{E_2}{E_1} = \frac{T_1T_2T_3p^3 + T_1(S_2 + T_3)p^2 + (T_1 + S_1)p + 1}{T_1T_2T_3p^3 + [T_1(S_2 + T_3) + T_2(T_1 + S_1 + T_3)]p^2 + (T_1 + S_1 + T_2 + S_2 + T_3)p + 1} \quad (1)$$

The appropriate form which this is made to assume in order to obtain the approximate proportional-derivative characteristic at carrier frequency ω_o is

$$\frac{E_2}{E_1} = \frac{(Up + 1)(\frac{1}{\omega_o^2} p^2 + \frac{2}{n\omega_o} p + 1)}{(Up + 1)(\frac{1}{\omega_o^2} p^2 + \frac{2}{m\omega_o} p + 1)} = \frac{m}{n} \frac{[1 + jT_d \frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)]}{[1 + j\ell \frac{\omega + \omega_o}{2\omega} (\omega - \omega_o)]} \qquad (2)$$

where in the second expression $j\omega$ has been substituted for p, and $T_d = n/\omega_o$ is the derivative time constant, $\ell = m/\omega_o$ is the time lag. The second expression is obtained by multiplying numerator and denominator of the first by $n\omega_o/2p$, and then factoring n/m out of the denominator. The choice of the time indeterminate U, the coefficient of p in the linear factor which cancels in the numerator and denominator, does not affect the transfer characteristic. (But it does affect the input and output impedances, as will be seen.) If $E_1 = E(t) \sin \omega_o t$, the approximation $(\omega + \omega_o)/2\omega \cong 1$ is valid when the modulation E(t) does not have high frequency components of large amplitude.

To realize the above transfer characteristic, the determining constants of the parallel "T" must have the following values:

$$T_1 = \frac{A^2 + B_1 U + U^2}{B_2 - B_1} = \frac{g(nu^2 + 2u + n)}{2\omega_o(1 - g)}$$

$$T_2 = \frac{U^2(B_2 - B_1)}{A^2 + B_1 U + U^2} = \frac{2(1 - g)u^2}{g\omega_o(nu^2 + 2u + n)} \qquad (3)$$

$$T_3 = \frac{A^2}{U} = \frac{1}{u\omega_o}$$

$$S_1 = B_1 + U - T_1 = \frac{1}{\omega_o} \frac{-gn^2u^2 + 2n(1 - 2g)u + 4(1 - g) - n^2g}{2n(1 - g)}$$

$$S_2 = B_2 - B_1 - \frac{A^2}{U} - T_2 = \frac{[4(1 - g) - gn^2]u^2 + 2(1 - 2g)nu - gn^2}{gnu\omega_o(nu^2 + 2u + n)}$$

where $A^2 = 1/\omega_o^2$, $B_1 = 2/n\omega_o = 2/T_d\omega_o^2$, $B_2 = 2/m\omega_o = 2/\ell\omega_o^2$, $u = U\omega_o$, and the gain $g = m/n$.

The condition for non-negative S_1 is that u be in the interval with end-points $\frac{1}{m} - \frac{2}{n} \pm \frac{1}{m}\sqrt{1 - m^2}$. The condition for non-negative S_2 is that u be in the interval with end-points $\frac{-n^2 + 2mn \pm n^2\sqrt{1 - m^2}}{4n - 4m - mn^2}$. Therefore for realizability, $m < 1$ and u must lie in the common portion of the two intervals. The possible values of u are indicated in Fig. 9. The dotted left and right hand curves are approached asymptotically by respectively the right and left boundaries of the regions for u, as m approaches 1.

For the general parallel "T", the input impedance, with the output open-circuited, is

$$Z_1 = \frac{T_1T_2T_3p^3 + [T_1(S_2 + T_3) + T_2(T_1 + S_1 + T_3)]p^2 + (T_1 + S_1 + T_2 + S_2 + T_3)p + 1}{(C_1T_2T_3 + C_2T_1T_3 + C_3T_1T_2)p^3 + [C_1(T_3 + T_2 + S_2) + C_2(T_3+T_1+S_1)]p^2 + (C_1+C_2+C_3)p} \tag{4}$$

The impedance looking back into the output, with the input short-circuited, is:

$$Z_o = \frac{R_3T_2(S_1 + T_1)p^2 + [R_3(S_1 + T_1 + T_2) + R_2(S_1 + T_1)]p + R_2 + R_3}{T_1T_2T_3p^3 + [T_1(S_2 + T_3) + T_2(T_1 + S_1 + T_3)]p^2 + (T_1 + S_1 + T_2 + S_2 + T_3)p + 1} \tag{5}$$

The impedances Z_1 and Z_o may be expressed in terms of U and one of the parallel "T" constants, say C:

$$Z_1(C_3,U) = \frac{1}{C_3}\,\frac{(Up + 1)\left(\frac{1}{\omega_o^2}p^2 + B_2p + 1\right)}{\left[\left(\frac{1}{S_1} + \frac{1}{S_2}\right)\frac{1}{\omega_o^2} + U\right]Up^3 + (B_2 - B_1)\left(\frac{T_1}{S_1} + \frac{U}{S_2}\right)p^2 + \left(\frac{T_1}{S_1} + \frac{T_2}{S_2} + 1\right)p} \tag{6}$$

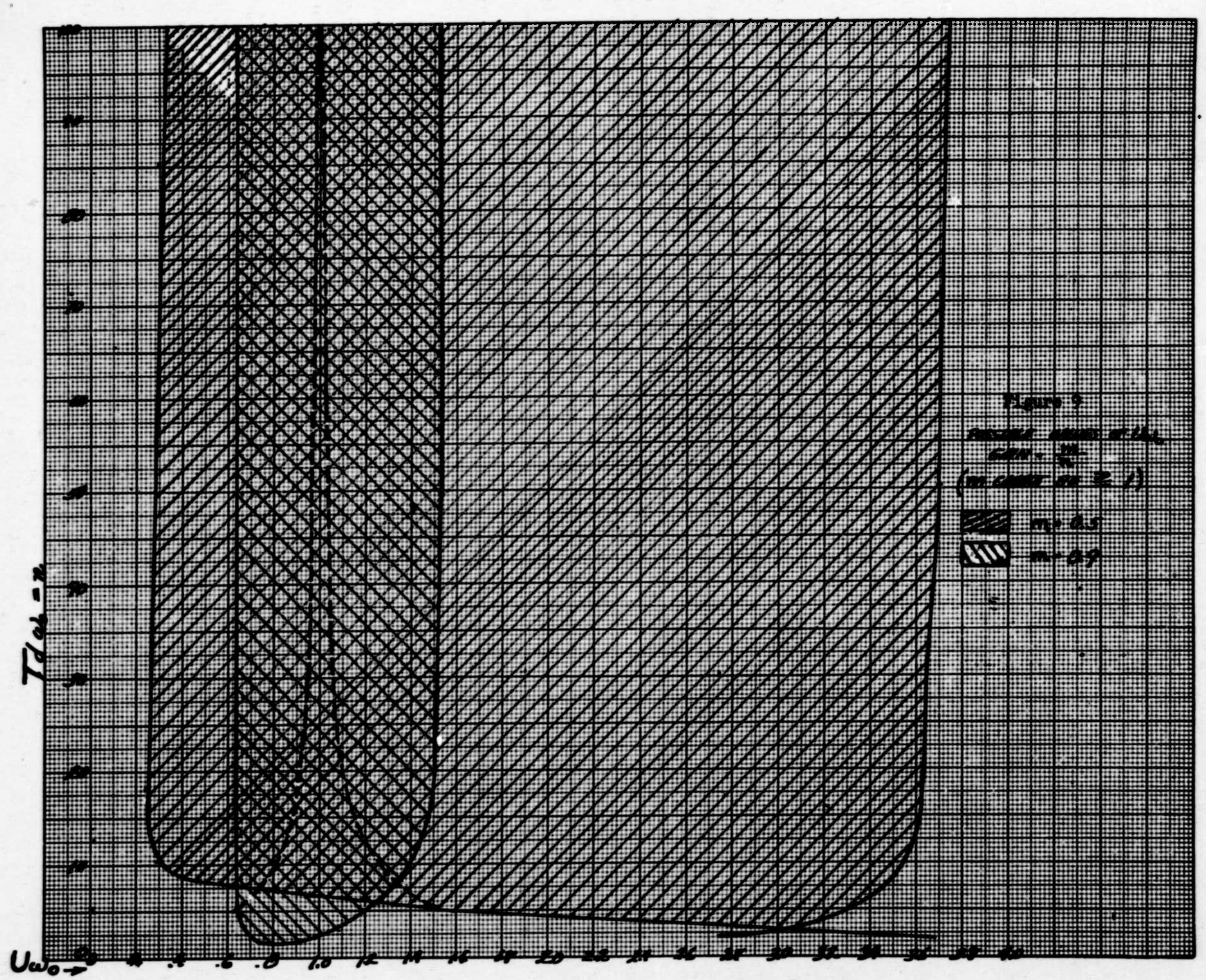


Figure 9
m = 0.5
m = 0.9

$$Z_o(C_3,U) = \frac{T_2(B_1 + U)p^2 + [(B_1 + U)(1 + S_2 U\omega_o^2) + T_2]p + (1 + S_2 U\omega_o^2)}{C_3 U\omega_o^2(Up + 1)(\frac{1}{\omega_o^2}p^2 + B_2 p + 1)} \qquad (7)$$

where T_1, T_2, S_1, S_2 are given in terms of U by formulae (3) above. From the expression for Z_1, it is evident that as U approaches an end-point of one of the critical intervals, so that S_1 or S_2 approaches zero, Z_1 approaches zero. Thus in order to avoid practically short-circuiting the source of the input voltage, U should not be near one of the end-points, and also one should not choose a voltage gain g too near the upper bound $1/T_d\omega_o$.

If it were desired to make a discriminate selection of U among the possible values, the basis might be to best match the angles and ratio of the input and output impedances Z_1 and Z_o to the angles and ratio of the source and load impedances to be used with the parallel "T" network. Charts for this purpose could be prepared from the expressions (6), (7) for $Z_1(C_3,U)$ and $Z_o(C_3,U)$.

After T_d, U, and g are selected, the time constants T_1, T_2, T_3, S_1, S_2 are determined by the formulae (3). The formulae in the next sections are obtained from (3) by imposing special conditions which fix the value of u, or the values of u and the gain g.

2. Reversible Parallel "T" Networks

A parallel "T" proportional-derivative network will be called reversible if it is also a proportional-derivative network when the input and output are interchanged. It is shown in Part VI that this requirement is equivalent to $C_3/C_1 = R_2/R_3$, or $S_1T_3 = S_2T_1$, and $u=1$. The derivative time constant T_d necessarily is the same in both directions, but unless the parallel "T" is symmetric, the gains and input and output impedances are different.

The design formulae for the reversible parallel "T" proportional-derivative network are as follows:

$$T_1 = \frac{g}{1 - g} \frac{(n + 1)}{\omega_o}$$

$$T_2 = \frac{1 - g}{g} \frac{1}{\omega_o} \frac{1}{(n + 1)}$$

$$T_3 = \frac{1}{\omega_o}$$

$$S_1 = \frac{1}{\omega_o} \left[\frac{(2 + n)(1 - g) - gn(n + 1)}{n(1 - g)}\right]$$

$$S_2 = \frac{1}{\omega_o} \left[\frac{2(1 - g)(n + 1) - n(gn + 1)}{gn(n + 1)}\right] \qquad (8)$$

where $T_d \omega_o = n$ and g is the gain in the forward direction. The gain g and n may be selected as desired; the gain g' in the reverse direction then may be seen to be

$$g' = \frac{g(n^2 + 2n + 2) - (n + 2)}{g(2n^2 + 3n + 2) - (n^2 + 2n + 2)} \qquad (9)$$

To express g in terms of g', it is necessary merely to interchange g and g'. Fig. 10 shows the values and relationship of the forward and reverse gains g and g', for several values of $T_d \omega_o = n$.

If the parallel "T" is symmetric, that is $C_3 = C_1$ and $R_2 = R_3$, or $S_1 = T_1$ and $S_2 = T_3$, the gain is

$$g = g' = g_{sym.} = \frac{1}{2n - \frac{n - 2}{n + 2}} \qquad (10)$$

3. <u>Parallel "T" with Arbitrary Condensers</u>

Let us assume that the values of the condensers C_1, C_2, C_3 in the parallel "T" are known or specified, and determine the values of R_1, R_2, R_3 to realize any desired $T_d = n/\omega_o$.

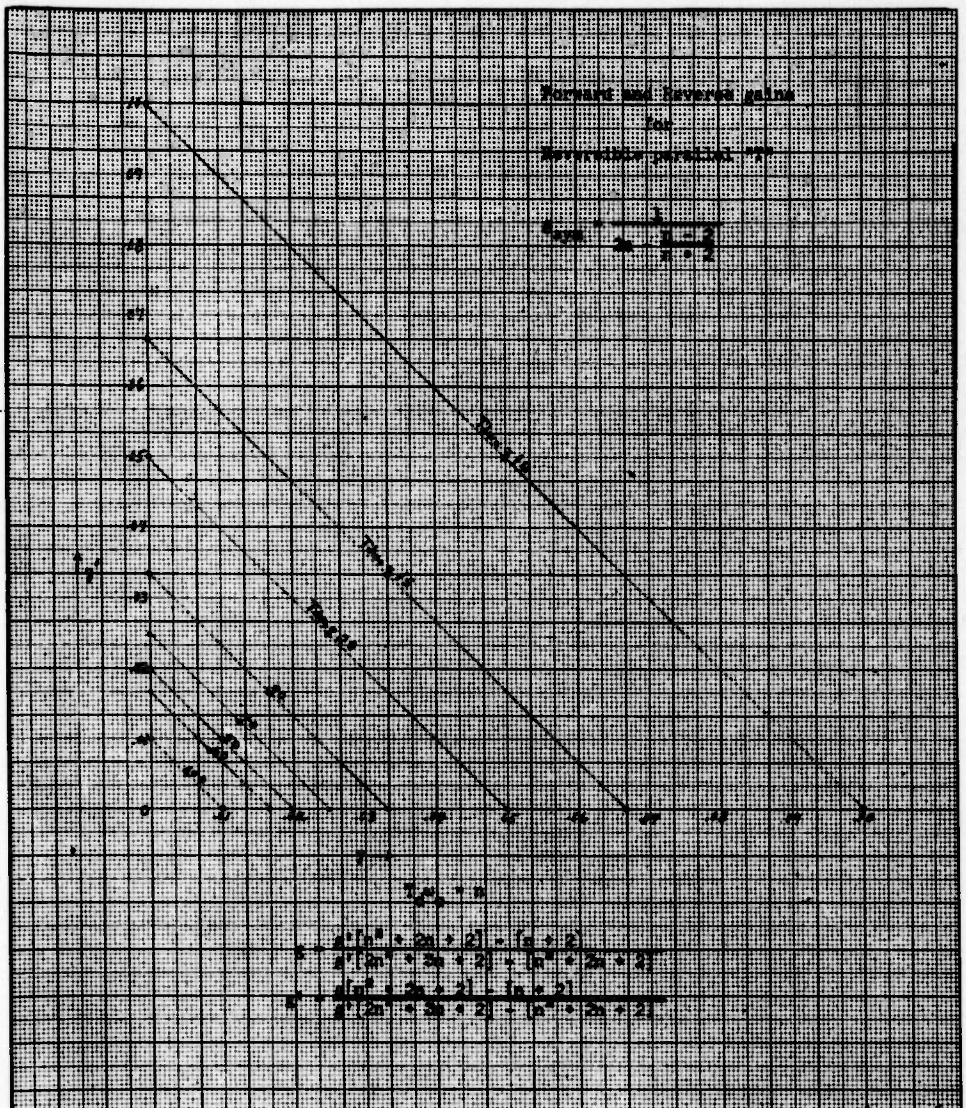


FIGURE 10

Eliminating g and u from equations (3), we find

$$R_1 = \frac{1}{\omega_o} \frac{(\frac{2}{n} + u)}{(C_1 + C_3)}$$

$$R_2 = \frac{r}{\omega_o} \frac{u^2}{(\frac{2}{n} + u)C_2} \qquad (11)$$

$$R_3 = \frac{1}{u\omega_o C_3}$$

where $r=(C_1 + C_3)/C_1$, and u is a solution of

$$ru^3 - \frac{2}{n}\frac{C_2}{C_3} ru^2 + \frac{C_2}{C_3}(1 - r)u + \frac{2}{n}\frac{C_2}{C_3} = 0 \qquad (12)$$

The gain g is given by

$$\frac{2}{g} = \frac{rn}{(\frac{2}{n} + u)} + rnu + 2 \qquad (13)$$

For $C_1 = C_2 = C_3$, the positive solutions for u are $u = 2^{-\frac{1}{2}}$ and $u = 2/n$.

4. Parallel "T" with Arbitrary Resistors

If the resistors in the parallel "T" are arbitrarily specified, then to realize the proportional-derivative characteristic with $T_d\omega_o = n$, the value of condenser C_3 must be a solution of the following equation:

$$R_1R_3(R_2 + R_3)C_3^3 - \frac{2}{n\omega_o} R_3(R_2 + R_3)C_3^2 - \frac{1}{\omega_o^2} R_2C_3 + \frac{2}{n\omega_o^3} = 0$$

The values of condensers C_1 and C_2 then are given by

$$C_1 = \frac{1}{R_1R_3C_3\omega_o^2} + \frac{2}{R_1n\omega_o} - C_3$$

and

$$C_1C_2 = \frac{1}{R_1R_2R_3^2C_3^2\omega_o^4} .$$

The gain is given by

$$\frac{2}{gn} = \frac{1}{\omega_oR_1C_1}\left(1 + \frac{2}{n\omega_oR_3C_3} + \frac{1}{\omega_o^2R_3^2C_3^2}\right) + \frac{2}{n} .$$

For the case of equal resistors, $R_1 = R_2 = R_3 = R$, the two positive solutions of the cubic are $RC_3 = 2^{-1/2}\omega_o^{-1}$ and $RC_3 = 2/n\omega_o$. The values of the condensers corresponding to the first solution are:

$$C_3 = \frac{1}{2^{1/2}R\omega_o}$$

$$C_1 = \frac{1}{R\omega_o}\left(2^{-1/2} + \frac{2}{n}\right)$$

$$C_2 = \frac{1}{R\omega_o}\ \frac{2}{\left(2^{-1/2} + \frac{2}{n}\right)}$$

The indeterminate U in the general transfer characteristic has the value $U = 2^{1/2}/\omega_o$. The gain is:

$$g = \frac{2^{1/2}n + 4}{3n^2 + 3\cdot 2^{1/2}n + 4}$$

The input impedance is:

$$Z_1 = \frac{(3n^2 + 3\sqrt{2}n + 4)jR}{(-5.25\sqrt{2}\,n^2 - 18n - 2\sqrt{2}) + j(2n^3 + \sqrt{2}n + 4)}$$

For the case of $n = T_d\omega_o = 15$, the value of the input impedance is $Z_1 = (.0882 - .3606j)R$.

5. Divider Circuits to Increase and Decrease T_d

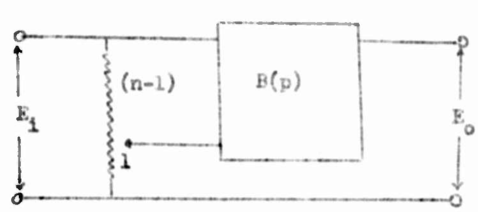


Fig. 11

Given a proportional-derivative characteristic B(p) with derivative time constant T_d', a characteristic with any lower value of T_d may be obtained with the circuit of Fig. 11, assuming a perfect voltage divider, that is neglecting the loading effect of the input impedance of B(p) on the divider.

(Actually the frequencies of minimum output and zero phase of B(p) will change; the amount of this shift may be readily calculated; or for fixed divider values B(p) may be designed to give exactly the desired characteristic in combination with the divider.) In particular, $T_d\omega_o = n$ may be taken to be ℓ in the formulae of the preceding sections, to obtain a "resonant" parallel "T", and then with suitable divider values any desired T_d may be obtained.

The transfer characteristic of a resonant parallel "T" is:

$$\frac{j\ell\frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)}{1 + j\ell\frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)} .$$

If $B(j\omega)$ in Fig. 11 is this transfer characteristic, assuming a perfect divider of ratio $1/n$, the transfer characteristic of Fig. 11 is

$$\frac{E_o}{E_i} = \frac{1}{n}\left[1 + \frac{j(n-1)\ell\frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)}{1 + j\ell\frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)}\right]$$

$$= \frac{\ell}{T_d}\left[\frac{1 + jT_d\frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)}{1 + j\ell\frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)}\right]$$

where $T_d = n/\omega_o$, which is the same as (8). Although the gain for the combination of resonant "T" and divider is the same as for the proportional-derivative parallel "T", the former has the disadvantage that the input impedance is lower for equivalent output impedances.

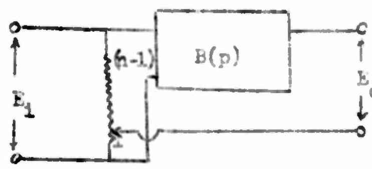


Fig. 12

Given a proportional-derivative characteristic B(p), a characteristic with any higher value of T_d may be obtained with the circuit of Fig. 12. In this case there is no loading effect on the divider, so it is unnecessary to make corrective calculations as it is in the case of Fig. 11. If

$$B(p) = [1 + jT_d' \frac{\omega + \omega_o}{2\omega} (\omega - \omega_o)] / [1 + j\ell \frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)],$$

then for Fig. 12,

$$\frac{E_o}{E_i} = \frac{n - 1}{n} \frac{\ell}{T_d} \frac{1 + jT_d \frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)}{1 + j\ell \frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)}$$

where $T_d = (n - 1)\ell / (n\ell/T_d' - 1)$, $n = (T_d/\ell - 1)/(T_d/T_d' - 1)$

This is the same as (8) except that the gain is lower by the factor $(n - 1)/n$. If $n < T_d'/\ell$, T_d is negative, and a servo using the circuit of Fig. 12 then naturally would be less stable than a proportional servo.

A scheme for increasing T_d, in which input and output have a common terminal (which may be grounded), is shown in Fig. 13.

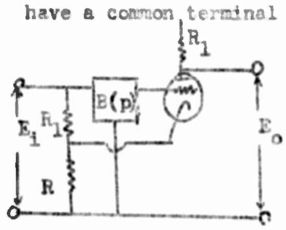


Fig. 13

If T_d' is the derivative time constant of B(p), it may be shown that the derivative time constant T_d of E_o/E_i is given by

$$T_d = \frac{T_d'}{1 - \frac{(\mu + 1)a}{\mu(1-b)}} \cong \frac{R + R_1}{R_1} T_d'$$

where μ is the amplification factor of the tube, r_p is its plate resistance, and $a = R(r_p + R_L)/[R(R_1 + r_p + R_L) + R_1(r_p + R_L)]$, $(1-b) = (R + R_1)(r_p + R_L) / [R(R_1 + r_p + R_L) + R_1(r_p + R_L)]$.

6. Numerical Values for 60-cycle Parallel "T"s

Following is a table of values of R's and C's for various parallel "T" networks of the types described in the preceding sections, for a carrier frequency of 60 cycles. The R's and C's are numbered as in Fig. 8.

The "notch width" of a band-rejection network is defined as the length of the frequency interval between the points at which the output is $\sqrt{2}$ times the minimum output. A proportional-derivative network may be characterized as well by the notch width, as by the derivative time constant T_d. The notch widths in the following table are for the ideal proportional-derivative characteristic $[1 + j\, T_d(\omega - \omega_c)]$; they are only slightly different from the actual ones for the parallel "T" networks. Evidently for the ideal characteristic at 60 cycles, the end points of the notch interval are at distances $\pm \frac{1}{2\pi T_d} = \pm \frac{60}{n}$ cps from 60 cps where $T_d \omega_o = n$.

The R's and C's in the tables are in corresponding units, as for example megohms and microfarads. As mentioned previously, a parallel "T" with any desired different impedance level may be obtained from the values in the table by multiplying R's by any constant and dividing C's by the same constant. For a carrier frequency f different from 60 cycles, if f = 60a, divide either the R's or C's by a, divide T_d by a, multiply notch width by a; and Z_i/R_3 and Z_o/R_3 at carrier frequency are unchanged.

Symmetric Parallel "T" Networks, f_o = 60 cps.

$R_2 = R_3 = .002652$ $C_1 = C_5 = 1.000$

$T_d\omega_o$	Notch Interval cps	$R_1 x10^3$	C_2	Forward Gain	Reverse Gain	Input Impedance $Z_i x10^3$	Output Impedance $Z_o x10^3$
2.5	±24	2.337	1.1111	.2045	.2045	1.667(1-j)	1.869(1-j)
5	±12	1.856	1.4286	.1045	.1045	1.431(1-j)	1.465(1-j)
7.5	±9	1.680	1.5790	.06934	.06934	1.425(1-j)	1.418(1-j)
10	±6	1.591	1.6667	.05172	.05172	1.399(1-j)	1.395(1-j)
15	±4	1.503	1.7647	.03421	.03421	1.373(1-j)	1.372(1-j)
20	±3	1.459	1.8182	.02552	.02552	1.361(1-j)	1.360(1-j)
30	±2	1.414	1.8750	.01691	.01691	1.349(1-j)	1.349(1-j)
40	±1.5	1.392	1.9048	.01264	.01264	1.343(1-j)	1.343(1-j)
50	±1.2	1.379	1.9231	.01009	.01009	1.340(1-j)	1.340(1-j)
60	±1.0	1.370	1.9355	.008400	.008400	1.337(1-j)	1.337(1-j)
∞	(Resonant)	1.326	2.0000			1.326(1-j)	1.326(1-j)

Reversible Parallel "T" Networks

$R_2 = .01326$, $R_3 = .002652$, $C_1 = .2000$, $C_3 = 1.000$

$T_d\omega_o$	Notch Interval cps	$R_1 x10^3$	C_2	Forward Gain	Reverse Gain	Input Impedance $Z_i x10^3$	Output Impedance $Z_o x10^3$
2.5	±24	3.978	.6667	.07395	.3000	7.200(1-j)	1.808(1-j)
5	±12	3.094	.8571	.03743	.1628	6.889(1-j)	1.574(1-j)
7.5	±9	2.799	.9474	.02423	.1105	6.796(1-j)	1.487(1-j)
10	±6	2.652	1.0000	.01786	.06417	6.752(1-j)	1.445(1-j)
15	±4	2.505	1.0588	.01167	.05574	6.710(1-j)	1.404(1-j)
20	±3	2.451	1.0909	.008654	.04183	6.689(1-j)	1.384(1-j)
30	±2	2.357	1.1250	.005702	.02787	6.669(1-j)	1.364(1-j)
40	±1.5	2.321	1.1429	.004250	.02090	6.660(1-j)	1.354(1-j)
50	±1.2	2.298	1.1539	.003387	.01671	6.654(1-j)	1.349(1-j)
60	±1.0	2.284	1.1613	.002815	.01392	6.650(1-j)	1.345(1-j)
∞	(Resonant)	2.210	1.2000			6.631(1-j)	1.326(1-j)

Parallel "T" Networks With Equal Condensers, $u = 1/\sqrt{2}$

$R_3 = .003751$ $C_1 = C_2 = C_3 = 1.000$

$T_d\omega_o$	Notch Interval cps	$R_1 \times 10^3$	$R_2 \times 10^3$	Forward Gain	Reverse Gain	Input Impedance $Z_i \times 10^3$	Output Impedance $Z_o \times 10^3$	Reverse Phase Shift at ω_o
2.5	±24	1.998	1.760	.2259	.17419	1.324-1.545j	1.798-1.768j	-5.00°
5	±12	1.468	2.395	.1105	.09455	1.303-1.573j	1.565-1.768j	-1.84°
7.5	±9	1.291	2.723	.07690	.06467	1.303-1.620j	1.469-1.768j	- .95°
10	±6	1.205	2.934	.05237	.04307	1.289-1.640j	1.418-1.768j	- .57°
15	±4	1.114	3.155	.03395	.03200	1.280-1.675j	1.364-1.768j	- .27°
20	±3	1.070	3.286	.02505	.02394	1.274-1.695j	1.337-1.768j	- .16°
30	±2	1.026	3.427	.01640	.01590	1.267-1.717j	1.306-1.768j	- .06°
40	±1.5	1.004	3.503	.01218	.01202	1.264-1.729j	1.293-1.768j	- .04°
50	± 1.2	.9907	3.550	.009682	.009500	1.261-1.737j	1.285-1.768j	- .02°
60	± 1.0	.9818	3.582	.008034	.007910	1.260-1.742j	1.279-1.768j	- .01°
∞	(Resonant)	.9378	3.751			1.250-1.768j	1.250-1.768j	

Part III

Bridge "T" Networks

1. Design Formulae

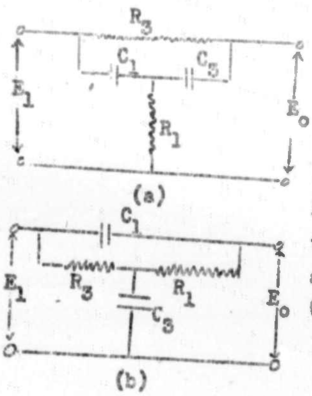


Fig. 14

The two forms of the bridge "T" proportional-derivative networks, Fig. 14(a) and (b), may be obtained as special cases of the parallel "T" in Fig. 8, (a) by taking $R_2 = 0$, and (b) by taking $C_1 = \infty$. In (b), R_2 of Fig. 8 is changed to R_3, R_3 is changed to R_1, C_3 to C_1, C_2 to C_3, for convenience, so that similar formulae will apply to both (a) and (b).

The transfer characteristic is

$$\frac{E_o}{E_1} = \frac{T_1 T_3 p^2 + (T_1 + S_1)p + 1}{T_1 T_3 p^2 + (T_1 + S_1 + T_3)p + 1}$$

where $T_1 = R_1C_1$, $T_3 = R_3C_3$; and $S_1 = R_1C_3$ in case (a), $S_1 = R_3C_1$ in case (b).

