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An overview of the relationship between approximation theory and filtration

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Abstract

This paper gives an overview of the similarities and differences between the requirements and techniques used in mathematical approximation theory and filtration in surface metrology. Although the two fields tend to use the same or similar mathematical objects to produce functions that simplify a function in a controlled manner, it is the way that this simplification is achieved which is the main difference between the two. Approximation theory uses norms to judge the closeness of the approximation while filtration uses the concept of wavelength to control the "smoothness" of the result of filtration. The new ISO definition of a filter is stated, together with a generalisation of the concept of wavelength through "brickwall" filters. This new ISO definition of a filter illustrates the closeness of approximation theory and filtration. The paper then proceeds to survey some recent developments in filtration in the hope that there can be some cross-fertilisation between approximation theory and filtration. These include wavelets, robust filters and non-linear filters such as the family of morphological filters, which includes envelope filters and alternating sequence filters (non-linear multiresolution). Examples from surface texture are used throughout the paper.

1 Introduction

This paper gives an overview of the similarities and differences between the requirements and techniques used in mathematical approximation theory and filtration in surface metrology. It is not the intention of this paper to give full mathematical detail but to survey recent developments in filtration in the hope that there can be some cross-fertilisation between approximation theory and filtration.

Although the two fields tend to use the same or similar mathematical objects to produce functions that simplify the original function in a controlled manner, it is the way that this simplification is achieved which is the main difference between the two.

Mathematical approximation theory is concerned with best and good approximation of a large family of functions from a smaller set (usually finitely generated, linear or non-linear) in certain normed spaces (such as L_p), the construction of good approximants (if possible) and the determination of approximation order. Classical tools to achieve this include polynomial tools and splines. More recent tools include wavelets and multiresolution that decompose the normed spaces.

Filtration uses the concept of "wavelength" to control the "smoothness" of the result of filtration. In surface metrology, filtration is concerned with the extraction of features within a prescribed "wavelength" band defined by "wavelength cut-offs". Classical tools to achieve this include Gaussian filters [1], polynomials and splines [4]. Recently there has been a resurgence of activity, both fundamentally and practically, in filtration for surface metrology.

The International Standards Organisation Technical Committee 213 (ISO TC/213), whose remit includes surface metrology, has recently set up an Advisory Group (AG9) to explore filtration for surface metrology. They are producing a series of technical specifications (ISO/TS 16610 series [2, 3, 4, 5, 6, 7, 8, 9, 10, 11]) to standardise filter terminology and to introduce to industry other filtration tools, which include spline wavelets [5], morphological filters [9] and scale-space techniques [10].

Other groups are also producing filtration for surface metrology. The University of Huddersfield has used second generation wavelets to produce an improved spline wavelet [12]. The University of Hanover is exploring robust Gaussian filtration [6]. PTB has developed a Robust Spline filter [7]. The rest of the paper surveys some of the results of this recent activity.

2 Basic concepts of filtration

This section is a summary of the basic concepts of filtration as given in ISO/TS 16610 part 1 [2].

Let $\mathcal{P}$ be the space of real surfaces.

Let $\mathcal{V}_\lambda$ be a set of nested subspaces indexed by $\lambda \in \mathcal{R}^+$ (here $\mathcal{R}^+$ is the set of positive reals which includes zero) such that

$$\forall \lambda > \mu \geq 0; \mathcal{V}_\lambda \subseteq \mathcal{V}_\mu \subseteq \mathcal{P} \text{ and } \mathcal{V}_0 \text{ is dense on } \mathcal{P}.$$

The nesting index λ a number indicating the relative level of nesting for a particular subspace in such a way that given a particular nesting index, subspaces with lower indices contain more surface information and subspaces with higher nesting indices contain less surface information. By convention, as the nesting index approaches zero there exists a surface in that indexed subspace that approximates the real surface to within any given measure of closeness as defined by a suitable norm. Thus approximation theory is used to define Filtration. The usual norm used in filtration is L_2 but others are used such as the one-sided Chebychev for morphological filters.

Let $\Phi_\lambda : \mathcal{P} \to \mathcal{V}_\lambda$ be a projection from the space of real surfaces onto the subspace indexed by $\lambda \geq 0$ which satisfies the following two properties.

