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EIGHTH PROGRESS REPORT

on the S.2-4

DEVELOPMENT OF A DIRIGIBLE BOMB

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April 30, 1943

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EIGHTH PROGRESS REPORT

on the

DEVELOPMENT OF A DIRIGIBLE BOMB

SECRET

Contract No. OEM sr 240

April 30, 1943.

Contractor: Division of Industrial Cooperation
Massachusetts Institute of Technology
Cambridge, Mass.

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I. INTRODUCTION

Heretofore, the principal object in having a controlled bomb carry a motion picture camera in its nose has been to study the film for evidence of roll and to determine by qualitative examination whether the bomb went through any large yawing or pitching motions. Up to the present, all "quantitative" measurements of yaw, pitch, and roll, as we shall define them, were made with the full realization that the geometry of the measuring method was only a crude approximation to what it should be.

The object of the film analyzer shown in Fig. 3 is to determine the trajectory of the bomb in a set of rectangular coordinates and to determine the orientation of the bomb at any desired point on this trajectory. Having the above data, some of the types of questions that can then be answered are:

(1) Does the bomb fall with a certain pitch angle even though the elevators are set at zero? In other words, how stiff are the bombs now under development?

(2) What is the roll torque produced when the bomb is falling in combined pitch and yaw?

(3) How long does it take for the bomb to reach equilibrium yaw and pitch after the controls are applied to give it this yaw and pitch? And do these equilibrium

values of yaw and pitch agree with those determined by wind tunnel measurements? What is the motion of the bomb from the time the controls are applied to the time equilibrium is reached?

(4) Summarizing: In what respect does the bomb's motion depart from the motion as predicted by trajectory calculations? How should the equations of motion be modified to more accurately predict the motion at high speed? What are the observed values of lift, drag, and moment coefficients at high speed?

Thus, whereas the theodolite records give only the trajectory of the bomb, the film analyzer will, in addition, give bomb orientation. It is only by having both pieces of information that aero-dynamic characteristics at high speed can be determined.

It is the opinion of the M. I. T. group that film analysis, as herein described, is a very potent tool for the determination of aero-dynamic characteristics at high speed. Hence, the elementary aspects of film analysis theory and technique are covered in detail in this report in the hope that such information will aid in further use and development of the method.

II. PRINCIPLE OF THE FILM ANALYSIS METHOD

The film from the nose of the bomb is shown in a projector system capable of motion in the six degrees of freedom. The projected frame is made to coincide with the appropriate portion of a map of the bombing range by adjusting these six variables. When coincidence is obtained, the position and orientation of the projector in a coordinate system erected on the map will bear a known relationship to the position and orientation of the bomb in a coordinate system erected on the bombing range.

III. NECESSARY EQUIPMENT FOR FILM ANALYSES

The bomb under study should carry a motion picture camera in its nose, the optical axis of the camera being coincident with the longitudinal axis of the bomb. The larger the field of view of the camera, the lower is the error in the final trajectory and orientation data obtained. High shutter speeds are desirable, being most important near the end of the bomb flight when slow shutter speeds permit blurring of the film image. If a bomb were dropped from 15000 feet, a shutter speed of 8 frames per second would be satisfactory down to about 10000 feet. From 10000 to 5000, 16 frames per second; from 5000 to 1000, 32 frames per second; from 1000 feet to the ground, 64 frames per second. With the present use of spring-driven cameras, the upper limit to the frame speed is determined mainly by the length of time the camera will run at this speed on one winding. This speed is about 25 frames per second for a bomb released from 15000 feet. Since the frame speed goes down during the drop of a spring-driven camera and lighting of the object can change considerably and high image resolution is desirable, a wide-latitude, fine-grain emulsion should be used.

There should be incorporated into the camera system a method of timing each frame, and also a system whereby it can be determined at which frame the controls were applied and the amount of deflection of the controls. In the Gulf System, a stop-watch dial, indicator lights and/or dials,

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and the terrain are photographed simultaneously onto each frame. In the M. I. T. converted M-44 Bomb, a series of timing spots were laid along the margin of the film by a resistance-condenser-neon tube circuit carried in the bomb nose. The timing of the applications of the control was made in the plane, this method thus making impossible the determination of the lag between the time when the control signal was sent from the plane to the bomb and the time the bomb control surfaces went into motion.

The bombing range should be thoroughly covered by roads and other contrasting landmarks. This insures that sufficient detail will always be present to effect a satisfactory alignment.

The processed film is shown from a projector carried in gimbal frames permitting the six degrees of freedom, the projector carrying a lens having approximately the same focal length as the camera in the bomb. Roughly, the greater the differences in the focal lengths, the greater will be the probable error in the trajectory and orientation data.

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IV. GEOMETRICAL CONSIDERATIONS IN PHOTOGRAPHING THE GROUND FROM THE NOSE OF THE BOMB AND IN ALIGNING THE PROJECTED PICTURE WITH A MAP OF THE GROUND

A. Photographing the Ground.

Assuming simple lens theory, let us examine the geometry in a plane determined by the vertical to the ground through the lens center and the optic axis of the lens system. Assume a distance on the ground "a" lying ahead of the axis, and a distance "e" behind. Let the altitude of the lens center be Z_c and let the optic axis make an angle θ with the ground, the distance from the lens center to the ground, measured along the axis, being u_c. Let the image of "a" on the film be c_f and the image of "e" on the film be b_f. See Fig. 1. If the object is very far away from the camera, then the image distance is very nearly f_c, where f_c is the focal length of the camera. Then:

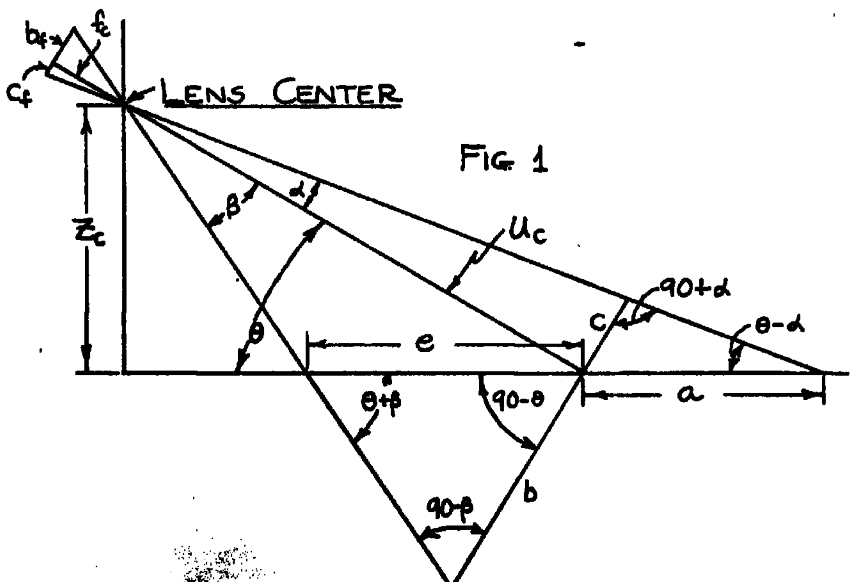


FIG 1

$$C_f = f_c \tan\alpha \qquad (IV-1)$$

$$b_f = f_c \tan\beta \qquad (IV-2)$$

$$b = U_c \tan\beta \qquad C = U_c \tan\alpha \qquad (IV-3, 4)$$

FROM THE LAW OF SINES:

$$\frac{e}{\sin(90-\beta)} = \frac{b}{\sin(\theta+\beta)} = \frac{e}{\cos\beta} \qquad (IV-5)$$

COMBINING (5) AND (3):

$$e = \frac{U_c \sin\beta}{\sin(\theta+\beta)} \qquad (IV-6)$$

ALSO:

$$\frac{a}{\sin(90+\alpha)} = \frac{c}{\sin(\theta-\alpha)} \qquad (IV-7)$$

COMBINING (4) AND (7)

$$a = \frac{U_c \sin\alpha}{\sin(\theta-\alpha)} \qquad (IV-8)$$

B. <u>The Projector Setup</u>.

We have a similar diagram. See Fig. 2. Let the scale of the map be "s". Thus a distance "d" on the ground is of length $\frac{d}{s}$ on the map. Let the projector focal length be f_p, image distance be v_p, and object distance be u_p.

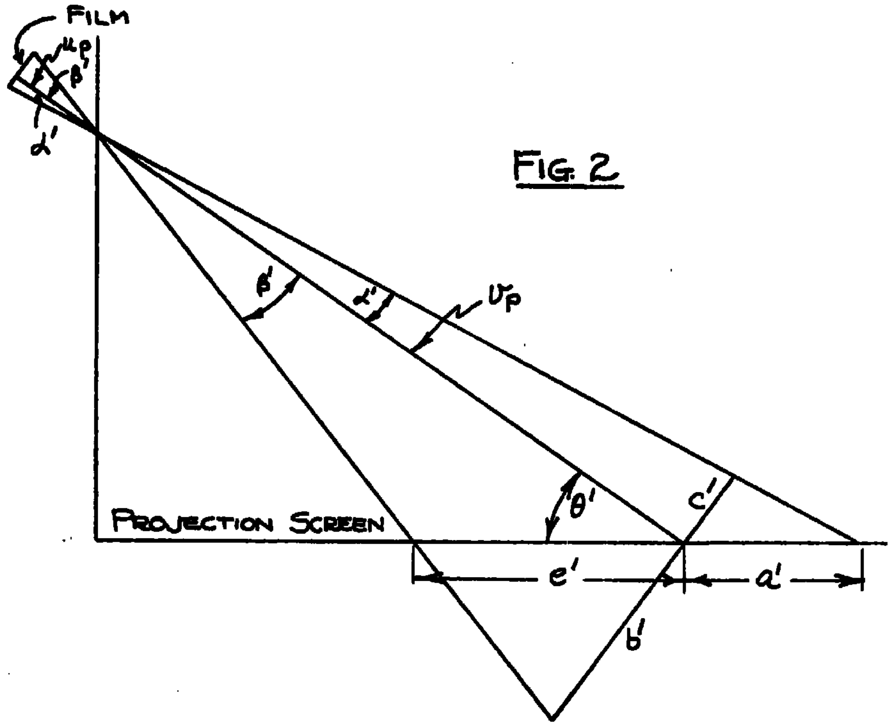


FIG. 2

THUS:

$$C' = v_p \tan \alpha' \qquad \text{(IV-9)}$$

$$a' = \frac{v_p \sin \alpha'}{\sin(\theta' - \alpha')} \qquad \text{(IV-10)}$$

$$b' = v_p \tan \beta' \qquad \text{(IV-11)}$$

$$e' = \frac{v_p \sin \beta'}{\sin(\theta' + \beta')} \qquad \text{(IV-12)}$$

[redacted]

Now in operating the film analyzer, we adjust our six variables until the projected picture is superimposed in exact alignment, as nearly as we can determine, on the appropriate portion of the map. Hence for the "a" and "e" lines our conditions of match become:

$$a' = \frac{a}{S} \qquad e' = \frac{e}{S}$$

HENCE FROM (10) AND (12):

$$e = \frac{S\,U_p \sin\beta'}{\sin(\theta' + \beta')} \qquad a = \frac{S\,U_p \sin\alpha'}{\sin(\theta' - \alpha')} \qquad \text{(IV-13,14)}$$

WE THEN EQUATE (13) TO (6) AND (14) TO (8):

$$\left.\begin{aligned} \frac{S\,U_p \sin\beta'}{\sin(\theta' + \beta')} &= \frac{U_c \sin\beta}{\sin(\theta + \beta)} \\ \frac{S\,U_p \sin\alpha'}{\sin(\theta' - \alpha')} &= \frac{U_c \sin\alpha}{\sin(\theta - \alpha)} \end{aligned}\right\} \text{CONDITIONS FOR "RANGE" MATCH} \qquad \begin{aligned} &\text{(IV-15)} \\ &\text{(IV-16)} \end{aligned}$$

Thus if we consider only the conditions for match in <u>range</u> we solve equations (15) and (16) simultaneously for θ, the glancing angle at which the frame was taken, in terms of θ', the glancing angle as read on the gimbals. Dividing (15) by (16):

$$\frac{\sin\beta'}{\sin\alpha'} \cdot \frac{\sin(\theta' - \alpha')}{\sin(\theta' + \beta')} = \frac{\sin\beta}{\sin\alpha} \cdot \frac{\sin(\theta - \alpha)}{\sin(\theta + \beta)}$$

Thus if $\alpha' = \alpha$ and

$\beta' = \beta$

then $\theta = \theta'$ for all values of θ'

Since $\tan \alpha' = \frac{c_f}{u_p}$ and $\tan \alpha = \frac{c_f}{f_c}$

then for α' to equal α:

$u_p = f_c$, which, as can be seen by similar treatment, is also the condition for β' to equal β.

Thus, if we consider match in "range" alone and since u_p varies during our film analysis to maintain focus on the screen, we draw the following conclusions:

1. We can get exact alignment for, at best, one position of the analyzer, that is when $u_p = f_c$. All other alignments will be approximate, the operator in someway distributing the distortions so as to give a "best" fit, or distributing the distortions in a predetermined manner.

2. There are three ways of getting around this difficulty of inherent mis-fit:

a. Use a projector lens having a tremendous depth of focus -- in the limit, a pin-hole projector. In this way u_p can be equated to f_c and held fixed; thus the projected pictures will always be in focus regardless of the projector-to-map distance.

b. The operator, in aligning the projector, can distribute the distortions in a way that permits an accurate calculation of θ through equations (15) and (16). This method will be discussed in detail later.

c. The error in Θ can be made negligible in comparison with other inherent errors by proper choice of the focal length of the projector. This method will also be discussed later.

Let us now assume there is another line of length "ℓ" on the ground, perpendicular to the "a" - "e" line and intersecting the optic axis of the camera.

Then, the way "ℓ" will photograph on the film will not depend on Θ, and will be of length $(\ell \cdot \frac{f_c}{u_c})$ on the film. In projection, this object on the film is magnified by the factor $(\frac{v_p}{u_p})$. Hence for "lateral" match:

$$\left[\ell \cdot \frac{f_c}{u_c}\right] \cdot \left[\frac{v_p}{u_p}\right] = \frac{\ell}{s} \quad \text{OR}$$

$$s f_c = \frac{u_c u_p}{v_p} \qquad \text{(IV-17)}$$

SINCE $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$u_c = s f_c v_p \left[\frac{1}{f_p} - \frac{1}{v_p}\right] \qquad \text{(IV-18)}$$

$$\text{NOW } \tan\alpha' = \frac{C_f}{u_p} = \frac{f_c \tan\alpha}{u_p} = f_c \tan\alpha \left(\frac{1}{f_p} - \frac{1}{v_p}\right) = \Delta \qquad \text{(IV-19)}$$

$$\alpha' = \tan^{-1}\left[f_c \tan\alpha \left(\frac{1}{f_p} - \frac{1}{v_p}\right)\right] \qquad \text{(IV-20)}$$

$$\sin\alpha' = \frac{f_c \tan\alpha \left(\frac{1}{f_p} - \frac{1}{v_p}\right)}{\sqrt{1 + f_c^2 \tan^2\alpha \left(\frac{1}{f_p} - \frac{1}{v_p}\right)^2}} \qquad \text{(IV-21)}$$

SUBSTITUTING (18), (19), (20), AND (21) IN (16) WE GET:

$$\frac{\sin\left[\theta' - \tan^{-1}\Delta\right]\sqrt{1+\Delta^2}}{\tan\alpha} = \frac{\sin(\theta-\alpha)}{\sin\alpha}$$

Expanding the above and simplifying:

$$\sin\theta' - \Delta\cos\theta' = \sin\theta - \tan\alpha\cos\theta \quad \text{(IV-22)}$$

which gives the relationship between θ' and θ in order to have <u>simultaneously</u> an exact <u>range</u> match for a particular point in the <u>upper</u> half of the picture and exact <u>lateral</u> match through the optic axis. Since $\theta' \cong \theta$ and Δ is never greater than about .1, Eq. (22) becomes:

$$\sin\theta' \cong \sin\theta - (\tan\alpha - \Delta)\cos\theta$$

or $$\sin\theta' \cong \sin\theta - \left[1 - f_c\left(\frac{1}{f_p} - \frac{1}{v_p}\right)\right]\tan\alpha\cos\theta \quad \text{(IV-23)}$$

From Eq. (23) it is apparent:

1. θ' is always less than θ for $f_c < f_p$ and practical values of v_p.

2. That the error in θ <u>increases with α</u>. (Unfortunately, we want a large α, since this insures plenty of ground detail.)

3. That the error in θ <u>decreases with increase in θ</u>. Hence, the percentage error is usually greater at high altitudes where the bomb is more nearly horizontal.

4. If $f_c = f_p$, the error in θ will decrease with increase in v_p.

Equation (22) indicates that the error in θ will be kept to a minimum if the projector focal length is slightly less than the camera focal length. Thus from equation (22), for θ' to equal θ:

$$v_p = \frac{f_c f_p}{f_c - f_p}$$

If our total range in v_p is from 10 inches to 100 inches and if f_c = 2.000" and f_p = 1.802" then for θ = 45 deg.:

θ' = 44.3 deg. at v_p = 10"

$\theta' = \theta$ = 45 deg. somewhere between v_p = 10" and 100"

θ' = 45.8 deg. at v_p = 100"

} Assuming $\Delta \cong .1$

Thus we can keep our error in θ' at a minimum by the proper choice of the focal length of the projector.

To get an idea of how far θ' is off from θ for the M. I. T. converted M-44 bombs, let us solve equation (23) for the following representative data:

θ = 43.8 deg. α = 7 deg. (α_{max} = 11 deg.)

f_c = 1" f_p = 2" v_p = 46.8"

Thus, $\sin\theta' = .692 - \left[1 - \left(\frac{1}{2} - \frac{1}{46.8}\right)\right] \times .1228 \times .722 = .646$

Therefore, θ' = 40.1 deg. Thus the operator would set the projector at a smaller glancing angle than that at which the picture was taken. If this difference in the θ's were not taken into account in computing the trajectory, a considerable error in altitude would result.

By a similar treatment using equations (18), (15), and the β' equivalent of (20), we get the relationship between θ' and θ in order to have <u>simultaneously</u> an exact range match for a particular point in the lower half of the picture and exact lateral match through the optic axis:

$$\sin \theta' + \nabla \cos \theta' = \sin \theta + \tan \beta \cos \theta \qquad \text{(IV-24)}$$

or: $$\sin \theta' \cong \sin \theta + \left[1 - f_c\left(\frac{1}{f_p} - \frac{1}{v_p}\right)\right] \tan \beta \cos \theta \qquad \text{(IV-25)}$$

where $\nabla = f_c \tan \beta\left(\frac{1}{f_p} - \frac{1}{v_p}\right) = \tan \beta'$

(24) and (25) are similar to (22) and (23) but demand that θ' always be larger than θ, contrary to (22) and (23) which demand that θ' always be smaller than θ for $f_c < f_p$.

Hence we have three inconsistent relationships between θ' and θ:

A. For simultaneous fit in upper and lower range:

$$\frac{\sin \beta'}{\sin \alpha'} \cdot \frac{\sin (\theta'-\alpha')}{\sin (\theta'+\beta')} = \frac{\sin \beta}{\sin \alpha} \cdot \frac{\sin (\theta-\alpha)}{\sin (\theta+\beta)}$$

B. For simultaneous fit in upper range and lateral distance:

$$\sin \theta' - \Delta \cos \theta' = \sin \theta - \tan \alpha \cos \theta$$

$$\left[\Delta = f_c \tan \alpha \left(\frac{1}{f_p} - \frac{1}{v_p}\right)\right]$$

C. For simultaneous fit in lower range and lateral distance:

$$\sin \theta' + \nabla \cos \theta' = \sin \theta + \tan \beta \cos \theta$$

$$\left[\nabla = f_c \tan \beta \left(\frac{1}{f_p} - \frac{1}{v_p}\right)\right]$$

Hence, in performing an alignment on the film analyzer, the operator attempts to get the "best" solution from the above three inconsistent relationships, and he has essentially but two variables by which to get this fit: θ' and v_p.

C. <u>Methods of Getting a "Best" Solution</u>:

Suppose our alignment is made at θ^* and v_p^*, then from Equations (6) and (12)

$$\frac{\dfrac{v_p^* \sin\beta'}{\sin(\theta^*+\beta')} - \dfrac{u_c \sin\beta}{s \sin(\theta+\beta)}}{\dfrac{v_p^* \sin\beta'}{\sin(\theta^*+\beta')}} = 1 - \frac{u_c \sin\beta \sin(\theta^*+\beta')}{s \sin(\theta+\beta)\, v_p^* \sin\beta'} \qquad \text{(IV-26)}$$

= fractional error in <u>lower range</u> match.

From Equations (3) and (10)

$$\frac{\dfrac{v_p^* \sin\alpha'}{\sin(\theta^*-\alpha')} - \dfrac{u_c \sin\alpha}{s \sin(\theta-\alpha)}}{\dfrac{v_p^* \sin\alpha'}{\sin(\theta^*-\alpha')}} = 1 - \frac{u_c \sin\alpha \sin(\theta^*-\alpha)}{s \sin(\theta-\alpha)\, v_p^* \sin\alpha'} \qquad \text{(IV-27)}$$

= fractional error in <u>upper range</u> match.

Now for a lateral image ℓ_f on the film, the projected image is: $\ell_p^* = \ell_f \dfrac{v_p}{u_p}$

and the correct projected image is:

$$\ell_p = \ell_f \frac{v_p}{u_p}$$

And the coincident figure on the map is given by:

$$\frac{\ell}{s} = \frac{\ell_f}{s}\frac{u_c}{f_c}$$

Thus:

$$\frac{\ell_f \dfrac{v_p^*}{u_p^*} - \ell_f \dfrac{v_p}{u_p}}{\ell_f \dfrac{v_p^*}{u_p^*}} = 1 - \frac{v_p\left(\dfrac{1}{f_p} - \dfrac{1}{v_p}\right)}{v_p^*\left(\dfrac{1}{f_p} - \dfrac{1}{v_p^*}\right)} \qquad \text{(IV-28)}$$

= fractional error in <u>lateral</u> match.

