TITLE: The Principle of Independence of the Cavity Sections Expansion [Logvinovich's Principle] as the Basis for Investigation on Cavitation Flows

The Principle of Independence of the Cavity Sections Expansion (Logvinovich's Principle) as the Basis for Investigation on Cavitation Flows

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Summary

For the investigation of complex non-steady cavitation flows in three dimensions it is extremely important to find a simple model which on one hand would correspond to the basic laws of a fluid stream (the laws of conservation of energy and momentum) and on the other hand would be simple enough for the numerical calculation. This model was presented by G. V. Logvinovich, and was called "The Principle of Independence of the Cavity Sections Expansion". In the present lecture the deduction of the principle of independence from the equation of conservation of energy is adduced. It is shown that for the stationary cavities the results obtained on the basis of the principle of independence agree with the results obtained on the basis of the slender body theory and the numerical methods as well as with the experimental data. It is shown that for the special case - the cavitation number equals zero - the cavity shape determined on the basis of the principle of independence agrees with the well-known Levinson-Gurevich asymptotic. The application of the principle of independence for the calculation of some types of the non-stationary cavities and cavities with variable external pressure (the vertical cavities in the gravitational field) is considered.

Introduction

In the case of high-speed motion in water, cavities filled with gas or vapor are formed behind the body-cavitator. The mathematical problem on the determination of the cavity shape is stated as the inverse problem on the flow around a body – the free boundary shape is found from the constant pressure condition. In case of the non-stationary cavities, the cavities with variable external pressure along their length or the asymmetrical cavities, the problem on the determination of the cavity shape becomes extremely complex. For the practical problems it is very important to have a simple method of calculation of non-stationary cavitation flows that corresponds with the basic laws of a fluid stream. Such a method was suggested by Logvinovich and was called "The Principle of Independence of the Cavity Sections Expansion".

The principle of independence is stated as following. Each cross section of a cavity expands relatively to the trajectory of the center of a cavitator which happens almost independently from the following or the previous body motion. The expansion occurs according to the definite law which is dependent only upon the difference between the pressure at infinity and the pressure within the cavity, the speed, the size, and the drag of a body at the moment when a body passes the plane of the considered section [1]. The law of expansion of the cavity fixed section can be determined using the principle of conservation of energy in the wake. The thin cavity in ideal fluid can be considered as the wake. Each unit of the wake length keeps the energy which was expended by the cavitator at the moment when it passes this unit of the length of the trajectory [2,3]. This theory applied to the cavity with the constant difference of pressures gives results which are very similar to the results observed in the experiments. Therefore, we can suggest that this theory is also applicable for the case of variable difference of pressures.

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1. The Deduction of the Principle of Independence from the Equation of Conservation of Energy

We designate the coordinate along the arc of the trajectory of the cavitator center as h (Fig. 1). This coordinate is fixed relatively to the motionless fluid. At the point h let us draw the plane Σ that is perpendicular to the trajectory. We will observe the development of the cavity cross section arisen on this plane at the moment t=0.

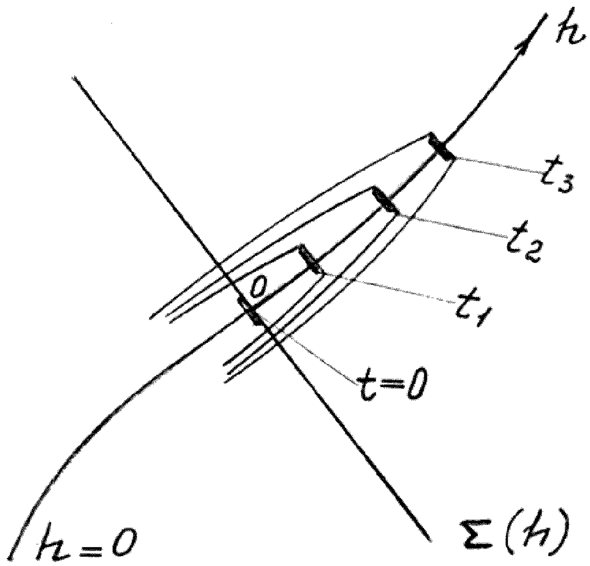

Fig. 1 The scheme of development of the non-stationary cavity

Let us introduce the following designations: the radius of the cavity section $R=R(h,t)$, the area of the cavity section $S(h,t)=\pi R^2$, the pressure within the cavity $P_k=P_k(h,t)$, the pressure at infinity $P_\infty(h,t)$. We can consider the pressure $P_\infty(h,t)$ to be the pressure at the point "0" of the intersection of the trajectory with the plane Σ when the cavity is absent or to be the hydrostatic pressure on the depth of the point '0'.

The cavitator passing the path Δh performs certain work and gives the energy $W\Delta h$ to the fluid (where W is the drag force acting on the cavitator at the moment t=0). Proceeding from the physical scheme of the flow we can accept that approximately the energy $W\Delta h$ is conserved in the considered section at the same segment Δh in the form of the kinetic energy $T\Delta h$ и potential energy $E\Delta h$ up to the moment of this section closure. Therefore, we can write an approximate equation for each cavity cross section

$$T(h,t)+E(h,t)=W(h,0) \tag{1.1}$$

Equation (1.1) can be applied to the whole cavity length and signifies that the energy that was imparted by the cavitator at the point of the trajectory h is conserved on the cavity at this point. Proceeding from the first Green's formula [4] we can express the kinetic energy that fits the unit of the cavity length by the following equation

$$T = -\frac{1}{2}\rho\varphi\, 2\pi R\,\frac{\partial\varphi}{\partial n} \tag{1.2}$$

where φ is the potential of the absolute velocity at the boundary, R and $\dot{R} \approx \partial\varphi / \partial n$ are the radius and the radial velocity of cavity boundary at the point of the trajectory h, ρ is the fluid density.

The potential energy of a cavity section is determined as follows

$$E = \int_o^t \Delta P(h,t) 2\pi R \dot{R}\, dt \tag{1.3}$$

where $\Delta P(h,t) = P_\infty(h,t) - P_k(h,t)$. After substitution of (1.2) and (1.3) in (1.1) we obtain an approximate equation of energy

$$-\frac{1}{2}\rho\varphi\dot{S} + \int_o^t \Delta P(h,t)\dot{S}dt = W(h,0) \tag{1.4}$$

where $\dot{S} = 2\pi R\dot{R}$ is the time derivative of the area of a cavity cross section.

The kinetic energy is determined along the stream tubes that lean on the expanding cavity hole in the motionless plane. The trajectory of a cavitator intersects this plane along normal at the point h. The dynamic boundary condition is written in a form of the general Bernoulli equation applied to the points of space that coincide with the cavity boundary at the considered moment (later on we will be writing ΔP instead of $\Delta P(h,t)$)

$$\frac{\partial\varphi}{\partial t} + \frac{v^2}{2} = \frac{\Delta P}{\rho} \tag{1.5}$$

where v is the absolute velocity of the fluid particles on the cavity boundary. In the middle section of the cavity $v \approx \dot{R}$ is the small quantity and we neglect the value of v^2 compared to the value of $\Delta P/\rho$. Then equation (1.5) is written in the form

$$\frac{\partial\varphi}{\partial t} = \frac{\Delta P}{\rho} \tag{1.6}$$

Let us differentiate equation (1.4) with respect to t and using expression (1.6) we obtain

$$\varphi\ddot{S} = \frac{\Delta P}{\rho}\dot{S} \tag{1.7}$$

From equation (1.6) the value of the potential on the cavity section boundary is determined as follows

$$\varphi = \varphi_n + \int_o^t \frac{\Delta \mathrm{P}}{\rho} dt \tag{1.8}$$

where φ_n is the value of the potential at the edge of a cavitator at the moment of generation of the cavity section *(t*=0*)*. After substitution of (1.8) in (1.7) we obtain the following differential equation

$$\left[\rho\varphi_n + \int_o^t \Delta \mathrm{P} dt \right] \ddot{S} = \Delta \mathrm{P} \dot{S} \tag{1.9}$$

