SCATTERING OF ELECTROMAGNETIC WAVES BY HOMOGENEOUS DIELECTRIC GRATINGS WITH PERFECTLY CONDUCTING STRIP

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INTRODUCTION

Recently, the refractive index can easily be controlled to make the periodic structures such as optoelectronic devices, photonic bandgap crystals , frequency selective devices, and other applications by the development of manufacturing technology of optical devices[1]. Thus, the scattering and guiding problems of the inhomogeneous gratings have been considerable interest, and many analytical and numerical methods which are applicable to the dielectric gratings having an arbitrarily periodic structures combination of dielectric and metallic materials[2].

In this paper, we proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings with perfectly conducting strip using the combination of improved Fourier series expansion method[3] and point matching method[4].

METHOD OF ANALYSIS

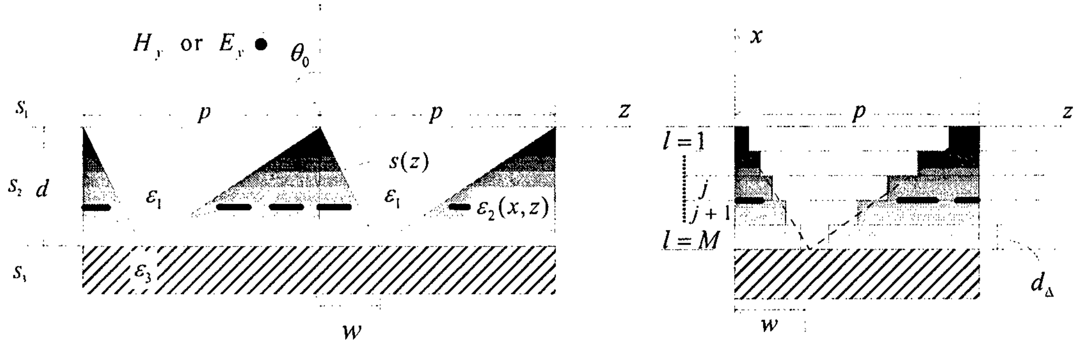
We consider inhomogeneous dielectric gratings with perfectly conducting strip as shown in Fig.1(a). The grating is uniform in the y-direction and the permittivity $\varepsilon(x,z)$ with respect to the position($= w$) is an arbitrary periodic function of z with period p.The permeability is assumed to be μ_0. The time dependence is $\exp(-i\omega t)$ and suppressed throughout. In the formulation, the TM wave is discussed. When the TM wave (the magnetic field has only the y-component)is assumed to be incident from $x>0$ at the angle θ_0, the magnetic fields in the regions $S_1\,(x\geq 0)$ and $S_3\,(x\lesssim -d)$ are expressed[3] as

$$\underline{S_1(x\geq 0)\ :}\quad H_y^{(1)}=e^{ik_1(z\sin\theta_0-x\cos\theta_0)}+e^{ik_1z\sin\theta_0}\sum_{n=-N}^{N}r_n^{(1)}e^{i\left(k_n^{(1)}x+2\pi nz/p\right)}\ ;\ k_1\triangleq\omega\sqrt{\varepsilon_1\mu_0} \tag{1}$$

$$\underline{S_3(x\leq -d)\ :}\quad H_y^{(3)}=e^{ik_1z\sin\theta_0}\sum_{n=-N}^{N}t_n^{(3)}e^{-i\left\{k_n^{(3)}(x+d)-2\pi nz/p\right\}} \tag{2}$$

$$k_n^{(j)}\triangleq\sqrt{k_0^2\varepsilon_j/\varepsilon_0-\gamma_n^2}\ ;\ \gamma_n\triangleq k_1\sin\theta_0+2\pi n/p\,,\ k_0\triangleq 2\pi/\lambda\,,\ j=1,3\,.$$

where $r_n^{(1)}$,and $t_n^{(3)}$ are unknown coefficients to be determined by boundary conditions.



(a)Coordinate system, (b) Approximated inhomogeneous layers.
Fig.1 Structure of inhomogeneous dielectric grating with perfectly conducting strip.

