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Energy Loss of Ions by Collisions with Magnetized Electrons

G. Zwicknagel* and C. Toepffer*

*Institut für Theoretische Physik, Universität Erlangen-Nürnberg,
Staudtstr. 7, D - 91058 Erlangen, Germany

Abstract. The interaction of ions with electrons in the presence of an external magnetic field is a basic process for the description of the transport properties of a magnetized plasma, as e.g. the energy loss of ions to the superimposed electron beam in the electron cooler of a storage ring. But a comprehensive description of binary collisions of ions and electrons in the presence an external magnetic field has still to be established. Here, we recently achieved some progress by calculating the energy transfer in binary ion-electron collisions both by fully numerical classical-trajectory-Monte-Carlo (CTMC) simulations and by perturbation theory. These two techniques and some first results are presented and discussed, in particular the strong dependencies of the energy loss on the direction of the ion motion with respect to the magnetic field and on the anisotropy of the velocity distribution of the electrons.

INTRODUCTION

One fundamental process for the description of the transport properties of a magnetized plasma is the interaction of ions and electrons in the presence of an external magnetic field. Prominent examples are the resulting cooling of ions by a surrounding electron plasma in a trap or by the energy loss of the ions to the superimposed electron beam in the electron cooler of a storage ring. Although electron cooling in a storage ring is a well established method, a lot of observations are not yet satisfactorily explained and understood. Here, an improvement of the theoretical understanding has to start with the energy loss of a highly charged heavy ion in a magnetized anisotropic electron plasma, which is the fundamental process in the electron cooler. Whereas the energy loss of ions in unmagnetized electron plasmas has already been studied extensively [1], a qualified and comprehensive description of the interaction of ions with magnetized electrons is still an open problem and a rather formidable task. The presence of a magnetic field considerably complicates the description of the energy loss mainly because of the loss of symmetries as compared to the case of an isotropic nonmagnetized electron plasma.

At weak coupling between the ion and the magnetized electrons a linear response description of the energy loss is applicable, where the energy loss is given in a closed from in terms of the dielectric function, see e.g. [1, 2]. The appropriate dielectric function which describes the response of an ideal plasma of magnetized electrons with an anisotropic velocity distribution can be derived analytically and is given in most textbooks on plasma physics, e.g. in [3]. Its rather complex structure implies, however, a very intricate numerical evaluation of the final expression for the energy loss. Such a

numerical analysis is in progress [4] and an important subject for an advanced understanding of the stopping power at these conditions.

Also the description of the energy loss as a result of subsequent binary ion-electron collisions is strongly complicated by the presence of a magnetic field as compared to the nonmagnetic case where the energy transfer in a collision is a function only of the relative velocity. With magnetic field, additional dependencies show up and the ion-electron motion no more separates in a center-of-mass and relative part, in general. This makes even binary ion-electron collisions to a rather exhausting and challenging problem. Thus the energy loss is typically treated on an approximative level only. Starting with the expression for the energy loss by nonmagnetized ion-electron collisions, the influence of the magnetic field enters only through certain cutoffs which divide the possible impact parameters into several classes corresponding to slow and fast and/or nonmagnetized and magnetized collisions. For each of these cases a separate contribution to the energy loss with a specific Coulomb logarithm is formulated, see e.g. [5] for details. More accurate descriptions of the energy loss have to start with a detailed treatment of ion-electron collisions in a magnetic field. Here we discuss two different approaches for the determination of the energy transfer in this type of binary collisions; first a fully numerical classical-trajectory-Monte-Carlo (CTMC) scheme and, additionally, a perturbation treatment up to 2nd order in the ion-electron interaction.

ION-ELECTRON COLLISIONS IN A MAGNETIC FIELD

We consider the interaction between an ion (mass M, charge Ze, position $\mathbf{R}$, velocity $\mathbf{V} = \dot{\mathbf{R}}$) and an electron ($m$, $-e$, $\mathbf{r}_e$, $\mathbf{v}_e = \dot{\mathbf{r}}_e$) in a static, homogeneous magnetic field $\mathbf{B}$ (and a vector potential $\mathbf{A} = c(\mathbf{B} \times \mathbf{r})/2$) in framework of classical mechanics. This is specified by the Lagrangian $\mathcal{L}$

$$\mathcal{L} = \frac{m}{2}v_e^2 + \frac{M}{2}V^2 - \frac{e}{2}(\mathbf{B} \times \mathbf{r}_e) \cdot \mathbf{v}_e + \frac{Ze}{2}(\mathbf{B} \times \mathbf{R}) \cdot \mathbf{V} - \Phi(|\mathbf{r}_e - \mathbf{R}|). \qquad (1)$$

