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## SYMBOLIC COMPUTATION TECHNIQUES FOR APERTURE ANTENNAS

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### ABSTRACT

Symbolic Computations are very important since to get closed formula solutions in many applications. One of the computer code is MACSYMA that is written in program language LISP for the performing symbolic and numeric mathematical manipulations. The purpose of this paper is to present a number of MACSYMA applications that show how the new MACSYMA possibilities can be used in electromagnetics. To understand the procedure easily, rectangular aperture antenna analysis has been studied and the results have been illustrated.

### SYMBOLIC COMPUTATION OF A RECTANGULAR APERTURE

The analysis of apertures begins by considering the radiation from the elemental area oriented in the  $x=0$  plane as shown in Figure 1. The elemental area is part of some arbitrary aperture bounded by the curve C. The spherical coordinates of the elemental area is  $(r', \pi/2, \theta')$  and the fields are to be evaluated at the point  $P(r, \phi, \theta)$ .

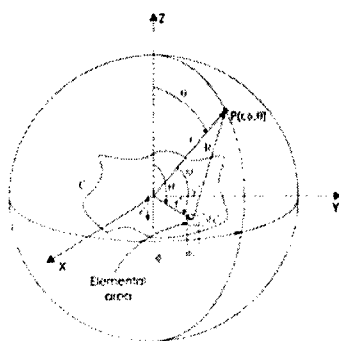


Figure 1. Elemental area

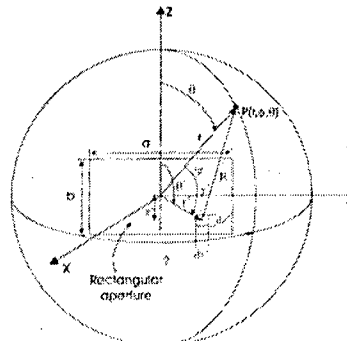


Figure 2. Rectangular aperture

Since the elemental area being analyzed can be excited by both electric and magnetic fields, It is convenient to use both the magnetic and electric vector potentials[2]. Therefore, the magnetic (**A**) and electric (**F**) vector potentials become, respectively

(c1) `(declare([a,v,y],m,n,v,e,n,e1,h1), nonscalar), depends([n,v], [rp,phip,thetap]))$`

(c2) `la: vect_express(mu*exp(-%i*k*r)*n/(4*%pi*r),spherical);`

(d2)

$$\left[ \frac{\mu e^{-ikr} N_r}{4\pi r}, \frac{\mu N_\phi e^{-ikr}}{4\pi r}, \frac{\mu e^{-ikr} N_\theta}{4\pi r} \right]$$

(c3) `lf: vect_express(epsilon*exp(-%i*k*r)*v/(4*%pi*r));`

(d3)

$$\left[ \frac{\epsilon e^{-ikr} L_r}{4\pi r}, \frac{\epsilon L_\phi e^{-ikr}}{4\pi r}, \frac{\epsilon e^{-ikr} L_\theta}{4\pi r} \right]$$

where **N** and **L** are the radiation vectors. The far electric field from the electric and magnetic vector potentials becomes

(c4) `le: vect_express(-%i*w*la-%i*w/k^2*grad(div(la))+curl(lf)/epsilon)$`

(c5) `(le:ev(le,diff),le:expand(le));`

(d5)

$$\left[ \begin{aligned} & \frac{\mu e^{-ikr} N_\theta \cos(\theta) w}{4\pi k^2 r^2 \sin(\theta)} + \frac{i\mu e^{-ikr} N_\theta \cos(\theta) w}{2\pi k^2 r^3 \sin(\theta)} + \frac{i\mu e^{-ikr} N_r w}{2\pi k^2 r^3} - \frac{L_\phi e^{-ikr} \cos(\theta)}{4\pi r^2 \sin(\theta)} \\ & \frac{ik e^{-ikr} L_\theta}{4\pi r} - \frac{i\mu N_\phi e^{-ikr} w}{4\pi r} \\ & \frac{i\mu e^{-ikr} N_\theta \cos^2(\theta) w}{4\pi k^2 r^3 \sin^2(\theta)} - \frac{i\mu e^{-ikr} N_\theta w}{4\pi r} + \frac{i\mu e^{-ikr} N_\theta w}{4\pi k^2 r^3} - \frac{ik L_\phi e^{-ikr}}{4\pi r} \end{aligned} \right]$$

Since the far field distance is large, terms which vary inversely with the distance can be ignored compared to the other terms so that for the far electric field

(c6) `(le:[0,le[2],part(le,3,[2,4])],le:subst(mu=k*z0/w,le),le:undistrib(le));`

(d6)

$$\left[ 0, \frac{ik e^{-ikr} (L_\theta - N_\phi Z_0)}{4\pi r}, -\frac{ik e^{-ikr} (N_\theta Z_0 + L_\phi)}{4\pi r} \right]$$

A rectangular aperture of finite dimensions can be analyzed in terms of the elemental area. Consider an aperture in the  $x=0$  plane with sides of lengths  $a$  and  $b$  in the  $y$  and  $z$  directions, respectively, as shown in Figure 2. Let the electric field be aligned with the  $y$  axis and the magnetic field be aligned with the  $z$  axis to give a plane wave traveling in the  $x$  direction. If the aperture is uniformly illuminated, the electric field is constant in amplitude and phase over the aperture. For this case, the electric and the magnetic surface current densities are