Now for a given θ^* and v_p^* these errors do not all have the same sign, hence, one of the possible methods for a best fit would require the <u>magnitudes</u> of these errors to be the same. Using equation (16), we get:

$$\left| 1 - \frac{v_p\left(\frac{1}{f_p} - \frac{1}{v_p}\right)}{v_p^*\left(\frac{1}{f_p} - \frac{1}{v_p^*}\right)} \right| = \left| 1 - \frac{f_c v_p\left(\frac{1}{f_p} - \frac{1}{v_p}\right) \sin\beta \, \sin(\theta^* + \beta)}{\sin(\theta + \beta)\, v_p^* \, \sin\beta'} \right|$$

$$= \left| 1 - \frac{f_c v_p\left(\frac{1}{f_p} - \frac{1}{v_p}\right) \sin\alpha \, \sin(\theta^* - \alpha)}{\sin(\theta - \alpha)\, v_p^* \, \sin\alpha'} \right| \qquad (IV-29)$$

Using some data from the film analysis of the M. I. T. converted K-44 let us compute the distortions accompanying some of the possible "best" alignments.

Let $\alpha = \beta = 7$ deg. $\alpha' = \beta'$ $f_c = 1"$ $f_p = 2"$ $v_p = 46.80"$ $\theta = 43^{\circ}48'$ where the above v_p and θ would be the observed values on the analyzer system, provided a perfect fit were possible.

Case 1. The analyzer is adjusted to the correct value of v_p and θ. Hence $v_p^* = v_p = 46.80"$ and $\theta^* = \theta = 43^{\circ}48'$.

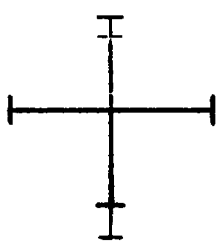
Error in lateral direction = $\epsilon_{LA} = 0\ \%$

Error in lower range fit = $\epsilon_{LO} = +\ 5.92\ \%$

Error in upper range fit = $\epsilon_{UP} = -\ 7.64\ \%$

and since $Z_c = s\ f_c\ v_p\left[\frac{1}{f_p} - \frac{1}{v_p}\right] \sin\theta$

letting s = 10400 our altitude would be computed as 13 437'. Then if the blue lines represent detail on the map and the red lines our projected picture our alignment would appear as follows:



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Case II. We diminish v_p and vary θ' until the magnitudes of the errors by equation (29) are the same.

Thus for $v_p^* = 44.27"$ $\theta^* = 40°24'$

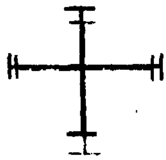
$\epsilon_{LL} = -5.99\%$

$\epsilon_{LO} = +5.99\%$

$\epsilon_{UP} = -5.99\%$

$Z_c = 11872'$

And the alignment would look like this:



Case III. We increase v_p and vary θ' until the magnitudes of the errors by equation (29) are the same.

Then for $v_p^* = 50.35"$ and $\theta^* = 47°50'$

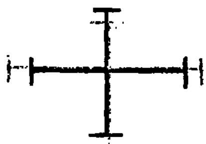
$\epsilon_{LL} = 7.34\%$

$\epsilon_{LO} = 7.34\%$

$\epsilon_{UP} = -7.34\%$

$Z_c = 15529'$

And the alignment would look like this:



Case IV. We keep v_p equal to its theoretically correct value but adjust θ' so that there is simultaneously an exact fit in lower range and lateral distance.

Then for $v_p^* = 46.80"$ $\theta^* = 47°51'$

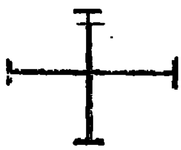
$\epsilon_{LL} = 0\%$

$\epsilon_{LO} = 0\%$

$\epsilon_{UP} = -16.3\%$

$Z_c = 14393'$

And the alignment would look like this:



Case V. We keep v_p equal to its theoretically correct value but adjust θ' so there is simultaneously an exact fit in upper range and lateral distance.

Then for $v_p^* = 46.80"$ $\theta^* = 40^\circ 25'$

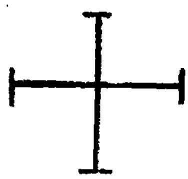
$\epsilon_{LA} = 0\%$

$\epsilon_{LO} = 11.23\%$

$\epsilon_{UP} = 0\%$

$Z_c = 12586'$

And our alignment would look like this:



D. Conclusions on the Geometrical Considerations

From the foregoing treatment, it is apparent that it is both desirable and practicable to adjust the focal lengths of the camera and of the projector to make negligible the uncertainty in θ and hence in v_p. From the calculations on page 13, it can be seen that the uncertainty in θ can be held to less than ½ degree throughout the range of v_p's and of θ's found from the analyzer as now designed. From the experience at M. I. T. with the analyzer, uncertainties of this magnitude are probably within the human errors introduced by the operator when he calls an alignment satisfactory when it theoretically is not. The proper relationship between the focal lengths would considerably speed up an analysis since the operator could always find a good alignment, while if there is a great difference in the focal lengths the operator searches a long time for a good alignment which is inherently non-existent.

In case the proper focal length projector is not available the best thing to do would be to perform each alignment the same, choosing one of the five cases above as an example. Thus, even though the values of θ and v_p obtained were not the correct values, the correct values could be obtained through the equations associated with the particular case used.

[redacted]

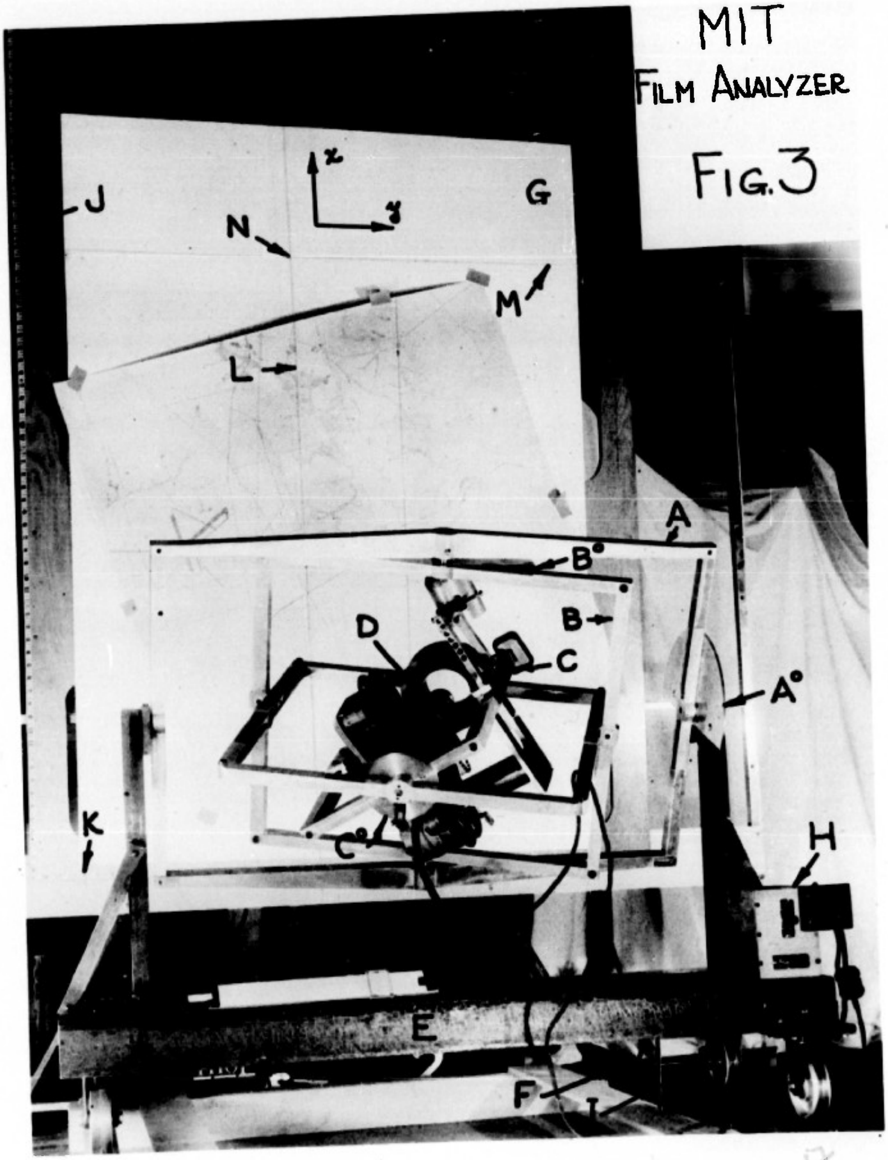
V. DESCRIPTION AND OPERATION OF M. I. T. ANALYZER

A. Description

See Fig. 3. The projector is carried on three gimbal frames, (A), (B), and (C), the axes of which intersect the optic axis of the projector in the plane of the film (D). Each gimbal carries a protractor, A°, B°, C°, by which bomb orientation can be determined.

The outer gimbal (A) is supported on a truck (E) which is pushed back and forth on tracks (F) by the operator. The tracks are perpendicular to the plane of the projecting screen (G), which carries the map of the bombing range. The screen is capable of motion in the x and y directions, being driven by motors controlled from panel (H) on the gimbal truck.

The gimbal truck carries a pointer (I) which runs along a tape divided into feet and inches, the tape being parallel to the tracks. The projecting screen carries two such tapes (J) and (K), one along the left edge for measuring x displacements, and one along the bottom edge for measuring y displacements. The vertical tape is parallel to the x-motion of the screen and also to the fixed string (L). The horizontal tape is parallel to the y motion of the screen and also to the fixed string (M). Thus, the intersection of the horizontal string (M) with the vertical tape (J) indicates the x displacement, and the intersection of the vertical string (L) with the



MIT
FILM ANALYZER
FIG. 3

[redacted]

horizontal tape (K) gives a reading for y displacement. Although the line along which (D) moves, the projector Z motion, does not intersect the string origin (N), the two strings and the line along which (D) moves, Z, form a fixed, rectangular coordinate system in which displacements are conveniently measured.

The particular form of gimbal system used was chosen because the roll angle could be read directly off the C^o protractor. As will be shown in the Ninth Progress Report, the directional gyro system used in both the Gulf and M.I.T. bombs requires, for a bomb in roll equilibrium, that the line perpendicular to the longitudinal axis of the bomb and lying in the plane of the vertical fins always lie in the plane represented by the vertical fins at the instant of release. With the usual method of release, this plane is coincident with the plane determined by the line perpendicular to the earth and the line represented by the bomb velocity vector at the instant of release.

The intersection of the three gimbals is capable of being moved from a position about 120" in front of the screen to within about 8" of the screen. The total motion of the screen in the x-direction is about 44"; in the y-direction about 22.5". The x and y motions can effectively be increased by an origin shift. This is done by either moving the map on the screen or by moving the tracks in the y-direction.

B. Performing an alignment

1. Place the center of the projected film in approximate coincidence with the map.

2. Make approximate alignment by adjusting x, y, z distances and A°, B°, C° gimbal angles.

3. Now study alignment and keeping x and y constant, adjust A°, B°, C°, and Z until the roads and landmarks in the projected picture are "parallel" to the corresponding map detail.

4. Now keeping A°, B°, C° and Z constant vary x and y until the formerly "parallel" detail is superimposed.

5. Repeat steps 3 and 4 until the alignment is considered satisfactory.

It is important that:

a. The map be flat and lie snugly against the projection screen.

b. The map scale be so adjusted that the operator can work as far back from the screen as possible; thus, misalignments, under this higher magnification, will be easily discernable.

c. The gimbal and screen system be disturbed between the alignments of consecutive frames so that each alignment will be as free as possible from bias on the part of the operator.

[redacted]

VI. REDUCTION OF DATA TO OBTAIN BOMB TRAJECTORY AND ORIENTATION

A. Treatment of Data Preparatory to Reduction

We first define two quantities associated with the bomb trajectory.* Assume the following thick-lens optical system:

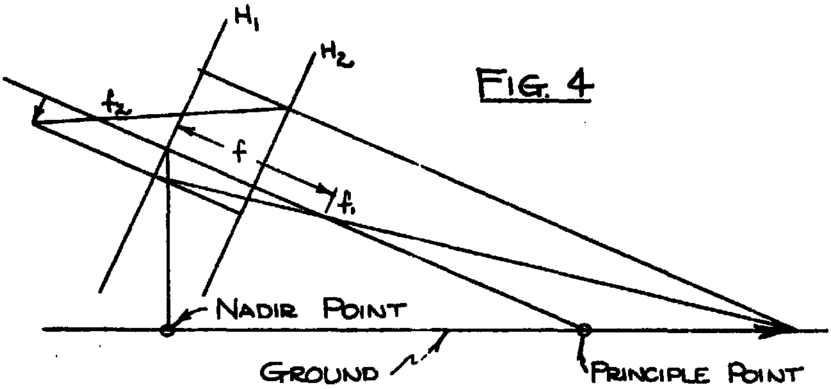


FIG. 4

where H_1 and H_2 are the principal planes of the lens. Then if light enters the system from the right, it appears to enter plane H_1 and to emerge from H_2. Then u is measured from H_1 to the ground and v is measured from H_2 to the image. The "nadir" point is the projection on the ground of the intersection of the "entrance" principal plane, H_1, with the optic axis. The "principal" point is the point of intersection of the optic axis with the ground. Then:

a. The nadir point is approximately the point on

---

* These definitions are based on the pamphlet "Supplemental Topics in Aerial Photogrammetry" by Earl Church, Syracuse University, November, 1933.

[redacted]

the ground directly beneath the c.g. of the bomb. Hence, the plot of the nadir points is the projection on the ground, or x-y plane, of the bomb trajectory.

b. The principal point is the point on the ground at which the bomb is looking, or the intersection of the $\vec{B}$ vector with the ground.

After the operator has achieved a satisfactory match, he records nine quantities:

1. Frame identification - either frame number or the time of the frame.

2. A number, R_3, read on the projection screen at the intersection of the horizontal string with the vertical tape.

3. A number, R_4, read on the projection screen at the intersection of the vertical string with the horizontal tape.

4. A number, R_5, read at the intersection of the pointer on the gimbal truck with the tape which runs along the track.

5. An angle A° from the protractor on the outer gimbal frame.

6. An angle B° from the protractor on the middle gimbal frame.

7. An angle C° (the roll angle) from the protractor on the inner gimbal.

8. The position of the "principal" point, or the intersection of the optic axis with the map, is marked on the map.

9. Comments, including the quality of the match. When the trajectories have been computed and plotted, this comment will aid in determining whether "off-trend" points should be ignored.

To convert the above data to displacements in the map coordinate system, and, incidentally, to scaled displacements in the bombing range coordinate system, since these are for convenience made coincident, we proceed as below. See Fig. 5.

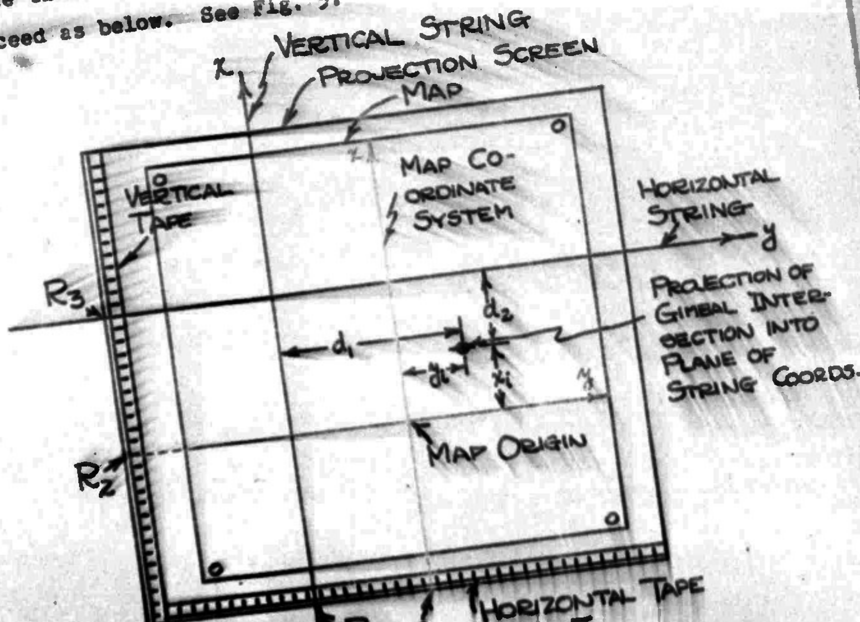


FIG. 5

It is desired to obtain the coordinates, (x_1, y_1), of the projected gimbal intersection point in the <u>map</u> coordinate system (red). Let the coordinates of the projected gimbal intersection point in the string coordinate system be d_1 and d_2. Let the map axes, extended, intersect readings R_1 and R_2 on the tapes. Note that R_1, R_2, d_1 and d_2 are constants throughout the analysis, unless there is an origin shift. Let R_3 be the x reading as recorded by operator and let R_4 be the y reading.

Then, since, on the M.I.T. analyzer the vertical tape increases from the top towards the bottom:

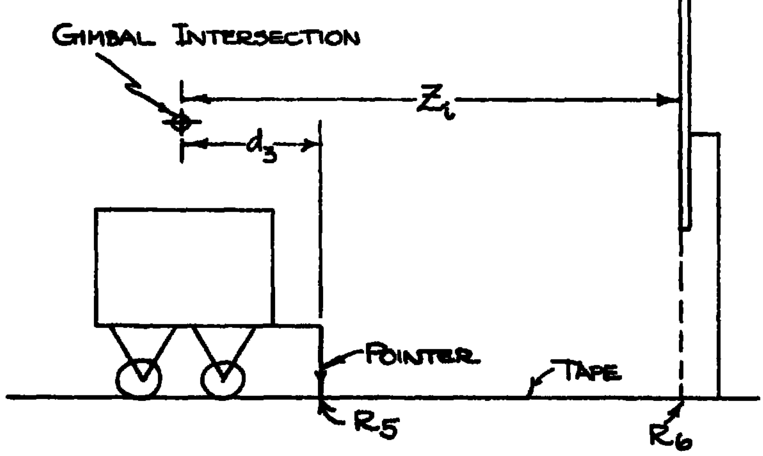
$$x_1 = (R_2 - R_3) + d_2 = (R_2 + d_2) - R_3 = \text{const} - R_3 \quad \text{(VI-1)}$$

and as the horizontal tape increases from left to right:

$$y_1 = d_1 - (R_1 - R_4) = R_4 - (R_1 - d_1) = R_4 - \text{const.} \quad \text{(VI-2)}$$

To get the distance from the gimbal intersection to the screen (z_1):

FIG. 6



Since the numbers on the track tape increase as we go toward the screen:

$$z_1 = (R_6 - R_5) + d_3 = (R_6 + d_3) - R_5 = \text{const.} - R_5 \quad \text{(VI-3)}$$

The A° protractor is so numbered that when the projector axis is pointed in the direction of positive x on the screen the protractor will read between 90° and 180°. When the projector is anywhere along a line parallel to the screen horizontal string axis and distance d_2 below it (see Fig. 5), the protractor will read 90°, and when the axis is pointing toward negative x the reading is between 0° and 90°. We use the angle: $\alpha = A^\circ - 90^\circ$ (VI-4)

When the projector optic axis is pointing towards positive y, B° reads between 0° and 90°. When the projector is pointing along a line parallel to the vertical string axis and distance d_1 to the right of it (See Fig. 5), B° reads 90°, and when the projector points toward negative y, B° reads between 90° and 180°. We use the angle: $\beta = 90^\circ - B^\circ$ (VI-5)

The roll angle protractor, C° is divided into 360° and reads 0° when there is no roll. Looking down the projector axis towards the screen, if the bomb rolls, say, 5° in the clockwise direction, the protractor will read 355°; if the bomb rolls 5° in the counter-clockwise direction, the protractor will read 5°. For the determination of yaw and pitch (see page 39), we need not distinguish between clockwise and counter-clockwise roll and, instead, we define the roll angle, Φ, by the equation:

$$\Phi = 360^\circ - C^\circ \quad \text{(VI-6)}$$

When the data have been taken, the map is removed from the screen and the coordinates of the principal points in the map coordinate system are measured with a ruler. We thus can measure directly $\frac{x_{pp}}{s}$ and $\frac{y_{pp}}{s}$. All data pertaining to distance are taken in inches; the computations are made in inches, to be later converted to feet.

[redacted]

B. <u>Reduction of Data for Trajectories</u>

We are interested in computing and plotting three associated trajectories:

a. The x-y plot of the principal points;

b. The x-y plot of the bomb trajectory, or the nadir point curve;

c. The x-z plot of the bomb trajectory.

<u>1. Principal Point Curve.</u>

If a complete set of data is taken, the map will contain at the end of a run the whole set of principal points without further treatment. If, for some reason, the principal points are not recorded, or if we wish to check some of the recorded points, we perform the following calculation: See Fig. 7.

