Defense Technical Information Center Compilation Part Notice

ADP013932

TITLE: Galerkin Method for Solving of Singular Integral Equation of Diffraction Problem

DISTRIBUTION: Approved for public release, distribution unlimited

This paper is part of the following report:

TITLE: 2002 International Conference on Mathematical Methods in Electromagnetic Theory [MMET 02]. Volume 2

To order the complete compilation report, use: ADA413455

The component part is provided here to allow users access to individually authored sections of proceedings, annals, symposia, etc. However, the component should be considered within the context of the overall compilation report and not as a stand-alone technical report.

The following component part numbers comprise the compilation report:
ADP013889 thru ADP013989

Galerkin Method for Solving of Singular Integral Equation of Diffraction Problem*

Smirnov Yu.G., Tsupak A.A.

1 The statement of the diffraction problem

Let $P = \{x : 0 \le x_1 \le a, 0 \le x_2 \le b, 0 \le x_3 \le c\}$ be a resonator with perfectly conducting boundary. Let Q be a three-dimensional body, located in P. Q is characterized by tensor permittivity $\hat{\varepsilon}$ and constant permeability μ_0. We suppose that components of $\hat{\varepsilon}$ are smooth functions in $\bar{Q}$ and $\left(\frac{\hat{\varepsilon}}{\varepsilon_0} - \hat{I}\right)$ is invertible in $\bar{Q}$; $Q \cap \partial P = \emptyset$. Let $P/\bar{Q}$ be homogeneous and isotropic medium. Incident and diffraction fields depend on time variable as $e^{-i\omega t}$.

We will find electromagnetic diffraction fields E and H, satisfying Maxwell's equations in $P \setminus \partial Q$:

$$\begin{aligned} rot\ \vec{H} &= -i\omega\hat{\varepsilon}\vec{E} + \vec{j}_E^{\,0} \\ rot\ \vec{E} &= i\omega\mu\vec{H} - \vec{j}_H^{\,0} \quad . \end{aligned} \tag{1}$$

The complete field should have continuous tangent components at ∂Q :

$$\left[\vec{n} \times \vec{E}^c\right]\Big|_{\partial Q} = \left[\vec{n} \times \vec{H}^c\right]\Big|_{\partial Q} = 0$$

and must satisfy the following boundary condition:

$$\vec{E}_\tau^c|_{\partial P} = 0 \ . \tag{2}$$

2 Integro-differential equations for the diffraction problem

We will express the solution of the stated problem in terms of vector potentials $\vec{A}_E$ and $\vec{A}_H$ [4]:

$$\begin{aligned} &\vec{A}_E = \int\limits_Q \hat{G}_E(x,y)\vec{j}_E(y)dy\ , \vec{A}_H = \int\limits_Q \hat{G}_H(x,y)\vec{j}_H(y)dy\ , \\ &\vec{E} = i\omega\mu_0\vec{A}_E - \tfrac{1}{i\omega\varepsilon_0} grad\ div\ \vec{A}_E - rot\ \vec{A}_H\ , \\ &\vec{H} = i\omega\,\varepsilon_0\,\vec{A}_H - \tfrac{1}{i\omega\mu_0} grad\ div\ \vec{A}_H + rot\ \vec{A}_E\ . \end{aligned} \tag{3}$$

Here $\vec{j}_E = \vec{j}_E^{\,0} + \vec{j}_E^{\,p}$, $\vec{j}_H = \vec{j}_H^{\,0} + \vec{j}_H^{\,p}$, ($\vec{j}_E^{\,p}$, $\vec{j}_H^{\,p}$ are polarization currents). $\hat{G}_E, \hat{G}_H$ are Green functions for Helmholtz equation, conforming to the arbitrary currents $\vec{j}_E^{\,0}, \vec{j}_H^{\,0}$.

