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Analysis of Scalar Datasets on Multi-Resolution Geometric Models

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Abstract. Recently, multi-resolution methods based on non-nested spaces were introduced to allow the visualization and approximation of functions defined on irregular triangulations [3,4,5]. This paper comes back to these methods and shows more precisely how the subdivision/prediction/correction scheme of ordinary wavelet-based multi-resolution analysis (MRA) is also present in that framework. As an illustration, it is demonstrated how it can be applied in two of the classical issues of MRA: compression and level-of-detail editing. We also show that the framework can be used for the analysis and approximation of scalar data defined on meshes with arbitrary topology, thus extending our previous results in the plane and the sphere. Here again, the link with the corresponding classical multi-resolution scheme of [6] as well as decimation methods is made.

§1. Introduction

In the last few years, the problem of simplifying huge 3D triangular meshes, for the purpose of *e.g.*, visualization, transmission or storage, has received considerable attention. Among those works, two major approaches can be found. In the case of regular meshes, the use of a wavelet-based framework has proven to be a powerful solution [6,9,14]. On the other hand, when meshes are not regular, the approach has been to simplify the mesh by applying a sequence of elementary geometric simplification operations, such as vertex removals, edge collapses or triangle collapses, the order of removal being driven by a greedy algorithm [1,8,11]. We refer to this latter approach as a decimation approach.

This paper is concerned with the simplification of data that is defined on a surface by means of a triangulation. This topic is closely related to surface simplification, since a triangular mesh can be seen, at least locally, as the graph of a piecewise linear function supported by a triangulation. Here again, when the surface is well-known, for example a plane square, a sphere, a cylinder, *etc.*, several wavelet based approaches have been employed [9,14], with regular underlying meshes.

In [3,4,5] the concept of non-nested MRA was introduced and applied to the approximation and progressive visualization of piecewise constant or linear functions defined on arbitrary planar or spherical meshes. In Section 2, we investigate the relationship between the non-nested framework and irregular subdivision. Examples of compression and level-of-detail editing in that framework are given in Section 3.

Section 4 focuses on the approximation and visualization of functions defined on triangular meshes with arbitrary topology. Here, an additional difficulty is that the surface which supports the function is also altered by the approximation process. Like in the case of surface simplification, a wavelet approach can successfully be applied when the original mesh has subdivision connectivity [6,12]. Otherwise, in an irregular setting, a decimation approach is usually employed, and the function is approximated during the simplification process [1,11]. We show how to apply our framework to functions defined on such general meshes. We will see that it makes the link between the wavelet-based approach available for subdivision surfaces and the decimation model. As in our previous papers, the function in its multi-resolution form is described by a coarse approximation defined on the simplified mesh, and a sequence of detail coefficients that are used for the reconstruction of the function on every LOD up to the original mesh. Our scheme is fully bijective: The function multi-resolution representation has the same size as the original one. The approximation process performs L_2 approximation of the data, but other types of approximation are also possible.

§2. Non-Nested Framework and Irregular Subdivision

In Section 2.1 we briefly review the non-nested decomposition scheme described in [3,5]. Section 2.2 makes the link with the notion of irregular subdivision.

2.1. Decomposition scheme

For simplicity, every space is supposed to have finite dimension. Let Ω be a measurable domain and V^i , $i = 0, \dots, N$, a sequence of subspaces of $L_2(\Omega)$. These spaces do not have to be nested but will in general be "growing" in the sense that $\dim(V^i) \leq \dim(V^{i+1})$. Now let $f = f_N$ be a function in the finest subspace V^N . In classical MRA, the spaces are nested and the link between V^{N-1} and V^N is made by taking a complementary space W^{N-1} of V^{N-1} in V^N , that is

$$V^N = V^{N-1} \oplus W^{N-1}.$$

Now if we write $f_N = f_{N-1} + g_{N-1}$ according to the space decomposition, f_{N-1} can be seen as an approximation of f_N in V^{N-1} , and g_{N-1} as the detail needed to recover the original function from its approximation. By repeating the decomposition, one obtains

$$f_N = f_0 + g_0 + \dots + g_{N-1}$$

which corresponds to the space decomposition $V^N = V^0 \oplus \bigoplus_0^{N-1} W^i$.