To realize the proportional derivative characteristic (8), with derivative time constant T_d and lag ℓ, or gain $g = \ell/T_d$, the time constants T_1, T_3, S_1 are given by the following formulae:

$$T_3 = \frac{2}{\omega_o^2}\left(\frac{1}{\ell} - \frac{1}{T_d}\right) = \frac{2}{\omega_o^2 T_d}\left(\frac{1}{g} - 1\right)$$

$$T_1 = \frac{1}{2\left(\frac{1}{\ell} - \frac{1}{T_d}\right)} = \frac{T_d}{2\left(\frac{1}{g} - 1\right)}$$

$$S_1 = \left[\frac{2}{T_d\omega_o^2} - \frac{1}{2\left(\frac{1}{\ell} - \frac{1}{T_d}\right)}\right] = \frac{2}{T_d\omega_o^2} - \frac{T_d}{2\left(\frac{1}{g} - 1\right)}$$

For positive S_1, $g < 4/(4 + n^2)$, where $n = T_d \omega_o$. For the parallel "T", the upper bound of gain is $1/n$. Thus for n greater than 2, the gains attainable with the bridge "T" are considerably smaller than those which can be obtained with the parallel "T". (Also, since the time lag ℓ is smaller, the relative amplification at frequency $3\omega_o$, the predominant frequency of synchro "hash", is greater for the bridge "T".)

The following formulae in terms of arbitrary C_1, C_3 for case (a), and in terms of arbitrary R_1, R_3 for case (b), may be more convenient. For case (a),

$$R_1 = \frac{2}{n\omega_o(C_1 + C_3)}$$

$$R_3 = \frac{n}{2\omega_o}\left(\frac{1}{C_1} + \frac{1}{C_3}\right)$$

$$g = \frac{4}{n^2\left(1 + \frac{C_3}{C_1}\right) + 4}$$

For case (b),

$$C_1 = \frac{2}{n\omega_o(R_1 + R_3)}$$

$$C_3 = \frac{n}{2\omega_o}\left(\frac{1}{R_1} + \frac{1}{R_3}\right)$$

$$g = \frac{4}{n^2\left(1 + \frac{R_3}{R_1}\right) + 4}$$

The formulae for (b) are obtained from those for (a) by interchanging R_1 with C_1, and R_3 with C_3.

From the above formulae, it is clear that any bridge "T" is reversible, with the same T_d but different gains in the forward and reverse directions, unless it is symmetric, that is $C_1 = C_3$ in case (a), $R_1 = R_3$ in case (b).

2. Input and Output Impedances

For bridge "T" (a), the input impedance (with the output open-circuited) is

$$Z_1 = R_1 + \frac{(T_3 p + 1)}{C_1 T_3 p^2 + (C_1 + C_3)p}$$

The output impedance (with the input short-circuited) is

$$Z_o = \frac{R_3[(S_1 + T_1)p + 1]}{T_1 T_3 p^2 + (T_1 + S_1 + T_3)p + 1}$$

where $S_1 = R_1 C_3$.

For bridge "T" (b), the input impedance (with the output open-circuited) is

$$Z_1 = \frac{1}{C_3 p} + \frac{R_3(T_1 p + 1)}{(T_1 + S_1)p + 1}$$

where $S_1 = R_3 C_1$. The output impedance (with the input short-circuited) is

$$Z_o = \frac{R_1 T_3 p + (R_1 + R_3)}{T_1 T_3 p^2 + (S_1 + T_1 + T_3)p + 1}$$

3. Formulae for Wien Bridge

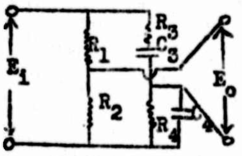


Fig. 15

The Wien Bridge proportional-derivative circuit is shown in Fig. 15. The transfer characteristic is:

$$\frac{E_o}{E_1} = \frac{R_2}{(R_1 + R_2)} \frac{T_3T_4p^2 + [T_3 + T_4 - (R_1/R_2)S_4]p + 1}{T_3T_4p^2 + (T_3 + T_4 + S_4)\,p + 1}$$

where $T_3 = R_3C_3$, $T_4 = R_4C_4$, $S_4 = R_4C_3$. The independent variables of the transfer characteristic may be taken to be T_3, T_4, S_4, and the divider ratio $\rho = R_2/(R_1 + R_2)$. Solving for T_3, T_4, S_4 in terms of $T_d\omega_o = n$, $\ell\omega_o = m$, and ρ, we obtain:

$$T_3 = \frac{1}{\omega_o}\left[\frac{\rho}{n} + \frac{(1-\rho)}{m}\right] \pm \frac{1}{\omega_o}\left[\left(\frac{\rho}{n} + \frac{(1-\rho)}{m}\right)^2 - 1\right]^{1/2}$$

$$T_4 = \frac{1}{\omega_o}\left[\frac{\rho}{n} + \frac{(1-\rho)}{m}\right] \mp \frac{1}{\omega_o}\left[\left(\frac{\rho}{n} + \frac{(1-\rho)}{m}\right)^2 - 1\right]^{1/2}$$

$$S_4 = \frac{2\rho}{\omega_o}\left(\frac{1}{m} - \frac{1}{n}\right)$$

For a given m and ρ, the condition for realizability is $n \leq \rho m/(m + \rho - 1)$. From the form of the transfer characteristic, the gain is $\rho m/n$. For maximum gain, $\rho = m = n - [n(n-1)]^{1/2} >n - [(n-0.5)^2]^{1/2} = 0.5$, the radical vanishes, and $T_3 = T_4 = 1/\omega_o$, $S_4 = 2(n-1)^{1/2}/\omega_o n^{1/2}$. The maximum gain is $2\rho - 1$. For the usual values of $T_d\omega_o = n$, the maximum obtainable gain is approximately 1/4n, which is one-half the gain of the symmetric parallel "T", one-fourth the upper bound of gains obtainable with the parallel "T".

The derivative time constant T_d may be varied in the Wien Bridge by changing the divider ratio ρ, without changing R_3, C_3, R_4, C_4. Solving for $T_d\omega_o = n$, we have

$$n = \frac{2m}{2 - \dfrac{\omega_o S_4 m}{\rho}}$$

Thus as ρ decreases to $1/2\ \omega_o S_4 m$, $n \to \infty$; if $\rho = 1/2\ \omega_o S_4 m$, the Wien Bridge is "resonant"; and as ρ increases to 1, by the realizability condition, n decreases to a value not greater than 1. For any R_3, C_3, R_4, C_4 such that $T_3 T_4 \omega_o{}^2 = 1$, any desired T_d may be obtained by taking ρ to be sufficiently small. Since $2/m\omega_o = T_3 + T_4 + S_4$, the maximum time lag $\ell = m/\omega_o$ (but not maximum gain) will be obtained if $T_3 = T_4 = 1/\omega_o$ and R_4 and C_3 are as small as possible. For $T_3 = T_4 = 1/\omega_o$, the realizability condition is $n \leq 2\rho/[(S_4\omega_o + 2)\rho - S_4\omega_o]$.

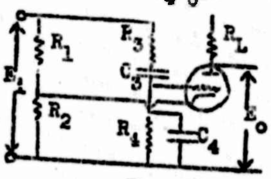


Fig. 16

A vacuum tube circuit similar to Fig. 13 may be used with the Wien Bridge in order to have a common input and output terminal (which may be grounded). See Fig. 16. It may be shown that the transfer characteristic is:

$$\frac{E_o}{E_1} = \frac{-R_L R_2(\mu - 1)\left[(T_3 p + 1)(T_4 p + 1) - \dfrac{(R_2 + R_1\mu)S_4 p}{R_2(\mu - 1)}\right]}{[R_2(R_1 + r_p + R_L) + R_1(r_p + R_L)]\,[(T_3 p + 1)(T_4 p + 1) + S_4 p]} .$$

If the amplification factor μ is very large compared to $R_2/R_1 = \rho/(1 - \rho)$, then except for the gain factor, this transfer characteristic is the same as the transfer characteristic for Fig. 15.

4. Numerical Values for Bridge "T" Networks

The remarks on the tables of values for parallel "T" networks apply also to the following tables for 60-cycle Bridge "T" networks.

Symmetrical Bridge "T" Networks (a)

$C_1 = C_3 = 1.000$

$T_d\omega_o$	Notch Interval cps	$R_1 \times 10^3$	$R_3 \times 10^3$	Gain	Input Impedance $Z_i \times 10^3$	Output Impedance $Z_o \times 10^3$
2.5	± 24	1.0610	6.63	.24242	1.7079-2.135j	1.6075-2.009j
5	± 12	.5305	13.26	.07407	.9878-2.470j	.9824-2.456j
7.5	± 9	.3536	19.89	.03433	.6838-2.564j	.6830-2.561j
10	± 6	.2653	26.53	.01961	.5203-2.602j	.5201-2.601j
15	± 4	.1768	39.79	.008811	.3505-2.629j	.3506-2.629j
20	± 3	.1326	53.05	.004975	.2639-2.639j	.2639-2.639j
30	± 2	.08841	79.58	.002217	.1764-2.647j	.1764-2.647j
40	± 1.5	.06631	106.1	.001248	.1325-2.649j	.1325-2.649j
50	± 1.2	.05305	132.6	.0007994	.1060-2.650j	.1060-2.650j
60	± 1.0	.04420	159.2	.0005552	.08836-2.651j	.08836-2.651j
100	± 0.6	.02653	265.5	.0002000	.05304-2.652j	.05304-2.652j

Bridge "T" Networks (a)

$C_1 = 1.000, C_3 = 2.500$

$T_d\omega_o$	Notch Interval cps	$R_1 \times 10^3$	$R_3 \times 10^3$	Forward Gain	Reverse Gain	Input Impedance $Z_i \times 10^3$	Output Impedance $Z_o \times 10^3$
2.5	± 24	.6062	4.642	.15456	.3137	1.5305-1.914j	.7175- .8979j
5	± 12	.3031	9.284	.04372	.1026	.9565-2.391j	.4059-1.015j
7.5	± 9	.2020	13.926	.01991	.04834	.6737-2.527j	.2772-1.040j
10	± 6	.1516	18.57	.01130	.02778	.5159-2.580j	.2098-1.049j
15	± 4	.1010	27.85	.005054	.01254	.3493-2.619j	.1408-1.056j
20	± 3	.07579	37.14	.002849	.007092	.2634-2.634j	.1058-1.058j
30	± 2	.05052	55.70	.001268	.003165	.1763-2.644j	.07064-1.0597j
40	± 1.5	.03789	74.27	.0007138	.001783	.1324-2.648j	.05301-1.0603j
50	± 1.2	.03031	92.84	.0004569	.001142	.1060-2.649j	.04242-1.06052j
60	± 1.0	.02526	111.4	.0003174	.0007930	.08835-2.650j	.03536-1.0607j
100	± 0.6	.01516	185.7	.0001143	.0002856	.05304-2.652j	.02122-1.0609j

To obtain values for Bridge "T" (b), in the tables above interchange R_1 with C_1, and R_3 with C_3.

Maximum Gain Wien Bridge Networks

$R_3 = .002653$, $C_3 = 1.000$

$Z_1 = (R_1+R_2)$ in parallel with Z_1'. $Z_o = R_1R_2/(R_1+R_2)$ in series with Z_o'.

$T_d\omega_o$	Notch Interval cps.	$R_2/(R_1+R_2) = \rho = m$	$R_4 \times 10^3$	C_4	Gain	$Z_1' \times 10^3$	$Z_o' \times 10^3$
5	± 12	.5279	4.745	.5590	.05572	5.024(1-j)	1.252(1-j)
10	± 6	.5162	5.033	.5270	.03234	5.168(1-j)	1.291(1-j)
15	± 4	.5086	5.125	.5175	.01724	5.214(1-j)	1.303(1-j)
20	± 3	.5064	5.171	.5130	.01282	5.237(1-j)	1.309(1-j)
30	± 2	.5042	5.216	.5086	.008476	5.259(1-j)	1.315(1-j)
40	± 1.5	.5032	5.238	.5064	.006330	5.271(1-j)	1.318(1-j)
50	± 1.2	.5025	5.252	.5051	.005050	5.277(1-j)	1.319(1-j)
60	± 1.0	.5021	5.260	.5042	.004220	5.282(1-j)	1.320(1-j)
100	± 0.6	.5013	5.278	.5025	.002512	5.291(1-j)	1.322(1-j)
∞	(resonant)	.5000	5.305	.5000		5.305(1-j)	1.326(1-j)

Wien Bridge, $R_3 = .002653 = 2R_4$, $C_4 = 2.0 = 2C_3$, $m = 0.8$

$T_d\omega_o$	$R_2/(R_1+R_2) = \rho$	Gain
5	.2381	.03810
10	.2174	.01739
15	.2113	.01127
20	.2083	.008333
30	.2055	.005479
40	.2041	.004031
50	.2033	.003252
60	.2027	.002702
100	.2016	.001612
∞	.2000	

Wien Bridge, $R_4 = .005305 = 2R_3$, $C_3 = 1.0 = 2C_4$, $m = 0.5$

$T_d\omega_o$	$R_2/(R_1+R_2) = \rho$	Gain
5	.5556	.05556
10	.5263	.02632
15	.5172	.01724
20	.5128	.01282
30	.5085	.008474
40	.5063	.006329
50	.5051	.005050
60	.5042	.004201
100	.5025	.002502
∞	.5000	

Part IV

Methods of Obtaining Required Carrier Phase Shift

As mentioned in Part I, it is necessary that the phase of the carrier of the modulated error signal applied to the control winding of the two-phase motor be 90° out of phase with the voltage on the fixed winding. All methods of obtaining phase shift of the carrier cause the frequency characteristic of the a.c. servo controller to become unsymmetrical about the carrier frequency, and also affect the phase characteristic. This distortion of the frequency-phase characteristics, and the question of its effect on servo performance, would not arise in a case where it was feasible to shift instead the phase of the voltage applied to the fixed winding. (Assuming the simplified motor, the effect of the distortion on stability and error in any case may be determined by the method of Part I.)

So far as obtaining the proportional-derivative effect on the modulation is concerned, the essential portion of the transfer characteristic of the parallel "T" is the factor $[1 + jT_d \frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)]$ in the numerator. Accordingly, in the following first section we investigate the phase shift which can be obtained from the remaining portion of the parallel "T" transfer characteristic, preserving the proportional-derivative factor in the numerator.

1. Phase Shift at ω_o In Terms of Independent Constants in the Parallel "T"

Let the numerator and denominator of the parallel "T" transfer characteristic [(1), Part II] be multiplied by $n\omega_o/2p$, and let $p = j\omega$. Then in order that the numerator consist of the proportional-derivative characteristic $[1 + jT_d \frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)]$, multiplied by a linear factor $(Uj\omega + 1)$, the following conditions must be satisfied by the fundamental time constants:

$$T_1T_2T_3 = \frac{u}{\omega_o^3} \qquad (1)$$

$$T_1(S_2 + T_3) = \frac{1}{\omega_o^2}(1 + \frac{2u}{n}) \qquad (2)$$

$$T_1 + S_1 = \frac{1}{\omega_o}(\frac{2}{n} + u) \qquad (3)$$

Suppose that these conditions are satisfied. Then we may consider T_1 and T_2 as independent variables, T_3, S_1 and S_2 as dependent variables: $T_3 = u/\omega_o^3 T_1T_2$ from (1), and then S_2 and S_1 may be expressed in terms of T_1, T_2 from (2) and (3). For positive S_1 and S_2, we see that T_1, T_2 must be restricted to lie in the region $T_2\omega_o > nu/(n + 2u)$, $T_1\omega_o < (2 + nu)/n$.

In terms of $T_1\omega_o$, $T_2\omega_o$, u, the phase shift ϕ at ω_o is given by:

$$\tan \phi = \frac{u^2 - z}{(u^2 + 1)[a + \frac{n}{2}(1 + z)] + u\,(1 + z)} \qquad (4)$$

where $a = T_1\omega_o$, $z = T_1T_2\omega_o^2$, $n = T_d\omega_o$. Thus if $T_1T_2\omega_o^2 < u^2$, the phase shift is a lead; and if $T_1T_2\omega_o^2 > u^2$, it is a lag. If $T_1T_2\omega_o^2 = u^2$, then not only is there zero phase shift at ω_o, but also it may be verified that the factor $(Up + 1)$ of the numerator is also a factor of the denominator, so that the transfer characteristic is of the form (2), Part II. The gain at ω_o is given by:

$$g = \frac{2a(u^2 + 1)^{1/2}}{[(2a + nz + n + 2u)^2 + (2au + nu + 2z + znu)^2]^{1/2}}$$

$$= \frac{2a}{2(a + u) + n(1 + u^2)} \qquad \text{in case } z = u^2 .$$

Let us examine the expression (4) for $\tan \phi$ to determine the maximum amount of phase shift which can be obtained. The expression:

$$\frac{u^2 - z}{K(u^2 + 1) + u(1 + z)}$$

is a monotonic function of u; its value increases from $-z/K$ at $u = 0$ to $1/K$ as $u \to \infty$. Therefore the tangent of the angle of phase lead cannot be greater than $\frac{1}{a + n(1 + z)/2} < \frac{2}{n}$. The tangent of the angle of phase lag cannot be greater than

$$\frac{z}{[a + \frac{n}{2}(1 + z)]} = \frac{1}{[\frac{a}{z} + \frac{n}{2}(\frac{1}{z} + 1)]} < \frac{2}{n}.$$

Thus for $T_d\omega_o = n = 15$, the upper bound of possible phase shift is less than 7.6°. Actually, the bounds as found by calculation, for the case of equal condensers, were 3.8° lag, 0.5° lead.

It may be verified that equations (1), (2), (3) are invariant when the input and output of the parallel "T" are reversed, that is, when $R_2 \rightleftarrows R_3$ and $C_1 \rightleftarrows C_3$. They are also invariant when one of the individual "T"'s of the two "T"'s in parallel is reversed, that is, when either $R_2 \rightleftarrows R_3$, or $C_1 \rightleftarrows C_3$. Thus any proportional-derivative parallel "T" is reversible, without changing T_d, and the individual "T"'s likewise are reversible, to within the practically negligible amount of phase shift of the carrier.

2. Load Impedance and Series Input Impedance

Though the intrinsic phase shift obtainable in the parallel "T", together with the proper proportional-derivative characteristic for usably high values of T_d, is very small, an appreciable phase shift at the carrier frequency may be obtained (1) by loading the output of the parallel "T", or (2) by inserting a resistor or other impedance in series with the input to the parallel "T". A third method is to insert a one or two stage R-C or C-R phase-lag or phase-lead network either preceding or following the proportional-derivative parallel "T". (With any of the three methods, there is of course an accompanying effect on the gain-phase margin diagram, which may be calculated as in Part I.)

In this section we consider (1) and (2). If the input impedance to the parallel "T" is A-jB, and the source of E_1 has series impedance Z, Fig. 17, the phase shift is $\angle[(A-jB)/(A+Z-jB)]$. Thus if Z is resistive, the phase shift approaches a lag of arc tan B/A as $Z \rightarrow \infty$; and if Z is inductive, a phase lag up to 90° + arc tan B/A may be obtained.

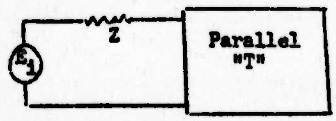


Fig. 17

If a load impedance Z_L is connected to the output of the parallel "T", by Thevenin's theorem the circuit is equivalent to Fig. 18, where B(p) is the parallel "T" transfer characteristic. If the output impedance is $Z_o = C-jD$, the phase shift is $Z_L/(C-jD+Z_L)$. Thus if Z_L is resistive, the phase shift approaches a lead of arc tan D/C as $Z_L \rightarrow 0$; and if $\angle Z_L = \theta > 0$, a phase lead up to θ+ arc tan D/C may be obtained by taking Z_L to be sufficiently small.

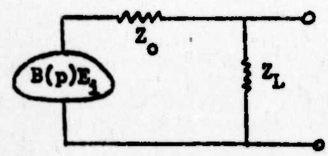


Fig. 18

Since the gain of the transfer characteristic is small, the effects of series input impedance and load impedance are nearly independent, and the total phase shift with both is approximately equal to the sum of the phase shifts which would be caused by each separately. Thus for example if Z is a large resistance, and Z_L a large capacity, a total phase lag of approximately $\angle Z_1 + (90° - \angle Z_o)$ may be obtained.

If series input impedance Z and load impedance Z_L are connected to a transfer characteristic B(p), Fig. 19, it may be shown that the resulting transfer characteristic is:

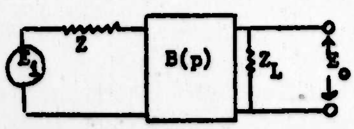


Fig. 19

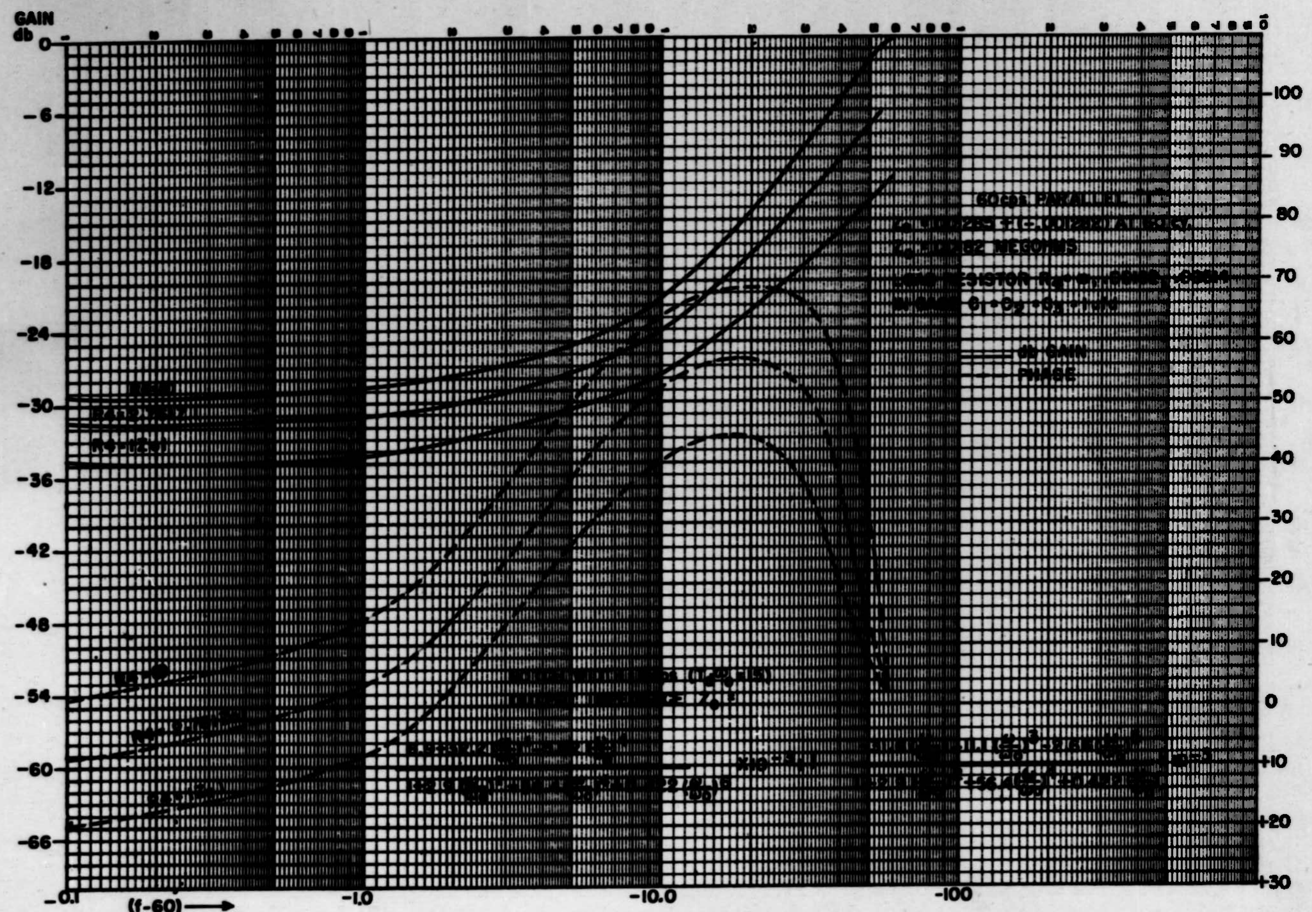
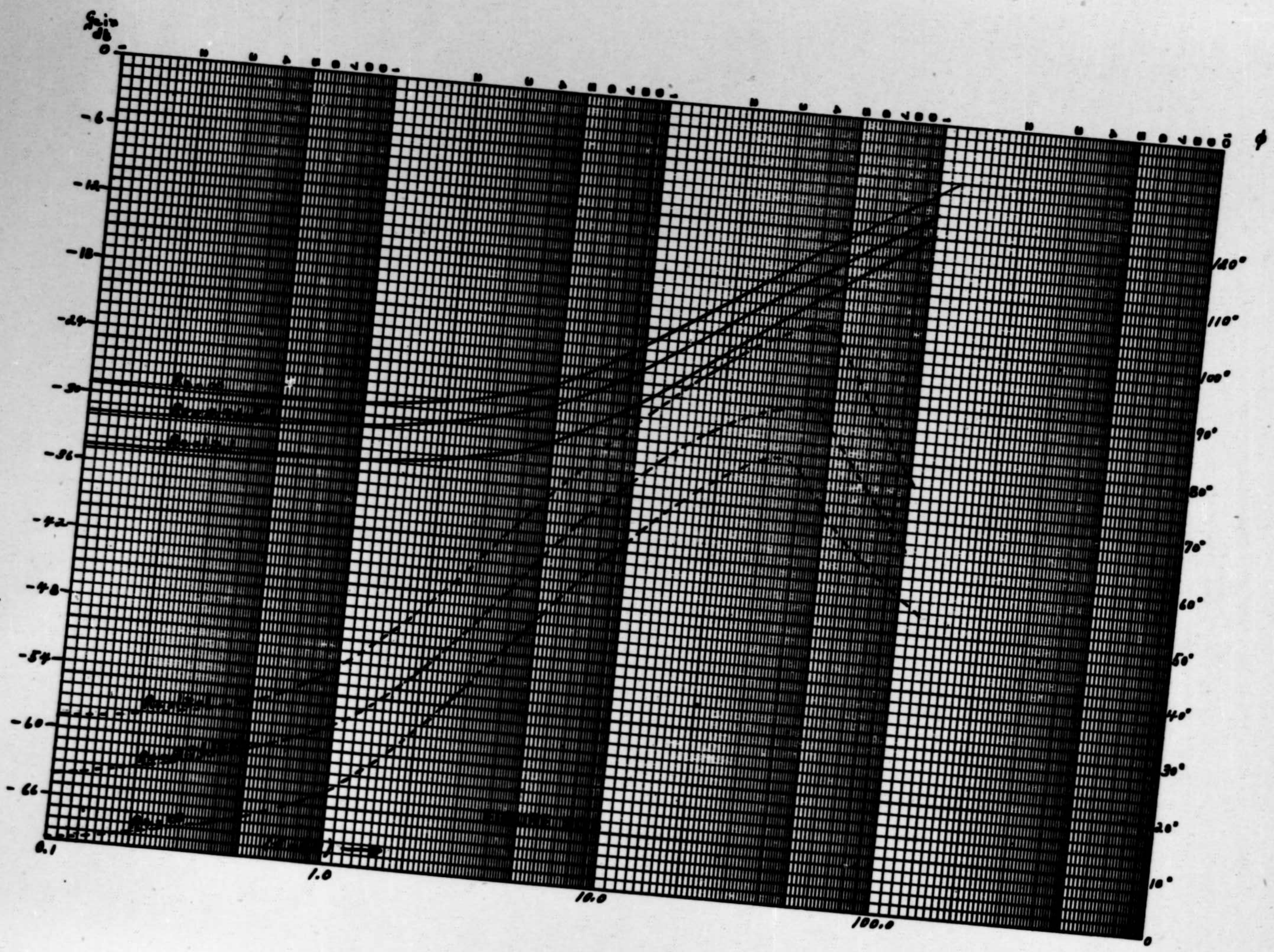

GAIN
db
0
-6
-12
-18
-24
-30
-36
-42
-48
-54
-60
-66
-0.1
(f-60) ⟶
-1.0
-10.0
-100
100
90
80
70
60
50
40
30
20
10
0
+10
+20
+30

FIGURE 20 A


Gain db
0
-6
-12
-18
-24
-30
-36
-42
-48
-54
-60
-66
0.1
1.0
10.0
100.0
φ
120°
110°
100°
90°
80°
70°
60°
50°
40°
30°
20°
10°
0