Because the two-body interaction $\Phi(\mathbf{r}_e, \mathbf{R}) = \Phi(|\mathbf{r}_e - \mathbf{R}|)$ depends only on the relative distance, we next perform a transformation to the cms $(\mathbf{R}_{cm}, \mathbf{V}_{cm})$ and relative the frame $(\mathbf{r}_r, \mathbf{v}_r)$, defined through $\mathbf{R}_{cm} = (m\mathbf{r}_e + M\mathbf{R})/(m+M)$, $\mathbf{V}_{cm} = (m\mathbf{v}_e + M\mathbf{V})/(m+M)$, and $\mathbf{r}_r = \mathbf{r}_e - \mathbf{R}$, $\mathbf{v}_r = \mathbf{v}_e - \mathbf{V}$. In these coordinates, the Lagrangian (1) takes then the form

$$\mathcal{L} = \frac{m+M}{2}V_{cm}^2 + \frac{\mu}{2}v_r^2 - \Phi(r_r) + \frac{(Ze - e)}{2}(\mathbf{B} \times \mathbf{R}_{cm}) \cdot \mathbf{V}_{cm} \qquad (2)$$

$$+ \frac{\mu^2}{2}(\frac{Ze}{M^2} - \frac{e}{m^2})(\mathbf{B} \times \mathbf{r}_r) \cdot \mathbf{v}_r - \frac{\mu}{2}(\frac{Ze}{M} + \frac{e}{m})\{(\mathbf{B} \times \mathbf{R}_{cm}) \cdot \mathbf{v}_r + (\mathbf{B} \times \mathbf{r}_r) \cdot \mathbf{V}_{cm}\},$$

where μ is the reduced mass $\mu^{-1} = M^{-1} + m^{-1}$. The resulting equations of motions in the cms and relative frame are:

$$\frac{d}{dt}(m+M)\mathbf{V}_{cm} = (Ze-e)(\mathbf{V}_{cm}\times\mathbf{B}) - \mu(\frac{Ze}{M}+\frac{e}{m})(\mathbf{v}_r\times\mathbf{B}) \tag{3}$$

$$\frac{d}{dt}\mu\mathbf{v}_r = \mu^2(\frac{Ze}{M^2}-\frac{e}{m^2})(\mathbf{v}_r\times\mathbf{B}) - \mu(\frac{Ze}{M}+\frac{e}{m})(\mathbf{V}_{cm}\times\mathbf{B}) - \nabla\Phi(r_r)$$

Expressions (2) and (3) already offer very useful insight in the complications which here appear compared to the case without a magnetic field. In the nonmagnetic case only the first terms of the Lagrangian (2) are present and the motion separates in a cm- and a relative part in the usual way. But with magnetic field the cms motion is coupled to the relative motion, as given by the last term of L (2) and the corresponding terms in Eqs.(3), and the cm-velocity $\mathbf{V}_{cm}$ is not conserved. While the total energy E, that is, the Hamiltonian $\mathcal{H} = (m+M)V_{cm}^2/2 + \mu v_r^2/2 + \Phi(r_r) = E = E_{cm}+E_r$, is a constant of motion $(d\mathcal{H}/dt = 0)$, the cm-energy $E_{cm} = (m+M)V_{cm}^2/2$ and the relative energy $E_r = \mu v_r^2/2 + \Phi(r_r)$ are not conserved separately and the problem cannot be reduced to three relevant degrees of freedom, in general. This is, however, possible for electron-electron collisions, which are of great interest as well when discussing transport properties of a magnetized plasma. Setting formally $Z = -1, M = m, \mu = m/2$, the prefactor of the coupling term in Eq. (2) cancels and the Lagrangian takes the form $\mathcal{L}_{ee} = \mathcal{L}_{cm}(\mathbf{V}_{cm}, \mathbf{R}_{cm}) + \mathcal{L}_r(\mathbf{v}_r, \mathbf{r}_r)$. This possible reduction to a much simpler system makes a very important difference between ion-electron collisions and electron-electron collisions in a magnetic field.