Notice that in this case $f_i = P_{V^i}(f_{i+1})$, where P_{V^i} is the projector on V^i with direction W^i . We return to the general case and suppose that a linear “projector” $P^i : V^{i+1} \rightarrow V^i$ is given. To avoid technical details, these projectors are required to be *surjective*, but the results in this paragraph also hold if this is not the case. Let W^i be the kernel of P^i , and $\tilde{V}^i$ be a complementary space of W^i in V^{i+1} . We now observe that the restriction of P^i to $\tilde{V}^i$ is a bijective operator, having the same range as P^i . Thus if $f_i = P^i(f_{i+1})$ and $g_i = Q^i(f_{i+1})$, where Q^i is the projector on W^i (defined by the choice of $\tilde{V}^i$), the following reconstruction formula holds:

$$f_{i+1} = \text{Inv}(P^i_{|\tilde{V}^i})(f_i) + g_i. \quad (1)$$

Again, by iterating this decomposition, we obtain a coarse approximation $f_0 = P^0 \circ \dots \circ P^{N-1}(f_N)$ and “detail” functions $g_0, \dots, g_{N-1}$.

We now take a look at the reconstruction process. Denote by S^i the inverse of $P^i_{|\tilde{V}^i}$, $S^i : V^i \rightarrow \tilde{V}^i \subset V_{i+1}$. The complete coarse-to-fine reconstruction formula is obtained by iterating the reconstruction formula (1):

$$f_N = S^{N-1} \circ \dots \circ S^0(f_0) + S^{N-1} \circ \dots \circ S^1(g_0) + \dots + S^{N-1}(g_{N-2}) + g_{N-1}. \quad (2)$$

2.2. Approximating spaces, subdivision spaces and scaling spaces

We are going to see that the previous scheme can actually be considered in two different ways. Until now, it was implicitly assumed that the spaces V^i were playing the role of the scaling spaces in classical MRA. Under this assumption, we conceptually have a really *non-nested* framework; if the spaces were nested, the operators S^i would be the identity (formally injecting V^i in V^{i+1}). However, we will not call them scaling spaces but approximation spaces, and keep the term “scaling” for other spaces that are going to be defined below.

In the non-nested framework, one loses the notion of subdivision (or cascade algorithm). However, looking at things slightly differently allows subdivision to fit into the non-nested scheme. To show this, consider the operators S^i as *subdivision* operators, and call the spaces V^i subdivision spaces accordingly. This means that we start from a function $f_0 \in V^0$ and iteratively subdivide it into $f_1, f_2, \dots, f_N$ using the formula $f_{i+1} = S^i(f_i)$. The notion of subdivision used here is very general. In that context, classical regular or semi-regular subdivision schemes would give rise to a nested sequence of subdivision spaces. But completely irregular schemes would require the non-nested framework to be fitted in. The use of non-nested MRA was introduced in [2], and was later applied to triangular schemes in [3,4,5]. Recently, another approach on general irregular schemes was proposed in the work of Sweldens and Guskov [10,13].

We now define the scaling spaces. Like in classical MRA, they are the spaces containing the limit functions resulting from the subdivision process. Since very little is known about the convergence of such schemes, we won't push the subdivision to infinity, but restrict ourselves to an integer N . This

makes sense since, when using such a framework, one usually starts from an initial triangulation $\mathcal{T}_N$ which is *coarsened* to $\mathcal{T}_0$. This contrasts with the traditional approach in subdivision where one starts from a base mesh and subdivides it according to a systematic rule.

The scaling spaces, for $i = 0, \dots, N$, are defined as

$$V^{i,N} = \{f_k^{i,N} \mid f_k \in V^i\},$$

where $f_k^{i,N} = S^{i,N}(f_k)$, and $S^{i,N} = S^{N-1} \circ \dots \circ S^i$. This operator carries functions in V^i through $N - i$ subdivision steps to functions in $V^{i,N}$.

