TITLE: Comparison of Radiation and Scattering Mechanisms for Objects Having Rayleigh Wave Velocities Greater than or Smaller Than the Speed of Sound in the Surrounding Water

COMPARISON OF RADIATION AND SCATTERING MECHANISMS FOR OBJECTS HAVING RAYLEIGH WAVE VELOCITIES GREATER THAN OR SMALLER THAN THE SPEED OF SOUND IN THE SURROUNDING WATER

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ABSTRACT

Metallic objects typically have material properties such that the characteristic Rayleigh wave velocity exceeds the speed of sound in water. As a consequence, over a wide range of frequencies, smooth objects (including empty shells) support surface guided waves having phase velocities exceeding the speed of sound in water. Such waves are effective in leaking radiation and give rise to various backscattering enhancements not necessarily associated with global resonances of the object. For example, high frequency meridional ray backscattering enhancements have been observed and modeled for tilted metallic solid cylinders [K. Gipson, Ph.D. Thesis, WSU (1998)] and shells [S. F. Morse, et al., J. Acoust. Soc. Am. **103**, 785-794 (1998)]. For "plastic" polymer objects, however, it is necessary to reexamine the significant radiation and scattering mechanisms because the intrinsic Rayleigh phase velocity is smaller than the speed of sound in water. Some novel scattering enhancement mechanisms for such objects are introduced including the caustic merging transition for waves transmitted through tilted cylinders and the tunneling to subsonic Rayleigh waves. The former has an optical analogy in the scattering of light by tilted dielectric fibers [C. M. Mount, D. B. Thiessen, and P. L. Marston, Appl. Opt. **37**, 1534-1539 (1998)] and has been observed in sound scattered by tilted truncated polystyrene cylinders [F. J. Blonigen and P. L. Marston, J. Acoust. Soc. Am. **102**, 3088 (A) (1997)]. [Work supported by the Office of Naval Research.]

TRANSCRIPT

DR. MARSTON: In contrast to most situations of interest in RUS, where you want to avoid fluid loading, our interest is scattering, where fluid loading plays a major role because we are interrogating objects with a sound wave.

We are going to look at some situations in both this talk and in the one this afternoon by Todd Hefner where fluid-loading effects are very important. In his talk resonances are also important. In this one, less so, but it serves to introduce the problem that he is going to address.

[Transparency 1]

We are going to start off by reviewing some situations where fluid-loading mechanisms for metallic objects that I touched upon at the last meeting are used. Then we are going to look at what happens if you consider plastic objects in water rather than metallic objects; there are some major differences if you want to consider scattering mechanisms for those. Then we will introduce some of these novel mechanisms, one of which Todd will develop this afternoon.

[Transparency 2]

This is a one-page review of what was discussed at the last meeting. If we consider a typical very stiff object in water, either metallic or, in this case, a tungsten carbide sphere, we can get large elastic contributions, because Rayleigh waves on this object have phase velocities exceeding that of the surrounding fluid.

There is a coupling condition, in this case location B, where there is a matching of the projection of the wave number of the incident wave with that of the Rayleigh wave on the object. There is radiation off, but when you get around to this point B′ there is radiation in the backward direction.

You can see these with this series of echoes and you can see these echoes are decaying very rapidly. Even though this is a very dense object, the Q's are not typically very high because of the large effective radiation loading. A Q of 10 would be a large Q for this particular scattering mechanism.

In what follows we are going to express frequencies in terms of the wave number radius product, the wave number being that of the sound in water, and we call that product ka.

[Transparency 3]

The mechanism I would like to start off with at the beginning is one where these leaky waves are important and are on elastic cylinders that are bluntly truncated. There is a mechanism for cylinders that is somewhat like the one in the diagram I just put up; that is, for a tilted cylinder you can have helical waves that run around.

It turns out at high frequencies a stronger backscattering mechanism is the meridional ray, the ray that is launched in the meridian defined by the incident wave vector and the axis of the cylinder, runs down this meridian, reflects from the edge, has a leaky wave and than radiates off, and some of that radiation goes back in the direction of the source and produces strong scattering.

This is an expanded diagram. One of the things that will come up in modeling this is the reflection coefficient of the leaky wave at the end of the cylinder. Another aspect that comes up in the modeling is the attenuation length just due to the leakage of this radiation.

For the high-frequency systems we are interested in, the attenuation length is short when compared with the length of the cylinder and, as a consequence, the resonances are actually not very important. In fact, the phenomenon is relatively frequency independent, and that is partly because the radiation damping is so large.