As $M \gg m, \mu \to m$ and for cases where the cyclotron period of the ion is large compared to the collision time, we may also consider a simpler problem by calculating the energy transfer in an ion-electron collision under the assumption that the ion velocity is constant up to terms $O(m/M)$, i.e. $\mathbf{V} = \text{const} + O(m/M)$. Neglecting terms of $O(m/M)$, the cm-velocity $\mathbf{V}_{cm} = \mathbf{V} + O(m/M)$ remains constant as well, whereas the time evolution of the cm-energy

$$\frac{dE_{cm}}{dt} = \mathbf{V}_{cm}\cdot\frac{d}{dt}(m+M)\mathbf{V}_{cm} = -e\,\mathbf{V}\cdot(\mathbf{v}_r\times\mathbf{B}) + O\left(\frac{m}{M}\right) \tag{4}$$

is still coupled to the relative motion, which is now determined by

$$\frac{d}{dt}m\mathbf{v}_r = -\nabla\Phi(r_r) - e\,(\mathbf{v}_r\times\mathbf{B}) - e\,(\mathbf{V}\times\mathbf{B}) + O\left(\frac{m}{M}\right). \tag{5}$$

There exists, up to terms $O(m/M)$, the conserved quantity

$$K = \frac{m}{2}v_r^2 + \Phi(r_r) + e\,(\mathbf{V}\times\mathbf{B})\cdot\mathbf{r}_r \tag{6}$$

which replaces the total energy as a constant of motion in the limit $M \gg m$. The magnetic term in K is here associated with the non-conservation of E_{cm}, Eq. (4).

ENERGY TRANSFER AND ENERGY LOSS

To calculate the energy loss of an ion in a magnetized electron plasma we have first to determine the energy transfer to the ion $\Delta E_M = M(V'^2 - V^2)/2$ from the velocities v'_r, V'_{cm} after a binary ion-electron collision with given initial velocities v_r, V_{cm} (and positions r_r, R_{cm}), where $V = V_{cm} - \mu/M v_r$ and $V' = V_{cm}' - \mu/M v_r'$. In the limit $M \gg m$, the energy change $dE_M/dt = M V \cdot dV/dt$ during the collision is in leading order given by $dE_M/dt = M V_{cm} \cdot dV_{cm}/dt + m V \cdot dv_r/dt$. This results, using Eqs. (4) - (6), in an energy transfer

$$\Delta E_M = e \, (V \times B) \cdot (r_r' - r_r) - m V \cdot (v_r' - v_r) = -\frac{m}{2}(v_r'^2 - v_r^2) - m V \cdot (v_r' - v_r). \quad (7)$$

The required final velocity v_r' is now calculated both within a fully numerical approach and by perturbation theory.

The fully numerical evaluation of the energy loss is based on a classical-trajectory-Monte-Carlo (CTMC) like treatment. Here the relevant equations of motion, see Eq. (5),

$$m\frac{dv_r}{dt} = -\nabla\left[-\frac{Ze^2}{4\pi\varepsilon_0 r_r}\exp(-\frac{r_r}{\lambda})\right] - e\,(v_r \times B) - e\,(V \times B)\,, \qquad \frac{dr_r}{dt} = v_r, \quad (8)$$

are integrated numerically for a given set of initial conditions using a modified Velocity-Verlet algorithm which has been specifically designed for particle propagations in a (strong) magnetic field. It is described in detail in Ref. [6]. The ion-electron interaction Φ is modeled by a Yukawa type screened Coulomb interaction with a screening length λ, which here represents an additional external parameter. The numerical integration starts, for cases with $r_c < \lambda$, at an ion-electron distance r_r of several λ and stops when r_r exceeds this distance again after the collision. If the cyclotron radius r_c of the electron exceeds λ, this distance is enlarged by r_c. To achieve the required accuracy the constant of motion K (6) is monitored and the actual time-step is adapted continuously. The resulting relative deviations of K are of the order of $10^{-6} \ldots 10^{-5}$. The initial conditions are, besides the fixed, constant ion velocity V, the relative velocity v_r, a phase φ of the helical motion of the electron in the magnetic field and the position r_r. This initial position is related to an impact parameter b which lies in a plane perpendicular to the initial velocity of the guiding center in the relative frame $g_r(t = 0) = -V_x e_x + v_r \cdot e_z$, where we assumed $B = B e_z$ and $V = V_x e_x + V_z e_z$. See also Ref. [7] for more details. From the changes of the relative velocity, $v_r \rightarrow v_r'$, for a specific trajectory we then obtain the energy transfer $\Delta E_M(v_r, V, b, \varphi)$ as a function of the initial values v_r, V, b, φ. As the next step, an averaging over the phases φ and the impact parameters b is performed by a Monte-Carlo sampling which results in the quantity

$$\langle\, \Delta E_M(v_r, V) \,\rangle = \int d^2b \int_0^{2\pi} \frac{d\varphi}{2\pi} \Delta E_M(v_r, V, b, \varphi). \qquad (9)$$