$\hat{G}_E, \hat{G}_H$ are known [3] to have the form of diagonal tensors (the components of $\hat{G}_E$ are written out below):

$$\begin{aligned} G_E^1 &= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} \tfrac{2\,\varepsilon_n}{ab\gamma \mathrm{sh}\gamma c} \cos(\tfrac{\pi n}{a}x_1)\sin(\tfrac{\pi m}{b}x_2)\cos(\tfrac{\pi n}{a}y_1)\sin(\tfrac{\pi m}{b}y_2) \begin{cases} \mathrm{sh}\gamma x_3\ \mathrm{sh}\gamma(c-y_3),\ x_3 < y_3 \\ \mathrm{sh}\gamma y_3\ \mathrm{sh}\gamma(c-x_3),\ x_3 > y_3 \end{cases} \\ G_E^2 &= \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} \tfrac{2\,\varepsilon_m}{ab\gamma \mathrm{sh}\gamma c} \sin(\tfrac{\pi n}{a}x_1)\cos(\tfrac{\pi m}{b}x_2)\sin(\tfrac{\pi n}{a}y_1)\cos(\tfrac{\pi m}{b}y_2) \begin{cases} \mathrm{sh}\gamma x_3\ \mathrm{sh}\gamma(c-y_3),\ x_3 < y_3 \\ \mathrm{sh}\gamma y_3\ \mathrm{sh}\gamma(c-x_3),\ x_3 > y_3 \end{cases} \\ G_E^3 &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \tfrac{4}{ab\gamma \mathrm{sh}\gamma c} \sin(\tfrac{\pi n}{a}x_1)\sin(\tfrac{\pi m}{b}x_2)\sin(\tfrac{\pi n}{a}y_1)\sin(\tfrac{\pi m}{b}y_2) \begin{cases} \mathrm{ch}\gamma x_3\ \mathrm{ch}\gamma(c-y_3),\ x_3 < y_3 \\ \mathrm{ch}\gamma y_3\ \mathrm{ch}\gamma(c-x_3),\ x_3 > y_3 \end{cases} \end{aligned} \tag{4}$$

*supported by Russian Foundation for Basic Research, grant 01-01-00053

Here $\gamma = \sqrt{\left(\frac{\pi n}{a}\right)^2 + \left(\frac{\pi m}{b}\right)^2 - k_0^2}$ (the proper branch for square root is chosen as in [2], §2.3), $\varepsilon_0 = 1$ and $\varepsilon_n = 2$ for $n = 1, 2, 3, \dots$.

We can obtain the following integro-differential equations (under the condition $\hat{\mu} = \mu_0 \hat{I}$ in P):

$$\vec{E}(x) = \vec{E}^0(x) + k_0^2 \int\limits_Q \hat{G}_E \left[\frac{\hat{\varepsilon}(y)}{\varepsilon_0} - \hat{I}\right] \vec{E}(y)dy + grad\ div \int\limits_Q \hat{G}_E \left[\frac{\hat{\varepsilon}(y)}{\varepsilon_0} - \hat{I}\right] \vec{E}(y)dy\ ,$$

and we have (5)

$$\vec{H}(x) = \vec{H}^0(x) - i\omega\varepsilon_o\, rot \int\limits_Q \hat{G}_E \left[\frac{\hat{\varepsilon}(y)}{\varepsilon_0} - \hat{I}\right] \vec{E}(y)dy,\ x \in Q.$$

We can extract singularity of Green function $\hat{G}$. Using Fourier transformation and interpolation polynomials we can obtain:

$$\hat{G}_E(x,y) = \frac{1}{4\pi}\frac{e^{ik_0|x-y|}}{|x-y|} \cdot \hat{I} + diag\{g_1(x,y), g_2(x,y), g_3(x,y)\},$$

where g_k are smooth functions.