- The sieve criterion: $\forall\lambda, \mu \geq 0$ and $\forall a \in \mathcal{P}; \Phi_\lambda(\Phi_\mu(a)) = \Phi_{\sup(\lambda,\mu)}(a)$.
- The projection criterion: $\forall\lambda \geq 0$ and $\forall a \in \mathcal{V}_\lambda; \Phi_\lambda(a) = a$.

Φ_λ is called the brickwall filter (or primary mapping) and is a method of choosing a particular surface belonging to a subspace with a specified nesting index, to represent the real surface, which satisfies the projection and sieve criteria [16].

The sieve criterion allows brickwall filters to have the property that once the surface has been brickwall filtered at a particular nesting index, subsequent brickwall filtering with a higher nesting index will produce the same surface as brickwall filtering the original surface with the brickwall filter with the higher nesting index.

The projection criterion is required in order that the nesting index is a scale or size. For define the set operator $\Psi_\lambda : \mathcal{P} \rightarrow \mathcal{P}$ as

$$\forall\lambda \geq 0 \text{ and } \forall P \subseteq \mathcal{P}; \Psi_\lambda(P) := \{p : p \in P \text{ and } \Phi_\lambda(p) = p\}.$$

That is to say $p \in \Psi_\lambda(P)$ if and only if $p \in P$ and $\Phi_\lambda(p) = p$. Then it is easily demonstrated that the set operator Ψ_λ is a granulometry [16] on $\mathcal{P}$ and λ is the scale/size of the granulometry.

Since the nesting index of brickwall filters is a scale/size and it satisfies the sieve criterion, it can be used to define the generalised concept of wavelength. An example of a brickwall filter is a morphological closing filter using a sphere as the structuring element. Here the nesting index is the radius of the sphere.

Other filters can be constructed using brickwall filters (e.g. weighted mean of brickwall filters, supremum of brickwall filters, etc.).

3 Wavelet filters

An important example of the concepts discussed in the previous section is wavelet filtration. The multiresolution form of the wavelet transform consists of constructing a ladder of smooth approximations to the profile. The first rung is the original profile. Each rung in the ladder consists of a filter bank where the profile A_i is split into two components giving, a smoother version A_{i+1} of the profile which becomes the next rung and a component D_{i+1} that is the "difference" between the two rungs.

The multiresolution ladder structure lends itself naturally to a set of nested mathematical models of the profile, with the ith model m_i, reconstructed from (D_1, D_2, D_3, ..., D_i, A_i). The nesting index is the order of the model, the higher the model the smoother the representation with less detail. Thus m_{i+1} is a smoother version of the profile than m_i.

As part of a research programme at the University of Huddersfield, the use of biorthogonal wavelets for surface analysis has been investigated because of their significant merits [12]. A very fast, second-generation, in-place algorithm, which uses the lifting scheme, has been developed at Bell Laboratories for biorthogonal wavelets [13]. One important property of biorthogonal wavelets is that they allow the construction of symmetric wavelets and thus linear phase filters that preserves the location of surface features with far less distortion than phase shift filters.

Surface texture analysis usually breaks down a surface into defined wavelength components of the surface called roughness, waviness and form. There are many well-known problems with the current standardised filter [14], i.e. Gaussian filter [1], including lost data at the edges, distortion due to form, retention of unwanted wavelengths, etc.. Huddersfield has investigated the possibility of using a 'lifting wavelet' model to overcome some of these problems and enhance the extraction accuracy for roughness, waviness and form. This is achieved by using the wavelet transform to break down the surface into subsets at different scales and recombining only those subsets of the scales of interest (i.e. setting all the other subsets to zero and applying the inverse wavelet transform). Figure 1 shows the application of the wavelet filtering technique a femoral head from an artificial hip joint. Full details of the particular biorthogonal wavelet and its associated lifting scheme together with some engineering applications are given in reference [12].