Taking into account that $d\int_o^t \Delta \mathrm{P} dt = \Delta \mathrm{P} dt$, as a result of dividing variables and integrating we obtain

$$\dot{S} = A\left[\rho\varphi_n + \int_o^t \Delta \mathrm{P} dt \right] \tag{1.10}$$

where A is the constant connected with the initial velocity of expansion of the cavity section by dependence $A = \dot{S}_o / \rho\varphi_n$; $\dot{S}_o$ is the initial velocity of expansion. As a result equation (1.10) is represented in the following form

$$\dot{S} = \dot{S}_o \left[1 + \frac{1}{\varphi_n} \int_o^t \frac{\Delta \mathrm{P}}{\rho} dt \right] \tag{1.11}$$

The value of the potential at the edge of a cavitator we can write as following: $\varphi_n = -\frac{1}{2} a R_n V(0)$, where a is some constant, R_n is the radius of a cavitator, *V(0)* is the velocity of a cavitator at the moment t=0. Since for the stationary cavity the flow is symmetrical the value of the potential on the mid-section equals zero, consequently for the stationary cavity equation (1.8) can be written in the following form

$$\varphi_n = -\int_o^{t_k} \frac{\Delta \mathrm{P}}{\rho} dt = -\frac{\Delta \mathrm{P}}{\rho} t_k = -\frac{1}{2} a R_n V(0) \tag{1.12}$$

where t_k is the time of expansion of a cavity section to the maximum size. In case of the stationary cavity we can obtain the expression of the constant a from equation (1.12)

$$a = \frac{\sigma L_k}{2R_n} \tag{1.13}$$

where $\sigma = \dfrac{2\Delta P}{\rho V^2}$ is the cavitation number, V is the velocity of stationary motion of a cavitator, $L_k = 2Vt_k$ is the cavity length. Let us determine the initial velocity of expansion $\dot{S}_o$ from the equation of energy (1.4) at the moment t=0. Taking into account that $W = C_x \pi R_n^2 \dfrac{\rho V^2(0)}{2}$, where C_x is the drag coefficient of a cavitator we obtain

$$\dot{S}_o = \frac{2\pi C_x R_n V(0)}{a}, k = -\frac{\dot{S}_o}{\varphi_n} = \frac{4\pi C_x}{a^2} \tag{1.14}$$

As a result we go on from equations (1.11) and (1.14) to the well-known mathematical entry expressing the principle of independence of the cavity sections expansion in the form of

$$\ddot{S} = -\frac{k\Delta P}{\rho} \tag{1.15}$$

Equation (1.15) with the coefficient (1.14) (the constant a is slightly dependent on the cavitation number and its value is selected from the range 1.5÷2) is widely applied for the investigation of non-stationary cavities and cavities with variable external pressure along their length. The velocity of expansion of a cavity section is determined from equation (1.11)

$$\dot{S} = \dot{S}_o - k\int_o^t \frac{\Delta P}{\rho} dt \tag{1.16}$$

Integrating equation (1.16) we obtain the dependence of the area of a cavity section on time

$$S = S_0 + \dot{S}_0 t - k\int_0^t \int_0^t \frac{\Delta P}{\rho} dt dt \tag{1.17}$$

where S_0 is the initial area of a cavity cross section. In particular for the stationary cavity the value $\dfrac{\Delta P}{\rho}$ is constant and from (1.17), (1.14) and (1.13) we obtain the well-known result

$$\frac{S-S_0}{S_k-S_0}=\frac{t}{t_k}\left(2-\frac{t}{t_k}\right) \tag{1.18}$$

where S_k is the maximum quantity of the area of a cavity cross section (the area of the mid-section).

If as the quantity S_0 we choose the quantity of the area of the cross section on which the incline of a cavity boundary is sufficiently small (slightly deviating from a disk-cavitator), then the calculations from formula (1.18) agree very well with the experimental data as shown in Fig. 2 (in this figure the points show the experimental data for the different cavitation numbers).

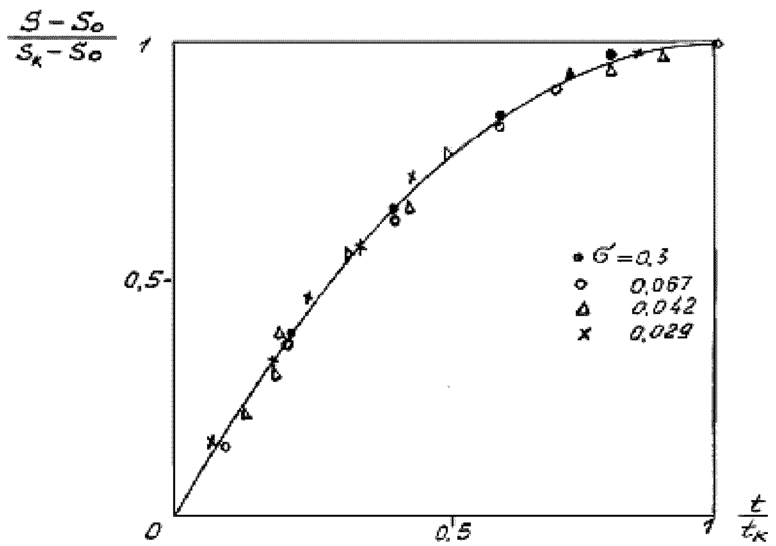

Fig. 2 The comparison of the stationary cavity profile obtained on the basis of the principle of independence with the experimental data

Usually the profile of the stationary cavity past a disk is constructed as follows. In the vicinity of the disk the cavity shape is not dependent on the cavitation number and can be expressed in the form [1]

$$R(x)=R_n\left(1+\frac{3x}{R_n}\right)^{1/3} \tag{1.19}$$

where x is the coordinate measured along the axis of symmetry from the center of a disk. The cavity shape determined from equation (1.18) joins with the shape expressed by the dependence $R(x)$ (1.19). We can consider the point with the coordinates: $x=2R_n$, $R(x)=1.92R_n$ to be the point of contact.

2. The comparison of the results obtained for stationary cavitation flows on the basis of the principle of independence with the results obtained on the basis of the slender body theory and the numerical methods

In this chapter we will demonstrate that the shape of the stationary cavity represented by expression (1.18) can be obtained from the slender body theory. As an example we use the cavitation flow past a thin cone. Let us consider a more general case - the supercavity past a thin cone in subsonic compressible fluid flow. We apply the Riabouchinsky scheme (Fig. 3, the cavity is closed by a body with the same dimensions as the caivtator).

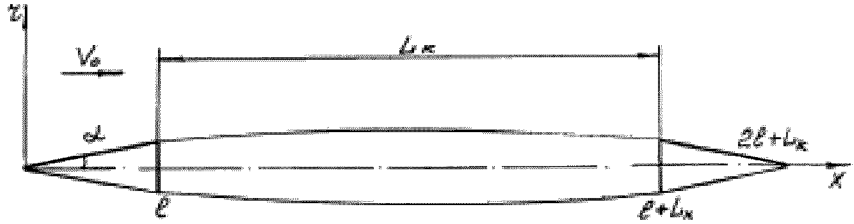

Fig. 3 The Riabouchinsky scheme for the cavitation flow past a thin cone

The origin of the orthogonal system *x, r* is placed at the apex of the cone, the geometric dimensions are scaled by the radius of the cone base (the radius of the cone base R_n equals unity, l is the altitude of the cone, α is the apex half-angle, L_k is the cavity length, $l+L_k$ and $2l+L_k$ are the coordinates of the base and the apex of the closing cone respectively, $L=2l+L_k$ is the total length).

Let us apply the linearized equation of the flow in the cylindrical coordinate system [5]

$$\left(1-M^2\right)\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}=0 \tag{2.1}$$

where $\Phi=\Phi^*/V_0R_n$ is the dimensionless flow velocity potential,

Φ^* is the flow velocity potential,

V_0 is the free stream velocity,

$M=V_0/a_0$ is the Mach number,

a_0 is the free stream speed of sound

We assume that the parameter of the cavity thinness has the same order as the parameter of the cone-cavitator thinness. The parameter of thinness ε equals the ratio of the cone radius to its altitude $(\varepsilon=1/l=tg\alpha)$. Let us represent the potential Φ in the following way: $\Phi=\varphi+x$, where φ is the dimensionless perturbed velocity potential which is aimed at zero when x and r are aimed at infinity. The dynamical boundary condition on the cavity surface is written as follows [6,7] (we neglect the members that have the order of trifle exceeding $\varepsilon^4 ln^2\varepsilon$)

$$2\frac{\partial\varphi}{\partial x}+\left(\frac{\partial\varphi}{\partial r}\right)^2=\sigma \tag{2.2}$$

For the slender axisymmetrical bodies the perturbed velocity potential is defined by the method of sources and sinks distributed along the axis of symmetry. For the subsonic flow the potential φ satisfying the equation (2.1) has the form [5]

$$\varphi = -\frac{1}{4\pi}\int_0^L \frac{q(x_1)dx_1}{\sqrt{(x-x_1)^2 + (1-M^2)r^2}} \tag{2.3}$$

where $q(x_1)$ is the intensity of sources and sinks on the axis of symmetry. Near the axis of symmetry the perturbed velocity potential is represented by the following asymptotic equation [6,7]

$$\varphi = \frac{1}{2\pi}S'(x)\ln r + g(x) \tag{2.4}$$

where $S'(x)$ is the derivative of the dimensionless area of the slender body cross section with respect to x coordinate.

