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# CONSTANT STRAIN ANALYSIS

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## ABSTRACT

The state of a linear system is independent of how hard it is driven. Not so, a nonlinear system. As you sweep a nonlinear system through resonance you change the state of the system. If you want to study a system in a particular state it is advantageous to maintain it in that state. The antagonism between conventional experimental practice, sweeping through a resonance, and the desire to maintain the system in a particular state is mitigated by constant strain analysis. The analysis of a sequence of resonance curves on a Berea sandstone will be used to illustrate application of constant strain analysis. The idea of constant strain analysis generalizes to constant field analysis. Resonance data can be treated so that the field (displacement, strain, velocity, ...) responsible for the nonlinear behavior of a system can be identified.

## TRANSCRIPT

DR. GUYER: The work that I am going to talk about is done experimentally by Jim Tencate at Los Alamos, the theoretical work by myself and Eric Smith, who is also at Los Alamos.

[Transparency 1 and return to Transparency 6 from his previous presentation]

Just to remind you, Katherine went over a bit of what I might have said about theory, so I will figure out some way to sort of pass over that. Here is another example -- Fontainebleu. This is just to remind you to think of the fabric of defects in the rock as the important thing that determines its elastic properties.

One of the things we are going to be talking about is the theoretical model that relates the quasi-static measurement of the dynamic measurement just to make it seem like this is making a deep statement. There is a traditional model of elasticity that involves Taylor series expansion of the free energy in terms of the strain field. Nonlinear elasticity would just have -- those are the $C_{ijkl}$ and there would be a whole bunch of other fancy coefficients, high-order terms in the strain field.

There would be other higher order terms in the strain field, all of it, basically, on the other hand, assuming analyticity of the free energy, so you develop notions of cubic nonlinearities and if you are a phonon physicist, you talk about the 3-phonon process and the 4-phonon process.

If you do acoustics, you talk about the coalescence of 2 acoustic waves to produce a third, 3 acoustic waves to produce a fourth, and so on, and these are well-studied phenomena in nonlinear elasticity.

If you try to make a theory at all involving hysteresis, discrete memory, the kind of magnitude and sign of the frequency shift we are going to talk about in resonance, this theory simply does not come anywhere near it. It simply cannot be used. In some sense, it would seem that for the kinds of materials we are talking about, or at least that I am talking about, this is simply not the way to start and it is not the way you would go about describing the materials.

[Transparencies 2-5]

In some sense, the traditional theory, which is this theory of strain fields and Taylor series expansions in strain fields, does not really work well. But we know it is not supposed to look at all these phenomena, extremely large nonlinearities, hysteresis, and discrete memory.

But there is a theoretical model and it is the one that I alluded to and I will now allude to it, again, that does describe the quasi-static measurements and I am going to tell you about the experiment that confirms how that theory also, seemingly, describes the dynamic measurements.

Remember, it was a theory where we paid attention to the elastic elements between the grains; the grains were thought of as essentially rigid. This comes with a very fancy name, it is called "PM space," and the reason for that is that the original description of hysteretic elements, whether they be elastic elements or magnetic domains or happen to be fluid configurations, is due to a man named Preisach, a German scientist from the 1930s, and it was elaborated in an engineering context by a man named Mayergoyz, so we called it PM space, but it is a place in which one tracks the hysteretic elements that are in your system.

In our case we will be talking about hysteretic elastic elements, and it is a space in which you track the way in which the hysteretic elastic elements are responding. If you are doing a quasi-static measurement, it is the response to the pressures or the stress fields you are putting on the system or, if you are doing an AC measurement, it is the response to the AC stress fields that happen to be in the system.

I am not going to say a great deal about that, except to remark that the theoretical model actually has a rather nice feature.

[Transparencies 6-7]

The feature is that it gives you the qualitative description of the quasi-static measurement but it also provides you with the recipe where you can take the quasi-static measurement and actually infer some of the quantitative features of the elastic elements that you are talking about, the hysteretic elastic elements, so there is a forward model where, if you happen to know how the hysteretic elastic elements behave, you can predict that curve and you can predict the details of the discrete memory and the various interior loops and so on, but it also has the property that if you have a suitable set of data of this type you can actually infer what the elastic elements that are in the system are like.

Then what you notice, of course, is that this is a discussion about quasi-static measurements, somebody squeezing with 200 atm up and down. Of course, when a sound wave goes through a piece of material, it squeezes it locally with a pressure that might be a fraction of an atmosphere. A sound wave does nothing more in a material than what the quasi-static pressure, except for issues of time scale.

If you have some understanding of the elastic elements in the material for these big 200-atm variations, you might also be able to give a discussion of basically how an elastic wave propagates through the material. That is what we hope we have done.