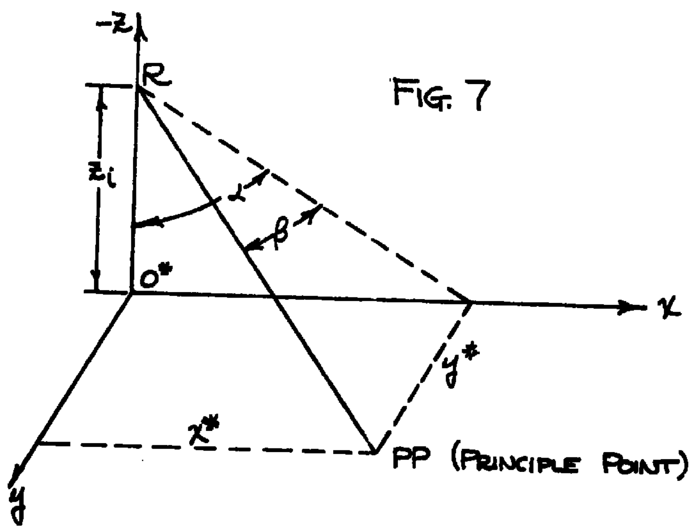


FIG. 7

Let x, y, z axes be erected on the map with origin at O^*, the projection of the gimbal intersection into the plane of the string coordinates. Let this coordinate system be parallel to the map coordinate system. Thus, in our new system, the projector is looking from the gimbal intersection point, R, on the -z axis to the principal point which has coordinates x^* and y^*. Hence:

$$x^* = z_1 \tan \alpha \qquad \text{(VI-7)}$$

$$y^* = z_1 \frac{\tan \beta}{\cos \alpha} \qquad \text{(VI-8)}$$

where z_1 is given by Eq. (VI-3), α by Eq. (VI-4), and β by Eq. (VI-5). Since the coordinates of O^* in the map system are given by Eq. (VI-1), (VI-2), the coordinates of the principal point in the <u>map</u> system are given by:

$$x_{pp} = x_1 + x^* \qquad \text{(VI-9)}$$

$$y_{pp} = y_1 + y^* \qquad \text{(VI-10)}$$

<u>2. Nadir Point and X-Z Plots</u>.

From the definition of a nadir point, it is apparent that the projection of the gimbal intersection into the plane of the string coordinates is <u>not</u> the nadir point of the projector system. We now wish to get the nadir point of the projector system.

[redacted]

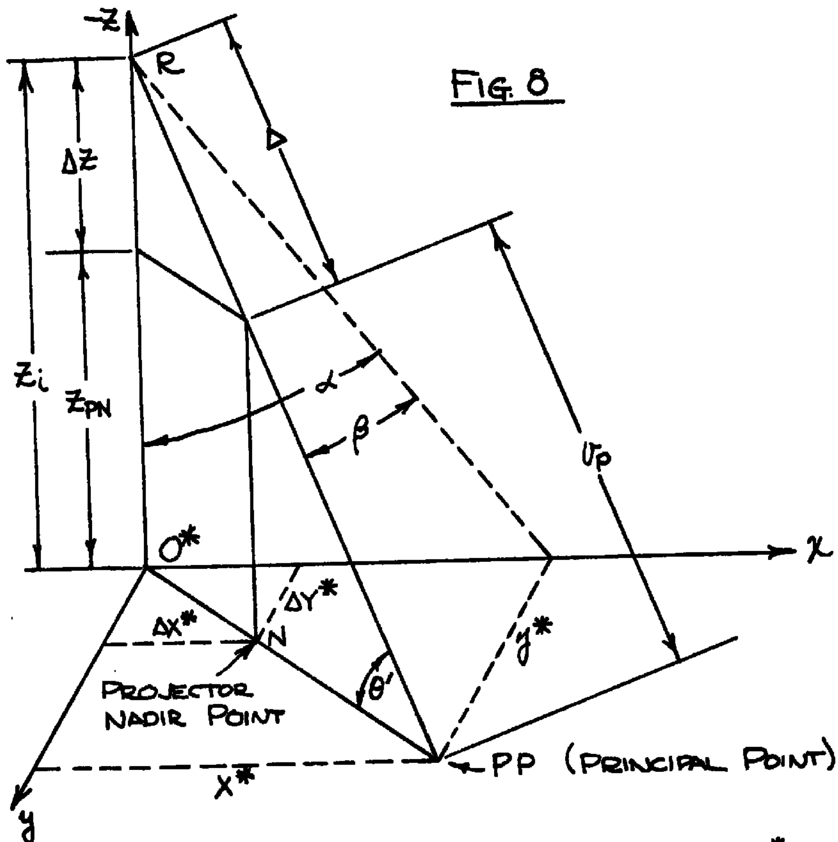


Fig. 8 is similar to Fig. 7. We wish to determine Δx^* and Δy^*, the coordinates of the nadir point in the coordinate system erected over O^*, the projection of the gimbal intersection into the plane of the string coordinates. The distance $\overline{R - PP}$ is the slant distance from the film to the screen, and in terms of known quantities is given by:

$$\overline{R - PP} = \frac{z_1}{\cos \alpha \cos \beta} = z_1 \times (\overline{R - PP}\ \text{factor}) \qquad \text{(VI-12)}$$

It will turn out that we do not need to know $\overline{R - PP}$ very accurately, hence prepare an ($\overline{R - PP}$ factor) chart by which determine the multiplier of z_1.

We now wish to determine Δ, the difference between $\overline{R - PP}$ and v_p, as a function of $\overline{R - PP}$. It will turn out that Δ does not change very rapidly with $\overline{R - PP}$, hence a rough value of $\overline{R - PP}$ will give us a sufficiently accurate value of Δ. Fig. 10 has been prepared from the 2" lens on the M.I.T. analyzer. The optical constants of this lens are as shown below:

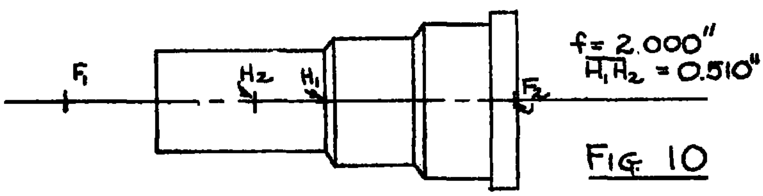


Fig. 10

θ', the glancing angle, can be got from α and β from the equation

$$\cos \theta' = \sqrt{\cos^2\beta \sin^2\alpha + \sin^2\beta} \qquad \text{(VI-13)}$$

From this equation, we prepare Fig. 11. We thus compute Δz, Δx^*, and Δy^* from the equations:

$$\Delta z = \Delta \cdot \sin \theta' \qquad \text{(VI-14)}$$

$$\Delta x^* = \Delta z \cdot \tan \alpha \qquad \text{(VI-15)}$$

$$\Delta y^* = \Delta \cdot \sin \beta \qquad \text{(VI-16)}$$

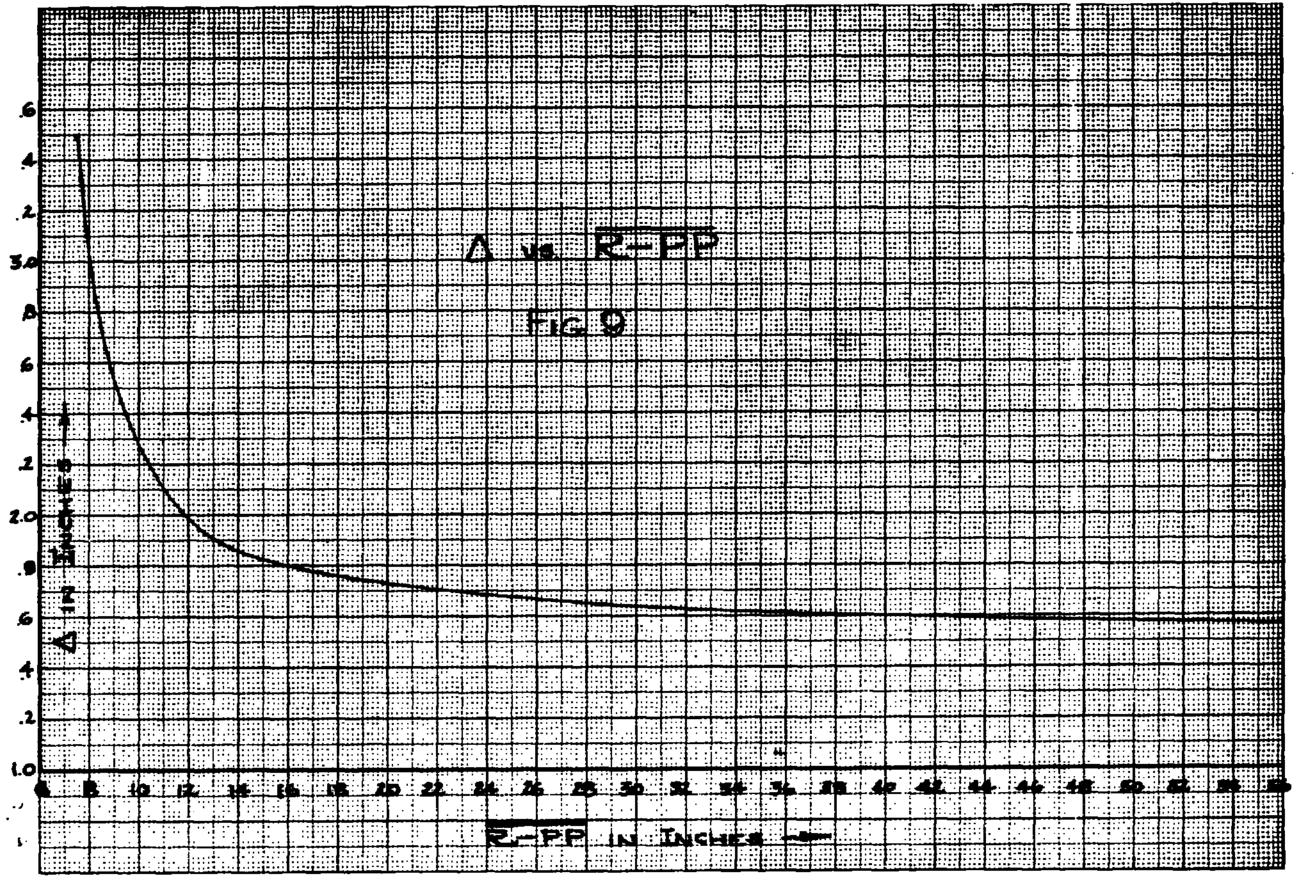
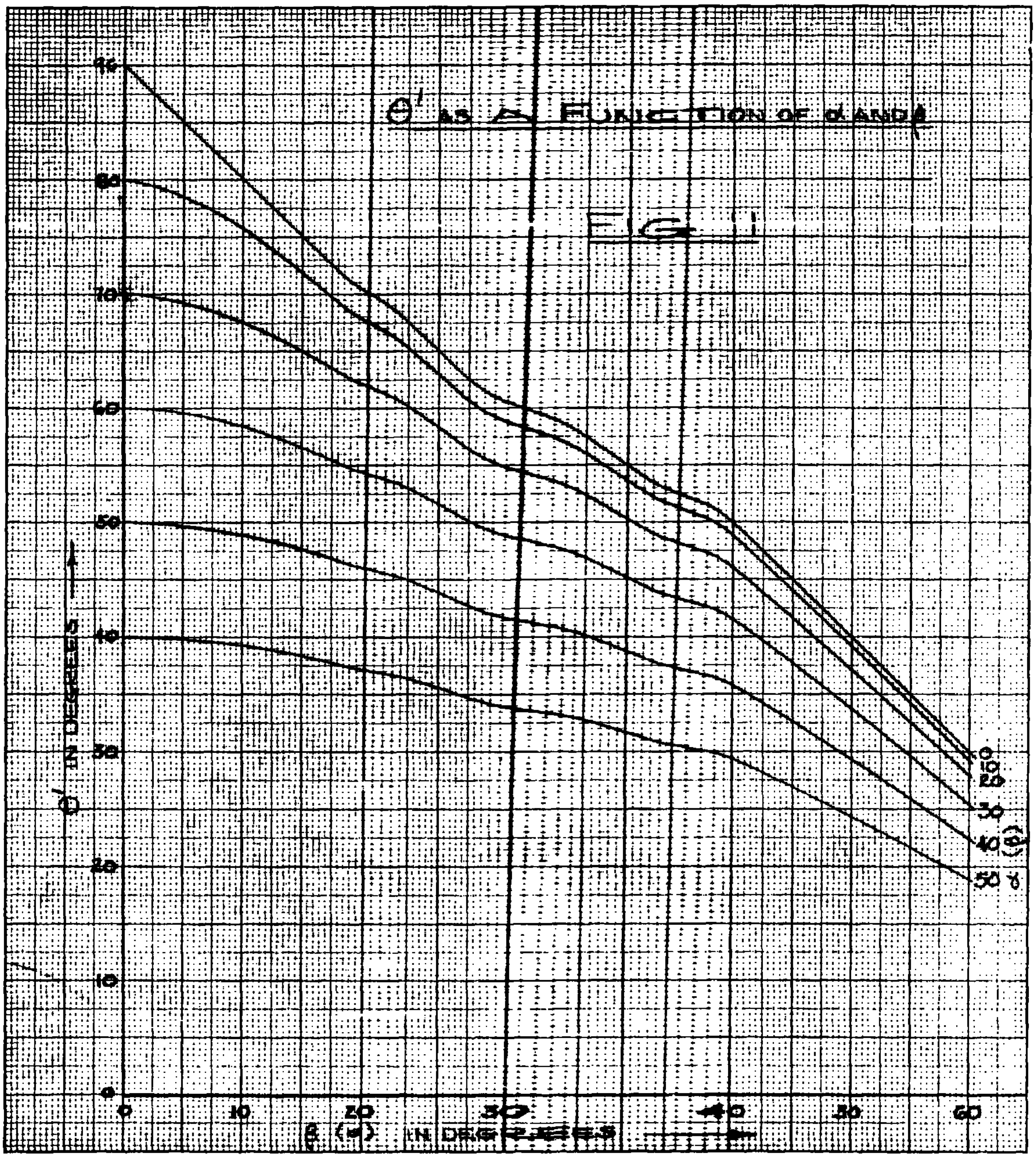

Δ vs. R-PP
Δ IN INCHES
R-PP IN INCHES
.6
.4
.2
3.0
.8
.6
.4
.2
2.0
.8
.6
.4
.2
1.0
6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56

FIG 9


θ' AS A FUNCTION OF α AND β
FIG. 11
θ' IN DEGREES
β (α) IN DEGREES
γ (β)

Thus the altitude of the point, the projection of which is the projector nadir point is given in the map coordinate system by:

$$z_{PN} = z_1 - \Delta z \qquad \text{(VI-17)}$$

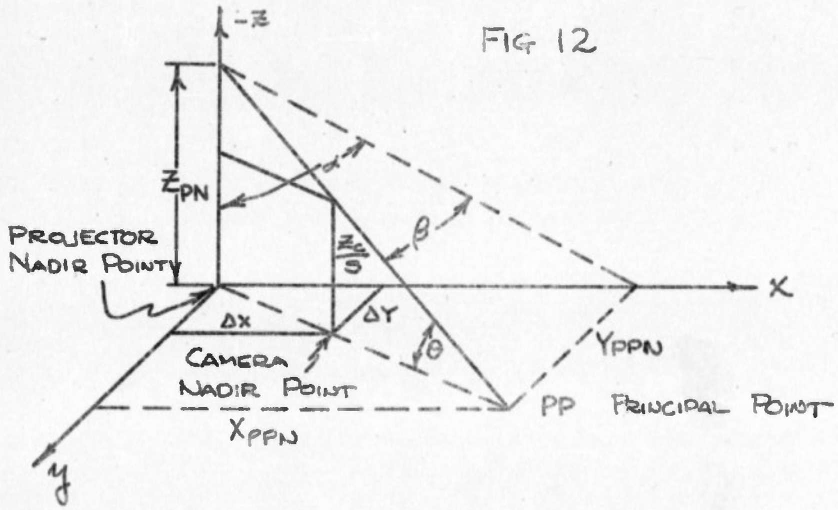
And the coordinates of the projector nadir point in the map coordinate system are

$$x_{PN} = x_1 + \Delta x^* \qquad \text{(VI-18)}$$

$$y_{PN} = y_1 + \Delta y^* \qquad \text{(VI-19)}$$

We now wish to find similar coordinates of the bomb-camera nadir point in the map coordinate system. We assume that the focal lengths of the camera and projector are different and that they are so related to each other that the observed glancing angle θ' is equal to the true glancing angle θ for all practical purposes. See Fig. 12.

FIG 12



[redacted]

Let us erect a coordinate system over our newly found projector nadir point where z_{PN} is the "altitude" of the projector nadir point and $\frac{z_c}{s}$ is the "altitude" of the camera nadir point in the map coordinate system. Now from Eq. (IV-18):

$$\frac{z_c}{s} = f_c \, v_p \left[\frac{1}{f_p} - \frac{1}{v_p}\right] \sin \theta$$

$$\frac{z_c}{s} = f_c \left[\frac{z_{PN}}{f_p} - \sin \theta\right] \qquad \text{(VI-20)}$$

since $v_p \sin \theta = z_{PN}$.

The corrections to the projector nadir point to get the bomb nadir point are then given by (From Fig. 12):

$$\Delta x = \left[z_{PN} - \frac{z_c}{s}\right] \tan \alpha \qquad \text{(VI-21)}$$

$$\Delta y = \frac{\left[z_{PN} - \frac{z_c}{s}\right]}{\cos \alpha} \tan \beta \qquad \text{(VI-22)}$$

And referred to the map system, the coordinates of the bomb nadir point become:

$$\frac{x_c}{s} = x_1 + \Delta x^{*} + \Delta x \qquad \text{(VI-23)}$$

$$\frac{y_c}{s} = y_1 + \Delta y^{*} + \Delta y \qquad \text{(VI-24)}$$

Thus, $\frac{x_c}{s}$, $\frac{y_c}{s}$, $\frac{z_c}{s}$ are the trajectory coordinates of the bomb in the map system, and if the bombing range system is coincident, the coordinates in this system are x_c, y_c, and z_c.

In case there is a considerable difference between the focal length of the projector and the focal length of the bomb camera, $(z_{PN} - \frac{z_c}{s})$, equations (VI-21), (VI-22), becomes large compared to $\frac{x_c}{s}$ and $\frac{y_c}{s}$. Thus,

[redacted]

small errors in α and β can cause considerable error in the x-y trajectory. Under these conditions, it is preferable to compute Δx and Δy by proportional triangles from Fig. 12. Since the coordinates of the principal point are known in the map system (by actual measurement), they can be converted into coordinates x_{PPN}, y_{PPN} in the system erected over the projector nadir point. Hence:

$$\Delta x = x_{PPN} \frac{z_{PN} - \frac{z_c}{s}}{z_{PN}} \qquad \text{(VI-25)}$$

$$\Delta y = y_{PPN} \frac{z_{PN} - \frac{z_c}{s}}{z_{PN}} \qquad \text{(VI-26)}$$

In the example to follow, equations (VI-25) and (VI-26) are used because the focal lengths are considerably different.

3. Example Case 1 fit. See page 16.

R_1 = 12' - 1 3/8" = 145.375"

R_2 = 18' - 1 1/4" = 217.250"

R_3 = 16' - 10 1/2" = 202.500"

R_4 = 11' - 2 1/2" = 134.500"

R_5 = 7' - 7 1/4" = 91.250"

R_6 = 9' - 8 1/2" = 116.500"

d_1 = 1.400"

d_2 = - 40.562"

d_3 = 8.000"

A° = 135.0°

B° = 79.0°

C° = 5.0°

[redacted]

$$\frac{x_{PP}}{s} = 8.80''$$

$$\frac{y_{PP}}{s} = -0.60''$$

$f_c = 1.00''$ $\quad f_p = 2.00''$ $\quad s = 10080$

From Figs. 5 and 6 and Eqs. (VI-1) through (VI-6)

$$x_i = (R_2 + d_2) - R_3 = (217.250 - 40.562) - 202.500 = -25.81''$$

$$y_i = R_4 - (R_1 - d_1) = 134.500 - (145.375 - 1.400) = -9.47''$$

$$z_i = (R_6 + d_3) - R_5 = (116.500 + 8.00) - 91.250 = 33.25''$$

$$\alpha = A^\circ - 90^\circ = 135.0^\circ - 90^\circ = 45.0^\circ$$

$$\beta = 90^\circ - B^\circ = 90^\circ - 79.0^\circ = 11.0^\circ$$

$$\varphi = 360^\circ - C^\circ = 355^\circ$$

For $\alpha = 45.0^\circ$, $\beta = 11.0^\circ$ the $\overline{R - PP}$ factor is 1.46. Hence from Eq. (VI-12):

$$\overline{R - PP} = 33.25 \times 1.46 = 48.5''$$

From Fig. 9 for $\overline{R - PP} = 48.5''$, $\Delta = 1.58''$

From Fig. 11 for $\alpha = 45.0^\circ$, $\beta = 11.0^\circ$ $\theta' = 43.8^\circ$

From Eqs. (VI-14, -15, -16):

$$\Delta z = \Delta \cdot \sin \theta' = 1.58 \sin 43.8^\circ = 1.09''$$

$$\Delta x^* = \Delta z \cdot \tan \alpha = 1.09 \tan 45.0^\circ = 1.09''$$

$$\Delta y^* = \Delta \cdot \sin \beta = 1.58 \sin 11.0^\circ = 0.31''$$

Therefore, from Eqs. (VI-17, -18, -19):

$$z_{PN} = z_i - \Delta z = 33.25 - 1.09 = 32.16''$$

$$x_{PN} = x_i + \Delta x^* = -25.81 + 1.09 = -24.72''$$

$$y_{PN} = y_i + \Delta y^* = -9.47 + 0.31 = -9.16''$$

[redacted]

From Eq. (VI-20):

$$\frac{z_c}{s} = f_c\left[\frac{z_{PN}}{f_p} - \sin\theta\right] = 1\left[\frac{32.16}{2} - \sin 43.8^\circ\right]$$

$$\frac{z_c}{s} = 15.39''$$

Since $\frac{x_{PP}}{s} = 8.80''$ and $x_{PN} = -24.72''$

then $x_{PPN} = 8.80 - (-24.72) = 33.52''$

similarly, $y_{PPN} = -0.60 - (-9.16) = 8.56''$

Thus, by Eqs. (VI-25, -26):

$$\Delta x = x_{PPN}\,\frac{z_{PN} - \frac{z_c}{s}}{z_{PN}} = 33.52 \times \frac{32.16 - 15.39}{32.16} = 17.46''$$

$$\Delta y = y_{PPN}\,\frac{z_{PN} - \frac{z_c}{s}}{z_{PN}} = 8.56 \times \frac{32.16 - 15.39}{32.16} = 4.46''$$

And by Eqs. (VI-23, -24):

$$\frac{x_c}{s} = x_1 + \Delta x^* + \Delta x = -25.81 + 1.09 + 17.46 = -7.26''$$

$$\frac{y_c}{s} = y_1 + \Delta y^* + \Delta y = -9.47 + 0.31 + 4.46 = -4.70''$$

Removing the scale factor:

$$z_c = \frac{15.39 \times 10080}{12} = 12920 \text{ feet}$$

$$x_c = -7.26 \times 840 = -6100 \text{ feet}$$

$$y_c = -4.70 \times 840 = -3950 \text{ feet}$$

[redacted]

C. Reduction of Data for Orientation

1. Determination of Bomb Orientation.

By "Determination of Bomb Orientation" we shall mean the determination of the direction cosines of the unit vector $\vec{B}$, representing the longitudinal axis of the bomb, in a rectangular coordinate system erected on the bombing range. This determination can be made in two ways:

Method 1. From the readings on the A° and B° protractors.