The logarithmic potential is the main part of expression (2.4), and some function $g(x)$ is added to it. The function $g(x)$ is determined from the condition of matching with the potential of the distributed sources (2.3) since the logarithmic potential does not satisfy the condition at infinity. Also, the function $g(x)$ takes into account the influence of compressibility. After substitution $g(x)$ we obtain the following expression for the perturbed velocity potential near the surface of the body and the cavity [7]

$$\varphi = \frac{1}{4\pi}S'(x)\ln\frac{(1-M^2)r^2}{4x(L-x)} - \frac{1}{4\pi}\int_0^L \frac{S'(x_1) - S'(x)}{|x-x_1|}dx_1 \tag{2.5}$$

Substituting equation (2.5) in (2.2) we obtain the integro-differential equation for the cavity profile [8]

$$\frac{u'^2}{2u} + u''\ln\frac{(1-M^2)u}{4x(L-x)} - \int_0^l \frac{u_1''(x_1) - u''(x)}{|x-x_1|}dx_1 -$$

$$-\int_l^{l+L_k} \frac{u''(x_1) - u''(x)}{|x-x_1|}dx_1 - \int_{l+L_k}^{L} \frac{u_1''(x_1) - u''(x)}{|x-x_1|}dx_1 = 2\sigma \tag{2.6}$$

$$u = R^2, u_1 = R_1^2, u_1'(0) = 0, u_1'(L) = 0$$

The cavity profile is determined by altering its radius along the coordinate x: $R(x)$. Along the coordinate x the radius R_1 of the cone-cavitator and the closing cone respectively change to $R_1=\varepsilon x$ and $R_1=\varepsilon(L-x)$. The following boundary conditions are added to the integro-differential equation (2.6)

$$x = l : R = 1, R' = \varepsilon$$
$$x = l + L_k : R = 1, R' = -\varepsilon$$

We seek the solution for the whole area of the cavity by expanding the asymptotic rows with the small parameter ε.

$$\begin{aligned} R^2 &= \varepsilon^2\left[R_0^2 + R_{-1}^2\left(\ln\frac{1}{\varepsilon^2}\right)^{-1} + R_{-2}^2\left(\ln\frac{1}{\varepsilon^2}\right)^{-2} + \ldots\right] \\ \sigma &= \varepsilon^2\left[\sigma_1\left(\ln\frac{1}{\varepsilon^2}\right) + \sigma_0 + \sigma_{-1}\left(\ln\frac{1}{\varepsilon^2}\right)^{-1} + \ldots\right] \end{aligned} \qquad (2.7)$$

After substitution of the rows (2.7) into (2.6) and conservation of the two members of the rows the integro-differential equation (2.6) is transformed to two differential equations. The first equation is obtained from the equality of the members at $\varepsilon^4 ln1/\varepsilon^2$, the second one follows from the equality of the members at ε^4. The first differential equation with the boundary conditions is written as follows

$$\begin{aligned} &\frac{d^2R_0^2}{dx^2} = -2\sigma_1 \\ &R_0^2(l) = l^2, R_0^2(l + L_k) = l^2, \frac{dR_0^2}{dx}\Big|_{x=l} = 2l \end{aligned} \qquad (2.8)$$

The solution of the equation (2.8) has the form

$$\begin{aligned} &R_0^2 = \sigma_1(a-x)(x-b) \\ &\sigma_1 = \frac{2l}{L_k}, a = \frac{L}{2} + \sqrt{\frac{L^2}{4} - \frac{lL}{2}}, b = \frac{L}{2} - \sqrt{\frac{L^2}{4} - \frac{lL}{2}} \end{aligned} \qquad (2.9)$$

After reduction of equations (2.7) and (2.9) to the dimensional form and some transformations the first approximation of the cavity shape is written as follows

$$R^2 = \frac{2R_n^2}{lL_k}\left(-x^2 + xL - \frac{lL}{2}\right) \qquad (2.10)$$

If we transfer the origin of the coordinate system from the apex of the cone to its base then the cavity profile takes the form of

$$R^2 = \frac{2R_n^2}{lL_k}\left(-x^2 + xL_k + \frac{lL_k}{2}\right) \tag{2.11}$$

For the stationary cavity we can write the coordinate x and the length L_k as following: $x = V_0 t, L_k = 2V_0 t_k$, where t is the time. Substituting x and L_k in (2.11) and taking into account that when $t=t_k$ the equality $R=R_k$ is satisfied, where R_k is the cavity mid-section radius, we obtain the following equation

$$\frac{R^2 - R_n^2}{R_k^2 - R_n^2} = \frac{t}{t_k}\left(2 - \frac{t}{t_k}\right) \tag{2.12}$$

Equation (2.12) corresponds with equation (1.18) if $S_0 = \pi R_n^2$. Thus, the principle of independence of the cavity sections expansion and the slender body theory give the same result - we can consider an ellipsoid of revolution to be the first approximation of the shape of the stationary cavity. Taking into account that the first approximation is obtained for the compressible fluid we can draw a more general conclusion – the principle of independence of the cavity sections expansion is satisfied in the subsonic flow of the compressible fluid. For the case of the non-stationary cavities in the incompressible fluid the conclusion of the principle of independence from the slender body theory has been performed in [9] where the so-called "circular model" was applied.

Let us consider the results obtained for the stationary cavities by the numerical methods. We can divide the numerical methods into three groups. The first group can be called 'the boundary integral equation method'. In this method the stream function is applied and the surfaces of a cavity and a cavitator are represented by the vortex layer. The cavitation flow is found with the help of the numerical solution of the vortex layer integral equations [10,11,12]. The boundary element method based on the Green's integral is related to the second group. In this method on the cavitator surface the sought function is the perturbed velocity potential; on the cavity surface the sought function is the normal derivative of the perturbed velocity potential [13,14]. The finite-difference method represents the third group [15,16].

The published works on the numerical calculations of the stationary cavitation flows contain the results on the cavitation drag coefficient of a cavitator, the cavity mid-section radius and the cavity length. In these works the difference in the results of the numerical calculations does not exceed several percent. The numerical calculations have shown that the stationary cavity shape is close to an ellipsoid of revolution for the cavitation numbers that have the order $10^{-2} \div 10^{-1}$. This result agrees with the principle of independence of the cavity sections expansion.

As an example, let us consider the results of the numerical calculations obtained by the author [16]. Using the finite-difference method the author has calculated the cavities past a disk in the subsonic flow of the compressible fluid. The Riabouchinsky scheme was applied and the Mach numbers were located on the range $0 \le M \le 0.95$. The numerical calculation has shown that on the whole range of the Mach numbers the cavity profile is close to an ellipsoid of revolution which is represented by the equation (1.18). An ellipsoid joins with the cavity profile in the vicinity of the disk; as the point of contact we can take the point with the coordinates: $x=2R_n$, $R=2\ R_n$. Figure 4 illustrates the cavity profile in the vicinity of the disk (the geometric dimensions are scaled by the radius of the disk R_n).

