TITLE: Dispersive and Diffraction Analysis of Integrated Periodic Waveguide Structure

# DISPERSIVE AND DIFFRACTION ANALYSIS OF INTEGRATED PERIODIC WAVEGUIDE STRUCTURE

T. I. Bugrova
Radio Engineering Department of Zaporozhye National Technical University
Bildg.64, Zhukovsky Street, Zaporozhye 69063, Ukraine
Tel.+38 (0612) 643281 fax 642141 e-mail: bugrova@zstu.edu.ua

## ABSTRACT

A semi - infinite homogenious nonmagnetic shielded slab with grating of thin metallic strips that are printed symmetrically on both sides of the slab was considered. The main wave of the dielectric slab falls under arbitrary angle on the boundary between slab and grating. The dispersive equation of periodic structure was obtained and solved by numerical-analytical method. The diffraction problem was formulated and solved by Viener-Hopf technique.

## THE EIGENWAVE PROBLEM

Let us consider a homogeneous nonmagnetic dielectric shielded slab of thickness 2d. Gratings of thin metallic strips with $2l \ll \lambda$ width, where $\lambda$ is a wavelength in slab medium, are printed symmetrically on both sides of the slab. Coordinates are assumed as shown in Fig.1.The eigenwaves of the structure considered in our case are assumed to be solutions of a boundary value problem for an electromagnetic field. These solutions exibit a harmonic dependence on the x axis of $\exp(-j\chi x)$ and quasiperiodic dependence on the z axis of type $E(z)=E(z+nP)\exp(j\beta nP)$, where $\chi$ and $\beta$ are spectral parameters determining the wave propagation direction, and n is the strip number. The electric field amplitude time dependence $\exp(j\omega t)$ is omitted for simplification. An electric or magnetic wall can be placed in the structure symmetry plane y=0. In the present article we shall limit ourselves to analysis of waves corresponding to the magnetic wall case. Because of thinness of the grating strips, the longitudinal components of currents are much more then cross components. So we can use only one boundary condition for the formulation of the problem. It is assumed that $E_x=0$ for perfectly conducting metallic strips.

Let us obtain the approximate dispersion equation. This equation couples the structure spectral parameters $\chi$ and $\beta$with nonspectral ones: $P/\lambda$, $2d/\lambda$, $2l/\lambda$, and $\varepsilon_r$ – relative dielectric permittivity of the slab. Keeping in mind the equal spacing of the grating, let us set periodic conditions for the strip currents at $I_{xn}=I_0\exp(-j\beta nP)$, where $I_0$ – current density on zero strip. It is known that the cross-strip current distribution is given by the Maxwell function $(1-(z'/l)^2)^{1/2}$ [1]. Taking into account the above approximation and using the boundary conditions, we shall formulate the integral equation as

$$\int_{-\infty}^{\infty}\int_{-l}^{l} G(x,x';z,z')I(x)\exp(-j\beta nP)(1-(z'/l)^2)^{-1/2}dx'dz'=0, \quad (1)$$

where

n=0,±1,±2,...- the strip number. The function G can be obtained by the Fourier integral

$$G(x,x';z,z')=\int_{-\infty}^{\infty} g(\xi,\alpha)\exp(-j\xi(x-x')-j\alpha(z-z'))d\xi d\alpha, \quad (2)$$

where $g(\xi,\alpha)$ is a known function. Let us evaluate an integral on $\alpha$ in (2) according to Cauchy theorem. An integral on $x'$ in (1) is evaluated trivially, assuming the current variability along the strip to be $I(x')=\exp(-j\chi x')$. It is equal to $2\pi\delta(\xi-\chi)$, where the symbol $\delta$ denotes the Dirac delta function. To avoid a $z'$ dependence in (1) let us use the Galerkin method. As a rule in a slab two modes are propagating ones. They are the $TE_1$ and $TM_2$ modes with $\alpha_1= \alpha_h$ and $\alpha_2= \alpha_e$. The term with n=0 is calculated numerically. As a result we obtain a rather simple relationship reducing the integrals on $z'$ and n $\alpha$ in (1) and (2) to a double series. Evaluating an integral on $\xi$ we obtain

$$ZD_eD_h + j\chi_e k_0 \beta_h^{-1} \sin(\chi_h P) D_e + j\chi\beta_e^{2} k_0^{-2} \eta_s^{2} \chi_e^{-1} \sin(\chi_e P) D_h = 0, \tag{3}$$

where $D_{e,h} = \cos(\chi_{e,h}P) - \cos(\beta P)$, $\eta_s$, $Z$ are definite functions of $\chi$, and $\chi_{e,h}=(\beta_{e,h}^2-\chi^2)^{1/2}$. For the grating considered, as a rule, single – mode conditions are not satisfied because even in a shielded slab without a grating $TE_1$ and $TM_2$ waves exist. In the grating they are converted into $HE_{11}$ and $EH_{21}$ modes with similar structure. However, by proper selection of structure parameters, we can create the situation where only the main $HE_{11}$ mode propagates. Cutoff conditions for $HE_{12}$ and $EH_{21}$ modes were obtained from the dispersion equation

$$\beta_{HE_{12}}^{cut} = P^{-1} \arccos\left[ \frac{k_0}{\beta_h Z(0)} \sin(\beta_h P) + \cos(\beta_h P)) \right]; \beta_{EH_{21}}^{cut} = \beta_e .$$

