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LATERAL STABILITY OF HOMING GLIDE-BOMBS
WITH APPLICATION TO NAVY SWOD MARK 7 AND MARK 9

E/NDRC-NBS

by

Harold K. Skramstad
National Bureau of Standards

A report to Division 5
from the
National Bureau of Standards

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Preface

The work described in this report is pertinent to the projects designated by the War Department Liaison officer as AC-1, AC-36, and AC-42 and to the projects designated by the Navy Department Liaison officer as NO-115, NO-174, and NO-235. This work was carried out and reported by National Bureau of Standards under a transfer of funds from OSRD with the co-operation of the Washington Radar Group of the Massachusetts Institute of Technology and Section Re4g of the Bureau of Ordnance, Navy Department.

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LATERAL STABILITY OF HOMING GLIDE-BOMBS WITH APPLICATION TO NAVY SWOD MARK 7 AND MARK 9

1. Lateral Stability Equations

The motion of a glider in flight is determined by two factors, the aerodynamic forces and moments due to the reaction of the air on various parts of the glider, and the forces and moments due to gravity. The resultant of the gravitational forces may be represented by a force equal to the weight of the glider acting vertically downwards through the center of gravity. The resultant of the aerodynamic reaction is thereby conveniently represented by three mutually perpendicular forces acting through the center of gravity and three moments acting about these three mutually perpendicular axes which meet at the center of gravity. In general, a glider has a plane of symmetry, which in normal steady flight includes the direction of motion. For convenience, we will choose axes as follows: The X-axis is taken in the plane of symmetry in the direction of the relative wind during the steady flight condition, the Y-axis perpendicular to the plane of symmetry, and the Z-axis in the plane of symmetry and perpendicular to the X-axis. Rotation about the X, Y, and Z axes are denoted by the angles ϕ, θ, and ψ, respectively, and angular rates of rotation about these axes by p, q, and r.

We may ordinarily consider the motion of a glider divided into two independent types of motion. One type includes motion that does not displace the plane of symmetry of the airplane, called longitudinal motion, and the stability of motion in this plane is termed "longitudinal stability". The other type of motion includes all components that do displace the plane of symmetry, called lateral motion, and the stability of this motion is termed "lateral stability". The longitudinal stability of homing glide-bombs is discussed in detail in reference (1), and will not be considered further here. The present discussion of lateral stability refers principally to gliders in which the aileron is the only lateral control surface, although the discussion of stability with fixed control surfaces applies equally well to gliders equipped with both rudder and ailerons.

The stability characteristics of a glider are studied from the standpoint of the motion obtained from small displacements from a state of equilibrium. Under equilibrium conditions the glider flies in a straight line at constant speed with the plane of symmetry vertical, - wings level. The resultant air reaction lies in the plane of symmetry and passes through the center of gravity. Let us define the following quantities:

γ = angle between flight path and horizontal

ϕ = angle of roll

ψ = angle of yaw

δ = angular displacement of ailerons

W = weight of glider

m = mass of glider

V = velocity along X-axis

v = velocity in Y-direction (side slip velocity)

ρ = air density

S = wing area

b = wing span

Y = force along Y-axis (lateral force)

L = moment about X-axis (rolling moment)

N = moment about Z-axis (yawing moment)

p = $\frac{d\phi}{dt}$ = rate of roll

r = $\frac{d\psi}{dt}$ = rate of yaw

A = moment of inertia of glider about X-axis

C = moment of inertia of glider about Z-axis

Let us also define the following coefficients:

$$C_L = \frac{\text{Lift}}{1/2\,\rho S V^2} \qquad C_D = \frac{\text{Drag}}{1/2\,\rho S V^2}$$

$$C_\ell = \frac{L}{1/2\,\rho S V^2 b} \qquad C_n = \frac{N}{1/2\,\rho S V^2 b} \qquad (1)$$

$$C_y = \frac{Y}{1/2\,\rho S V^2} \qquad \beta = \frac{v}{V}$$

At equilibrium, the quantities L, N, Y, p, r, ϕ, ψ, and v are all equal to zero.

Let us now assume a small displacement from equilibrium, and determine the equations which govern the motion. In order to reduce the complexity of the problem to enable a solution to be obtained without a prohibitive amount of calculation, the following assumptions are made:

(1) Forces and moments on lifting surfaces are assumed proportional to the square of the airspeed.

(2) Forces are unaffected by angular velocities and angular accelerations and moments by angular accelerations.

(3) The glider is assumed symmetrical, and thus lateral motions and longitudinal motions are assumed to be independent.

(4) The combined effect of two or more forces or moments is assumed proportional to the algebraic sum of the

separate components.

(5) The changes in aerodynamic forces and moments due to a deviation are assumed proportional to the deviation.

(6) Secondary effects involving the product of two small quantities are neglected.

(7) The principal axes of inertia of the glider are assumed coincident with the reference axes.

The lateral equations of motion are given by:

$$m \frac{dv}{dt} + mV \frac{d\psi}{dt} = \frac{\partial Y}{\partial v} v + mg \sin\phi \cos\gamma + mg \sin\psi \sin\gamma, \tag{2}$$

$$A \frac{d^2\phi}{dt^2} = \frac{\partial L}{\partial v} v + \frac{\partial L}{\partial p} p + \frac{\partial L}{\partial r} r + \frac{\partial L}{\partial \delta} \delta, \tag{3}$$

$$C \frac{d^2\psi}{dt^2} = \frac{\partial N}{\partial v} v + \frac{\partial N}{\partial p} p + \frac{\partial N}{\partial r} r + \frac{\partial N}{\partial \delta} \delta. \tag{4}$$

Let us define the following quantities:

$$L_p = \frac{1}{A}\frac{\partial L}{\partial p} \qquad L_r = \frac{1}{A}\frac{\partial L}{\partial r} \qquad L_v = \frac{1}{A}\frac{\partial L}{\partial v} \qquad L_\delta = \frac{1}{A}\frac{\partial L}{\partial \delta}$$

$$N_p = \frac{1}{C}\frac{\partial N}{\partial p} \qquad N_r = \frac{1}{C}\frac{\partial N}{\partial r} \qquad N_v = \frac{1}{C}\frac{\partial N}{\partial v} \qquad N_\delta = \frac{1}{C}\frac{\partial N}{\partial \delta} \tag{5}$$

$$Y_v = \frac{1}{m}\frac{\partial Y}{\partial v}$$

Remembering that ϕ and ψ are assumed to be small quantities, equations (2), (3), and (4) become:

$$\frac{dv}{dt} = - V \frac{d\psi}{dt} + Y_v v + (g \cos\gamma)\phi + (g \sin\gamma)\psi. \tag{6}$$

$$\frac{d^2\phi}{dt^2} = L_v v + L_p \frac{d\phi}{dt} + L_r \frac{d\psi}{dt} + L_\delta \delta \qquad (7)$$

$$\frac{d^2\psi}{dt^2} = N_v v + N_p \frac{d\phi}{dt} + N_r \frac{d\psi}{dt} + N_\delta \delta \qquad (8)$$

Let us assume that v, ϕ , ψ , and δ vary with time according to laws of the form:

$$v = v_o \exp(\lambda t), \qquad \phi = \phi_o \exp(\lambda t),$$
$$\psi = \psi_o \exp(\lambda t), \qquad \delta = \delta_o \exp(\lambda t) \qquad (9)$$

and determine what values of λ will satisfy the above equations. The stability of the various motions will be determined by the nature of the values of λ that satisfy these equations. If λ is positive, small displacements in these quantities will continuously increase with time, and the motion will be unstable. If λ is negative, small displacements will decrease with the time, and the motion will be stable. If λ is complex, the motions will be oscillatory, with increasing or decreasing amplitude depending upon whether the real part of λ is positive or negative.

If we substitute the values of v, ϕ , ψ , and δ given by equation (9) into equations (6), (7), and (8), we obtain:

$$(\lambda - Y_v) v - (g \cos \gamma)\phi + (\lambda V - g \sin \gamma)\psi = 0 \qquad (10)$$

$$- L_v v + (\lambda^2 - \lambda L_p)\phi - \lambda L_r \psi - L_\delta \delta = 0 \qquad (11)$$

$$- N_v v - \lambda N_p \phi + (\lambda^2 - \lambda N_r)\psi - N_\delta \delta = 0 \qquad (12)$$

Here we have three equations involving four variables, since we so far have made no mention of the variation of δ with the time.

2. Stability with Fixed Control Surfaces

If we assume fixed control surfaces, we set δ equal to zero, and we have three equations in three variables. To determine what values of λ satisfy the above equations, it is only necessary to determine what values of λ make the following determinant vanish.

$$\begin{vmatrix} \lambda - Y_v & - g \cos \gamma & \lambda V - g \sin \gamma \\ - L_v & \lambda^2 - \lambda L_p & - \lambda L_r \\ - N_v & - \lambda N_p & \lambda^2 - \lambda N_r \end{vmatrix} = 0 \qquad (13)$$

Solving the above determinant, we obtain the following equation for λ :

$$\lambda^5 + (-N_r - L_p - Y_v)\lambda^4 + (L_pN_r - L_rN_p + L_pY_v + N_rY_v + VN_v)\lambda^3$$

$$+ (-L_pN_rY_v + L_rN_pY_v - L_v\, g\cos\gamma - N_v\, g\sin\gamma + VL_vN_p - VL_pN_v)\lambda^2 \qquad (14)$$

$$+ (L_vN_r\, g\cos\gamma - L_rN_v\, g\cos\gamma - L_vN_p\, g\sin\gamma + L_pN_v\, g\sin\gamma)\lambda = 0$$

This equation may be written in the form:

$$A\lambda^5 + B\lambda^4 + C\lambda^3 + D\lambda^2 + E\lambda + F = 0 \qquad (15)$$

where

$A = 1$

$B = -L_p - N_r - Y_v$

$C = L_pN_r - L_rN_p + Y_v(L_p + N_r) + VN_v \qquad (16)$

$D = (L_rN_p - L_pN_r)Y_v - L_v\, g\cos\gamma - N_v\, g\sin\gamma + V(L_vN_p - L_pN_v)$

$E = (L_vN_r - L_rN_v)g\cos\gamma + (L_pN_v - L_vN_p)g\sin\gamma$

$F = 0$

The determination of the values of λ which satisfy equation (15) depends in general upon the solution of a fifth degree equation. Since in this case, one root is zero, it reduces to a fourth degree equation. The values of the above quantities for a conventional type glider are such that the roots of equation (15) are one pair of conjugate complex roots, two negative roots, and, of course, one zero root. In general, the real parts of all the roots will be negative corresponding to a stable condition if all coefficients are positive and Rouths' Discriminant $(BCD-D^2-B^2E)$ is positive.