Since it leads to simple reduction formulas, the operator chooses the various coordinate systems involved so the coordinates of the analyzer system are parallel to both the coordinates erected on the map and the coordinates erected on the bombing range. Thus the direction cosines of a line are the same in all three systems. In Fig. 13, the gimbal center is at O on the z' axis with the optic axis or $\vec{B}$ vector pointing at the point P on the map, which lies in the x-y plane. Angles α and β are obtained from the A° and B° gimbal protractors.

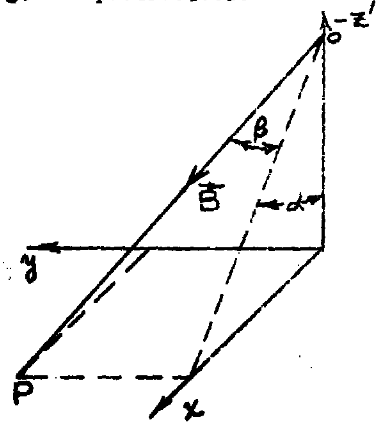


FIG. 13

[redacted]

Then if ℓ <u>is the direction angle of $\vec{B}$ with x</u>:

$$\cos \ell = \cos \beta \sin \alpha \qquad \text{(VI-27)}$$

If <u>m is the direction angle of $\vec{B}$ with y</u>:

$$\cos m = \sin \beta \qquad \text{(VI-28)}$$

If <u>n is the direction angle of $\vec{B}$ with z'</u>:

$$\cos n = \cos \beta \cos \alpha \qquad \text{(VI-29)}$$

Hence $\vec{B} = [\cos \beta \sin \alpha]i + [\sin \beta]j + [\cos \beta \cos \alpha]k$ (VI-30)

<u>Method 2</u>. <u>From the trajectory plots</u>.

The determination of $\vec{B}$ along with other useful quantities by this method are outlined in the following paragraphs.

2. <u>Determination of Yaw, Pitch and Angle of Attack</u>.

After preparing a set of trajectories comparable to the one below, we measure the indicated angles by a protractor from the drawing.

[redacted]

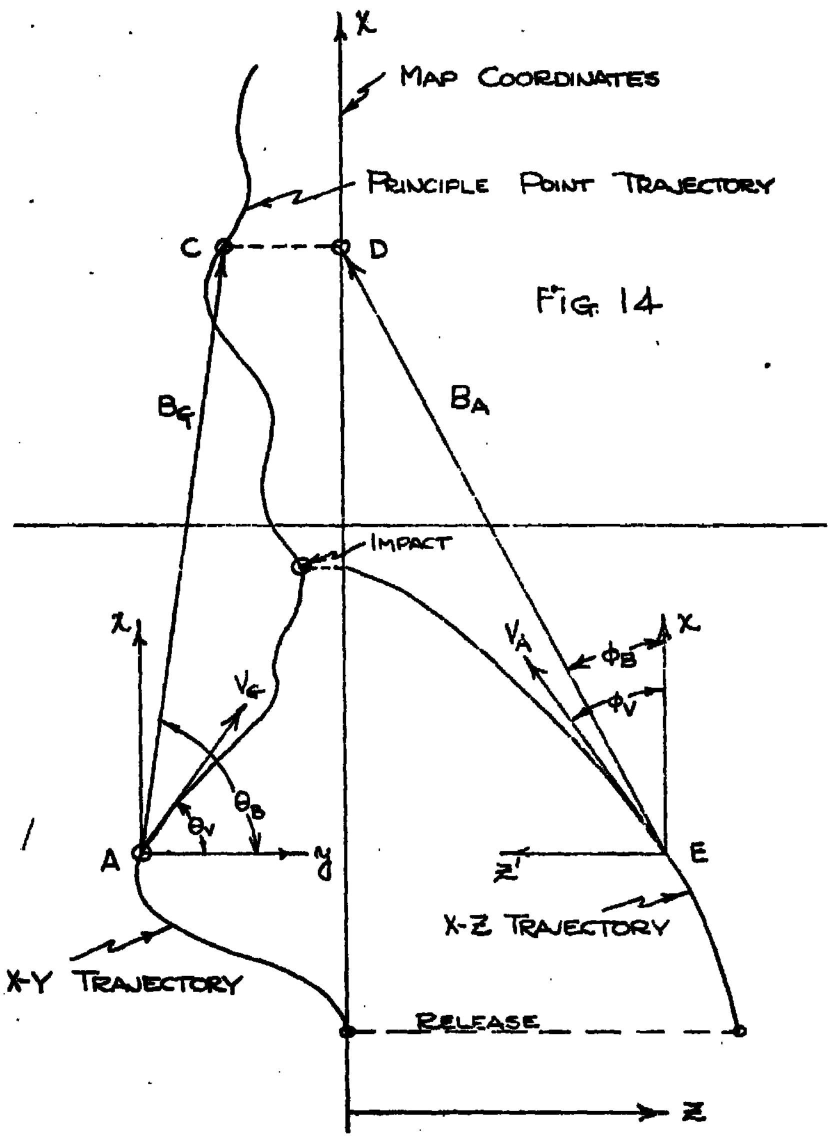


FIG. 14

[redacted]

In making the above plot, an origin is chosen on the map (usually the intended target), the x axis is oriented so as to be the ground projection of the bombing run, or more accurately, the projection on the ground of the longitudinal axis of the bomb (the $\vec{B}$ vector) at the instant of release. The y axis is erected perpendicular to the x and the x-z plane is folded down so as to be coplanar with the x-y plane. Hence x corresponds to range, y to position in azimuth or side deflection and z to altitude. Due to choice of origin and positive direction of x and y, all the x coordinates of the bomb will be negative, provided the bomb does not overshoot the target, y coordinates will be negative if the bomb veers to the left of the bombing run line, positive if the bomb veers to the right. z, of course, will always be positive.

In the diagram, points A, B, C, D refer to the bomb at a particular time instant. A is position of the bomb on the ground, or the x-y coordinates of the nadir point, B is the position of the bomb in the folded x-z plane, C is the point on the ground at which the bomb is looking at this particular time (the coordinates of the "principal" point), and D is the position of the principal point in the folded x-z plane. Connect the points as shown and draw in the trajectory tangents at A and B. x-y axes are then erected at A with positive directions as shown, and x-z' axes are erected at B with positive direction as shown. With this layout x, y, and z' form a right-handed system.

[redacted]

The labeled vectors are thus:

B_G is the "ground" projection of the longitudinal axis of the bomb or $\vec{B}$ vector.

V_G is the ground projection of the trajectory tangent or the instantaneous direction of the bomb velocity vector.

B_A is the "air" projection of the $\vec{B}$ vector.

V_A is the air projection of the bomb velocity vector.

To determine the angle of attack, we first determine the direction cosines of $\vec{B}$ and $\vec{V}$ in the x,y,z' system. Thus given any unit vector L making angles θ and ϕ as shown:

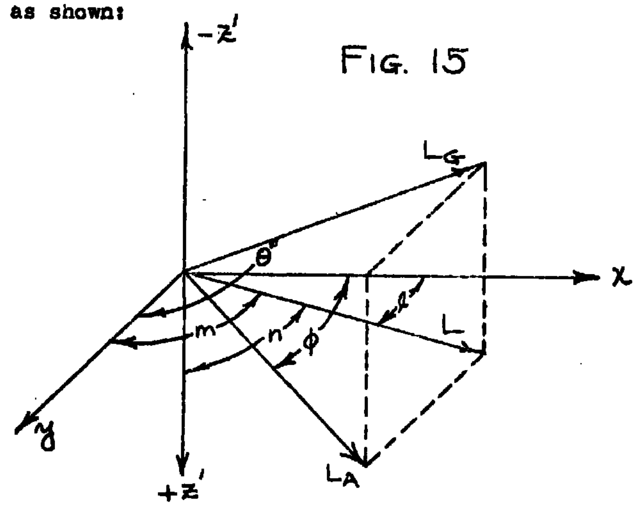


FIG. 15

and letting the direction angles with x, y, and z' be ℓ, m, and n.

[redacted]

We solve the following equations:

$$L_A \sin\varphi = \cos n$$

$$L_A \cos\varphi = \cos \ell$$

$$L_G \sin\theta = \cos \ell$$

$$L_G \cos\theta = \cos m$$

$$\cos^2\ell + \cos^2 m + \cos^2 n = 1$$

Thus:

$$\cos n = \tan\varphi \cos\ell \qquad \text{(VI-31)}$$

$$\cos m = \cot\theta \cos\ell \qquad \text{(VI-32)}$$

$$\cos\ell = \frac{1}{\sqrt{1 + \cot^2\theta + \tan^2\varphi}} \qquad \text{(VI-33)}$$

Thus we solve equation (33) for ℓ and then determine n and m through equations (31) and (32).

After getting $\cos n_B$, $\cos m_B$ and $\cos \ell_B$ for the $\vec{B}$ vector and $\cos n_V$, $\cos m_V$, and $\cos \ell_V$ for the $\vec{V}$ vector, we get the angle between them (or what we shall call the angle of attack) by the following equation:

$$\text{cosine (angle of attack)} = \cos\gamma = \cos\ell_V \cos\ell_B + \cos m_V \cos m_B + \cos n_V \cos n_B \qquad \text{(VI-34)}$$

To be able to compare the results of our film trajectory analysis with the trajectory calculations we must determine the angles of pitch and yaw, which, combined, produce the angle of attack γ. The pitch angle θ is defined as the angle between the longitudinal axis of the bomb, the $\vec{B}$ vector, and the bomb velocity vector (anti-parallel to the wind vector) projected into the plane of the vertical fins of the bomb. The yaw angle ψ is the angle between the $\vec{B}$ vector and the $\vec{V}$ vector projected into the plane of the horizontal fins. To give the pitch and yaw angles the proper

[redacted]

signs, we set up the following rectangular coordinate system:

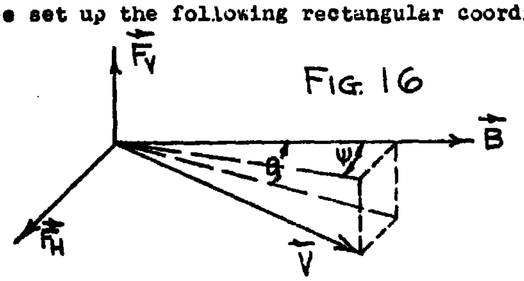


FIG. 16

where $\vec{F}_V$ lies in the plane of the vertical fins and $\vec{F}_H$ lies in the plane of the horizontal fins. Let us look along the $\vec{B}$ axis from the <u>positive end toward the origin</u>:

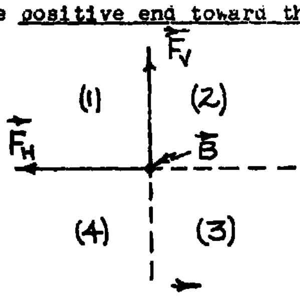


FIG. 17

Then if the terminus of the $\vec{V}$ vector lies in quadrant (1):

- Pitch angle θ is <u>negative</u> (Bomb diving)
- Yaw angle ψ is <u>negative</u> (Bomb veering towards left as we look down on trajectory)

If the terminus of the $\vec{V}$ vector lies in quadrant (2):

- Pitch angle θ is <u>negative</u> (Bomb diving)
- Yaw angle ψ is <u>positive</u> (Bomb veering towards the right as we look down on trajectory)

If the $\vec{V}$ vector lies in quadrant (3):

- Pitch angle θ is <u>positive</u> (Bomb sailing)
- Yaw angle ψ is <u>positive</u> (Bomb veering towards right)

[redacted]

If the $\vec{V}$ vector lies in quadrant (4):

Pitch angle θ is <u>positive</u> (Bomb sailing)
Yaw angle ψ is <u>negative</u> (Bomb veering towards left)

The problem now is to determine θ and ψ both in magnitude and sign having been given the x, y, z' components of $\vec{B}$ and $\vec{V}$ and having been given the angle of roll φ as read off the film analyzer.

From the method of roll control that the directional gyro exerts, we define a "zero roll" plane as follows.

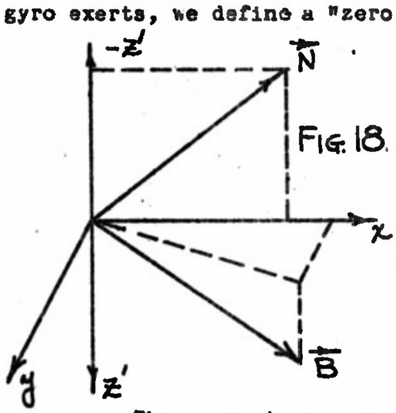


FIG. 18

We construct in xz' plane a unit vector $\vec{N}$ perpendicular to $\vec{B}$, $\vec{N}$ being so directed as to obey the equation

$$\vec{N} = \frac{\vec{V} \times \vec{B}}{|\vec{V} \times \vec{B}|}$$

The vectors $\vec{N}$ and $\vec{B}$ determine the "zero roll" plane, which has the following significance:

The present gyro system attempts to keep the plane of the vertical fins coincident with the $\vec{N} - \vec{B}$ plane. If the vertical fins are, for some reason, rolled out of this plane, the gyro-system attempts to restore coplanarity. We call the dihedral angle between the plane of the vertical fins and the $\vec{N} - \vec{B}$ plane the roll angle, φ. Since $\vec{F}_V$ is also perpendicular to $\vec{B}$, ϕ is the angle between $\vec{N}$ and $\vec{F}_V$, and is measured from $\vec{N}$ proceeding clockwise to $\vec{F}_V$, while looking from the origin out towards the terminus of $\vec{B}$. Thus φ is always positive. To determine θ and ψ, we shall determine the components of $\vec{V}$ in the $\vec{B}$, $\vec{F}_V$, $\vec{F}_H$ system of axes. To do this, we shall obtain first the x, y, z' components of $\vec{V}$, $\vec{B}$, $\vec{F}_V$, and $\vec{F}_H$.

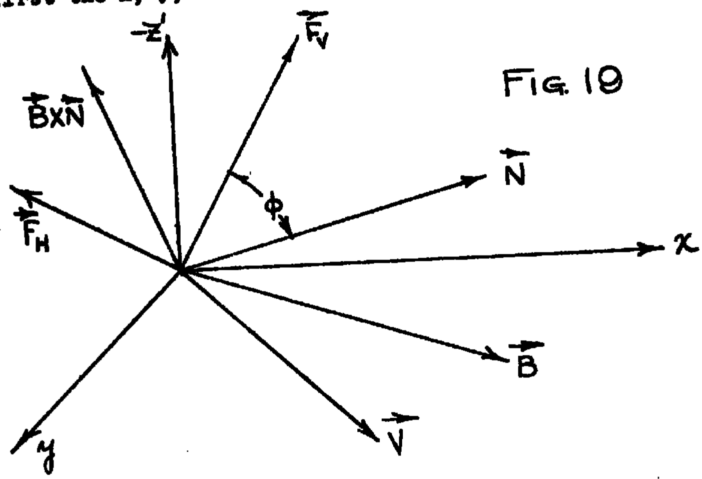


FIG. 10

We deal with three orthogonal systems:

x, y, z'; $\vec{B}$, $\vec{N}$, $\vec{B}\times\vec{N}$; $\vec{B}$, $\vec{F}_V$, $\vec{F}_H$.

$$\vec{B} = B_x i + B_y j + B_z k \quad \text{(Unit vector)}$$

$$\vec{N} = \frac{\vec{y} \times \vec{B}}{|\vec{y} \times \vec{B}|} = \frac{B_z}{\sqrt{B_x^2 + B_z^2}}\, i - \frac{B_x}{\sqrt{B_x^2 + B_z^2}}\, k \quad \text{(Unit vector)}$$

$$\vec{B} \times \vec{N} = -\frac{B_x B_y}{\sqrt{B_x^2 + B_z^2}}\, i + \frac{B_x^2 + B_z^2}{\sqrt{B_x^2 + B_z^2}}\, j - \frac{B_y B_z}{\sqrt{B_x^2 + B_z^2}}\, k \quad \text{(Unit vector)}$$

Now the component of $\vec{F}_V$ on $\vec{N}$ is cos φ, hence the components <u>of this component</u> in the x, y, z' system are:

on x: $\cos\varphi \dfrac{B_z}{\sqrt{B_x^2 + B_z^2}}$

on y: $\cos\varphi \cdot 0 = 0$

on z': $-\cos\varphi \dfrac{B_x}{\sqrt{B_x^2 + B_z^2}}$

The component of $\vec{F}_V$ on $\vec{B} \times \vec{N}$ is $\sin\varphi$, hence the components <u>of this component</u> in the x, y, z' system are:

on x: $-\sin\varphi \dfrac{B_x B_y}{\sqrt{B_x^2 + B_z^2}}$

on y: $\sin\varphi \dfrac{B_x^2 + B_z^2}{\sqrt{B_x^2 + B_z^2}}$

on z': $-\sin\varphi \dfrac{B_y B_z}{\sqrt{B_x^2 + B_z^2}}$

And since $\vec{F}_V$ has no components on $\vec{B}$, the components of $\vec{F}_V$ in the x, y, z' system are then:

$$\left.\begin{aligned}\vec{F}_V = &\left[\cos\varphi \frac{B_z}{\sqrt{B_x^2 + B_z^2}} - \sin\varphi \frac{B_x B_y}{\sqrt{B_x^2 + B_z^2}}\right] i + \left[\sin\varphi \frac{B_x^2 + B_z^2}{\sqrt{B_x^2 + B_z^2}}\right] j \\ &- \left[\sin\varphi \frac{B_y B_z}{\sqrt{B_x^2 + B_z^2}} + \cos\varphi \frac{B_x}{\sqrt{B_x^2 + B_z^2}}\right] k\end{aligned}\right\} \text{Unit Vector}$$

$$\vec{F}_H = \vec{B} \times \vec{F}_V$$

$$\left.\begin{aligned}\vec{F}_H = &- i\left[\frac{\sin\varphi}{\sqrt{B_x^2 + B_z^2}} B_z + \frac{\cos\varphi}{\sqrt{B_x^2 + B_z^2}} B_x B_y\right] + j\left[\cos\varphi \sqrt{B_x^2 + B_z^2}\right] \\ &+ k\left[\frac{\sin\varphi}{\sqrt{B_x^2 + B_z^2}} B_x - \frac{\cos\varphi}{\sqrt{B_x^2 + B_z^2}} B_y B_z\right]\end{aligned}\right\} \text{Unit Vector}$$

Determination of ψ

In accordance with our sign convention, we define ψ by the following equation:

$$\tan \psi = - \frac{V_{FH}}{V_B} \qquad \text{(VI-35)}$$

where

$$V_B = \vec{V} \cdot \vec{B} = V_x B_x + V_y B_y + V_z B_z = \cos \gamma \qquad \text{(VI-36)}$$

and

$$V_{FH} = - V_x \left[\frac{\sin \phi}{\sqrt{B_x^2 + B_z^2}} B_z + \frac{\cos \phi}{\sqrt{B_x^2 + B_z^2}} B_x B_y \right] + V_y \left[\cos \varphi \sqrt{B_x^2 + B_z^2} \right]$$

$$+ V_z \left[\frac{\sin \varphi}{\sqrt{B_x^2 + B_z^2}} B_x - \frac{\cos \varphi}{\sqrt{B_x^2 + B_z^2}} B_y B_z \right] \qquad \text{(VI-37)}$$

Determination of θ

Similarly, we define θ by the following equation:

$$\tan \theta = - \frac{V_{FV}}{V_B} \qquad \text{(VI-38)}$$

and

$$V_{FV} = V_x \left[\frac{\cos \varphi}{\sqrt{B_x^2 + B_z^2}} B_z - \frac{\sin \varphi}{\sqrt{B_x^2 + B_z^2}} B_x B_y \right] + V_y \left[\sin \varphi \sqrt{B_x^2 + B_z^2} \right]$$

$$- V_z \left[\frac{\sin \varphi}{\sqrt{B_x^2 + B_z^2}} B_y B_z + \frac{\cos \varphi}{\sqrt{B_x^2 + B_z^2}} B_x \right] \qquad \text{(VI-39)}$$

Numerical Example:

We draw up our plot as shown on page 40 and get the following angles by measurement from the drawing:

$\theta_B = 109^\circ \qquad \theta_V = 78^\circ$

$\varphi_B = 57^\circ \qquad \varphi_V = 60^\circ$ roll angle $\varphi = 349^\circ = (-11^\circ)$

$\cot \theta_B = \cot 109^\circ = -.3443 \qquad \cot \theta_V = \cot 78^\circ = .2125$

$\tan \varphi_B = \tan 57^\circ = 1.541 \qquad \tan \varphi_B = 1.732$

from equation (VI-33) $\cos \ell_B = \dfrac{1}{\sqrt{1 + .1192 + 2.382}} = .535 = B_x$

from equation (VI-31) $\cos n_B = 1.541 \times .535 = .824 = B_z$

from equation (VI-32) $\cos m_B = -.3443 \times .535 = -.1842 = B_y$

$$\cos \ell_V = \frac{1}{\sqrt{1 + .0454 + 3.000}} = \underline{.498} = V_x$$

$$\cos n_V = \underline{.863} = V_z$$

$$\cos m_V = \underline{.1058} = V_y$$

from equation (VI-34):

$$\cos \gamma = .535 \times .498 - .1842 \times .1058 + .824 \times .863 = \underline{.9572}$$

Therefore, $\gamma = 16°49'$ <u>Angle of attack</u>

Now $\dfrac{\sin \varphi}{\sqrt{B_x^2 + B_z^2}} = -\ .1942$; $\dfrac{\cos \varphi}{\sqrt{B_x^2 + B_z^2}} = .999$

and substituting into equation (VI-37):

$$V_{FH} = -\ .498 \left[-\ .1942 \times .824 + .999 \times .535\ (-\ .1842)\right] + .1058 \left[.98163 \times .982\right] + .863 \left[-\ .1942 \times .535 - .999 \times (-\ .1842) \times .824\right]$$

$$V_{FH} = .2718$$

From equation (VI-35) $\tan \psi = -\dfrac{.2718}{.9572} = -\ .284$

Therefore $\psi = -\ 15°52'$ <u>Yaw angle</u>

And from equation (VI-39):

$$V_{FV} = .498 \left[.999 \times .824 - (-\ .1942)(.525)(-\ .1842)\right] + .1058 \left[-\ .19081 \times .982\right] - .863 \left[-\ .1942 \times (-\ .1842) \times .824 + .999 \times .535\right]$$

$$V_{FV} = -\ .1058$$

And from equation (VI-38) $\tan \theta = -\dfrac{-\ .1058}{.9572} = .1105$

Therefore, $\theta = 6°18'$ <u>Pitch angle</u>

Hence the bomb is <u>sailing</u> with a pitch angle of <u>6°18'</u> and is veering towards the <u>left</u> with a yaw angle of <u>15°52'</u>.