3 Galerkin method

Let us introduce the following auxilary function

$$\tilde{G}(x,y) = -\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{4}{ab\gamma \mathrm{sh}\gamma c} \sin(\tfrac{\pi n}{a}x_1)\sin(\tfrac{\pi m}{b}x_2)\sin(\tfrac{\pi n}{a}y_1)\sin(\tfrac{\pi m}{b}y_2)\times \\ \times \begin{cases} \mathrm{sh}\gamma \mathrm{x}_3\ \mathrm{sh}\gamma(\mathrm{c}-\mathrm{y}_3),\ \mathrm{x}_3 < \mathrm{y}_3 \\ \mathrm{sh}\gamma \mathrm{y}_3\ \mathrm{sh}\gamma(\mathrm{c}-\mathrm{x}_3),\ \mathrm{x}_3 > \mathrm{y}_3 \end{cases}. \tag{6}$$

The derivatives of $\tilde{G}$ are connected to the derivatives of G_E^i through the equalities:

$$\frac{\partial G_E^i}{\partial x_i} = \frac{\partial \tilde{G}}{\partial y_i}, \quad i = 1,2,3. \tag{7}$$

Before describing the method itself we should make some transformations of equation (5). Denoting $\left(\frac{\hat{\varepsilon}(x)}{\varepsilon_0} - \hat{I}\right)^{-1}$ as $\hat{\xi}$ and $\left(\frac{\hat{\varepsilon}(x)}{\varepsilon_0} - \hat{I}\right)\vec{E}$ as $\vec{J}$ we obtain the following equation

$$A\vec{J} := \hat{\xi}\vec{J}(x) - k_0^2 \int\limits_Q \hat{G}_E \vec{J}(y)dy - grad\ div \int\limits_Q \hat{G}_E \vec{J}(y)dy = \vec{E}_0(x) \tag{8}$$

We can write vector equation (8) as a system of three scalar equations:

$$\sum_{i=1}^{3} \xi_{li} J^i(x) - k_0^2 \int\limits_Q G_E^l(x,y) J^l(y)dy - \frac{\partial}{\partial x_l} div_x \int\limits_Q \hat{G}(x,y)\vec{J}(y)dy = E_0^l(x), \quad l = 1,2,3. \tag{9}$$

We will determine the components of approximate solution $\vec{\bar{J}}$ in the following way:

$$\bar{J}^1 = \sum_{k=1}^{N} a_k f_k^1(x),\ \bar{J}^2 = \sum_{k=1}^{N} b_k f_k^2(x),\ \bar{J}^3 = \sum_{k=1}^{N} c_k f_k^3(x), \tag{10}$$

where f_k^i are basis "hat"-functions dependent essentially on x^i. The explicit form of f_k^1 is given below.

Let Q be a parallelepiped: $Q = \{x : a_1 \le x^1 \le a_2, b_1 \le x^2 \le b_2, c_1 \le x^3 \le c_2\}$, $Q \subset P$. We will cover Q with smaller parallelepipeds

$$\Pi^1_{klm} = \{x : x^1_{k-1} \le x^1 \le x^1_{k+1},\ x^2_l \le x^2 \le x^2_{l+1},\ x^3_m \le x^3 \le x^3_{m+1}\} \\ x_k^1 = a_1 + \frac{a_2 - a_1}{n}k,\ x_l^2 = b_1 + 2\frac{b_2 - b_1}{n}l,\ x_m^3 = c_1 + 2\frac{c_2 - c_1}{n}m; \tag{11}$$

where $k = 1, \dots, n-1$; $\quad l, m = 0, 1, \dots, \frac{n}{2} - 1$.