Notice that, for every i , $V^{i,N} \subset V^N$ and moreover, that $V^{i,N} \subset V^{i+1,N}$. Now fix a basis (φ^i) for V^i and (ψ^i) for W^i . If (a^i) and (b^i) denote the coordinates of $f_i \in V^i$ and $g_i \in W^i$ with respect to these basis, the reconstruction formula can be re-written as

$$f_N = \sum_k a_k^j \varphi_k^{j,N} + \sum_{i=j}^{N-1} \sum_k b_k^i \psi_k^{i,N},$$

for any $j = 0, \dots, N$, which corresponds to the decomposition of f_N in $V^{N,N} = V^N$ (using the same notations as above: $\varphi_k^{j,N} = S^{j,N}(\varphi_k^j)$, and $\psi_k^{i,N} = S^{i+1,N}(\psi_k^i)$).

Although formally identical, considering the V^i 's as approximation or subdivision spaces changes the aspect of several questions. For example, in the problem of error measure in the context of approximation spaces, we are interested in $\|f_N - f_i\|$, whereas in the other context we are looking for $\|f_N - f_i^{i,N}\|$.

§3. Application to Data Compression and LOD Editing

In this section we show how the framework can be used on functions defined over irregular triangulations to achieve data compression and LOD editing, which are both standard applications of MRA. The context here is the planar or spherical setting of [4,5], only the filters need to be changed. Indeed, the analysis operator used in those papers was the orthogonal projector (the goal being progressive visualization). However, for compression it is often useful to know in advance the error between the original function and its approximation, in terms of the wavelet coefficients that were used in the reconstruction. This will be achieved by designing new filters.

3.1. Isometric subdivision

In order for the detail coefficients to have the error measure property, the synthesis operator is required to be an isometry. Indeed, suppose that

- $S^i : V^i \rightarrow V^{i+1}$ is an isometric operator, $\forall i = 0, \dots, N - 1$,
- the complementary spaces $\tilde{V}^i$ are chosen orthogonal to the W^i , $\forall i = 0, \dots, N - 1$.

When the latter condition is fulfilled, we say that we are in a semi-orthogonal framework. If f_N denotes the original function, then, according to the global reconstruction formula (2), the quantity $\|f_N - f_0^{0,N}\|^2$ which measures the contribution of the correction steps in the reconstruction process is

$$\|S^{N-1} \circ \dots \circ S^1(g_0) + \dots + S^{N-1}(g_{N-2}) + g_{N-1}\|^2.$$

Because of semi-orthogonality, it is equal to

$$\|S^{N-1} \circ \dots \circ S^1(g_0) + \dots + S^{N-1}(g_{N-2})\|^2 + \|g_{N-1}\|^2,$$

and because S^{N-1} is an isometric operator, we can factorize and remove it from the first term above, and then iterate the operation to get

$$\|f_N - f_0^{0,N}\|^2 = \sum_{i=0}^{N-1} \|g_i\|^2.$$

Notice that even if the subdivision operators are not isometric,

$$\|f_N - f_0^{0,N}\|^2 \leq \|S^{N-1}\|^2 \dots \|S^1\|^2 \|g_0\|^2 + \dots + \|S^{N-1}\|^2 \|g_{N-2}\|^2 + \|g_{N-1}\|^2$$

still holds in the semi-orthogonal setting. Let $\hat{f}_N$ denote the partially reconstructed function, and suppose in addition that we are in an orthonormal framework, that is, the functions ψ^i form an orthonormal basis of W^i . Proceeding as above leads to

$$\|f_N - \hat{f}_N\|^2 = \sum_{i=0}^{N-1} \sum_k \varepsilon_k^i (b_k^i)^2,$$

where the b_k^i 's are the wavelet coefficients and ε_k^i equals 1 whenever b_k^i is taken in the reconstruction and 0 otherwise. Consequently, in this setting we have an error measure in terms of wavelet coefficients.