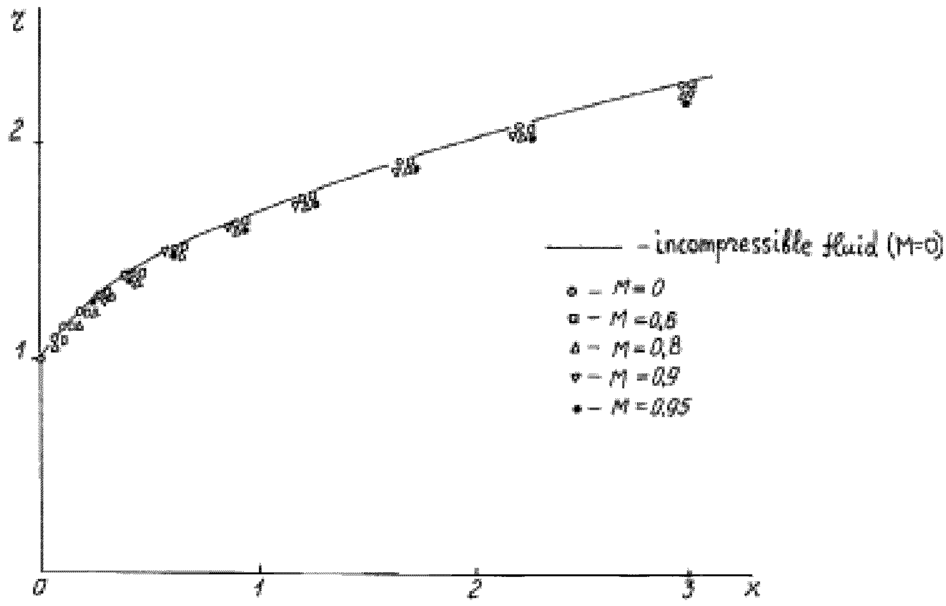

Fig. 4 The front of the caivty

In Fig. 4 the continuous curve represents the result of the numerical calculation by the boundary integral equation method for the incompressible fluid [10], the author's numerical results in the subsonic flow for the different Mach numbers are shown by the points [16]. Figure 5 shows the cavity profiles in the compressible fluid (M=0.8) and in the incompressible fluid (M=0) for the same cavitation number σ=0.0235 [16].

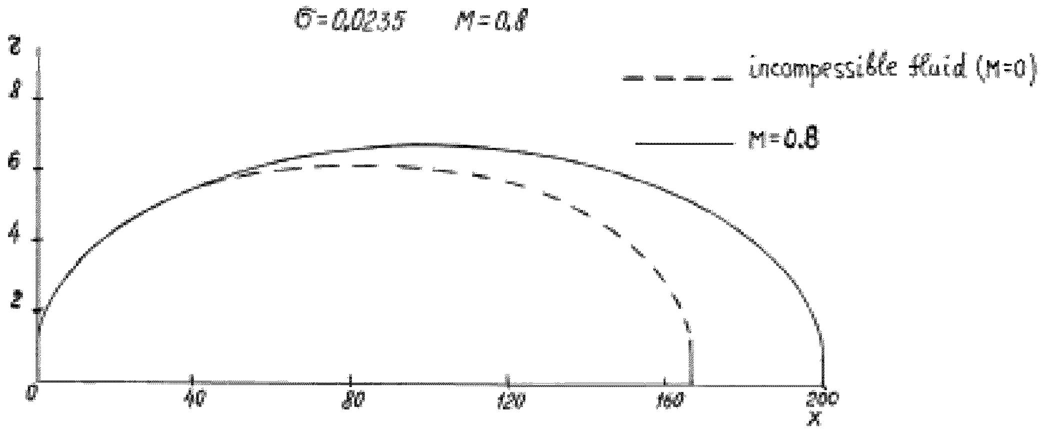

Fig. 5 Cavity profile

Both of the cavity profiles are close to an ellipsoid of revolution. The dimensions of an ellipsoid for M=0.8 exceed the analogous dimensions for M=0 since the cavitation drag coefficient for M=0.8 exceeds that for M=0.

3. The application of the equation of conservation of energy for determining the stationary cavity shape at $\sigma \to 0$

The equation of conservation of energy on the given cavity section (1.1) is equivalent to the principle of independence of the cavity sections expansion since all the three quantities that construct equation (1.1) are independent from both the following and the previous cavitator motion. The sum of the kinetic and potential energies on the section Σ (Fig. 1) is determined by the value of the cavitator drag W at the moment when a cavitator passes the section Σ. The law of the cavity section expansion (1.15) obtained from the energy equation (1.1) is independent from the following and the previous cavitator motion. This law represents the principle of independence. It should be noted that the energy equation similar to equation (1.1) was applied in [17] in order to estimate the axisymmetric cavity shape. However, in [17] the kinetic energy in the wake was determined for the radial flow on the plane circular layer. In chapter 1 in contrast to [17] the kinetic energy (formula (1.2)) is determined along the stream tube leaning on the expanding cavity hole.

The principle of independence is some approximation to the reality. However, a large number of experiments have confirmed its accuracy for both the stationary and non-stationary cavities. The principle of independence agrees with the results obtained from the slender body theory for the stationary and non-stationary cavitation flows. Furthermore, the calculation results of the stationary cavities using the numerical methods show that the cavity shape is close to an ellipsoid of revolution for the cavitation number that have the order $10^{-2} \div 10^{-1}$. Also, the ellipsoidal form of the stationary cavity has been obtained from the principle of independence as we neglect the value of $v^2/2$ compared to the value of $\Delta P/\rho$ in equation (1.5). Let us prove that the principle of independence of the cavity sections expansion or the energy equation (1.1) are applicable for the special case. That is the cavitation number is equal to zero (the infinite cavity length).

When the cavitation number equals to zero then $\Delta \mathrm{P} = P_\infty - P_k = 0$ and the potential energy in the wake E (equation (1.1)) also equals zero. The kinetic energy in the wake is determined by formula (1.2). Let us determine the quantities of the velocities on the cavity surface at the zero cavitation number (Fig. 6).

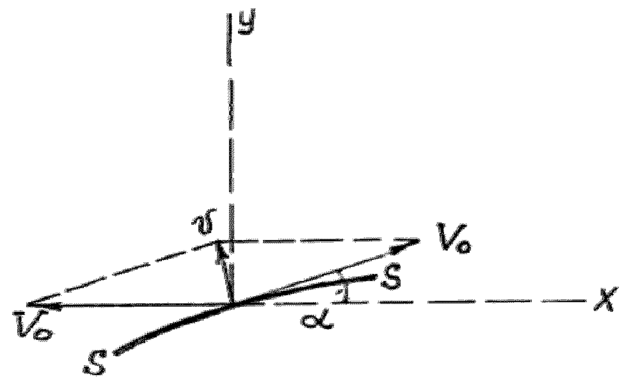

Fig. 6 The absolute velocity of the fluid particle on the cavity surface at σ=0

The absolute velocity of the fluid particles on the cavity surface v is determined as the vector sum of the transport velocity V_0 directed along the x axis and the relative velocity directed along the tangent to the cavity surface (for the zero cavitation number the relative velocity is equal to V_0). In Fig. 6 the section of the cavity surface is designated as s-s. From Fig. 6 we obtain the following correlations for the velocity components.

$$v = 2V_0 \sin\frac{\alpha}{2}, \quad v_y = \dot{R} = V_0 \sin\alpha, \quad v_n = \frac{\partial\varphi}{\partial n} = V_0 \sin\alpha$$
$$v_n = v_y = \dot{R}, \quad v \approx \dot{R} \tag{3.1}$$

where α is the angle of inclination of the cavity surface to x axis. Correlations (3.1) are approximately satisfied for the small cavitation numbers different from zero. They were used in chapter 1. For the zero cavitation number the energy equation (1.1) we can write in the following form (taking into account equation (1.2) and correlations (3.1))

$$-\rho\varphi\pi R\dot{R} = W \tag{3.2}$$

Let us substitute φ from equation (3.2) in the boundary condition (1.5). We assume that the approximate equality $v \approx \dot{R}$ is satisfied as α is a small quantity (correlations (3.1)). As a result we obtain the equation

$$-\frac{W}{\pi\rho}\frac{d}{dt}\left(\frac{1}{R\dot{R}}\right) + \frac{\dot{R}^2}{2} = 0 \tag{3.3}$$

The first integral of the differential equation (3.3) has the form [1]

$$\dot{R} = \frac{1}{R}\sqrt{\frac{W}{\pi\rho(\ln R/R_n + A)}} \tag{3.4}$$

where A is some constant.