Let us now investigate which are the important terms in the equation in λ and thus show how the nature of the solutions depends upon the values of the aerodynamic coefficients involved. In C, the quantity VN_v is large compared to the other terms present, and in D, the quantity $V(L_vN_p - L_pN_v)$ is large compared to other terms. Let us, then, for the present, neglect the other quantities involved in these terms, and let C and D be given as follows:

$$C = VN_v \qquad D = V(L_vN_p - L_pN_v) \tag{17}$$

In general, C and D will be large compared to B and E. Let us assume then that we may approximately reduce equation (15) to the following form:

$$\left[\lambda^2 + (B - \tfrac{D}{C})\lambda + C\right]\left[\lambda + \tfrac{E}{D}\right]\left[\lambda + \tfrac{D}{C}\right]\lambda = 0 \tag{18}$$

Multiplying, we obtain:

$$\lambda^5 + (B + \tfrac{E}{D})\lambda^4 + \left[C + \tfrac{D}{C}(B - \tfrac{D}{C}) + \tfrac{BE}{D}\right]\lambda^3 \tag{19}$$

$$+ \left[D + \tfrac{E}{D}\left\{C + \tfrac{D}{C}(B - \tfrac{D}{C})\right\}\right]\lambda^2 + E\lambda = 0$$

Thus, in order that equation (15) may be approximately represented by an equation of form (18), the following relations must hold:

$$B \gg \frac{E}{D} \qquad C \gg \frac{D}{C}\left(B - \frac{D}{C}\right) + \frac{BE}{D} \qquad D \gg \frac{EC}{D} \tag{20}$$

In general, the values of the aerodynamic coefficients for normal flight conditions of aircraft type missiles are such that these conditions are satisfied. Inserting the values of B, C, D, and E in equation (18), assuming the approximate values of C and D given by equation (17), we obtain:

$$\left[\lambda^2 + \left(- N_r - Y_v - \frac{L_v N_p}{N_v}\right)\lambda + VN_v\right]\left[\lambda + \frac{L_v N_r - L_r N_v}{L_v N_p - L_p N_v}\frac{g}{V}\cos\gamma - \frac{g}{V}\sin\gamma\right]\left[\lambda - L_p + \frac{L_v N_p}{N_v}\right]\lambda = 0 \tag{21}$$

With these approximations, λ is given by:

$$\lambda_1 = L_p - \frac{L_v N_p}{N_v}$$

$$\lambda_2 = -\frac{g\cos\gamma}{V}\left(\frac{L_v N_r - L_r N_v}{L_v N_p - L_p N_v}\right) + \frac{g\sin\gamma}{V} \tag{22}$$

$$\lambda_{3,4} = \frac{1}{2}\left(N_r + Y_v + \frac{L_v N_p}{N_v}\right) \pm i\sqrt{VN_v - \frac{1}{4}\left(N_r + Y_v + \frac{L_v N_p}{N_v}\right)^2}$$

$$\lambda_5 = 0$$

Thus, v, ϕ, and ψ can be represented by equations of the form

$$v, \phi, \psi = C_1 \exp\left\{L_p - \frac{L_v N_p}{N_v}\right\}$$
$$+ C_2 \exp\left\{-\frac{g\cos\gamma}{V}\left(\frac{L_v N_r - L_r N_v}{L_v N_p - L_p N_v}\right) + \frac{g\sin\gamma}{V}\right\}$$
$$+ C_3 \exp\left\{\frac{1}{2}\left(N_r + Y_v + \frac{L_v N_p}{N_v}\right)\right\} \tag{23}$$
$$\cos\left\{\sqrt{VN_v - \frac{1}{4}\left(N_r + Y_v + \frac{L_v N_p}{N_v}\right)^2} + C_4\right\}$$
$$+ C_5$$

where C_1, C_2, C_3, C_4, and C_5 are arbitrary constants, which have values depending upon which of the quantities v, ϕ, and ψ is being represented, and the particular boundary conditions for that quantity.

In general, the first term above represents a rapid subsidence, whose rate is a function mainly of L_p, the roll damping term. This is the most important term in roll motion.

The second term is a slow subsidence or divergence whose rate depends primarily upon the magnitude of $L_v N_r$ compared to $L_r N_v$, and upon the glide angle γ. This term determines the spiral stability characteristics of the glider; if the exponent is negative, the glider is spirally stable, if positive, spirally unstable. In general, $L_v N_r$ is larger than $L_r N_v$, and spiral stability is obtained although the stability is always small. As seen from the second term of equation (23), spiral stability is increased greatly at steep angles of descent $(\sin \gamma \rightarrow -1)$.

The third term in equation (23) represents an oscillation whose period is determined largely by the term VN_v, and is given approximately by $T = 2\pi/\sqrt{VN_v}$. As is to be expected, this period depends mainly on N_v, the yawing moment due to sideslip, and the damping depends mainly on the term N_r, the yawing moment due to rate of yaw. These are the main terms in the yaw motion. The constant C_5 occurs because of the fact that in a non-homing missile, the zero point for measuring angle of yaw is arbitrary; there is no preferred direction in space.

3. Stability of SWOD Mark 12 and Mark 13 Air Stabilizers

The following is a table of values of the lateral coefficients applicable to the SWOD gliders:

	Mark 12	Mark 13
W (lbs)	900	1500
S (ft^2)	18.3	24.4
b (ft)	8.4	10
A (lb ft sec^2)	23	45
C (lb ft sec^2)	75	200

$$\frac{\partial C_l}{\partial (\frac{pb}{2V})} = -.5$$

$$\frac{\partial C_l}{\partial (\frac{rb}{2V})} = .2C_L$$

$$\frac{\partial C_l}{\partial \beta} = -.17$$

$$\frac{\partial C_n}{\partial (\frac{pb}{2V})} = (-.03C_L)$$

$$\frac{\partial C_n}{\partial (\frac{rb}{2V})} = -.2$$

$$\frac{\partial C_n}{\partial \beta} = .17$$

$$\frac{\partial C_y}{\partial \beta} = -.6$$

The values of W, S, b, A, and C are measured values determined at the National Bureau of Standards. The values of $\frac{\partial C_l}{\partial \beta}$, $\frac{\partial C_n}{\partial \beta}$, $\frac{\partial C_y}{\partial \beta}$ given in the table were obtained from a wind tunnel test at the California Institute of Technology of an early model glider somewhat similar to the

Mark 12 glider (reference 2). However, since there are some marked differences between this model and the present Mark 12 and Mark 13 gliders, these values should be considered as only approximate. From page 20, table 3, of reference (2), the following values are given:

$$\frac{\partial C_l}{\partial \psi} = .003 \qquad \frac{\partial C_n}{\partial \psi} = -.003 , \qquad (24)$$

and from figure 12 of reference (2), we obtain

$$\frac{\partial C_c}{\partial \psi} = .010$$

where C_c is the crosswind force coefficient, which for small angles of yaw may be considered equal to C_y, the lateral force coefficient. The values of the damping terms

$$\frac{\partial C_l}{\partial (\frac{pb}{2V})}, \quad \frac{\partial C_l}{\partial (\frac{rb}{2V})}, \quad \frac{\partial C_n}{\partial (\frac{pb}{2V})}, \text{ and } \frac{\partial C_n}{\partial (\frac{rb}{2V})}$$

are estimated by the methods described in references (3) and (4) and should be considered as only very approximate.

Let us substitute the above values into the stability equations for a typical condition of flight of Mark 13, with fixed controls in pitch. Assume $C_L = .3$, which corresponds approximately to an average position of the elevons of 10° up from neutral. At equilibrium this gives a velocity of flight at sea level of $V = 415$ ft/sec.

Under this condition of flight, the following quantities have the values listed below:

$$\left.\begin{aligned}
V &= 415 \text{ ft sec}^{-1} \\
\gamma &= -12.5^\circ \\
L_p &= \frac{\partial C_l}{\partial\left(\frac{pb}{2V}\right)} \cdot \frac{\frac{1}{4}\rho S V b^2}{A} = -4.0 \text{ sec}^{-1} \\
L_r &= \frac{\partial C_l}{\partial\left(\frac{rb}{2V}\right)} \cdot \frac{\frac{1}{4}\rho S V b^2}{A} = +0.8 \text{ sec}^{-1} \\
L_v &= \frac{\partial C_l}{\partial \beta} \cdot \frac{\frac{1}{2}\rho S V b}{A} = -0.5 \text{ ft}^{-1} \text{ sec}^{-1} \\
N_p &= \frac{\partial C_n}{\partial\left(\frac{pb}{2V}\right)} \cdot \frac{\frac{1}{4}\rho S V b^2}{C} = -0.03 \text{ sec}^{-1} \\
N_r &= \frac{\partial C_n}{\partial\left(\frac{rb}{2V}\right)} \cdot \frac{\frac{1}{4}\rho S V b^2}{C} = -0.6 \text{ sec}^{-1} \\
N_v &= \frac{\partial C_n}{\partial \beta} \cdot \frac{\frac{1}{2}\rho S V b}{C} = +0.1 \text{ ft}^{-1} \text{ sec}^{-1} \\
Y_v &= \frac{\partial C_y}{\partial \beta} \cdot \frac{\frac{1}{2}\rho S V}{m} = -0.15 \text{ sec}^{-1}
\end{aligned}\right\} \quad (25)$$

Let us now substitute these values into equations (16), and thus determine the coefficients in equation (15) for λ We thus obtain the following equation for λ :

$$\lambda^5 + 4.75\lambda^4 + 44.6\lambda^3 + 189.0\lambda^2 + 9.81\lambda = 0 \qquad (26)$$

$$\lambda_1 = -4.35 \qquad \lambda_2 = -.053$$

$$\lambda_{3,4} = -.172 \pm 6.55i \qquad \lambda_5 = 0 \qquad (27)$$

λ_1 represents a rapid subsidence which corresponds to the high damping in roll. λ_2 is a slow subsidence determining the combined roll and yaw spiral motion. λ_3 and λ_4 are a pair of conjugate complex roots which represent a damped oscillation in yaw.