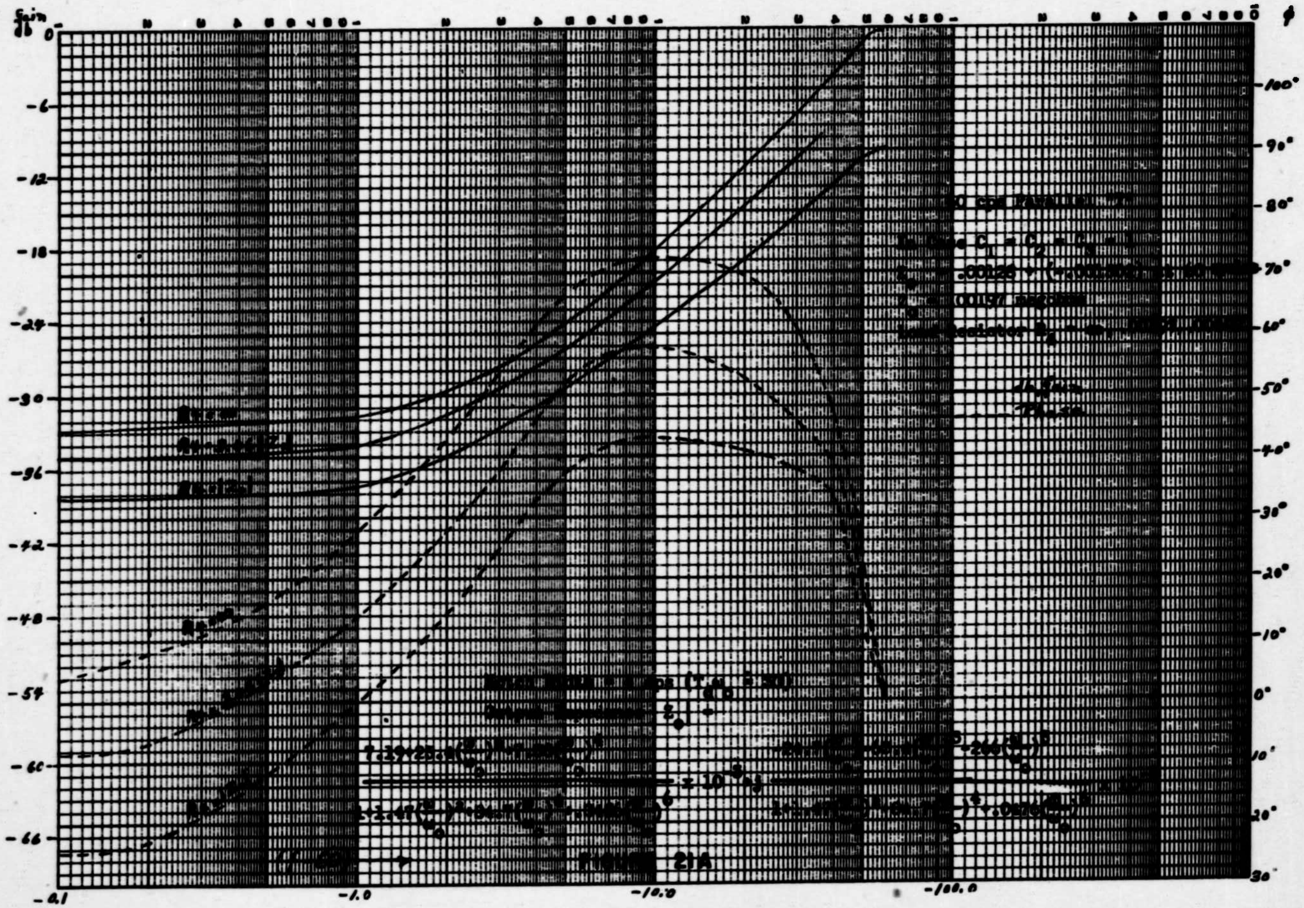
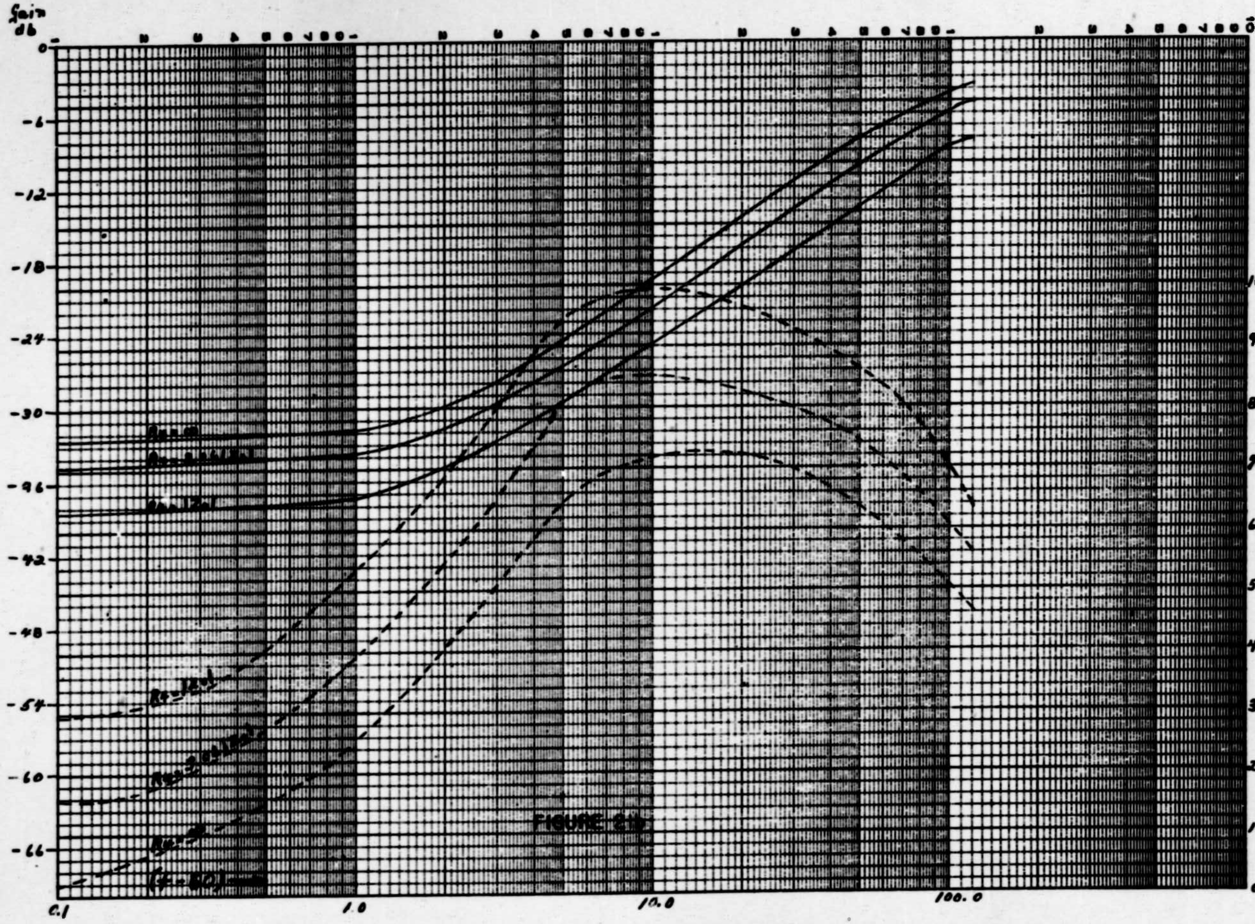

Gain db
φ
0
-6
-12
-18
-24
-30
-36
-42
-48
-54
-60
-66
100°
90°
80°
70°
60°
50°
40°
30°
20°
10°
0°
10°
20°
30°
-0.1
-1.0
-10.0
-100.0
× 10⁻³ s

FIGURE 21A



Gain db
0
-6
-12
-18
-24
-30
-36
-42
-48
-54
-60
-66
0.1
1.0
10.0
100.0
100°
90°
80°
70°
60°
50°
40°
30°
20°
10°
0

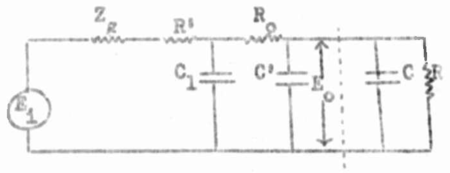
FIGURE 21

$$\frac{E_o}{E_i} = \frac{B(p)}{(1 + \frac{Z}{Z_i})(1 + \frac{z_{22}}{Z_L}) - \frac{Z_i}{Z_L}[B(p)]^2}$$

where $z_{22} = Z_o + Z_i\,[B(p)]^2$ is the output impedance with the input open-circuited.

Figures 20 and 21 show the effect of a load resistor on the parallel "T" transfer characteristic, for the case of equal condensers, with $T_d\omega_o = 15$ and 30.

3. <u>Formulae for Phase-Lag Networks</u>



$R_1 = R_g + R'$
$C_o = C + C'$
Fig. 22

Let the load on the phase-lag network at carrier frequency ω_o be replaced by an equivalent resistor R and condenser C in parallel, as in Fig. 22. The capacity C is included in C_o; the source impedance Zg, if purely resistive, is included in R_1. (A procedure for dealing with the case where Zg is complex is indicated later.) The transfer characteristic is:

$$\frac{E_o}{E_i} = \frac{R}{R\,T_1T_op^2 + [R_oT_1 + R(T_o + S_1 + T_1)]p + R + R_o + R_1}$$

where $T_o = R_oC_o$, $T_1 = R_1C_1$. Thus the angle of phase <u>lag</u> through the network is:

$$\emptyset = \text{arc tan}\,\frac{\omega_o[R(R_o + R_1)C_o + T_1(R_o + R)]}{R + R_o + R_1 - RT_oT_1\omega_o^2}\ .$$

Let $\tan\emptyset = -k$. Then the gain is:

$$g = \frac{R}{\pm(RT_oT_1\omega_o^2 - R - R_o - R_1)(1 + k^2)^{1/2}}$$

where the + sign corresponds to $\emptyset > 90°$, the - sign to $\emptyset < 90°$.

Suppose that k, R, C_o, C_1 are given. Then the following formulae determine R_o, R_1 for maximum gain.

$$c(1-kb)mR_o^2 + [b(m + 1) + c(n + mr-kbn) + k(1 + m)]R_o + n(b+cR)+k(R+n) = 0$$

$$R_1 = mR_o + n$$

where $b = \omega_o RC_o$, $c = C_1\omega_o$, $m = (b^2 + 1)/[(b^2 + 1) + bcR]$,
$n = R/[(b^2 + 1) + bcR]$.

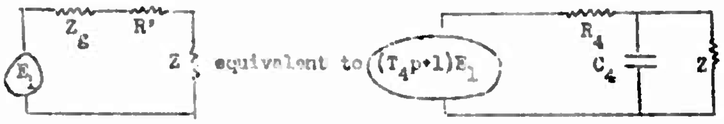
If k, R, C_o, C_1 are given, the following formulae determine R_o, R_1 for a specified gain g:

$$c(1-kb)R_o^2 - c(1-kb)AR_o - kR - (k + b)A = 0$$

$$R_1 = A-R_o$$

where $A = R\,[(kb - 1)g^{-1}\,(1+k^2)^{-1/2} + kb - 1 - k]/(b + k + 1 - kb)$.

In case Zg is complex, let $Zg + R' = x + jy$. Then the voltage source E_1 with impedance $x + jy$, at frequency ω, is equivalent to a source $(T_4p + 1)E_1$ with resistance R_4, paralleled by a (positive or negative) capacity C_4, where $x = R_4/(1 + T_4^2\omega^2)$, $y = -R_4T_4\omega/(1 + T_4^2\omega^2)$, $T_4 = R_4C_4$, or $R_4 = x + y^2/x$, $T_4\omega = -y/x$.



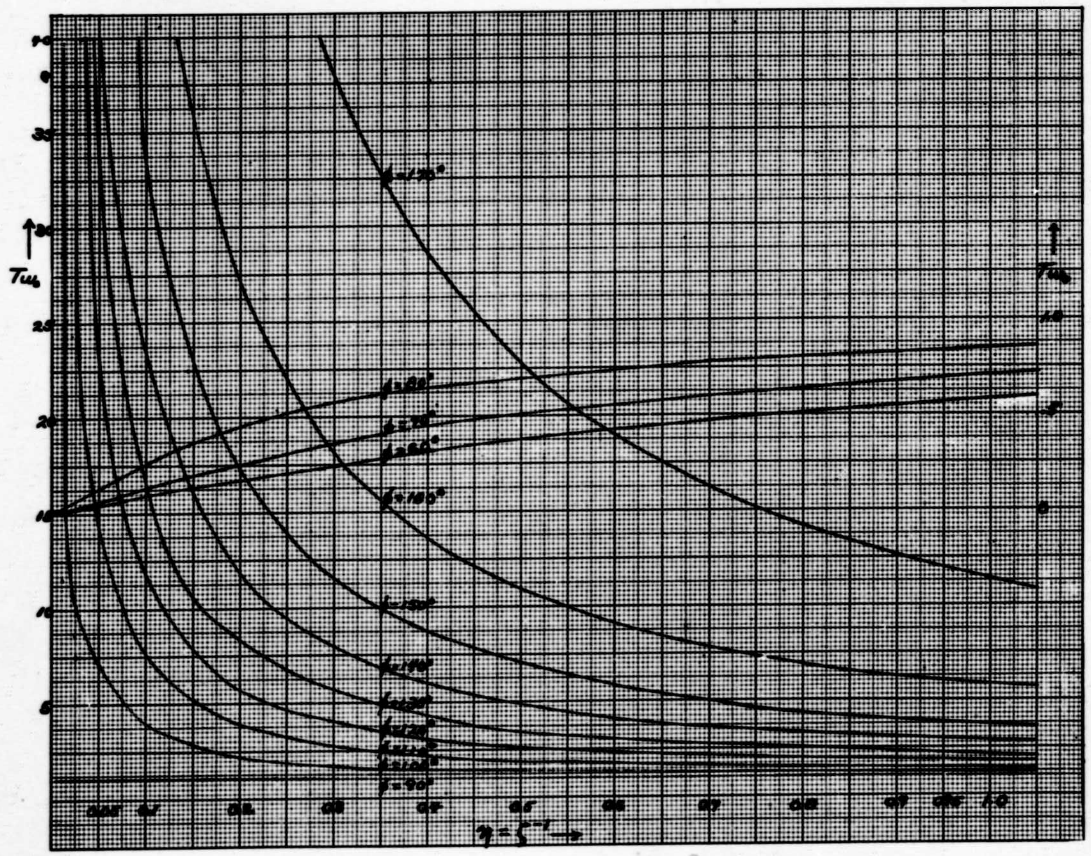


FIGURE 23

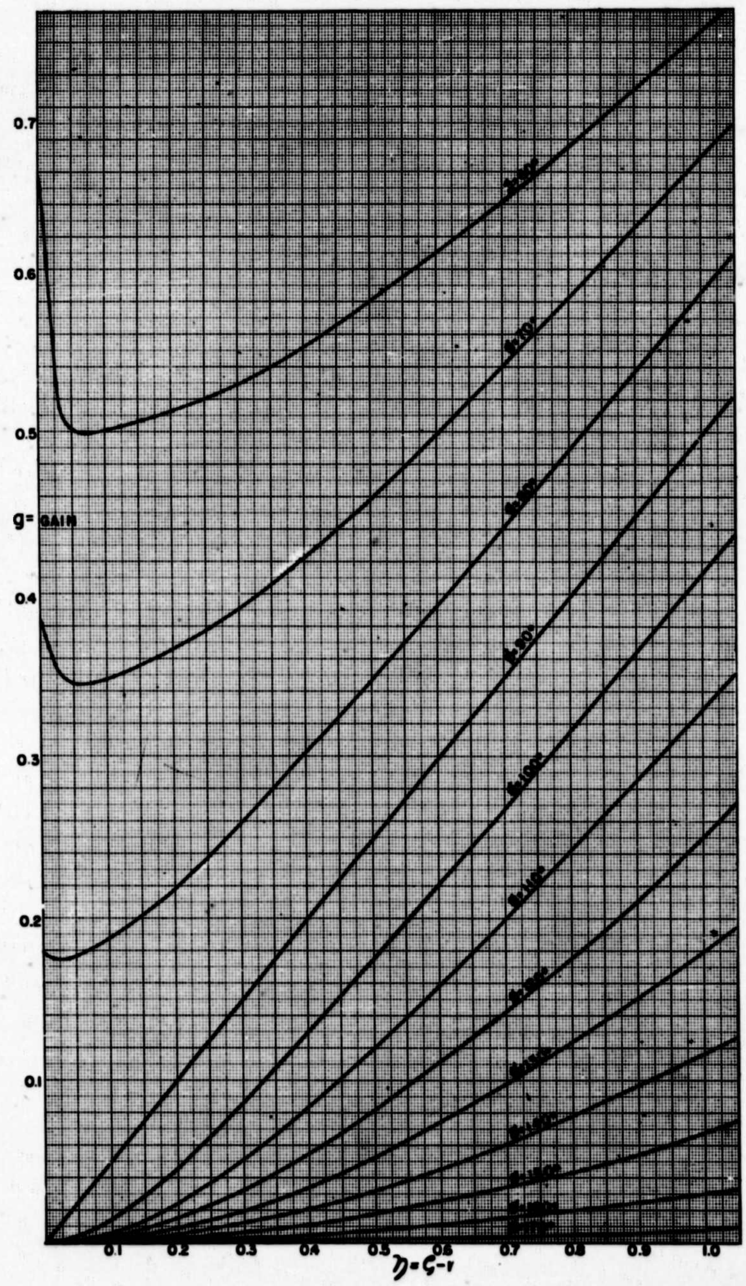


FIGURE 24

Thus to compute the phase shift, R_4 may be substituted for R_1, and C_1 modified by the addition of C_4. Then the phase lag with respect to E_1 is ϕ - arc tan $T_4\omega_o$, and the gain is $g(1 + T_4^2\omega^2)^{1/2}$.

4. Numerical Values for Phase Lag Networks

Consider the transfer characteristic

$$\frac{1}{T^2p^2 + 2\zeta Tp + 1}$$

The phase lag ϕ of this transfer characteristic at frequency ω_o is

$$\phi = \text{arc tan} \frac{2\zeta T\omega_o}{(1 - T^2\omega_o^2)} .$$

Fig. 23 is a graph of the values of ζ^{-1} and $T\omega_o$ for phase lag angles ϕ from 60° to 170°, and Figure 24 is a graph of corresponding values of gain at ω_o as a function of $\zeta^{-1} = \eta$. The values are also tabulated below, in convenient form for the design of phase lag networks.

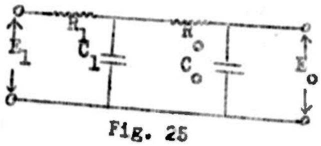


Fig. 25

The transfer characteristic for the two-section phase lag network with no load resistor, Fig. 25, is:

$$\frac{E_o}{E_1} = \frac{1}{T_1T_op^2 + (T_o + S_1 + T_1)p + 1}$$

where $T_1 = R_1C_1$, $T_o = R_oC_o$, $S_1 = R_1C_o$. The input impedance is

$$Z_1 = \frac{T_1T_op^2 + (T_o + S_1 + T_1)p + 1}{C_1T_op^2 + (C_o + C_1)p}$$

The output impedance (with the input short-circuited) is

$$Z_o = \frac{R_oT_1p + R_o + R_1}{T_1T_op^2 + (T_o + S_1 + T_1)p + 1} .$$

If the condensers C_o, C_1 are specified, the following formulae give the two solutions for R_o, R_1 to realize any desired ζ and T:

$$R_1 = \frac{\zeta T \pm T\,[\zeta^2 - 1 - (C_o/C_1)]^{1/2}}{(C_o + C_1)}$$

$$R_o = \frac{\zeta T \mp T[\zeta^2 - 1 - (C_o/C_1)]^{1/2}}{C_o}$$

These solutions are tabulated below, together with the impedences Z_1 and Z_o, for several sets of values for C_o, C_1.

If a load resistor R is added to Fig. 25, as in section 3, the transfer characteristic is:

$$\frac{E_o}{E_1} = \frac{R/(R + R_o + R_1)}{T_1T_op^2R/(R+R_o+R_1) + [R_oT_1 + R(T_o+S_1+T_1)]p/(R+R_o+R_1) + 1}$$

Thus T and ζ become T' and ζ', where $T' = [R/(R+R_o+R_1)]^{1/2}T$, $2\zeta'T' = [R_oT_1 + 2\zeta TR]/(R+R_o+R_1)$. If T' and ζ' are calculated, the new phase shift may be determined from Fig. 23, and the new gain is $R/(R + R_o + R_1)$ times the value corresponding to ζ' and T' in Fig. 24. Or, if the output impedance Z_o of the phase lag network is at hand, the change of phase due to any load impedance may be easily calculated.

ζ^{-1}	60° T	60° ζT	60° g	70° T	70° ζT	70° g	80° T	80° ζT	80° g	90° T	90° ζT	90° g
0.05	.000115	.00229	.501	.000181	.00363	.344	.000369	.00738	.177	.00265	.0531	.025
0.1	.000228	.00228	.504	.000358	.00358	.348	.000700	.00700	.187	.00265	.0265	.05
0.2	.000446	.00223	.515	.000681	.00341	.366	.001198	.00599	.218	.00265	.0133	.1
0.3	.000648	.00216	.532	.000952	.00317	.393	.001518	.00506	.258	.00265	.00884	.15
0.4	.000829	.00207	.554	.001173	.00293	.425	.001730	.00433	.302	.00265	.00663	.2
0.5	.000989	.00198	.581	.001350	.00270	.462	.001877	.00375	.348	.00265	.00531	.25
0.6	.001129	.00188	.611	.001493	.00249	.501	.001985	.00331	.395	.00265	.00442	.3
0.7	.001251	.00179	.643	.001610	.00230	.542	.002067	.00295	.442	.00265	.00379	.35
0.8	.001357	.00170	.677	.001707	.00213	.584	.002132	.00267	.490	.00265	.00332	.4
0.9	.001450	.00161	.713	.001789	.00199	.627	.002183	.00243	.538	.00265	.00295	.45
0.95	.001492	.00157	.731	.001824	.00192	.649	.002205	.00232	.563	.00265	.00279	.475

ζ^{-1}	100° T	100° ζT	100° g	110° T	110° ζT	110° g	120° T	120° ζT	120° g	130° T	130° ζT	130° g
0.05	.019077	.38154	.0034	.038797	.77594	.0016	.061370	1.2274	.00094	.089105	1.78209	.00057
0.1	.010054	.10054	.013	.019666	.19666	.0065	.030856	.30856	.0037	.044670	.44670	.0023
0.2	.005875	.02938	.044	.010335	.05167	.024	.015760	.07880	.015	.022568	.11284	.0090
0.3	.004636	.01545	.085	.007388	.02463	.051	.010857	.03619	.032	.015298	.05099	.020
0.4	.004068	.01017	.123	.006000	.01500	.083	.008486	.02122	.054	.011728	.02932	.035
0.5	.003748	.00750	.174	.005212	.01042	.120	.007114	.01423	.081	.009633	.01927	.053
0.6	.003544	.00591	.221	.004711	.00785	.159	.006233	.01039	.111	.008270	.01378	.074
0.7	.003404	.00486	.269	.004369	.00624	.200	.005625	.00804	.143	.007320	.01046	.097
0.8	.003301	.00413	.317	.004121	.00515	.242	.005185	.00648	.177	.006626	.00828	.123
0.9	.003223	.00358	.355	.003934	.00437	.285	.004853	.00539	.213	.006099	.00678	.150
0.95	.003190	.00336	.389	.003857	.00406	.307	.004716	.00496	.231	.005832	.00619	.164

ζ^{-1}	140° T	140° ζT	140° g	150° T	150° ζT	150° g	160° T	160° ζT	160° g	170° T	170° ζT	170° g
0.05	.12650	2.5300	.00034	.18379	3.6759	.00018	.23804	5.76080	.000079	.60169	12.034	.000019
0.1	.063334	.63334	.0013	.091963	.91963	.00072	.145803	1.45803	.00031	.300882	3.00862	.000077
0.2	.031833	.15917	.0054	.046095	.23048	.0029	.072105	.36053	.0013	.150476	.75238	.00031
0.3	.021403	.07134	.012	.030857	.10286	.0064	.048729	.16243	.0028	.100356	.33452	.00069
0.4	.016239	.04060	.021	.023274	.05819	.011	.036198	.09050	.0050	.075308	.18827	.0012
0.5	.013178	.02636	.032	.018752	.03750	.018	.029045	.05809	.0078	.060288	.12058	.0019
0.6	.011167	.01861	.046	.015787	.02631	.025	.024292	.04049	.0112	.050283	.08381	.0027
0.7	.009753	.01393	.061	.013642	.01949	.034	.021163	.03023	.0150	.043143	.06163	.0037
0.8	.008711	.01089	.078	.012069	.01509	.044	.018384	.02298	.020	.037793	.04724	.0049
0.9	.007914	.00879	.097	.010858	.01206	.055	.016430	.01326	.025	.033638	.03738	.0062
0.95	.007583	.00798	.107	.010352	.01090	.061	.015788	.01662	.027	.031891	.03357	.0069

Critical Values

ζ^{-1}	60°			70°			80°			90°		
	T	ζT	g	T	ζT	g	T	ζT	g	T	ζT	g
.953	.001495	.00157	.733	.001827	.00192	.651	.002207	.00231	.564	.00265	.00278	.477
.913	.001461	.00160	.718	.001798	.00197	[illegible]	.002189	.00240	.545	.00265	.00291	.456
.816	.001373	.00168	.683	.001722	.00211	.591	.002141	.00262	.498	.00265	.00325	.408
.707	.001259	.00178	.645	.001618	.00229	.545	.002072	.00293	.446	.00265	.00375	.354
.577	.001099	.00190	.604	.001463	.00253	.492	.001963	.00340	.384	.00265	.00459	.289
.408	.000843	.00207	.556	.001139	.00291	.428	.001744	.00427	.306	.00265	.00650	.204
.302	.000651	.00216	.532	.000956	.00317	.393	.001522	.00505	.259	.00265	.00830	.151

ζ^{-1}	100°			110°			120°			130°		
	T	ζT	g	T	ζT	g	T	ζT	g	T	ζT	g
.953	.003188	.00334	.391	.003852	.00404	[illegible]	.004707	.00494	.233	.005868	.00615	.165
.913	.003214	.00352	.371	.003913	.00429	.291	.004816	.00528	.218	.006041	.00662	.154
.816	.003286	.00403	.324	.004086	.00500	.249	.005124	.00628	.183	.006529	.00800	.127
.707	.003395	.00480	.272	.004349	.00615	.203	.005590	.00791	.145	.007264	.01027	.099
.577	.003584	.00621	.210	.004808	.00833	.150	.006404	.01109	.104	.008534	.01478	.069
.408	.004035	.00988	.132	.005918	.01450	.086	.008345	.02044	.056	.011514	.02820	.039
.302	.004624	.01534	.085	.007360	.02441	.051	.010809	.03585	.03[illegible]	.015225	.05050	.020

ζ^{-1}	140°			150°			160°			170°		
	T	ζT	g	T	ζT	g	T	ζT	g	T	ζT	g
.953	.007561	.00793	.108	.01032	.01082	.0613	.01573	.01650	.0275	.03176	.0333	.00691
.913	.007825	.00857	.099	.01072	.01174	.0565	.01640	.01796	.02526	.03317	.0363	.00634
.816	.008565	.01049	.081	.01185	.01451	.0457	.01824	.02234	.02031	.03704	.0454	.00508
.707	.009669	.01367	.0[illegible]2	.01352	.01911	.0347	.02095	.02962	.01531	.04271	.0604	.00381
.577	.011559	.02002	.043	.01635	.02831	.0234	.02552	.04420	.01026	.05225	.0905	.00254
.408	.015928	.03902	.022	.02232	.05589	.0119	.03590	.08793	.00516	.07379	.1808	.00127
.302	.021299	.07064	.012	.03070	.10133	.0065	.04849	.16031	.00282	.09986	.3312	.00070

$C_1 = 1.0$, $C_o = 1.0$

	60°				70°			
ζ^{-1}	$R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$	$R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$
0.05	2.292 0	0 4.584	2.293-i1.324 1.968-i1.325	.572-i1.995 1.150-i1.989	3.629 .0096	.0191 7.241	3.627-i1.325 1.896-i1.326	.435-i1.174 .859-i2.340
0.1	2.269 .0115	.0229 4.537	2.286-i1.321 1.967-i1.324	.586-i1.000 1.165-i1.984	3.562 .018	.0360 7.124	3.586-i1.314 1.900-i1.321	.444-i1.177 .866-i2.357
0.2	2.189 .0456	.0911 4.369	2.254-i1.307 1.962-i1.316	.621-i1.017 1.171-i1.968	3.338 .072	.1441 6.676	3.435-i1.284 1.921-i1.308	.497-i1.196 .905-i2.318
0.3	2.058 .1020	.2040 4.116	2.205-i1.267 1.952-i1.302	.674-i1.042 1.194-i1.942	3.022 .148	.2964 6.044	3.229-i1.240 1.954-i1.290	.571-i1.225 .958-i2.286
0.4	1.890 .1305	.3609 3.779	2.144-i1.262 1.940-i1.283	.748-i1.081 1.223-i1.901	2.674 .256	.5117 5.348	3.005-i1.192 2.054-i1.286	.675-i1.260 1.040-i2.215
0.5	1.684 .2907	.5814 3.379	2.078-i1.235 1.924-i1.260	.842-i1.138 1.250-i1.844	2.305 .395	.7908 4.609	2.785-i1.159 2.065-i1.229	.787-i1.342 1.088-i2.170
0.6	1.438 .4422	.8843 2.876	2.007-i1.209 1.909-i1.234	.963-i1.223 1.268-i1.755	1.903 .587	1.173 3.807	2.564-i1.126 2.072-i1.187	.922-i1.447 1.148-i2.066
0.7	1.021 .7687	1.587 2.043	1.924-i1.190 1.902-i1.190	1.149-i1.422 1.231-i1.563	1.313 1.987	1.975 2.625	2.299-i1.110 2.181-i1.130	1.107-i1.674 1.167-i1.839
$2^{-1/2}$	.859	1.760	1.910-i1.192	1.194-i1.491	1.144	2.288	2.232-i1.118	1.143-i1.755

$C_1 = 1.0$, $C_o = 1.0$

	80°				90°			
ζ^{-1}	$R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$	$R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$
0.05	7.348 .0093	.018 14.696	7.381-i1.313 1.266-i1.328	.235-i1.284 .461-i2.553	52.990 .117	.233 105.970	52.95 -i1.163 .133-i1.325	.116-i1.310 .117-i2.811
0.1	6.965 .035	.070 13.930	7.013-i1.280 1.297-i1.320	.262-i1.293 .486-i2.566	26.367 .133	.266 52.734	26.46 -i .839 .266-i1.321	.132-i1.333 .132-i2.646
0.2	5.868 .122	.245 11.735	6.024-i1.192 1.412-i1.303	.348-i1.318 .565-i2.546	12.991 .309	.618 25.982	13.09 -i .762 .531-i1.300	.302-i1.357 .302-i2.622
0.3	4.821 .239	.476 9.642	5.098-i1.111 1.552-i1.276	.460-i1.347 .664-i2.512	8.420 .420	.840 16.839	8.625-i .683 .798-i1.267	.400-i1.389 .400-i2.589
0.4	3.948 .382	.763 7.897	4.344-i1.052 1.698-i1.238	5.84-i1.399 .771-i2.459	6.047 .584	1.167 12.093	6.325-i .696 1.070-i1.220	.532-i1.443 .533-i2.536
0.5	3.202 .548	1.096 6.404	3.732-i1.007 1.853-i1.192	.714-i1.474 .875-i2.384	4.529 .731	1.562 9.058	4.882-i .719 1.355-i1.157	.666-i1.521 .666-i2.457
0.6	2.530 .780	1.559 5.061	3.203-i .985 2.024-i1.132	.858-i1.588 .985-i2.269	3.379 1.042	2.083 6.757	3.830-i .752 1.686-i1.074	.796-i1.639 .796-i2.340
0.7	1.634 1.266	2.532 3.368	2.608-i .978 2.340-i1.039	1.025-i1.833 1.057-i2.020	2.163 1.627	3.255 4.325	2.753-i .845 2.276-i .928	.929-i1.896 .928-i2.083
$2^{-1/2}$	1.465	2.931	2.464-i1.016	1.051-i1.929	1.876	3.751	2.501-i .884	.938-i1.990

