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**TITLE: VALIDATION OF PROJECTILE TRANSIENT RESPONSE MODELS FOR THE STUDY OF PRESSURE OSCILLATIONS IN GUNS**

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**ABSTRACT:**

This paper presents results that provide a validated simulation approach to study the behavior of projectiles subjected to transient in-bore loadings. In this study, the PXR6353 instrumented round was modeled. The approach used features vibration studies of the components of this projectile. This includes a comparison of the vibration modes determined by finite element techniques with those modes determined using experimental modal analysis techniques. The ANSYS finite element code was used to generate the analytic model. Both impact and shaker excitation methods were used in the experimental modal analysis phase. The direct comparison between analytical and experimental data showed that the finite element model correctly captured the dominant modes of vibration of the projectile.

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VALIDATION OF PROJECTILE TRANSIENT RESPONSE MODELS FOR  
THE STUDY OF PRESSURE OSCILLATIONS IN GUNS

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INTRODUCTION

The design of artillery projectiles has been based upon the presumed use of solid propellant as the propelling charge [1]. The pressure profile across the base is typically uniform for solid propellants. Also, the pressure-time history is smooth. However, with the nonconventional propelling charges now under consideration, neither of these conditions is necessarily true. Consequently, it is vital that the capability of existing munitions to survive these new launch environments be determined.

One means of addressing this issue is to conduct large scale firing tests using representative samples from all current munitions. This approach is time consuming. Also, limited information concerning the causes of failure is obtainable from this approach for munitions which do not survive the launch environment. Another approach is to use finite element (FE) methods to analytically determine the probable response of a projectile to the expected launch environment. This approach allows the determination of potential deleterious effects of the launch environment provided the FE model accurately represents the essential structural dynamic characteristics of the real projectile. For simple structures it is relatively straightforward to generate an appropriate FE model. However, complex structures such as artillery projectiles which have numerous internal components are a much more difficult task. Validation of these FE models is essential if confidence in the analytical results is expected.

Experimental Modal Analysis (EMA) has proven to be an extremely useful tool for validating complex FE models. In its simplest function, EMA determines the basic structural characteristics such as natural frequencies and modes of vibration of a structure. These measured characteristics can be compared with the predicted values obtained from an FE analysis to determine the accuracy of the model. Good agreement indicates that the FE model can be expected to realistically capture the basic dynamic behavior of the actual structure. For many FE models, this is all that is required for validation. Additional concerns such as the refinement of the FE model to account for stress concentrations are a separate issue and cannot be addressed by EMA. However, if the FE model cannot even reproduce the gross dynamic behavior of the structure correctly, then these additional concerns are moot.

Prior analyses [2,3,4,5] of projectiles typically used either the same FE model for the dynamic analysis as was used for the static analysis or relied upon the experience of the FE analyst to generate an acceptable FE model. For conventional solid propellant these approaches normally produced reasonable results. This success is due in part to the fact that the response of the projectile can be approximated as quasi-static when the loading is due to solid propellant. This is not true for analyses which attempt to simulate the liquid propellant (LP) launch environment. As is seen in Figure 1, the pressure-time curve exhibits high frequency oscillations. These oscillations can cause significant structural response if the frequency of the oscillations is near a fundamental frequency of some component of the projectile. Consequently, it is imperative for dynamic analysis that the FE model faithfully reproduces the correct dynamic response of the projectile. This paper examines the use of EMA in the development of an FE model of the PXR6353 instrumented projectile for use in dynamic analysis.

BACKGROUND

The basic goal of dynamic analysis is the determination of the dynamic response of a structure to a defined forcing function. For complex structures this often entails the development of an appropriate FE model. The discretized FE model yields a system of n equations describing the dynamic behavior of the structure. These equations can be written in the form