VII. FIELD TESTS, AND RESULTS OF FILM ANALYSES AND THEODOLITE RECORDS

A. Introduction and Abstract of M. I. T. Tests.

In the drop tests of December 22, 1942, through January 12, 1943, at Eglin Field, the M. I. T. group dropped five bombs as described below:

a. Two Army M-44's, converted for direct-sight, azimuth only. They were gyro-stabilized and under radio control. See Fig. 20.

b. One "replica" bomb containing television equipment and a parachute salvaging mechanism, but neither gyroscopes nor radio-control link, was dropped twice. On the first drop, the bomb was recovered undamaged, so it was dropped again, this time without the parachute.

c. One bomb containing only television equipment and gyro-stabilizer.

d. One bomb containing television equipment, gyro-stabilization, and radio control link.

The above bombs were described in detail in Sections 5 and 6 of the Sixth Progress Report.

The direct-sight bombs were successful to the extent that when control was applied, a momentary deflection of the bomb was observed, but in neither case was the bomb steered continuously to the ground. The film from one of these bombs was recovered in sufficiently good shape to permit a film analysis to be made from which the probable

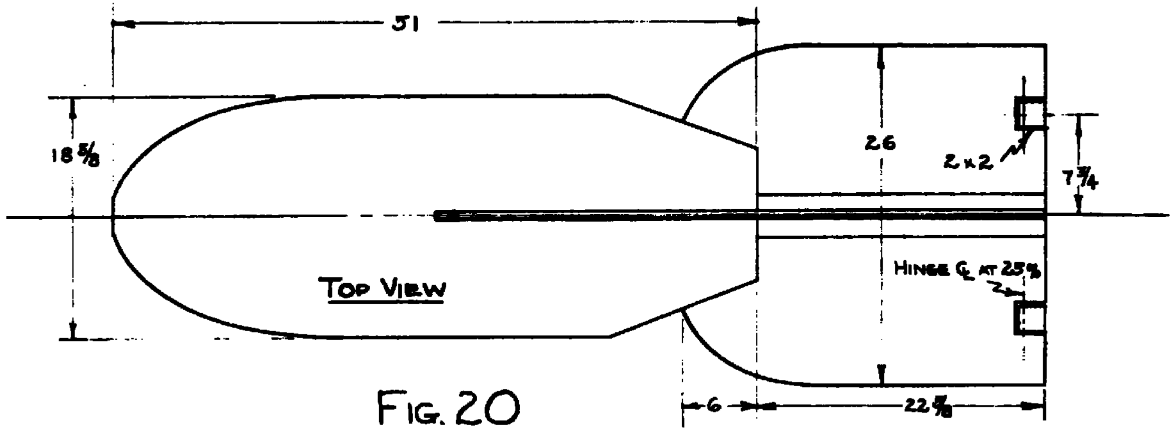
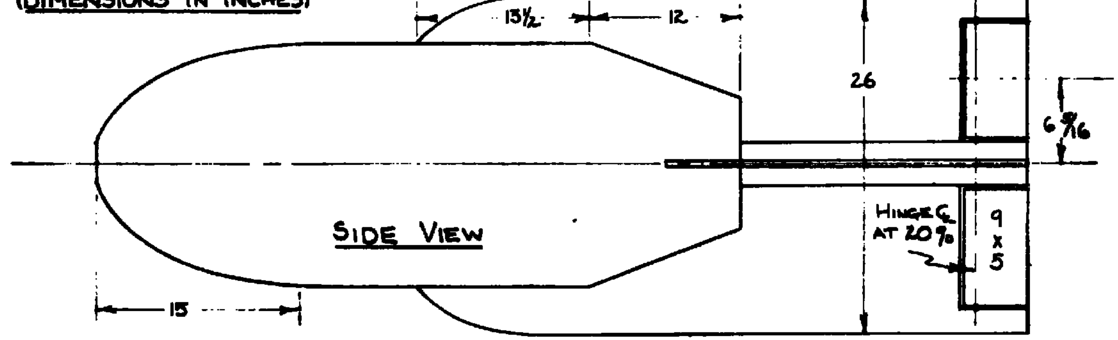


Fig. 20

M-44 Converted for Direct Sight, Azimuth Only

(Dimensions in inches)



[redacted]

cause of failure is determined. It was observed in the field that the operator can judge fairly well the error in azimuth and to some extent the error in range.

The images received in the plane from the television-equipped bombs appeared to be more than adequate for the purpose. Objects on the ground stood out sharply and it was evident that a bombardier could readily recognize his target and steer the bomb for it. Although the television was satisfactory, the bomb which fell under radio control went into a spin and, hence, brought failure to the proposed steering.

B. M-44's Converted for Direct-Sight, Azimuth Only.

On the first drop, the plane came in from the north-east, parallel to the north-east leg of the bombing triangle. This is an equilateral triangle, 2600 feet on a side, and containing a target at the center. See Fig. 21. The bomb weighed 920 lb., was released from 15200' at an airspeed of 150 m.p.h. The computed range for this bomb was 6430 feet and it was released this distance behind the center of the leg running north-east. The object of the test was to steer the bomb into any portion of the north-east leg.

The flare, with which the bomb was equipped, failed to ignite after release; nevertheless, the bomb could be seen from the plane for about 20 sec. As long as the detailed structure of the bomb could be seen from the plane (6-8 secs.), it appeared to be held in proper

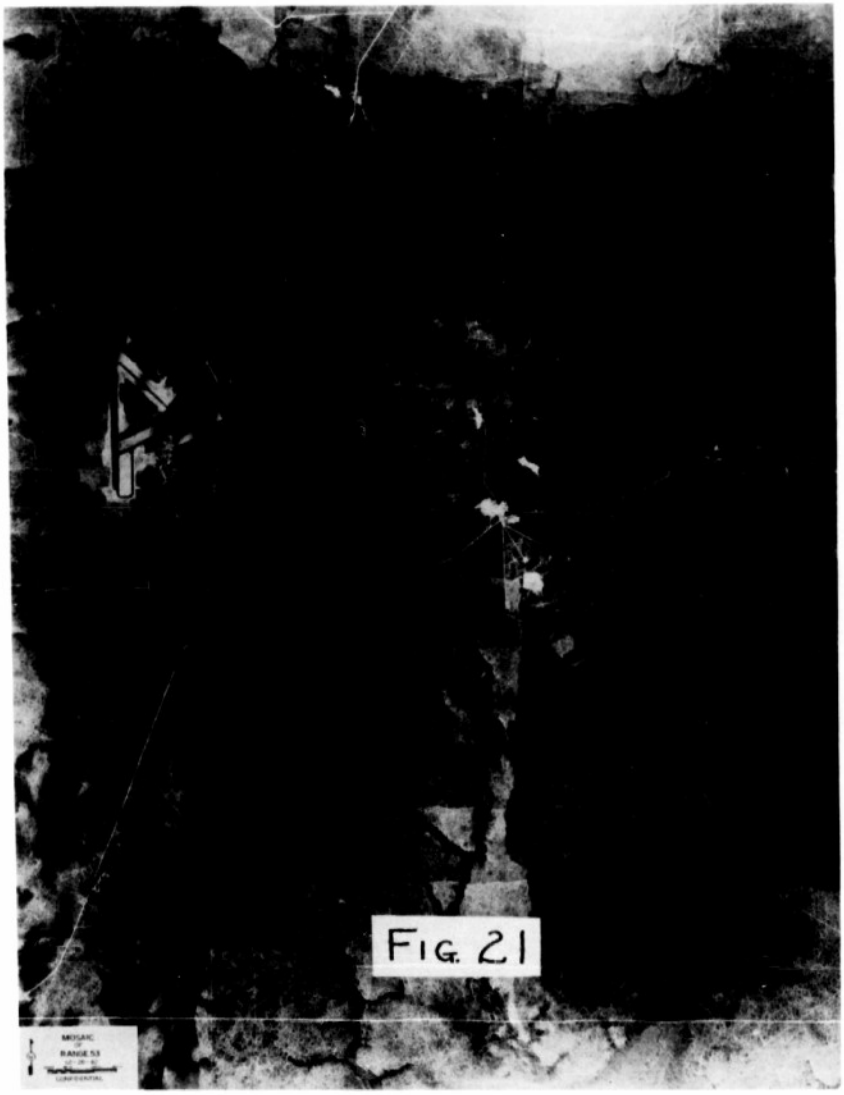


FIG. 21
MOSAIC
OF
RANGE 53
CONFIDENTIAL

[redacted]

orientation by the gyro stabilizer. At the time of release, the bomb appeared from the plane to be not only right of the north-east leg, but slightly to the right of the central target, and after 15 seconds of free fall the operator attempted to correct this azimuthal error by applying left control. A momentary deviation in the proper direction was seen by all observers, including the bombardier. Though this deviation ceased to exist after a few seconds, left control was applied throughout the rest of the flight.

The bomb landed inside the triangle between the central target and the north-east leg, about half-way between the two in azimuth and considerably short of the central target in range. Thus, ignoring the effect of misleading perspective, the azimuthal deflection was approximately 400 feet. Trajectory calculations, which have been shown to be substantially correct, predict a change in azimuth of about 3500 feet for this case. Hence, we conclude that, though the bomb started to veer in the proper direction, it went into a spin shortly after control application. The camera in the nose was so badly damaged that the film was lost for analysis. No theodolite record was taken.

For the drop of the second M-44, the plane came in at 15100 feet from south -- 42° west, heading for the southern triangle corner. It was released at about 22° with the vertical at an airspeed of 150 m.p.h. (ground-speed

[redacted]

172 m.p.h.). The object of this test was again to bomb the north-east triangle leg. The flare lit this time, and after 15 seconds of free fall it appeared to the operator that left control was necessary to steer the bomb into the leg. A momentary deviation in the left direction was observed from both the plane and the theodolite stations. Shortly afterwards, the bomb went into a spiral course, from which it never recovered. The camera in this bomb was also badly damaged though a number of short pieces of film were recovered, thus enabling a fairly satisfactory analysis to be made.

1. <u>Film Analysis and Theodolite Record of Converted M-44</u>.

The analysis of this bomb and the analyses of two other bombs, to be later described, conform only crudely to the principles laid down in Chapter IV.

a. Since the focal lengths of the projector and bomb camera did not satisfy the equations of Chapter IV, excellent fits were impossible.

b. The theory of matching was not completely developed at the time the analyses were made. Hence, probably all five cases of "best" fit were used at one time or another, the operator being unaware of which particular case he was using. Hence our only assumption can be that on the average, $\theta' = \theta$.

c. In two out of the three analyses, it would have been possible to work further back from the screen, thus increasing the accuracy.

[redacted]

d. Accurate data on the orientation of the bomb at the time of release were not always available. This will cause error in the roll angle readings because the projector-map setup is determined by the initial orientation of the bomb.

e. Since the M-44's were timed by light flashes on the margin of the film, the partial loss of the film record made necessary another method of timing the frames, which left much to be desired.

Fig. 22 shows the principal point curve, the x-y plot, and the x-z plot from the film analysis along with the x-y and x-z plots as obtained from the theodolite records. Neither film nor theodolite record is complete as can be seen from the plot. There was a considerable scattering of points on the film curves which masked the spiralling effects, which were supposedly present. There is excellent correspondence between the theodolite ground plot and the film ground plot, but the altitude plots check very poorly. The reason for the discrepancy is not very clear, but the difference in focal lengths is probably a contributing factor.

To arrive at a possible time scale, the accumulated roll angle was plotted against altitude. See Fig. 23. The projector reads only the roll angle, and since there was only a fragmentary record, it was necessary to add the appropriate number of 2π's to the roll angles from other considerations. Since the approximate frame speed was known, it was possible to associate an angular velocity with each roll angle. Also, the altitude and, hence, the

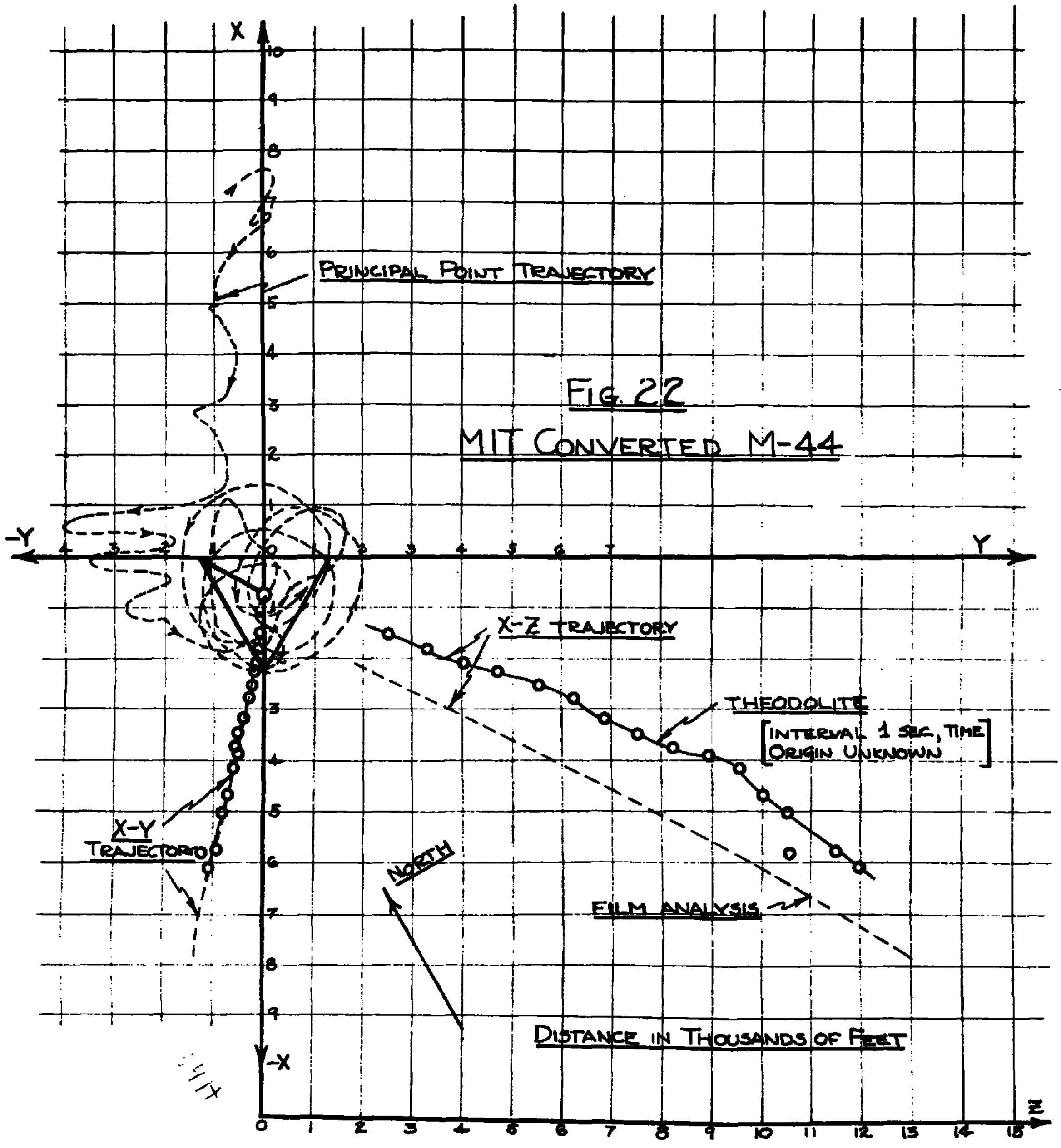
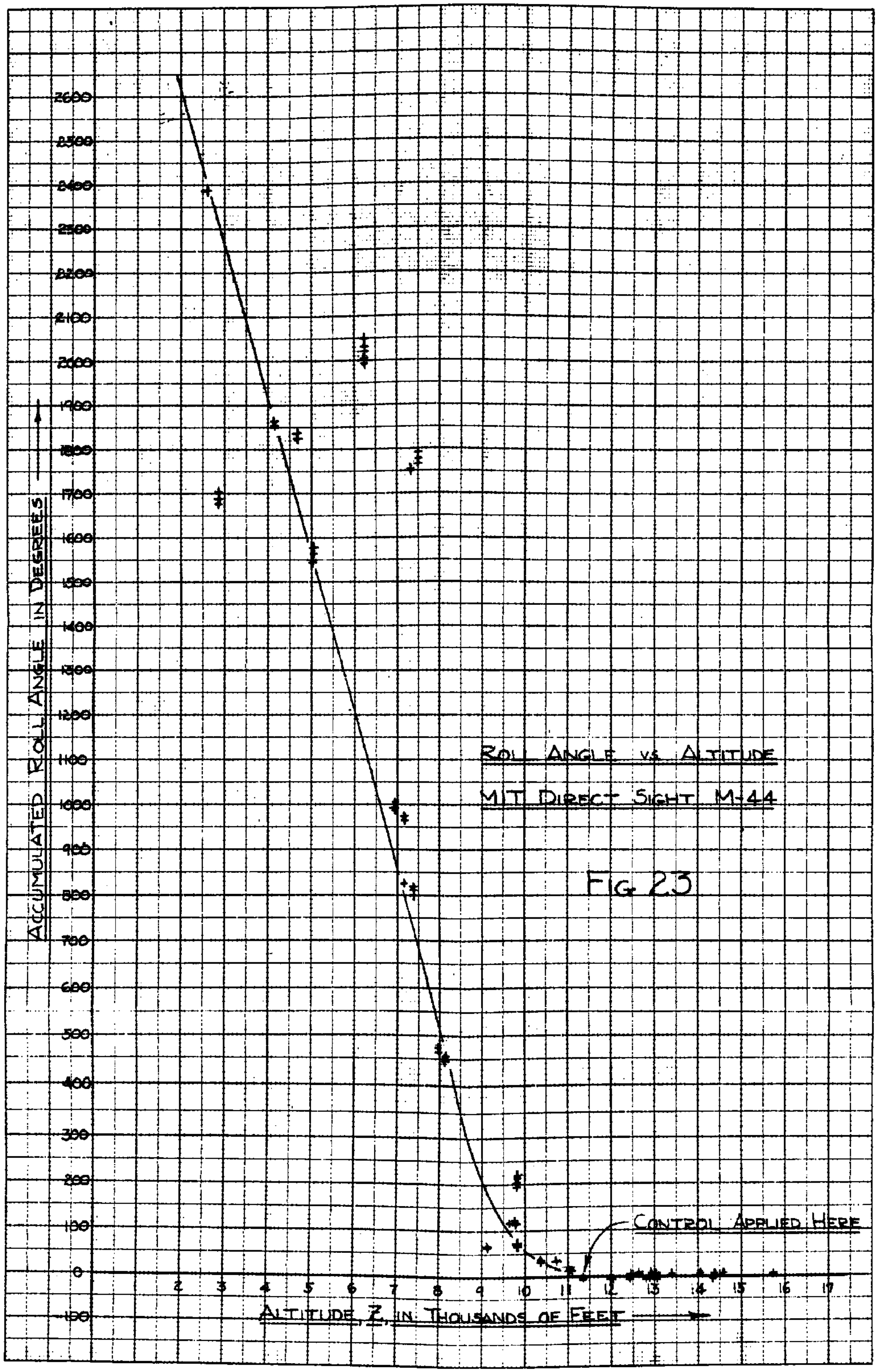


Fig 22
MIT Converted M-44



Roll Angle vs Altitude
M.I.T. Direct Sight M-44
Accumulated Roll Angle in Degrees
Altitude, Z, in Thousands of Feet
Control Applied Here
2600
2500
2400
2300
2200
2100
2000
1900
1800
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
-100
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

Fig 23

[redacted]

approximate time (from the trajectory calculations) associated with a particular roll angle was known. It was also noted that the angular velocity tended to increase as the bomb fell. Thus Fig. 23 was constructed by adding the right number of 2π's to each roll angle to have the angular velocity increase as the bomb fell, and by having the accumulated roll angle difference between two points agree roughly with the product of the average angular velocity between the points and the time difference between the points. Thus, though the absolute times associated with the frames may be off by a matter of seconds, it is probable that the time difference between two frames not too far separated is suffuciently accurate to merit calculation of roll forces. The approximate validity of the assumptions made in obtaining Fig. 23 is demonstrated by the fairly smooth relationship obtained.