[Transparency 4]

One way to interrogate this and get a lot of spectral information is to use a very wide bandwidth source. We use a PVDF sheet source in water that is also sufficiently transparent that the sound that is radiated by the target reaches the hydrophone on the other side.

If you apply a voltage step to this source, it produces a pressure impulse that has broad spectral characteristics and one can then plot the spectrum of the frequencies along this axis as a function of the tilt of the cylinder, where gamma expresses the tilt, 0° is broad side, and 90° is end on.

For our purposes, we will be interested today in this meridional ridge, this ridge here that arises because of this meridional ray, and this is potentially useful because this extends all the way to the near end-on illumination. It gives you a backscattering enhancement for a wide range of angles. If one considers, say, randomly oriented cylinders that you might be looking for with a high-frequency sonar system, this is a potentially important mechanism.

The class of leaky wave of interest for shells turns out to be that which is like a_0, that is, the lowest antisymmetric Lamb wave that runs down the shell.

[Transparency 5]

To remind you of the dispersion curve for this, if you consider a plate or a cylinder by itself, in the absence of fluid loading you would get this fine dotted line here, but this line crosses the situation where the velocity matches that of water -- that is, in these units, c_ℓ/c is equal to unity. There is a splitting of this mode and one has a supersonic part, which is the part that is relevant to the present discussion.

Notice this part exists because the Rayleigh speed for the material exceeds that of water. We know that for the a_0, the limit of the c_ℓ is the Rayleigh phase velocity and that issue will come up when we return to plastics at the end.

[Transparency 6]

Scott Morse was able to collect data on this as a function of angle and of frequency, so this shows the tilt in our units, it is a form function. These are actually rather large form functions, because if you consider, say, just the end diffraction from a perfectly rigid cylinder, the end-diffraction contributions scale as 1/root ka and are at least an order of magnitude smaller than these peaks.

We also show on here the data are the points -- there is a ray model that is a smooth curve and then there is a somewhat structured curve that is an approximate partial wave series model that does not incorporate the proper mechanics of the radiation processes at the ends and makes a number of other assumptions, but it is a useful sanity check on what we are doing.

You will notice that we recover, really, the right behavior in the ray model but as we go up in frequency there is a discrepancy between both the predictions of the ray model and the data. We will see why this is on the next transparency.

[Transparency 7]

What we are going to look at now is walking along this ridge, that is to say, adjust the tilt angle to maximize the scattering. In this region the ray model works very well as does the approximate partial wave series. However, there is a deviation and finally there is a large difference up here.

We understand this large difference and, in fact, the dashed curve is the ray model without corrections for the reflection coefficient at the end. When the a_0 wave hits the end, we first assume that there is a unimodular reflection coefficient. What Scott Morris was able to put in the model here is the mode threshold for generation of a propagating a_1 wave, that is, the next higher antisymmetric Lamb wave and there is mode conversion to that Lamb wave.

That clearly is evident if you just consider the simple plate problem and that is also evident in the data. We see that there is actually a precursor to that mode conversion that extends into this region. We would like to at least understand why that precursor is there and it tells us something about radiation for fluid-loaded objects.

[Transparency 8]

We can get an idea of that by considering the results of Mindlin's analysis for plate modes. On the axis are the wave numbers, the real wave number along here, frequency vertically, and the imaginary component wave number in this plane. This is for a plate in a vacuum.

The a_0 wave we have been talking about is the one that runs along this curve. The a_1 wave, if you are in the propagating region, that is to say you are above the threshold, you are in this region here. But if you go below the propagating threshold, Mindlin found that there is a purely imaginary branch -- that is this part.

Why is that relevant to radiation? When you are near the threshold, this wave number is purely imaginary but small in magnitude, which means the scale is large in magnitude and that corresponds to an exponential flopping of the end of the plate and that would give rise to a large fluid-loading effect and hence the depression of the reflectivity and hence the form function for a broad region before you get to this threshold.

[Transparency 9]

This is also evident in these very broad spectral data that Scot Morse obtained. This dip here extends a little bit below this threshold.

[Transparency 10]

I can say more about the model to you privately, later, if you interested. It is somewhat analogous to the path integral approach of quantum mechanics in that you have to take into account defective paths in addition to rays, which are the stationary phase paths.