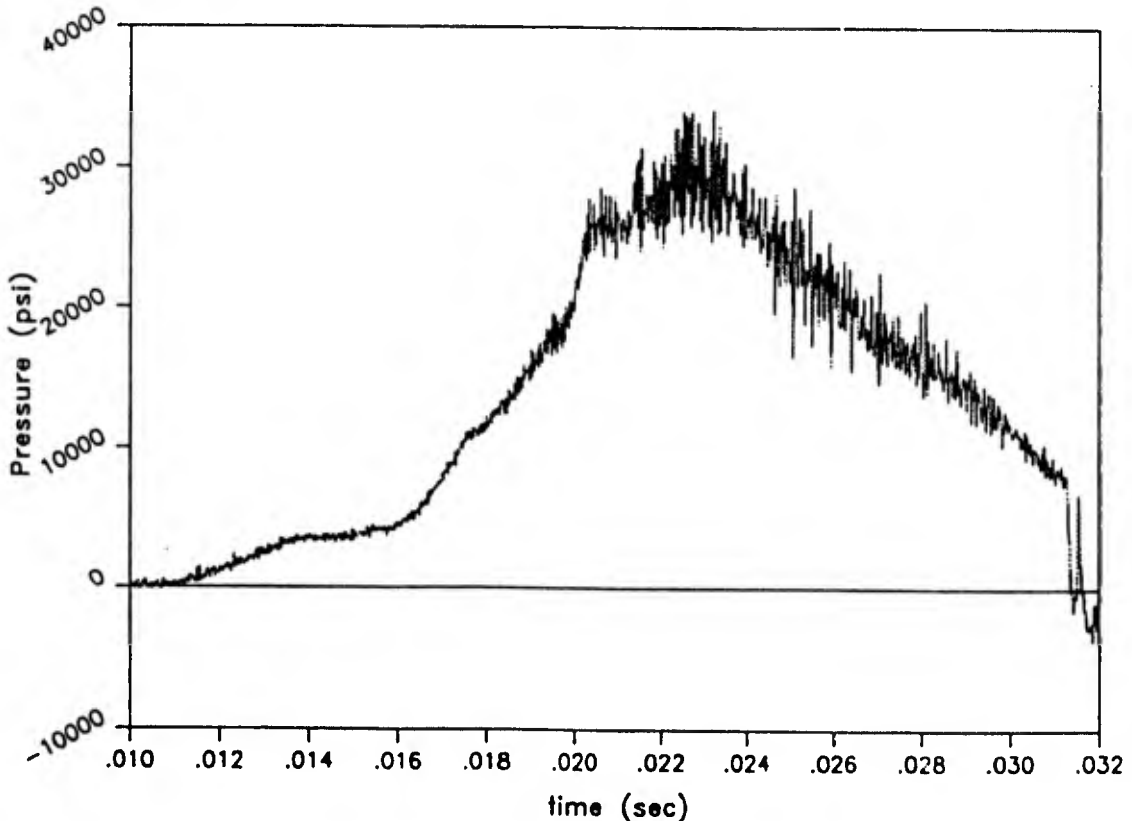


Figure 1. LP Pressure History

$$M \ddot{\mathbf{x}}(t) + C \dot{\mathbf{x}}(t) + K \mathbf{x}(t) = \mathbf{f}(t) \quad (1)$$

where  $M$ ,  $C$ , and  $K$  are the mass, damping and stiffness matrices,  $\mathbf{f}(t)$  is a defined forcing function, and  $\mathbf{x}(t)$  is the response. For linear structural analysis  $M$ ,  $C$ , and  $K$  are symmetric and time invariant. Equation 1 can be rewritten in the Laplace domain as

$$s^2 M \mathbf{X}(s) + s C \mathbf{X}(s) + K \mathbf{X}(s) = \mathbf{F}(s) \quad (2)$$

$\mathbf{X}(s)$  and  $\mathbf{F}(s)$  are the transformed responses and forces, respectively. This equation represents an eigenvalue problem. For lightly damped structures, it is not unusual to neglect the damping and instead consider undamped free vibration response. The nontrivial solution of for this case is given by

$$|s^2 M + K| = 0 \quad (3)$$

Solution of Equation 3 yields  $n$  natural frequencies,  $s_n = j\omega_n$ , and  $n$  modal vectors  $\mathbf{Y}_n$ . These can then be used in a normal mode analysis to solve the forced vibration problem given by Equation 1. Also, the predicted natural frequencies and mode shapes can be compared with the corresponding experimentally determined quantities to determine the accuracy of the FE model.

The experimental determination of the natural frequencies and modal vectors constitutes EMA [6]. As with FEA, EMA starts with a system of equations written as in Equation 2. However,  $M$ ,  $C$ , and  $K$  are now unknown. Instead, the response,  $\mathbf{x}(t)$ , and the applied load,  $\mathbf{f}(t)$ , vectors are the known quantities. Accordingly, neglecting damping, Equation 2 is rewritten as

$$B(s) \mathbf{X}(s) = \mathbf{F}(s) \quad (4)$$

where  $B(s) = Ms^2 + K$ .

The transfer function  $H(s)$  is then defined as

$$H(s) = [B(s)]^{-1} \quad (5)$$

Therefore, Equation 4 can be expressed as

$$H(s) \mathbf{F}(s) = \mathbf{X}(s) \quad (6)$$

The transfer function  $H(s)$  relates the input to the system,  $\mathbf{F}(s)$ , to the output,  $\mathbf{X}(s)$ . In component form, Equation 6 relates the input at some point  $q$  to the output at a point  $p$  by the relation

$$H_{pq}(s) = \frac{X_p}{F_q} \quad (7)$$

The individual components of  $H(s)$ ,  $H_{pq}$ , are assumed to have the form

$$H_{pq} = \sum_{r=1}^n \left[ \frac{Q_r \Psi_r \Psi_r^T}{s-s_n} + \frac{Q_r^* \Psi_r^* \Psi_r^{*T}}{s-s_n^*} \right] \quad (8)$$

where  $Q_r$  is a scaling factor,  $\Psi_r$  is the  $r$ th modal vector, and  $s_r = j\omega_r$  is the  $r$ th pole. This representation is based upon a simple one degree-of-freedom oscillator [7].

Assumptions in the derivation of Equation 1, and consequently Equation 6 imply that the entire transfer function matrix  $H(s)$  can be reconstructed by measuring the transfer functions, Equation 7, of a single row or column of  $H(s)$ . To increase accuracy, it is common practice though to measure several rows or columns. These data are then used to determine the natural frequencies and modal vectors by curve fitting the data. Also, if multiple modes at a single frequency are to be resolved, then multiple rows or columns must be measured. As mentioned, agreement between the predicted and measured natural frequencies,  $\omega_r$ , and the corresponding mode shapes  $\Psi_r$  is an indication of the accuracy of the FE model.