The actual number of computed trajectories needed for $\langle \Delta E_M \rangle$ is adjusted by monitoring the convergence of the averaging procedure. Typically, around 10^5 trajectories are required for one set of parameters v_r, V, B. To finally obtain the energy loss, i.e. the energy

change of the ion per unit path length, the averaged energy transfer $\langle \Delta E_M \rangle$ is multiplied with the flux density of electrons $n|\mathbf{g}_r|$, which is directed parallel to the normal vector of the area element d^2b, and integrated over the electron distribution, that is,

$$\frac{dE}{ds} = \frac{1}{V}\frac{dE}{dt} = \frac{n}{V}\int d^3 v_e f(\mathbf{v}_e)|\mathbf{g}_r| \int d^2 b \int_0^{2\pi} \frac{d\varphi}{2\pi} \Delta E_M(\mathbf{v}_r, \mathbf{V}, b, \varphi) \tag{10}$$

The energy loss dE/ds (10) basically represents the energy change per time dE/dt as an average over the number of collisions per time and the energy transfer per collision.

In the second approach the velocity transfer $\Delta\mathbf{v}_r = \mathbf{v'}_r - \mathbf{v}_r$ is calculated in a 2nd-order perturbation treatment of the relative motion

$$m\frac{d\mathbf{v}_r}{dt} = -\nabla\left[-\frac{Ze^2}{4\pi\varepsilon_0 r_r}\right] - e\,(\mathbf{v}_r \times \mathbf{B}) - e\,(\mathbf{V} \times \mathbf{B}), \qquad \frac{d\mathbf{r}_r}{dt} = \mathbf{v}_r, \tag{11}$$

using the bare Coulomb interaction $\Phi = -Ze^2/4\pi\varepsilon_0 r_r$. The approximate solution starts with the analytically given trajectory $\mathbf{r}_0(t)$, $\mathbf{v}_0(t) = \dot{\mathbf{r}}_0(t)$ for the unperturbed electron motion, i.e. $\Phi = 0$ and

$$m\frac{d\mathbf{v}_0}{dt} + e(\mathbf{v}_0 \times \mathbf{B}) = -e(\mathbf{V} \times \mathbf{B}).$$

Using the trajectory $\mathbf{r}_0(t)$ for calculating the force between ion and electron, a 1st-order correction $\mathbf{r}_1, \mathbf{v}_1$ with

$$m\frac{d\mathbf{v}_1}{dt} + e(\mathbf{v}_1 \times \mathbf{B}) = -\nabla\Phi(r)|_{\mathbf{r}=\mathbf{r}_0}, \qquad \frac{d\mathbf{r}_1}{dt} = \mathbf{v}_1$$

provides the velocity transfer $\delta\mathbf{v} = \mathbf{v}_1' - \mathbf{v}_1$ which is $O(Z)$. In the next step a $O(Z^2)$ velocity transfer $\delta^{(2)}\mathbf{v} = \mathbf{v}_2' - \mathbf{v}_2$ is obtained from a 2nd-order correction to the unperturbed motion by taking into account the additional force due the 1st-order displacement of the trajectory $\mathbf{r}_1$. Here

$$m\frac{d\mathbf{v}_2}{dt} + e(\mathbf{v}_2 \times \mathbf{B}) = -[\nabla\Phi(r)|_{\mathbf{r}=\mathbf{r}_0-\mathbf{r}_1} - \nabla\Phi(r)|_{\mathbf{r}=\mathbf{r}_0}],$$

where the parenthesized force term is expanded up to contributions linear in $\mathbf{r}_1$.

Three regimes for the velocity transfer can be distinguished:

(i) The Coulomb field is dominant when the cyclotron radius r_c of the electrons is larger than the distance of closest approach b_0. Then, for an initial velocity v_r,

$$\delta\mathbf{v} = -\frac{Ze^2}{4\pi\varepsilon_0 m}\frac{2}{v_r b_0^2}\mathbf{b}_0 \qquad \delta^{(2)}\mathbf{v} = -\left(\frac{Ze^2}{4\pi\varepsilon_0 m}\right)^2 \frac{2}{v_r^4 b_0^2}\mathbf{v}_r \tag{12}$$

(ii) For the case of a large magnetic field with $r_c < b_0$ the transversal motion of the electron is quenched and there remains only the relative motion with respect to the guiding center, $\mathbf{g}_r = -V_x \mathbf{e}_x + \mathbf{v}_r \cdot \mathbf{e}_z$.

(a) If the pitch δ is larger than b_0 the helix is stretched and there apply the same expressions (12) with $\mathbf{v}_r \to \mathbf{g}_r$.