If we put the values of the constants in equation (25) in the simplified expressions given by equation (21), we obtain:

$$(\lambda + 4.15)(\lambda + .057)(\lambda^2 + .6\lambda + 41.5) = 0 \qquad (28)$$

$$\lambda_1 = -4.15 \qquad \lambda_2 = -.057 \qquad \lambda_{3,4} = -.30 \pm 6.44i \qquad (29)$$

These values are a fairly good approximation to those given by equation (27), except for the damping of the yaw oscillation.

4. Lateral Control System of SWOD Mark 7 and Mark 9

In the case of homing gliders, that is, the case where the control surfaces are moved in such a way as to direct the glider toward a target, the manner in which the control surfaces are caused to move in response to the homing signals depends upon the characteristics of the particular servomechanism used. It is not possible to compute the effect of a general functional relation, and to consider all possible specific relations which have been used as a basis for servomechanisms. We will consider here only the case of the lateral control system used in SWOD Mark 7 and Mark 9. The complete theory, taking into account the off-on link between the gyro and the servo, time lags in the servo, time lags in the homing control, and the complete set of lateral stability equations would be a very unwieldy calculation, and it is thought that by treating these various factors separately as to their effects, the discussion may be made clearer.

The gyros and servos used in SWOD Mark 7 and Mark 9 are discussed in references (5) and (6). Lateral stabilization is obtained through the "turn gyro", which is essentially a rate gyro equipped with electromagnet coils and electrical contacts. The electromagnets are connected to apply torques to the gimbal frame which are proportional to the error angle in yaw as obtained by the homing device. The electrical contacts are arranged on opposite sides of the gimbal frame so that one contact or the other is closed, depending upon the sign of the sum of the torque applied by the electromagnets and the torque due to precession of the gyro wheel. Closing of a contact causes the servo to move the ailerons. The gyro is mounted in the glider at an angle so as to be sensitive to both roll and yaw.

It was assumed in the derivation of the equations of motion that the principal axes of inertia of the glider were coincident with the reference axes. In general, this is not true, for in the case of SWOD Mark 7 and Mark 9, the roll axis under normal equilibrium flight conditions is inclined to the direction of the relative wind by about 3°. This difference has only a very small effect on the stability calculations for free flight, but must be taken into account in the case of homing flight.

Since the flight path is on the average toward the target on a true homing course, the error angle as measured by the homing device will differ from the error angle referred to coordinates where the X-axis is in the direction of the axis of roll by an amount given for small angles by the following equation:

$$\psi' = \psi - \beta\phi \qquad (30)$$

where ψ' is the error determined by the homing device, ψ is the error angle, referred to coordinates with X-axis along the roll axis, β is the angle between the roll axis and a line from the glider to the target, and ϕ is the angle of bank.

Let α represent the angle the axis of the gyro makes with the axis of roll. Let c represent the rate of yaw in degrees per second that produces the same torque on the gimbal frame as an angular error of one degree. The particular contact on the gyro which is closed depends on the sign of the quantity ω defined by

$$\omega = \frac{d\psi}{dt} + \tan\alpha \ \frac{d\phi}{dt} + c\ (\psi - \beta\phi) \qquad (31)$$

When ω is positive, that contact on the gyro will be closed that causes the servo-control unit to move the ailerons differentially at a constant speed to produce a rolling moment to the left; when ω is negative, the other contact will be closed, causing the ailerons to move at constant speed to give a rolling moment to the right. Thus the movement of the ailerons is always in such a direction as to reduce the value of ω to zero. A hunting motion is set up, the gyro contacts alternately closing and the ailerons moving alternately for right and left differential.

If we neglect this hunting motion, and assume that ω is, on the average, zero, we have

$$\frac{d\psi}{dt} + \frac{d\phi}{dt}\tan\alpha + c\psi - \beta c\phi = 0 \qquad (32)$$

If we neglect the sideslip motion of the glider and assume, for the moment, that equilibrium of forces always exists along the lateral axis, equation (6) reduces to:

$$0 = -V\frac{d\psi}{dt} + (g\cos\gamma)\phi + (g\sin\gamma)\psi. \tag{33}$$

In general, γ is sufficiently small so that the term involving $\sin\gamma$ may be neglected in equation (33) and $\cos\gamma$ set equal to unity. Equation (33) thus becomes:

$$\frac{d\psi}{dt} = \frac{g}{V}\phi. \tag{34}$$

Substituting equation (33) and its time derivative in equation (32), we obtain:

$$\frac{d^2\psi}{dt^2} + \frac{d\psi}{dt}\frac{1}{\tan\alpha}\left(\frac{g\cos\gamma}{V} - \frac{g\sin\gamma\tan\alpha}{V} - \beta c\right) + \frac{(1+\beta\tan\gamma)\,cg\cos\gamma}{V\tan\alpha}\psi = 0. \tag{35}$$

If we use the simplified equation (34), which neglects terms in $\sin\gamma$ and $\tan\gamma$, and assume $\cos\gamma$ equal to unity, we obtain:

$$\frac{d^2\psi}{dt^2} + \frac{d\psi}{dt}\left(\frac{g}{V\tan\alpha} - \frac{\beta c}{\tan\alpha}\right) + \frac{cg\psi}{V\tan\alpha} = 0 \tag{36}$$

This equation has the following approximate solution:

$$\psi = \psi_o \exp\left(-\frac{t}{2\tan\alpha}\left(\frac{g}{V} - \beta c\right)\right) \cos\left(\sqrt{\frac{cg}{V\tan\alpha}}\,t + \epsilon\right). \quad (37)$$

This equation represents a damped oscillation in yaw.

It is seen that the damping is influenced considerably by the value of βc. The effect of this quantity, in general, is to reduce the damping. If the line from the glider to the target is below the roll axis (β positive), the damping of the oscillation is decreased; if it is above the roll axis, the damping is increased. As discussed on page 18, the average value of β for SWOD Mark 9 is about 3°. β occurs in the damping term multiplied by c, the ratio of the rate of turn of the gyro to the error angle. The effect of β on the damping is thus emphasized by a large value of c. In the case of high sensitivity homing information, that is, a large signal for a small error angle, the damping may become negative, and the oscillations become undamped and increase in amplitude to a limit where the homing information saturates on each oscillation and these assumed equations no longer hold.

Let us assume typical values of the constants in this equation. Let α = 20°, β = .05, c = .5, V = 415 ft/sec, we obtain:

$$\psi = \psi_o \exp(-.103\,t) \cos(.326\,t + \epsilon). \quad (38)$$

This shows that an oscillation in yaw should occur with a period of about 19 seconds and damping to $1/e$ of its initial value in about 10 seconds.

Although this simplified theory predicts quite closely the period of the oscillation in yaw actually obtained in flight, the predicted damping is not obtained, and sustained hunting of about this period and of an amplitude of about 4° is obtained. In order to increase this damping in yaw sufficiently, a bias gyro has been added to the control systems of SWOD Mark 7 and Mark 9. This gyro is similar to the turn gyro except that it does not contain the electromagnets, but only contacts, one or the other of which closes, depending upon the sense of the rate of turn. Its design and operation are described in detail in reference (5). This gyro is mounted in the glider along the average roll axis so that it will be sensitive chiefly to the yawing motion of the glider. Rolling motion will affect it slightly because the axis of roll does not remain fixed for all conditions of flight.

The bias gyro is connected in the circuit of the turn gyro so that when the rate of yaw of the glider is to the right, the right coil in the turn gyro is shunted by a resistor, and when the rate of yaw is to the left, the left coil in the turn gyro is shunted. The effect of the shunting current in the coils is shown in reference (5), figure 11. It is seen that a bias is given to the signals put into the electromagnets in such a direction as to oppose the yawing motion of the glider, and thus increases the damping of the yaw oscillations. Experimentally, it has been found necessary to use this bias gyro to obtain sufficient damping in yaw.

Figure 1 shows a typical flight of SWOD Mark 9. The curve labeled "Apparent Horizontal Motion of Reflector" shows that an oscillation in yaw of period of about 20 seconds is evident, but is of very small amplitude.

Obviously the reason the damping predicted by the simplified theory above is not obtained is due to the approximations made, which, in effect, neglect the effects of sideslip velocity, the time lags present in the homing signals, and the roll hunting motion involving time lags in the gyro and servo system.

Let us now consider the effect of time lag in the homing intelligence. The homing intelligence has a time lag which is equivalent to that produced in an RC circuit of time constant of approximately 0.3 second. If we assume that the homing information has a lag corresponding to an RC circuit, equation (32) becomes

$$\frac{d\psi}{dt} + \frac{d\phi}{dt}\tan\alpha + c\exp\left(-\frac{t}{RC}\right)\int\frac{(\psi - \beta c)}{RC}\exp\left(\frac{t}{RC}\right)dt = 0. \qquad (39)$$

Combining this equation with equation (34), we obtain:

$$\frac{d^3\psi}{dt^3} + \left(\frac{g}{V\tan\alpha} + \frac{1}{RC}\right)\frac{d^2\psi}{dt^2} + \left(\frac{g}{VRC\tan\alpha} - \frac{\beta c}{RC\tan\alpha}\right)\frac{d\psi}{dt}$$

$$+ \frac{cg\psi}{VRC\tan\alpha} = 0. \qquad (40)$$

If RC is zero, this reduces, of course, to equation (36). If we substitute the values of β, c, α, and V used in equation (38), and in addition let RC = 0.3, equation (40) becomes:

$$\frac{d^3\psi}{dt^3} + 3.55\frac{d^2\psi}{dt^2} + .687\frac{d\psi}{dt} + .355\psi = 0. \qquad (41)$$

Solving, we obtain:

$$\psi = \psi_1 \exp(-3.38\ t) + \psi_2 \exp(-.086\ t)\cos(.324\ t + \epsilon). \qquad (42)$$

It is seen that the period of the oscillations is virtually unchanged, but that the damping is decreased about 20 percent. Sideslip effects and the hunting in roll with its various time lags involved are still neglected.