Fig. 23 has two important features:

1. The bomb held its roll orientation satisfactorily down to an altitude slightly greater than 11000 feet. From then on it went into a continuous spin with apparently ever increasing angular velocity. It should be recalled in this conjunction that for all the M. I. T. bombs, the directional gyro was limited in its motion to about $\pm 45°$ by stops. Hence, once the bomb rolls more than 45°, the gyro hits the stop, and the stabilizing system loses its "memory."

[redacted]

2. The break-away point at slightly better than 11000 feet checks with the trajectory calculations and with the time control was applied. The control signal was sent to the bomb 15 seconds after release, and the trajectory calculations put the altitude at this time at 11400 feet. Hence, the bomb went into the roll shortly after the controls were applied.

Assuming the correlation between accumulated roll angle and altitude to be as smooth as we've drawn it, we can then get the approximate velocity in the z direction by computing first the quantity $\frac{\Delta z}{\Delta \phi}$, and then computing $\frac{\Delta z}{\Delta t}$ by using the frame speed of 22 per second. Fig. 24 compares the velocities obtained by the film analysis to the velocities computed from trajectory calculations. Considering the assumptions made in getting this relationship, the correlation is not bad. The cross-over effect can be explained if the camera started out the time interval at faster than 22 frames per second and ended up going somewhat less than 22.

Let us now associate times (through the trajectory calculations) with even the off-curve points in Fig. 23 by moving these horizontally into the curve, and plot angular velocity against time. Fig. 25 shows a smooth relationship and indicates the bomb was probably spinning a revolution per second by the time it hit.

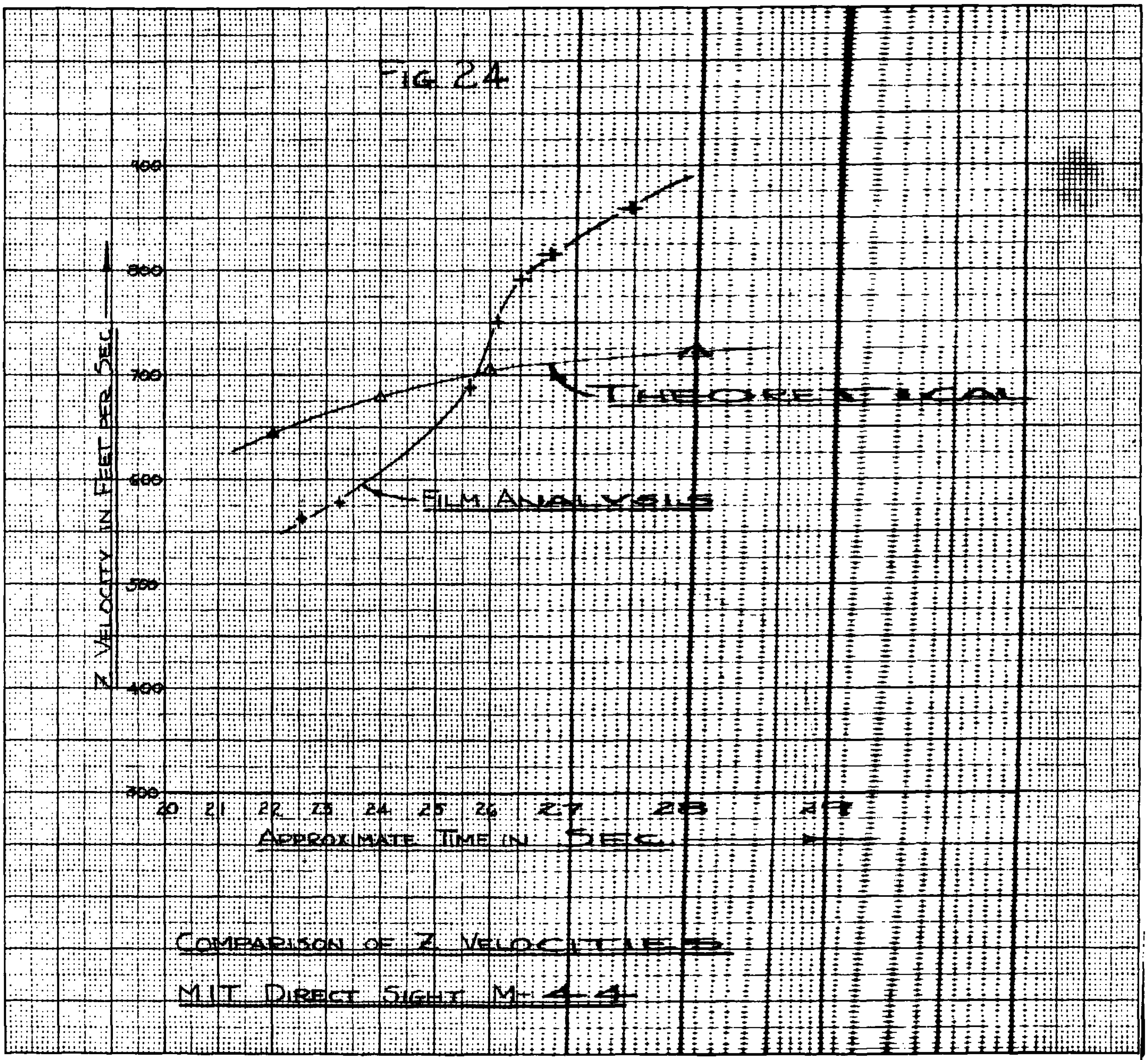
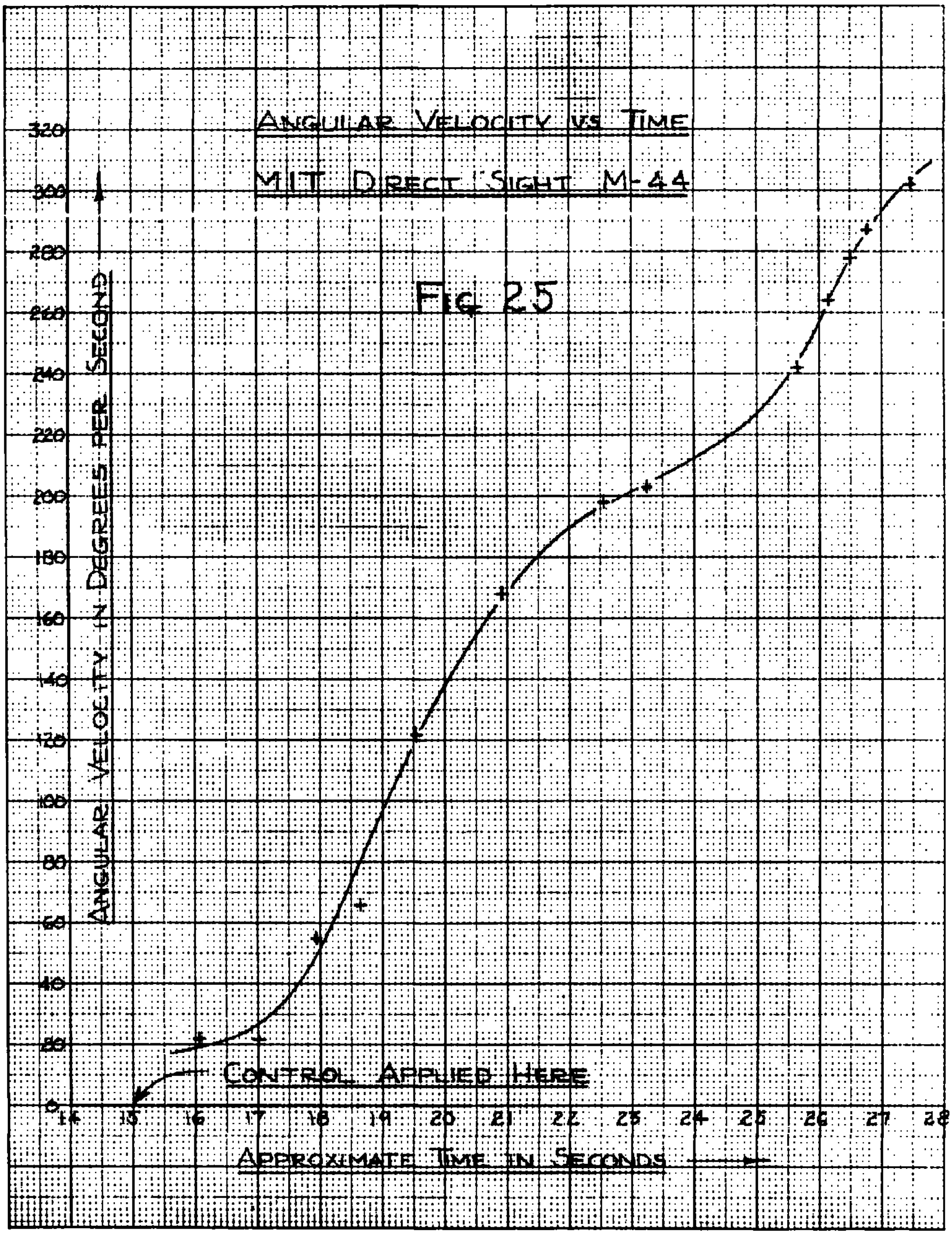


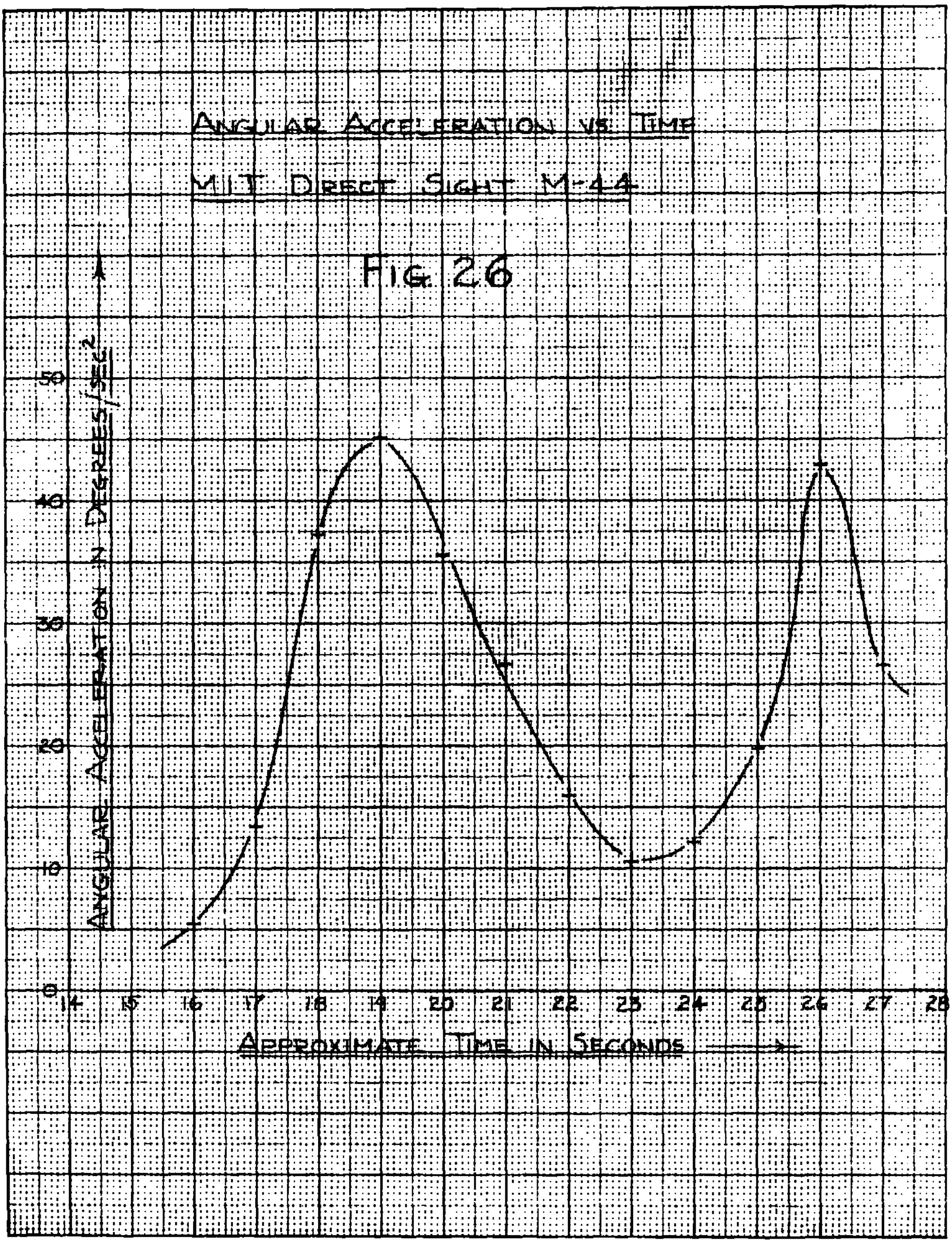
FIG. 24
Z VELOCITY IN FEET PER SEC.
900
800
700
600
500
400
300
20
21
22
23
24
25
26
27
28
APPROXIMATE TIME IN SEC.
THEORETICAL
FILM ANALYSIS
COMPARISON OF Z VELOCITIES
MIT DIRECT SIGHT M-4-4


ANGULAR VELOCITY VS TIME
MIT DIRECT SIGHT M-44
FIG 25
ANGULAR VELOCITY IN DEGREES PER SECOND
CONTROL APPLIED HERE
APPROXIMATE TIME IN SECONDS
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28

[redacted]

Differentiating the curve of Fig. 25, we get Fig. 26. Fig. 26 is admittedly rough, the height of the first maximum being subject to considerable error, and the very existence of the second maximum being questionable. However, using the rough value of 7.2 slug-ft.^{2} for the bomb moment of inertia about the longitudinal axis, let us convert the accelerations to roll torques. The roll torques are then reduced to the value they would have at 150 m.p.h., through the squares of the velocity ratios using total velocities as obtained from the theoretical calculations. This "uncontrolled" roll torque is the upper shaded area in Fig. 27, the lower area being the estimated stabilizing torque built into the bomb. From Fig. 27, we thus conclude the bomb would have been held under roll control if the ailerons had been capable of delivering 3.2 lb.-ft. of torque at 150 m.p.h. instead of 2.4 lb.-ft. Due to the apparent sharpness of the transient roll acceleration which accompanies the application of control, and, hence, due to the difficulty of measuring the acceleration accurately, a torque of from 6 to 7 lb.-ft. at 150 m.p.h. is recommended for this bomb. Thus the torque should be greater than the present design by a factor of from 2.5 to 3.

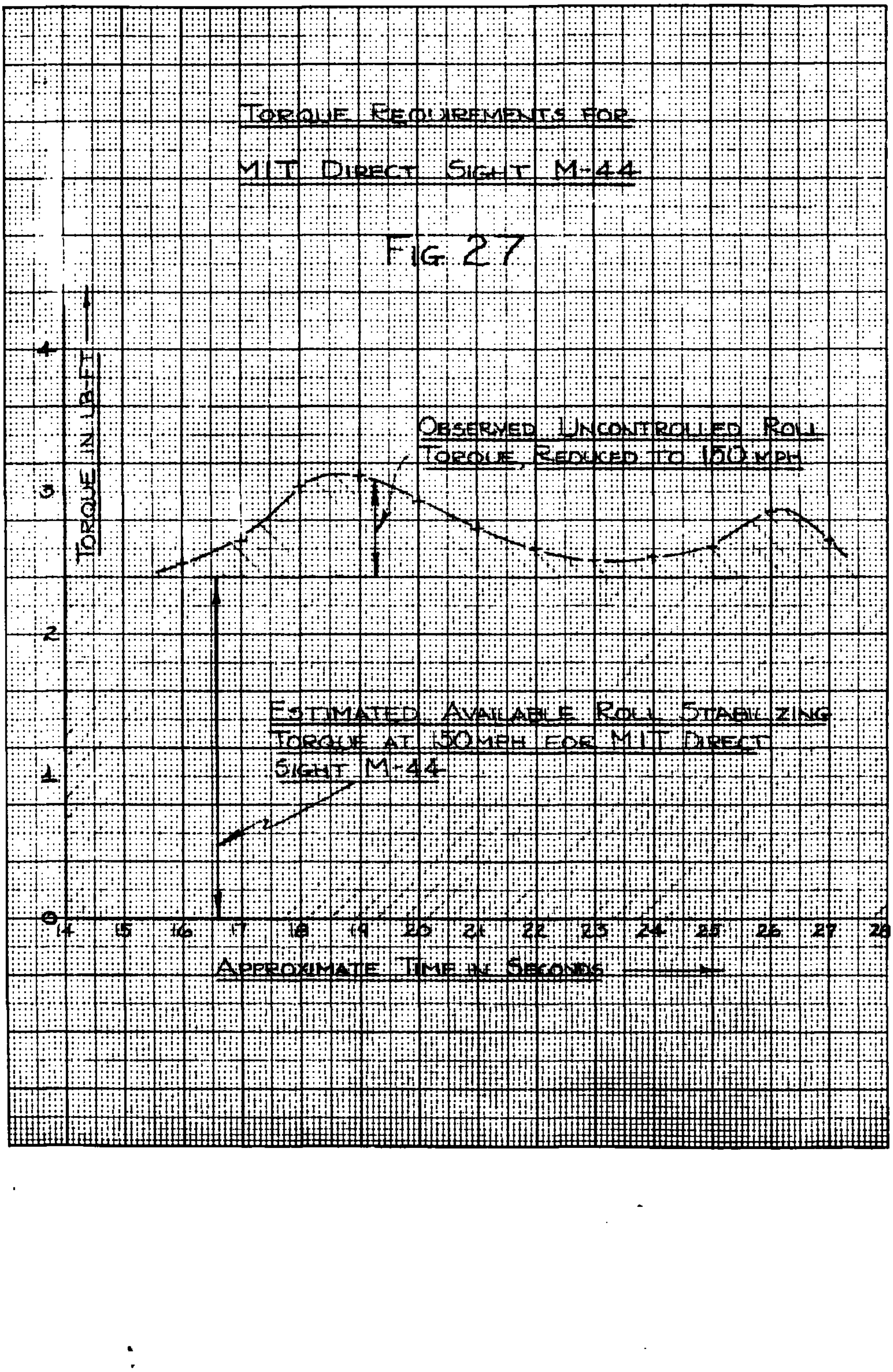
Finally, from the methods outlined in Chapter VI, Section C, we plot the yaw, pitch, and attack angles against time for the first few seconds following the application of control. See Fig. 28. Since the pitch angle should remain constant, it can be seen that control was completely lost



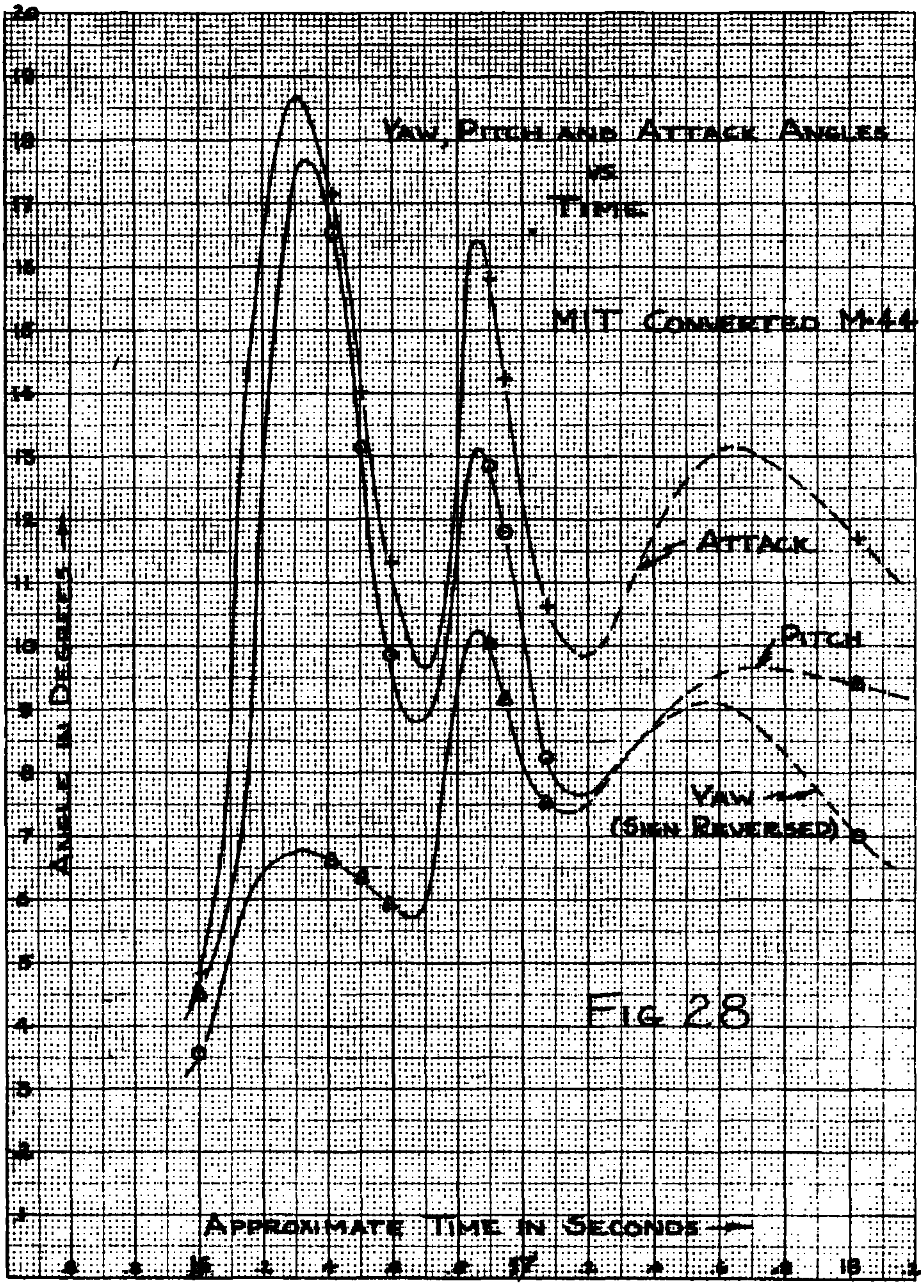
Angular Acceleration vs Time

MIT Direct Sight M-44

Fig. 26



TORQUE REQUIREMENTS FOR
MIT DIRECT SIGHT M-44
FIG 27
TORQUE IN LB-FT
4
3
2
1
0
OBSERVED UNCONTROLLED ROLL
TORQUE, REDUCED TO 150 MPH
ESTIMATED AVAILABLE ROLL STABILIZING
TORQUE AT 50 MPH FOR MIT DIRECT
SIGHT M-44
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
APPROXIMATE TIME IN SECONDS


Yaw, Pitch and Attack Angles
vs
Time.
M.I.T. Converted M-44
Attack
Pitch
Yaw
(Sign Reversed)
Angle in Degrees
Approximate Time in Seconds
Fig 28

[redacted]

after about 16.6 seconds, or about 1.6 seconds after the control was applied. Other conclusions that can be drawn are:

1. The period of oscillation about any lateral axis is about .6 sec. at 350 m.p.h. From Fig. 22, Sixth Progress Report, Fin Type I, this period is calculated as 0.56 seconds. Thus, this bomb would have a period of about 1.4 sec. at 150 m.p.h. and about .36 sec. at 590 m.p.h., the speed at impact.