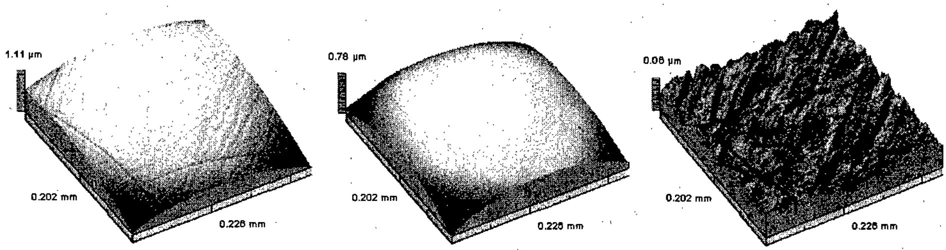

FIG. 1. Metallic femoral head showing original, reference and roughness surfaces.

4 Envelope filters

Traditional linear filters, such as the Gaussian filter [1], produce a smoothed mean surface through a measured surface. Many engineering applications of functional surfaces involve mechanical contact where the envelope of the surface is of interest rather than the mean surface. But what exactly is the envelope of a surface?

The following are defining properties of the envelope of a surface used by ISO TC/213 AG9 [8]:

- the envelope filter must be Extensive, i.e., $\forall A, F(A) \geq A$,
- the envelope filter must be Increasing, i.e., $A \leq B$ implies $F(A) \leq F(B)$,
- the envelope filter must be Idempotent, i.e., $F(F(A)) = F(A)$,

where A, B are surfaces and F(A) is the filtered surface of surface A.

But these are also the defining properties of a morphological closing filter [15]; hence all envelope filters are morphological closing filters. A morphological closing filter using a disk as the structuring element is illustrated in Figure 2.

Unfortunately, envelope filters, by definition, are not very robust to outliers, consisting of large spikes, in the surface. Scale-space is an attempt to overcome this problem with the morphological closing filter.

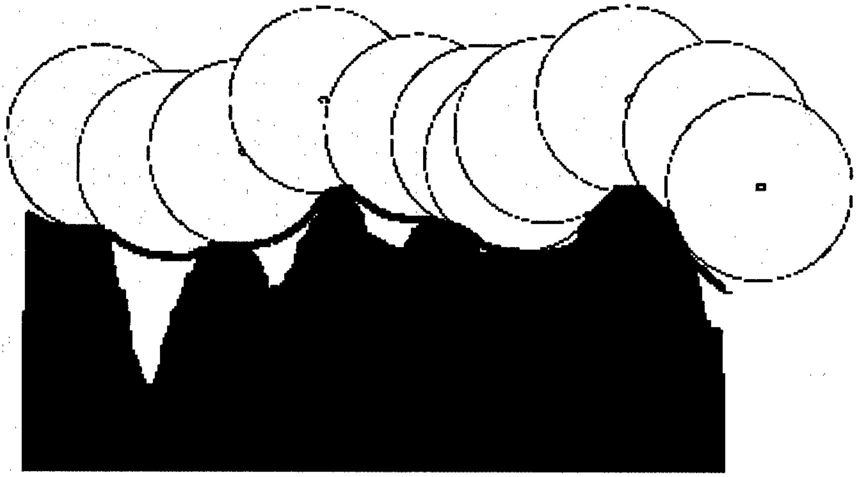

FIG. 2. An envelope filter using a closing filter with a disk as a structural element.

5 Scale-space

Scale-space is a way of breaking down a signal or image into objects of different scales. To define scale-space we need to define the size of objects in a signal or image. This is achieved using Alternating Sequence Filters [10].

Alternating Sequence Filters (ASFs) are defined in terms of matched pairs of closing and opening filters. A closing followed by an opening both at a given scale (radius of the circle, length of the horizontal segment, etc.) will eliminate features of the surface whose "scales" are smaller than the given scale.

ASFs begin by eliminating very small features, then eliminating slightly larger features, and then eliminating slightly larger features still etc., in a systematic way up to a given scale. Usually there is a constant ratio between successive scales. This process produces a ladder structure similar to wavelet analysis. At each rung in the ladder the profile is filtered by a matched pair of closing and opening filters at a given scale to obtain the next rung profile and a component that is the "difference" between the two rungs. The ladder structure leads to a multiresolution analysis, similar to wavelet analysis, with all of the associated analysis techniques. An example of scale space of a profile from a ceramic surface is given in Figure 3. The top part of this figure shows the original non-smoothed profile with the final smoothed profile.