$C_1 = 1.0,\ C_o = 1.0$ 100° / 110°

ζ^{-1}	$R_1 \times 10^3$ (100°)	$R_o \times 10^3$ (100°)	$Z_1 \times 10^3$ (100°)	$Z_o \times 10^3$ (100°)	$R_1 \times 10^3$ (110°)	$R_o \times 10^3$ (110°)	$Z_1 \times 10^3$ (110°)	$Z_o \times 10^3$ (110°)
0.05	381.06	.960	357.8+i 31.71	.239-i1.295	774.97	1.94	598.3 +i124.0	.429-i1.173
	.48	762.12	-.450-i1.331	.012-i2.571	.97	1549.94	-.958-i1.325	.0035-i2.342
0.1	100.04	1.01	93.77 +i7.872	.259-i1.293	195.67	1.93	149.2+i30.64	.401-i1.177
	.51	200.07	-.399-i1.320	.040-i2.566	.99	391.34	-.934-i1.320	.019-i2.336
0.2	28.78	1.20	26.83 +i1.767	.339-i1.313	55.62	2.11	38.16 +i7.366	.527-i1.209
	.60	57.56	-.256-i1.299	.121-i2.547	1.06	111.23	-.778-i1.287	.079-i2.312
0.3	14.72	1.46	13.67 +i .556	.435-i1.347	23.46	2.33	17.34 +i2.981	.534-i1.227
	.73	29.44	.0104-i1.263	.230-i2.512	1.16	46.93	-.690-i1.261	.126-i2.288
0.4	9.28	1.78	8.600-i .078	.541-i1.399	13.78	2.63	9.868+i1.398	.600-i1.282
	.88	18.56	.294-i1.203	.444-i2.459	1.32	27.37	-.478-i1.203	.248-i2.241
0.5	6.40	2.20	5.928-i .197	.650-i1.475	8.90	3.05	6.195+i .599	.661-i1.344
	1.10	12.80	.631-i1.132	.470-i2.384	1.52	17.79	-.188-i1.117	.356-i2.171
0.6	4.52	2.73	4.206-i .397	.751-i1.589	6.00	3.70	3.942+i .031	.722-i1.448
	1.39	9.04	1.059-i1.022	.631-i2.271	1.85	12.00	.176-i .765	.496-i2.067
0.7	2.77	4.17	2.589-i .651	.822-i1.837	3.56	5.36	1.953-i .436	.741-i1.675
	2.09	5.55	1.893-i .805	.801-i2.023	2.68	7.12	1.140-i .684	.681-i1.841
$2^{-1/2}$	2.401	4.802	2.218-i .729	.8[illegible]-i1.931	3.07	6.15	1.517-i .566	.716-i1.757

$C_1 = 1.0,\ C_o = 1.0$ 120° / 130°

ζ^{1}	$R_1 \times 10^3$ (120°)	$R_o \times 10^3$ (120°)	$Z_1 \times 10^3$ (120°)	$Z_o \times 10^3$ (120°)	$R_1 \times 10^3$ (130°)	$R_o \times 10^3$ (130°)	$Z_1 \times 10^3$ (130°)	$Z_o \times 10^3$ (130°)
0.05	1225.86	3.07	612.3+i264.8	.575-i1.001	1779.86	4.45	309.8+i438.9	.650-i.782
	1.54	2451.73	-1.528-i1.325	.0057-i2.006	2.22	3559.73	-2.224-i1.325	.0003-i1.559
0.1	307.01	3.10	161.9+i 65.59	.580-i1.005	444.46	4.49	75.42+i108.2	.654-i.785
	1.55	614.02	-1.509-i1.320	.011-i1.986	2.24	888.91	-2.221-i1.320	.0051-i1.556
0.2	77.19	3.22	37.41+i15.77	.610-i1.017	110.54	4.61	17.41+i26.01	.659-i.797
	1.61	154.38	-1.469-i1.299	.069-i1.971	2.36	221.07	-2.206-i1.299	.064-i1.544
0.3	34.48	3.42	16.05+i 6.519	.621-i1.044	48.58	4.81	6.695+i10.79	.661-i.818
	1.71	68.96	-1.385-i1.261	.100-i1.944	2.40	97.17	-2.177-i1.261	.067-i1.523
0.4	19.36	3.73	6.434+i3.205	.651-i1.084	26.75	5.14	2.829+i5.363	.666-i.850
	1.86	38.71	-1.257-i1.201	.175-i1.904	2.57	53.50	-2.130-i1.200	.127-i1.491
0.5	12.14	4.17	4.804+i1.512	.676-i1.142	16.44	5.65	.960+i2.767	.666-i.896
	2.03	24.29	-1.065-i1.109	.267-i1.845	2.82	32.89	-2.053-i1.104	.202-i1.446
0.6	7.94	4.89	2.628+i .654	.692-i1.231	10.54	6.49	-.146+i1.255	.649-i.965
	2.44	15.89	-.760-i .959	.386-i1.757	3.24	21.07	-1.917-i .942	.301-i1.376
0.7	4.59	6.90	.744-i .218	.657-i1.424	5.97	8.98	-1.137-i .020	.571-i1.016
	3.45	9.18	.0019-i.579	.576-i1.565	4.49	11.94	-1.532-i .494	.479-i1.226
$2^{-1/2}$	3.95	7.90	.345-i .411	.619-i1.493	5.14	10.27	-1.351-i .279	.528-i1.171

$C_1 = 1.0$, $C_o = 1.0$ 140° 150°

ρ^{-1}	$R_1 x10^3$	$R_o x10^3$	$Z_i x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_i x10^3$	$Z_o x10^3$
0.05	2526.83	6.54	-442.0+i619.1	.757-i.553	3671.24	9.31	-1855.5+i769.8	.587-i.337
	3.67	5053.66	-3.162-i1.328	.0011-i1.100	4.66	7342.49	-4.598-i1.325	.0016-i.669
0.1	630.16	6.37	-120.4+i158.5	.662-i.555	915.01	9.24	-459.8+i196.3	.572-i.339
	3.18	1260.31	-3.166-i1.316	.0045-i1.097	4.62	1830.02	-4.610-i1.320	.0035-i.667
0.2	155.92	6.51	-29.37+i36.72	.651-i.563	222.78	9.41	-115.2+i 46.98	.558-i.339
	3.26	311.83	-3.180-i1.299	.027-i1.089	4.76	451.55	-4.658-i1.299	.029-i.662
0.3	67.97	6.74	-14.03+i15.19	.642-i.578	98.00	9.72	-51.31+i19.26	.557-i.353
	3.37	135.94	-3.204-i1.261	.051-i1.074	4.86	196.00	-4.743-i1.261	.036-i.653
0.4	37.04	7.12	-8.688+i7.546	.632-i.600	53.08	10.21	-28.86+i 9.521	.540-i.366
	3.56	74.08	-3.240-i1.199	.093-i1.052	5.10	106.17	-4.878-i1.199	.065-i.639
0.5	22.50	7.72	-6.208+i3.924	.613-i.633	32.01	10.98	-18.35+i 4.949	.513-i.385
	3.36	45.00	-3.291-i1.102	.151-i1.020	5.49	64.02	-5.090-i1.100	.107-i.620
0.6	14.23	8.76	-4.834+i1.829	.577-i.681	2[illegible].12	12.39	-12.39+i 2.296	.477-i.416
	4.33	28.46	-3.371-i .932	.231-i.971	6.20	40.23	-5.448-i .925	.174-i.592
0.7	7.95	11.96	-3.863+i .147	.476-i.713	11.12	16.73	-7.945+i .276	.373-i.479
	5.98	15.90	-3.589-i .429	.334-i.942	3.36	22.25	-6.567-i .383	.291-i.526
$2^{-1/2}$	6.84	13.67	-3.706-i .173	.433-i.826	9.56	19.11	-7.658-i .010	.333-i.503

$C_1 = 1.0$, $C_o = 1.0$ 160° 170°

ρ^{-1}	$R_1 x10^3$	$R_o x10^3$	$Z_i x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_i x10^3$	$Z_o x10^3$
0.05	5753.58	14.43	-4378.8+i929.1	.430-i.162	12026.42	29.12	-12031.3+i163.3	.231-i.047
	7.22	11507.17	-7.208-i1.328	.0018-i.325	14.56	24052.83	-15.05-i1.324	.0015-i.087
0.01	1450.70	14.65	-1113.4+i231.0	.424-i.165	2993.40	29.64	-2865.9+i268.8	.227-i.047
	7.32	2901.41	-7.320-i1.320	.0026-i.323	14.82	5986.80	-15.10-i1.320	.0029-i.086
0.2	353.16	14.73	-270.6+i 54.90	.423-i.168	737.02	30.73	-673.6+i60.43	.222-i.048
	7.36	706.33	-7.330-i1.299	.0099-i.320	15.36	1474.03	-15.32-i1.299	.0047-i.086
0.3	154.76	15.34	-118.3+i 22.52	.412-i.172	318.72	31.60	-300.6+i24.69	.216-i.049
	7.67	309.52	-7.507-i1.262	.023-i.316	15.80	637.44	-15.76-i1.261	.011-i.085
0.4	82.56	15.87	-64.76+i11.89	.397-i.179	171.76	33.02	-162.5+i12.12	.208-i.050
	7.94	165.13	-7.791-i1.199	.043-i.309	16.51	343.52	-16.44-i1.199	.020-i.083
0.5	49.58	17.01	-39.67+i 5.723	.374-i.188	102.92	35.32	-97.78+i6.231	.195-i.053
	8.50	99.17	-8.246-i1.099	.070-i.300	17.66	205.84	-17.53-i1.099	.034-i.081
0.6	30.96	19.07	-25.47+i 2.668	.340-i.202	64.08	39.46	-61.27+i2.904	.175-i.056
	9.54	61.91	- 9.054-i .921	.112-i.286	19.73	128.16	-19.50-i .919	.055-i.077
0.7	17.25	25.96	-15.24+i .411	.257-i.228	35.17	52.92	-34.08+i .419	.131-i.064
	12.99	34.50	-11.77-i .328	.197-i.250	26.46	70.34	-25.86-i .334	.099-i.070
$2^{-1/2}$	14.81	29.62	-13.20+i .039	.227-i.239	30.20	60.41	-29.39-i .010	.115-i.067

$C_1 = 1.0$, $C_o = 0.5$ 60° 70°

ς^{-1}	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	3.06	0	3.057-i1.765	.768-i1.328	4.83	.034	4.843-i1.770	.591-i1.570
	0	9.17	2.187-i1.5146	2.301-i3.981	.011	14.49	1.863-i1.4277	1.721-i4.679
0.1	3.03	.034	3.048-i1.762	.789-i1.337	4.76	.054	4.782-i1.752	.605-i1.575
	.011	9.09	2.189-i1.5140	2.309-i3.970	.018	14.27	1.870-i1.4264	1.732-i4.675
0.2	2.93	.14	3.005-i1.744	.860-i1.369	4.47	.22	4.581-i1.713	.713-i1.611
	.045	8.78	2.197-i1.5133	2.340-i3.935	.072	13.42	1.905-i1.4253	1.809-i4.635
0.3	2.78	.30	2.939-i1.715	.964-i1.418	4.08	.44	4.304-i1.652	.858-i1.669
	.10	8.34	2.203-i1.5116	2.389-i3.884	.15	12.24	1.961-i1.4247	1.923-i4.576
0.4	2.58	.53	2.857-i1.680	1.111-i1.493	3.66	.75	4.005-i1.587	1.055-i1.741
	.18	7.75	2.216-i1.5095	2.454-i3.808	.25	10.97	2.115-i1.4201	2.082-i4.433
0.5	2.36	.83	2.766-i1.642	1.296-i1.603	3.22	1.13	3.711-i1.535	1.282-i1.886
	.28	7.09	2.226-i1.5037	2.513-i3.704	.38	9.67	2.096-i1.4122	2.187-i4.359
0.6	2.10	1.21	2.667-i1.602	1.518-i1.750	2.79	1.60	3.423-i1.482	1.543-i2.064
	.40	6.31	2.245-i1.4997	2.554-i3.548	.53	8.36	2.181-i1.4023	2.307-i4.181
0.7	1.81	1.74	2.560-i1.561	1.798-i1.974	2.32	2.23	3.134-i1.434	1.836-i2.316
	.58	5.42	2.267-i1.4936	2.582-i3.336	.74	6.97	2.289-i1.3920	2.419-i3.925
0.8	1.36	2.72	2.424-i1.515	2.194-i2.389	1.70	4.01	2.686-i1.398	2.557-i2.537
	.91	4.08	2.321-i1.4929	2.504-i2.922	1.34	5.11	2.486-i1.3802	2.711-i3.543
$1.5^{-1/2}$	1.12	3.36	2.367-i1.500	2.572-i2.650	1.41	4.22	2.618-i1.381	2.691-i3.124

$C_1 = 1.0$, $C_o = 0.5$ 80° 90°

ς^{-1}	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	9.83	.028	9.841-i1.751	.321-i .718	70.67	.40	70.56-i1.773	.266-i1.775
	.0092	29.49	1.104-i1.3546	.924-i5.142	.13	212.00	-.0995-i1.326	.260-i5.298
0.1	9.30	.11	9.352-i1.709	.376-i1.729	35.20	.40	35.28-i .887	.266-i1.782
	.035	27.89	1.136-i1.3543	.972-i5.131	.13	105.60	.200-i1.325	.259-i5.292
0.2	7.86	.36	8.032-i1.585	.540-i1.765	17.43	.91	17.46-i .998	.596-i1.829
	.12	23.60	1.241-i1.3524	1.127-i5.094	.30	52.29	.400-i1.315	.590-i5.244
0.3	6.51	.71	6.797-i1.479	.767-i1.835	11.37	1.25	11.51-i .904	.804-i1.893
	.24	19.53	1.376-i1.3477	1.335-i5.022	.42	34.11	.608-i1.304	.810-i5.180
0.4	5.40	1.12	5.798-i1.395	1.018-i1.937	3.27	1.71	8.451-i .908	1.066-i1.996
	.37	16.20	1.523-i1.3383	1.538-i4.922	.57	24.81	.825-i1.286	1.066-i5.077
0.5	4.48	1.56	4.987-i1.323	1.269-i2.071	6.33	2.24	6.554-i .922	1.335-i2.140
	.52	13.44	1.686-i1.3264	1.749-i4.738	.75	19.00	1.056-i1.260	1.342-i4.933
0.6	3.70	2.13	4.311-i1.277	1.551-i2265	4.94	2.85	5.224-i .936	1.593-i2.337
	.71	11.11	1.864-i1.3079	1.965-i4.593	.95	14.83	1.318-i1.228	1.594-i4.735
0.7	2.98	2.86	3.829-i1.242	1.850-i2.546	3.83	3.68	4.162-i .962	1.859-i2.628
	.95	8.94	2.274-i1.2847	2.157-i4.311	1.23	11.48	1.640-i1.183	1.863-i4.448
0.8	2.14	4.27	3.019-i1.226	2.203-i3.088	2.66	5.32	3.055-i1.021	2.131-i3.188
	1.42	6.41	2.461-i1.2477	2.317-i3.722	1.77	7.96	2.210-i1.105	2.122-i3.887
$1.5^{-1/2}$	1.75	5.24	2.725-i1.234	2.303-i3431	2.17	6.50	2.598-i1.061	2.171-i3.538

$C_1 = 1.0,\ C_o = 0.5$

ξ^{-2}	100° $R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$	110° $R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$
0.05	508.24	1.43	477.2+i42.61	.621-i1.719	033.61	2.92	790.0 +i164.6	1.148-i1.566
	.48	1524.72	-.454-i1.325	.024-i5.142	.97	3100.84	-.959-i1.325	.0076-i4.683
0.1	133.55	1.51	125.1+i 10.54	.668-i1.728	261.23	2.96	227.2+i41.03	1.163-i1.575
	.50	400.64	-.416-i1.321	.061-i5.132	.99	783.68	-.942-i 1.321	.040-i4.674
0.2	38.57	1.80	35.77+i 2.356	.834-i1.768	67.85	3.14	51.06+i 9.936	1.235-i1.609
	.60	115.72	-.288-i1.308	.248-i5.093	1.05	203.54	-.874-i 1.307	.138-i4.637
0.3	19.88	2.16	18.27+i .779	1.024-i1.836	31.69	3.45	23.28+i 4.069	1.352-i1.671
	.72	59.64	-.116-i1.287	.462-i5.026	[illegible]	93.07	-.765-i 1.282	.295-i4.575
0.4	12.69	2.61	11.54+i .151	1.238-i1.936	13.72	3.85	13.35+i 1.978	1.480-i1.762
	.87	38.07	.089-i1.258	.711-i4.926	1.28	56.15	-.618-i 1.246	.484-i4.484
0.5	8.95	3.15	8.037-i .184	1.457-i2.076	12.44	4.36	8.557+i .962	1.611-i1.888
	1.05	26.85	.326-i1.219	.978-i4.785	1.45	37.32	-.427-i 1.197	.706-i4.357
0.6	6.61	3.81	5.866-i .404	1.721-i2.269	8.78	5.05	5.740+i .348	1.394-i2.063
	1.27	19.84	.699-i1.135	1.258-i4.596	1.68	26.35	-.180-i 1.130	.967-i4.182
0.7	4.91	4.71	4.300-i .574	1.858-i2.547	6.30	6.05	3.814-i .085	1.853-i2.318
	1.57	14.73	.987-i1.097	1.646-i4.315	2.02	18.91	.169-i 1.032	1.273-i3.928
0.8	3.30	6.61	2.778-i .778	2.018-i3.088	4.12	8.24	2.065-i .510	1.887-i2.811
	2.20	9.91	1.683-i .963	1.896-i3.773	2.75	12.36	.875-i .831	1.663-i3.436
$1.5^{-1/2}$	2.68	8.05	2.179-i .874	1.993-i3.426	3.34	10.01	1.401-i .685	1.813-i3.127

$C_1 = 1.0,\ C_o = 0.5$

ξ^{-2}	120° $R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$	130° $R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$
0.05	1635.00	4.60	817.3+i353.6	1.531-i1.232	373.89	6.68	412.1+i584.5	1.739-i1.045
	1.53	4905.00	-1.529-i1.325	.0022-i3.980	2.23	7121.68	-2.253-i1.325	.0043-i3.117
0.1	409.87	4.64	203.7+i87.74	1.540-i1.340	593.36	6.72	101.3+i144.9	1.742-i1.051
	1.55	1229.60	-1.520-i1.321	.025-i3.972	2.24	1780.08	-2.222-i1.321	.013-i3.111
0.2	103.47	4.80	50.21+i21.24	1.574-i1.370	48.16	6.88	23.64+i35.02	1.753-i1.075
	1.60	310.40	-1.485-i1.306	.090-i3.942	2.29	444.48	-2.211-i1.306	.061-i3.087
0.3	46.57	5.06	21.75+i 8.913	1.622-i1.423	65.61	7.13	9.360+i16.48	1.760-i1.116
	1.69	139.70	-1.426-i1.280	.203-i3.891	2.38	196.83	-2.192-i 1.279	.144-i3.047
0.4	26.48	5.45	11.64+i 4.523	1.687-i1.501	36.59	7.52	4.276+i 7.570	1.771-i1.177
	1.82	79.43	-1.339-i1.241	.356-i3.813	2.51	109.76	-2.161-i 1.238	.256-i2.986
0.5	16.98	5.96	6.899+i2.465	1.741-i1.608	23.00	8.08	1.872+i 4.191	1.777-i1.262
	1.99	50.96	-1.218-i1.185	.573-i3.706	2.69	69.00	-2.116-i 1.179	.397-i2.902
0.6	11.62	6.68	4.188+i1.281	1.790-i1.756	15.42	8.86	.540+i 2.317	1.758-i1.378
	2.23	34.88	-1.049-i1.106	.756-i3.557	2.95	46.26	02.048-i 1.052	.573-i2.785
0.7	8.14	7.73	2.421+i .505	1.802-i1.968	10.56	10.15	-.373+i 1.084	1.714-i1.548
	2.58	24.43	-.794-i .984	1.011-i3.347	3.38	31.69	-1.936-i .945	.816-i2.617
0.8	5.18	10.37	.793-i .238	1.730-i2.391	6.62	13.25	-1.162+i .019	1.540-i1.874
	3.46	15.55	.230-i .717	1.428-i2.923	4.42	19.87	-1.672-i .625	1.196-i2.290
$1.5^{-1/2}$	4.18	12.55	.512-i .507	1.604-i2.656	5.33	15.99	-1.452-i .351	1.387-i2.082

811-42A

$C_1 = 1.0, \quad C_o = 0.5$ 140° — 150°

ρ^{-2}	$R_1 \times 10^3$ (140°)	$R_o \times 10^3$ (140°)	$Z_1 \times 10^3$ (140°)	$Z_o \times 10^3$ (140°)	$R_1 \times 10^3$ (150°)	$R_o \times 10^3$ (150°)	$Z_1 \times 10^3$ (150°)	$Z_o \times 10^3$ (150°)
0.05	3370.17 3.17	9.50 10110.50	-588.0+i827.4 -2.962-i1.325	1.742-i.740 .0065+i2.192	4896.53 4.67	14.00 14689.60	-2449.8+i1020.0 -4.597-i 1.325	1.564-i.454 .0037+i1.336
0.1	841.27 3.18	9.54 2523.82	-160.0+i212.1 -3.081-i1.273	1.741-i.744 .012-i2.195	1221.56 4.62	13.85 3664.68	-613.1+i262.3 -4.606-i1.321	1.528-i.461 .0099+i1.334
0.2	208.99 3.23	9.70 626.98	-38.65+i49.71 -3.175-i1.306	1.734-i.762 .043-i2.180	302.63 4.68	14.04 907.88	-153.6+i 63.57 - 4.641-i1.306	1.515-i.466 .025-i1.323
0.3	91.80 3.32	9.97 275.38	-18.25+i20.93 -3.194-i1.279	1.718-i.790 .096-i2.149	132.35 4.80	14.40 397.04	-68.47+i26.54 -4.704-i1.278	1.476-i.484 .073-i1.306
0.4	50.66 [illegible].47	10.42 151.98	-11.18+i10.71 -3.219-i1.236	1.704-i.833 .183-i2.107	72.61 4.98	14.93 217.82	-38.61+i13.63 -4.803-i1.236	1.465-i.510 .132-i1.280
0.5	31.46 [illegible].68	11.05 93.39	-7.859+i5.970 -3.290-i1.185	1.672-i.892 .289-i2.027	44.77 5.23	15.70 134.30	-24.71+i 7.620 -4.939-i1.173	1.417-i.545 .204-i1.244
0.6	20.82 [illegible].99	11.97 62.47	-6.029+i3.344 -3.311-i1.083	1.616-i.974 .434-i1.965	29.44 5.64	16.92 88.32	-17.02+i 4.706 -5.140-i1.077	1.360-i.597 .320-i1.193
0.7	14.07 4.51	13.52 42.20	-4.897+i1.635 -3.393-i .932	1.527-i1.094 .632-i1.846	19.68 6.31	18.92 59.04	-10.9+i 1.590 -5.526-i .980	1.251-i.668 .464-i1.123
0.8	8.71 5.81	17.42 26.16	-3.955+i .876 -3.588-i .556	1.312-i1.324 .967-i1.618	12.07 8.05	24.15 36.21	-8.385+i .423 -6.377-i .503	1.037-i.807 .731-i.984
$1.5^{-1/2}$	6.99	20.98	-3.768-i .223	1.147-i1.470	9.67	29.02	-7.193-i .124	.886-i.895

$C_1 = 1.0, \quad C_o = 0.5$ 160° — 170°

ρ^{-2}	$R_1 \times 10^3$ (160°)	$R_o \times 10^3$ (160°)	$Z_1 \times 10^3$ (160°)	$Z_o \times 10^3$ (160°)	$R_1 \times 10^3$ (170°)	$R_o \times 10^3$ (170°)	$Z_1 \times 10^3$ (170°)	$Z_o \times 10^3$ (170°)
0.05	7673.86 7.21	21.62 23021.58	-5842.5+i1242.8 -7.206-i1.325	1.146-i.225 .0016-i.646	16030.14 15.19	45.58 48090.42	-14941+i1332.1 -15.09-i1.325	.613-i.065 .0086-i.175
0.1	1936.72 7.32	21.96 5810.16	-1485.2+i 308.6 -7.312-i1.321	1.134-i.228 .0054-i.642	3996.26 14.70	44.10 11988.78	-3849.4+i 366.9 -15.10-i1.321	.608-i.067 .0090-i.173
0.2	473.38 7.33	21.98 1420.14	-362.0+i 74.29 -7.397-i1.306	1.133-i.230 .022-i.640	987.89 15.28	45.84 2963.68	-929.4+i81.90 -15.27-i1.306	.596-i.069 .0095-i.172
0.3	209.00 7.57	22.72 627.00	-159.1+i 31.19 -7.425-i1.278	1.114-i.238 .044-i.631	430.43 15.60	46.80 1291.28	-405.5+i34.22 -15.57-i1.278	.584-i.071 .022-i.170
0.4	112.93 7.74	23.22 338.78	-87.77+i15.92 -7.623-i1.236	1.084-i.251 .084-i.619	234.93 16.09	48.28 704.80	-221.8+i17.51 -16.04-i1.235	.567-i.074 .040-i.167
0.5	69.34 8.11	24.33 208.02	-54.17+i 8.876 -7.857-i1.172	1.041-i.268 .134-i.602	143.94 16.84	50.51 431.80	-136.3+i 9.724 -16.74-i1.171	.543-i.073 .065-i.162
0.6	45.30 8.68	26.04 135.92	-36.35+i 4.972 -8.366-i1.074	.983-i.292 .205-i.570	93.78 17.97	53.91 281.34	-89.16+i5.442 -17.82-i1.072	.510-i.084 .100-i.157
0.7	30.53 9.78	29.33 91.59	-25.23+i 2.503 -9.252-i .908	.887-i.320 .303-i.532	62.24 19.94	59.81 186.71	-59.50+i2.735 -19.68-i .903	.461-i.093 .150-i.148
0.8	18.38 12.26	36.77 55.15	-15.82+i .554 -11.13-i .409	.725-i.393 .496-i.480	37.79 25.20	75.58 113.38	-36.51+i .606 -24.64-i .449	.367-i.110 .247-i.121
1.5-1/2	14.89	44.67	-13.21-i .054	.605-i.426	30.24	90.72	-29.40-i .013	.308-i.120

$C_1 = 0.5$, $C_o = 1.0$ 60° / 70°

-2	60° $R_1 x10^3$	60° $R_o x10^3$	60° $Z_1 x10^3$	60° $Z_o x10^3$	70° $R_1 x10^3$	70° $R_o x10^3$	70° $Z_1 x10^3$	70° $Z_o x10^3$
0.05	3.05 0	0 4.56	3.057-i1.765 4.073-i1.987	.765-i1.324 1.149-i1.988	4.82 .016	.024 7.24	4.841-i1.767 3.522-i2.167	.583-i1.563 .866-i2.341
0.1	3.02 .023	.034 4.53	3.048-i1.763 3.051-i1.979	.736-i1.332 1.164-i1.986	4.74 .036	.054 7.11	4.762-i1.752 3.515-i2.150	.602-i1.569 .881-i2.335
0.2	2.86 .092	.14 4.32	3.005-i1.744 3.020-i1.951	.846-i1.346 1.204-i1.967	4.40 .14	.22 6.60	4.561-i1.713 3.501-i2.094	.702-i1.587 .961-i2.316
0.3	2.67 .21	.31 4.01	2.940-i1.716 2.965-i1.907	.936-i1.372 1.268-i1.943	3.92 .31	.46 5.88	4.308-i1.658 3.474-i2.008	.841-i1.617 1.087-i2.286
0.4	2.38 .38	.58 3.56	2.864-i1.687 2.897-i1.845	1.072-i1.421 1.339-i1.892	3.36 .54	.32 5.04	4.010-i1.599 3.450-i1.907	1.019-i1.650 1.232-i2.208
0.5	1.98 .66	.99 2.97	2.734-i1.661 2.813-i1.768	1.232-i1.492 1.424-i1.824	2.70 .90	1.35 4.05	3.715-i1.570 3.389-i1.768	1.235-i1.756 1.377-i2.146
$3^{-1/2}$	1.26	1.90	2.796-i1.672	1.428-i1.652	1.69	2.53	3.425-i1.605	1.458-i1.921

$C_1 = 0.5$, $C_o = 1.0$ 80° / 90°

-2	80° $R_1 x10^3$	80° $R_o x10^3$	80° $Z_1 x10^3$	80° $Z_o x10^3$	90° $R_1 x10^3$	90° $R_o x10^3$	90° $Z_1 x10^3$	90° $Z_o x10^3$
0.05	9.82 .018	.028 14.73	9.841-i1.751 2.947-i2.448	.321-i1.716 .470-i2.570	70.42 .38	.56 105.64	70.39-i2.477 .399-i2.649	.371-i1.773 .378-i2.648
0.1	9.26 .071	.11 13.89	9.352-i1.709 2.935-i2.395	.374-i1.722 .521-i2.565	35.07 .27	.40 52.60	35.28-i .887 .791-i2.615	.265-i1.775 .268-i2.646
0.2	7.74 .25	.37 11.61	8.032-i1.590 3.209-i2.312	.539-i1.741 .684-i2.547	17.15 .58	.87 25.73	17.47-i .955 1.544-i2.496	.561-i1.797 .561-i2.624
0.3	6.26 .49	.74 9.38	6.797-i1.490 3.410-i2.162	.758-i1.779 .88?-i2.509	10.92 .86	1.29 16.39	11.49-i .931 2.249-i2.316	.797-i1.832 .797-i2.588
0.4	4.97 .81	1.21 7.45	5.786-i1.42? 3.579-i1.930	.997-i1.836 1.108-i2.454	7.60 1.24	1.85 11.41	8.388-i.975 2.893-i2.074	1.060-i1.890 1.067-i2.531
0.5	3.75 1.25	1.37 5.63	4.940-i1.395 3.752-i1.768	1.247-i1.927 1.326-i2.359	5.31 1.77	2.66 7.96	6.362-i1.063 3.536-i1.768	1.330-i1.991 1.327-i2.430
$3^{-1/2}$	2.27	3.40	4.123-i1.495	1.479-i2.144	3.06	4.59	4.596-i1.326	1.530-i2.210