[Transparency 8]

Here is the program. We have from Greg Boitnott at New England Research a spectacular set of quasi-static measurements. We are able to analyze those quasi-static measurements and actually say something quantitative about the nature of the hysteretic elastic elements that are in the system.

We are then able to take our understanding of those hysteretic elastic elements and imagine now that there is a sound wave going through the system. This measurement goes over 200 atm and that space where we have an understanding of the way in which the hysteretic elastic elements behave is sort of 200 atm from top to bottom --

DR. SACHSE: On that second plot, what are the axes? I have no idea what you are plotting there.

DR. GUYER: The behavior of the elastic elements in this material, how they behave. You squeeze on the material and the elastic elements might, for example, make a discontinuous change from having one spring constant to another at some point in the pressure protocol that is going from, let's say, 0 to 200 atm. I did not want to allude to this in great detail but just to say that it was possible to do this, so this is supposed to be a picture that says I understand how the elastic elements respond to pressure.

DR. LEVY: What are the coordinates?

DR. GUYER: You do want me to talk about this, don't you?

The elastic elements are hysteretic, they behave something like this. The elastic element is in state one, you squeeze with a pressure, it stays in state one. At a certain pressure, it makes a transition to a different state and it stays in that state, i.e., you continue to increase the pressure, it stays in that state. You decrease the pressure, it stays in that state until a second pressure, when it goes, boop, back to the other state.

The coordinates in this so-called PM space are the pressures, the two pressures, at which the elastic elements respond. It is somewhere between 0 and 200 atm, because that is the range over which the measurement was made.

From an understanding of the elastic elements in the quasi-static measurement we can predict the behavior we expect, for now a very small pressure variation in the material, much more modest than the 200 atm, so we can predict the behavior here.

DR. LEVY: On the vertical coordinate, that is, again, number density, probability?

DR. GUYER: That is a density of elastic elements in a space where you track their response to pressure and the two coordinates are two pressure coordinates and the density is the number of elastic elements that respond in a particular way to a particular pressure. That is the part I was trying to avoid talking about in detail.

You would not normally start out describing elasticity this way, you would write down some Taylor series expansion. Who the hell would write down a density of elastic elements in a space where you track their hysteretic behavior? But if you are going to describe this kind of experiment, you cannot get away without doing that.

Here is what traditional theory predicts. Traditional theory is the Landau-Lifschitz-Taylor series expansion. It says, for example, for the fundamental resonance in a bar you would find a shift in the frequency of the maximum going as the square of the strain field in the bar and in proportion to something which is the quartic anharmonicity -- it is usually called delta among the people who do this for a living. You would find a nonlinear attenuation, a Q, differing from the Q at a basically arbitrarily small strain field, that goes as the square of the strain field.

For the strain fields we have been talking about, (they go to, at most, on the order of $10^{-6}$) the coefficient delta is thought to be of the order of $10^6$ and would suggest a frequency shift of one part in $10^6$ and a one part in $10^6$ change in Q.

What is observed experimentally is that -- well, let's go to the other theory, first, I guess. It predicts is a frequency shift proportional to the magnitude of the strain field; that is, this is this nonanalytic function, a coefficient that is related to the density of elastic elements in that space up there. It is related somehow to that density; again, there is a theoretical model that is rather elaborate that has to be carried out to do this.

A change in Q, also proportional to the magnitude of the strain field, again, a nonanalytic function, with a coefficient not quite the same as C1 but related to it by
$4\pi/3$, so of the same order of magnitude and, furthermore, from an understanding of the density in that space you can actually say that these numbers are of the order of 1000. That is a prediction.

Now, what I am going to talk to you about is an experiment to try to confirm it. The experiment, of course, is just a resonance experiment, but if you are doing nonlinear resonance, you have to do things in a rather -- you do not have to -- you ought to do things in a rather different way.

[Transparency 9]

This experiment was done by Tencate. It is the data you have seen now about 4.5 to 7 times. This experiment was done after the Berea sandstone, that long and that big in diameter, had been in a vacuum chamber for about seven months, and it was done during the 8th month. The experiment is still in the vacuum chamber; it has now been about 18 months. It is in a temperature-controlled environment, so we are trying very much to -- in fact, you have to, you have to worry about humidity, as somebody pointed out.

You have to worry about temperature, a rock is a very good thermometer. The measurement is to look at the resonance as a function of the drive amplitude. If you are used to doing linear resonance, then you do not think about this too hard. If you do nonlinear resonance, then in some sense you think about it in this way, you fix the voltage of the drive amplitude, let's say 81, and you measure a certain number of frequencies that take you through the resonance.

You change the drive amplitude. You go through the same set of frequencies again, change the drive amplitude, go through the same set of frequencies again, that is, generate this set of curves. What you are actually doing is filling out a matrix; that is, there are n values of the voltage, m values of the frequency, so each experiment is, in some sense, a matrix of response at a set of points that are characterized by the voltage you are driving with and by the frequency.