## ANALYSIS

### Experimental Modal Analysis

The PXR6353 instrumented projectile was analyzed both as a complete structure and as three separate components. In this paper discussion is limited to the results obtained for the separate components. The subdivision of the projectile into three components was made because these components represent logical substructures of the overall projectile. Section A consisted of the boattail and motor body, Figure 2. Section B was the body section, Figure 3. Section C consisted of all parts between the antenna section and nose inclusive, Figure 4. Section B was the simplest component consisting essentially of a cylinder with one end closed, while Section C was the most complex. All modal tests conducted simulated "free-free" boundary conditions by suspending the test sections by elastic cords with the axis of symmetry of the sections oriented horizontally. The "rigid body" frequencies for the test sections are substantially below 150 Hz which is well below the first flexural modes of all the sections. Data was collected in the frequency range of 0 to 10,000 Hz. Mode Indicator Functions (MIF) are shown in Figures 5-7 for Sections A, B, and C, respectively. Minimums in the value of the MIF indicate the location of a potential modes. The results of the modal test are summarized in Table 1.

### Finite Element Analysis

The EMA results were used to validate both 2D and 3D FE models of the PXR6353 instrumented projectile. For the 2D model, four-node axisymmetric harmonic elements were used, while eight-node trilinear hexahedron elements were used in the 3D FE model [8]. The 2D and 3D FE meshes were generated such that in the  $r$ - $z$  plane the mesh pattern was

Table 1. EMA and FEA Modal Results

	Mode	Frequency (Hz)			% Error		Damping (%)
		EMA	FEA		2D	3D	
			2D	3D			
Section A	1	1823	1941	1966	6.5	7.8	0.145
	2	4265	4748	4900	11.3	14.9	0.087
	3	5489	5717	5820	4.2	6.0	0.232
Section B	1	1897	1957	1984	3.2	4.6	0.207
	2	4679	4902	5075	4.8	8.5	0.142
	3	6111	6199	6332	1.4	3.6	0.122
Section C	1	1613	1696	1731	5.1	7.3	0.246
	2	3448	-	2508	-	-27.	4.00
	3	4034	4225	4345	4.73	7.7	0.583