$C_1 = 0.5$, $C_o = 1.0$ 100°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	507.77 .95	1.43 761.65	477.2+i42.61 -.882-i2.647	.620-i1.717 .466-i2.571
0.1	133.04 1.01	1.52 199.56	125.0+i10.45 -.728-i2.631	.672-i1.722 .520-i2.566
0.2	37.96 1.22	1.82 56.94	35.72+i2.306 -.196-i2.552	.833-i1.742 .698-i2.547
0.3	19.10 1.50	2.25 28.65	18.13+i .667 .539-i2.396	1.035-i1.778 .906-i2.511
0.4	11.67 1.89	2.84 17.50	11.27-i .025 1.422-i2.145	1.257-i1.836 1.145-i2.455
0.5	7.50 2.50	3.75 11.25	7.493-i .483 2.501-i1.768	1.478-i1.950 1.402-i2.359
$3^{-1/2}$	4.14	6.21	4.436-i1.100	1.630-i2.145

110°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	1032.64 1.95	2.92 1548.96	790.0+i164.6 -1.907-i2.648	1.147-i1.565 .867-i2.342
0.1	260.23 1.98	2.97 390.35	198.9+i40.85 -1.837-i2.631	1.170-i1.569 .884-i2.338
0.2	66.76 2.13	3.20 100.14	50.63+i9.662 -1.548-i2.563	1.248-i1.587 .979-i2.320
0.3	30.45 2.39	3.59 45.67	22.80+i3.776 -1.054-i2.424	1.371-i1.620 1.125-i2.287
0.4	17.21 2.79	4.18 25.82	12.68+i1.588 -.314-i2.183	1.510-i1.672 1.307-i2.236
0.5	10.42 3.47	5.21 15.65	7.410+i .356 .811-i1.768	1.661-i1.759 1.518-i2.151
$3^{-1/2}$	5.55	8.32	3.286-i .842	1.738-i1.953

$C_1 = 0.5$, $C_o = 1.0$ 120°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	1633.46 3.07	4.61 2450.19	815.5+i352.5 -3.052-i2.648	1.534-i1.281 1.150-i1.992
0.1	408.31 3.11	4.66 612.46	202.3+i 87.17 -3.016-i2.637	1.544-i1.334 1.169-i1.988
0.2	101.81 3.25	4.88 152.72	49.32+i20.67 -2.868-i2.566	1.587-i1.350 1.225-i1.972
0.3	44.74 3.51	5.27 67.11	20.72+i 8.258 -2.619-i2.464	1.650-i1.378 1.320-i1.945
0.4	24.34 3.95	5.92 36.52	10.39+i 3.733 -2.115-i2.202	1.728-i1.423 1.454-i1.901
0.5	14.23 4.74	7.12 21.34	5.094+i1.717 -1.270-i1.768	1.809-i1.497 1.613-i1.827
$3^{-1/2}$	7.39	11.09	.982-i .601	1.814-i1.662

130°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	2371.66 4.46	6.69 3557.49	410.1+i582.9 -4.448-i2.648	1.741-i1.044 1.307-i1.561
0.1	591.10 4.50	6.75 886.65	99.85+i143.7 -4.437-i2.632	1.747-i1.047 1.318-i1.559
0.2	145.79 4.66	6.99 218.69	22.33+i33.96 -4.359-i2.567	1.766-i1.060 1.357-i1.546
0.3	63.04 4.95	7.42 94.56	7.893+i13.56 -4.289-i 2.439	1.789-i1.081 1.419-i1.525
0.4	33.64 5.45	8.18 50.46	2.613+i 6.179 -4.099-i 2.212	1.822-i1.116 1.507-i1.490
0.5	19.27 6.42	9.64 28.90	-.171+i 2.429 -3.706-i 1.763	1.849-i1.174 1.629-i1.433
$3^{-1/2}$	9.85	14.78	-2.473-i .397	1.802-i1.303

C_1 = 0.5, C_o = 1.0   14[illegible]   150°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	3366.99 6.34	9.51 5050.49	-589.4[illegible] -6.[illegible].648	1.743-i.740 1.309-i1.104	4891.93 9.27	13.91 7337.89	-2070.1+i876.0 -9.199-i2.648	1.547-i.453 1.154-i.673
0.1	838.07 6.38	9.57 1257.11	-161.0+i210.4 -6.337-i2.632	1.742-i.742 1.313-i1.101	1216.91 9.27	13.90 1825.36	-613.1+i259.9 -9.235-i2.632	1.551-i.454 1.158-i.672
0.2	205.65 6.57	9.86 308.48	-39.90+i47.90 -6.379-i 2.567	1.748-i.749 1.308-i1.09[illegible]	297.79 9.51	14.27 446.69	-153.5+i 60.89 -9.382-i2.568	1.527-i.460 1.167-i.669
0.3	88.20 6.92	10.38 132.30	-19.5[illegible].00 -6.4[illegible].442	1.748-i.765 1.376-i1.076	127.16 9.99	14.98 190.74	-68.23+i23.87 -9.654-i2.443	1.521-i.469 1.195-i.658
0.4	46.58 7.55	11.32 69.88	-12.45[illegible].602 -6.55[illegible]18	1.752-i.790 1.439-i1.054	66.76 10.82	16.23 100.15	-38.08+i10.71 -10.12-i 2.220	1.509-i.484 1.233-i.643
0.5	26.36 8.79	13.18 39.54	-9.143+i3.414 -6.73[illegible]1.768	1.744-i.831 1.527-i1.0[illegible]	37.50 12.50	18.75 56.25	-23.52+i 4.228 -11.03-i 1.768	1.482-i.508 1.291-i.619
$3^{-1/2}$	13.35	20.02	-[illegible]78	1.669-i.922	18.88	28.31	-14.27-i 1.292	1.403-i.563

C_1 = 0.5, C_o = 1.0   16[illegible]   170°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	7666.64 14.43	21.64 11499.96	-5839.[illegible]+i1239.6 -[illegible].648	1.146-i.224 .861-i.329	16015.06 30.27	45.41 24022.59	-14991.8+i1346.9 -30.14-i2.648	.609-i.067 .438-i.094
0.1	1929.35 14.69	22.04 2894.02	-14[illegible] -1[illegible]	1.145-i.226 [illegible]-i.328	3961.04 29.92	44.38 5971.56	-3788.8+i348.6 -30.31-i2.633	.605-i.067 .443-i.093
0.2	465.82 14.89	22.33 698.73	-3[illegible].04 -14[illegible]68	1.141-i.227 .873-i.326	972.12 31.05	46.58 1458.18	-915.9+i78.07 -31.02-i2.568	.600-i.068 .458-i.093
0.3	200.81 15.77	23.65 301.21	-155[illegible]27.71 -1[illegible].444	1.132-i.231 .886-i.322	413.55 32.47	48.71 620.33	-390.9+i30.27 -32.37-i2.444	.595-i.069 .466-i.092
0.4	103.84 16.83	25.24 155.76	[illegible]2.311 -16.58-i 2.221	1.110-i.239 .908-i.315	216.02 35.01	52.51 324.03	-205.5+i13.37 -34.83-i2.222	.585-i.071 .476-i.090
0.5	58.09 19.36	29.04 87.14	-48.18+i 4.314 -18.40-i 1.768	1.090-i.260 .916-i.3[illegible]	120.58 40.20	60.29 180.87	-115.5+i 5.183 -39.73-i1.768	.569-i.074 .493-i.087
$3^{-1/2}$	29.47	44.20	-27.23-i .057	1.013-i.271	60.33	90.49	-58.79-i .013	.532-i.080

$C_1 = 1.0$, $C_o = 0.2$

-1	60° $R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$	70° $R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$
0.05	3.82	0	3.821-i2.206	.959-i1.658	6.04	.077	6.055-i2.215	.774-i1.974
	0	22.93	2.276-i1.602	5.753-i9.954	.013	36.22	1.784-i1.449	4.309-i11.69
0.1	3.79	.068	3.810-i2.202	.976-i1.683	5.95	.11	5.977-i2.191	.802-i1.983
	.011	22.73	2.281-i1.608	5.732-i9.921	.018	35.69	1.796-i1.469	4.329-i11.68
0.2	3.67	.27	3.755-i2.179	1.182-i1.756	5.61	.44	5.726-i2.142	1.075-i2.078
	.045	22.03	2.295-i1.619	5.854-i9.842	.073	33.66	1.834-i1.473	4.529-i11.59
0.3	3.50	.60	3.673-i2.147	1.455-i1.887	5.14	.86	5.379-i2.062	1.428-i2.215
	.10	21.00	2.314-i1.625	5.975-i9.716	.14	30.84	1.897-i1.483	4.771-i11.44
0.4	3.28	1.04	3.569-i2.096	1.817-i2.074	4.64	1.47	5.005-i1.981	1.917-i2.441
	.17	19.66	2.343-i1.634	6.110-i9.521	.25	27.83	2.023-i1.496	5.135-i11.22
0.5	3.03	1.63	3.455-i2.050	2.237-i2.341	4.13	2.20	4.638-i1.912	2.485-i2.744
	.27	18.17	2.374-i1.643	6.275-i9.267	.37	24.80	2.057-i1.500	5.474-i10.92
0.6	2.75	2.31	3.367-i1.996	2.813-i2.674	3.64	3.07	4.280-i1.841	3.130-i3.154
	.38	16.49	2.417-i1.656	6.404-i8.921	.51	21.83	2.158-i1.511	5.797-i10.51
0.7	2.45	3.21	3.194-i1.940	3.463-i3.146	3.15	4.12	3.932-i1.773	3.839-i3.699
	.54	14.69	2.465-i1.670	6.545-i8.469	.69	18.88	2.281-i1.525	6.128-i9.962
0.8	2.10	4.41	3.045-i1.879	4.242-i3.810	2.63	5.51	3.577-i1.706	4.634-i4.471
	.74	12.58	2.534-i1.692	6.550-i8.102	.92	15.79	2.440-i1.542	6.359-i9.180
0.9	1.57	6.70	2.835-i1.797	5.400-i5.104	1.94	8.29	3.099-i1.627	5.758-i6.019
	1.12	9.40	2.669-i1.737	6.205-i6.494	1.38	11.61	2.736-i1.575	6.335-i7.644
$1.2^{-1/2}$	1.33	8.00	2.747-i1.764	5.843-i5.793	1.64	9.85	2.909-i1.598	6.106-i6.827

$C_1 = 1.0$, $C_o = 0.2$

-1	80° $R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$	90° $R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$
0.05	12.29	.056	12.30-i2.189	.424-i2.152	88.25	.70	88.25-i1.941	.582-i2.224
	.0092	73.74	.994-i1.364	2.310-i12.85	.15	530.10	.0796-i1.326	.749-i13.24
0.1	11.63	.21	11.69-i2.133	.552-i2.176	44.05	.80	44.10-i1.108	.665-i2.244
	.035	69.79	1.022-i1.366	2.431-i12.83	.13	264.20	.1593-i1.327	.648-i13.23
0.2	9.86	.73	10.04-i1.985	.984-i2.274	21.86	1.86	21.81-i1.274	1.528-i2.365
	.12	59.18	1.114-i1.370	2.820-i12.73	.31	131.14	.3208-i1.322	1.528-i13.11
0.3	8.20	1.40	8.497-i1.843	1.530-i2.440	14.32	2.49	14.38-i1.125	2.017-i2.522
	.23	49.20	1.234-i1.375	3.300-i12.56	.41	85.92	.4888-i1.320	1.992-i12.95
0.4	6.89	2.48	7.226-i1.788	2.404-i2.783	10.49	3.37	10.58-i1.120	2.666-i2.772
	.41	41.32	1.357-i1.380	4.080-i10.74	.56	62.92	.6646-i1.315	2.657-i12.70
0.5	5.74	3.05	6.240-i1.643	2.796-i3.014	8.12	4.38	8.219-i1.131	3.349-i3.122
	.51	34.45	1.521-i1.385	4.389-i10.58	.73	48.72	.8525-i1.305	3.349-i12.35
0.6	4.84	4.08	5.403-i1.576	3.504-i3.521	6.46	5.46	6.592-i1.131	3.990-i3.577
	.68	29.02	1.685-i1.387	4.925-i10.29	.91	38.74	1.064-i1.295	3.990-i11.90
0.7	4.04	5.27	4.695-i1.516	4.216-i4.058	5.18	6.80	5.092-i1.246	4.642-i4.192
	.88	24.23	1.832-i1.391	5.437-i9.907	1.13	31.10	1.311-i1.278	4.636-i11.28
0.8	3.29	6.93	4.020-i1.471	5.008-i4.922	4.10	8.62	4.275-i1.157	5.323-i5.081
	1.16	19.77	2.129-i1.392	5.940-i9.176	1.44	24.58	1.632-i1.261	5.331-i10.39
0.9	2.36	10.12	3.202-i1.423	5.929-i6.604	2.87	12.29	3.077-i1.168	5.978-i6.816
	1.69	14.18	2.608-i1.401	6.262-i8.399	2.05	17.21	2.264-i1.224	5.980-i8.659
$1.2^{-1/2}$	2.00	11.99	2.888-i1.410	6.167-i7.496	2.42	14.53	2.641-i1.206	6.042-i7.725

811-43A

$C_1 = 1.0,\ C_o = 0.2$ 100°

-1	$R_1 \times 10^3$	$R_o \times 10^3$	$Z_i \times 10^3$	$Z_o \times 10^3$
0.05	635.42 .48	2.85 3812.55	596.7+i53.47 .457-i1.325	1.688-i2.153 .059-i12.86
0.1	167.06 .50	3.02 1002.40	156.3+i13.17 -.427-i1.322	2.050-i2.177 .155-i12.83
0.2	48.37 .60	3.60 290.20	45.65+i2.945 -.325-i1.313	2.473-i2.277 .629-i12.73
0.3	25.04 .71	4.28 150.22	22.88+i1.005 -.190-i1.299	2.933-i2.443 1.132-i12.57
0.4	16.09 .86	5.14 96.56	14.48+i .228 -.031-i1.281	3.470-i2.687 1.377-i12.32
0.5	11.48 1.02	6.14 68.86	10.13-i .177 .150-i1.259	4.017-i3.024 2.426-i11.99
0.6	8.63 1.22	7.29 51.81	7.480-i.429 .362-i1.232	4.552-i3.467 3.152-i11.54
0.7	6.65 1.45	8.69 39.91	5.642-i.604 .620-i1.198	5.050-i4.062 3.846-i10.95
0.8	5.10 1.79	10.71 30.59	4.172-i.758 .968-i1.150	5.546-i4.927 4.653-i10.09
0.9	3.48 2.48	14.90 20.90	2.655-i.928 1.683-i1.052	5.859-i6.607 5.539-i8.403
$1.2^{-1/2}$	2.93	17.60	2.125-i.994	5.782-i7.505

110°

-1	$R_1 \times 10^3$	$R_o \times 10^3$	$Z_i \times 10^3$	$Z_o \times 10^3$
0.05	1292.27 .97	5.80 7753.60	990.2+i207.5 -.961-i1.325	3.552-i1.964 .020-i11.71
0.1	326.73 .93	5.90 1960.70	249.3+i51.51 -.947-i1.322	3.605-i1.985 .060-i11.68
0.2	85.08 1.04	6.25 510.45	63.95+i12.51 -.893-i1.311	13.790-i2.074 .316-i11.60
0.3	39.91 1.14	6.86 239.44	29.19+i 5.139 -.808-i1.292	4.098-i2.229 .730-i11.44
0.4	23.74 1.26	7.58 142.42	16.84+i2.564 -.696-i1.267	4.407-i2.450 1.199-i11.22
0.5	15.95 1.42	8.49 95.76	10.90+i1.315 -.556-i1.235	4.737-i2.752 1.785-i10.93
0.6	11.47 1.61	9.66 68.84	7.451+i.578 -.381-i1.194	5.084-i3.158 2.407-i10.52
0.7	8.54 1.86	11.17 51.23	5.189+i .091 -.157-i1.140	5.402-i3.691 3.115-i9.970
0.8	6.36 2.22	13.34 38.16	3.504-i .278 .162-i1.062	5.694-i4.345 3.920-i9.111
0.9	4.25 3.03	18.19 25.51	1.845-i .655 .850-i .393	5.621-i6.021 5.025-i7.656
$1.2^{-1/2}$	3.57	21.43	1.295-i .786	5.393-i6.834

$C_1 = 1.0,\ C_o = 0.2$ 120°

-1	$R_1 \times 10^3$	$R_o \times 10^3$	$Z_i \times 10^3$	$Z_o \times 10^3$
0.05	2044.13 1.53	9.20 12264.80	1021.6+i442.1 -1.529-i1.325	4.785-i1.673 .0065-i9.950
0.1	512.72 1.55	9.30 3076.30	254.0+i109.3 -1.522-i1.322	4.824-i1.692 .067-i9.931
0.2	129.74 1.60	9.58 778.40	62.58+i26.64 -1.495-i1.310	4.931-i1.768 .242-i9.855
0.3	58.64 1.68	10.05 351.85	27.40+i11.28 -1.449-i1.290	5.017-i1.897 .515-i9.726
0.4 ~~0.5~~	33.58 1.79	10.75 201.45	14.81+i5.818 -1.383-i1.261	5.131-i2.088 .874-i9.538
0.5	21.78 1.94	11.63 130.67	8.937+i3.278 -1.297-i1.223	5.327-i2.346 1.329-i9.280
0.6	15.18 2.13	12.80 91.10	5.632+i1.842 -1.182-i1.172	5.467-i2.690 1.851-i8.937
0.7	11.00 2.40	14.41 66.00	3.513+i .917 -1.026-i1.150	5.632-i3.155 2.644-i8.475
0.8	8.00 2.80	16.79 48.01	1.999-i .255 -.793-i .996	5.575-i3.817 3.275-i7.810
0.9	5.24 3.74	22.44 31.46	.559-i .384 -.259-i .753	5.230-i5.120 4.433-i6.577
1.2-1/2	4.40	26.38	.101-i .590	4.901-i5.814

130°

-1	$R_1 \times 10^3$	$R_o \times 10^3$	$Z_i \times 10^3$	$Z_o \times 10^3$
0.05	2967.93 2.22	13.30 17807.60	522.6+i736.7 -2.225-i1.325	5.407-i1.316 .0031+i7.793
0.1	742.26 2.24	13.45 4453.55	126.5+i180.9 -2.223-i1.322	5.453-i1.331 .039-i7.776
0.2	185.78 2.28	13.70 1114.70	30.00+i44.14 -2.214-i1.310	5.464-i1.391 .143-i7.719
0.3	82.63 2.36	14.14 495.76	12.09+i18.75 -2.199-i1.289	5.484-i1.492 .353-i7.618
0.4	46.40 2.47	14.88 278.38	5.619+i9.689 -2.177-i1.259	5.545-i1.642 .626-i7.470
0.5	29.49 2.63	15.76 176.94	2.742+i5.576 -2.146-i1.216	5.545-i1.844 1.002-i7.270
0.6	20.14 2.83	16.96 120.84	1.139+i3.294 -2.104-i1.158	5.514-i2.111 1.420-i7.000
0.7	14.31 3.12	18.73 85.86	.092+i1.832 -2.042-i1.076	5.457-i2.476 1.961-i6.639
0.8	10.22 3.58	21.45 61.34	.641+i .810 -1.945-i .947	5.269-i2.994 2.660-i6.118
0.9	6.59 4.71	28.23 39.57	1.330-i .130 -1.708-i .636	4.705-i4.016 3.810-i5.103
1.2-1/2	5.51	33.09	-1.543-i .415	4.276-i4.558

$C_1 = 1.0$, C_o = [illegible] 150°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_i x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_i x10^3$	$Z_o x10^3$
0.05	4213.50	19.00	-735.0+i1034.2	5.444-i.937	6121.82	27.10	-3062.1+i1383.3	4.682-i.581
	3.17	25281.00	-3.162-i1.325	.027-i5.497	4.68	36730.90	-4.596-i1.325	0.089-i3.341
0.1	1052.39	19.05	-199.4+i266.0	5.435-i.946	1528.11	27.65	-766.3+i329.3	4.772-i.587
	3.18	6314.35	-3.165-i1.322	.036-i5.487	4.61	9168.65	-4.604-i1.322	.017-i3.334
0.2	262.06	19.35	-48.06+i162.49	5.427-i.990	379.47	28.00	-192.0+i179.95	4.747-i.613
	3.22	1572.35	-3.172-i1.310	.107-i6.165	4.67	2276.80	-4.632-i1.310	.082-i3.309
0.3	115.60	19.80	-22.54+i126.60	5.387-i1.061	166.67	28.58	-85.64+i133.85	4.699-i.656
	3.30	693.60	-3.188-i1.289	.247-i5.373	4.76	1000.00	-4.681-i1.289	.175-i3.265
0.4	64.24	20.54	-13.66+i113.87	5.351-i1.166	92.08	29.44	-48.34+i117.70	4.625-i.719
	3.42	385.46	-3.209-i1.257	.453-i5.270	4.91	552.45	-4.754-i1.256	.327-i3.202
0.5	40.34	21.54	-9.509+i7.990	5.278-i1.308	57.40	30.61	-31.01+i110.25	4.508-i.805
	3.59	242.06	-3.238-i1.213	.725-i5.129	5.10	344.39	-4.859-i1.211	.502-i3.115
0.6	27.20	22.92	-7.228+i4.760	5.161-i1.497	38.45	32.40	-21.50+i6.073	4.385-i.922
	3.82	163.18	-3.279-i1.150	1.059-i4.939	5.40	230.70	-4.999-i1.144	.775-i3.010
0.7	19.06	24.93	-5.827+i2.746	4.983-i1.752	26.67	34.90	-13.98+i 2.793	4.156-i1.075
	4.16	114.36	-3.337-i1.060	1.497-i4.685	5.82	160.00	-5.213-i1.092	1.087-i2.849
0.8	13.45	28.23	-4.875+i1.334	4.705-i2.120	18.63	39.12	-11.63+i1.796	3.842-i1.297
	4.70	80.68	-3.427-i .913	2.075-i4.320	6.52	111.78	-5.578-i.889	1.537-i2.623
0.9	8.55	36.59	-4.057+i .098	3.996-i2.836	11.75	50.20	-8.196+i.285	3.152-i1.730
	6.10	51.30	-3.656-i .540	3.087-i3.600	8.37	70.40	-6.513-i.469	2.354-i2.193
$1.2^{-1/2}$	7.14	42.86	-3.825-i .268	3.565-i3.220	9.79	58.72	-7.221-i.151	2.766+i1.962

$C_1 = 1.0$, $C_o = 0.2$ 160° 170°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_i x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_i x10^3$	$Z_o x10^3$
0.05	9594.12	43.25	-7302.1+i1552.5	3.584-i.296	20041.45	87.30	-19422.7+i1689.1	1.813-i.100
	7.21	57564.75	-7.205-i1.325	.0051-i1.615	15.22	120248.70	-15.05-i1.325	.027-i.434
0.1	2422.74	43.85	-1858.3+i387.5	3.541-i.298	4999.11	87.55	-4844.0+i468.5	1.809-i.101
	7.31	14536.45	-7.211-i1.322	.011-i1.613	14.59	29994.65	-15.09-i1.322	-i.432
0.2	593.58	43.89	-452.9+i93.27	3.556-i.311	1238.73	91.40	-1165.0+i103.5	1.868-i.105
	7.30	3561.50	-7.277-i1.310	.050-i1.600	15.23	7432.40	-15.22-i1.310	.023-i.430
0.3	263.20	45.10	-199.8+i39.81	3.484-i.325	542.05	92.90	-510.3+i43.76	1.839-i.110
	7.52	1579.20	-7.378-i1.288	.211-i1.546	15.48	3252.30	-15.46-i1.288	.054-i.425
0.4	143.20	45.73	-110.7+i20.76	3.427-i.362	297.92	95.20	280.9+i22.89	1.798-i.118
	7.63	859.20	-7.530-i1.256	.198-i1.529	15.87	1787.50	-15.32-i1.256	.099-i.417
0.5	88.91	47.44	-69.46+i11.99	3.328-i.403	184.55	98.50	-174.3+i13.17	1.741-i.128
	7.91	533.45	-7.748-i1.209	.331-i1.508	16.42	1107.30	-16.34+i1.209	.160-i.407
0.6	59.17	49.88	-46.61+i7.155	3.197-i.520	122.48	103.25	-116.0+i7.869	1.664-i.142
	8.31	355.02	-8.060-i1.142	.493-i1.454	17.21	734.35	-17.08-i1.140	.240-i.393
0.7	41.36	54.12	-33.47+i4.204	2.981-i.571	84.32	110.35	-80.21+i4.614	1.560-i.161
	9.02	248.18	-8.638-i1.042	.702-i1.350	18.39	505.95	-18.21-i1.038	.346-i.374
0.8	28.37	59.56	-23.38+i 2.153	2.747-i.633	58.33	122.43	-55.77+i2.368	1.411-i.187
	9.93	170.24	-9.304-i.874	1.022-i1.274	20.40	349.97	-20.10-i .865	.500-i.348
0.9	17.76	76.02	-15.29+i .417	2.209-i.846	36.36	155.63	-35.11+i .504	1.118-i.240
	12.67	106.58	-11.40-i .421	1.610-i1.067	25.94	213.17	-25.30-i .390	.802-i.296
$1.2^{-1/2}$	14.97	89.86	-13.21+i .069	1.893+i.937	30.28	181.68	-29.39-i .017	.949+i.268

811-430

$C_1 = 0.2, \quad C_o = 1.0$ 60° 70°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	3.81 .0060	.0073 4.57	3.[illegible]2-12.207 4.[illegible]6-12.545	.960-11.657 1.1[illegible]3-11.926	6.02 .031	.037 7.22	6.050-12.207 5.841-12.962	.738-11.955 .873-12.340
0.1	3.74 .058	.070 4.49	3.[illegible]10-12.202 4.[illegible]94-12.529	1.0[illegible]3-11.662 1.188-11.984	5.88 .091	.11 7.05	5.977-12.191 5.788-12.918	.793-11.960 .929-12.336
0.2	3.48 .238	.29 4.17	3.[illegible]58-12.181 4.[illegible]8-12.466	1.141-11.681 1.303-11.965	5.32 .37	.44 6.38	5.726-12.142 5.601-12.759	1.022-11.979 1.148-12.319
0.3	3.02 .579	.70 3.62	3.6[illegible]4-12.154 3.8[illegible]3-12.366	1.356-11.712 1.498-11.933	4.44 .85	1.02 5.32	5.392-12.086 5.338-12.527	1.365-12.016 1.460-12.278
0.4	2.07 1.38	1.66 2.48	3.6[illegible]3-12.181 3.6[illegible]5-12.210	1.658-11.790 1.690-11.853	2.93 1.95	2.34 3.52	5.037-12.081 5.037-12.196	1.800-12.105 1.832-12.185
$6^{-1/2}$	1.72	2.06	3.640-12.181	1.691-11.818	2.43	2.91	5.009-12.145	1.855-12.148

$C_1 = 0.2, \quad C_o = 1.0$ 80° 90°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	12.25 .046	.056 14.70	12.30-12.169 7.715-14.234	.423-12.098 .497-12.570	88.08 .42	.50 105.70	88.33-11.388 1.960-16.509	.415-12.212 .418-12.651
0.1	11.49 .177	.21 13.79	11.69-12.133 7.680-14.061	.646-12.150 .622-12.566	43.49 .67	.81 52.19	44.09-11.122 3.761-16.167	.665-12.217 .660-12.646
0.2	9.34 .64	.77 11.21	10.04-11.997 7.499-13.526	.965-12.171 1.029-12.544	20.71 1.46	1.75 24.85	21.84-11.201 6.429-15.019	1.363-12.240 1.404-12.624
0.3	7.08 1.36	1.63 8.49	8.495-11.899 7.181-12.908	1.496-12.214 1.548-12.503	12.36 2.37	2.85 14.83	14.28-11.278 7.888-13.675	1.992-12.282 1.988-12.580
0.4	4.33 2.88	3.46 5.20	7.145-11.861 6.886-12.212	2.085-12.313 2.100-12.400	6.63 4.42	5.30 7.96	9.910-11.650 8.841-12.211	2.650-12.386 2.653-12.476
$6^{-1/2}$	3.56	4.27	6.955-12.066	2.142-12.358	5.41	6.50	9.281-11.895	2.705-12.430

$C_1 = 0.2,\ C_o = 1.0$ 100°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	633.51 2.39	2.87 760.21	596.3+i53.06 -2.068-i6.604	1.932-i2.146 1.855-i2.572
0.1	165.02 2.55	3.06 198.02	156.1+i12.96 -1.284-i6.501	2.056-i2.151 1.933-i2.567
0.2	45.82 3.14	3.77 54.99	44.42+i2.683 1.479-i5.870	2.468-i2.172 2.401-i2.546
0.3	21.61 4.14	4.97 25.93	22.17+i.474 5.023-i4.498	2.981-i2.214 2.928-i2.505
0.4	10.17 6.78	8.14 12.20	8.811-i1.067 9.233-i2.094	3.539-i2.318 3.521-i2.402
$6^{-1/2}$	8.24	9.88	10.59-i1.591	3.580-i2.361

110°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	1288.37 4.87	5.84 1546.04	987.5+i1205.7 -4.708-i6.606	3.570-i1.957 3.432-i2.345
0.1	322.78 4.99	5.99 387.33	247.8+i50.51 -4.343-i6.521	3.639-i1.962 3.488-i2.340
0.2	80.60 5.52	6.62 96.72	62.31+i11.45 -2.752-i6.073	3.840-i1.981 3.718-i2.322
0.3	34.44 6.606	7.93 41.33	26.92+i3.776 .291-i4.953	4.163-i2.019 4.065-i2.284
0.4	15.00 10.00	12.00 18.00	11.56-i.208 6.453-i2.211	4.531-i2.113 4.503-i2.191
$6^{-1/2}$	12.08	14.50	8.816-i1.209	4.522-i2.152

$C_1 = 0.2,\ C_o = 1.0$ 120°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	2037.77 7.70	9.24 2445.56	311.2+i134.3 -7.599-i6.606	4.795-i1.668 4.605-i1.997
0.1	506.43 7.83	9.40 607.72	251.3+i1.075 -7.418-i6.526	4.826-i1.672 4.638-i1.993
0.2	122.92 8.42	10.10 147.50	59.32+i24.35 -6.576-i6.131	4.936-i1.688 4.771-i1.977
0.3	50.61 9.70	11.64 60.74	22.70+i8.317 -4.663-i5.158	5.116-i1.720 4.987-i1.945
0.4	21.22 14.15	16.98 25.46	6.350+i.726 .759-i2.211	5.335-i1.801 5.295-i1.867
$6^{-1/2}$	17.04	20.44	3.324-i.834	5.337-i1.833