Because of the known delicacy of this system to things like temperature, or whatever, when we actually do this experiment, we accept an experimental run if we go to this voltage and run through the frequency, go to that next voltage and run through the frequency, go through the whole series of resonance curves up and down, as the voltage goes up to the very highest voltage, come back down, do the resonance up and down and --

[Transparency 10]

-- compare the whole set of resonant frequencies you have been finding the whole way and, if they are not consistent with one another by some sensible measure of their value, we generally do not use the experiment, because there are very slow time evolutionary drifts to these systems that one simply cannot avoid, so there is a great deal of effort put into being sure that there are no systematic events taking place at the same time that we are trying to give a discussion about very precise analysis.

What you actually do is you measure the in-phase and out-of-phase component of the response at every voltage and frequency. You actually measure 2 matrices. I call the in-phase matrix A and the out-of-phase matrix B, and if you want to think about how to think about it, plot the frequencies at which you make the measurements this way and then stack the set of voltages back that way.

Why are we doing this? We are doing this because the nonlinear state of a rock is a function of some internal field in the rock, its strain field, the velocity field, something like that. When you go through a resonance curve you are continually changing the strain field, so you are continually changing the very field you are interested in trying to understand; that is, you sweep a resonance and the strain is very small here, it is very large there, very small there, you change the voltage -- the same thing, again.

If you are going to try to talk about the way in which there are various internal fields in a rock (in anything, in anything that is nonlinear), that cause consequences in the behavior of the rock, you might want to look at it at constant values of the field that causes the consequences. That is, you might want to study these data not by studying resonance curves but, rather, by taking a family of resonance curves and studying them on trajectories that cut through the whole set of resonance curves at constant field so that the rock is in the same state along this trajectory the whole time. It is in a continually varying state along that trajectory.

The things we have done in order to be able to make rather careful quantitative measurements is to analyze data --this was called "constant strain analysis" -- sets, if you like, cutting this matrix like that as opposed to tracking it like that.

[Transparency 11]

This is how you do it. You take a bunch of data, there are all kinds of prefacing related to determining internal phase shifts and so on in the instruments. It is supposed to be a set of resonance curves and you essentially cut them that way and produce -- I am going to show you a contour plot of what a nonlinear resonance looks like, and you study it along contours of constant strain to understand its behavior.

[Transparency 12]

There is one other wrinkle -- well, it is not really a wrinkle, but another little thing we do when we do this study -- we do not fit Lorentzians to resonance curves. The Lorentzians actually drive to zero the edges of a resonance curve.

What you ought to do -- there is the in-phase component, it has got this big denominator, so as soon as you get off by a Q-and-a-half to the left or right, you basically drive the amplitude, so you do not want to fit A to a Lorentzian, or the out-of-phase component to a Lorentzian. You can experimentally get rid of this denominator, because it is just the sum of the squares of the 2 fields you are experimentally measuring.

Of course, again, I said these are matrices. You have a matrix entry $A_{ij}$ at each voltage and frequency, a matrix entry $B_{ij}$ and a corresponding magnitude of U, so just manipulate your data like this. What you are studying is simply -- I used a bunch of dimensionless measures here – omega squared minus one, and you can understand where the resonance is very simply in this thing. If you study that quantity, you are basically simply studying omega divided by Q.

If you have got a nonlinear resonance, it has got a very peculiar shape; you certainly do not want to say the width has anything to do with Q. You learn about Q by studying the way in which the amplitude at resonance evolves or you can, if you like, study this function and Q will be the quantity in the denominator.

[Transparency 13]

Here is a sequence of resonance curves. These were the ones that were studied. They were on Berea sandstone and there were about 121 frequencies -- do not ask me what the voltage was but I will tell you that the strains went from $2 \times 10^{-8}$ for the lowest curve to basically $6 \times 10^{-7}$ for the highest curve.

You almost cannot see it here, but if you look extremely carefully, you will see that the resonances here (and it is shifted slightly to that side of this dashed line), so this is the very low-strain region of the nonlinearity, and that is what we are going to study to try to see how that frequency shift moves as a function of strain field.

If you take that same set of data and spread it out and do the constant strain-field cut and look at it as a set of contours, now you can actually notice the resonance very cleanly, notice the change in the resonance, because the contour of constant strain actually -- it is not symmetric, but the important quality is that along this trajectory the system is in the same elastic state.

A normal way of doing resonance cuts it this way and then moves from elastic state to elastic state the whole time you are looking at it. This way you look at it along a constant elastic state and you do the various things I described.

[Transparency 14]

I will show you what the answer is. If you do very careful analysis of the behavior of the resonance at constant strain field along each of these various trajectories, you can say what resonance frequency characterizes the system on a particular one of those trajectories and, as you go from trajectory to trajectory, therefore changing the strain, how does the resonant frequency change with strain?

