TITLE: Mathematical Models of Electromagnetic Wave Scattering by Two-Element Strip Grating with a Perpendicularly Magnetized Gyrotropic Medium

MATHEMATICAL MODELS OF ELECTROMAGNETIC WAVE SCATTERING BY TWO-ELEMENT STRIP GRATING WITH A PERPENDICULARLY MAGNETIZED GYROTROPIC MEDIUM

Y.V. Gandel, V.V. Khoroshun
Kharkov National University
Pl. Svobody 4, Kharkov 61077, Ukraine
E-mail: gandel@ilt.kharkov.ua

ABSTRACT

Two mathematical models are proposed for analyzing a linearly polarized plane wave scattering by a strip grating placed on isotropic-gyrotropic media interface in the case of oblique incidence. the first mathematical model is based on reducing the original boundary value problem to the Riemann-Hilbert problem. The second model is based on reducing the same problem to a singular integral equation of the first kind with Cauchy kernel and its numerical solving by the discrete singularities method.

In this paper, the results obtained in [1-3] are generalized for the case of periodic structure consisting of two strips of different widths per period (two-element grating). This results in richer diffraction phenomena in comparison to simple grating because of additional control parameters. Moreover, unlike papers [1-3] here the case of oblique incidence of an H-polarized plane wave on a two-element grating is considered. The center of coordinate system is chosen in the middle of one of the strips.

The following set of dual series equations is mathematical model of a structure

$$\sum_n A_n\gamma_{n1}e^{in\varphi}=\gamma_{01},\quad \theta_m<\varphi<\theta_m^{(1)}; \tag{1}$$

$$\sum_n A_n r_n e^{in\varphi}=-p_0,\quad \theta_m^{(1)}<\varphi<\theta_{m+1}^{(1)}, \tag{2}$$

where $\gamma_{n1}=\sqrt{k_0^2n_1^2-h_n^2}$, $n_1=\sqrt{\varepsilon_1\mu_1}$, $h_n=k_0n_1\sin\zeta+\dfrac{2\pi}{l}n$,

$$r_n=1+\frac{\gamma_{n1}}{\varepsilon_1(R\gamma_{n2}-iLh_n)},\qquad \gamma_{n2}=\sqrt{k_0^2\varepsilon_\perp\mu_\parallel-h_n^2},\quad \varepsilon_\perp=\frac{\varepsilon^2-\varepsilon_a^2}{\varepsilon}$$

$$R=\varepsilon_\perp^{-1},\quad L=-\frac{\varepsilon_a}{\varepsilon^2-\varepsilon_a^2},\qquad k_0=\frac{2\pi}{\lambda},$$

$$p_0=1-\frac{\gamma_{01}}{\varepsilon_1(R\gamma_{02}-iLh_0)},\ \varphi=\frac{2\pi}{l}y,\ \theta_m=2\pi\frac{y_m}{l},\quad (m=1,2),$$

l is the grating period, d is the slot width, λ is the wavelength, ζ is the incidence angle.
Denote $\widetilde{A}_n=A_nr_n+p_0\delta_{0n}$, and δ_{0n} for the Kronecker symbol.
Then initial set of dual series equations takes the form:

$$\sum_n \widetilde{A}_n e^{in\varphi}=0,\qquad \theta_m<\varphi<\theta_m^{(1)}; \tag{3}$$

$$\sum_{n>0} n\widetilde{A}_n \frac{\eta_{n1}}{r_n^+} e^{in\varphi} + \sum_{n<0} |n| \widetilde{A}_n \frac{\eta_{n1}}{r_n^-} e^{in\varphi} = (1 - \frac{\widetilde{A}_0 - p_0}{r_0}) \kappa_1 \cos\zeta ,$$

$$\theta_m^{(1)} < \varphi < \theta_{m+1}^{(1)}, \tag{4}$$

$$\sum_{n \neq 0} \widetilde{A}_n e^{in\delta_m} = -\widetilde{A}_0 , \qquad \theta_m^{(1)} < \delta_m < \theta_{m+1}^{(1)}, \tag{5}$$

The set of equations (3)-(5) can be reduced to non-homogeneous conjugation problem (Riemann-Hilbert's problem) with a complex-valued coefficient in the case of account of dissipative losses.

To calculate matrix elements of final matrix equation, it is necessary to introduce polynomials $Q_n(u_m, \rho)$ [3], where $\rho = \frac{\ln|G|}{2\pi}$, $u_m = \cos\theta_m$, (m=1,2).