130°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	2958.98 11.18	13.41 3550.77	508.9+i725.7 -12.01-i6.608	5.444-i1.313 5.231-i1.570
0.1	733.16 11.34	13.61 879.79	121.5+i177.0 -11.05-i6.527	5.463-i1.316 5.251-i1.567
0.2	176.01 12.06	14.47 211.21	24.41+i39.58 -10.76-i6.155	5.520-i1.329 5.331-i1.555
0.3	71.31 13.67	16.40 85.78	5.580+i13.35 -9.962-i5.249	5.606-i1.354 5.473-i1.533
0.4	29.32 19.55	23.46 35.18	-4.675+i1.571 -7.926-i2.210	5.708-i1.416 5.664-i1.468
$6^{-1/2}$	23.50	28.20	-5.524-i.532	5.697-i1.442

$C_1 = 0.2$, $C_o = 1.0$ 140° / 150°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	4200.79 15.88	19.05 5040.95	-738.2+i1029.8 -15.82-i6.607	5.445-i.935 4.650-i1.116	6103.36 23.14	27.77 7324.03	-3062.7+i1301.8 -23.03-i6.606	4.806-i.580 4.616-i.689
0.1	1039.49 16.08	19.29 1247.39	-204.1+i259.0 -15.88-i6.305	5.449-i.937 5.240-i1.114	1509.37 23.35	28.02 1811.24	-766.2+i318.4 -23.21-i6.528	4.789-i.581 4.603-i.688
0.2	248.28 17.01	20.41 297.93	-53.00+i55.22 -16.08-i6.166	5.475-i.946 5.286-i1.105	359.13 25.01	30.01 430.95	-191.1+i66.55 -24.01-i6.177	4.876-i.587 4.709-i.683
0.3	99.78 19.13	22.95 119.73	-27.76+i18.18 -16.45-i5.305	5.506-i.963 5.360-i1.087	143.85 27.58	33.10 172.62	-83.77+i22.28 -25.70-i5.333	4.799-i.597 4.668-i.672
0.4	40.60 27.07	32.44 48.72	-18.76+i2.239 -17.28-i2.210	5.537-i1.008 5.492-i1.014	58.19 38.79	46.55 69.83	-41.27+i2.736 -31.60-i2.211	4.786-i.624 4.747-i.646
$6^{-1/2}$	32.51	39.02	-17.83-i.031	5.520-i1.026	46.57	55.89	-35.51-i.166	4.769-i.635

$C_1 = 0.2$, $C_o = 1.0$ 160° / 170°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	9565.19 36.14	43.37 11478.23	-7289.9+i1540.7 -36.12-i6.606	3.585-i.295 3.422-i.348	19981.0 75.67	90.80 23977.20	-18743.5+i1684.8 -75.49-i6.606	1.893-i.083 1.820-i.114
0.1	2393.03 37.02	44.42 2871.64	-1843.5+i373.8 -36.76-i6.529	3.552-i.290 3.414-i.341	4937.82 75.88	91.06 5925.38	-4674.5+i417.7 -76.35-i6.529	1.874-i.101 1.800-i.114
0.2	562.36 38.53	46.23 674.83	-438.4+i80.36 -39.69-i6.392	3.582-i.299 3.457-i.345	1173.58 80.39	96.47 1408.29	-1108.9+i87.73 -81.20-i6.177	1.886-i.102 1.820-i.113
0.3	227.17 43.55	52.26 272.60	-181.2+i25.35 -41.83-i5.349	3.574-i.304 3.477-i.339	467.83 89.70	107.64 561.40	-445.6+i27.31 -89.12-i5.357	1.879-i.103 1.828-i.112
0.4	90.50 60.33	72.40 108.60	-77.33+i2.999 -55.57-i2.211	3.551-i.316 3.522-i.327	188.27 125.52	150.62 225.92	-182.5+i3.254 -123.2-i2.210	1.863-i.104 1.848-i.109
$6^{-1/2}$	73.28	87.93	-65.92-i.070	3.504-i.318	150.63	180.75	-147.0-i.017	1.814-i.105

$C_1 = 1.0$, $C_0 = 0.1$

-1	60° $R_1 \times 10^3$	60° $R_0 \times 10^2$	60° $Z_1 \times 10^3$	60° $Z_0 \times 10^3$	70° $R_1 \times 10^3$	70° $R_0 \times 10^3$	70° $Z_1 \times 10^3$	70° $Z_0 \times 10^3$
0.05	4.17 0	0 45.87	4.169-12.407 2.291-11.635	1.046-11.810 11.511-119.912	6.59 .014	.15 72.45	6.606-12.417 1.752-11.477	.911-12.175 8.627-123.384
0.1	4.13 .011	.126 45.47	4.156-12.402 2.297-11.640	1.158-11.857 11.538-119.841	6.49 .018	.20 71.40	6.521-12.390 1.760-11.478	.956-12.189 8.658-123.366
0.2	4.01 .045	.50 44.10	4.097-12.377 2.314-11.647	1.499-12.010 11.707-119.681	6.13 .073	.81 67.39	6.247-12.337 1.798-11.484	1.509-12.384 9.058-123.179
0.3	3.83 .10	1.10 42.10	4.007-12.338 2.336-11.655	2.042-12.269 11.951-119.432	5.62 .14	1.58 61.82	5.868-12.250 1.862-11.497	2.205-12.658 9.545-122.883
0.4	3.59 .17	1.89 39.51	3.893-12.288 2.370-11.668	2.750-12.630 12.232-119.053	5.08 .24	2.68 55.92	5.459-12.159 2.041-11.567	3.181-13.099 10.196-122.442
0.5	3.33 .27	2.96 36.64	3.768-12.236 2.407-11.682	3.691-13.159 12.574-118.550	4.54 .36	4.01 49.99	5.059-12.085 2.025-11.523	4.327-13.706 10.892-121.836
0.6	3.04 .38	4.18 33.42	3.629-12.155 2.458-11.693	4.740-13.815 12.861-117.874	4.02 .51	5.56 44.24	4.670-12.005 2.129-11.539	5.603-14.502 11.659-121.045
0.7	2.73 .52	5.77 30.03	3.480-12.115 2.515-11.722	6.028-14.723 13.066-116.988	3.51 .67	7.38 38.62	4.293-11.928 2.252-11.559	6.991-15.540 12.250-120.000
0.8	2.38 .71	7.77 26.23	3.322-12.046 2.591-11.751	7.509-15.939 13.203-115.775	2.99 .88	9.69 32.91	3.917-11.852 2.409-11.585	8.544-16.963 12.773-118,564
0.9	1.95 .98	10.78 21.42	3.127-11.964 2.710-11.797	9.437-17.855 12.880-113.828	2.41 1.21	13.34 26.46	3.486-11.771 2.637-11.621	10.505-19.265 13.026-116.282
0.95	1.55 1.31	14.36 17.04	2.952-11.893 2.877-11.871	11.136-110.071 12.044-111.616	1.89 1.60	17.57 20.83	3.124-11.704 2.914-11.667	11.957-111.862 12.644-113.673
$1.1^{-1/2}$	1.43	15.68	2.898-11.871	12.041-111.239	1.74	19.16	3.016-11.685	12.302-112.747

$C_1 = 1.0$, $C_0 = 0.1$

-1	80° $R_1 \times 10^3$	80° $R_0 \times 10^3$	80° $Z_1 \times 10^3$	80° $Z_0 \times 10^3$	90° $R_1 \times 10^3$	90° $R_0 \times 10^3$	90° $Z_1 \times 10^3$	90° $Z_0 \times 10^3$
0.05	13.41 .0093	.10 147.50	13.42-12.388 .955-11.366	.503-12.355 4.621-125.710	96.39 .16	1.32 1060.3	96.26-12.177 .073-11.326	1.198-12.440 1.597-126.49
0.1	12.69 .035	.39 139.61	12.75-12.328 .982-11.368	.767-12.403 4.862-125.661	48.05 .13	1.46 528.54	48.11-11.204 .147-11.327	1.324-12.478 1.296-126.46
0.2	10.77 .12	1.33 118.47	10.95-12.163 1.062-11.376	1.618-12.598 5.640-125.466	23.87 .31	3.45 262.55	23.78-11.406 .294-11.324	3.096-12.724 3.060-126.21
0.3	8.97 .23	2.57 98.63	9.269-12.011 1.181-11.382	2.726-12.933 6.604-125.129	15.66 .41	4.55 172.25	15.69-11.224 .448-11.324	4.030-13.032 3.994-125.90
0.4	7.51 .37	4.04 82.56	7.902-11.894 1.308-11.389	4.002-13.430 7.759-124.636	11.49 .56	6.16 126.44	11.54-11.218 .610-11.321	5.338-13.531 5.340-125.41
0.5	6.31 .50	5.54 69.46	6.809-11.787 1.454-11.399	5.238-14.062 8.727-123.993	8.93 .72	7.97 98.23	8.976-11.226 .782-11.316	6.702-14.221 6.659-124.71
0.6	5.35 .67	7.39 58.81	5.905-11.713 1.611-11.407	6.665-14.918 9.847-123.112	7.14 .90	9.87 78.52	7.213-11.220 .975-11.312	7.972-15.104 7.994-123.83
0.7	4.50 .86	9.45 49.55	5.137-11.644 1.796-11.418	8.063-16.079 10.884-121.976	5.78 1.11	12.20 63.60	5.864-11.226 1.198-11.306	9.302-16.292 9.314-122.64
0.8	3.75 1.11	12.20 41.20	4.428-11.569 2.021-11.428	9.662-17.681 11.894-120.382	4.66 1.38	15.18 51.22	4.744-11.234 1.477-11.296	10.65-17.926 10.65-121.35
0.9	2.94 1.48	16.29 32.31	3.694-11.532 2.368-11.446	11.360-110.176 12.692-117.883	3.57 1.80	19.78 39.22	3.671-11.244 1.909-11.283	11.97-110.50 11.97-118.44
0.95	2.29 1.93	21.22 25.18	3.115-11.491 2.787-11.470	12.457-113.029 12.809-115.025	2.75 2.32	25.52 30.28	2.876-11.258 2.444-11.268	12.58-113.44 12.69-115.50
$1.1^{-1/2}$	2.10	23.15	2.944-11.480	12.688-114.053	2.53	27.82	2.650-11.263	12.66-114.48

$C_1 = 1.0$, $C_o = 0.1$ 100° 110° 811-44C

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	693.24	5.20	651.2+i58.64	4.166-i2.357	1409.83	10.70	1077.6+i224.6	7.803-i2.152
	.47	7625.60	-.458-i1.325	.022-i25.71	.97	15508.10	-.961-i1.325	.040-i23.42
0.1	182.30	5.64	170.5+i14.36	4.447-i2.405	356.58	10.80	272.1+i56.31	7.848-i2.196
	.50	2005.30	-.430-i1.323	.438-i25.66	.98	3922.40	-.949-i1.323	.123-i23.37
0.2	52.82	6.59	48.79+i3.222	5.287-i2.604	92.90	10.95	71.11+i14.52	8.240-i2.351
	.60	581.01	-.337-i1.314	1.265-i25.47	.95	1022.20	-.899-i1.312	.251-i23.20
0.3	27.38	7.82	24.97+i1.109	6.203-i2.934	43.64	12.54	31.88+i5.635	8.838-i2.681
	.71	301.18	-.214-i1.302	2.279-i25.14	1.14	480.06	-.822-i1.296	1.483-i22.89
0.4	17.64	9.38	15.81+i.261	7.267-i3.421	26.02	13.84	18.42+i2.823	9.467-i3.122
	.85	194.02	-.071-i1.288	3.518-i24.65	1.26	286.16	-.721-i1.274	2.446-i22.45
0.5	12.62	11.17	11.08-i.176	8.394-i4.084	17.54	15.44	11.95+i1.471	10.12-i3.717
	1.02	138.83	.092-i1.270	4.923-i23.99	1.40	192.96	-.596-i1.247	3.493-i21.85
0.6	9.55	13.20	8.204-i.445	9.471-i4.956	12.68	17.48	8.215+i.677	10.83-i4.511
	1.20	105.00	.281-i1.250	6.265-i23.12	1.59	139.52	-.443-i1.213	4.812-i21.06
0.7	7.42	15.58	6.231-i.624	10.46-i6.093	9.52	20.02	5.789+i.161	11.47-i5.556
	1.42	81.62	.508-i1.224	7.687-i21.98	1.82	104.78	-.251-i1.169	6.210-i20.02
0.8	5.80	18.85	4.690-i.776	11.49-i7.684	7.23	23.47	4.022-i.217	12.01-i6.995
	1.71	63.75	.799-i1.190	9.209-i20.39	2.13	79.53	.009-i1.110	7.762-i18.58
0.9	4.33	23.98	3.323-i.912	12.27-i10.18	5.28	29.27	2.515-i.545	12.24-i9.274
	2.18	47.62	1.266-i1.137	10.91-i17.90	2.66	58.13	.440-i1.011	9.691-i16.31
0.95	3.31	30.74	2.356-i1.014	12.39-i13.04	4.00	37.14	1.512-i.768	11.84-i11.88
	2.79	36.46	1.858-i1.069	12.03-i15.03	3.38	44.06	1.013-i.880	11.21-i13.70
$1.1^{-1/2}$	3.04	33.44	2.097-i1.043	12.28-i14.05	3.67	40.40	1.249-i.827	11.55-i12.79

$C_1 = 1.0$, $C_o = 0.1$ 120° 130°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	2230.10	16.90	1112.2+i480.8	10.46-i1.839	3237.95	24.40	568.8+i802.6	11.81-i1.454
	1.54	24531.10	-1.529-i1.325	.064-i19.90	2.22	35617.40	-2.225-i1.325	.041-i15.59
0.1	559.47	17.00	277.9+i119.8	10.48-i1.876	809.95	24.60	138.9+i198.2	11.87-i1.483
	1.55	6154.20	-1.523-i1.323	.136-i19.86	2.24	8909.40	-2.223-i1.323	.083-i15.56
0.2	141.68	17.53	68.76+i29.15	10.66-i2.029	202.88	25.10	32.92+i48.22	11.92-i1.602
	1.59	1558.50	-1.498-i1.312	.423-i19.71	2.28	2231.70	-2.215-i1.312	.302-i15.44
0.3	64.13	18.38	29.98+i12.36	10.90-i2.287	90.36	25.87	13.29+i20.54	11.96-i1.804
	1.67	705.42	-1.456-i1.294	.986-i19.45	2.35	993.93	-2.202-i1.293	.681-i15.24
0.4	36.80	19.62	16.25+i6.405	11.23-i2.665	50.85	27.05	6.368+i10.77	12.03-i2.098
	1.78	404.78	-1.398-i1.268	1.733-i19.08	2.46	559.35	-2.182-i1.265	1.257-i14.94
0.5	23.95	21.15	9.862+i3.645	11.53-i3.173	32.43	28.66	3.134+i6.555	12.08-i2.499
	1.92	263.45	-1.321-i1.234	2.621-i18.57	2.61	356.74	-2.155-i1.228	2.002-i14.55
0.6	16.78	23.16	6.278+i2.091	11.82-i3.845	22.26	30.68	1.401+i3.729	12.03-i3.022
	2.11	184.64	-1.221-i1.190	3.741-i17.90	2.79	244.92	-2.119-i1.178	2.808-i14.02
0.7	12.27	25.83	4.005+i1.104	12.06-i4.737	15.96	33.60	.276+i2.152	11.95-i3.722
	2.35	134.97	-1.090-i1.132	4.990-i17.02	3.05	175.60	-2.069-i1.109	3.874-i13.33
0.8	9.10	29.54	2.418+i.414	12.13-i5.954	11.62	37.74	-.481+i1.083	11.64-i4.673
	2.69	100.06	-.905-i1.050	6.467-i15.79	3.43	127.86	-1.995-i1.008	5.182-i12.37
0.9	6.52	36.10	1.107-i.160	11.83-i7.889	3.19	45.43	-1.103+i.218	10.94-i6.193
	3.28	71.70	-.583-i.907	8.369-i13.86	4.13	90.17	-1.848-i.658	7.040-i10.86
0.95	4.89	45.26	.277-i.525	10.92-i10.08	6.11	56.63	-1.484-i.310	9.779-i7.924
	4.11	53.83	-.133-i.707	10.01-i11.64	5.15	67.17	-1.665-i.557	8.765-i9.128
$1.1^{-1/2}$	4.49	49.37	.054-i.624	10.57-i10.87	5.59	61.54	-1.581-i.443	9.275-i8.527

$C_1 = 1.0$, $C_o = 0.1$ 140° / 160°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	4596.84	34.80	-800.2+11130.6	11.27-11.04	6678.76	50.60	-3341.3+11442.3	10.67-1.660
	3.16	50565.20	-3.162-11.325	.019-110.99	4.69	73466.40	-4.596-11.325	.024-16.682
0.1	1148.35	34.90	-217.2+1290.6	11.85-11.06	1667.45	50.70	-836.0+1359.1	10.418-1.672
	3.17	12631.90	-3.164-11.323	.033-110.97	4.61	18341.90	-4.603-11.323	.0363-16.668
0.2	286.17	35.50	-52.50+168.07	11.87-11.149	414.39	51.30	-209.5+187.37	10.369-1.723
	3.23	3147.90	-3.172-11.311	.269-110.89	4.66	4558.30	-4.629-11.311	.151-16.617
0.3	126.42	36.20	-29.82+130.51	11.76-11.291	182.27	52.28	-93.43+137.14	10.271-1.809
	3.29	1390.60	-3.186-11.292	.486-110.75	4.75	2004.90	-4.673-11.292	.353-16.532
0.4	70.41	37.48	-14.79+115.31	11.69-11.499	100.92	53.73	-52.75+119.55	10.123-1.935
	3.41	774.52	-3.205-11.264	.929-110.54	4.88	1110.10	-4.740-11.263	.634-16.407
0.5	44.36	39.19	-10.27+18.890	11.56-11.780	63.12	55.66	-33.87+111.44	9.890-11.104
	3.56	488.01	-3.232-11.224	1.439-110.26	5.06	694.34	-4.834-11.222	.999-16.237
0.6	30.07	41.45	-7.770+15.398	11.32-12.149	42.51	58.61	-23.52+16.903	9.657-11.332
	3.77	330.75	-3.269-11.170	2.102-19.893	5.33	467.59	-4.957-11.163	1.538-16.029
0.7	21.26	44.70	-6.249+13.221	11.00-12.639	29.75	62.58	-15.33+13.313	9.221-11.626
	4.06	233.90	-3.320-11.094	2.924-19.401	5.69	327.22	-5.123-11.121	2.133-15.720
0.8	15.29	49.66	-5.224+11.731	10.51-13.314	21.18	68.82	-12.89+12.310	8.654-12.034
	4.51	168.14	-3.396-1.978	4.034-18.735	6.26	232.98	-5.434-1.957	2.980-15.314
0.9	10.63	58.87	-4.428+1.577	9.538-14.377	14.58	80.77	-9.608+1.881	7.660-13.024
	5.35	116.93	-3.539-1.761	5.612-17.660	7.34	160.43	-5.986-1.714	4.211-14.666
0.95	7.87	73.00	-3.962-1.118	8.251-15.599	10.75	99.72	-7.687+1.036	6.478-13.418
	6.64	86.60	-3.754-1.432	7.252-16.443	9.07	118.28	-6.845-1.335	5.597-13.932
$1.1^{-1/2}$	7.21	79.30	-3.850-1.287	7.770-16.025	9.84	108.23	-7.232-1.163	6.033-13.673

$C_1 = 1.0$, $C_o = 0.1$ 160° / 170°

-1	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$	$R_1 x10^3$	$R_o x10^3$	$Z_1 x10^3$	$Z_o x10^3$
0.05	10466.97	79.30	-7965.5+11693.2	7.820-1.282	21864.76	157.60	-21485.0+12153.7	4.203-1.138
	7.21	115136.70	-7.205-11.325	.007-13.229	14.33	240512.40	-15.05-11.325	.00594-1.869
0.1	2643.65	80.15	-2029.6+1426.2	7.704-1.263	5455.50	165.90	-5302.5+1516.7	4.116-1.140
	7.29	29030.20	-7.308-11.323	.0238-13.217	15.08	60010.30	-15.08-11.323	.010-1.863
0.2	648.21	80.30	-494.7+1102.3	7.736-1.382	1352.75	167.40	-1272.0+1115.2	4.080-1.149
	7.30	7130.30	-7.329-11.311	.109-13.199	15.22	14880.20	-15.21-11.311	.0482-1.860
0.3	287.83	82.50	218.2+143.70	7.579-1.416	592.77	169.90	-558.0+148.09	4.022-1.159
	7.50	3166.10	-7.362-11.292	.222-13.142	15.44	6520.50	-15.42-11.292	.106-1.843
0.4	156.95	83.54	-121.2+122.95	7.513-1.483	362.52	173.70	-307.8+125.31	3.997-1.190
	7.59	1726.50	-7.499-11.262	.408-13.099	15.79	3591.70	-15.75-11.262	.196-1.835
0.5	97.78	86.28	-83.92+114.76	7.313-1.565	202.95	179.10	-191.6+114.75	3.827-1.195
	7.84	1075.50	-7.695-11.221	.649-13.018	16.28	2232.50	-16.21-11.220	.315-1.814
0.6	65.41	90.25	-51.53+18.137	7.057-1.672	135.40	186.79	-128.1+18,966	3.678-1.222
	8.20	719.55	-7.969-11.162	.975-12.911	16.98	1489.40	-16.86-11.160	.473-1.787
0.7	46.14	97.03	-37.11+14.949	6.635-1.797	94.70	197.83	-89.35+15.442	3.506-1.259
	8.82	507.57	-8.475-11.077	1.388-12.711	17.98	1034.80	-17.82-11.073	.677-1.752
0.8	32.26	104.78	-26.31+12.764	6.222-11.008	66.31	215.38	-63.25+13.065	3.206-1.307
	9.53	354.82	-8.984-1.945	1.970-12.576	19.58	729.42	-19.32-1.937	.962-1.703
0.9	22.08	122.33	-18.55+11.105	5.438-11.317	45.20	250.41	-43.39+11.256	2.773-1.385
	11.12	242.87	-10.20-1.646	2.855-12.269	22.76	497.19	-22.31-1.664	1.410-1.625
0.95	16.40	152.05	-14.29+1.152	4.456-11.636	33.12	307.12	-32.54+1.223	2.270-1.474
	13.82	180.35	-12.32-1.264	3.792-11.877	27.92	364.28	-27.16-1.222	1.913-1.536
$1.1^{-1/2}$	15.00	165.02	-13.22-1.073	4.122-11.756	30.28	333.10	-29.38-1.018	2.093-1.505

C_1 = 0.1, C_o = 1.0

-1	60° R_1x10^3	R_ox10^3	Z_ix10^3	Z_ox10^3	70° R_1x10^3	R_ox10^3	Z_ix10^3	Z_ox10^3
0.05	4.14	.0 1	4.170-i2.408	1.058-i1.806	6.56	.058	6.642-i2.407	.823-i2.136
	.019	4.5	4.436-i2.667	1.163-i1.988	.053	7.20	6.774-i3.029	.898-i2.340
0.1	4.03	.1	4.156-i2.403	1.134-i1.815	6.32	.20	6.521-i2.390	.932-i2.145
	.12	4.43	4.408-i2.647	1.233-i1.985	.18	6.96	6.689-i2.974	1.004-i2.336
0.2	3.54	.56	4.102-i2.389	1.390-i1.827	5.42	.86	6.250-i2.372	1.388-i2.157
	.51	3.90	4.295-i2.556	1.470-i1.966	.78	5.96	6.650-i2.885	1.444-i2.315
0.3	2.16	1.94	4.060-i2.387	1.818-i1.887	3.17	2.85	5.920-i2.360	2.068-i2.222
	1.77	2.38	4.085-i2.412	1.835-i1.910	2.59	3.49	5.938-i2.411	2.077-i2.244
$1.1^{-1/2}$	1.96	2.16	4.071-i2.398	1.830-i1.897	2.38	3.17	5.923-i2.383	2.085-i2.234

C_1 = 0.1, C_o = 1.0

-1	80° R_1x10^3	R_ox10^3	Z_ix10^3	Z_ox10^3	90° R_1x10^3	R_ox10^3	Z_ix10^3	Z_ox10^3
0.05	13.33	.10	13.42-i2.388	.5001-i2.341	95.81	.81	96.38-i1.338	.731-i2.417
	.090	14.66	11.66-i4.578	.540-i2.571	.74	105.39	6.867-i12.40	.734-i2.651
0.1	12.37	.40	12.75-i2.330	.756-i2.346	46.82	1.50	48.10-i1.236	1.325-i2.418
	.36	13.60	11.31-i4.279	.791-i2.565	1.36	51.50	11.72-i10.34	1.322-i2.646
0.2	9.52	1.51	10.96-i2.305	1.581-i2.368	21.10	3.38	23.79-i1.378	2.681-i2.441
	1.37	10.47	10.26-i3.431	1.609-i2.542	3.08	23.22	14.76-i5.875	2.689-i2.622
0.3	5.06	4.55	9.227-i2.284	2.637-i2.442	8.84	7.96	14.96-i2.041	3.980-i2.521
	4.14	5.57	9.174-i2.412	2.645-i2.467	7.23	9.72	14.47-i2.410	3.975-i2.543
$1.1^{-1/2}$	4.59	5.05	9.181-i2.342	2.659-i2.454	8.00	8.80	14.66-i2.211	4.000-i2.532

C_1 = 0.1, C_o = 1.0

-1	100° R_1x10^3	R_ox10^3	Z_ix10^3	Z_ox10^3	110° R_1x10^3	R_ox10^3	Z_ix10^3	Z_ox10^3
0.05	689.06	5.11	652.3+i59.74	4.063-i2.342	1401.35	10.39	1089.8+i232.8	7.509-i2.139
	4.65	757.97	-3.675-i13.15	4.026-i2.[illegible]	9.45	1541.49	-9.210-i3.16	7.437-i2.349
0.1	177.63	5.69	170.0+i13.88	4.461-i2.348	347.45	11.13	268.8+i54.12	7.904-i2.144
	5.17	195.39	-.721-i12.64	4.420-i2.563	10.12	382.19	-7.810-i12.82	7.832-i2.344
0.2	46.69	7.40	47.94+i2.485	5.285-i2.371	82.13	13.00	65.68+i11.07	8.338-i2.165
	6.73	51.36	8.986-i9.170	5.257-i2.546	11.82	90.34	-1.027-i10.51	8.281-i2.324
0.3	15.45	13.90	24.15-i1.417	6.309-i2.447	24.63	22.17	20.54-i.453	8.979-i2.235
	12.64	17.00	18.82-i2.412	6.309-i2.471	20.15	27.09	15.92-i2.411	8.968-i2.256
$1.1^{-1/2}$	13.94	15.34	19.82-i1.891	6.327-i2.458	22.19	24.41	18.16-i1.418	9.134-i2.239

$C_1 = 0.1, \; C_o = 1.0$ 120° / 130°

-1	$R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$	$R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$
	120°				130°			
0.05	2214.24	16.44	1142.9+i500.7	10.078-i1.825	3218.47	23.86	609.4+i835.3	11.456-i1.444
	14.94	2435.66	-15.11-i13.18	9.971-i2.003	21.69	3540.32	-22.18-i13.17	11.338-i1.584
0.1	545.14	17.47	270.2+i114.8	10.527-i1.831	789.20	25.28	126.7+i133.3	11.914-i1.448
	15.88	599.65	-14.40-i12.85	10.427-i2.002	22.98	868.12	-21.96-i12.86	11.801-i1.582
0.2	125.24	19.83	59.55+i23.28	10.762-i1.849	179.35	28.40	19.31+i37.13	12.042-i1.462
	18.03	137.77	-10.62-i10.97	10.686-i1.985	25.82	197.28	-20.52-i11.16	11.954-i1.568
0.3	36.19	32.57	10.39+i1.561	11.122-i1.909	50.99	45.89	-8.532+i1.406	12.224-i1.509
	29.61	39.81	4.882-i2.411	11.112-i1.927	41.72	56.10	-12.11-i2.414	12.211-i1.523
$1.1^{-1/2}$	32.59	35.85	7.549-i.961	11.125-i1.918	45.90	50.50	-10.39-i.605	12.222-i1.516

$C_1 = 0.1, \; C_o = 1.0$ 140° / 150°

-1	$R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$	$R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$
	140°				150°			
0.05	4569.19	33.84	-749.7+i1200.2	11.457-i1.037	6633.60	49.34	-3339.8+i1534.9	10.095-i.655
	30.81	5026.11	-31.66-i13.17	11.339-i1.135	44.86	7302.46	-46.36-i13.23	9.992-i.714
0.1	1118.94	35.85	-228.2+i274.3	11.894-i1.041	1624.74	52.05	-835.5+i335.9	10.446-i.657
	32.59	1230.83	-31.89-i12.42	11.782-i1.134	47.32	1787.21	-46.82-i12.87	10.350-i.714
0.2	252.98	40.06	-63.82+i50.68	11.912-i1.050	366.33	58.00	-206.8+i62.47	10.466-i.663
	36.42	278.28	-32.62-i11.26	11.864-i1.128	52.73	402.96	-50.10-i11.31	10.388-i.708
0.3	71.34	64.21	-36.03+i2.034	12.021-i1.088	102.86	92.57	-75.97+i2.463	10.481-i.683
	58.37	78.47	-34.75-i2.407	12.008-i1.07[illegible]	84.16	113.15	-66.66-i2.413	10.472-i.689
$1.1^{-1/2}$	64.22	70.64	-35.29-i.351	12.020-i1.085	92.57	101.83	-70.87-i.182	10.475-i.686

$C_1 = 0.1, \; C_o = 1.0$ 160° / 170°

-1	$R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$	$R_1 \times 10^3$	$R_o \times 10^3$	$Z_1 \times 10^3$	$Z_o \times 10^3$
	160°				170°			
0.05	10404.03	77.17	-8095.9+i1823.9	7.544-i.348	21733.27	161.40	21028.3+i2011.5	3.983-i.139
	70.16	11444.43	-72.42-i13.17	7.467-i.377	146.73	23906.60	-151.4-i13.17	3.942-i.146
0.1	2575.94	82.53	-1992.4+i392.7	7.751-i.345	5315.21	169.71	-5023.8+i434.0	4.106-i.141
	75.03	2833.53	-79.28-i12.88	7.678-i.372	154.28	5846.73	-154.7-i12.88	4.067-i.148
0.2	573.02	90.74	457.1+i7123	7.824-i.354	1195.83	189.35	-1136.0+i76.99	4.120-i.142
	82.49	630.32	-80.79-i11.33	7.766-i.375	172.14	1315.41	-171.3-i11.35	4.089-i.147
0.3	162.43	146.19	-142.3+i2.738	7.738-i.357	334.52	301.07	-326.5+i2.897	4.108-i.144
	132.90	178.67	-119.7-i2.411	7.730-i.360	273.70	367.97	-268.0-i2.411	4.104-i.145
$1.1^{-1/2}$	146.19	160.81	-131.8-i.076	7.740-i.358	301.07	331.18	-293.9-i.053	4.107-i.145

5. <u>Overall Transfer Characteristic for Bridge "T" and Phase Lag Network</u>

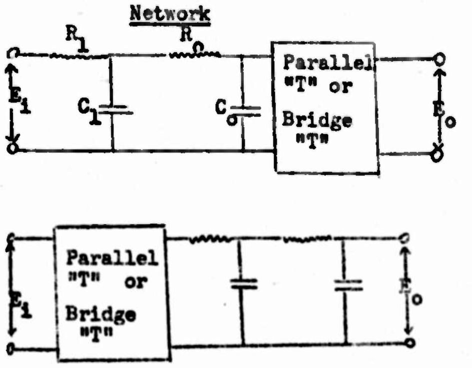


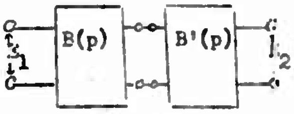
Fig. 26

From the form of the input and output impedances of the bridge "T" and of the parallel "T", it may be seen that the essential proportional-derivative numerator factor, $[\omega_o^{-2}p^2 + 2(n\omega_o)^{-1}p + 1]$, is preserved in the overall transfer characteristic for either of the phase-lag circuits of Fig. 26. This is true also for the methods of series input impedance or load impedance. In either case of Fig. 26, and for non-inductive series input or load impedances, the overall proportional derivative characteristic is not very seriously disturbed since it is known that the denominator of the transfer characteristic for any RC network can have no complex roots. For the parallel "T", the transfer characteristic for either circuit of Fig. 26 will be of the form:

$$\frac{E_o}{E_i} = \frac{(U_3p + 1)\left(\frac{1}{\omega_o^2}p^2 + \frac{2}{n\omega_o}p + 1\right)}{(U_4p + 1)(U_1^2p^2 + 2\zeta_1U_1p + 1)(U_2^2p^2 + 2\zeta_2U_2p + 1)} \qquad (1)$$

where $\zeta_1 > 1$ and $\zeta_2 > 1$. For the bridge "T", the transfer characteristic will be of the same form with $U_3 = U_4 = 0$.