Main process of our method to treat these problems is as follows (see Fig.1(b)):
(1) First, the grating layer ($-d < x < 0$) is approximated by an assembly of M stratified layer of modulated index profile with step size $d_\Delta (\triangleq d/M)$ approximated to step index profile $\varepsilon^{(l)}(z)[\triangleq \varepsilon((l+0.5)d_\Delta, z);\ l = 1 \sim M]$, and the magnetic fields are expanded appropriately by a finite Fourier series.

$$\underline{S_2(-d<x<0)\ :}\ H_y^{(l,2)} = \sum_{\nu=1}^{2N+1}\left[A_\nu^{(l)} e^{ih_\nu^{(l)}\{x+(l-1)d_\Delta\}} + B_\nu^{(l)} e^{ih_\nu^{(l)}(x+ld_\Delta)}\right] e^{ik_1 z\sin\theta_0} \sum_{n=-N}^{N} u_{\nu,n}^{(l)} e^{i2\pi nz/p} \tag{3}$$

where $h_\nu^{(l)}$ is the propagation constant in the x-direction.We get the following eigenvalue equation in regard to $h^{(l)}$ [3] .

$$\Lambda_1 \mathbf{U}^{(l)} = \left\{ h^{(l)} \right\}^2 \Lambda_2 \mathbf{U}^{(l)}\ ;\ \Lambda_1 \triangleq \left[\eta_{m,n}^{(l)}\right],\ \Lambda_2 \triangleq \left[\zeta_{m,n}^{(l)}\right],\ l = 1 \sim M\ , \tag{4}$$

where, $\mathbf{U}^{(l)} \triangleq \left[u_{-N}^{(l)}, \cdots, u_0^{(l)}, \cdots u_N^{(l)}\right]^T$, T : *transpose* ,

$$\zeta_{n,m}^{(l)} \triangleq k_0^2 \xi_{n,m}^{(l)} - \gamma_n \left\{\gamma_n \eta_{n,m}^{(l)} + 2\pi(n-m)\eta_{n,m}^{(l)}/p\right\}, \gamma_n^{(l)} \triangleq \left(k_1 \sin\theta_0 + 2\pi n/p\right), \tag{5}$$

$$\eta_{n,m}^{(l)} \triangleq \frac{1}{p}\int_0^p \left\{\varepsilon^{(l)}(z)\right\} e^{i2\pi(n-m)z/p} dz\ ,\ \xi_{n,m}^{(l)} \triangleq \frac{1}{p}\int_0^p \left\{\varepsilon^{(l)}(z)\right\}^2 e^{i2\pi(n-m)z/p} dz\ ,\ \ m,n = -N,\cdots,0,\cdots N\ .$$

For the TM case, the permittivity profile approximated by a Fourier series of N_f terms[3] and N_f is related to the modal truncation number N($N = 1.5N_f$) [3]..

(2) Second, the strip region ($j < l < j+1$), see Fig.1(b), we obtain the matrix form combination of metallic region C and the dielectric region $\overline{C}$ using boundary condition at the matching points $z_k (= pk/(2N+1), k = 0, \cdots 2\mathrm{N})$ on $x = -l \cdot d_\Delta$ ($l = j$).Boundary condition are as follows:

$$z_k \in C : \left[E_z^{(2,j)} = E_z^{(2,j+1)} = 0\right] \sum_{\nu=1}^{2N+1} h_\nu^{(j)} \left[A_\nu^{(j)} e^{ih_\nu^{(j)} d_\Delta} - B_\nu^{(j)}\right] \cdot \sum_{n=-N}^{N} u_{n,\nu}^{(j)} e^{inz_k} = 0, \tag{6}$$

$$\sum_{\nu=1}^{2N+1} h_\nu^{(j+1)} \left[A_\nu^{(j+1)} + B_\nu^{(j+1)} e^{ih_\nu^{(j+1)} d_\Delta}\right] \cdot \sum_{n=-N}^{N} u_{n,\nu}^{(j+1)} e^{inz_k} = 0$$

$$z_k \in \overline{C} : \left[H_y^{(2,l)} = H_y^{(2,l+1)}\right]$$

$$\sum_{\nu=1}^{2N+1} \left[A_\nu^{(j)} e^{ih_\nu^{(j)} d_\Delta} + B_\nu^{(j)}\right] \cdot \sum_{n=-N}^{N} u_{n,\nu}^{(j)} e^{inz_k} = \sum_{\nu=1}^{2N+1} \left[A_\nu^{(j+1)} + B_\nu^{(j+1)} e^{ih_\nu^{(j+1)} d_\Delta}\right] \cdot \sum_{n=-N}^{N} u_{n,\nu}^{(j+1)} e^{inz_k} \tag{7}$$

$$z_k \in \overline{c}\left[E_z^{(2,j)} = E_z^{(2,j+1)}\right]: \quad \frac{1}{\varepsilon^{(j)}(z)} \sum_{\nu=1}^{2N+1} h_\nu^{(j)} \left[A_\nu^{(j)} e^{ih_\nu^{(j)} d_\Delta} - B_\nu^{(j)}\right] \cdot \sum_{n=-N}^{N} u_{n,\nu}^{(j)} e^{inz_k} =$$

$$\frac{1}{\varepsilon^{(j+1)}(z)} \sum_{\nu=1}^{2N+1} h_\nu^{(j+1)} \left[A_\nu^{(j+1)} + B_\nu^{(j+1)} e^{ih_\nu^{(j+1)} d_\Delta}\right] \cdot \sum_{n=-N}^{N} u_{n,\nu}^{(j+1)} e^{inz_k} \tag{8}$$