For the stationary cavity the equality $dx=Vdt$ is satisfied. Let us introduce the following designations: $x^*=x/R_n$, $R^*=R/R_n$. Equation (3.4) is written as follows

$$\frac{dR^*}{dx^*} = \sqrt{\frac{C_x}{2}}\frac{1}{R^*\sqrt{\ln R^* + A}} \tag{3.5}$$

Let us join the cavity profile (3.5) with the cavity profile in the vicinity of the disk (1.19). As the point of contact we take the point with the coordinates x^*=2, R^*=1.92. At the point of contact we determine the derivative dR^*/dx^* from equation (1.19) and substitute it in equation (3.5). As a result we determine the constant A. According to the data presented in [1] the constant A equals 0.845. We obtain the cavity profile from equation (3.5) in the form of

$$x^* = 2 + \sqrt{\frac{2}{C_x}}\int_{\sqrt[3]{7}}^{R^*} u\sqrt{\ln u + A}du \tag{3.6}$$

where u is the variable of integration. The asymptotic law of the axisymmetric cavity expansion has been obtained for the zero cavitation number in [18,19]. This law can be presented as follows

$$R^* \sim \sqrt[4]{4C_x}\,\frac{\sqrt{x^*}}{\sqrt[4]{\ln x^*}} \tag{3.7}$$

In [20] the law of the cavity expansion was generalized for the case of subsonic compressible fluid flow. In [20] it is shown that equation (3.7) has the same form for both incompressible and compressible fluid. The compressibility exerts influence on the law of the cavity expansion by means of the drag coefficient C_x, which is dependent upon the Mach number.

Let us compare the cavity profiles past a disk in incompressible fluid for the zero cavitation number (C_x=0.82). We compare the profile obtained from the law of conservation of energy in the wake (equation (3.6)) with the profile expressed by the asymptotic law (3.7). The results of this comparison are shown in the table.

TABLE

x^*	R^* eq. (3.7)	R^* eq. (3.6)
5	2.67	2.59
10	3.45	3.36
25	5.02	4.90
50	6.77	6.63
100	9.19	9.03
250	13.88	13.70
500	19.06	18.86
1000	26.25	26.04
1500	31.69	31.48

It is evident from the table that the difference in the cavity profiles does not exceed several percent. We can conclude here that for the zero cavitation number the results obtained based on the energy equation in the wake (the principle of independence) agree with the well-known Levinson-Gurevich asymptotic (3.7).

In chapters 1 and 3 we have determined the cavitation profiles based on the energy equation (the principle of independence) and have applied the dynamical boundary condition (1.5) for two cases:

1. when we can neglect the value of $v^2/2$ compared to the value of $\Delta P/\rho$ in the middle section of the cavity
2. when the value ΔP equals zero (the zero cavitation number).

In the first case the cavity shape agrees with the experimental data and the theoretical results for the cavitation numbers that have the order $10^{-2} \div 10^{-1}$. In the second case the cavity shape agrees with the Levinson-Gurevich asymptotic at σ=0. Thus, we do not investigate the cavities on the range $0<\sigma<0.005$. Let us extend the application of the principle of independence to this range of the cavitation numbers.

Recently G. V. Logvinovich has considered the general case – two members $v^2/2$ and $\Delta P/\rho$ are taken into account in the dynamical boundary condition (1.5) [21]. Let us determine the stationary cavity shape for this case. The potential energy is defined as $E=\pi R^2 \Delta P$ and equation (1.4) is written in the following form

$$-\rho\varphi\pi R\dot{R} + \pi R^2 \Delta\mathrm{P} = W \tag{3.8}$$

After substitution of φ from equation (3.8) into the dynamical boundary condition (1.5) using the approximate equality $v \approx \dot{R}$ we obtain the following equation

$$-\frac{W}{\pi\rho}\frac{d}{dt}\left(\frac{1}{R\dot{R}}\right) + \frac{\Delta\mathrm{P}}{\rho}\frac{d}{dt}\left(\frac{R}{\dot{R}}\right) = \frac{\Delta\mathrm{P}}{\rho} - \frac{\dot{R}^2}{2} \tag{3.9}$$

As a result of the transformations the first integral of differential equation (3.9) for the stationary cavity can be written as follows [21]

$$\frac{dR^*}{dx^*} = \sqrt{\frac{C_x}{2}}\frac{1}{R^*}\sqrt{\frac{1-\sigma R^{*2}/C_x}{\ln R^* + A}} \tag{3.10}$$

At $\sigma=0$ differential equation (3.10) is transformed to equation (3.5). Using equation (3.10) the cavity profiles were calculated for the wide range of cavitation numbers [21]. For the cavitation numbers that have the order 10^{-2} the calculations on the basis of equation (3.10) have shown that the cavity shape is close to an ellipsoid of revolution. This result agrees with the well-known experimental and theoretical results.

Thus, on the basis of the carried out investigations we can conclude that the principle of independence of the cavity sections expansion deduced from the equation of conservation of energy in the wake is applicable to the wide range of the cavitation numbers $0 \le \sigma \le 0.1$. On this range the stationary cavity profiles can be calculated by the single formula (3.10).

4. The calculations of the cavities in the gravitational field on the basis of the principle of independence of the cavity sections expansion

The principle of independence of the cavity sections expansion makes the calculation of the cavity profile easier in the arbitrary variable pressure fields. Based on this principle the cavity shape is determined if we know the law of the cavitator motion and the values of the pressure at infinity and within the cavity. Let us apply the principle of independence for the calculation of the vertical cavities in the gravitational field [22].

Let us consider the vertical axisymmetrical cavity formed by the stream around the motionless disk-cavitator in the gravitational field. The stream can be descending or ascending, the value of the stream velocity V is the constant (Fig. 7).

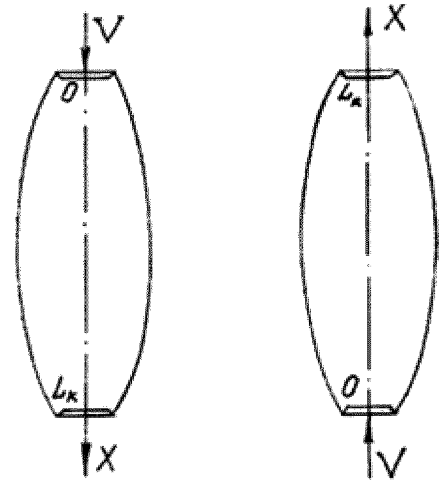

Fig. 7 The coordinate systems for the calculation of the cavity profile in the gravitational field

At the level of the cavitator the free stream pressure P_n (the pressure at infinity) is the constant. Let us introduce the vertical coordinate axis x directed along the stream velocity and the symmetry axis of the cavity from the cavitator center. On the section with coordinate x the free stream pressure (the pressure at infinity) is expressed as following

$$P_\infty(x) = P_n \pm \rho g x \tag{4.1}$$

where g is the gravitational acceleration, the sign 'plus' corresponds with the descending stream, the sign 'minus' corresponds to the ascending stream.

We determine the profile of the stationary vertical cavity. For the stationary cavity the pressure within it is the constant - $P_K = const.$ The area of the cavity cross section fixed relatively the motionless fluid changes according to equation (1.15). Integral of equation (1.15) is equation (1.17) which describes the change of the area of the cavity fixed cross section. In this example we consider the cavitator to be motionless and observe the cavity section which has been formed at the edge of the cavitator at the moment t=0 and moves at the velocity V along with the stream. The area change of this section can be represented by the following expression

$$S(t) = S_0 + \dot{S}_0 t - \frac{k}{\rho}\int_0^t du \int_0^u \Delta\mathrm{P}(v)\,dv \tag{4.2}$$

where v and u are the variables of integration, S_0 is the section initial area equal to the cavitator area. Let us substitute the expression of the external pressure (4.1) in dependence (4.2). Note that in (4.1) x is written as $x(v)=Vv$. After integration of equation (4.2) we obtain

$$S(t)=S_0+\dot{S}_0t-\frac{kt^2}{2\rho}\left(\Delta P_0\pm\frac{1}{3}\rho gVt\right) \tag{4.3}$$

Expression (4.3) contains the difference ΔP_0 of the pressure at infinity and the pressure within the cavity $\Delta P_0=P_n-P_k$. This difference is related to the section at the level of the cavitator.

Let us consider the case of the descending stream. For the stationary cavity the variable x is written as $x=Vt$. After substitution of (1.14) into (4.3) and certain transformations we obtain the expression for the vertical cavity profile in the dimensionless form

$$R^{*2}=1+\frac{2C_x}{a}x^*-\frac{C_xx^{*2}}{a^2}\left(\sigma_0+\frac{x^*}{3Fr^2}\right) \tag{4.4}$$

where R^*, x^* are the dimensionless cavity radius and coordinate x scaled by the disk radius R_n,

$\sigma_0=2\Delta P_0/\rho V^2$ is the cavitation number at the level of the cavitator,

$Fr=V/\sqrt{2R_ng}$ is the Froude number.