To investigate the effect of sideslip velocity, let us now consider the general stability equations, with an ideal servomechanism, where the differential of the ailerons δ, instead of increasing or decreasing at a constant rate depending upon the sign of ω, will actually be proportional to ω.

Thus let us write

$$\delta = -K\omega = -K\left(\frac{d\psi}{dt} + \frac{d\phi}{dt}\tan\alpha + c\psi - \beta c\phi\right) \qquad (43)$$

If we assume δ is given by an expression of the type $\delta = \delta_0 \exp(\lambda t)$, we may write equation (43) as follows:

$$\delta + K(\lambda\tan\alpha - \beta c) + K(\lambda + c)\psi = 0 \qquad (44)$$

Combining this equation with equations (10), (11), (12), the values of λ which satisfy these four equations are those that make the following determinant vanish:

$$\begin{vmatrix} \lambda - Y_v & -g\cos\gamma & \lambda V - g\sin\gamma & 0 \\ -L_v & \lambda^2 - \lambda L_p & -\lambda L_r & -L_\delta \\ -N_v & -\lambda N_p & \lambda^2 - \lambda N_r & -N_\delta \\ 0 & K(\lambda\tan\alpha - \beta c) & K(\lambda + c) & 1 \end{vmatrix} = 0 \qquad (45)$$

Solving the above determinant, an equation of the fifth degree in λ with 55 terms is obtained. This may be written in the form given by equation (15) with coefficients given in the following table:

$$A = 1,$$

$$B = -N_r - L_p - Y_v + K\left[N_\delta + L_\delta \tan\alpha\right],$$

$$C = L_p N_r - L_r N_p + Y_v (L_p + N_r) + VN_v + KL_\delta\left[-N_r \tan\alpha - \beta c + N_p - Y_v \tan\alpha\right] + KN_\delta\left[L_r \tan\alpha - L_p + c - Y_v\right],$$

$$\begin{aligned} D = {} & Y_v(-L_p N_r + L_r N_p) - L_v g \cos\gamma - N_v g \sin\gamma + V(L_v N_p - L_p N_v) \\ & + KL_\delta\left[\beta c N_r + c N_p + N_r Y_v \tan\alpha + \beta c Y_v - N_p Y_v + VN_v \tan\alpha\right] + KN_\delta\left[-\beta c L_r - c L_p - L_r Y_v \tan\alpha \right. \\ & \left. + L_p Y_v - c Y_v - VL_v \tan\alpha\right], \end{aligned} \quad (46)$$

$$\begin{aligned} E = {} & (L_v N_r - L_r N_v) g \cos\gamma + (-L_v N_p + L_p N_v) g \sin\gamma \\ & + KL_\delta\left[-\beta c N_r Y_v - c N_p Y_v + N_v g \cos\gamma - \beta c VN_v - (N_v \tan\alpha) g \sin\gamma\right] + KN_\delta\left[\beta c L_r Y_v + c L_p Y_v \right. \\ & \left. - L_v g \cos\gamma + \beta c VL_v + (L_v \tan\alpha) g \sin\gamma\right], \end{aligned}$$

$$F = KL_\delta\left[c N_v g \cos\gamma + \beta c N_v g \sin\gamma\right] + KN_\delta\left[-cL_v g \cos\gamma - \beta c L_v g \sin\gamma\right].$$

Let us now substitute values for the coefficients in this equation for the typical flight condition given by equation (25). We need values for the additional quantities L_δ, N_δ, K, α, and β. From wind tunnel tests (reference 2), we may take $\frac{\partial C_l}{\partial \delta} = .072$, which gives $L_\delta = 80$. We will assume for this flight condition that no yaw moments are produced by the ailerons, that is, $N_\delta = 0$. Let K = 5, $\alpha = 20°$, and $\beta = .05$, which are typical values. The equation for λ becomes:

$$\lambda^5 + 150.4\lambda^4 + 138.1\lambda^3 + 6229\lambda^2 + 953\lambda + 622 = 0. \quad (47)$$

This equation has the following approximate roots:

$$\lambda_1 = -149.8, \quad \lambda_{2,3} = -.225 \pm 6.43i, \quad \lambda_{4,5} = -.0765 \pm .316i. \quad (48)$$

Comparing this result with the free flight results, equation (27), we see that instead of two subsidences, a damped oscillation, and a zero root, we now have a very rapid subsidence and two damped oscillations. The roots λ_2 and λ_3 represent the natural yaw oscillation similar to that for the free flight condition, and whose period and damping are mainly a function of the aerodynamic constants. The roots λ_4 and λ_5 represent a damped oscillation whose period and damping depend primarily upon the constants α and c assumed for the ideal servo system. The period of this oscillation is about the same and the damping somewhat less than that obtained from equation (38), in which sideslip velocity was neglected. While the effect of sideslip is sometimes taken into account by introducing an "aerodynamic lag" in the simplified equation, it is important to note that there is no evidence of a true aerodynamic lag when the complete equations of motion are used.

By examination of the solutions of equations (36), (40), or (45), it is seen that the damping in yaw should increase as α decreases. If we let $\alpha = 0$ in equation (36), we obtain:

$$\frac{d\psi}{dt} + \frac{c\,\psi}{1 - \frac{\beta c V}{g}} = 0 . \qquad (49)$$

Solving for ψ, we obtain:

$$\psi = \psi_0 \exp\left(- \frac{ct}{1 - \frac{\beta c V}{g}}\right) . \qquad (50)$$

This equation shows that the yaw oscillation should reduce to a subsidence when $\alpha = 0$. However, if we let $\alpha = 0$ in equation (46), which takes into account sideslip velocity effects, we obtain the following equation for λ :

$$\lambda^5 + 4.75\lambda^4 + 22.6\lambda^3 + 173.7\lambda^2 + 851\lambda + 622 = 0, \qquad (51)$$

which has the following roots:

$$\lambda_1 = -.871, \qquad \lambda_{2,3} = -4.08 \pm 2.49i, \qquad (52)$$

$$\lambda_{4,5} = +2.14 \pm 5.16i.$$

We obtain the rapid subsidence, the damped natural yaw oscillation as in the case $\alpha = 20°$, but now λ_4 and λ_5 represent a rapidly divergent oscillation, and thus an unstable condition results. At some small value of α, the real parts of the roots λ_4 and λ_5 change from negative to positive, change the oscillation from a damped one to a divergent one.

5. Roll Stabilization System

In order to study the effects of the roll hunting motion on the lateral stability, a more detailed discussion of the roll stabilization system will be given, taking into account the off-on character of the link between the gyros and the servo-clutches, and the time lag in the response of the servo. Let us assume that the glider is flying straight and level, that the hunting motion in roll has reached a steady state condition, and that the motion is periodic.

Neglecting the effects of rolling moment due to rate of yaw, and rolling moments due to sideslip velocity, equation (7) for the motion in roll becomes:

$$\frac{d^2\phi}{dt^2} = L_p \frac{d\phi}{dt} + L_\delta \delta. \tag{53}$$

At time $t = 0$, let the rate of roll p equal p_1, the angle of bank ϕ equal ϕ_1, and the rolling acceleration produced by the ailerons equal to $L_\delta \frac{d\delta}{dt} t_1$. Let us define $K = L_\delta \frac{d\delta}{dt}$, and thus K becomes the acceleration in roll produced by the amount of differential on the ailerons developed by the servo in moving the ailerons at constant speed $\frac{d\delta}{dt}$ for one second.

At the instant $t = 0$, the ailerons start to move differentially with a constant speed and in such a direction as to reduce the rolling acceleration. At time $t = t_1$, it is easily seen that the rolling acceleration will become zero, and at $t = 2t_1$, it will become equal in magnitude and opposite in sign to its value at $t = 0$. At $t = 2t_1$, it is assumed that the direction of motion of the ailerons reverses, so that at time $t = 3t_1$, the rolling acceleration is again reduced to zero, and at $t = 4t_1$, is equal to its value at $t = 0$. Thus a periodic hunting in the aileron motion is obtained of "saw-toothed" wave form, and with a period $4t_1$.