When two networks are connected in cascade, as for example the parallel "T" and phase lag networks in Fig. 26, there will be no phase shift due to the connection of the output of the first network to the input of the second, if the angle of the output impedance of the first network is equal to the angle of the input impedance of the second. This of course does not mean that the overall transfer characteristic is the product of a constant gain ≤ 1 and the separate transfer characteristics for each network. In fact if $B(p)$, $B'(p)$ are the separate transfer characteristics for each of the two networks, Fig. 27, the overall transfer characteristic is:



$$\frac{E_2}{E_1} = \frac{B(p)B'(p)}{1 + \frac{z22}{z11'} - [B(p)]^2\frac{z11}{z11'}} = \frac{z11'}{(z11' + Z_o)} B(p)B'(p) \qquad (2)$$

Fig. 27

where $z11$, $z11'$ are the separate input impedances, and $z22 = Z_o + z11\,[B(p)]^2$ is the impedance looking into the output of the first network with the input open-circuited. This follows from the transfer characteristic of Fig. 19, where $Z = 0$; or immediately by Thevenin's theorem.

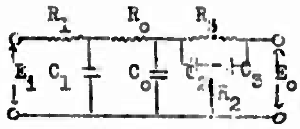


Fig. 28

For the combination of phase lag network and bridge "T" as in Fig. 28, the transfer characteristic is:

$$\frac{E_o}{E_i} = \frac{\frac{1}{\omega_o^2}p^2 + \frac{2}{n\omega_o}p + 1}{Ap^4 + Bp^3 + Cp^2 + Dp + 1} \qquad (3)$$

where

$$A = \frac{T_oT_1}{\omega_o^2} = \frac{T^2}{\omega_o^2}$$

$$B = \frac{(T_o + S_1)}{\omega_o^2} + \frac{2T_1T_o}{n\omega_o} + \frac{T_1(R_o/R_2 + 1)}{\omega_o^2} = \frac{2\zeta T}{\omega_o^2} + \frac{2T^2}{n\omega_o} + \frac{T^2}{R_2C_o\omega_o^2}$$

$$C = \frac{2(T_o + S_1 + T_1)}{n\omega_o} + \frac{(R_o/R_2 + 1)}{\omega_o^2} + \frac{R_1}{R_2\omega_o^2} + T_oT_1 + \frac{2T_1R_o/R_2}{n\omega_o}$$

$$= \frac{4\zeta T}{n\omega_o} + \frac{1}{\omega_o^2} + \frac{(2\zeta T - T_1)}{R_2C_o\omega_o^2} + T^2 + \frac{2T^2}{R_2C_on\omega_o}$$

$$D = (T_o + S_1 + T_1) + \frac{2(R_o + R_1)/R_2}{n\omega_o} + \frac{2}{n\omega_o}$$

$$= 2\zeta T + \frac{2(2\zeta T - T_1)}{R_2C_on\omega_o} + \frac{2}{n\omega_o} .$$

The transfer characteristic $B(p)$ of the phase lag network is:

$$B(p) = \frac{1}{T^2p^2 + 2\zeta Tp + 1} = \frac{1}{T_1T_op^2 + (T_o + S_1 + T_1)p + 1}$$

where $T_1 = R_1C_1$, $T_o = R_cC_o$, $S_1 = R_1C_o$. The transfer characteristic $B'(p)$ of the bridge "T" is:

$$B'(p) = \frac{\frac{1}{\omega_o^2}p^2 + \frac{2}{n\omega_o}p + 1}{\frac{1}{\omega_o^2}p^2 + \frac{2}{m\omega_o}p + 1} = \frac{T_2T_3p^2 + (T_2 + S_2)p + 1}{T_2T_3p^2 + (T_2 + S_2 + T_3)p + 1}$$

where $T_2 = R_2C_2$, $T_3 = R_3C_3$, $S_2 = R_2C_3$.

The transfer characteristic (3) is of the form (1) with $U_3 = U_4 = 0$. The denominator consists of a product of four linear factors:

$$(U_1^2p^2 + 2\zeta_1U_1p + 1)(U_2^2p^2 + 2\zeta_2U_2p + 1) = (\frac{\mu_1}{\omega_o}p + 1)(\frac{\mu_2}{\omega_o}p + 1)(\frac{\mu_3}{\omega_o}p + 1)\cdot(\frac{\mu_4}{\omega_o}p + 1).$$

(The phase lag ϕ_1 at ω_o due to a linear factor $(\frac{\mu_1}{\omega_o}p + 1)$ is given by $\tan \phi_1 = \mu_1$.) Since there are several combinations of products of pairs of the linear factors, U_1, ζ_1, U_2, ζ_2 of course are not unique for specified A,B,C,D. The relations between U_1,U_2, ζ_1, ζ_2 and A,B,C,D are:

$$A = U_1^2U_2^2$$

$$B = 2U_1U_2(\zeta_1U_2 + \zeta_2U_1)$$

$$C = 4\zeta_1\zeta_2U_1U_2 + U_1^2 + U_2^2$$

$$D = 2(\zeta_1U_1 + \zeta_2U_2)$$

The procedure now will be indicated for answering the following two questions: (1) Given a bridge "T", and a phase lag network either determined experimentally or designed by the method of a previous section to give the appropriate carrier phase shift, what are the values of U_1, ζ_1, U_2, ζ_2? (2) Given a bridge "T", what should be the values of the components of a phase lag network in order to realize a specified set of values of U_1, U_2, ζ_1, ζ_2 or A,B,C,D?

To answer (1), the values of A,B,C,D may be calculated by the above formulae, and the roots for $x = (p/\omega_o)$ of the equation

$$A\omega_o^4x^4 + B\omega_o^3x^3 + C\omega_o^2x^2 + D\omega_o x + 1 = 0$$

found by numerical methods. Each of the four quantities μ_1 in the factored form of the denominator then is given by $\mu_1 = -1/x_1$, where x_1 is one of the roots. The effect of the phase lag network on stability of the bridge "T" servo then may be determined by referring to Fig. 29 in the next section.

To answer (2), we solve the second of the above sets of expressions for A,B,C,D for T, ζ, $S_2 = R_2C_o$, $T_1 = R_1C_1$ in terms of A,B,C,D. Since the bridge "T" is given, R_2 is known, and S_2 determines C_o. Formulae for the design of an unloaded phase lag network to realize any T, ζ have been given in section 3; from the formula for R_1, it follows that the ratio $r = C_o/C_1$ may be found from T_1 by solving the quadratic equation:

$$T_1^2r^2 + (2T_1^2 - 2\zeta TT_1 + T^2)r + (T^2 - 2\zeta TT_1 + T_1^2) = 0.$$

The values of C_o and r determine C_1; and R_o is given by the formula of section 3 for R_o in terms of ζ, T,r, and C_o. Thus if T, ζ, S_2, T_1 to realize a given set of A,B,C,D are known, the phase lag network components R_1, R_o, C_1, C_o are determined.

The solutions for T, ζ, S_2, T_1 in terms of A,B,C,D are as follows:

$$T = \omega_o A^{1/2},$$

ζ is a root of $a\zeta^2 + b\zeta + c = 0,$

where

$$a = \left[\frac{8T^2}{m\omega_o^2}\left(\frac{4}{m} - \frac{4}{n} - n\right)\right]$$

$$b = \left[\frac{4nTD''}{m\omega_o^2} + \frac{16TB'}{mn} + \frac{2TC''}{\omega_o}\left(\frac{4}{n} - \frac{8}{m} + n\right)\right]$$

$$c = \left[2(C'')^2 - \frac{n}{\omega_o}C''D'' - \frac{4\omega_o}{n}B'C''\right]$$

$$C'' = C - \frac{1}{\omega_o^2} - T^2, \quad D'' = D - \frac{2}{m\omega_o}, \quad B' = B - \frac{2T^2}{m\omega_o};$$

$$T_1 = 2\zeta T + \frac{2T^2D'\omega_o}{(nD' - 2C'\omega_o)},$$

$$S_2 = \frac{2T^2D'}{(2C'\omega_o - nD')C'\omega_o} + \frac{2T^2}{nC'\omega_o}$$

where

$$C' = C'' - \frac{4\zeta T}{m\omega_o}, \quad D' = D'' - 2\zeta T.$$

6. <u>Multi-section Phase Lag Networks. Gain-Phase Margin Diagrams</u>

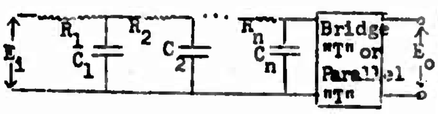


Fig. 28a

The transfer characteristic for an n-section R-C phase lag network followed by a bridge "T", or for a bridge "T" followed by an n-section phase lag network, is of the form:

$$\frac{E_o}{E_1} = \frac{\frac{1}{\omega_o^2}p^2 + \frac{2}{n\omega_o}p + 1}{(U_1p + 1)(U_2p + 1)\cdots(U_np + 1)(T_1p + 1)(T_2p + 1)}.$$

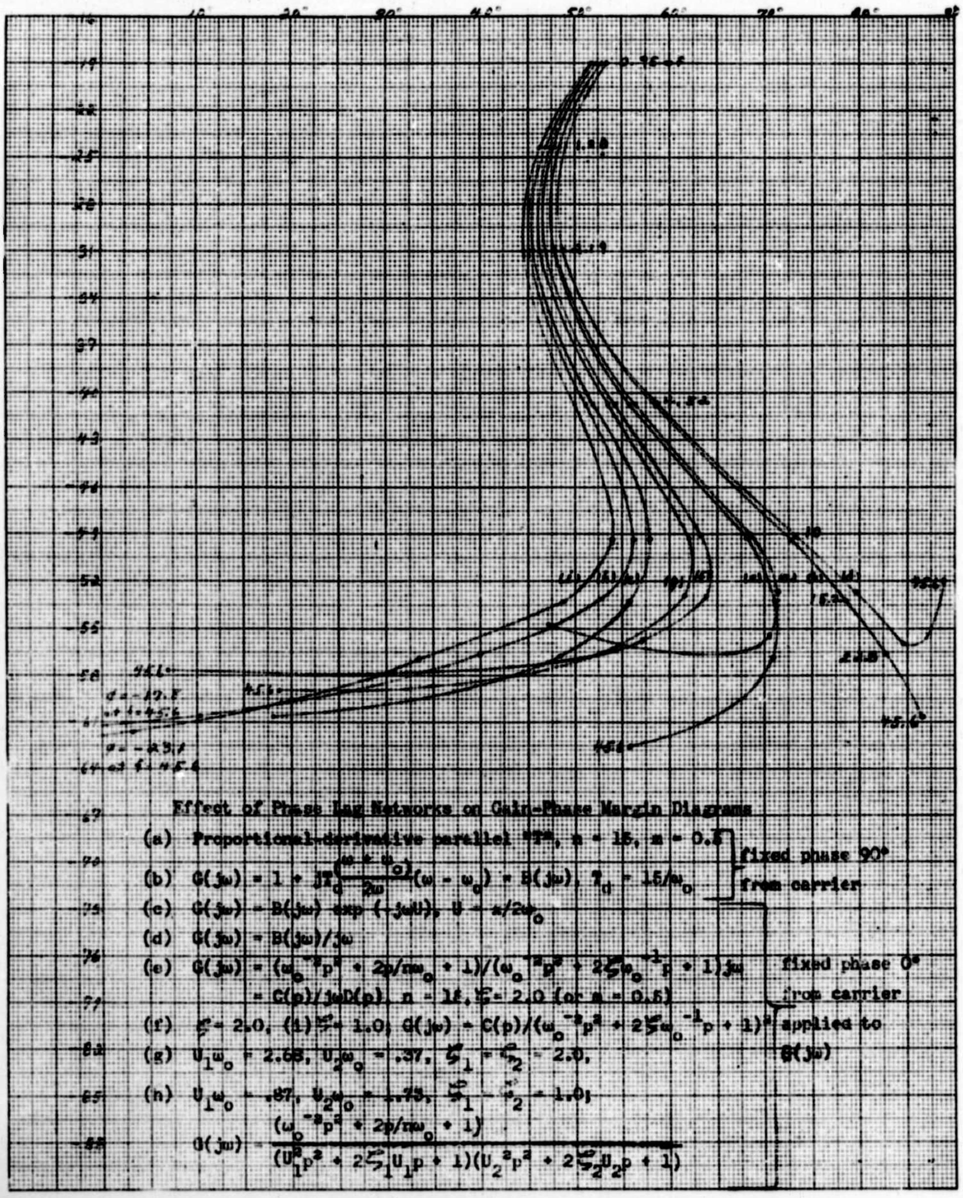
Excluding the proper proportional-derivative factor $(\omega_o^{-2}p^2 + 2p/n\omega_o + 1)$ $(n\omega_o/2p)$, the phase lag at any frequency ω of the remaining portion of the transfer characteristic is:

$$\phi = \Sigma \arctan U_1\omega + \arctan T_1\omega + \arctan T_2\omega - 90°.$$

Clearly, as $\omega \to \infty$, $\phi \to (n + 1)90°$, so that for a phase lag ϕ_o at ω_o, it is best to keep the number of sections n as small as possible, in order to minimize the change of phase with frequency in the vicinity of $\omega = \omega_o$. The frequency characteristic, on the other hand, is most flat, and the gain is largest, when the number of sections is large. For n R-C sections of equal time constant, as $n \to \infty$, the transfer characteristic $1/(Tp/n + 1)^n \longrightarrow \exp(-Tp)$. Thus the limiting frequency characteristic is $|\exp(-j\omega T)| = 1$ and the limiting phase lag is $\phi = \omega T$.

The gain-phase margin diagrams in Fig. 29, together with the M and E curves of Fig. 5, show the effect on error and stability of various methods of obtaining 90° phase lag, for a servo using the simplified motor with time constant $T = 0.2$ second. Curve (a) is the decibel-phase margin plot for the proportional-derivative parallel "T" transfer characteristic $(\omega_o^{-2}p^2 + 2p/n\omega_o + 1)/(\omega_o^{-2}p^2 + 2\zeta\omega_o^{-1}p + 1)$, where $n = T_d\omega_o = 15$, $\zeta = m^{-1} = 2.0$. (This is the same as curve (d) in Fig. 7.) Curve (b) is the decibel-phase margin plot for $G(j\omega) = 1 + jT_d \frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)$. (For curves (a) and (b) the fixed phase excitation is 90° out of phase with the carrier; for curves (c) through (i) the fixed phase excitation is in phase with the carrier applied to $G(j\omega)$.) Curve (c) is the plot for $G(j\omega) = [1 + jT_d \frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)]e^{-j\omega U}$, where $U = \pi/2\omega_o$. This corresponds to a 90° phase lag network having a very large number of sections and gain nearly 1. Curve (d) is the plot for $G(j\omega) = [1 + jT_d \frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)]\frac{1}{j\omega}$; curve (e) for $[1 + jT_d \frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)]/[1 + j\ell \frac{\omega + \omega_o}{2\omega}(\omega - \omega_o)]j\omega = (\omega_o^{-2}p^2 + 2p/n\omega_o + 1)/(\omega_o^{-2}p^2 + 2\zeta\omega_o^{-1}p + 1)j\omega$. The latter characteristic is approached by a single-section R-C network as its phase lag approaches 90° and its gain approaches 0. Curves (f) and (i) are the plots for $G(j\omega) = (\omega_o^{-2}p^2 + 2p/n\omega_o + 1)/(\omega_o^{-2}p^2 + 2\zeta\omega_o^{-1}p + 1)^2$, where $\zeta = 2.0$ for (f), $\zeta = 1.0$ for (i); and curves (g) and (h) for

$$G(j\omega) = \frac{(\omega_o^{-2}p^2 + 2p/n\omega_o + 1)}{(U_1^2p^2 + 2\zeta_1 U_1p + 1)(U_2^2p^2 + 2\zeta_2U_2p + 1)}$$



Effect of Phase Lag Networks on Gain-Phase Margin Diagrams
(a) Proportional-derivative parallel "T", n = 16, m = 0.5
(b) G(jω) = 1 + jT_d ((ω + ω_o)/2ω)(ω - ω_o) = B(jω), T_d = 16/ω_o
fixed phase 90° from carrier
(c) G(jω) = B(jω) exp (-jωU), U = π/2ω_o
(d) G(jω) = B(jω)/jω
(e) G(jω) = (ω_o^-2 p^2 + 2p/nω_o + 1)/(ω_o^-2 p^2 + 2ζω_o^-1 p + 1)jω
= C(p)/jωD(p), n = 16, ζ = 2.0 (or m = 0.5)
(f) ζ = 2.0, (1) ζ = 1.0, G(jω) = C(p)/(ω_o^-2 p^2 + 2ζω_o^-1 p + 1)^2
(g) U_1ω_o = 2.68, U_2ω_o = .37, ζ_1 = ζ_2 = 2.0,
(h) U_1ω_o = .87, U_2ω_o = 1.15, ζ_1 = ζ_2 = 1.0;
G(jω) = (ω_o^-2 p^2 + 2p/nω_o + 1) / (U_1^2 p^2 + 2ζ_1U_1p + 1)(U_2^2 p^2 + 2ζ_2U_2p + 1)
fixed phase 0° from carrier applied to B(jω)

FIGURE 29

where $U_1\omega_o = 2.682$, $U_2\omega_o = .373$, $\zeta_1 = \zeta_2 = 2.0$ for (g), and $U_1\omega_o = 3^{-1/2}$, $U_2\omega_o = 3^{1/2}$, $\zeta_1 = \zeta_2 = 1.0$ for (h). The first factor produces 60° phase lag, and the second factor 120° phase lag so that with the numerator lead of 90°, the net phase lag is 90°.

Part V

Tolerance Requirements For Parallel "T", Bridge "T", and Phase Lag Networks

In this part formulae are given for the variations (1) of parallel "T" resonant frequency ω_o, (2) of derivative time constant T_d (or, equivalently, of notch width $1/\pi T_d$), (3) of phase shift at carrier frequency, (4) of input and output impedance, in terms of the variations of the individual components of the networks from their correct values. From these formulae are derived percentage tolerances for the components. Gain-phase margin diagrams are given to show the effect of varying T_d, and of incorrect resonant frequency ω_o. The effect of incorrect carrier phase has been discussed in Part I.

1. Variation of Resonant Frequency ω_o of Parallel "T" Network

For any parallel "T", the numerator of the transfer characteristic is of the form:

$$\left(\frac{u}{\omega_1}p + 1\right)\left(\frac{1}{\omega_1^2}p^2 + \frac{2}{n\omega_1}p + 1\right)$$

for some ω_1, u, and n. As in section 1, Part IV, the relations between these quantities and the parallel "T" constants are:

$$\frac{u}{\omega_1^3} = T_1T_2T_3 = a$$

$$\frac{1}{\omega_1^2}\left(1 + \frac{2u}{n}\right) = T_1(S_2 + T_3) = b \qquad (1)$$

$$\frac{1}{\omega_1}\left(\frac{2}{n} + u\right) = (T_1 + S_1) = c$$

Eliminating u and n, we obtain:

$$a^2x^3 - acx^2 + bx - 1 = 0$$

where $x = \omega_1^2$. If $\Delta_i = dR_i/R_i$, $\varepsilon_i = dC_i/C_i$, we have

$$da = a(\Delta_1 + \Delta_2 + \Delta_3 + \varepsilon_1 + \varepsilon_2 + \varepsilon_3)$$

$$db = b\Delta_1 + T_1S_2\Delta_2 + T_1T_3\Delta_3 + b(\varepsilon_1 + \varepsilon_3)$$

$$dc = c\Delta_1 + T_1\varepsilon_1 + S_1\varepsilon_3$$

and

$$\begin{aligned} Qdx = \ & \Delta_1[(cx^2 - 2ax^3)a - bx + acx^2] \\ & + \Delta_2[(cx^2 - 2ax^3)a - xS_2T_1] \\ & + \Delta_3[(cx^2 - 2ax^3)a - xT_1T_3] \\ & + \varepsilon_1[(cx^2 - 2ax^3)a - bx + ax^2T_1] \\ & + \varepsilon_2[(cx^2 - 2ax^3)a] \\ & + \varepsilon_3[(cx^2 - 2ax^3)a - bx + ax^2S_1] \end{aligned}$$

where $Q = (3a^2x^2 - 2acx + b)$.

For the parallel "T" with equal condensers, section 3, Part II, $T_1 = S_1 = (2/n + u)/2\omega_o$, $T_2 = S_2 = 1/\omega_o(2/n + u)$, $T_3 = 1/u\omega_o$. Substituting these values and $\omega_1 = \omega_o$ in the expression for dx, we obtain:

$$\left(1 + \frac{2\sqrt{2}}{n}\right)\frac{d\omega_o}{\omega_o} = \left(1 - \frac{\sqrt{2}}{n}\right)(\Delta_1 + \Delta_2) + \Delta_3 + \left(\frac{5}{4} - \frac{1}{\sqrt{2}n}\right)(\varepsilon_1 + \varepsilon_3) + \left(\frac{1}{2} - \frac{\sqrt{2}}{n}\right)\varepsilon_2.$$

For fairly large n, this is approximately:

$$\frac{d\omega_o}{\omega_o} = \Delta_1 + \Delta_2 + \Delta_3 + \frac{5}{4}(\varepsilon_1 + \varepsilon_3) + \frac{1}{2}\varepsilon_2.$$

For 5% tolerance on resonant frequency ω_o of the parallel "T" with equal condensers, taking $\Delta_1 = \Delta_2 = \Delta_3 = \Delta$ and $\varepsilon_1 = \varepsilon_3 = \varepsilon$, $.05 = 3\Delta + 10\varepsilon/4 + \varepsilon_2/2$. Therefore in order to insure that ω_o will fall in the range $2\pi 57$ to $2\pi 63$ (see curves (b) and (c), Fig. 30), the appropriate tolerances on the parallel "T" components are $\pm$ 0.83% for R_1, R_2, R_3, $\pm$ 0.67% for C_1, C_3, and $\pm$ 1.67% for C_2. Uniform tolerance of $\pm$ 1% will insure that the resonant frequency will fall within a band of $\pm$ 6% from the correct value.

2. Variation of Notch Width of Parallel "T" Transfer Characteristic

The notch width is $1/\pi T_d = y/2\pi$, where $y = 2/T_d$. Eliminating ω_1 and u from the relations (1), we obtain:

$$y(b-ay)^2 - c(b-ay) + a = 0.$$

Differentiating, we have:

$$\begin{aligned} Rdy = {} & \Delta_1[2ay^2(b-ay)-acy - a + b(2ay^2 - 2by + c) + (b-ay)c] \\ & + \Delta_2[2ay^2(b-ay) - acy - a + T_1S_2(2ay^2 - 2by + c)] \\ & + \Delta_3[2ay^2(b-ay) - acy - a + T_1T_3(2ay^2 - 2by + c)] \\ & + \varepsilon_1[2ay^2(b-ay)-acy - a + b(2ay^2 - 2by + c) + (b-ay)T_1] \\ & + \varepsilon_2[2ay^2(b-ay) - acy - a] \\ & + \varepsilon_3[2ay^2(b-ay) - acy - a + b(2ay^2 - 2by + c) + (b-ay)S_1] \end{aligned}$$

where $R = b^2 + ac - 4aby + 3a^2y^2$.

For the parallel "T" with equal condensers, the expression for Rdy becomes:

$$\left(\frac{3}{2} - \frac{\sqrt{2}}{n}\right)\frac{dy}{\omega_o} = \frac{1}{\sqrt{2}}\Delta_1 + \left(\frac{2\sqrt{2}}{n^2} - \frac{2}{n} - \frac{1}{2\sqrt{2}}\right)\Delta_2 - \left(\frac{1}{n} + \frac{1}{2\sqrt{2}}\right)\Delta_3$$

$$+\left(\frac{1}{2\sqrt{2}} - \frac{1}{n}\right)(\varepsilon_1 + \varepsilon_3) - \left(\frac{1}{2\sqrt{2}} + \frac{2}{n} - \frac{2\sqrt{2}}{n^2}\right)\varepsilon_2 .$$

For fairly large n, this is approximately:

$$\frac{3}{2}\frac{dy}{\omega_o} = \frac{1}{\sqrt{2}}\Delta_1 + \frac{1}{2\sqrt{2}}(-\Delta_2 - \Delta_3 + \varepsilon_1 - \varepsilon_2 + \varepsilon_3).$$

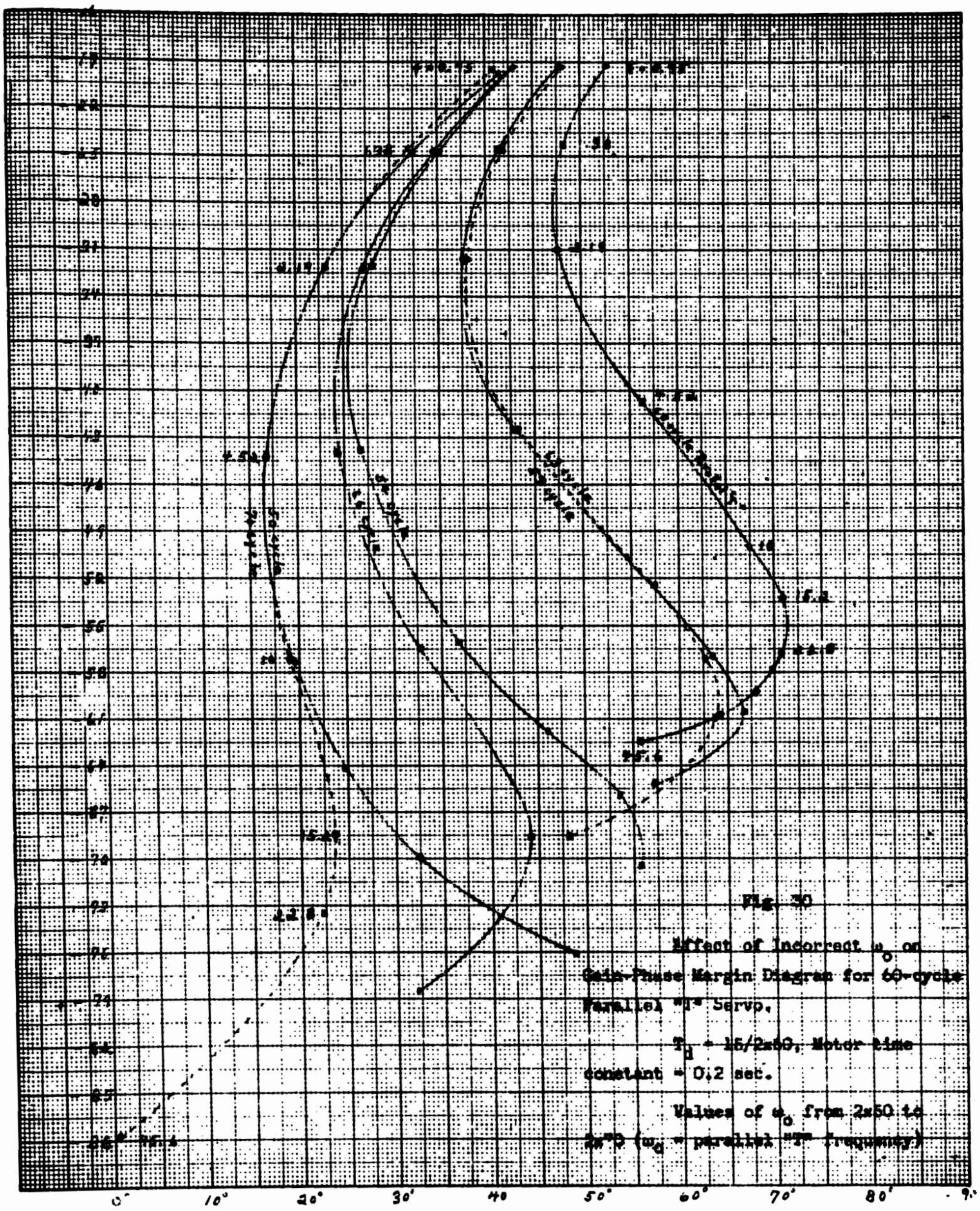


Fig. 30

Effect of Incorrect ω_o on Gain-Phase Margin Diagram for 60-cycle Parallel "T" Servo.