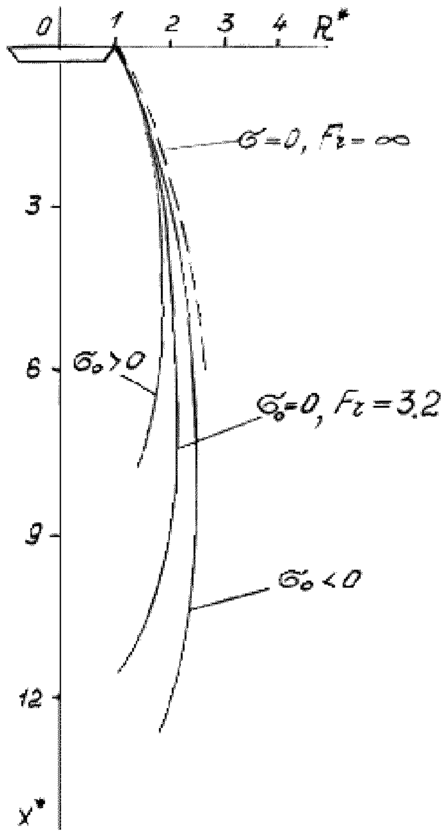

Fig. 8 The vertical cavity profiles in the descending stream

As an example, in Fig. 8 the cavity profiles calculated by formula (4.4) are represented by the continuous curves for the three values of the cavitation number σ_0: –0.1; 0; 0.1. The Froude number equals 3.2 and it remains the same for all the three cavities, the constant a equals 1.5. The Levinson-Gurevich asymptotic (3.7) for the zero cavitation number and weightless fluid ($Fr=\infty$) is represented by the broken curve.

Let us determine the length L_k. of the stationary vertical cavity for the Riabouchinsky scheme (Fig. 7). We assume that the cavity is closed by the disk of the same dimensions as the disk cavitator, i.e. $R^*=1$ at $x^*=L_k/R_n$. From expression (4.4) we obtain the quadratic equation for determining the cavity length

$$\left(\frac{L_k}{R_n}\right)^2 \frac{1}{3aFr^2} + \left(\frac{L_k}{R_n}\right)\frac{\sigma_0}{a} - 2 = 0 \tag{4.5}$$

The solution of equation (4.5) can be represented in the following form

$$\frac{L_k}{2R_n Fr} = -\frac{3}{4}\sigma_0 Fr + \sqrt{\frac{9}{16}\sigma_0^2 Fr^2 + \frac{3a}{2}} \tag{4.6}$$

In Fig. 9 dependence (4.6) is represented by the continuous curve, the points show the experimental data obtained for different values of the Froude and the cavitation numbers [22]. Figure 9 shows good agreement between the theoretical dependence (4.6) and the experimental data.

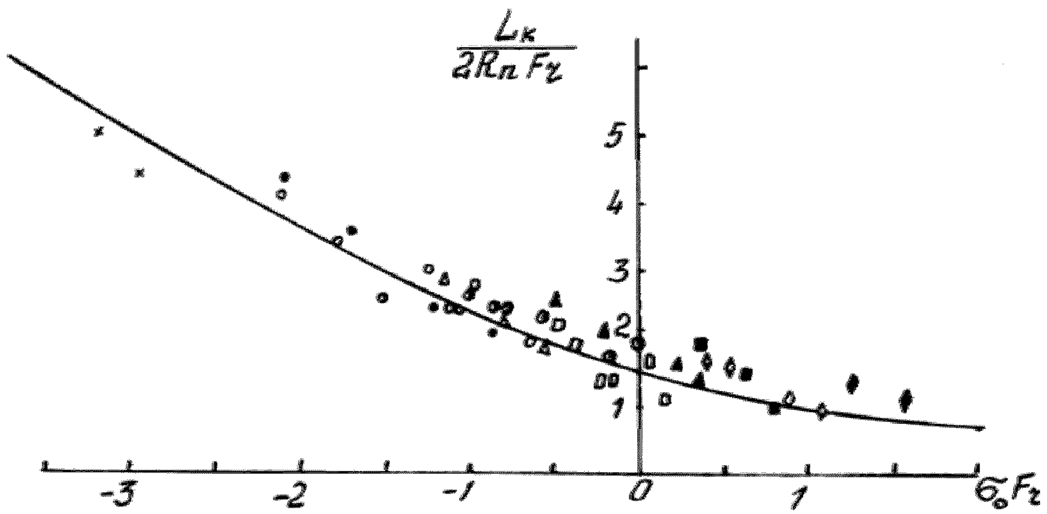

Fig. 9 The comparison of the calculation of the vertical cavity length based on formula (4.6) with the experimental data

Let us apply the principle of independence for the investigation of the cavity deep closure which occurs at the water entry of bodies [23]. At the moment of intersection of the water surface the cavity is formed past a body. This cavity communicates with the atmosphere, consequently in the initial period of submersion the gas pressure within the cavity is close to atmospheric pressure and the cavitation number is close to zero. However, as the body sinks the external hydrostatic pressure increases and under the influence of the excess pressure the cavity section closes. The cavity detaches from the atmosphere and the so-called deep closure occurs (Fig. 10).

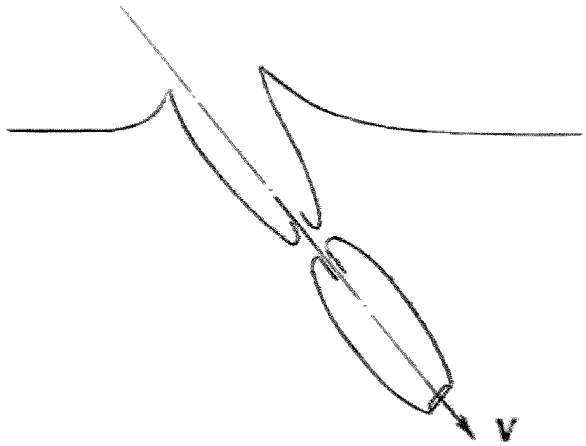

Fig. 10 The cavity deep closure at the water entry of a cavitator

Let us consider the inclined rectilinear water entry of the cavitator at the constant velocity V. We designate the entry angle as θ (the angle of inclination of the trajectory to the free surface). The time t is measured from the instant of the cavitator's initial contact with the water surface. We observe the section which has been formed at the moment t_1 on the depth H_1 (Fig. 11).

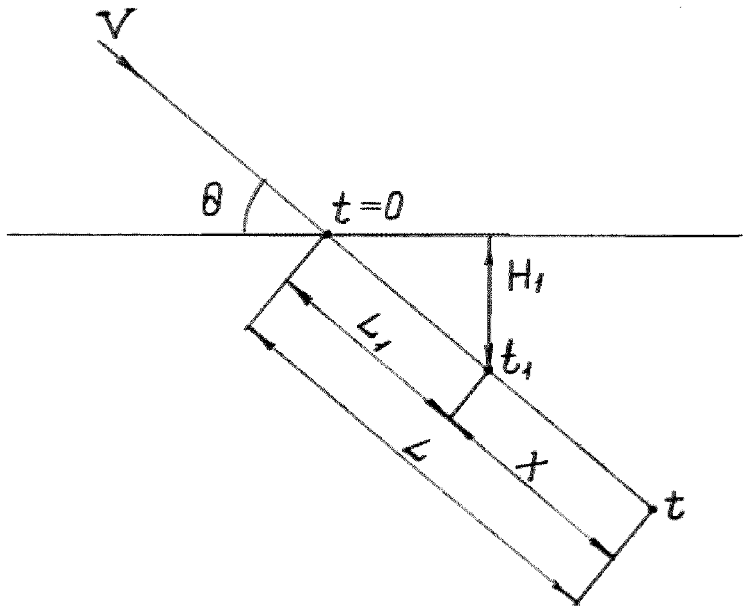

Fig. 11 The scheme for the calculation of the deep closure

We consider the pressure within the cavity equal to the atmospheric pressure then the excess pressure on the considered section is the constant and can be written in the form of

$$\Delta \mathrm{P} = \rho g H_1 = \rho g L_1 \sin\theta \tag{4.7}$$

The equation of the cavity section expansion is described by equation (1.15). The integral of equation (1.15) is written as follows

$$S(H_1,t)=S_0+\dot{S}_0(t-t_1)-\frac{k\Delta P}{2\rho}(t-t_1)^2 \tag{4.8}$$

where S_0 is the cross section initial area equal to the cavitator area. From equations (4.7) and (4.8) we can determine the cavity profile at the water entry of the disk-cavitator if we apply the obvious correlations - $x=V(t-t_1)$; $L_1=L-x$ where x is the coordinate measured from the cavitator center along the trajectory of the cavitator, L is the cavitator path passed after the intersection of the water surface.