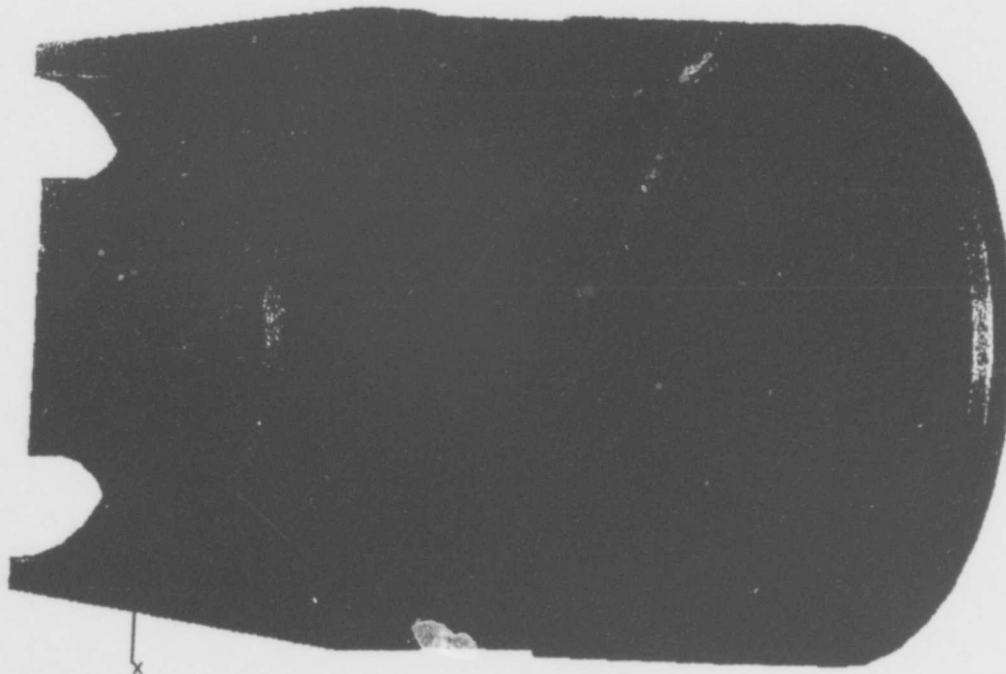


Figure 2. FEA Solid Model of Section A

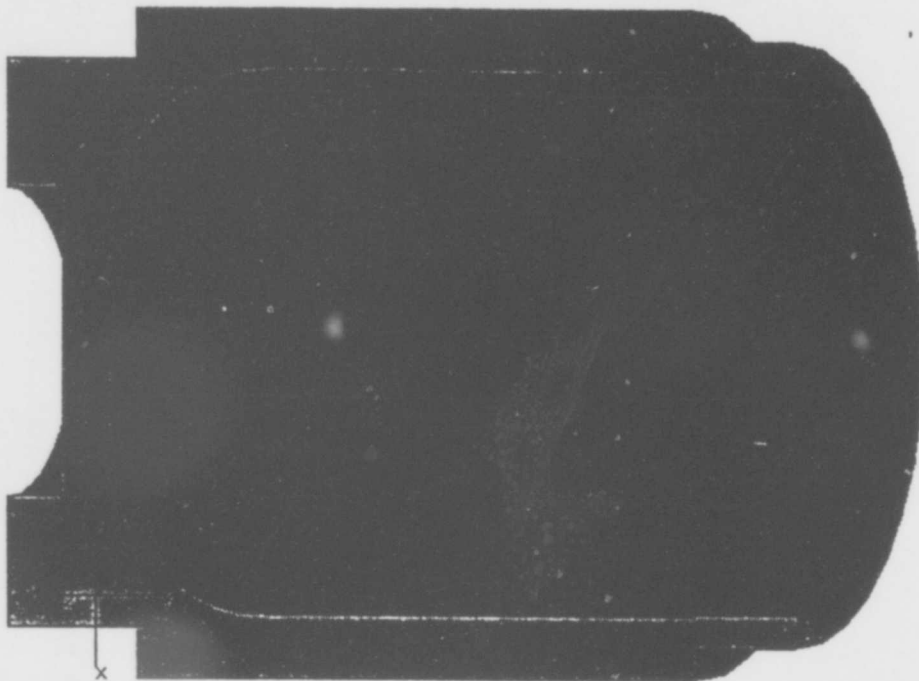


Figure 3. FEA Solid Model of Section B



Figure 4. FEA Solid Model of Section C

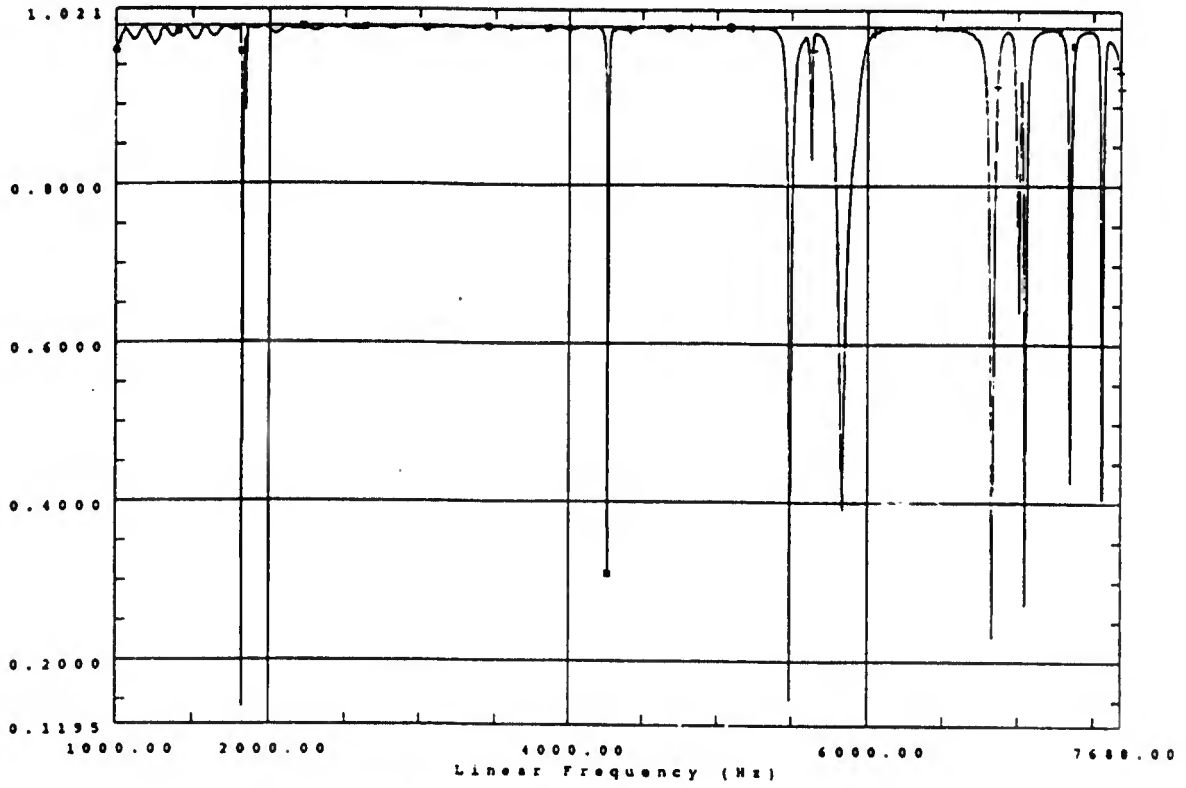


Figure 5. EMA MIF for Section A

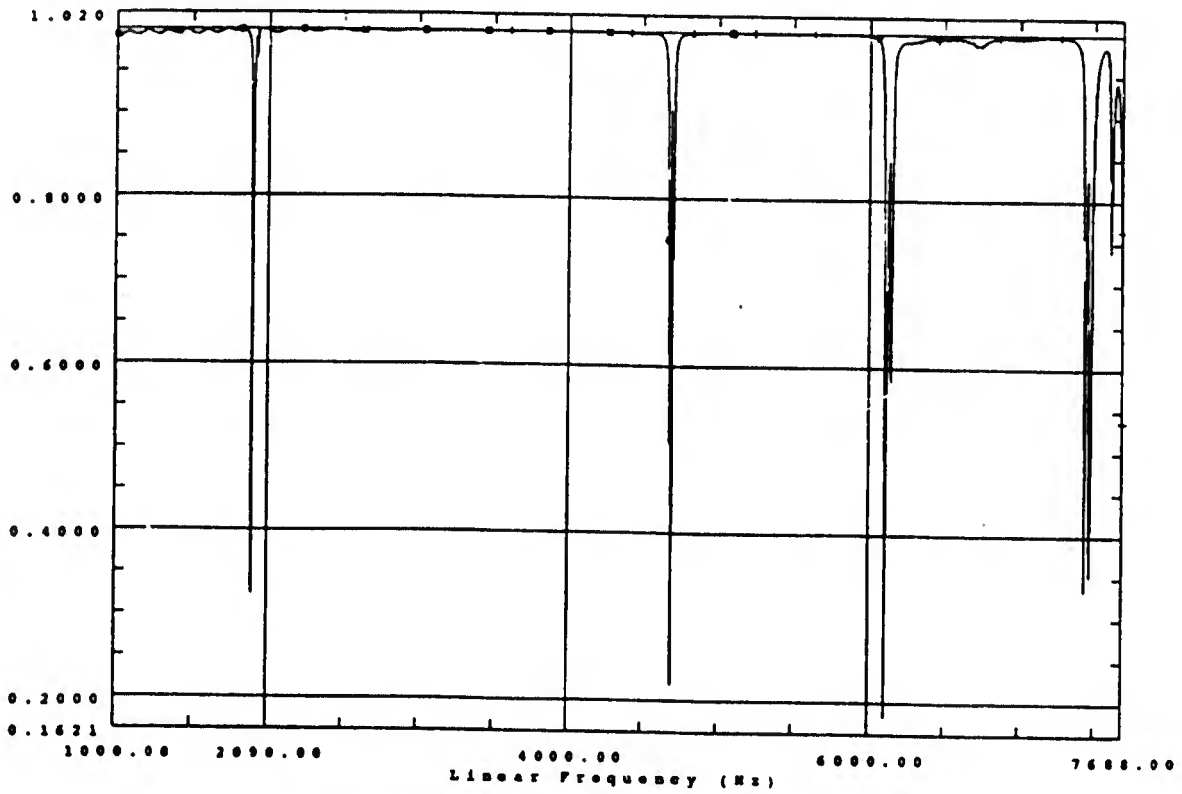


Figure 6. EMA MIF for Section B

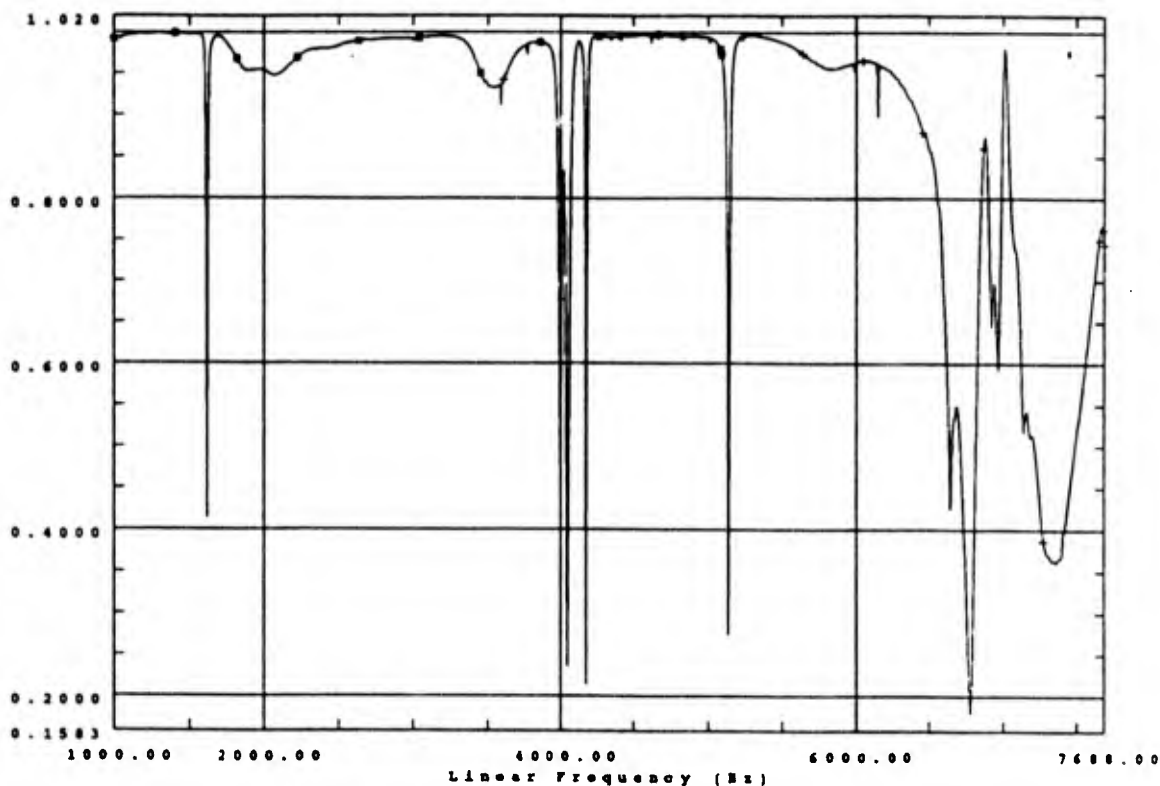


Figure 7. EMA MIF for Section C

the same. The mesh for Section B is shown in Figure 8 for illustration. The harmonic elements allow the specification of the mode shape in the circumferential direction by specifying the number of circumferential waves. The 3D FE model used 24 elements per 180 degrees in the circumferential direction. This number was arrived at by considering the effect of increasing the number of circumferential elements upon the frequency predictions for a simple hollow cylinder. The results of this cylinder problem are summarized in Table 2. It is seen that 24 circumferential elements give reasonable agreement with the same problem using quadratic elements. While increasing the number of circumferential elements would increase the accuracy, the results of this simple test problem indicate that the gains in accuracy would be minimal.