(c7) `(n:[1,0,0], le:[0,le0,0], h1:[0,0,le0/az0],j: vect_express(mu-h1), m: vect_express(-n-le1))$`

The radiation vectors for the rectangular aperture become

(c8) `(rp:[0,yp,zp], ar:[sin(theta)*cos(phi),sin(theta)*sin(phi),cos(theta)])$`

(c9) `in: integrate(integrate(y*exp(%i*k*vect_express(ar,rp)),yp,-a/2,a/2),zp,-b/2,b/2)$`

Is  $k \sin(\phi) \sin(\theta)$  zero or nonzero?

nonzero

(c10) `lf: integrate(integrate(m*exp(%i*k*vect_express(ar,rp)),yp,-a/2,a/2),zp,-b/2,b/2)$`

Is  $k \sin(\phi) \sin(\theta)$  zero or nonzero?

''

(c11) `t:[sin(theta)*cos(phi),-sin(phi),cos(theta)*cos(phi);sin(theta)*sin(phi),cos(phi),sin(phi)*cos(theta);cos(theta),0,-sin(theta)]$`

(c12) `{n:n,t,v:v,y}$`

The far electric field vector is obtained as

(c13)  $\text{le}:=\{0, \%i^{\wedge}k^{\wedge}r \exp(-\%i^{\wedge}k^{\wedge}r) \sqrt{4/3} \sqrt{2} \sqrt{z0} / (4^{\wedge} \%pi^{\wedge}3 r), -\%i^{\wedge}k^{\wedge}r \exp(-\%i^{\wedge}k^{\wedge}r) \sqrt{4/3} \sqrt{2} \sqrt{z0} + \sqrt{4/3} \sqrt{2} \sqrt{z0} \%pi^{\wedge}3 r\} / S$

(c14)  $\{ \text{le}::\text{expand}(\text{le}), \text{le}::\text{trigexpand}(\text{le}), \text{le}::\text{expand}(\text{le}), \text{le}::\text{undistrib}(\text{le}) \};$

$$(d14) \left[ \begin{array}{l} i e^{-i k r} \left( \frac{\cos(\phi)}{\sin(\theta)} + 1 \right) \sin\left(\frac{b k \cos(\theta)}{2}\right) \sin\left(\frac{a k \sin(\phi) \sin(\theta)}{2}\right) E_0 \\ 0, \frac{\pi k \sin(\phi) r \cos(\theta)}{\pi k r \sin(\theta)} \\ i e^{-i k r} \sin\left(\frac{b k \cos(\theta)}{2}\right) \sin\left(\frac{a k \sin(\phi) \sin(\theta)}{2}\right) E_0 \\ \pi k r \sin(\theta) \end{array} \right]$$

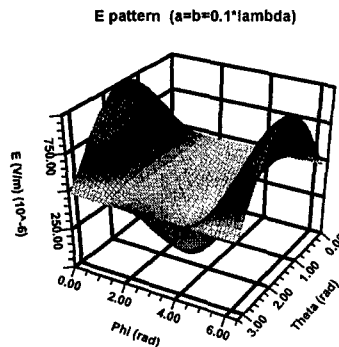
The electric field pattern is

(c15)  $(e:\text{sqrt}(\text{abs}(\text{part}(\text{le}, 2))^{\wedge}2 + \text{abs}(\text{part}(\text{le}, 3))^{\wedge}2), e:\text{trigsimp}(e))\$$

(c16)  $\text{define}(\text{le}(\text{phi}, \text{theta}), \text{subst}(\text{part}(\text{e}, 1, 1, 1) = (1 + \cos(\text{phi}))^{\wedge}2 \sin(\text{theta}))^{\wedge}2, \text{e}))\$$

(c17)  $(\text{lambda}:1, \text{le}0:1, \text{r}:10^{\wedge} \%lambda, \text{k}:2^{\wedge} \%pi / \%lambda, \text{a}:1^{\wedge} \%lambda, \text{b}:1^{\wedge} \%lambda) \$$

(c18)  $\text{plot3d}(\text{le}(\text{phi}, \text{theta}), \text{theta}, .0001, \%pi-.0001, \text{phi}, .0001, 2^{\wedge} \%pi-.0001, "Theta (rad)", "Phi (rad)", "E pattern (a=b=0.1^{\wedge} \%lambda)"), \text{zlabel}:"E (V/m)", \text{plotnumb}=50, \text{plotnum}=50, \text{plot\_style}=\text{scientific3d} \$$



## CONCLUSION

Symbolic computation results of a uniformly illuminated rectangular aperture have been obtained and the electric field pattern have been illustrated as a numerical example. So, how the symbolic computation techniques can be applied to electromagnetics has been shown.

## REFERENCES

- [1] "Macsyma Mathematics and System Reference Manual", 16<sup>th</sup> ed., Macsyma, Inc., USA, 1996.
- [2] WOLFF, Edward A., "Antenna Analysis", John Wiley & Sons, Inc., USA, 1966.