3.2. An isometric subdivision operator

The idea behind this construction is that the corresponding analysis operator should have reasonable approximation quality, which seems intuitively required to achieve compression. Accordingly, the projection operator used in [4,5] is taken as a starting point, the problem being to approximate it by means of an isometry. Let $P^i : V^{i+1} \rightarrow V^i$ be that operator. The first step is to find the matrix of P^i with respect to some orthonormal basis (e^{i+1}) and (e^i) of V^{i+1} and V^i . Now let UDV be the singular value decomposition of that matrix. This decomposition admits the following interpretation:

- V is the matrix of an isometric operator of V^{i+1} in the basis (e^{i+1}) since it is unitary and the basis is orthonormal.

- D is a $\ell \times c$ matrix ($\ell < c$) whose diagonal coefficients are all non-negative and ≤ 1 . The diagonal matrix formed by the positive entries of D is the matrix of a bijective operator mapping V^i onto $\text{Ran}(P^i)$.
- Like V , the matrix U is an isometric operator of V^i .

Let $\tilde{D}$ be the matrix obtained by replacing every positive diagonal element of D by 1. This amounts to turning the bijective operator above into an isometric one, and thus $U\tilde{D}V$ is the matrix of an operator $\tilde{P}^i$ whose corresponding subdivision operator S^i is isometric. A few remarks can be made about this construction:

- $\tilde{P}^i$ does not depend on a particular choice of orthonormal basis.
- $\tilde{P}^i$ is not the best isometric approximation of P^i in terms of L_2 norm of operators, but it can be shown that it is the best with respect to the Frobenius norm.
- The diagonal coefficient of D are by definition the cosines of the angles between the spaces V^i and V^{i+1} (see, *e.g.*, [7] Chapter 1). In the nested case, they would all be equal to 1, and the corresponding subdivision operator would be the identity.
- This method could be used to approximate operators by means of similarities, by replacing the entries of D by an appropriate scalar α instead of 1. Although better in terms of approximation quality, this leads to bad visual results since the resulting subdivision operator doesn't reproduce constants if $\alpha \neq 1$.

Notice that in the context of [4,5], this approximation is always computed locally, leading to a global algorithm in linear time.

3.3. Examples

As it is mentioned in the beginning of this section, these examples were created using the setting described in [3,4,5]; the reader is invited to look there for details. The initial triangulation is completely irregular, generated by random vertex insertion. In Figure 1, a piecewise linear setting is used for LOD editing. The function is edited at a coarse resolution, by pulling values up ($\rightarrow$ white) at some vertices, and then adding detail coefficients back. Figure 2 shows an example of data compression when the approximation spaces are spaces of piecewise constant functions over triangulations generated from the original one by homogeneous decimation. This last setting is the full generalization of Haar wavelets to irregular grids, as it would lead to them in the regular case.

§4. Scalar Datasets on Irregular Meshes with Arbitrary Topology

In this section we describe how the non-nested framework can be used to handle scalar attributes defined on meshes with any topology. As a starting point, we take a multi-resolution decimation model, based on the vertex-removal (VR) operation to simplify the geometry. This means we assume that an initial fine mesh is given along with its associated sequence of VR's

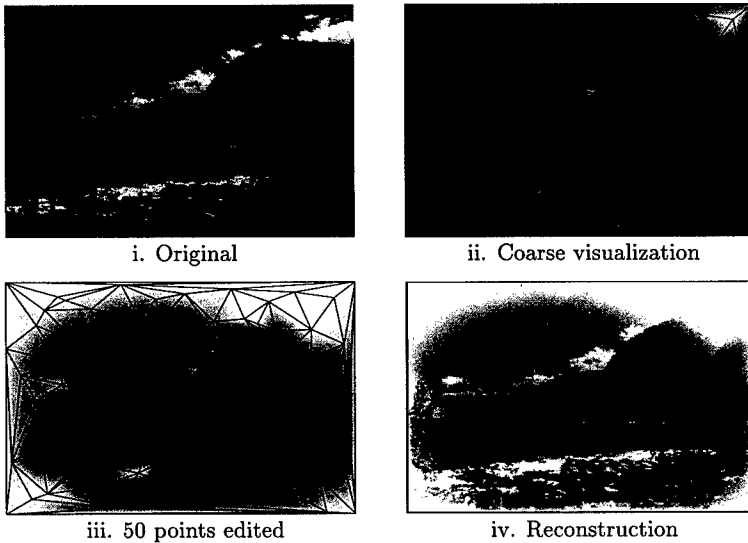


Fig. 1. Picture design using level-of-detail editing.