In the Eq.(8), the boundary condition at $E_z^{(2,j)} = E_z^{(2,j+1)}$, it is satisfied in all matching points. Therefor, rearranging after multiplying both sides $\varepsilon_j(z) \cdot \varepsilon_{j+1}(z)$ in Eq.(8) by using the orthogonality properties of $\{e^{i2\pi nz/p}\}$,we get following equation.

$$\sum_{\nu=1}^{2N+1} h_\nu^{(j)} \left[A_\nu^{(j)} e^{ih_\nu^{(j)} d_\Delta} - B_\nu^{(j)}\right] \cdot \phi_{n,\nu}^{(j)} = \sum_{\nu=1}^{2N+1} h_\nu^{(j+1)} \left[A_\nu^{(j+1)} - B_\nu^{(j+1)} e^{ih_\nu^{(j+1)} d_\Delta}\right] \cdot \psi_{n,\nu}^{(j+1)}\ , \tag{9}$$

where $\quad \phi_{n,\nu}^{(j)} \triangleq \sum_{m=-N}^{N} u_{\nu,m}^{(j)} \eta_{m,n}^{(j+1)} \quad C\psi_{n,\nu}^{(j+1)} \triangleq \sum_{m=-N}^{N} u_{\nu,m}^{(j+1)} \eta_{m,n}^{(j)},\ \ n = -N, \cdot\cdot, 0, \cdot\cdot N\ .$

By using matrix algebra in Eq.(9), we get following matrix form.

$$\mathbf{\Phi}^{(j)} \mathbf{C}^{(j)} [\mathbf{D}^{(j)} \mathbf{A}^{(j)} - \mathbf{B}^{(j)}] = \mathbf{\Psi}^{(j+1)} \mathbf{C}^{(j+1)} [\mathbf{A}^{(j+1)} - \mathbf{D}^{(j+1)} \mathbf{B}^{(j+1)}], \tag{10}$$

where $\mathbf{\Phi}^{(j)} \triangleq \left[\phi_{n,\nu}^{(j)}\right], \mathbf{\Psi}^{(j+1)} \triangleq \left[\psi_{n,\nu}^{(j+1)}\right], \mathbf{C}^{(j)} \triangleq [h_\nu^{(j)} \cdot \delta_{(n+N+1),\nu}], \mathbf{D}^{(j)} \triangleq [e^{ih_\nu^{(j)} d_\Delta} \cdot \delta_{(n+N+1),\nu}], \delta_{(n+N+1),\nu}$:

Kronecker's delta.We get following matrix form combined with Eq.(6) and Eq.(7).

$$\mathbf{H}'^{(j)}[\mathbf{D}'^{(j)}\mathbf{A}^{(j)}+\mathbf{D}''^{(j)}\mathbf{B}^{(j)}]=\mathbf{H}'^{(j+1)}[\mathbf{A}^{(j+1)}+\mathbf{D}'^{(j+1)}\mathbf{B}^{(j+1)}] \quad @ \quad @ \quad @ \quad @ \quad @ \quad @ \quad @ \quad @(11)$$

$$\mathbf{H}'^{(j)}=\begin{bmatrix} \vdots & \vdots & \vdots & & \vdots & \vdots \\ e^{-iNzk} & \cdots & e^{i0zk} & \cdots & & e^{iNzk} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ e^{-iNzk} & \cdots & e^{i0zk} & \cdots & & e^{iNzk} \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} z_k \in c \\ \left.\begin{matrix} \\ \end{matrix}\right\} z_k \in \overline{c} \end{matrix} \bullet \mathbf{U}^{(j)}, \quad \begin{bmatrix} \mathbf{D}'^{(j)} \triangleq [\eta e^{ih_\nu^{(j)}d_\Delta} \cdot \delta_{(n+N+1),\nu}], \\ \eta = h_\nu^{(j)}\ ;\ z_k \in c,\ \ \eta = 1\ ;\ z_k \in \overline{c} \\ \mathbf{D}''^{(j)} \triangleq [\eta \cdot \delta_{(n+N+1),\nu}] \\ \eta = -h_\nu^{(j)}\ ;\ z_k \in c\ ,\ \eta = 1\ ;\ z_k \in \overline{c} \end{bmatrix}$$