## DISCUSSION

### Frequency Comparisons

As mentioned, Section A consisted of the boattail and the motor body components of the projectile. The mass of the model for this section is 7,424.1 grams while the actual mass is 8,065 grams. The difference in mass is primarily due to the absence of the rotating band in the FE model. Inclusion of the rotating band mass in the FE model increases the model's mass to 7,935 grams. This indicates the model is approximately 1.6% lighter than the actual structure. In this study, this is considered an acceptable error. The FE frequency predictions

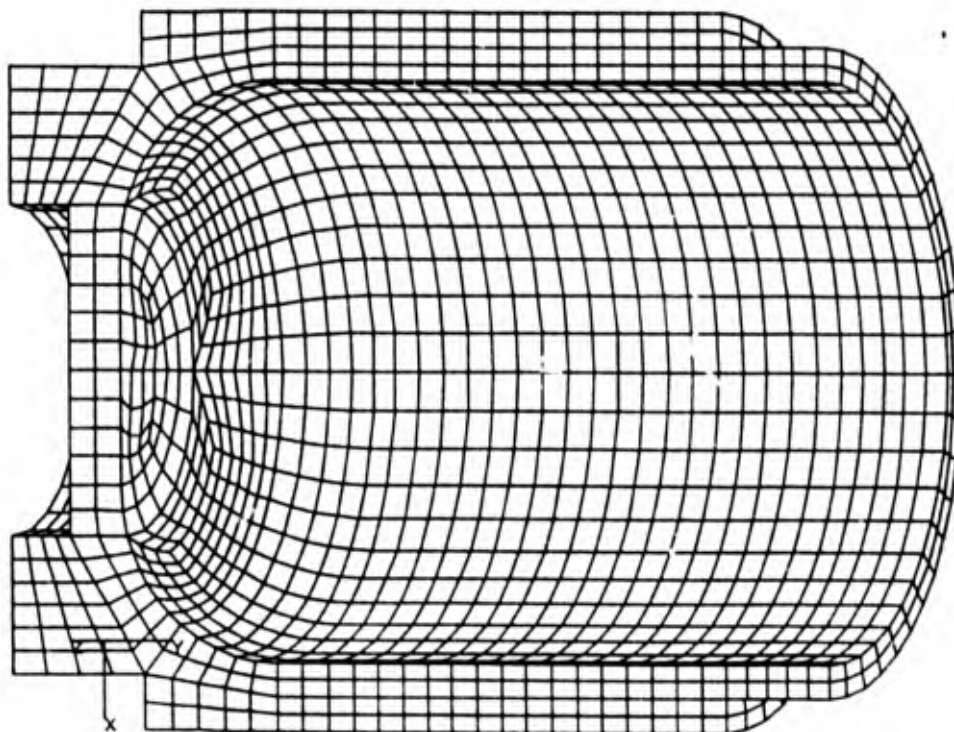


Figure 8. FE Mesh of Section B

Table 2. Convergence Examples

Element Type	# Circ. Elements	Frequency					
		Mode					
		1	2	3	4	5	6
Linear	6	1291	1335	1516	999	1959	2760
	12	853	873	1007	1012	1439	2236
	24	820	831	943	1015	1360	2145
Quadratic	6	1093	1098	1172	1001	1481	2085
	12	835	843	944	1003	1316	1983
	24	814	823	926	1004	1304	1976

are also included in Table 1. It is seen that the predicted frequencies are in good agreement with the experimental values.

The mass of the FE model of Section B is 11,024.2 grams while the actual mass of this section is 10,735 grams which constitutes an error of 2.7%. The difference in mass is due to the presence of transverse thru-holes in the base of Section B which are not included in the FE model. However, because the location of this extra mass in the FE model is in the base which is relatively rigid compared to the rest of the structure, the predicted natural frequencies are in good agreement with the experimentally measured frequencies.

Section C was the most complex section of the three sections. Each individual component of Section C though could be easily modelled. Consequently, the mass estimate is very good. The estimated mass is 10,033 grams while the measured mass is 10,007 grams which is an error of only 0.26%. This agreement is fortuitous since it is impossible to disassemble Section C to determine exactly which internal components are present. The close agreement between the FE model's mass and the actual structure's mass provides a measure of confidence that all internal components are included in the FE model. Despite the very good mass estimate, the predictions of the natural frequencies of Section C are not as accurate as might be expected. In particular, the second natural frequency is in error by -27.3%. It has since been determined that Section C was obtained from a round which had been used in a prior firing test and had sustained some internal damage. Therefore, it is highly unlikely that the FE model represents the actual internal boundary conditions between the internal components of this particular Section C correctly. Such errors in the internal boundary conditions can easily cause the large error in the predicted value of the second natural frequency since the second mode involves primarily components in the potentially damaged part of Section C.

### Mode Shapes

One method of assessing the accuracy of the mode shapes determined using EMA is the modal assurance criterion (MAC) [9]. Ideally, for linearly dependent modal vectors, the MAC value should approach unity, while for linearly independent modal vectors the MAC value approaches zero. However, there are other considerations that can also lead to MAC values of zero or one [9]. Some of these reasons include non-stationarity of the structure, nonlinearity of the structure, noise, and an invalid modal parameter estimation. During the EMA, appropriate precautions were taken to minimize these possibilities so that the computed MAC values should indicate whether two modal vectors are linearly independent. The MAC values for the first three modes of Sections A, B, and C are shown in Tables 4-6. It is seen that the experimentally determined mode shapes used to validate the FE model are orthogonal. The fundamental mode shapes determined by EMA for these sections are shown Figures 9-11. The corresponding mode shapes predicted by the FEA are shown in Figures 12-14. The close agreement between the predicted and measured mode shapes is evident.