[Transparency 11]

This same model we applied to meridional ray for Rayleigh waves on a solid cylinder. This is in the frequency range where the waves that would run down the cylinder are well approximated by, actually, Rayleigh waves for a flat half space, and the comparison between the data and the ray theory is shown and the magnitude is also correctly recovered, and that was work of Karen Gipson.

[Transparency 12]

I would like to now transition to the problem of what happens if, instead of looking for a metallic or fairly stiff object, you want to look for a plastic object in water or perhaps in a sediment. You see that you are in trouble if you compare the properties here of water and sediment, say, with those of polymers or a rubbery object. Then you find that the Rayleigh speeds, given over here in this column, are less than that of the water.

So you do not have the same kind of leaky Rayleigh wave mechanisms and associated leaky a_0 wave mechanisms that we have for stiffer objects, so we would like to consider what some of the other mechanisms would be for giving you large backscattering.

[Transparency 13]

Reflection just is not going to do it. These are reflectivity plots with aluminum water and, say, polystyrene water, if you modeled the materials for the purpose of this plot as lossless. A typical metallic object's reflection coefficient in normal incidence exceeds 85%, but if you go down here for the plastic object we have a reflection that is quite small, about 25%, and you can understand that because the impedance is much closer to that of water.

We can learn some other things from this diagram if we consider the phase behavior. This is a 0 to 35° scale here. Here we have looked at a 0 to 90° scale for the phase behavior. There is a Rayleigh pole in the reflection coefficient that shows up beyond the shear wave critical angle -- it is about 30 degrees -- and this corresponds to this region here, where there is a 2π phase evolution of the reflection coefficient.

There is no such phase evolution in the polystyrene water case. There is about a π phase evolution but there is no corresponding Rayleigh contribution, as we have indicated, because the Rayleigh wave is subsonic.

[Transparency 14]

In addition, in working with plastics we should keep in mind that they are really very different from normal elastic solids. They have a glass transition and an associated glass transition temperature. We are interested in plastics that are relatively stiff with the temperature of our experiments typically around 20° C, well below this glass transition temperature, and there are some consequences, particularly for the plastics we have worked with, namely, that over the frequency ranges of our experiments, even though the plastics are dispersive, the variation of the phase velocity for shear waves is much less than 1%.

Secondly, we have to take into account the material loss mechanisms. It has been found that you can model the loss by considering the ratio of the imaginary to the real parts of the wave number, denoted here by delta, taking that to be a constant independent of frequency. That is the work of Hartman and others.

[Transparencies 15 and 16]

What are some possible mechanisms? One of them was motivated by the following observation that I made a few years ago. In the winter I looked out my window and I saw some icicles. If you study the glints from these icicles, one realizes, in fact, they are rainbow rays within the icicles; that is, this very bright glint here comes from an exceptionally flat wave front and if one moves one's head around, one finds that these glints can disappear.

This is in contrast to there is a glint along here that is much weaker that is associated with a simple specular reflection from that.

[Transparency 17]

This raises the issue of how are rays transmitted within a tilted cylinder, and how do you describe those rays? It is easy to understand the case of normal incidence, but if you tilt the cylinder, it becomes a somewhat more complicated problem. However, you use the fact that the projection of the wave vector along the axis of the cylinder is an invariant.

From that Bravais was able to show, 150 years ago, that you can describe the refraction in such an object by looking at the projection of the rays along the orthogonal base plane and introducing an effective refractive index. It is scaled by the tilt of the object.

When you do this, you find, for example, the Descartes rays' (these are for the case of our icicle, which you model with a cylinder or for a plastic fiber that I will show you results from in a minute) angle varies with tilt and eventually gets to 0°, meaning that the Descartes rays have merged in the meridional plane that corresponds to a caustic regime transition.

[Transparency 18]

A student, Catherine Mount, did her masters thesis on this, where she took a fiber and tilted it. If you look at it at any given angle and illuminate it with light, there is a conic fan of rays that leave this and there is a resulting arc. This appears on a screen and then we videotaped the screen and then varied the tilt.

[Transparency 19]

What you find as you do this is there is an evolution as you vary the tilt of the Airy caustic and then at a critical tilt these merge into the meridional plane. You might say this could have nothing to do with anything you are ever going to see with sound.

[Transparency 20]

This is perhaps something I should have done before the student did the experiment I will describe in a minute but, in fact, I did this last week. I realized that I could take a code that I wrote and modify it to run the outgoing radiation as a function of azimuthal angle in the base plane as a function of the tilt of the cylinder for the infinite cylinder case, which is exactly solvable.