$$S(x)=S_0+\frac{\dot{S}_0}{V}x-\frac{kgL\sin\theta}{2V^2}x^2+\frac{kg\sin\theta}{2V^2}x^3 \tag{4.9}$$

We determine the cavity length from the condition that the cavity is closed by the disk with the same dimensions as the disk-cavitator. From (4.9) we obtain the quadratic equation for determining the cavity length

$$\frac{kg\sin\theta}{2V^2}L_k^2-\frac{kgL\sin\theta}{2V^2}L_k+\frac{\dot{S}_0}{V}=0 \tag{4.10}$$

The solution of equation (4.10) is written in the form of

$$L_k=\frac{L}{2}-\frac{L}{2}\sqrt{1-\frac{8\dot{S}_0V}{kgL^2\sin\theta}} \tag{4.11}$$

It is obvious that the maximum cavity length corresponds to the deep closure. It follows from (4.11) that the value of L_k is maximum when the radicand expression equals zero. Then the value of L_k equals $L/2$, i.e. the cavity closes halfway passed by the cavitator. The moment of the deep closure is determined from the equation

$$\frac{8\dot{S}_0V}{kgL^2\sin\theta}=1 \tag{4.12}$$

After substitution of correlations (1.14) and the equality $L=Vt_d$ in (4.12), for the dimensionless deep closure time we obtain the expression

$$t_d^*=2Fr\sqrt{\frac{2a}{\sin\theta}} \tag{4.13}$$

where $t_d^* = Vt_d / R_n$ is the dimensionless deep closure time, t_d is the deep closure time.

Previously expression (4.13) was obtained in a different way in [24]. The path passed by the cavitator at the moment of the deep closure is determined from the equality $L = R_n t_d^*$.

5. The application of the principle of independence for the calculation of the non-stationary cavities with variable internal pressure

The area of the cavity cross section fixed relatively to the motionless fluid changes according to equation (1.15). Equation (1.15) expresses the principle of independence. The integral of equation (1.15) is equation (1.17) that describes the area change of the cavity fixed cross section. Let us go on from equation (1.17) to the expression that represents the cavity profile at the fixed time [25]. In order to do that in equation (1.17) we introduce the time t which is common for all the sections. At the moment t the cavitator has the coordinate H. Let us designate the moment of the section formation with the coordinate h as t_1 ($t_1=t_1(h)<t=t_1(H)$). Then equation (1.17) transforms to

$$S(h,t) = S_0 + \dot{S}_0(t - t_1) - \frac{k}{\rho}\int_{t_1}^{t} du \int_{t_1}^{u} \Delta P(h,v)dv \tag{5.1}$$

where S_0 is the cavitator area, $\dot{S}_0$ is the constant that defines the initial velocity of the cavity expansion, v and u are the variables of integration. At the fixed moment t equation (5.1) expresses the cavity profile $S(h)$. At the fixed moment t_1, i.e. h=const, equation (5.1) describes the expansion of the fixed cavity cross section. We can transform the double integral in (5.1) to the line one as follows. Let us change the order of integration in the double integral

$$\int_{t_1}^{t} du \int_{t_1}^{u} \Delta P(h,v)dv = \int_{t_1}^{t} \Delta P(h,v)dv \int_{v}^{t} du = \int_{t_1}^{t} (t - v)\Delta P(h,v)dv \tag{5.2}$$

After substitution of (5.2) in (5.1) we obtain

$$S(h,t) = S_0 + \dot{S}_0(t - t_1) - \frac{k}{\rho}\int_{t_1}^{t} (t - v)\Delta P(h,v)dv \tag{5.3}$$

Equation (5.3) is especially convenient for the numerical calculation of the cavity profile when the pressure within it changes arbitrarily. As an example we consider the calculation of the cavity profile when the gas pressure within it changes according to the harmonic oscillations [26]. The time dependence of the gas pressure is written as follows

$$P_k(t) = P_{k_0} + A\sin\omega t \tag{5.4}$$

where P_{k_0} is some constant pressure, ω is the circular frequency, A is the amplitude of oscillations. We consider the external pressure $P_\infty(h,t)$ to be a constant and equal to P_∞. After substitution of (5.4) in (5.3) or (5.1) we obtain the profile of the pulsating cavity

$$S(x,t)=S_0+\frac{\dot{S}_0}{V}x-\frac{k\Delta P_0}{2\rho V^2}x^2-\frac{kA}{\rho\omega^2}F\left(\frac{\omega x}{V}\right)\sin[\omega t+\varphi(x)] \tag{5.5}$$

where $x=V(t-t_1)$ is the coordinate measured from the cavitator along the stream velocity, $\Delta P_0=P_\infty-P_{k_0}$ is the stationary excess pressure

$$F^2\left(\frac{\omega x}{V}\right)=2\left(1-\cos\frac{\omega x}{V}\right)+\frac{\omega x}{V}\left(\frac{\omega x}{V}-2\sin\frac{\omega x}{V}\right) \tag{5.6}$$

The first three members in (5.5) express the profile of the stationary cavity, the last member describes the perturbation of the cavity profile. The amplitude of the perturbation of the cavity profile increases to the downstream end of the cavity. The increase of the perturbations to the downstream end of the cavity is illustrated by Fig. 12 which shows the dependence (5.6).

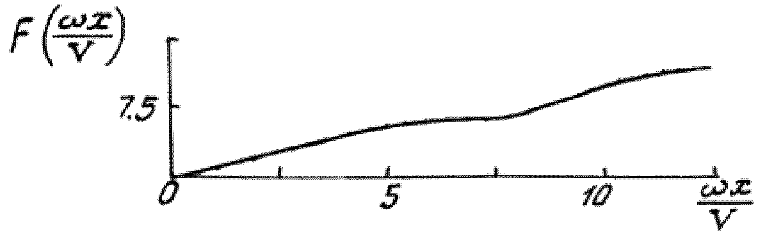

FIG. 12 THE INCREASE OF THE PERTURBATIONS TO THE DOWNSTREAM END OF THE CAVITY

Also, the increase of the perturbations to the downstream end of the cavity is confirmed by the experiments performed with the pulsating cavities [26].

If beforehand the gas pressure within the cavity is not known then it is necessary to add the subsidiary equations to equation (5.3) or (1.15). The equation for calculating the gas volume within the cavity is one of them

$$Q(t)=\int_{H(t)}^{h_c(t)}[S(h,t)-S_b(h,t)]dh \tag{5.7}$$

where $Q(t)$ is the volume contained between the cavity boundary and the body within the cavity,

$S_b(h,t)$ is the area of the body cross section;

$H(t)$ is the coordinate of the cavitator,

$h_c(t)$ is the coordinate of the cavity closure.

The equation of the gas mass change within the cavity has the form of

$$\frac{dm(t)}{dt} = \dot{m}_s - \dot{m}_l \tag{5.8}$$

$m(t)$ is the gas mass within the cavity;

$\dot{m}_s$ is the mass gas-supply rate to the cavity;

$\dot{m}_l$ is the mass-leakage rate from the cavity.

The following is the equation of the gas state within the cavity

$$\frac{P_k(t) \quad Q(t)}{m(t)} = RT(t) \tag{5.9}$$

where R is the gas constant,

$T(t)$ is the gas temperature within the cavity. This temperature is dependent upon the thermodynamic process. In most cases we can consider the thermodynamic processes within the cavity to be isothermal and assume that the gas temperature within the cavity is equal to the temperature of the surrounding fluid.

In general the equation of the cavitator motion appears like

$$\frac{dV}{dt} = F(V, H, P_k, h_c, \ldots) \tag{5.10}$$

where V is the velocity vector.

The system of the equations (5.3), (5.7)-(5.10) describes the dynamics of the non-stationary cavity in the closed form. In fact, we can solve this system by the numerical method at each time step substituting the cavity profile for the discrete set of the cross sections. Also, in this case using the numerical method we can solve equation (1.15) instead of equation (5.3).