$$\mathbf{H}'^{(j+1)}=\begin{bmatrix} \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & \cdots & 0 & \cdots & & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ e^{-iNzk} & \cdots & e^{i0zk} & \cdots & & e^{iNzk} \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} z_k \in c \\ \left.\begin{matrix} \\ \end{matrix}\right\} z_k \in \overline{c} \end{matrix} \bullet \mathbf{U}^{(j+1)}, \ \mathbf{D}'^{(j+1)} \triangleq [h_\nu^{(j+1)} e^{ih_\nu^{(j+1)}d_\Delta} \cdot \delta_{(n+N+1),\nu}]$$

(3) Finally, we obtain the relationship between $\mathbf{A}^{(1)},\mathbf{B}^{(1)}$ and $\mathbf{A}^{(M)},\mathbf{B}^{(M)}$.

$$\begin{pmatrix}\mathbf{A}^{(1)}\\ \mathbf{B}^{(1)}\end{pmatrix} \triangleq \begin{pmatrix}\mathbf{S}_1^{(1)} & \mathbf{S}_2^{(1)}\\ \mathbf{S}_3^{(1)} & \mathbf{S}_4^{(1)}\end{pmatrix} \cdot \begin{pmatrix}\mathbf{S}_1^{(2)} & \mathbf{S}_2^{(2)}\\ \mathbf{S}_3^{(2)} & \mathbf{S}_4^{(2)}\end{pmatrix} \cdots \begin{pmatrix}\mathbf{S}_1^{(j)} & \mathbf{S}_2^{(j)}\\ \mathbf{S}_3^{(j)} & \mathbf{S}_4^{(j)}\end{pmatrix} \cdot\cdot \begin{pmatrix}\mathbf{S}_1^{(M)} & \mathbf{S}_2^{(M)}\\ \mathbf{S}_3^{(M)} & \mathbf{S}_4^{(M)}\end{pmatrix} \begin{pmatrix}\mathbf{A}^{(M)}\\ \mathbf{B}^{(M)}\end{pmatrix} = \begin{pmatrix}\mathbf{S}_1 & \mathbf{S}_2\\ \mathbf{S}_3 & \mathbf{S}_4\end{pmatrix} \begin{pmatrix}\mathbf{A}^{(M)}\\ \mathbf{B}^{(M)}\end{pmatrix} \quad (12)$$

where $l \neq j\ \mathbf{S}_k^{(l)} \triangleq \left[{}^{(l)}s_{n,\nu}^{(k)}\right]\ k=1\sim 4,\ l=1\sim M$,

$${}^{(l)}s_{n,\nu}^{(1)} \triangleq \frac{1}{2}\left[\nu_{n,\nu}^{(l)} + \theta_{n,\nu}^{(l)} h_{n+N+1}^{(l+1)} / h_\nu^{(l)}\right] e^{-ih_\nu^{(l)}d_\Delta}\ {}^{(l)}s_{n,\nu}^{(2)} \triangleq {}^{(l)}s_{n,\nu}^{(3)} \cdot e^{i\left\{h_{n+N+1}^{(l+1)} - h_\nu^{(l)}\right\} d_\Delta},$$

$${}^{(l)}s_{n,\nu}^{(3)} \triangleq \left[\nu_{n,\nu}^{(l)} - \theta_{n,\nu}^{(l)} h_{n+N+1}^{(l+1)} / h_\nu^{(l)}\right]/2,\ {}^{(l)}s_{n,\nu}^{(4)} \triangleq \frac{1}{2}\left[\nu_{n,\nu}^{(l)} + \theta_{n,\nu}^{(l)} h_{n+N+1}^{(l+1)} / h_\nu^{(l)}\right] e^{ih_{n+N+1}^{(1)}d_\Delta}$$

$$\mathbf{V} \triangleq \left[\mathbf{v}_{n,\nu}^{(l)}\right] = \left[\mathbf{U}^{(l)}\right]^{-1} \cdot \left[\mathbf{U}^{(l+1)}\right], \quad \Theta \triangleq \left[\theta_{n,\nu}^{(l)}\right] = \left[\Phi^{(l)}\right]^{-1} \cdot \left[\Psi^{(l+1)}\right],$$

$$l = j \quad \mathrm{F}\mathbf{S}_k^{(j)} \triangleq \left[{}^{(j)}s_{n,\nu}^{(k)}\right] \quad \mathrm{C}^{(l)}s_{n,\nu}^{(1)} \triangleq \left[\nu_{n,\nu}^{(1)} + d_{n,\nu}^{(j)}\right]^{-1} \cdot \left[\nu_{n,\nu}^{(2)} + \theta_{n,\nu}\right],\ D^{(j)} \triangleq \left[d_{n,\nu}^{(j)}\right],$$