(b) Most interesting is the case of tight helices, where $\delta \ll b_0$. Here remains only a velocity transfer parallel to the magnetic field $\mathbf{B} = B\mathbf{e}_z$,

$$\delta \mathbf{v} = \frac{Ze^2}{4\pi\varepsilon_0 m} \frac{2V_x \cos(\psi)}{g_r^2 b_0} \mathbf{e}_z \qquad \delta^{(2)}\mathbf{v} = -\left(\frac{Ze^2}{4\pi\varepsilon_0 m}\right)^2 \frac{2V_x^2 \mathbf{g}_r \cdot \mathbf{e}_z}{g_r^6 b_0^2} \mathbf{e}_z \qquad (13)$$

provided that the ion has a velocity component V_x perpendicular to the magnetic field.

The energy loss of the ion is now obtained by integrating the energy transfer ΔE_M (7) first over an area element $\hat{\mathbf{v}}_r b_0 db_0 d\psi$ parallel to the relative current density $n\mathbf{v}_r$ and then over the velocity distribution $f(\mathbf{v}_e)$, i.e.

$$\frac{dE}{ds} = \frac{1}{V}\frac{dE}{dt} = \frac{n}{V}\int d^3 v_e f(\mathbf{v}_e)\, \mathbf{v}_r \cdot \hat{\mathbf{v}}_r\, 2\pi \int_{b_{min}}^{b_{max}} db_0 b_0 \langle \Delta E_M \rangle_\psi. \qquad (14)$$

In the ψ-averaged energy transfer $\langle \Delta E_M \rangle_\psi$ all terms $O(Z)$ vanish because of symmetry and up to $O(Z^2)$ we have

$$\langle \Delta E_M \rangle_\psi = -m\mathbf{V}\cdot\langle \delta^{(2)}\mathbf{v}\rangle_\psi - \frac{m}{2}(-2\mathbf{v}_r\cdot\langle\delta^{(2)}\mathbf{v}\rangle_\psi + \langle(\delta\mathbf{v})^2\rangle_\psi). \qquad (15)$$

The b_0 integration exceeds from $b_{min} = Ze^2/4\pi\varepsilon_0 m v_r^2$ to the screening length $b_{max} = \lambda$. Hard collisions are taken into account by regularizing the emerging b_0-integrals according to $\int_{b_{min}} db_0/b_0 \to \int_0 db_0 b_0/(b_0^2 + b_{min}^2)$. This procedure yields the exact result for the bare Coulomb case.

RESULTS

A comprehensive analysis of the energy loss of ion-electron collisions in a magnetic field is still in progress. For our current calculations we have been mainly interested in the specific situation in the electron cooler at the TSR storage ring at Heidelberg [8] and thus concentrated on the corresponding parameters, that is, an ion charge state $Z = 6$, an electron density $n = 8 \times 10^6$ cm^{-3}, magnetic field strengths around $B = 0.01$ T and an anisotropic velocity distribution $f(\mathbf{v}_e)$ of the electrons. It is modeled as a product of two Maxwell distributions for the parallel and the transversal degrees of freedom (with respect to $\mathbf{B}$), i.e.

$$f(\mathbf{v}_e) = f(v_\perp, v_\parallel) = \frac{m}{2\pi k_B T_\perp}\left(\frac{m}{2\pi k_B T_\parallel}\right)^{1/2}\exp\left(-\frac{mv_\perp^2}{2k_B T_\perp}\right)\exp\left(-\frac{mv_\parallel^2}{2k_B T_\parallel}\right) \qquad (16)$$

with temperatures $k_B T_\perp = 11.5$ meV, $k_B T_\parallel = 0.1$ meV, an anisotropy $T_\perp/T_\parallel = 115$ and respective thermal velocities $v_{th,\perp} = (k_B T_\perp/m)^{1/2} = 4.5 \times 10^4$ m/s, $v_{th,\parallel} = (k_B T_\parallel/m)^{1/2} =$