E. V. Parishev [25] has transformed the integral correlations (5.3) and (5.7) in the nonlinear differential equations with the lagging argument. It turned out that in general the dynamics of the non-stationary cavity is described by the system of the nonlinear differential equations of the sixth order with the variable lag (excluding the equations of the cavitator motion). In particular cases these equations are simplified. For example, in case of the axisymmetric cavity formed past a cavitator which is moving at the constant velocity in the weightless fluid and with the assumption that the gas mass within the cavity is the constant (gas-supply and gas-leakage are absent), the equation for the small pressure oscillations of the gas within the cavity has the form [27]

$$P_k'''(t^*) + P_k'(t^*) + P_k'(t^* - \tau^*) - \frac{2}{\tau^*} P_k(t^*) + \frac{2}{\tau^*} P_k(t^* - \tau^*) = 0 \tag{5.11}$$

where P_k is the gas pressure within the cavity (small oscillations relatively to the equilibrium), $t^*=t/T$ – is the dimensionless time, $\tau^*=\tau/T$ is the dimensionless lag, $\tau=L_k/V$ is the dimensional lag, L_k is the cavity length at the equilibrium, V is the cavitator velocity. The time scale T is determined by the expression

$$T=\sqrt{\frac{nQ_0\rho}{P_{k_0}kV\tau}} \tag{5.12}$$

where Q_0 is the cavity volume at the equilibrium, P_{k_0} is the gas pressure within the cavity at the equilibrium, $1/n$ is the polytropic index of the thermodynamic process within the cavity, for the isothermal process n equals 1.

Equation (5.11) has the single basic parameter equal to the dimensionless lag τ^*. As a result of transforming equation (5.12) this parameter can be represented as following

$$\tau^*=\sqrt{\frac{12}{n}\left(\frac{Eu}{\sigma}-1\right)}$$

where $Eu=2P_\infty/\rho V^2$ is the Euler number, σ is the average cavitation number defined by the pressure P_{k_0}.

Thus, all the cavity properties described by equation (5.11) are defined by the value of the ratio Eu/σ. Also the experimental works [28,29] show that the phenomenon of the artificially inflated cavity pulsation is defined by the ratio σ/Eu. Equation (5.11) has both the steady and unsteady (pulsating) solutions. For the vapor cavities ($\sigma\approx Eu$) and the ones close to them the solutions are steady. However, when $\tau^*>\pi\sqrt{2}$ the solutions are unsteady. This result corresponds to the well-known features of the vapor cavity which does not pulsate. Also, the experiments in [28,29] show that the cavities with a weak gas-supply, i.e. those that are close to the vapor ones, are steady.

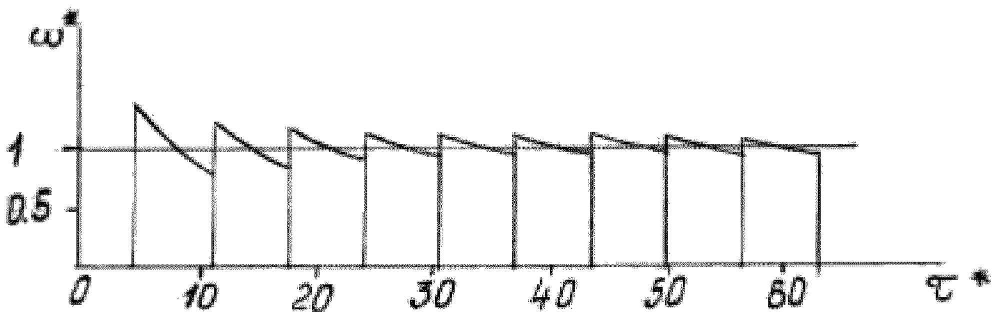

Fig. 13 The dependence of the dimensionless circular frequency of cavity pulsation ω^* on the parameter τ^*

In Fig. 13 the dependence of the dimensionless circular frequency of the cavity pulsation ω^* ($\omega^*=\omega L_k/\tau^* V$, where ω is the dimensional circular frequency) on τ^* is represented by the discontinuous serrate function with the average value equal to 1. Knowing the dimensionless frequency of pulsation we can determine the number of the waves N going into the cavity length

$$N = \frac{L_k}{\lambda} = \frac{\omega * \tau *}{2\pi}$$

where λ is the wave length. Since the dependence $\omega*(\tau*)$ is discontinuous the dependence $N(\tau*)$ is discontinuous too and the number of the waves going into the cavity length is close to integer. At the continuous change of $\tau*$ the value of N changes as the stepped function. The dependence $N(\tau*)$ is represented in Fig. 14. The points show the experimental data [26].

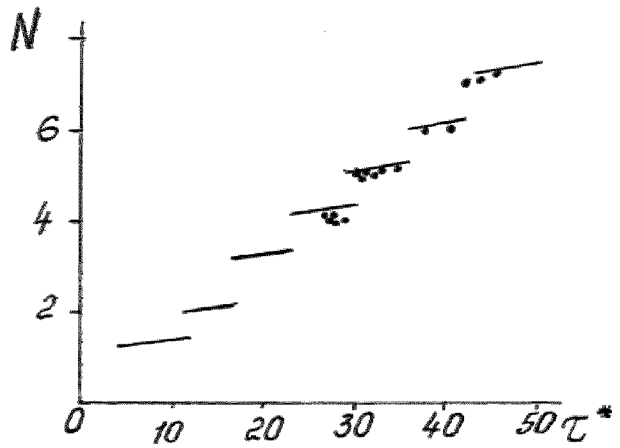

Fig. 14 The dependence of the number of the waves going into the cavity length N on the parameter $\tau*$

The discontinuous properties of the dependences $\omega*(\tau*)$ and $N(\tau*)$ agree with the well-known experimental data [29]. The experiments have shown [29] that the pulsating cavities have the regimes with the fixed number of waves close to integer. The experiments have also shown [29] that at the smooth value change of the gas-supply the number of waves changes as the stepped function.

Thus, the cavity instability is connected with the gas elasticity within it. The underwater bubble has the analogous instability when it is internally supplied by gas. We can consider the cavity to be a dynamic system with infinite degrees of freedom. The cavity cross section changes influenced by the difference between the free stream pressure and the pressure within the cavity; the cavity volume changes as well. The cavity volume change leads to the pressure change within the cavity, etc. The cavity being a dynamic system with these properties is analogous to the vibration of a body on a spring. The coordinate of a body is analogous to the cavity cross section area. The elastic force of a spring is analogous to the pressure within the cavity. However, in case of a cavity we have an infinite quantity of these bodies (sections) and the pressures within the cavity is a function of the whole cavity volume, i.e. the cross sections are connected by means of the gas that fills the cavity. The gas-leakage from the cavity is the factor of stability. When this factor is active the parameter $\tau*$ that corresponds to the cavity stability, exceeds the value of $\pi\sqrt{2}$. In addition to the mathematical model Parishev removed the gas along the bound vortexes and obtained the accordance with Epshtein's experimental data [26].

Conclusions

The principle of independence of the cavity sections expansion stated by G. V. Logvinovich provides an easier way for investigating the non-stationary cavities and the cavities with the variable external pressure. This principle corresponds to the flow's property around slender bodies and it is stated as following – each cavity cross section expands relatively to the trajectory of the cavitator center according to the certain law which is dependent on the conditions at the moment as a cavitator passes the plane of the considered section. The cross section expands almost independently from the following or the previous cavitator motion.

The principle of independence is equivalent to the equation of conservation of energy applied to the given cavity section. The sum of the kinetic and potential energies on the cavity section is determined only by the value of the cavitation drag at the moment of intersection with this section. The law of the cavity section expansion (1.15) obtained from the energy equation (1.1) is independent from the following and the previous cavitation motion.

The principle of independence is some approximation to the reality. However, the numerous experiments have confirmed it being accurate for both the stationary and non-stationary cavities. The principle of independence agrees with the results obtained from the slender body theory for the stationary and non-stationary cavitation flows.

Furthermore, the results of the stationary cavity calculation by the numerical methods demonstrate that the cavity shape is close to an ellipsoid of revolution for the cavitation numbers that have the order $10^{2}\div10^{-1}$. The ellipsoidal form of the stationary cavity has also been obtained from the principle of independence. The principle of independence of the cavity section expansion and the energy equation are applicable for a special case – the cavitation number equals zero (the infinite cavity length).

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