The aileron differential δ as a function of the time is shown in figure (2)

We may express the motion in roll between $t = 0$ and $t = 2t_1$ by the equation:

$$\frac{d^2\phi}{dt^2} = L_p \frac{d\phi}{dt} + Kt_1 - Kt \, . \tag{54}$$

Solving this differential equation for p and ϕ, including the initial conditions that $p = p_1$, and $\phi = \phi_1$, at $t = 0$, we obtain:

$$p = -\frac{K}{L_p}\left[t_1\left(1 - \exp\left(L_p t\right)\right) - t - \frac{1}{L_p}\left(1 - \exp\left(L_p t\right)\right)\right] + p_1 \exp\left(L_p t\right), \tag{55}$$

$$\phi = -\frac{K}{L_p}\left[t_1 t + \frac{t_1}{L_p}\left(1 - \exp\left(L_p t\right)\right) - \frac{t^2}{2} - \frac{t}{L_p} - \frac{1}{L_p^2}\left(1 - \exp\left(L_p t\right)\right)\right] - \frac{p_1}{L_p}\left(1 - \exp\left(L_p t\right)\right) + \phi_1 \tag{56}$$

At $t = 2t_1$, let $p = p_2$, and $\phi = \phi_2$. Then p_2 and ϕ_2 will be given by the following equations:

$$p_2 = \frac{K}{L_p}\left[t_1 + \frac{1}{L_p}\frac{\left(1 - \exp\left(2\,L_p t_1\right)\right)}{\left(1 + \exp\left(2\,L_p t_1\right)\right)}\right], \tag{57}$$

$$\phi_2 = \frac{K}{L_p^2}\left[t_1 + \frac{1}{L_p}\frac{\left(1 - \exp\left(2\,L_p t_1\right)\right)}{\left(1 + \exp\left(2\,L_p t_1\right)\right)}\right]. \tag{58}$$

Since we have assumed a steady state condition, the values of p and ϕ at $t = 2t_1$ must be equal in magnitude and opposite in sign to their values at $t = 0$. Putting $p_1 = -p_2$ and $\phi_1 = -\phi_2$, we obtain the following equations for p and ϕ :

$$p = -\frac{K}{L_p}\left[t_1 - t - \frac{1}{L_p}\frac{\left(1 + \exp\left(2\,L_p t_1\right) - 2\exp\left(L_p t\right)\right)}{\left(1 + \exp\left(2\,L_p t\right)\right)}\right], \tag{59}$$

$$\phi = \frac{K}{L_p^2}\left[t\left(1 - L_p t_1\right) + \frac{L_p t^2}{2} - t_1 + \frac{1}{L_p}\frac{\left(1 + \exp\left(2\,L_p t_1\right) - 2\exp\left(L_p t\right)\right)}{\left(1 + \exp\left(2\,L_p t_1\right)\right)}\right]. \tag{60}$$

Let us assume that there is a time lag Δt between the time the angular velocity to which the gyro is sensitive $\left(\frac{d\psi}{dt} + \frac{d\phi}{dt}\tan\alpha\right)$ is reduced to zero and the time the ailerons reverse their motion. This time lag includes the time it takes for the gyro contacts to reverse, the servo clutches to engage and reverse the direction of motion of the ailerons. If reversal of the aileron motion takes place at $t = 2t_1$, the angular velocity to which the gyro is sensitive must reduce to zero at time $t = 2t_1 - \Delta t$. Let the values of p and ϕ at this time be denoted by p_3 and ϕ_3, which are given by the following equations:

$$p_3 = -\frac{K}{L_p}\left[-t_1 + \Delta t - \frac{1}{L_p}\left(\frac{1 - 2\exp\left(L_p(2t_1 - t)\right) + \exp\left(2L_p t_1\right)}{1 + \exp\left(2L_p t_1\right)}\right)\right], \tag{61}$$

$$\phi_3 = \frac{K}{L_p^2}\left[t_1\left(1 - L_p\Delta t\right) - \Delta t + L_p\frac{\Delta t^2}{2} + \frac{1}{L_p}\left(\frac{1 - 2\exp\left(L_p(2t_1 - t)\right) + \exp\left(2L_p t_1\right)}{1 + \exp\left(2L_p t_1\right)}\right)\right]. \tag{62}$$

Since at time $t = 2t_1 - \Delta t$, the angular velocity to which the gyro is sensitive is reduced to zero, we have the following relation:

$$\frac{d\psi}{dt} + \frac{d\phi}{dt}\tan\alpha = 0 \quad \text{at} \quad t = 2t_1 - \Delta t. \tag{63}$$

If again we neglect the effects of sideslip velocity, we may make use of equation (34), and thus we obtain the following relation between p_3 and ϕ_3:

$$\frac{g}{V}\phi_3 + (\tan\alpha)\,p_3 = 0. \tag{64}$$

Due to the high damping in roll, the terms $\exp(2 L_p t_1)$ and $\exp\left(L_p (2 t_1 - \Delta t)\right)$ are negligible compared to unity for the observed values of t_1 and Δt, and will be neglected in what follows. Substituting equations (60) and (61) in equation (64), we obtain:

$$\frac{Kg}{VL_p^2}\left[t_1 - L_p t_1 \Delta t - \Delta t + L_p \frac{\Delta t^2}{2} + \frac{1}{L_p}\right]$$

$$+ \frac{K \tan\alpha}{-L_p}\left[-t_1 + \Delta t - \frac{1}{L_p}\right] = 0. \tag{65}$$

Solving for t_1, we obtain:

$$t_1 = \frac{\Delta t - \frac{1}{L_p} + \frac{g}{VL_p \tan\alpha}\left(\Delta t - L_p \frac{\Delta t^2}{2} - \frac{1}{L_p}\right)}{1 + \frac{g}{VL_p \tan\alpha}\left(1 - L_p \Delta t\right)}. \tag{66}$$

It is seen from the above equation that for values of

α such that $\frac{-g}{VL_p \tan} \ll 1$, the period of the roll hunt is determined chiefly by the damping acceleration in roll L_p and the time lag Δt. The amplitude of the roll hunt from equation (60) is seen to be a linear function of the period and directly proportional to the ratio of the rolling acceleration produced per second by the elevons. Thus to keep the roll amplitude small, time lags in the gyro and servo units must be kept to a minimum, and the rate of application of restoring moment small.

As α takes on smaller and smaller values so that $\frac{-g}{VL_p \tan \alpha}$ becomes of the order of unity, the denominator in equation (66) becomes very small, increasing the period and amplitude of the roll hunting motion, until at the value of α determined by the equation

$$\frac{g}{V \tan} \left(\frac{-1}{L_p} + \Delta t\right) = 1, \tag{67}$$

the denominator of equation (66) becomes zero, and the hunting motion becomes unstable.

Let us substitute appropriate values of L_p, g, V, α, and Δt for SWOD Mark 9, and determine the resultant values of the amplitude and period of the roll motion. Let us use the values of L_p and V given by equations (25), and, in addition, set $\alpha = 20°$ and $\Delta t = 0.1$ second. Substituting these values into equation (66), we obtain $t_1 = 0.35$ second.

Thus the roll hunting motion should have a period $T = 4t_1 = 1.4$ second. The amplitude will be approximately given by ϕ_3, which has a value of 0.095 radians or 5.4 degrees. These calculated values check [redacted] experiment as is seen by referring to the curve of figure (1), showing the angle of roll as a function of the time for a typical flight of SWOD Mark 9.

From equation (67), the smallest value of α that may be used before the motion becomes unstable is about 3.5°. This equation, however, neglects rolling moments due to sideslip and to rate of yaw which, if taken into account, have the effect of increasing this minimum angle.

6. Model Tests of Lateral Control System

To study the effect of time lags and speed of the servo system on the roll hunting motion, a mechani[redacted]to represent equation (53) was constructed. A photograph of the model is shown in figure (3). It consists of a table free to rotate about a vertical axis, and carries the "turn gyro" used in the control system. On the same axis is a cylinder which rotates inside a concentric cylinder with a small separation. The space between the cylinders is filled with oil to produce viscous damping of the motion of the table. Since the spacing of the cylinders is small and the oil is sufficiently viscous, the damping force is quite accurately proportional to the first power of the angular velocity of the table. A cord is wound around the shaft which supports the table, and each end is connected to a spring. The other ends of the springs are connected by cords around pulleys to the servo arms. Thus, if we neglect the small spring displacement caused by the motion of the table, the torque applied to the table will be proportional to the displacement of the servo arms and to the constants of the springs. If the ratio of the torque applied per radian displacement of the servo to the moment of inertia of the table is made equal to the value of L_{δ} for the missile, and the ratio of the damping torque to the moment of inertia of the table is made equal to the value of L_p for the missile, the motion of the table will be governed by equation (53). If the "turn gyro" mounted on the table is connected to the servo, as in the missile, a hunting motion will be set up simulating the roll hunting motion of the glider. Some records obtained with the model are shown in figure (4), which were taken to study the effect of time lag in the system and rate of movement of ailerons on the amplitude of the roll hunt. The results are in accord with the theoretical treatment above.

The Servomechanisms Laboratory of the Massachusetts Institute of Technology, in connection with the development of an alternate control system for SWOD Mark 7 and Mark 9, constructed a device to simulate the roll and yaw motions of a glider. A platform was arranged so as to be free to rotate about a horizontal axis (representing roll motion) and to be driven in rotation about a vertical axis. A torque motor and generator were connected to the roll axis, and synchros attached to the arms of the servo under test. Electrical circuits were arranged so that a torque was applied about the roll axis proportional to the differential displacement of the ailerons and to the rate of roll, with proper constants of proportionality so that the motion in roll would be represented by equation (33). The platform was driven in rotation about a vertical axis at an angular velocity proportional to the angular displacement about the horizontal or roll axis, and the constant adjusted so that its motion would be represented by equation (34). All effects due to sideslip and the cross derivatives L_r and N_p were neglected.

The turn and bias gyros and antenna system used in SWOD Mark 7 were mounted on the platform and a beacon for homing signals placed at a distance from the platform. The system was operated so that the lateral motions of the glider in flight would be simulated. Records obtained of the angle of bank ϕ and the angle of yaw ψ of the platform as a function of the time for various adjustments of the control system are shown in figures (5), (6), and (7). In all records shown, the platform was initially set 5° off the axis of homing. The vertical scale for ϕ is four times that for ψ.

Figure 5 shows a series of runs in which the angle of the gyro is varied from 2.5° to 20°. For 2.5° the motion is unstable, as predicted by the roll stabilization theory in section 5, the platform hitting its limit stops during the test. The value of 15° found to be most satisfactory from flight tests is somewhat larger than the best result from the model tests, the difference being due to the terms neglected in the equation governing the model tests.

Figure (6) shows a series of runs in which the angle designated as ψ_0, which represents the minimum error angle that produces saturation of the differential amplifier which feeds into the turn gyro coils, is varied. This, in effect, is the same as varying the value of $\dot{c}$, the rate of yaw in degrees per second that produces the same torque on the gimbal frame of the gyro as an angular error of one degree. The value $\psi_0 = 6°$ was found most suitable from flight tests, and this value is seen to be satisfactory by this model test.