In the case of multi-element gratings, an efficient numerical-analytical method for solving these dual series equations was suggested in [4]. The method consists in reducing them to a singular integral equation of the first kind with the Cauchy kernel on the set of segments, and its following solution by the method of discrete singularities [4,5]. Integral equation is of the following form

$$\frac{1}{\pi} \int_L \frac{F(\xi)}{\xi - x} d\xi + \frac{1}{\pi} \int_L K(x,\xi) F(\xi) d\xi = f(x), \quad x \in L \tag{6}$$

where $L = \bigcup_{q=1}^{m} (a_q, b_q)$, $-\infty < a_1 < b_1 < \ldots < a_m < b_m < +\infty$;

$f(x), x \in \overline{L}$; $K(x,\xi), x \in \overline{L}, \xi \in \overline{L}$ are known smooth functions, and function $F(\xi), \xi \in L$ is sought in the functional class whose restriction on interval (a_q, b_q):

$$F_q(\xi) = F(\xi), \qquad a_q < \xi < b_q, q = 1, \ldots, m$$

can be represented in the form

$$F_q(\xi) = \frac{v_q(\xi)}{\sqrt{(\xi - a_q)(b_q - \xi)}}, \quad a_q < \xi < b_q,$$

where $v_q(\xi)$, $\xi \in [a_q, b_q]$ is a smooth function.

The sought function $F(\xi), \xi \in L$ satisfies additional conditions, which in general case are of the following form:

$$\frac{1}{\pi} \int_L S_p(\xi) F(\xi) d\xi = C_p, \quad p = 1, \ldots, m, \tag{7}$$

where $S_p(\xi)$, $\xi \in [a_p, b_p]$ is a known smooth function, and C_p is a known constant.

In conclusion we shall present the discrete mathematical model that is a set of linear algebraic equations for numerical solution of the integral equation (6) with additional condition (7).

Denote

$$t_i^n = \cos\frac{2i-1}{2n}\pi, \quad i = 1, \ldots, n; \qquad t_{0j}^n = \cos\frac{j}{n}\pi, \quad j = 1, \ldots, n-1;$$

$$g_k(\tau)=\frac{b_k-a_k}{2}\tau+\frac{b_k+a_k}{2};\quad \xi_{qi}^{n_q}=g_q(t_i^{n_q}),\quad i=1,\dots,n_q;\,q=1,\dots,m$$

$$\chi_{pj}^{n_p}=g_p(t_{0j}^{n_p}),\quad j=1,\dots,n_p-1;\,p=1,\dots,m$$

To calculate approximate values $\left\{v_{qn_q}(\xi)\right\}_{q=1}^{m}$ of the desired functions $v_q(\xi)$, $q=1,\dots,m$ in principal points $\left\{t_i^{n_q}\right\}_{i=1}^{n_q}$, we have a set of linear algebraic equations (where $R(x,\xi)=\dfrac{1}{\xi-x}+K(x,\xi)$)

$$\sum_{q=1}^{m}\sum_{i=1}^{n_q}R(\chi_{pj}^{n_p},\xi_{qi}^{n_q})v_{qn_q}(\xi_{qi}^{n_q})\frac{1}{n_q}=f(\chi_{pj}^{n_p}),\qquad j=1,\dots,n_p-1;\quad p=1,\dots,m,$$

$$\sum_{i=1}^{n_p}S_p(\xi_{pi}^{n_p})v_{pn_p}(\xi_{pi}^{n_p})\frac{1}{n_p}=C_p,\quad (j=n_p),\quad p=1,\dots,m$$

The values of the physical characteristic of scattered field,

$$H=\int_L H(\xi)F(\xi)d\xi=\sum_{q=1}^{m}\int_{a_q}^{b_q}H_q(\xi)v_q(\xi)\frac{d\xi}{\sqrt{(\xi-a_q)(b_q-\xi)}}.$$

are expressed in terms of the functions $v_q(\xi)$, $\xi\in[a_q,b_q]$, $q=1,\dots,m$,
where $H_q(\xi)$, $\xi\in[a_q,b_q]$ are known functions.
Approximate values of

$$H_{\bar n}=\sum_{q=1}^{m}\sum_{i=1}^{n_q}H_q(\xi_{qi}^{n_q})v_{qn_q}(\xi_{qi}^{n_q})\frac{1}{n_q},\quad \bar n=(n_1,\dots,n_m)$$

are calculated in numerical experiments.
Obtained results can be applied in the design and elaboration of various devices containing periodic structures with ferrite substrates or in plasma.

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