2. Using the attack angle curve, the damping constant is about .8 sec^{-1} at 350 m.p.h., where the damping constant is used in the equation:

$$\theta_{(t+\tau)} = \theta_{(t)} e^{-.8\tau} \qquad \tau = \text{period}$$

3. Equilibrium yaw angle appears to be about 13 degrees, checking roughly with the wind tunnel value of 11.6°.

4. From other measurements on the trajectory, there appears to be a pitch angle of about 3° or 4° (sailing). This is to be expected, since the bomb rolled counter-clockwise (looking along the bomb, toward the earth) when negative yaw was applied. See Seventh Progress Report for discussion of this effect.

[redacted]

C. Television Dirigible Bombs.

In all of four drops using television, an adequate picture was received in the plane, and motion pictures of the screen are available for inspection. More detail on television performance can be obtained from N. B. C. No theodolite records were taken of any of the television drops.

For preliminary testing, each bomb was set up on the ground so it could project its signal vertically to an airplane flying at 15000 ft. It was found that a good quality picture could be received for about 10 miles either side of the bomb in the correct polarization plane and for about 5 miles either side of the bomb in the incorrect polarization plane. Both receiver and transmitter used matched dipole antennas. On the basis of these preliminary tests and the drop results, it can be said that satisfactory propagation is now possible.

Before the tests at Eglin, similar tests were made at M. I. T. at altitudes up to 10000 ft. Considerable trouble was encountered with the television reception at this test because of interference from a Radar station in the Boston area. The result was that the picture failed when the plane flew more than several miles away from the transmitter. During these tests, the control transmitter was tried out, improved, and was found to work satisfactorily. It will be recalled that the control transmitter operated on 84 megacycles with four audio modulations corresponding to up-down and left-right of the bomb's control surfaces, and radiated 25 watts. It was found that it produced no interference in the television receiver and the control receiver got no interference from the television transmitter.

At Eglin Field, the first bomb dropped was equipped with a parachute and contained nothing but a television transmitter. General Gardener, in charge of Eglin Field, and General Davidson were Air Corps observers. The bomb was released at 110 m.p.h. from 10000 ft. and while it was falling the airplane circled and climbed to 13500 ft. Throughout the 5 minute - 37 second flight of the bomb, excellent detail of roads, trees, etc., was received in the plane. The test showed that there are no serious ground reflections, that the plane of polarization is not too critical, and that the equipment is quite rugged as it withstood both the impact of the parachute opening at 110 m.p.h. and the impact on landing. The bomb was recovered unharmed. It is worthy of note that this equipment had been adjusted on the ground before take-off and was not touched again until the tests were completed.

This same bomb was dropped a second time without the chute. It was released at 150 m.p.h. from 15000 ft., and, again, during the 35 second flight an excellent picture was received in the plane. On observing the

[redacted]

picture, one does not get the feeling of high speed nor does he feel the need for haste in making a decision. The observers were able to tell from the television receiver within 100 ft. of where the bomb landed. Acoustical disturbances associated with the high speed apparently do not affect seriously the picture quality. Not having any gyroscopic roll control, this bomb made several revolutions about its longitudinal axis during its flight.

The third television drop was made with a bomb containing a television transmitter and gyro roll control but no control receiver. Prior to the drop, the bomb was flown in the plane and allowed to view the ground by means of a 45 deg. mirror. A minor contrast adjustment was then made at 10000 ft. The bomb was launched at 150 m.p.h. from 15000 ft. at an angle of 24.5 deg. between the line pointing to the target and the line pointing down the vertical. The bomb weighed 500 lbs. and was the 1000 lb. size. The signal from it was received both in the plane and by a receiver on the ground and it was felt by observers at both receivers that picture contrast and detail was better than the requirements for which television bombing call. There was apparently little acoustical disturbance during the 32 seconds of flight.

The motion pictures of the receiver in plane were run through the M. I. T. film analyzer to try to determine why the bomb rolled. To determine the effective

[redacted]

focal length at which the motion pictures were taken, N. B. C. supplied the following data:

1. Focal length of lens in front of iconoscope: 8 1/4".
2. Electrical magnification from iconoscope mosaic to kinescope surface: approx. x 1.50.
3. Focal length of recording movie camera: 1".
4. Distance of movie camera from kinescope: 15".

From the above data, the effective focal length is about 0.88". If this focal length is used to reduce the analyzer data there is a very poor check between computed altitude of release and the observed altitude of release. Hence an effective focal length of 1.412" was used to bring the two altitudes into coincidence. From the known time of flight and trajectory calculations, a frame speed of 21.6 frames per second was used. A comparison between altitude vs. time as found by the analyzer and the same relation as determined by trajectory calculations is shown in Fig. 29. From Fig. 29 it can be seen that trajectory results from the analyzer should be subjected to some sort of "smoothing function."

Fig. 30 gives roll angle against time and can only be interpreted as showing that there was no directional gyro roll control at all, the slight return starting at 19 seconds being purely accidental. It should be recalled that the directional gyro in this bomb was limited in its motion to about ± 45 deg. by stops, hence all "memory" was lost after the 11th second.

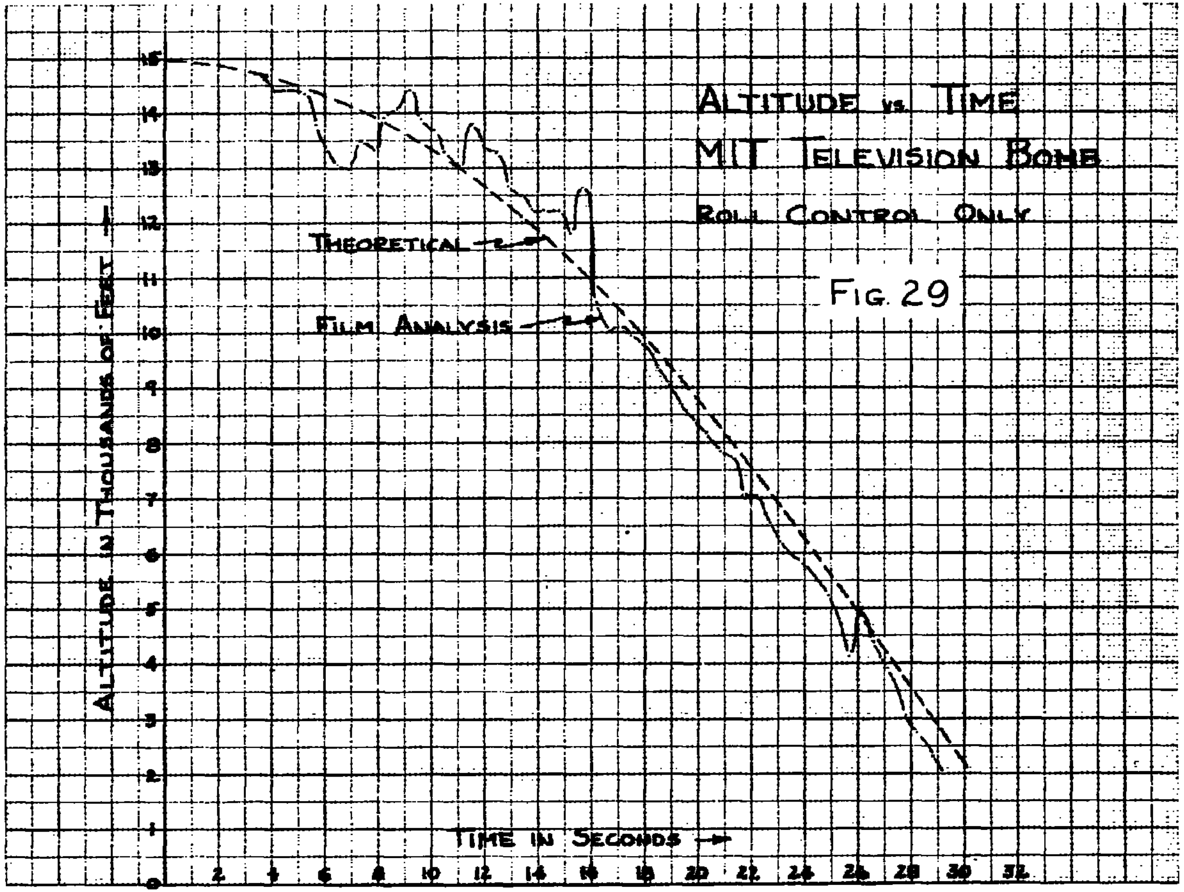

ALTITUDE vs. TIME
MIT TELEVISION BOMB
ROLL CONTROL ONLY
THEORETICAL
FILM ANALYSIS
ALTITUDE IN THOUSANDS OF FEET
TIME IN SECONDS
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32

FIG. 29

627

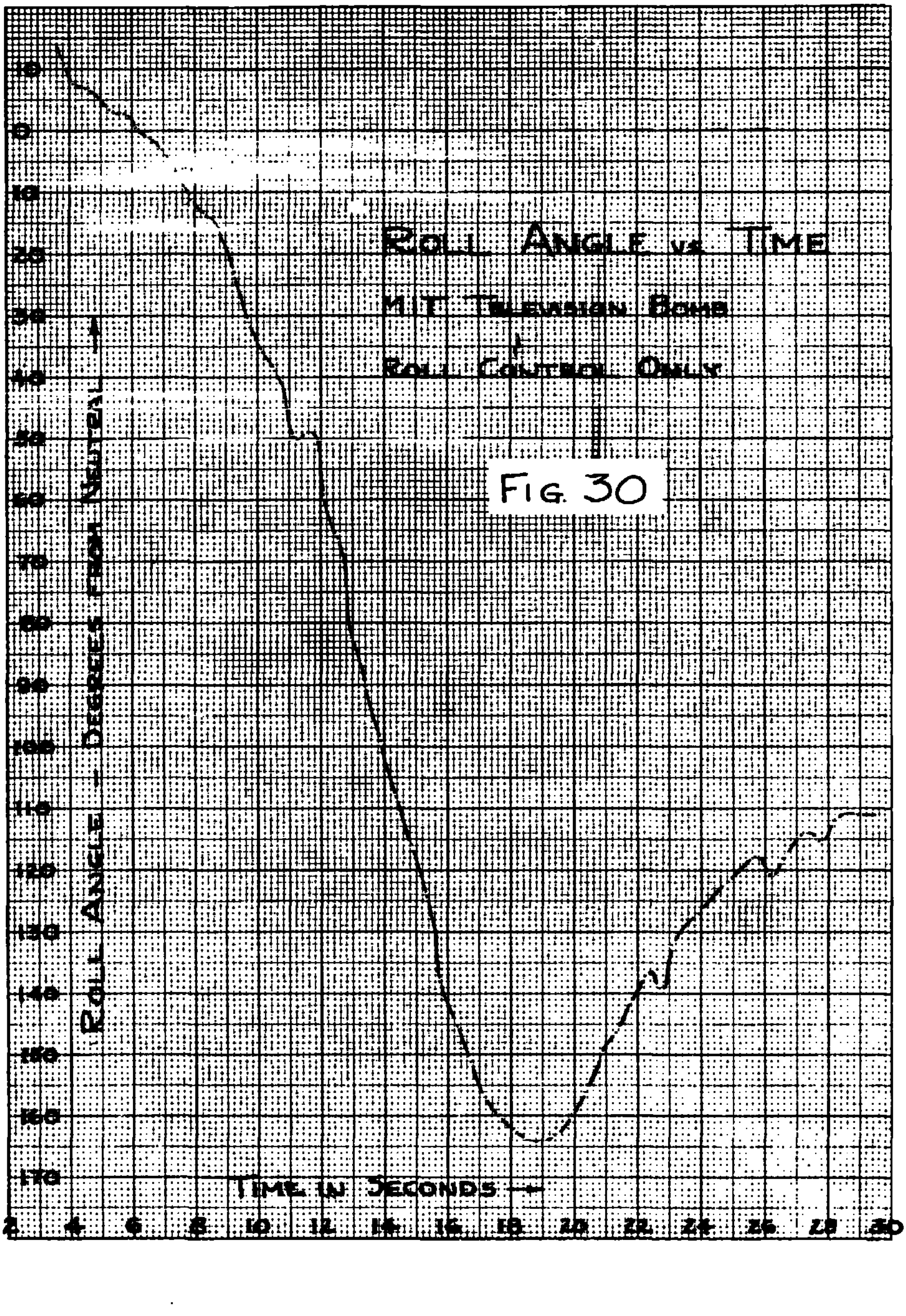

ROLL ANGLE vs TIME
M.I.T. TELEVISION BOMB
ROLL CONTROL ONLY
ROLL ANGLE — DEGREES FROM NEUTRAL
TIME IN SECONDS
10
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30

FIG. 30

[redacted]

The last drop was made with a bomb containing a television transmitter, gyro roll control, and control receiver. The bomb was released at 150 m.p.h. from 15000 ft. with an angle of 20.5 deg. between the direction to the target and the direction downward. There were clouds which sometimes obscured the ground and some electrical interference with the television signal was observed. This bomb rolled from the start and rolled even more violently when the controls were applied. However, the target was clearly recognized and for a few moments the application of control tended to bring the target area nearer the center of the picture. Since the gyro control was obviously inoperative, no film analysis was made on this bomb.

D. Conclusions on Television Tests.

Since the problem of roll and remote control has been solved (See Gulf reports on recent drops), and the television aspects of the tests described above appear well under control, the problem of constructing a television high-angle dirigible bomb appears merely to be the inclusion of television equipment of the type described above into the Gulf vehicle.

E. Theodolite Records.

At the time of the above tests, the Army set up two theodolite stations at Eglin Field for the purpose of determining the trajectories of some of the bombs dropped

[redacted]

by the groups from Gulf, the Bureau of Standards, and M. I. T. Twelve theodolite records were taken and two members of the M. I. T. group went to Camp Davis, N. C., to observe and assist in the reduction of the films. Out of these 12 records, 4 trajectories are presented in this report, the other 8 being rejected for various reasons as explained below. It should be remembered that the records were not taken under the most favorable conditions: the base-line was probably too short (4460'); trees between the stations made difficult the determinations from the films of the azimuth orientation corrections for the cameras; there was not always sufficient contrast between the bomb and sky, thus making ambiguous the location of the bomb on the film. It should be recalled that an unsatisfactory record from one station is sufficient to make useless the run from the other station regardless of the excellence of the film from this second station. Besides the difficulties inherent in taking the theodolite records, a number of blunders were made by the Army and by people from the projects whose bombs were being tested, these blunders either making the film useless, or so difficult to read that the resultant trajectories were obviously wrong.

The table below covers all the films examined and the disposition made of them. The following explanations are necessary for a complete understanding of the table:

[redacted]

GRD--Gulf Research and Development Co.-- followed by a bomb number.

NBS--National Bureau of Standards--followed by a bomb number.

MIT--Massachusetts Institute of Technology--followed by bomb number (both are direct-sight). The one marked "flare" was a 100 lb. practice bomb equipped with a flare to test visibility.

Under "Counter Start" and "Counter Stop" are given the times in seconds the theodolites started and stopped. Arbitrary origin. In the case of GRD No. 1, the wrong times were recorded. However, the correct times (denoted by asterisk) were later ascertained and are inserted.

Filter numbers are Army designations of filters used in the theodolites to increase contrast.

"Speed" is the camera speed in frames per second.

Under "Army's Remarks" are recorded the length of film shot, their opinions of the record, and orientation information. All remarks attributed to the Army were copied from the notebook kept by one of the Army men sent from Camp Davis to man one of the cameras.

Under "Our Remarks" are given our opinion of the runs, the disposition of the record, and the reason for the disposition.

The north theodolite station is also referred to as "O_1" or camera 73.

The south theodolite station is also referred to as "O_2" or camera 111.

Bomb No.	Date Dropped	Army's Data				Army's Remarks	Our Remarks
		Counter Start	C't'r Stop	Filter	Speed		
GRD No. 1	12-19-42 PM	1130 1305*	1171 1371*	25A	16	55' (Good)	Stat. 73 - Excellent Run (O_1) Stat. 111 - Didn't track at all (O_2) (One of the members of a civilian group manned camera) Not computed or plotted.
GRD No. 2	12-19-42 PM	1385	1422	25A	16	O_2 Lost O_1 Good	O_1 - Good run O_2 - Due to fogging no plane shots of extended interval. Tracked exact release time, first 8 seconds, and last 9 seconds. Analyzed and plotted first 8 secs. and last 9 secs. Checked very poorly with movies taken from plane. Not submitted.
GRD No. 3	12-20-42 AM	1975	2020	25A	16	O_1 Good, O_2 Lost. O_1 Camera was not exactly on 0 for orientation at the end of mission 12-20-42 AM O_2 Missed first bomb 12-20-42 AM	O_1 - Good run. O_2 - Completely lost -- bomb not visible. Not computed or plotted.

FAO* = First Altitude Obtained   LAO* = Last Altitude Obtained

Bomb No.	Date Dropped	Army's Date				Army's Remarks	Our Remarks
		Counter Start	C't'r Stop	Filter	Speed		
GRD No. 4	12-20-42 PM	2020	2071	5N5	16	Good O_1 Camera was not exactly on O for orientation at the end of mission 12-20-42 AM	O_1 - Last part only O_2 - Last 13 seconds only. Times on cameras not synchronized. FAO* = 3396 yds. Time 2055 LAO* = 638 yds. Time 2066 Hits at approx. 2068. After synchronization straightened out, last 11 seconds analyzed, plotted, and submitted. Hence, unknown quantity of beginning and time of release missed. One of the tracking cameras couldn't find bomb until near the end of the flight.
GRD No. 5		NO	RECORD.	Theodolites not set up.			
GRD No. 6		NO	RECORD.	Theodolites not set up.			
GRD No. 7	12-18-42	14	90	25A	20	130' film (Good) Time interval device was not adjusted on the 18th.	O_1 - Good. O_2 - Good. 8 seconds of tracking plane. Exact release unknown due to poor film development. FAO* = 5055 yds. at 30.05 secs. LAO* = 73 yds. at 82.00 secs. 37-42.05 secs. frames not computable. Most satisfactory all-around tracking job. Entire bomb plotted and submitted.

Bomb No.	Date Dropped	Army's Data: Counter Start	C't'r Stop	Filter	Speed	Army's Remarks	Our Remarks
GRD No. 8	12-21-42 PM	2657	2710	5N5	16	Good	O_1 - First 33 secs. of flight. O_2 - Last 17 secs. of flight. Last part of O_1 gone because at the request of one of the civilians portion torn off, developed at Eglin, and lost. Only 1 point in common -- not computed or plotted.
GRD No. 9	12-20-42 PM	2344	2402	5N5	16	Good	O_1 - Good run O_2 - Lost first 10 secs. (approx.) poor tracking. Computed and plotted all but first 18-19 secs. Plane 2370-2373 (secs.) Bomb 2374-2399 (secs.) Computations start 2387 (secs.) Plot does not agree with triangulated impact point or plane movies. Hence not submitted.
GRD No. 10	12-21-42 AM	2554	2598	5N5	16	Good	O_1 - Portion of plane flight -- but bomb does not show at all. No good. O_2 - Last 19 sec. of bomb flight. For O_1: 2554-2570 (secs.) Plane in sight. 2570 (secs.) Plane disappears. 2570-2579 (secs.) Nothing in sight. 2579 (secs.) Film ends. Not computed or plotted.

Bomb No.	Date Dropped	Army's Data Counter Start	C't'r Stop	Filter	Speed	Army's Remarks	Our Remarks
NBS No. 1	12-19-42 AM	1001	1083	39	16	95' (Good)	O_1 - Good run - but no tracking of plane or time of release. O_2 - Counter does not start on time. Time of release and unknown quantity of beginning missed. Last 74 secs. of flight obtained. They were computed, plotted, and submitted.
NBS No. 2	12-20-42 PM	NO RECORD				First bomb in afternoon 12-20-42 both cameras were off target. Plane obscured by clouds and unable to pick him up after he came out of the cloud.	
MIT No. 1	1-6-43 PM	2944	3047	25A	16	Good Orientation shots: 1-6-43 PM 2898	O_1 - Good run except that fog (photographic?) obscures exact release point. O_2 - Good run. Almost complete flight computed and plotted. First (1) or (2) seconds of bomb flight missing, probably. Bears no relation at all to plane movie. Possibly confused with a practice flare bomb. Not submitted.