Figure (7) shows the effect of the bias gyro on the damping of the yaw oscillations. If a bias gyro is not used ($R = \infty$), sustained oscillations in yaw are obtained both in flight tests and in the model test. The most suitable value for the biasing resistor determined from both flight tests and model tests was found to be about 5000 ohms.

April 12, 1946

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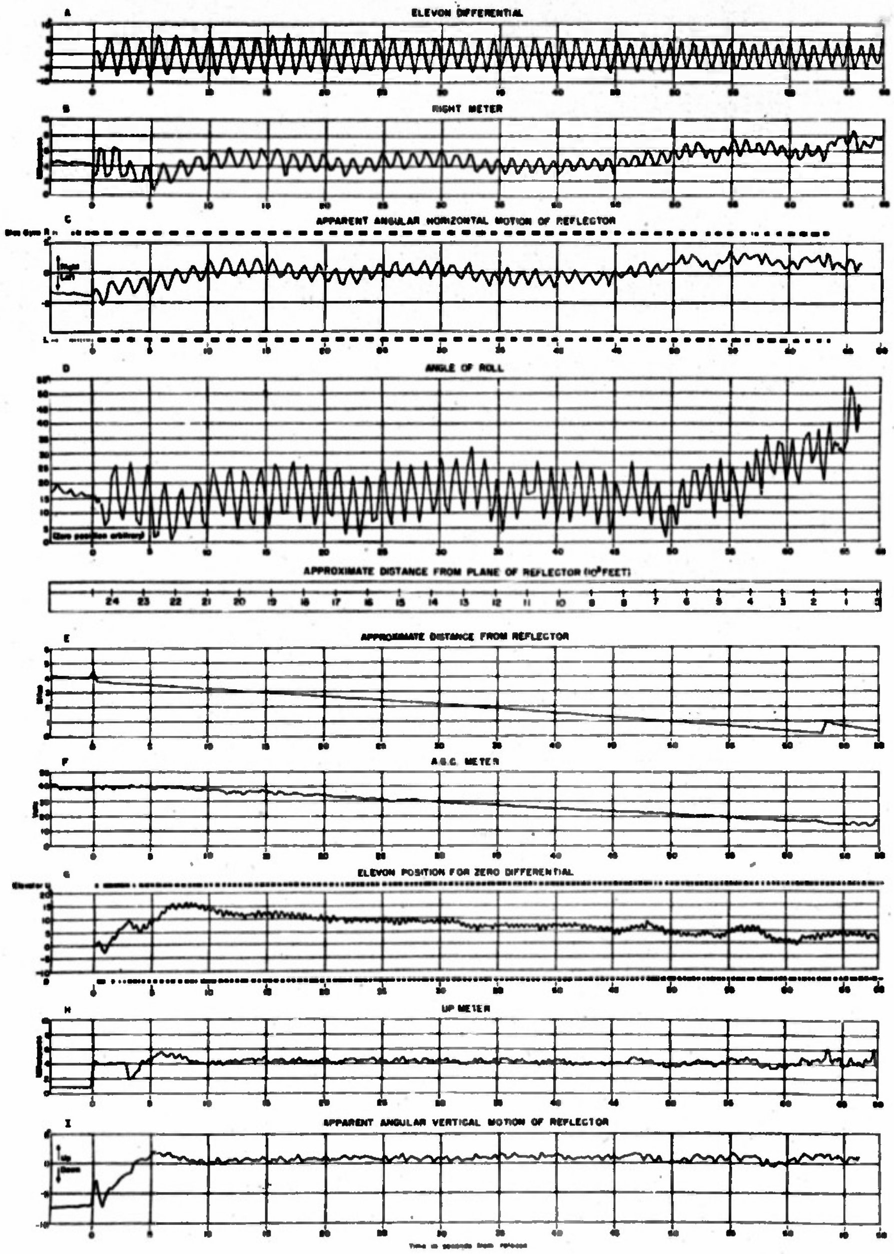

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FLIGHT B61
GLIDER 2006
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FIG. I