T_d = 15/2π60, Motor time constant = 0.2 sec.

Values of ω_o from 2π50 to 2π70 (ω_o = parallel "T" frequency)

Therefore if all components of the parallel "T" are correct within ± 1%, dy/ω_o will not exceed $7\sqrt{2}/6 = 1.65\%$ in numerical value. That is, the notch width will be within ± 1.65% of the carrier frequency of its correct value. For 60 cycle carrier frequency and ± 1% tolerances, the variation in notch width cannot exceed .0165 x 60 = 0.99 cps. (For $T_d\omega_o = 15$, the notch width then would be (8 ± .99) cps.; for $T_d\omega_o = 30$, (4 ± .99)cps.)

Fig. 31 shows the effect of variation of T_d (and also the effect of variation of motor time constant T).

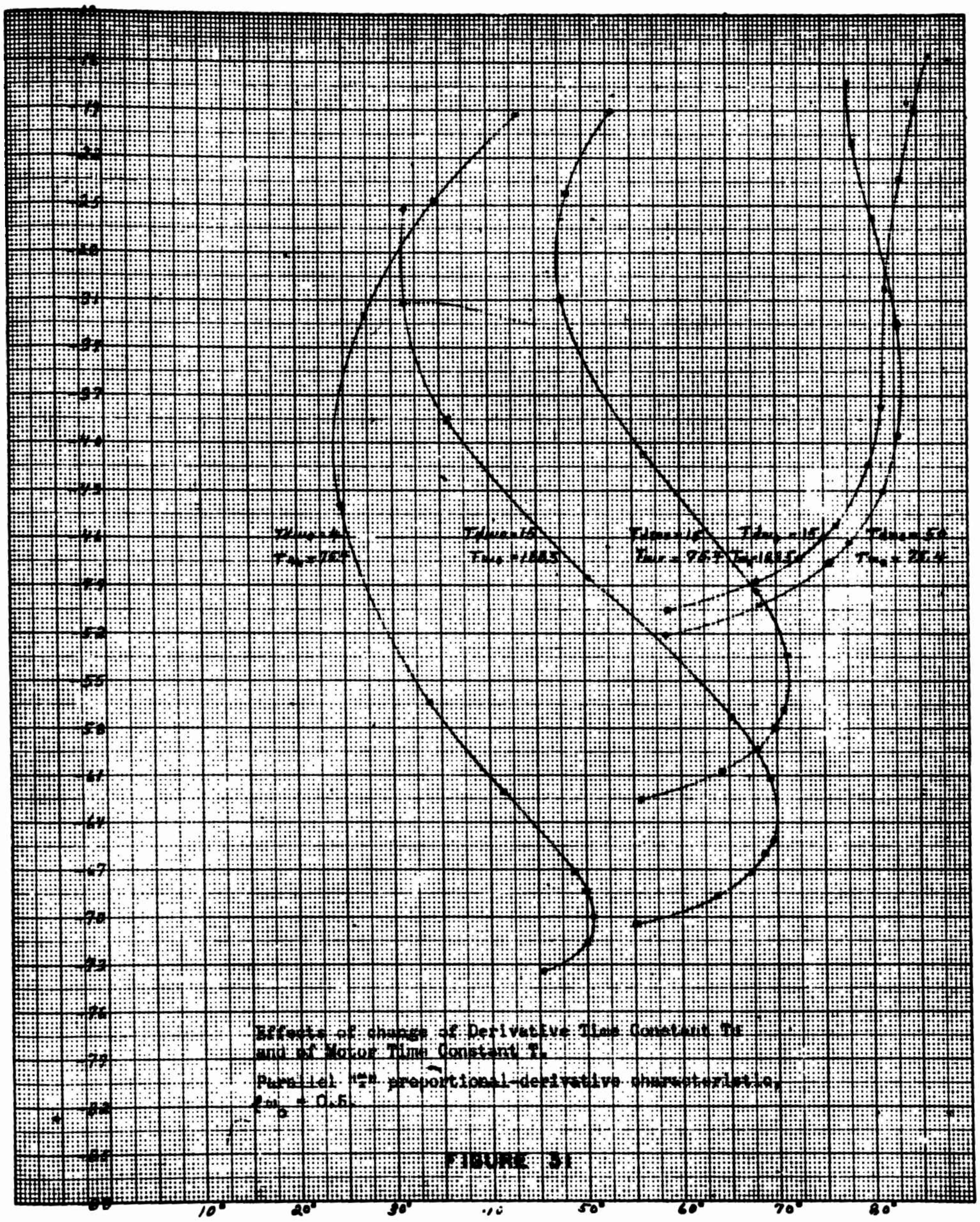
3. Variation of Phase Shift at Carrier Frequency

The phase shift at carrier frequency ω_o is

$$\phi = \arctan \frac{b\omega_o^2 - 1}{c\omega_o - a\omega_o^3} - \arctan \frac{(b + T_2T_3 + T_2c)\omega_o^2 - 1}{(c + T_2 + S_2 + T_3)\omega_o - a\omega_o^3} .$$

For fairly large n, for small variations of the parts from their correct values, the major part of the variation of ϕ is the variation of the first term; the second term is practically constant and equal to arc tan U. Accordingly for simplicity we calculate only the variation of the first term. Let the first term be denoted by ψ. Then we have:

$$\begin{aligned} A^2 d\tan\psi &= \Delta_1[\omega_o(c - a\,\omega_o^2)] \\ &+ \Delta_2[\omega_o^3(cT_1S_2 - a + aT_1T_3\omega_o^2)] \\ &+ \Delta_3[\omega_o^3(cT_1T_3 - a + a\,T_1S_2\omega_o^2)] \\ &+ \varepsilon_1[\omega_o(bS_1\omega_o^2 - a\omega_o^2 + T_1)] \\ &+ \varepsilon_2[a\omega_o^3(b\omega_o^2 - 1)] \\ &+ \varepsilon_3[\omega_o(bT_1\omega_o^2 - a\omega_o^2 + S_1)] \end{aligned}$$



Effects of change of Derivative Time Constant T_f and of Motor Time Constant T_L

Parallel "T" proportional-derivative characteristic, $\zeta_{\omega_0} = 0.5$.

FIGURE 31

where $A = (c\omega_o - a\omega_o^3)$. For the parallel "T" with equal condensers,

$$d \tan \psi = -\frac{n}{2}(\Delta_1 + \Delta_2) + (\frac{n}{2} + \frac{1}{\sqrt{2}})\Delta_3 + (\frac{5n}{8} + \frac{1}{2\sqrt{2}})(\varepsilon_1 + \varepsilon_3) + \frac{n}{4}\varepsilon_2,$$

and arc tan U = arc tan $2^{-1/2}$ = 35.26°. Since tan 45.26° = 1.0094 and tan 25.26° = .4720, the conditions for phase shift within ± 10° are $d \tan\psi <$ .3023 and $- .2351 < d \tan \psi$. Thus in case $n = T_d\omega_o = 15$, the appropriate tolerances are + .67%, - .52% for R_1, R_2; + .51%, - .48% for R_3; + .52%, - .40% for C_1, C_3; + 1.34%, - 1.05%, for C_2; or a uniform tolerance of + .65%, - .51% on all parts.

For the symmetric parallel "T", the corresponding conditions are $- .0572 < 1.433\,\Delta_1 + 1.311(\Delta_2 + \Delta_3) + 2.055(\varepsilon_1 + \varepsilon_3) + \varepsilon_2$

for $- 10° < \phi$,

and $.66\Delta_1 + 1.397\,(\Delta_2 + \Delta_3) + 1.227\,(\varepsilon_1 + \varepsilon_3) + \varepsilon_2 < .04$ for $\phi < + 10°$, which requires a uniform tolerance of + .58%, - .56% on all parts. (For n different than 15, the tolerances should be multiplied by approximately 15/n.)

For the resonant symmetric parallel "T" with perfect divider, the conditions are:

$$-45.7\Delta_1 - 54.16\Delta_2 - 53.3\Delta_3 - 59.3\varepsilon_1 - 33.72\varepsilon_2 - 60.2\varepsilon_3 < 1.72 \text{ for } - 10° < \phi,$$

$$22.4\Delta_1 + 40.3\Delta_2 + 40.9\Delta_3 + 36.7\varepsilon_1 + 30.8\varepsilon_2 + 36.1\varepsilon_3 < 1.2 \text{ for } \phi < 10°,$$

which requires the same uniform tolerance of + .58%, - .56% on all parts.

For the symmetric bridge "T", the corresponding conditions are:

$$- .0235 < \Delta_1 + \varepsilon_1 + \Delta_3 + \varepsilon_3 < .0235 \quad \text{for } -10° < \phi < 10°,$$

which requires a uniform tolerance of ± .59% on all parts. (For n different than 15, multiply tolerances by 15/n.)

For the general bridge "T", if ϕ is the phase shift at carrier frequency,

$$\tan \phi = -(\Delta_1 + \varepsilon_1 + \Delta_3 + \varepsilon_3)\frac{nr}{2(\frac{4}{n^2} + r)}$$

where $r = (C_1 + C_3)/C_1$ (R_1, C_1, R_3, C_3 are as in Fig. 14(a), Part III; C_1 is the condenser on the input side).

For the bridge "T" with values as in Fig. 32, $T_d\omega_o = 11$, and the conditions for phase shift at 60 cycles within $\pm$ 10° are:

$$-0.32 < \Delta_1 + \varepsilon_1 + \Delta_3 + \varepsilon_3 < 0.32,$$

which requires $\pm$ 0.8% tolerance on all parts, or for example $\pm$1.5% tolerance on C's if the R's are held to within $\pm$0.1%. The input impedance at 60 cycles in k is $Z_1 = 4.24 - 26.16j$. The fractional change of this impedance as a function of the variations of the components is:

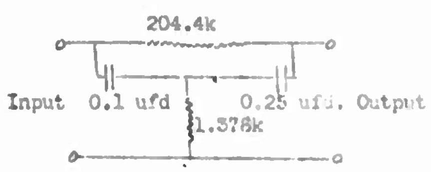


Fig. 32

$$\frac{\Delta Z_1}{Z_1} = (0.0211 + 0.0895j)\Delta_1$$

$$-(0.607 + 0.416j)\Delta_3$$

$$-(0.927 + 0.117j)\varepsilon_1 - (0.659 + 0.209j)\varepsilon_3.$$

For calculation of a phase lag network to precede this bridge "T", the input impedance at 60 cycles is replaced by an equivalent parallel condenser C = 0.099 ufd. and resistor R = 0.149 megohm. A network to give 120° phase lag and a gain of 0.293 to the bridge "T" input is as follows: input resistor R_2 = 4.767k, R_o = 27.2k, input condenser C_2 = 1.0 ufd., output condenser C' = 0.1 ufd. For this phase lag network, the tolerance requirements for phase shift of -120° correct to within $\pm$ 5° are:

$$-0.168 < 0.29\Delta + 0.85\,\Delta_o + 0.89\Delta_2 + 0.91\varepsilon_o + 0.95\varepsilon_2,$$

$$0.24\Delta + 0.43\Delta_o + 0.45\Delta_2 + 0.47\varepsilon_o + 0.51\varepsilon_2 < 0.121$$

where $C_o = C' + C$, $\varepsilon_c = \Delta C_c/C_o$, $\Delta = \Delta R/R$. If Z is the impedance of R and C in parallel, $Z = R/(T_j\omega + 1)$, where $T = RC$, and

$$\frac{dZ}{Z} = \frac{dR}{R} - \frac{T\omega^2 dT}{(1 + T^2\omega^2)} - \frac{j\omega dT}{(1 + T^2\omega^2)}$$

$$= - 4.45\Delta - 5.45\varepsilon - .985j(\Delta + \varepsilon).$$

From this and the above expression for $\Delta Z_1/Z_1$, assuming 0.6% tolerance on the bridge "T" components, it follows that $(\Delta C/C_o) = (a - 4.45b)/1.99$, $\Delta = (5.45b - a)$, where a and b in percent may have any values in the intervals $-.507 \leq b \leq +.507$, $-1.327 \leq a \leq +1.327$. Thus $\Delta C/C_o$ cannot exceed 1.8% in numerical value, and Δ cannot exceed 4.09%. (These are the bounds when a and b vary independently in the expressions for $\Delta C/C_o$ and $\Delta = \Delta R/R$; we do not pause for closer examination of the expressions and of the conditions for phase lag of 120° ± 5°, to determine the actual bounds.) Substituting $(\Delta C/C_o) = 1.8\%$ and $\Delta = 4.09\%$ in the conditions for phase lag of 120° ± 5°, we obtain the following tolerance conditions on the components of the phase lag network:

$$-.140 < 0.35\Delta_o + 0.39\Delta_2 + 0.457\varepsilon' + 0.95\varepsilon_2$$

$$0.43\Delta_o + 0.45\Delta_2 + 0.236\varepsilon' + 0.51\varepsilon_2 < .103 .$$

These conditions require a uniform tolerance of -4.45%, +6.33% on R_o, R_2, C_2, C'; or for example if R_o, R_2 are held to within 0.1%, the appropriate tolerances on C', C_2 are respectively -15.1%, +21.6% and -7.27%, + 10.0%. Thus if the bridge "T" components are held to ± 0.6%, to insure phase shift through the bridge "T" within ± 10°, these tolerances on the phase lag network components will insure that the phase lag through the network preceding the bridge "T" is 120° ± 5°.

Part VI

Analysis of the Parallel "T" Network

In this final part the method will be indicated by means of which the formulae of Part II were derived.

1. Transfer Characteristic and Input and Output Impedances

It is convenient to transform each "T" (or "Y") of the parallel "T" to its equivalent "π" (or "Δ") of impedances. The formulae for transformation from Δ to Y, and from Y to Δ, (see Fig. 33), are as follows:

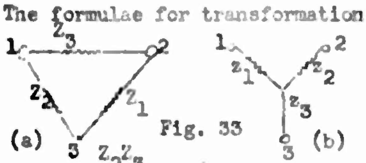


Fig. 33

$$z_1 = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3} \qquad Z_2 = \frac{z_1 z_2 + z_1 z_3 + z_2 z_3}{z_2}$$

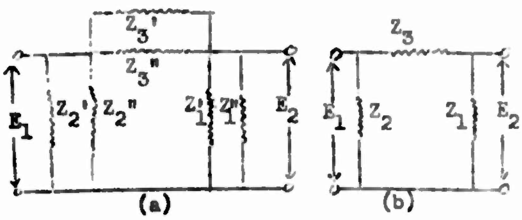
$$z_2 = \frac{Z_3 Z_1}{Z_1 + Z_2 + Z_3} \qquad Z_3 = \frac{z_1 z_2 + z_1 z_3 + z_2 z_3}{z_3}$$

$$z_3 = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3} \qquad Z_1 = \frac{z_1 z_2 + z_1 z_3 + z_2 z_3}{z_1}$$

These formulae are well-known. If unfamiliar to the reader, they may be established easily by a simple application of Kirchoff's laws.

In the parallel "T" as in Fig. 8, let $z_1' = R_2$, $z_2' = R_3$, $z_3' = 1/C_2p$; $z_1'' = 1/C_1p$, $z_2'' = 1/C_3p$, $z_3'' = R_1$.

Then after the transformation of each "T" to its equivalent "π", the parallel "T" network becomes the network shown in Fig. 34(a).



Networke Equivalent to the Parallel "T" Network

Fig. 34

For $k = 1, 2, 3$, let Z_k be the impedance of the parallel combination of Z_k' and Z_k''. Then it may be seen that

$$Z_2 = \frac{(T_2p + a)[(S_1 + T_1)p + 1]}{(T_2p + a)C_1p + [(S_1 + T_1)p + 1]C_2p}$$

$$Z_3 = \frac{R_3(T_2p + a)[(S_1 + T_1)p + 1]}{T_3T_1(T_2p + a)p^2 + [(S_1 + T_1)p + 1]}$$

$$Z_1 = \frac{(R_3C_2p + b)[(S_1 + T_1)p + 1]}{(R_3C_2p + b)C_3p + [(S_1 + T_1)p + 1]C_2p}$$

where the time constants are as previously defined, and $a = 1 + R_2/R_3$, $b = 1 + R_3/R_2$.

The impedance looking back into the output, with the input short-circuited, (formula (5), Part II), is the parallel combination of Z_3 and Z_1:

$$Z_o = \frac{R_3(T_2p + a)[(S_1 + T_1)p + 1]}{T_3p(T_1p + 1)(T_2p + a) + (T_2p + 1)[(S_1 + T_1)p + 1]}$$

$$= \frac{R_3\{T_2(S_1 + T_1)p^2 + [a(S_1 + T_1) + T_2]p + a\}}{T_1T_2T_3p^3 + [T_1(S_2 + T_3) + T_2T_3 + T_2(T_1 + S_1)]p^2 + (T_1 + S_1 + T_2 + S_2 + T_3)p + 1}$$

The input impedance with the output open-circuited, (formula (4), Part II), is the parallel combination of Z_2 and $(Z_3 + Z_1)$:

$$Z_i = \frac{T_1T_2T_3p^3 + [T_2(T_1 + S_1 + T_3) + T_1(T_3 + S_2)]p^2 + (T_1 + S_1 + T_2 + S_2 + T_3)p + 1}{(C_1T_2T_3 + C_2T_1T_3 + C_3T_1T_2)p^3 + [C_1(T_3 + T_2 + S_2) + C_2(T_3 + T_1 + S_1)]p^2 + (C_1 + C_2 + C_3)p}$$

The input impedance with any load similarly may be calculated as the parallel combination of Z_2 and $(Z_3 + \bar{Z}_1)$ where $\bar{Z}_1$ is the parallel combination of Z_1 and the load.

The transfer characteristic (formula (1), Part II) is:

$$\frac{E_2}{E_1} = \frac{Z_1}{Z_3+Z_1}$$

$$= \frac{T_1T_2T_3p^3 + T_1(S_2+T_3)p^2 + (T_1+S_1)p + 1}{T_1T_2T_3p^3 + [T_1(S_2+T_3) + T_2T_3 + T_2(T_1+S_1)]p^2 + (T_1+S_1+T_2+S_2+T_3)p + 1} \quad (1)$$

2. Determination of Constants to Realize the Proportional-Derivative Characteristic

The transfer characteristic may be written in factored form as follows:

$$\frac{E_2}{E_1} = \frac{(U_1p+1)(A_1^2p^2+B_1p+1)}{(U_2p+1)(A_2^2p^2+B_2p+1)} \quad (2)$$

where $U_1A_1^2 = U_2A_2^2 = T_1T_2T_3$. To obtain a proportional-derivative network at frequency ω_o, we take $U_1 = U_2 = U$, $A_1 = A_2 = A = 1/\omega_o$. Then if we let $B_1 = 2/T_d\omega_o^2$, $B_2 = 2/\ell\omega_o^2$, divide numerator and denominator of (2) by B_1p/ℓ, and replace p by $j\omega$, we obtain the characteristic:

$$\frac{E_2}{E_1} = \frac{\ell}{T_d}\,\frac{1+jT_d\frac{\omega+\omega_o}{2\omega}(\omega-\omega_o)}{1+j\ell\frac{\omega+\omega_o}{2\omega}(\omega-\omega_o)} = g\,\frac{1+jT_d\frac{\omega+\omega_o}{2\omega}(\omega-\omega_o)}{1+jgT_d\frac{\omega+\omega_o}{2\omega}(\omega-\omega_o)} \quad (3)$$

To solve for the constants in the parallel "T" network which will give a transfer characteristic of the form (3), we compare (2) with (1).
We thus obtain:

$T_1T_2T_3 = A^2U$ (a)

$T_1(S_2+T_3) = A^2 + B_1U$ (b)

$T_1 + S_1 = B_1 + U$ (c)

$T_2(T_3 + T_1 + S_1) = (B_2 - B_1)U$ (d)

$T_2 + S_2 + T_3 = B_2 - B_1$ (e)

Substituting from (a), (b), (c) into (d) and (e), we obtain $T_1 T_2 = U^2$. Then from (a), $T_3 = A^2/U$. Substituting the value of T_3 in (c) and (b), we are able to solve for T_2 and S_2. Then $T_1 = U^2/T_2$, and we solve (c) for S_1. Collecting results, we obtain the group of formulae (3) of Part II.

3. Proportional-Derivative Characteristic with Phase Shift of Carrier

If $T_d\omega_o = n$ is quite small (in the vicinity of 2 or 3), the upper bound arc tan 2/n will permit a useful amount of intrinsic phase shift. To obtain the proportional-derivative effect on the modulation, with phase shift of the carrier, the parallel "T" transfer characteristic should have the form:

$$\frac{E_2}{E_1} = \frac{Up + 1}{\alpha^{-2} Up + 1} \frac{(A^2p^2 + B_1p + 1)}{(A^2\alpha^{-2}p^2 + B_2p + 1)} = \left(\frac{\ell}{Td}\right) \frac{Uj\omega + 1}{\alpha^{-2} Uj\omega + 1} \frac{1 + jT_d \frac{\omega+\omega_o}{2\omega}(\omega-\omega_o)}{1 + j\ell \frac{\alpha\omega+\omega_o}{2\omega}(\alpha\omega-\omega_o)}$$

with $\alpha \neq 1$, where as before $A = 1/\omega_o$, $B_1 = 2/T_d\omega_o^2$, $B_2 = 2/\ell\omega_o^2$.

Comparing the above transfer characteristic with the parallel "T" characteristic (1), we have:

$$T_1T_2T_3 = A^2U$$

$$T_1(S_2 + T_3) = A^2 + B_1U$$

$$T_1 + S_1 = B_1 + U$$

$$T_2(T_3 + T_1 + S_1) = (\alpha^{-2}B_2 - B_1)U + A^2(\alpha^2 - 1)$$

$$T_2 + S_2 + T_3 = (\alpha^{-2} - 1)U + (B_2 - B_1).$$

Solving for T_1, T_2, T_3, S_1, S_2, we obtain the following formulae for determination of the constants in the parallel "T" network to give the phase-shifting proportional-derivative characteristic:

$$T_1 = \frac{B_1(A^2 + B_1 U + U^2)}{[(B_2 - B_1)B_1 - A^2(\alpha^2 - 1)] + (B_2 - B_1)(1 - \alpha^{-2})U + (\alpha^{-2} - 1)U^2}$$

$$T_2 = \frac{A^4(\alpha^2 - 1) + A^2[B_2(\alpha^{-2} - 1) + B_1(\alpha^2 - 1)]U + [B_1(\alpha^{-2}B_2 - B_1) - A^2(\alpha^{-2} - 1)]U^2}{B_1(A^2 + B_1 U + U^2)}$$

$$T_3 = \frac{A^2 U \left\{[(B_2 - B_1)B_1 - A^2(\alpha^2 - 1)] + (B_2 - B_1)(1 - \alpha^{-2})U + (\alpha^{-2} - 1)U^2\right\}}{A^4(\alpha^2 - 1) + A^2[B_2(\alpha^{-2} - 1) + B_1(\alpha^2 - 1)]U + [B_1(\alpha^{-2}B_2 - B_1) - A^2(\alpha^{-2} - 1)]U^2}$$

$$S_1 = B_1 + U - T_1$$

$$S_2 = (\alpha^{-2} - 1)U + (B_2 - B_1) - T_2 - T_3 .$$

Since, as was observed in preceding parts, the numerator of the parallel "T" transfer characteristic is invariant under reversal of input and output, when a phase-shifting proportional-derivative parallel "T" is reversed, it is still a phase-shifting proportional-derivative parallel "T" with the same T_d, but in general with a different gain and phase-shift.

A. Sobczyk
December 1, 1945

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13787

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ORIG. AGENCY NO.	R-811
PUBLISHING AGENCY NO.	(None)

TITLE: Parellel " T" Stabilizing Networks for AC Servos

AUTHOR(S): Sobczyk, A.
ORIGINATING AGENCY: Massachustts Institute of Technology, Cambridge, Mass.
PUBLISHED BY: Office of Scientific Research and Development, NDRC, Washington, D. C.

DATE	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
Dec ' 45	Unclass.	U.S.	Eng.	98	diagrs, graphs

ABSTRACT:

All the necessary equations for the parallel " T" stabilizing networks as used in AC servos are presented. The Diehl two-phase control motor is used in the servo-mechanism. Design formulas and tables of values for various forms of the parallel " T" network, of the bridge " T," and the Wien Bridge proportional derivative networks are developed from the simplest basic assumptions. Tolerance requirements on the components of the parallel " T," the bridge " T," and phase lag networks are given.

DISTRIBUTION: Copies of this report obtainable from Air Documents Division; Attn: MCIDXD

DIVISION: Electronics (3)
SECTION: Navigational Aids (3)

SUBJECT HEADINGS: Servos, Electrical (85000); Frequency stabilization (42150)

ATI SHEET NO.: R-3-3-28

TITLE: Part I - Carrier Frequency Systems

AUTHOR(S): Sobczyk, A.
ORIGINATING AGENCY: Massachusetts Institute of Technology, Cambridge, Mass.
PUBLISHED BY: (Same)

ATI- 13788

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DATE	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
Dec '45	Unclass.	U.S.	Eng.	17	diagrs, graphs

ABSTRACT:

A method of analysis of linear carrier frequency systems is indicated, and applied in particular to parallel "T" servos using the Diehl two-phase control motor. Improvements are described by the Nyquist Bode procedure for analysis of a feed-back amplifier or servo mechanism. The calculations are achieved by the Laplace transformation theory. Equations are developed for one-, two-, and three-phase motors. Time lags and their effects on the stability are discussed, for they have a direct effect of the calculations.

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DIVISION: Electronics (3)
SECTION: Components (10)

SUBJECT HEADINGS: Attenuators, Electronic (12850); Servos (84700); Frequency modulation systems (42144)

ATI SHEET NO.: R-3-10-93

~~[illegible]~~ 3

TITLE: Part II - Design Formulas for Parallel "T" Networks

AUTHOR(S): Sobczyk, A.

ORIGINATING AGENCY: Massachusetts Institute of Technology, Cambridge, Mass.

PUBLISHED BY: (Same)

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DIVISION (None)

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ABSTRACT:

Detailed design procedures and tables of values for various forms of parallel "T" networks are presented. The several forms arise from the fact that there are five time constants in the parallel "T," four of which are independent. The remaining degree of freedom may be used to obtain the most suitable input impedances for the source and load impedances with which the parallel "T" is to be used. If the constants are known, and if any one of the components is arbitrarily specified, then the other components may be determined.

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DIVISION: Electronics (3)

SECTION: Components (10)

SUBJECT HEADINGS: Attenuators, Electronic (12850); Impedance - Measurement (50927); Electric measurements (31490)

ATI SHEET NO.: R-3-10-97

ATI 13790

R-3-12-35

ORIG. AGENCY NUMBER

REVISION

AUTHOR(S): Sobczyk, A.

DIVISION: Electronics (3)

SECTION: Electronic Theory (12) 10

CROSS REFERENCES: Net works, Bridged parallel - T (66586)

AMER. TITLE: Part III - Bridge "T" networks

FORG'N. TITLE:

ORIGINATING AGENCY: Massachusetts Inst. of Technology, Radiation Lab., Cambridge

TRANSLATION:

COUNTRY	LANGUAGE	FORG'N.CLASS	U. S.CLASS.	DATE	PAGES	ILLUS.	FEATURES
U.S.	Eng.		Unclass.	Dec'45	6	3	diagrs

ABSTRACT

The two forms of the bridge "T" proportional-derivative networks may be obtained by assigning specific values to certain constants. From the formulas derived it is evident that any bridge "T" is reversible, but with different gains in the forward and reverse directions, unless it is symmetrical. The design formulas and tables of values for bridge "T" and Wien Bridge proportional-derivative networks are presented. A vacuum-tube circuit is used with the Wien Bridge in order to have a common input and output terminal, which may be grounded.

TSDIN FORM 60 (13 MAR 47)

Sobczyk, A. AUTHOR(S)	DIVISION: Electronics (3) SECTION: Components (10) CROSS REFERENCES: Phase shifters (70483)	ATI- 13791 ORIG. AGENCY NUMBER REVISION

AMER. TITLE: Part IV - Methods of obtaining required carrier phase shift

FORG'N. TITLE:

U

ORIGINATING AGENCY: Massachusetts Inst. of Technology, Radiation Lab., Cambridge

TRANSLATION:

COUNTRY	LANGUAGE	FORG'N.CLASS	U. S.CLASS.	DATE	PAGES	ILLUS.	FEATURES
U.S.	Eng.		Unclass.	Dec '45	44	7	diagr, graphs

ABSTRACT

Various method of obtaining the required 90° phase difference of the 60-cycle voltages on the fixed and control windings of the Diehl motor are discussed and compared. In order to obtain a large phase shift it is necessary to add either a series input or a load impedance to the parallel "T", or to use a phase-shifting network preceding or following the parallel "T". Formulas and tables of values for phase lag networks are given, which simplify the calculations to a great extent.

TITLE: Tolerance Requirements for Parallel "T", Bridge "T", and Phase Lag Networks	ATI- 13792
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ORIGINATING AGENCY: Massachusetts Institute of Technology, Cambridge, Mass.	ORIG. AGENCY NO. (None)
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Dec '45	Unclass.	U.S.	Eng.	9	graphs

ABSTRACT:

Formulas are given for the variations of parallel "T" resonant frequency of derivative time constant, of phase shift at carrier frequency, and of input and output impedance, in terms of the variations of the individual components of the networks from their correct values. From these formulas are derived percentage tolerances for the components. Gain-phase margin diagrams are given to show the effect of varying time constant, and of incorrect resonant frequency.

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DIVISION: Electronics (3)	SUBJECT HEADINGS: Attenuators, Electronic (12850); Networks, Bridged parallel - T (66586); Impedance - Measurement (50927)
SECTION: Components (10)	
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ABSTRACT:

The methods by means of which the design formulas for parallel "T" networks were derived are indicated. The transfer characteristic and output and input impedances are developed from an application of Kirchhoff's laws. The determination of constants to realize the proportional-derivative characteristic is accomplished by solving simultaneous equations. The numerator of the parallel "T" transfer characteristic is invariant under reversal of input and output, when a phase-shifting proportional-derivative parallel "T" is reversed, but the gain and phase-shifts vary.

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