4.2×10^3 m/s, i.e. $v_{th,\perp}/v_{th,\parallel} = 10.7$ and screening lengths $\lambda_{D,\perp} = (\varepsilon_0 k_B T_\perp/e^2 n)^{1/2} = 2.8 \times 10^{-4}$ m, $\lambda_{D,\parallel} = (\varepsilon_0 k_B T_\parallel/e^2 n)^{1/2} = 2.6 \times 10^{-5}$ m. Some examples for the energy loss dE/ds as obtained from the CTMC procedure, Eqs. (8)-(10), and the 2nd-order perturbation treatment, Eqs. (11)-(15), are shown in Figs. 1 and 2 as function of the ion velocity V in units of $v_{th,\parallel}$; in Fig. 1 for a magnetic field $B = 10$ mT and different directions $\alpha = 0^o, 30^o, 60^o$ and 90° of the ion motion, and in Fig. 2 for $\alpha = 0^o, 60^o$ and various $B = 2.5$ mT, 10 mT and 40 mT, where $\cos(\alpha) = \mathbf{B} \cdot \mathbf{V}/BV$. These results are compared with the energy loss of an ion in an anisotropic electron plasma without magnetic field $(B = 0)$, where the equations of motion (8) can be solved analytically and the
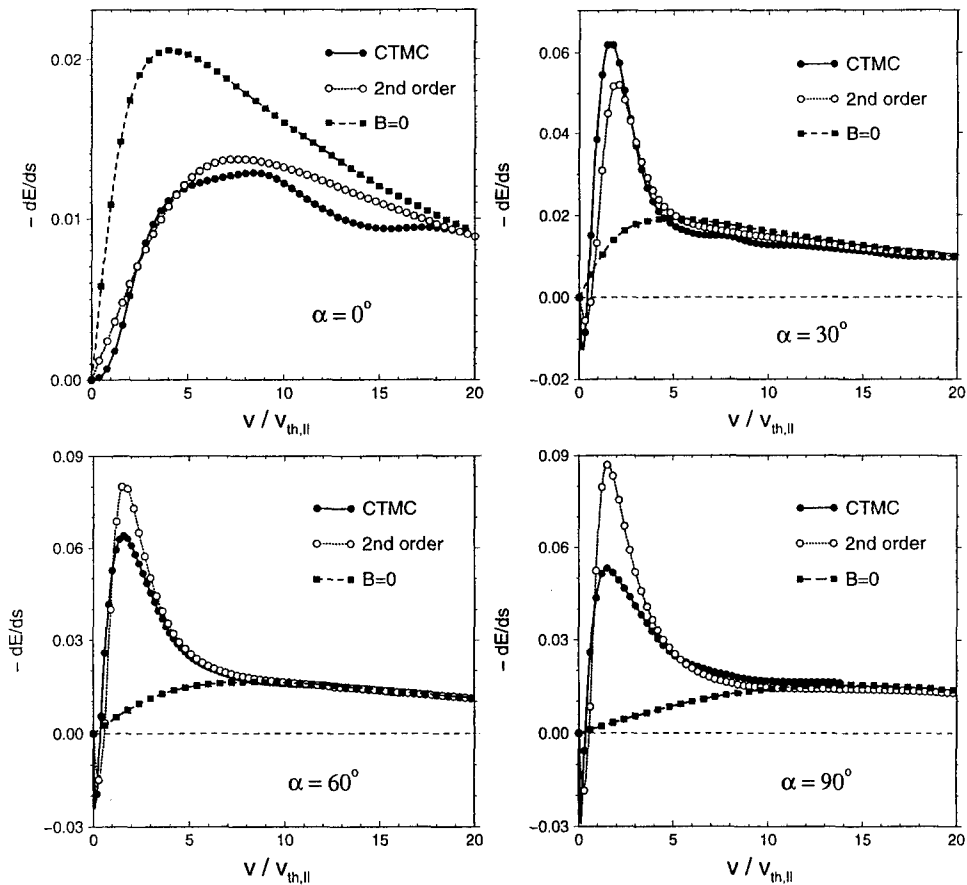

FIGURE 1. Energy loss dE/ds scaled in units of $4\pi n(Ze^2/4\pi\varepsilon_0)^2/mv_{th,\parallel}^2$ as function of the ion velocity V in units of $v_{th,\parallel} = 4.2 \times 10^3$ m/s and for different directions of the ion motion $\alpha = 0^o, 30^o, 60^o$ and 90°. Results for $B = 10$ mT as obtained from the CTMC method and the 2nd-order perturbation treatment are compared with the respective energy loss without magnetic field $(B = 0)$ [1]. In all cases, the used screening length is $\lambda = 1 \times 10^{-4}$ m and $T_\perp/T_\parallel = 115$.

energy loss (10) is thus given in closed from [1]. In all cases dE/ds is scaled in units of $4\pi n(Ze^2/4\pi\varepsilon_0)^2/mv_{th,\parallel}^2$ which equals 0.75 eV/cm for the given parameters.