Bomb No.	Date Dropped	Army's Data				Army's Remarks	Our Remarks
		Counter Start	C't'r Stop	Filter	Speed		
MIT No. 2	1-9-43 PM	3453	3489	x	16	Good. Orientation shots: 1-9-43 AM 3078	O_1 - Fogging kills release point. O_2 - Poor tracking. Computed and plotted after adjusting times on cameras -- got out of step by 1 sec. Unknown portion of middle computed (3466-3487 secs.)
MIT Flare	1-9-43 PM	3330	3373	x	16	Good	O_1 - OK O_2 - Time counter didn't work according to Army. Not computed or plotted.

[redacted]

Fig. 31 is the result of the theodolite analysis on Gulf Bomb No. 4. Only the last portion of the flight is available. The method of showing the trajectories is explained under Chap. VI, Section C. See Fig. 14 for comparison. This bomb lost roll control at about the 21st second, checking roughly with the theodolite-derived trajectory, which starts exhibiting "wiggle" at about the third second (different time origin). For comparison see Fig. 11, Gulf Progress Report of March 15, 1943; and Fig. 9, Division 5 Bimonthly Report of February 15, 1943.

Fig. 32 is the trajectory of Gulf Bomb No. 7 as obtained from the theodolite records. From the film analyzer were obtained Fig. 33, the trajectory of principal points, and Fig. 34, giving the roll angle and aileron action as functions of time.

Fig. 34 gives a good explanation of the peculiar trajectory this bomb took. It should be recalled that this bomb had the long "side-burn" fins and, hence, was very susceptable to the roll torques developed in combined pitch and yaw. Aileron lights 1, 2, 3, and 4 were photographed on the film in the nose of the bomb through a partially silvered mirror. If we define clockwise rotation as that which seems clockwise to an observer riding in the bomb and looking out through the nose, then the lights indicate the following:

No. 1. The clockwise rotational velocity is greater

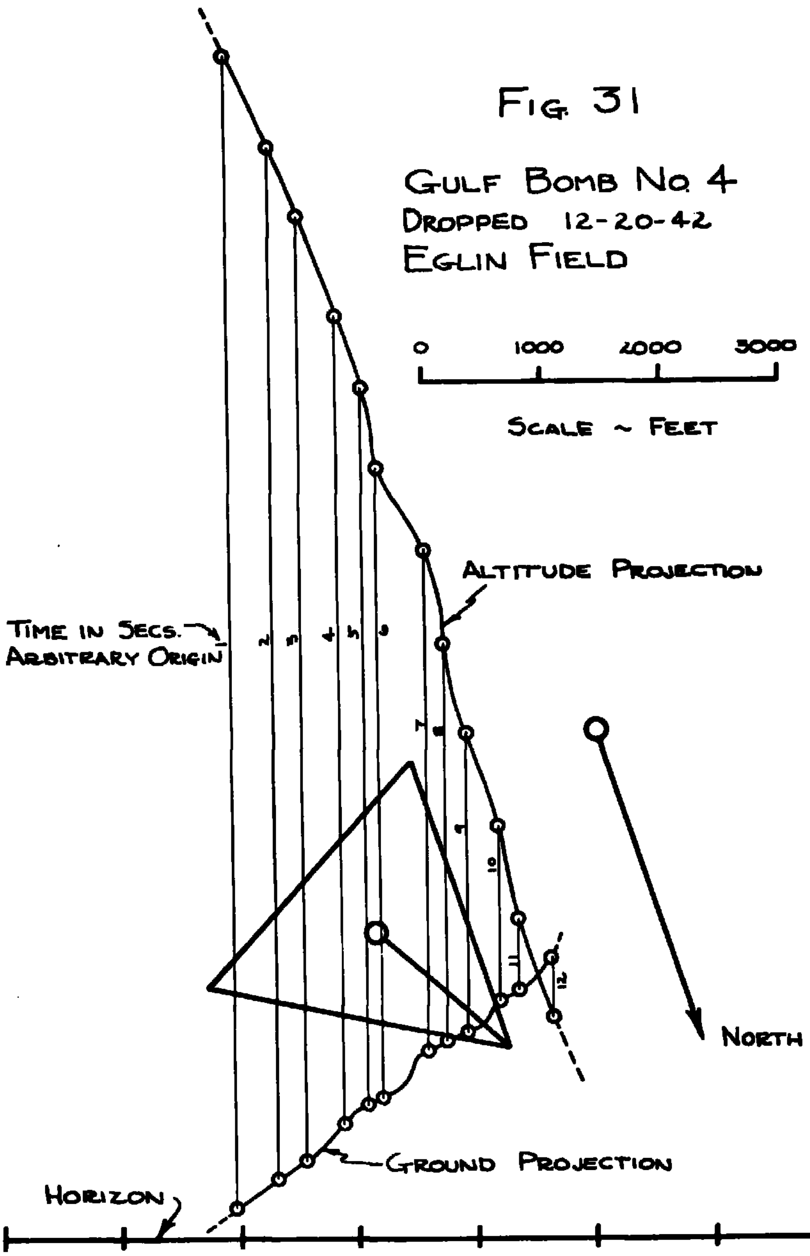
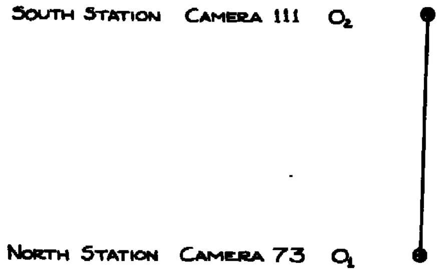


FIG. 31

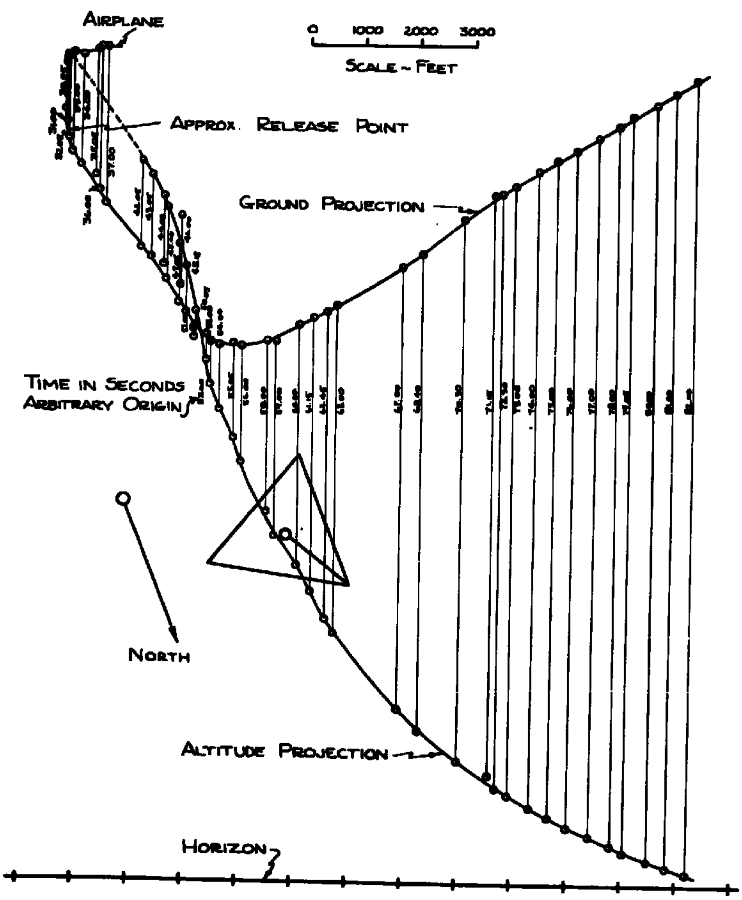
GULF BOMB NO. 4
DROPPED 12-20-42
EGLIN FIELD

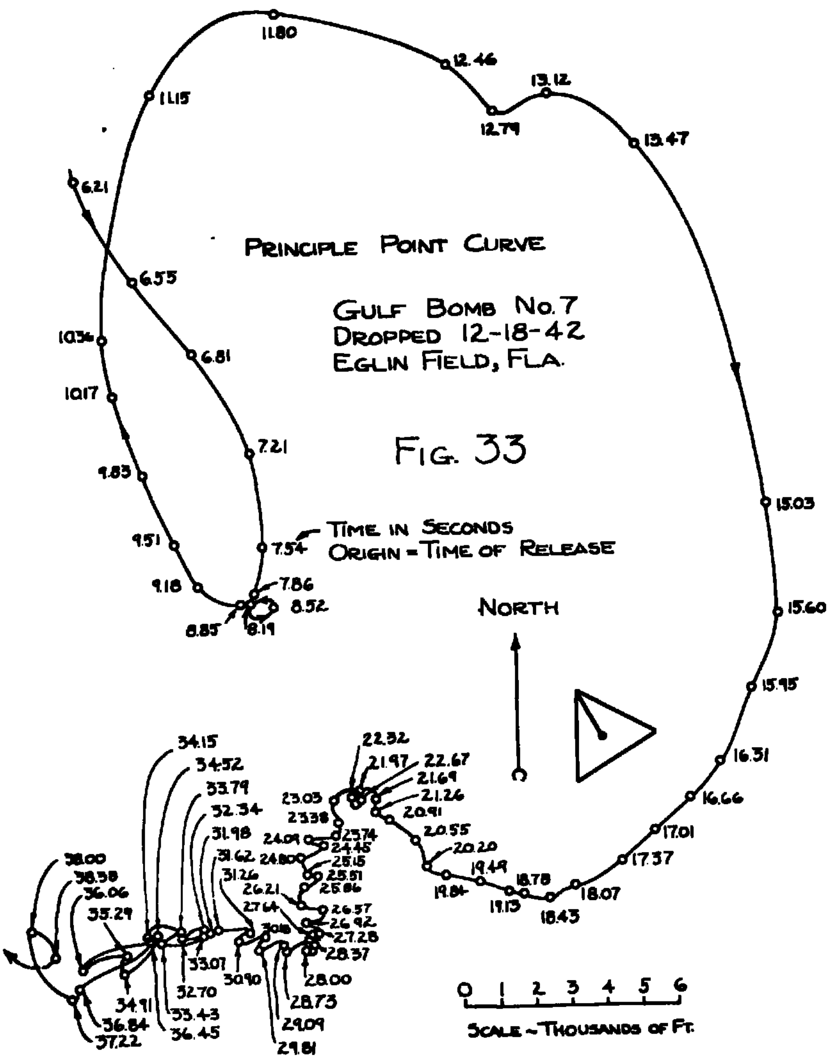
7113



GULF BOMB NO. 7
DROPPED 12-18-42
EGLIN FIELD, FLA.

FIG. 32





Principle Point Curve
Gulf Bomb No. 7
Dropped 12-18-42
Eglin Field, Fla.
Time in Seconds
Origin = Time of Release
North
Scale ~ Thousands of Ft.
0 1 2 3 4 5 6
6.21
6.55
6.81
7.21
7.54
7.86
8.19
8.52
8.85
9.18
9.51
9.83
10.17
10.34
11.15
11.80
12.46
12.79
13.12
13.47
15.03
15.60
15.95
16.31
16.66
17.01
17.37
18.07
18.43
18.78
19.13
19.49
19.84
20.20
20.55
20.91
21.26
21.69
21.97
22.32
22.67
23.03
23.38
23.74
24.09
24.45
24.80
25.15
25.51
25.86
26.21
26.57
26.92
27.28
27.64
28.00
28.37
28.73
29.09
29.81
30.18
30.90
31.26
31.62
31.98
32.34
32.70
33.07
33.43
33.79
34.15
34.52
34.91
35.29
36.06
36.45
36.84
37.22
38.00
38.38

Fig. 33

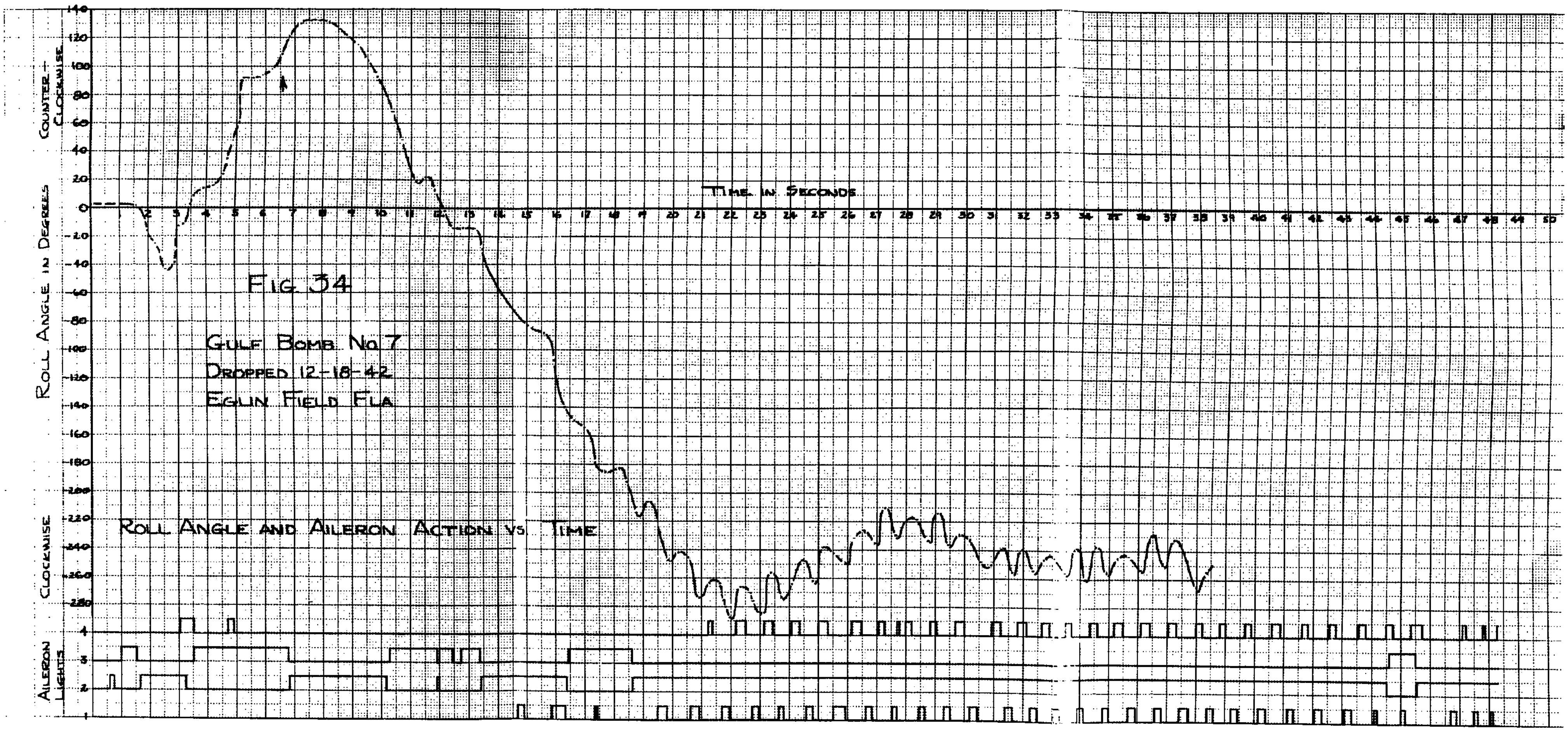


Fig. 34
Gulf Bomb No. 7
Dropped 12-18-42
Eglin Field Fla
Roll Angle and Aileron Action vs Time
Time in Seconds
Roll Angle in Degrees
Counter-Clockwise
Clockwise
Aileron Lights

[redacted]

than about 7 deg. per second.

No. 2. The clockwise displacement is greater than about 1 1/2 deg.

No. 3. The counter-clockwise displacement is greater than about 1 1/2 deg.

No. 4. The counter-clockwise rotational velocity is greater than about 7 deg. per second.

In all of the above cases when the lights go on they indicate also that the ailerons are being actuated to oppose the motion indicated by the lights.

Referring to Fig. 34, not until about 6.5 seconds was the ground distinct enough to make alignments, indicated by arrow, hence all alignments up to this time were performed by making the horizon, as seen on the film, coincident with a line on the map and normal to the original direction of release. For about 1 1/2 seconds, roll is adequately controlled, but shortly afterwards a clockwise torque sets in (due presumably to some momentary conditions of combined pitch and yaw) which the ailerons are unable to oppose effectively. The bomb continues to roll to approximately 45 deg. at which point the directional gyro hits the stop and loses its orientation. The subsequent rolling of the bomb resulted in the elevators, originally set for sailing, becoming rudders every quarter revolution. The sharp left turn is accounted for by the 270 deg. roll of the bomb. Since the bomb is in a nearly vertical position at the time it makes the turn, it is apparent that after

[redacted]

the turn the bomb will find itself again in a sailing position. As can be seen from the roll angle record, no transient conditions of combining pitch and yaw existed in the latter part of the flight, hence the bomb maintained fairly good roll orientation, even though the directional gyro was not operating properly. Since the rate gyro is not affected by roll displacement, it can be seen from the record that it was operating satisfactorily during the last portion of the flight. Apparently there was sufficient time lag in the response of the ailerons to the demands of the rate gyro that a sustained roll oscillation was set up, the rate gyro thus keeping the roll displacement constant to within $\pm$ 10 deg. (neglecting the drift). For additional information on this bomb, see Fig. 14, Gulf Progress Report of March 15, 1943; and Fig. 9, Division 5 Bimonthly Report of February 15, 1943.

Fig. 35 is the theodolite-derived trajectory of one of the Bureau of Standards bombs. As can be seen from the scattering of points, the last portion of the flight is extremely uncertain. Since this is not a high-angle bomb, the trajectory is presented without comment.

The fourth theodolite trajectory is that of one of the M. I. T. direct-sight bombs and has already been presented and discussed in Chap. VII, Section B.

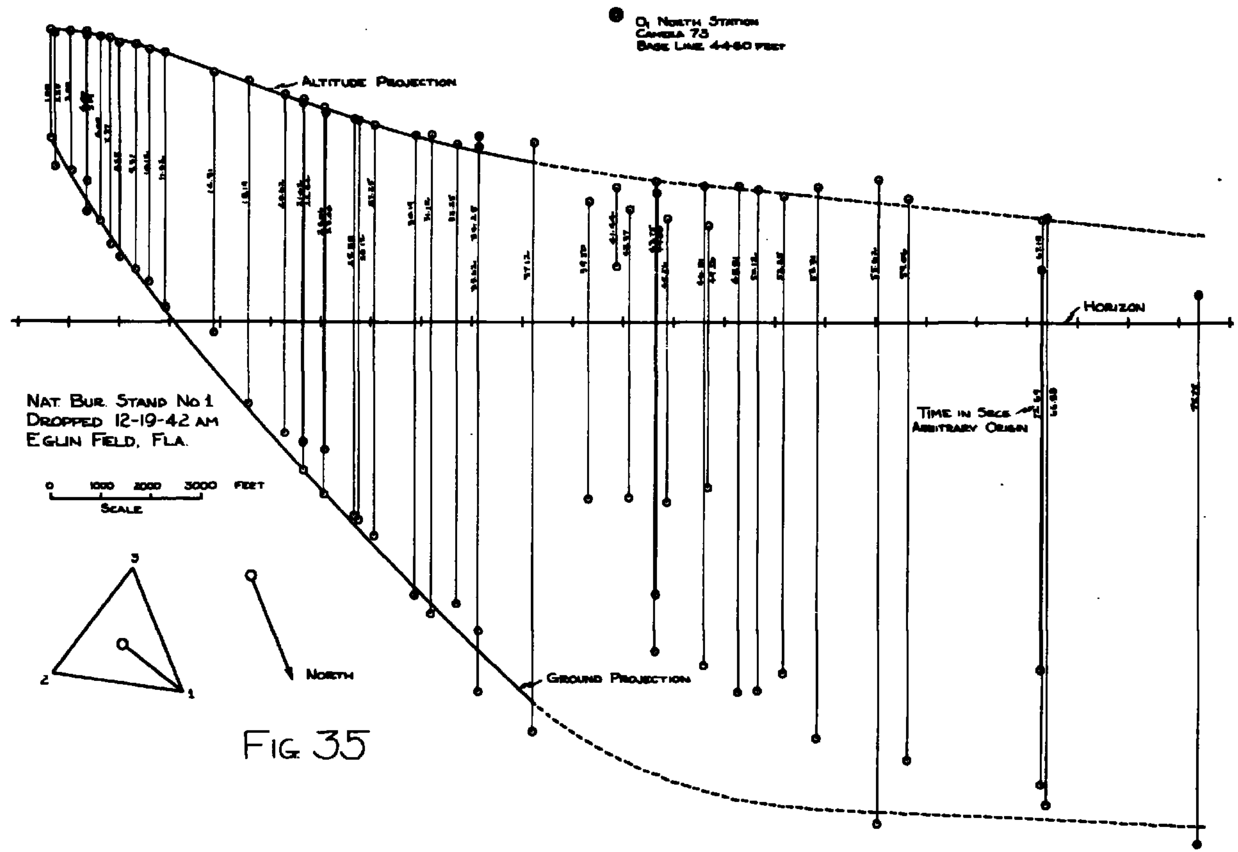

O1 North Station
Camera 73
Base Line 4460 feet
Altitude Projection
Horizon
Time in Secs
Arbitrary Origin
Ground Projection
Nat. Bur. Stand No 1
Dropped 12-19-42 AM
Eglin Field, Fla.
0 1000 2000 3000 feet
Scale
1
2
3
North

Fig. 35

Eighth Progress Report:

Written and submitted by

J. W. Fitzwilliam

Approved by

B. E. Warren

April 30, 1943.

SECRET

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REEL – C
68
A.T.I.

2074

TSDIN FORM 68 (13 FEB 47)

Fitzwilliam, J.

AUTHOR(S)

DIVISION: Guided Missiles (1)
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ABSTRACT

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P19/13 x Guided Bombs, Bomb Trajectories

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SECTION: Instrumentation (9)	
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