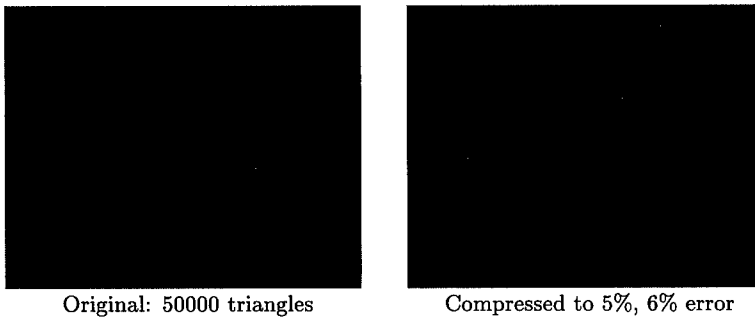


Fig. 2. Compression of a piece-wise constant function.

that can be progressively applied to decimate the mesh down to a base mesh. In addition, we suppose that the scalar attributes are defined by means of piecewise constant and/or linear functions parameterized on the initial mesh. In what follows, $PI(\mathcal{V})$ denotes the polygon of influence of a vertex $\mathcal{V}$; it is the polygonal area delimited by the 1-neighbours of $\mathcal{V}$. In order to apply the non-nested framework, approximation spaces and approximation operators need to be defined.

4.1. Local mapping

Let M^i , $i = N_0, \dots, N$, denote the triangular mesh consisting of i vertices (the original mesh after $N - i$ VR operations). Let $\mathcal{F}(M^i)$ be the space of real-valued functions defined on M^i , and $\mathcal{C}^i$ (resp. $\mathcal{L}^i$) be the subspace of functions of $\mathcal{F}(M^i)$ that are piecewise constant (resp. linear) on each triangle of M^i . We

refer to these spaces as the upper approximation spaces. Each VR alters locally the surface, thus functions of $\mathcal{F}(M^{i+1})$ and $\mathcal{F}(M^i)$ are defined on different domains. To define an approximation problem, a common parameterization for these functions is required. To that end, we assume that for every VR of vertex $\mathcal{V}_i$, a *local one to one projection* Π^i of $\text{PI}(\mathcal{V}_i)$ onto a plane is also known. The reader can refer to [8] for a study of the existence and determination of such a projection. We use Π^i to consider the *change of parameterization* $H^i : M^i \rightarrow M^{i+1}$ as the mapping defined by $\Pi^i \circ H^i = \Pi^i$ over $\text{PI}(\mathcal{V}_i)$ and by the identity outside.

4.2. Scaling spaces and data decomposition

Let $\mathcal{K}$ stand for $\mathcal{C}$ or $\mathcal{L}$. To a function $f \in \mathcal{K}^i$ we associate the function $f^{i,i-1} = f \circ H^{i-1}$, and let $\mathcal{K}^{i,i-1} \subset \mathcal{F}(M^{i-1})$ be the space of all these functions. This can be iterated: The local mappings also define a *global* mapping from the base mesh to the mesh M^i by $H^{N_0,i} = H^{i-1} \circ \dots \circ H^{N_0}$, for each $i = N_0 + 1, \dots, N$. This allows to define the approximation spaces $\bar{\mathcal{K}}^i$ from the upper approximation spaces as

$$\bar{\mathcal{K}}^i = \{f \circ H^{N_0,i} \mid f \in \mathcal{K}^i\} \subset \mathcal{F}(M^{N_0}).$$