For an ion motion along the magnetic field, i.e. $\alpha = 0^o$, a significant suppression of the energy loss compared to the nonmagnetized case occurs, which increases with increasing magnetic field. This feature has also been found in a perturbative treatment of the equivalent case of an ion at rest [9] and for the closely related equilibration of the transversal and longitudinal temperatures in a magnetized electron plasma by electron–electron collisions [10]. In all these case the cm-energy E_{cm} is conserved and a complete separation in cms and relative motion is possible, in contrast to the case
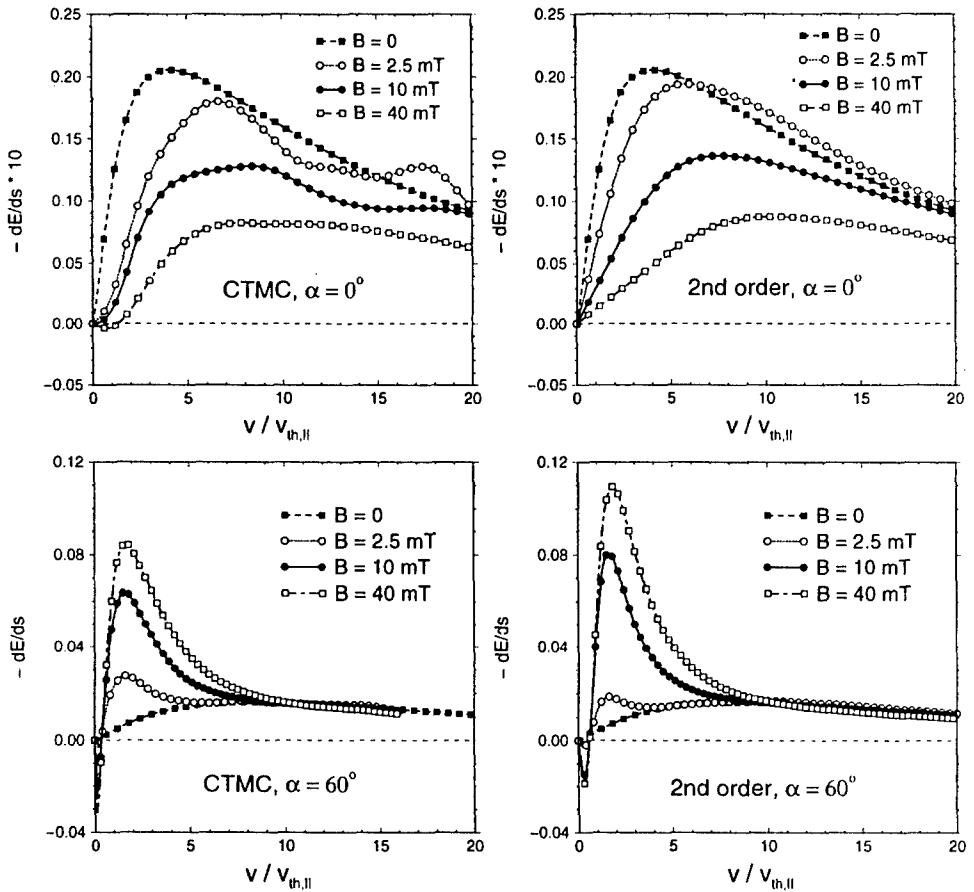

FIGURE 2. Energy loss dE/ds scaled in units of $4\pi n(Ze^2/4\pi\varepsilon_0)^2/mv_{th,\parallel}^2$ as function of the ion velocity V in units of $v_{th,\parallel} = 4.2 \times 10^3$ m/s. Shown are results of the CTMC method (left) and the 2nd-order perturbation treatment (right) for different magnetic field strength $B = 2.5$ mT, 10 mT and 40 mT and the two directions $\alpha = 0^o$ (top) and 60° (bottom) together with the analytically obtained dE/ds for $B = 0$. The screening length is again $\lambda = 1 \times 10^{-4}$ m and $T_\perp/T_\parallel = 115$.