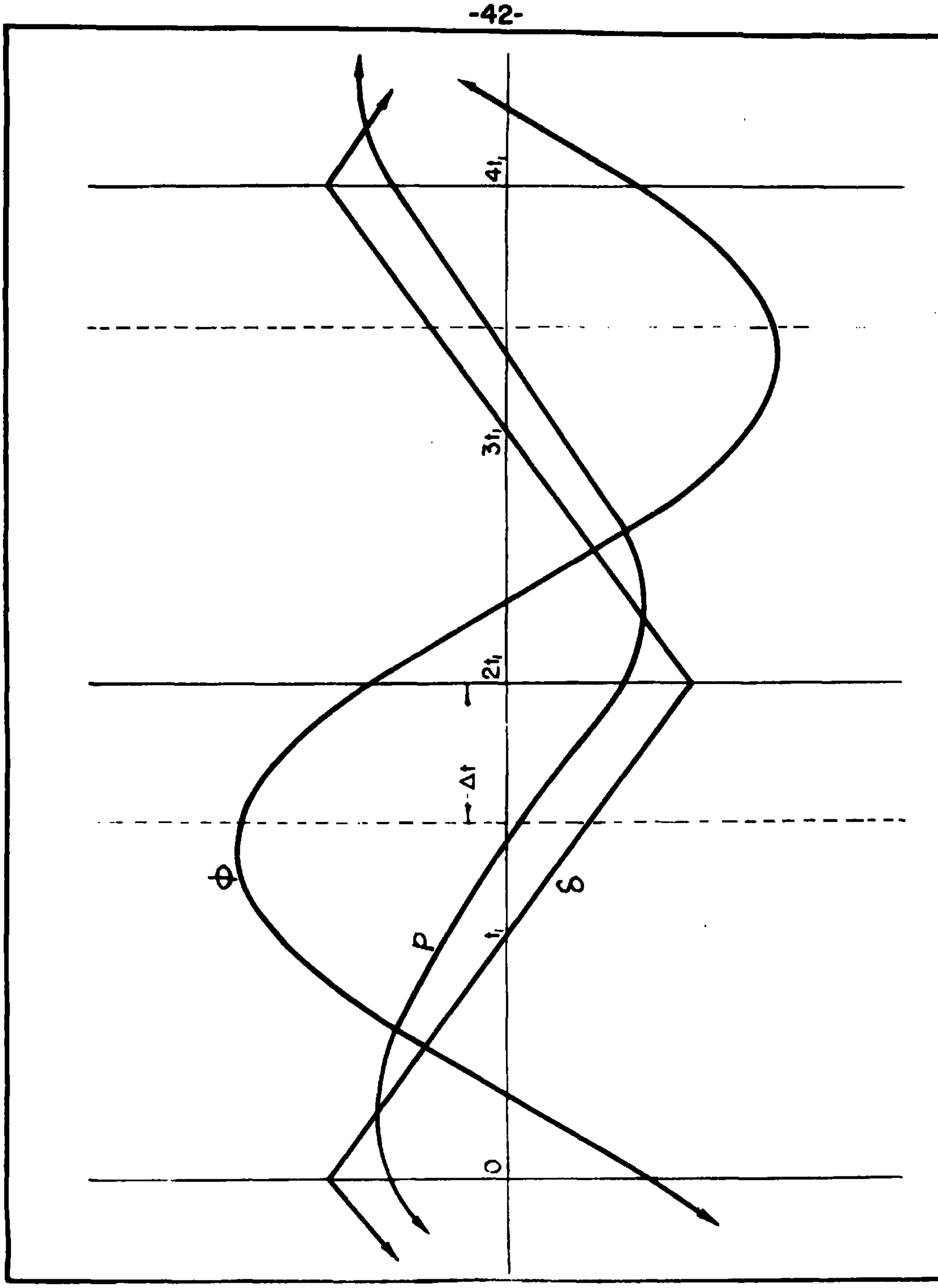


FIG. 2

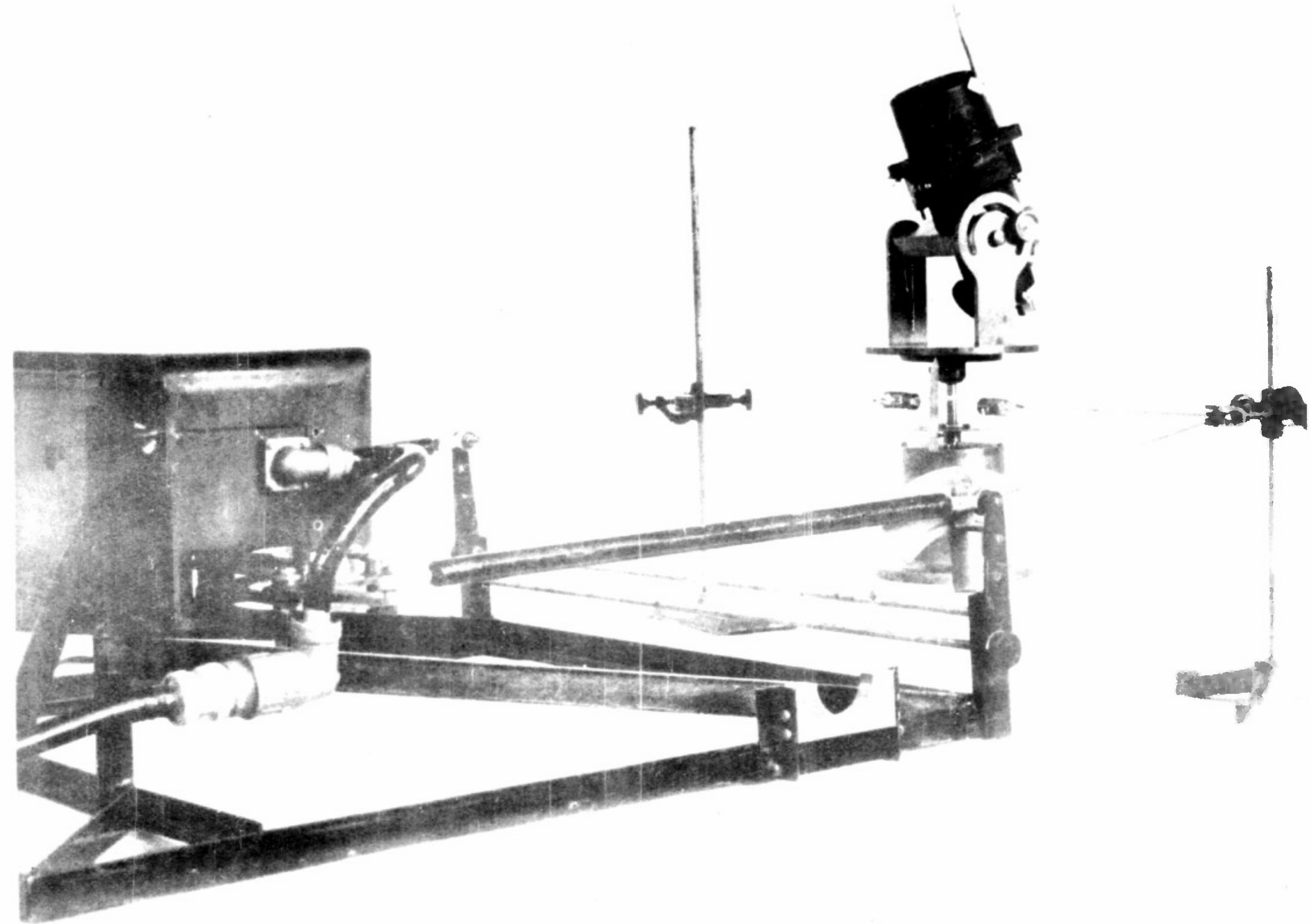
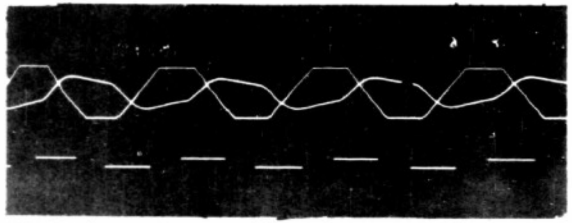
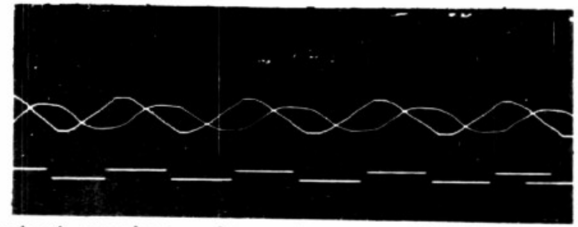


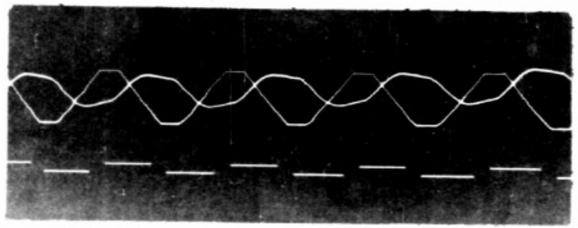
FIG 3



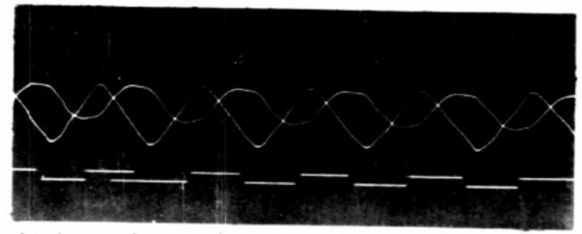
Contact spacing - wide Servo speed - normal



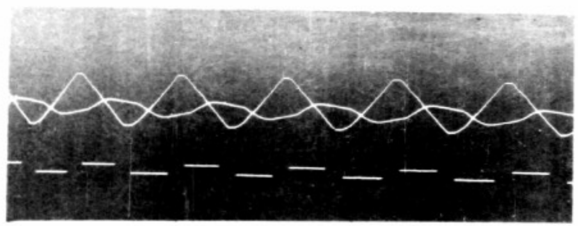
Contact spacing - close Servo speed - 0.7x normal



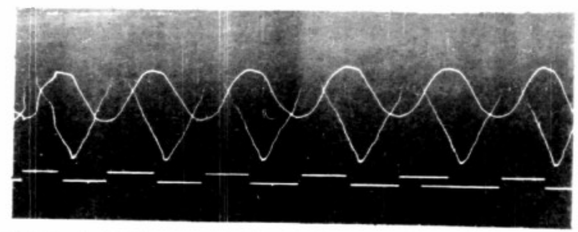
Contact spacing - moderate Servo speed - normal



Contact spacing - close Servo speed - 1.0x normal



Contact spacing - close Servo speed - normal



Contact spacing - close Servo speed - 1.3x normal

Roll Simulator records showing effects of gyro contact spacing and servo motor speed on the amplitude of the roll hunting motion. The "saw toothed" type curves represent the differential motion of the elevons, the sinusoidal type curves represent the angular motion of the table, and the broken lines the operation of the right and left servo clutches, respectively. The film speed is approximately one inch per second.

FIG. 4

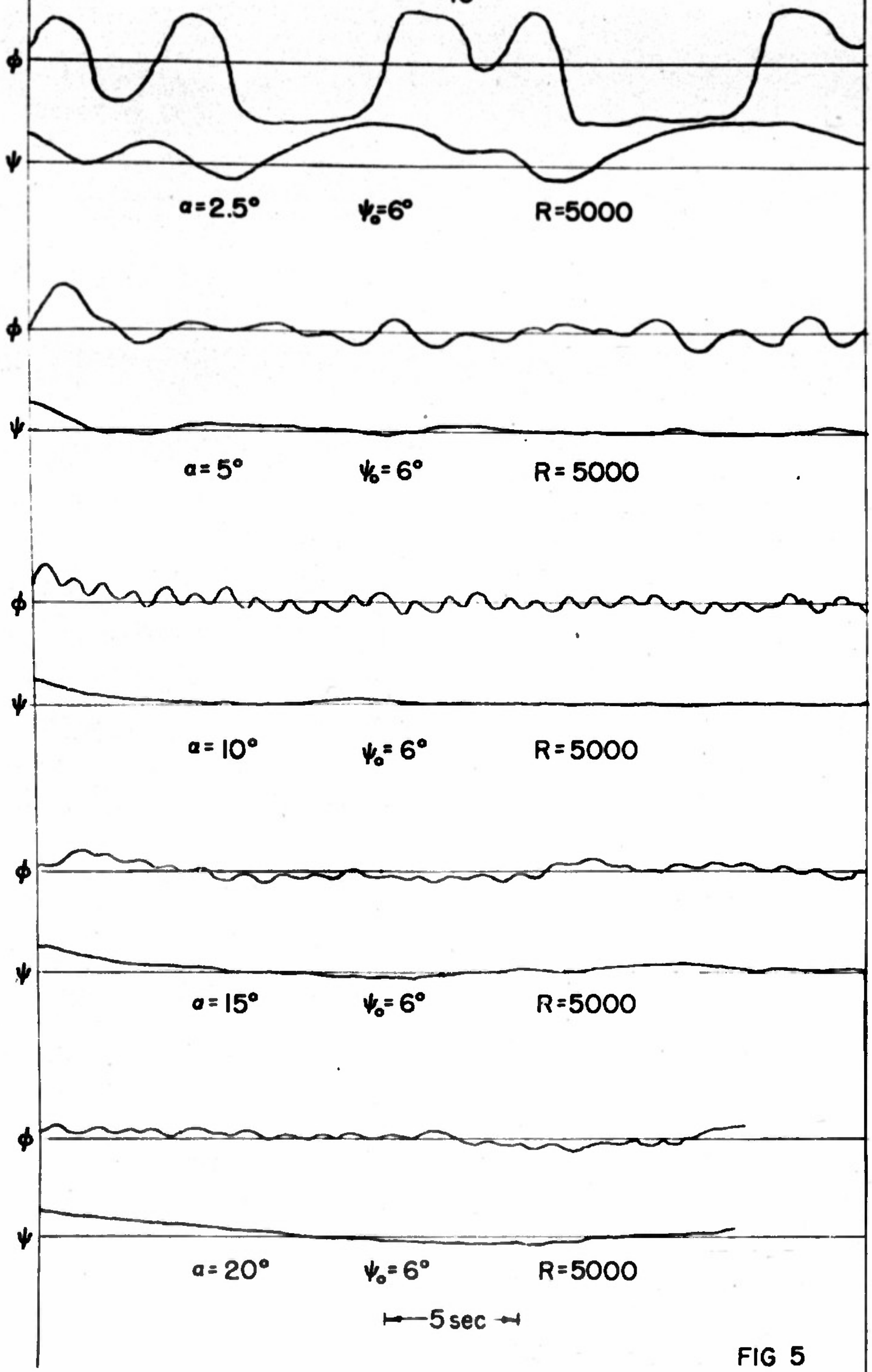

φ
ψ
a=2.5° ψ₀=6° R=5000
φ
ψ
a=5° ψ₀=6° R=5000
φ
ψ
a=10° ψ₀=6° R=5000
φ
ψ
a=15° ψ₀=6° R=5000
φ
ψ
a=20° ψ₀=6° R=5000
5 sec