## THE DIFFRACTION PROBLEM

The diffraction problem was solved by Viener-Hopf technique. A semi-infinite homogenious nonmagnetic shielded slab with grating of thin metallic strips that are printed symmetrically on both sides of the slab was considered. The main wave of the dielectric slab falls under arbitrary angle on the boundary between slab and grating. The transfer strip current approximation was taken as Maxwell function. The longitudinal component of strip current was found from the integral equation formulated for boundary conditions. Using the field expression through the Green's function G and using zero boundary conditions we obtain an integral equation:

$$\int I_t(s')\, Z(s-s')\, ds' + E_t(s) = 0, \tag{4}$$

where $E_t$ and $I_t$ are the electrical field and current components that are tangential to the strip axes. The function $Z(s - s')$ can be defined from the function $G$.The evaluation of an integral in (4) is carried out along the strip axes. Using the Galerkin technique and the transfer current approximation by the Maxwell function, we succeeded in obtaining a one-dimensional equation from the two-dimensional one. It is convenient to solve equations like (4) by using the Viener-Hopf technique [1]. Hence, we can obtain an expression for $I_t(s)$ in the Fourier integral form. Evaluating the residue of the integrand at the point $\alpha= \chi_h$, coincides with the root corresponding to the $HE_{11}$ wave, and describes the current component of our interest ($I_{tHE11}$).

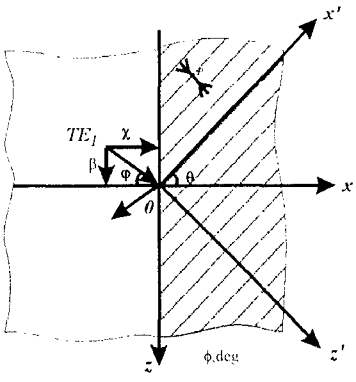


Fig.1

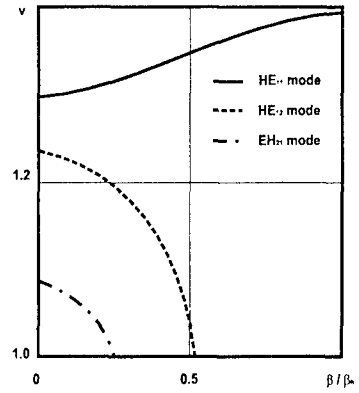


Fig.2

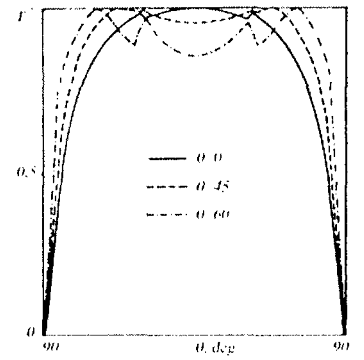

Fig.3

## NUMERICAL RESULTS

Dispersive equation (3) was solved numerically. Figure 3 shows variation of the $HE_{11}$-wave longitudinal normalized propagation constant $v=\chi_0/k_0$ versus cross-strip normalized transverse propagating constant $\beta/\beta_h$ for relative dielectric permittivity $\varepsilon_r=9.8$. It is shown that when $\beta=0$ (longitudinal wave propagation) the retardation factor is equal to that of a dielectric slab $H_1$ – wave for any structure parameters, because in this case an electric field has no longitudinal components ($E_x=0$) and is not perturbed by thin grating strips. If the direction of wave propagation varies ($\beta\neq0$), the retardation factor increases. It is characteristic for the dependence of $HE_{12}$ and $EH_{21}$ – wave factors on $\beta/\beta_h$ for various structure periods that from the onset of some critical P value, the $HE_{12}$ – wave retardation factor decreases rapidly.

Due to solving the diffraction problem we can obtain the conversion factor $T^2$ of an $H_l$ wave into an $HE_{11}$ wave as a ratio of wave powers transmitted normally to boundary. In Figure 2 the dependences of $T^2$ $(\phi)$ for various θ angles and the period $P/\lambda$ = 0.45 are shown. High values of $T^2$ is seen in the range of angles $\phi$ from -45° to +45°. At some angular points $T^2(\phi)$ is equal to unity. It takes place when the incidence angle φ is equal to the angle between the direction normal to an array boundary and strip axes θ. The main-mode incident wave does not interact with the grating. We observe complete transmission for $\phi$ = - θ. In this case the transmitted wave is perturbed by the grating strips. This effect is similar to the whole transmission with the Brewster angle in the case of wave diffraction on the boundary between the two dielectric media. In some angles the $T^2(\phi)$ dependence has sharp breaks. They appear when $E_2$ and $EH_{21}$ waves become nonpropagating ones, turning into an attenuating mode from a propatating one. This is characteristic for so-called Wood's anomalies, when the derivative on θ for transmission factor and for reflectivity is striving to infinity at some points.

The proposed structure can be used as a basis for integrated beam-forming networks for multibeam antennas.

## REFERENCES

[1] R. Mittra, S. W. Lee, Analytical Techniques in the Theory of Guided Waves. McMillan, New York, London, 1971.