Similarly, you can measure the out-of-phase component and see how the Q varies as you go from one strain field to the next.

[Transparency 15]

This is the result. The Q is the amplitude at resonance. It is not related to the width -- of course, it is related to the width, but since the system does not have a constant strain, you cannot quite decide how to handle that -- so find Q from the amplitude.

The Q dependence is that 1/Q starts out at a value of $1/Q_O$ and rises linearly in the amplitude of the strain. The units here are arbitrary units but they are close enough to say this is $10^{-6}$ in strain. This happens to be a linear plot, so $10^{-7}$ is actually down in there.

DR. MCCALL: What are the triangles?

DR. GUYER: Up and down. Are you asking why are there triangles versus squares? At the very lowest strains we are down at basically $2 \times 10^{-8}$ and the noise in the system is great enough that there is crazy behavior down here and then there is not so-bad behavior there, and once the strain gets above about $10^{-7}$, things start to straighten out and you have to just imagine there are systematics that make it very difficult to pin things down.

I tried to put some error bars up in here, but I did not put the corresponding error bars there, so you get a sense of -- one is talking about fractions of a tenth of a hertz on top of roughly 3000 Hz when you are talking about these kinds of numbers.

The frequency shift is experimentally directly proportional to the magnitude of the strain field. That is confirming the prediction of the fancy theory and, if you like, "disconfirming" traditional nonlinear elasticity.

[Transparency 16]

The only thing that remains -- not the only thing -- is are the numbers right? It is one thing to get a qualitative answer and the answer is, yes, the numbers are right. As we go to Greg Boitnott's experiment, we do the analysis for that stuff that is on the graph without coordinates but, nonetheless, I will tell you that somehow I can show you how it is quantitatively related and it predicts a numerical value for the coefficients that tell you how big the actual linear change in the frequency, the linear change in the 1/Q will be, and this is not fantastic agreement, but the fact that it is the same order of magnitude is gratifying, it is within roughly a factor of two of what is observed experimentally.

We take this to mean that you can take an experiment like this one -- this one goes up to strain fields of $10^{-3}$ and involves pressures of up to, let's say, 200 atm and learn about the elastic elements that are in the system and it also involves times scales (this is a quasi-static measurement, so the time scales here are minutes to hours, multiples of seconds), you learn about the elastic elements in the system, then propagate a wave, which tickles the system with strain fields of order of $10^{-7}$, so four to five orders of magnitude smaller on time scales that are probably four to five orders of magnitude faster and, nonetheless, find that you have learned something about the elastic elements over there that will also tell you, in a quantitative way, about the way in which the elastic elements in the system respond to dynamic measurements.

Here is the conclusion. I said it in words, but there it is in pictures relating -- here is the quasi-static measurement made out here, relatively high strain, relatively low frequency, quantitatively related to the dynamic measurement, which is done at relatively higher frequency and relatively low strain fields.

DR. MAYNARD: Do you have a reference for that PM space analysis?

DR. GUYER: Yes, Jay, there was one in *Physical Review Letters* about 1995. Also, there was a *Physical Review Letters* about a month ago describing this experiment that I just talked about and it makes reference back to this set of papers that are more or less behind it.

DR. SACHSE: Has anyone repeated that curve, that discrete hysteretic type of curve going up and down and then doing it a second time?

DR. GUYER: Oh, yes. You go up and you come down and you do not go back to where you were. You go up and you do not quite come back, and then you go up and about the third time you come back. This all relates to the fact that you pick up a rock off the ground and it has had a pressure experience in its life; you have no idea what elastic state it is in.

Some of the elastic elements in it are going to break, crack, or do whatever they like, the first time you touch it or squeeze it, but after you have fussed with it just a little bit, it will achieve an elastic state that still has hysteresis, discrete memory, and so on, but is repeatable and reversible, and that is, in fact, the elastic state we have been talking about.

PARTICIPANT: Have you tried to add any pore fluids in there to see if there is an --

DR. GUYER: Of course, as I said, this was done in vacuum and one of the things we are very aware of is that pore fluids have a great deal to do with the way in which -- there is a preliminary measurement from France, from Paul Johnson, in fact, that seems to show that saturation has an influence, a striking influence, on nonlinear behavior.

I come from the school that says that saturation is not -- what we have learned here is that the stress field, or the pressure, is not a good variable for characterizing a rock. If you have any experience with pore fluids and pore spaces, you also know that the saturation is a not a good variable. You have to study the history -- you say the history of the chemical potential, how has the chemical potential of the pore fluid been evolving, what is the state it is in at the moment, what have you done?

Both of those are hysteretic things, the pore-fluid configurations are hysteretic and so is the elasticity, and it makes it a really difficult system to study.

Thank you.