with a transversal component of the ion motion, that is, $\alpha \neq 0^o$. Here the presence of a magnetic field causes a strong enhancement of the energy loss at low ion velocities which increases with B. For even lower velocities $V < v_{th,\parallel}$ and $\alpha \neq 0^o$ we observe negative values of the energy loss which correspond to an energy gain and thus an acceleration of the ion. Both features, the strong enhancement and the negative values, are related to the nonequilibrium situation of a highly anisotropic velocity distribution. This can be concluded from calculations of the energy loss for a much lower anisotropy of $T_\perp/T_\parallel = 4$, as presented in Fig. 3. Here almost no negative values of the energy loss appear and only rather small enhancements can be found at large angles α and very low velocities. In general a reduction of the energy loss compared to the nonmagnetized case
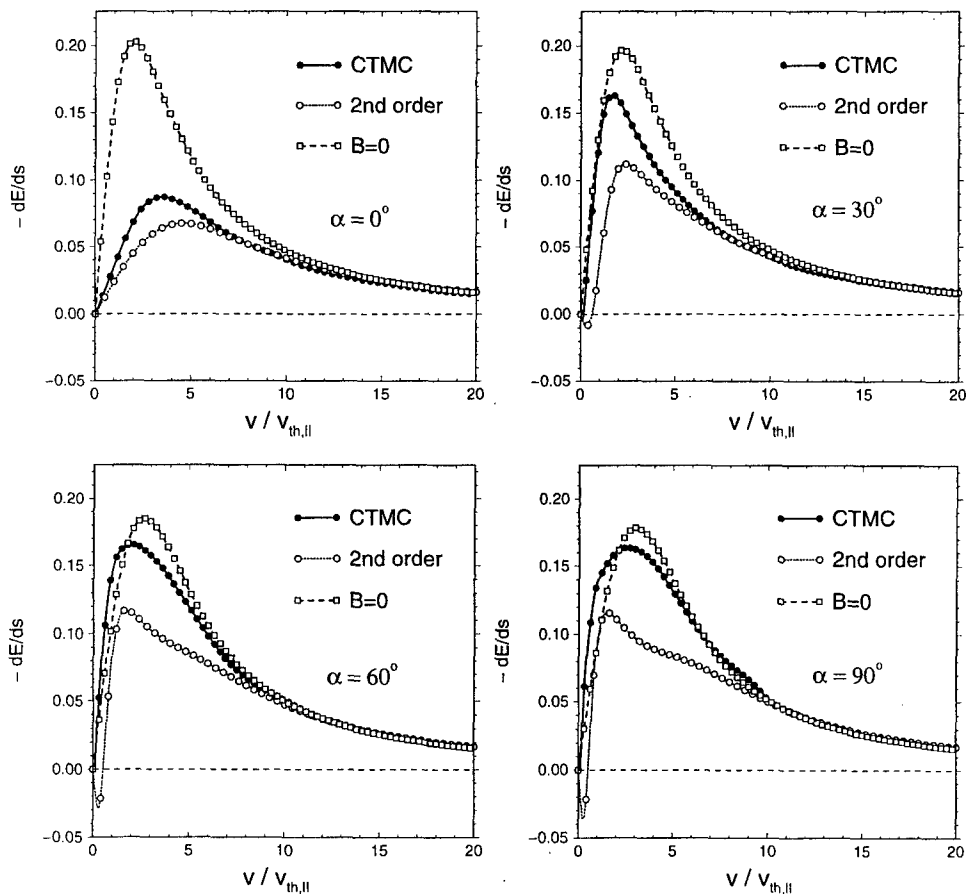

FIGURE 3. Energy loss dE/ds scaled in units of $4\pi n(Ze^2/4\pi\varepsilon_0)^2/mv_{th,\parallel}^2$ as function of the ion velocity V in units of $v_{th,\parallel} = 1.2 \times 10^4$ m/s and for different directions of the ion motion $\alpha = 0^o, 30^o, 60^o$ and 90°. Results for $B = 50$ mT as obtained from the CTMC method and the 2nd-order perturbation treatment are compared with the energy loss for $B = 0$ [1]. The used screening length is $\lambda = 2.5 \times 10^{-5}$ m and $T_\perp/T_\parallel = 4$.

is observed, which is strongest at $\alpha = 0^o$ but only very weak at $\alpha = 90^o$. Compared to the CTMC results, the 2nd-order perturbation treatment here predicts a much stronger reduction of dE/ds and much bigger negative values. It reproduces, however, very well the essential general features of the energy loss by magnetized ion-electron collisions as found by the CTMC method, in particular, in the previous case of the highly anisotropic electron distribution.

OBSERVATIONS AND CONCLUSIONS

The presented results for the energy loss by ion-electron collisions in a magnetic field, as obtained from both a fully numerical investigation and a 2nd-order perturbation treatment, give an impression of the characteristic and important effects. One essential observation is the strong directional dependence with respect to $\mathbf{B}$, i.e. on α, in particular for a highly anisotropic electron distribution with $T_\perp/T_\parallel \gg 1$. This situation is characterized by a strong reduction of dE/ds with increasing B for $\alpha = 0$, a substantial enhancement of dE/ds with increasing B for $\alpha \neq 0$, and an energy gain of the ion at $v \lesssim v_{th,\parallel}$, at variance with the case of vanishing anisotropy, $T_\perp/T_\parallel \sim 1$. There we found an overall reduction of dE/ds which increases with B and decreases with α. But in all cases, the magnetic field significantly affects the energy loss only at low ion velocities V. These features and the results of the CTMC calculations are generally well reproduced by the outlined 2nd-order perturbation treatment. In a further step, after an additional averaging over the distribution of the ion velocities, the obtained results will be compared to the experimentally measured cooling forces [8]. This work is in progress.

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