The second step is to define the operators $\bar{P}^i : \bar{\mathcal{K}}^{i+1} \rightarrow \bar{\mathcal{K}}^i$. Fortunately, working directly in the approximation spaces is not required: Because they are isomorphic (by construction) to the upper approximation spaces, it suffices to define some operators $P^i : \mathcal{K}^{i+1,i} \rightarrow \mathcal{K}^i$, and for the purpose of visualization, the least-square projection operators will be used. The operator $\bar{P}^i$ is thus defined by

$$\bar{P}^i : \bar{f}_{i+1} = f_{i+1} \circ H^{N_0,i+1} \longmapsto P^i(f_{i+1}^{i+1,i}) \circ H^{N_0,i}.$$

$f_{i+1} \in \mathcal{K}^{i+1}$ defines $f_{i+1}^{i+1,i}$ to which we apply one decomposition step to get an approximation $f_i \in \mathcal{K}^i$ and detail coefficients (1 in the linear case, and 2 in the constant case). The entire operation is then repeatedly applied to f_i , f_{i-1} , etc. Consequently, in practice, everything happens in the upper approximation spaces which have a much simpler structure than the corresponding approximation spaces.

4.3. Results and remarks

The output of this algorithm is a coarse function f_{N_0} defined on the base mesh and a list of detail coefficients that allow the exact reconstruction of the original function through the hierarchy of LODs. Figure 3 shows some results in the linear setting. In these examples, the geometric criterion guiding the decimation priority-queue is simply the distance from the 1-neighbours of a candidate vertex to their least-square approximation plane. On the upper right snapshot, we see the drawback of a geometric-only driven priority queue: Some quasi-planar areas on the object have been severely decimated, leading

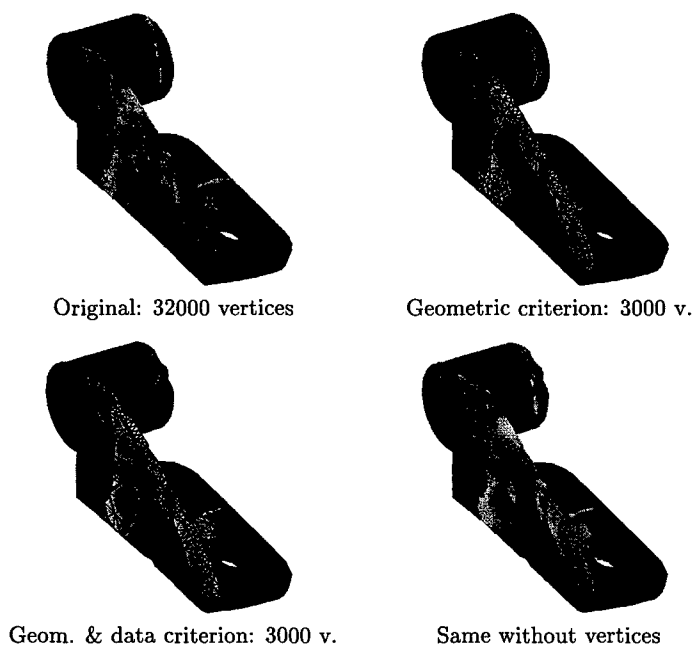


Fig. 3. Bracket: linear approximation examples.

to a quite coarse approximation of the function. The lower snapshots show the result using the geometric($\frac{2}{3}$) and data($\frac{1}{3}$) based criterion. The resulting approximations are better, but the geometry presents some visible deformations (this is also partially due to our simple geometric criterion). Finding a compromise in an automated way seems to be a difficult task. Moreover, if the simplification is just a process prior to other computations, such as can be the case, *e.g.* in mechanics, then a high accuracy in the approximated function might be the primary interest. Thus, it seems better to let the weights depend on the application, under user control.

4.4. Comparison to classical MRA

In [6], a MRA for subdivision surfaces is used to handle both the geometry and the scalar attributes of a mesh. The presentation given above makes the link between these methods and the decimation approach. Indeed, from the “upper” point of view — the decomposition using the operators P^i — it compares to decimation in many respects, whereas it is also a decimation step corresponding to the approximation spaces, which is exactly what is done in [6] in the nested case. However, parameterizations are then obtained without a local projection hypothesis, thanks to the particular 1-to-4 splitting strategy that is performed on the base mesh.

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