This is a negative of the results, so bright regions appear dark. The relevant ray diagram is shown here, where this happens to be a shear wave for a polystyrene cylinder and what we see is the rainbow caustic merging into the meridional plane at the critical tilt angle. You can verify that by superimposing the calculated caustic positions.

[Transparencies 21 and 22]

How can this have anything to do with backscattering and fluid loading? If you consider a bluntly truncated cylinder, then there is a path of rays like this in the meridional plane that go back to the source. The game is under what angles does this produce a lot of radiation coming back at you, namely, at this critical tilt angle.

[Transparency 23]

This was verified by Mr. Blonigen. Here are data he has taken with tone bursts at 300 kHz for a polystyrene cylinder where he has varied the tilt angle and there is this region near the critical tilt where there is a strong backscattering, whereas if this effect were not there, you would estimate the signal to be at least an order of magnitude smaller.

[Transparency 24]

In that case the relevant ray was the shear wave within the polystyrene. If you consider another low-velocity material, RTV rubber, which you might consider to sort of mimic an explosive -- explosives have low propagation velocities. We have, also, the enhancement in the expected region.

[Transparency 25]

The way you model this is by considering the shape of the outgoing wave front in the orthogonal plane and this becomes a fourth power when this Bravais index goes to 2. Because it is fourth power, the resulting diffraction integral contains a Pearcey function, which is one of the canonical forms of diffraction integrals, and one can work out a prediction for the backscattering amplitude.

[Transparency 26]

We have the slightly embarrassing result that the observed backscattering amplitude at the caustic exceeds our prediction, so we were looking for an effect and so far it has turned out to be a little bigger than what we had predicted it to be. Here is the prediction right on the caustic.

[Transparencies 27 and 28]

The student has argued that his data are right on the caustic. If I do not believe him and say, well -- I should remind you that the maximum for a scattering pattern for caustics that do not have perfect symmetry is shifted slightly from the caustic -- if I assume that, instead, he is right at the maximum of the pattern, that would give this upper line, which more or less bounds the data; that is to say, on the caustic corresponds to being here, the maximum corresponds to being here on this cut through the Pearcey function.

[Transparency 29]

On the other hand, with RTV rubber we have results that slightly are below the predictions but, again, it is a large effect compared with what you would have in the absence of this mechanism. We were only able to estimate the attenuation constants for the RTV rubber.

[Transparency 30]

Another test is considering an infinite cylinder, say, of rubber in the absence of attenuation and one can then solve this two-dimensional problem on both the computer and also with an analogous ray theory, and this shows that the two approach each other at high frequencies.

[Transparencies 31 and 32]

That is a geometrical mechanism. This afternoon Todd Hefner is going to describe a resonance-related mechanism but it is different in character from our previous resonances in that we are working with Rayleigh waves that are subsonic, so we had to worry about how to couple on to those Rayleigh waves that were subsonic.

He finds the resonances for a solid lucite sphere in water are readily observable through this kind of a ray model where there is tunneling. It happens that we had previously developed tunneling models for shells, metallic shells, and we could really lift all the mathematics of the metallic shell over to the solid-sphere model.

[Transparency 33]

This is a summary of what was presented. Thank you for your attention.

DR. MEHL: In that a_0 region where there was a precursor, there was an earlier kink.

[return to Transparency 31]

DR. MARSTON: You mean in here? I am astonished the student was able to get data as good as these, so I am not sure I would put any significance on the kink. (Laughter)

ADDENDUM

Subsequent to this presentation the following items have been published (or accepted for publication) pertaining to the research presented.

1. K. Gipson and P.L. Marston, "Backscattering enhancements due to reflection of meridional leaky Rayleigh waves at the blunt truncation of a tilted solid cylinder in water: Observations and theory," J. Acoust. Soc. Am. **106**, 1673-1680 (1999).
2. S.F. Morse and P.L. Marston, "Meridional ray contributions to scattering by tilted cylindrical shells above the coincidence frequency: ray theory and computations," J. Acoust. Soc. Am **106**, 2595-2600 (1999).
3. F.J. Blonigen and P.L. Marston, "Backscattering enhancements for tilted solid plastic cylinders in water due to the caustic merging transition: Observations and theory," J. Acoust. Soc. Am **107**, 689-698 (2000).
4. B.T. Hefner and P.L. Marston, "Backscattering enhancements associated with subsonic Rayleigh waves in polymer spheres in water: Observation and modeling for acrylic spheres," J. Acoust. Soc. Am. **107**, 1930-1936 (2000).