Denoting $(x_k - x_{k-1})$ as h^1 we get the formulas for f^1_{klm} :

$$f^1_{klm} = \begin{cases} \frac{x^1 - x^1_{k-1}}{x^1_k - x^1_{k-1}}, \; if \; x^1 \in [x^1_{k-1}; x^1_k] \; and \; x \in \Pi^1_{klm} \\ \frac{x^1_{k+1} - x^1}{x^1_{k+1} - x^1_k}, \; if \; x^1 \in [x^1_k; x^1_{k+1}] \; and \; x \in \Pi^1_{klm} \\ 0, \qquad if \; x \notin \Pi^1_{klm} \end{cases} \tag{12}$$

or

$$f^1_{klm} = \begin{cases} 1 - \frac{1}{h^1}|x^1 - x^1_k|, \; if \; x \in \Pi^1_{klm} \\ 0, \qquad if \; x \notin \Pi^1_{klm} \end{cases} . \tag{13}$$

Functions f^2_{klm} and f^3_{klm} should be determined by similar formulas. Since

$$f^1_{klm}|_{x^1 \in \{x^1_{k-1}, x^1_{k+1}\}} = 0, \; f^2_{klm}|_{x^2 \in \{x^2_{l-1}, x^2_{l+1}\}} = 0, \; f^3_{klm}|_{x^3 \in \{x^3_{m-1}, x^3_{m+1}\}} = 0 \; , \tag{14}$$

every component of approximate vector solution vanishes at some side of Q. However the constructed set of basis functions does satisfy the necessary approximation condition.

Introducing total enumeration for basis functions we get

$$f^1_k, f^2_k, f^3_k; \quad k = 1, \dots, N,$$

where $N = \frac{1}{4}(n^3 - n^2)$.

It is convenient to represent the augmented matrix for determining unknown coefficients a_k, b_k, c_k in block form:

$$\left(\begin{array}{ccc|c} A_{11} & A_{12} & A_{13} & B_1 \\ A_{21} & A_{22} & A_{23} & B_1 \\ A_{31} & A_{32} & A_{33} & B_1 \end{array} \right) \tag{15}$$

where columns B_k and matrices A_{kl} are determined by formulas:

$$B^i_k = (E^k_0, f^k_i); \tag{16}$$

$$\begin{aligned} A^{ij}_{kl} = (\xi_{kl} f^l_j, f^k_i) - \delta_{kl} k_0^2 \left(\int_Q G^k_E(x,y) f^l_j(y) dy, f^k_i(x) \right) - \\ - \left(\frac{\partial}{\partial x_k} \int_Q \frac{\partial}{\partial x_l} G^k_E(x,y) f^l_j(y) dy, f^k_i(x) \right) , \end{aligned} \tag{17}$$

$k = 1, 2, 3; \quad i = 1, \dots, N$. (f,g) determines the scalar product in L_2, $(f, g) = \int_Q f(x) g(x) dx$.

Applying the formulas of integration by parts to both internal and external integrals and taking into account (7) and (14) we obtain:

$$\begin{aligned} A^{ij}_{kl} = \int_{\Pi^l_j \cap \Pi^k_i} \xi_{kl} f^l_j(x) f^k_i(x) dx - \delta_{kl} k_0^2 \int_{\Pi^k_i} \int_{\Pi^l_j} G^k_E(x,y) f^l_j(y) f^k_i(x) dy dx - \\ - \int_{\Pi^k_i} \int_{\Pi^l_j} \tilde{G}(x,y) \frac{\partial}{\partial x_l} f^l_j(y) \frac{\partial}{\partial x_k} f^k_i(x) dy dx. \end{aligned} \tag{18}$$

References

[1] Ilinski, A.S., Smirnov, Yu.G., " *Electromagnetic Wave diffraction by Conducting Screens*", VSP, The Netherlands, Utrecht, 1998

[2] Markov, G.T., Panchenko, B.A., *"Tensor Green functions of rectangular resonators"*, Izvestiya Vuzov, Radiotechnika, Moscow, 1964, V. 7, pp. 34–41. (in Russian)

[3] Samohin, A.B., *"Integral Equations and Iteration Methods in Electromagnetic Scattering"*, Radio & Sviaz, Moscow, 1998. (in Russian)