$${}^{(j)}s_{n,\nu}^{(2)} \triangleq \left[\nu_{n,\nu}^{(1)} + d_{n,\nu}^{(j)}\right]^{-1} \cdot \left[\nu_{n,\nu}^{(3)} - \theta_{n,\nu} d_{n,\nu}^{(J+1)}\right] \quad \mathrm{C}^{(j)}s_{n,\nu}^{(3)} \triangleq \left[V_1^{-1} + \mathrm{D}^{-1(j)}\right]^{-1} \cdot \left[V_1^{-1}V_2 - \mathrm{D}^{-1(j)}\theta\right],$$

$${}^{(j)}s_{n,\nu}^{(4)} \triangleq \left[V_1^{-1} + \mathrm{D}^{-1(j)}\right]^{-1} \cdot \left[V_1^{-1}V_3 - \mathrm{D}^{-1(j)}\theta \mathrm{D}^{(j+1)}\right] \quad \mathrm{G}\mathbf{V}_1 \triangleq \left[\nu_{n,\nu}^{(1)}\right] = \left[\mathrm{H}'^{(j)}\mathrm{D}''^{(j)}\right]^{-1} \cdot \left[\mathrm{H}'^{(j)}\mathrm{D}'^{(j)}\right] \quad \mathrm{C}$$

$$\mathbf{V}_2 \triangleq \left[\nu_{n,\nu}^{(2)}\right] = \left[\mathrm{H}'^{(j)}\mathrm{D}''^{(j)}\right]^{-1} \cdot \left[\mathrm{H}'^{(j+1)}\mathrm{D}'^{(j+1)}\right], \mathbf{V}_3 \triangleq \left[\nu_{n,\nu}^{(3)}\right] = \left[\mathrm{H}'^{(j)}\mathrm{D}''^{(j)}\right]^{-1} \cdot \left[\mathrm{H}'^{(j+1)}\mathrm{D}'^{(j+1)}\right]$$

$$\Theta \triangleq \left[\theta_{n,\nu}^{(j)}\right] = \left[\mathrm{C}^{(j)}\right]^{-1} \cdot \left[\Phi^{(j)}\right]^{-1} \cdot \left[\Psi^{(j+1)}\right] \cdot \left[\mathrm{C}^{(j+1)}\right].$$

Using Eq.(12), we get the following homogeneous matrix equation in regard to $A^{(M)}$.

$$\mathbf{W} \cdot \mathbf{A}^{(M)} = \mathbf{F}, \quad \mathbf{W} \triangleq \lfloor \mathbf{Q}_1\mathbf{S}_1 + \mathbf{Q}_2\mathbf{S}_3 - (\mathbf{Q}_1\mathbf{S}_2 + \mathbf{Q}_2\mathbf{S}_4)\mathbf{Q}_4^{-1}\mathbf{Q}_3 \rfloor, \quad (13)$$

The mode power transmission coefficients $|T_n^{(TM)}|^2$ is given by

$$|T_n^{(TM)}|^2 \triangleq \varepsilon_1 \operatorname{Re}\{\ k_n^{(3)}\}|t_n^{(3)}|^2 / \left(\varepsilon_3 k_0^{(1)}\right) \quad (14)$$

CONCLUSION

In this paper, we have proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings with perfectly conducting strip using the combination of improved Fourier series expansion method and point matching method.

REFERENCES

[1] *Tamir, T., ed.,* Integrated Optics. Springer-Verlag, - p.110-118, 1979.

[2] *Tamir.T.M.Jiang and K.M.Leung.*,Modal Transmission-Line Theory of Composite Periodic Structures:I.Multilayerd Lamellar Gratings,URSI International Symposium on Electromagnetic Theory,MIN-T2.25,pp.332-334, 2001.

[3] *Yamasaki, T., Tanaka, T., Hinata, T. and Hosono, T.,* Analysis of Electromagnetic Fields Inhomogeneous Dielectric Gratings with Periodic Surface Relief, Radio Science, Vol.31, No.6, pp.1931-1939, 1996.

[4]*Yamasaki, T., Hinata, T. and Hosono, T.,* Scattering of Electromagnetic waves by Plane Grating with Reflector, IECE Trans. in Japan., Vol.J61-B, No.11. p.953-961, 1978 (in Japanese). [translated Scripta Technica, INC., Vol.61, No.11 pp.52-60,1978.