6 Robustness

Robustness of filtration is an increasingly important area of interest in surface metrology. Robustness is not in general an absolute property of a filter but a relative one. One can only say that a particular filter is more robust than an alternative filter against a particular phenomenon if there is less distortion in that filter's response to that phenomenon than in the alternative filter's response.

To make robustness an absolute property of filters we need to define a reference class

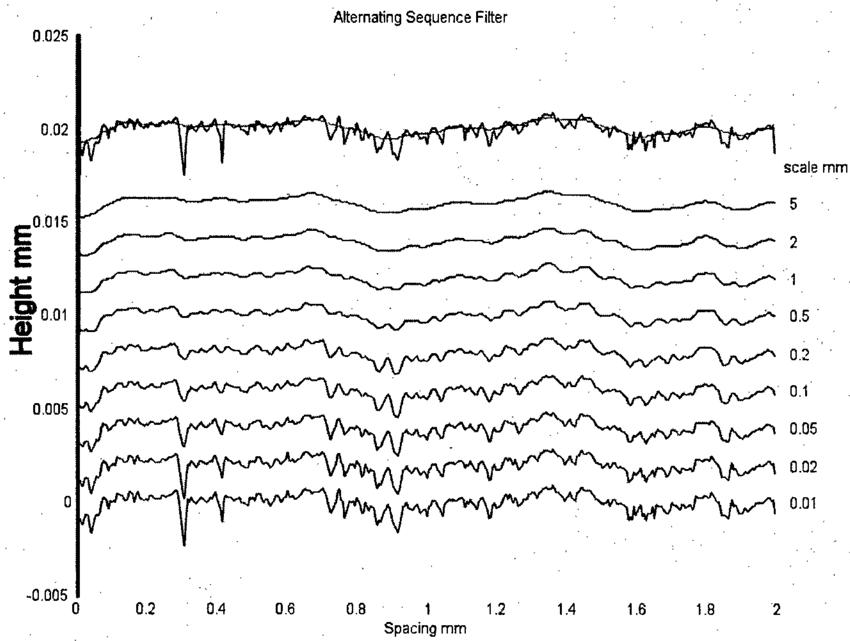

FIG. 3. Successively smooth profiles of a ceramic profile using an ASF with a disk.

of profile filters with which to compare. The reference class of filters defined in ISO TC/213 AG9 is the class of linear filters [3]. Hence by this definition all robust filters must be non-linear. There are several well-known techniques (all non-linear) which can produce robust filters for a particular phenomenon. These are indicated in the next sections.

6.1 Metric based

Here the metric used to fit the filter to the surface is altered to a more "robust" metric.

For example the metric based on the L_1 norm is more robust against spike discontinuities than the metric based on the least square norm (L_2 norm), which in turn is more robust then the metric based on the Chebychev norm (L_∞ norm).

The Robust Spline Filter given in ISO/TS 16610 part 32 uses an L_1 metric rather than the usual L_2 norm to make it more robust [7].

6.2 Robust statistics

Here each point on the surface is weighted according to its relative height position to the filter's smooth response, with points further away being given less influence on the filter response than points nearer in height. This is an attempt to make the filter more robust against spike discontinuities. There are several standard functions used to allocate the weights to points (Huber, Beaton functions, etc.) which can be found in any standard book on robust statistics [17].

The Robust Gaussian regression filter given in ISO/TS 16610 part 31 uses a Beaton function to alter the influence of outliers [6].

6.3 Pre-filtering

Pre-filtering is a technique where a phenomenon (such as spikes, form, etc.) in the surface are removed or greatly reduced, by other means, before filtration, thus removing or greatly reducing any effect the phenomenon can have on the filter's response. This approach has the advantage that once a method has been found to remove unwanted phenomenon then this method will work with any filter.

Form pre-filtering, involving removing the form of the surface before filtration, is a very common technique used in surface metrology. Less common is using scale space pre-filtering which involves removing singularities and other features of a certain size before filtration.

7 Conclusions

The paper has given an overview of the similarities and differences between the requirements and techniques used in mathematical approximation theory and filtration in surface metrology. Some recent work on filtration has been reported. It is hoped that this paper can generate some cross-fertilisation between the two areas of approximation theory and filtration.

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