FIG 5

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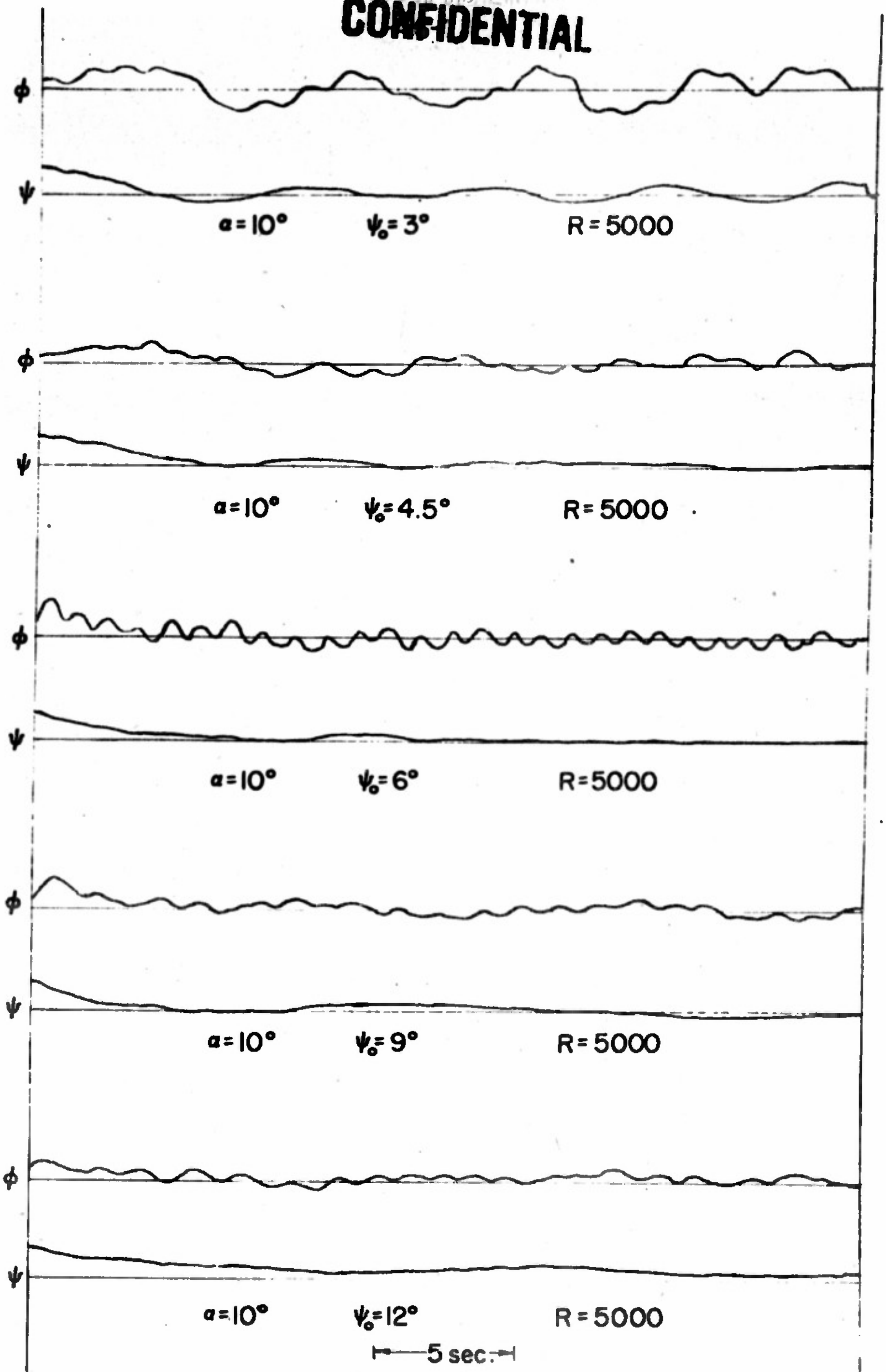

FIG. 6

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-7-

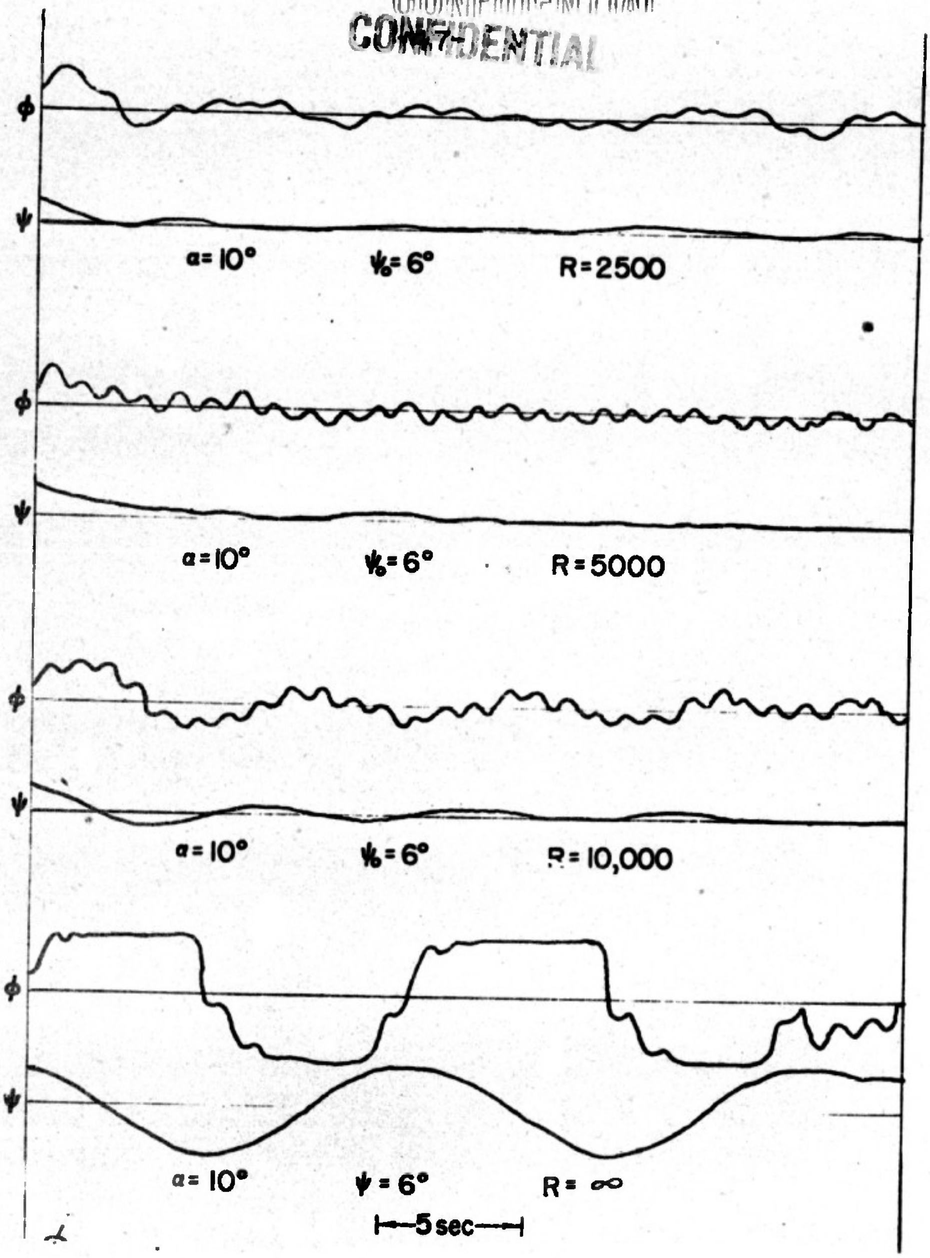
φ
ψ
α = 10°
ψ_0 = 6°
R = 2500
φ
ψ
α = 10°
ψ_0 = 6°
R = 5000
φ
ψ
α = 10°
ψ_0 = 6°
R = 10,000
φ
ψ
α = 10°
ψ = 6°
R = ∞
5 sec

FIG 7

REEL

C 38

FRAME

946

CONFIDENTIAL

TSDIN FORM 60 B (19 FEB 47)

AUTHOR(S)		ATI- 646
Skramstad, H. K.	DIVISION: Guided Missiles (1) SECTION: Aerodynamics and Ballistics (4) CROSS REFERENCES: Missiles, Guided - Stability (63650); Missiles, Guided - Control systems (62800); Swod Mark 7 (63650); Swod Mark 9 (63650)	ORIG. AGENCY NUMBER REVISION

AMER. TITLE: Lateral stability of homing glide-bombs with application to Navy Swod Mark 7 and Mark 9

FORG'N. TITLE:

ORIGINATING AGENCY: National Bureau of Standards, Washington, D. C.

TRANSLATION: PA: OSRD Div. 5

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U.S.	Eng.		Confd'l	Apr '46	52	7	photos, diagrs

ABSTRACT

Lateral stability equations are developed. A study is made of stability with fixed control surfaces, of the stability of the Swod Mark 12 and Mark 13, of the lateral control system of Swod Mark 7 and Mark 9, and of a roll stabilization system. The results obtained in model tests of lateral control system are discussed. Graphs show the elevon differential, the angle of roll and other data. Roll simulator records show the effects of gyro contact spacing and servo motor speed on the amplitude of the roll hunting motion.

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[redacted]

18 JAN 2013

Subject: OSD MDR Case 12-M-1571

Dear [redacted]:

We have reviewed the enclosed document in consultation with the Department of the Navy and have declassified it in full. If you have any questions, contact me by e-mail at Records.Declassification@whs.mil or by phone at 571-372-0483.

Sincerely,

Robert Storer

Robert Storer
Chief, Records and Declassification Division

Enclosures:
1. MDR request
2. Document 10

DEPARTMENT OF DEFENSE
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MEMORANDUM FOR DEFENSE TECHNICAL INFORMATION CENTER
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FT. BELVIOR, VA 22060-6218

28 JAN 2013

SUBJECT: OSD MDR Case 12-M-1571

At the request of [redacted], we have conducted a Mandatory Declassification Review of the attached document under the provisions of Executive Order 13526, section 3.5, for public release. We have declassified the document in full. We have attached a copy of our response to the requester on the attached Compact Disc (CD). If you have any questions, contact me by e-mail at storer.robert@whs.mil, robert.storer@whs.smil.mil, or robert.storer@osdj.ic.gov or by phone at 571-372-0483.

Robert Storer

Robert Storer
Chief, Records and Declassification Division

Attachment:
CD