Table 3. MAC for Section A

Mode	1	2	3
1	1		
2	.001	1	
3	.331	.004	1

Table 4. MAC for Section B

Mode	1	2	3
1	1		
2	.001	1	
3	.331	.004	1

Table 5. MAC for Section C

Mode	1	2	3
1	1		
2	.001	1	
3	.061	.001	1

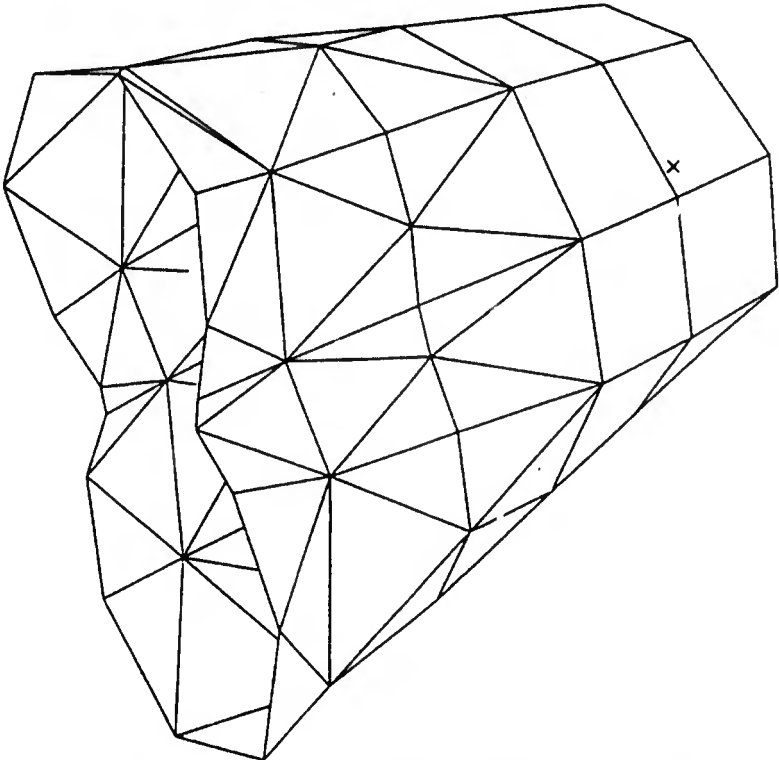


Figure 9. EMA Section A, Mode 1

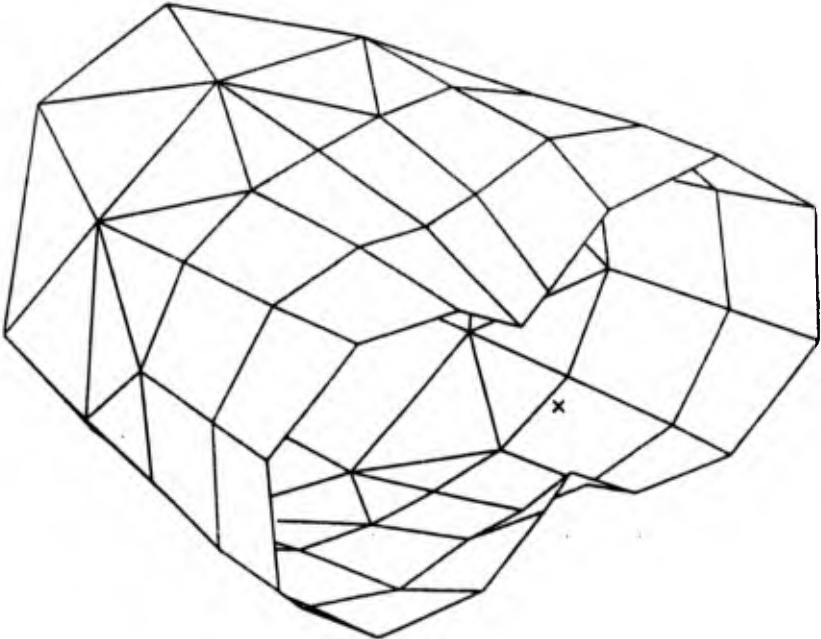


Figure 10. EMA Section B, Mode 1

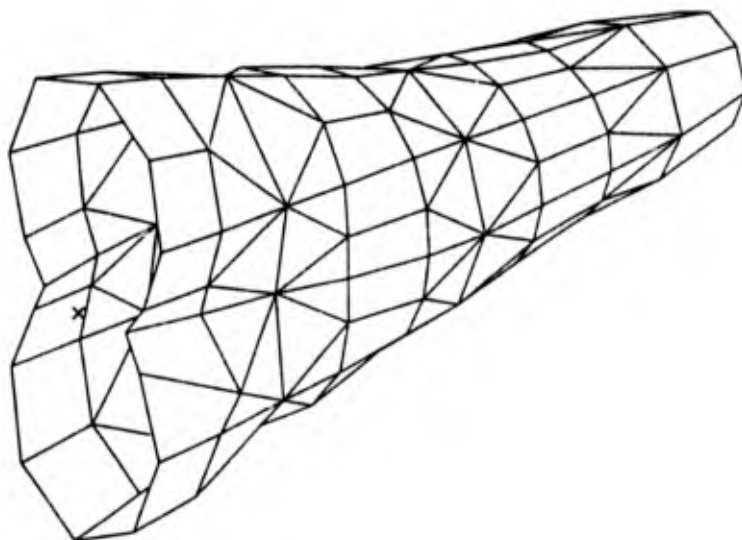


Figure 11. EMA Section C, Mode 1

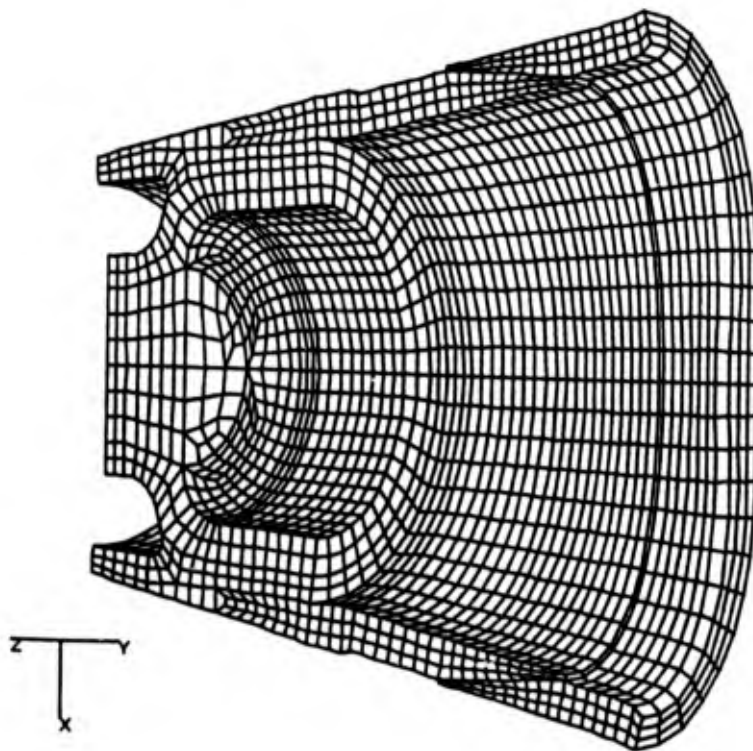


Figure 12. FEA Section A, Mode 1

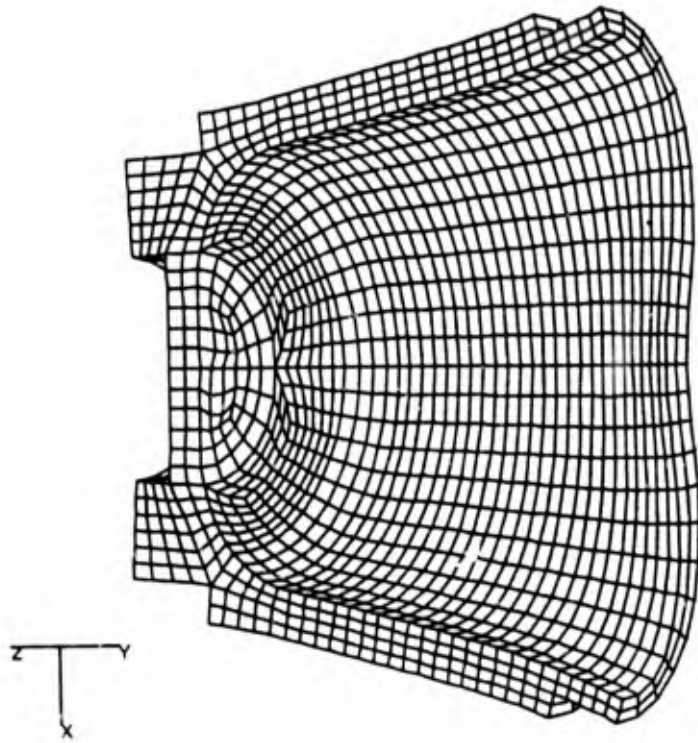


Figure 13. FEA Section B, Mode 1

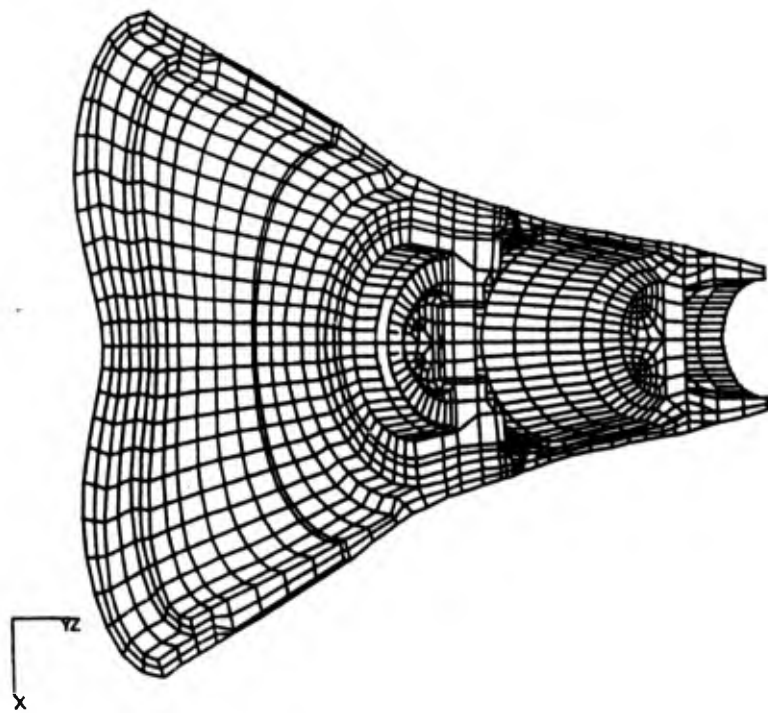


Figure 14. FEA Section C, Mode 1

## CONCLUSIONS

The natural frequencies and mode shapes of the PXR6353 instrumented projectile and its components have been determined using EMA. Linear independence of the mode shapes obtained by EMA has been verified. These experimentally determined natural frequencies and mode shapes have been used to validate an FE model of this projectile. The agreement between the FEA and EMA results indicate that an acceptable FE model has been developed. The FE model can therefore be used with confidence to predict the structural response of the projectile when subjected to the LP interior ballistic environment.

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