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THE PLANE H-POLARIZED WAVE DIFFRACTION BY A METAL GRATING WITH A MAGNETOACTIVE PLASMA

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A periodic grating of infinitely thin, perfectly conducting metal strips is considered in the yOz plane. The d -spaced strips are extending along the oZ axis, the grating period is l . The subspace $x < 0$ is occupied by a magnetoactive plasma with the magnetic field having the oZ direction. The plasma is characterized by the tensor

$$\hat{\epsilon} = \begin{pmatrix} \epsilon_1 & i\epsilon_2 & 0 \\ -i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix},$$

where $\epsilon_1 = 1 - \frac{\chi_p^2}{\chi^2 - \chi_c^2}$, $\epsilon_2 = -\frac{\chi_p^2 \chi_c}{\chi(\chi^2 - \chi_c^2)}$, $\epsilon_3 = 1 - \frac{\chi_p^2}{\chi^2}$, and $\chi = \frac{\omega l}{2\pi c}$, $\chi_p = \frac{\omega_p l}{2\pi c}$,

$\chi_c = \frac{\omega_c l}{2\pi c}$; $\omega = kc$ is the incident field frequency, ω_p and ω_c are, respectively, the plasma and electron cyclotron frequencies, c is the velocity of light in vacuum.

From above ($x > 0$), a plane H -polarized electromagnetic wave e^{-ikx} is normally incident on the grating plate. The time dependence is chosen to be $e^{-i\omega t}$. The field of the wave diffraction by the grating-plasma structure is necessary to find.

The diffraction field (function $V_1(x, y)$ and $V_2(x, y)$) sought in terms of the boundary value problem:

$$\epsilon_1 \Delta V_2(x, y) + k^2 (\epsilon_1^2 - \epsilon_2^2) V_2(x, y) = 0, \quad x < 0; \quad (1)$$

$$\Delta V_1(x, y) + k^2 V_1(x, y) = 0, \quad x > 0;$$

$$V_j(x, y \pm l) = V_j(x, y), \quad j = 1, 2; \quad (2)$$

$$\left(\frac{\partial V_2(0, y)}{\partial x} - i\tau \frac{\partial V_2(0, y)}{\partial y} \right) = 0, \text{ on metal,} \quad (3.a)$$

$$\frac{\partial V_1(0, y)}{\partial x} = ik, \text{ on metal,} \quad (3.b)$$

$$\left(\lambda \frac{\partial V_1(0, y)}{\partial x} - \frac{\partial V_2(0, y)}{\partial x} + i\tau \frac{\partial V_2(0, y)}{\partial y} + i\lambda k \right) = 0, \text{ over grating period,} \quad (4)$$

$$(V_2(0, y) - V_1(0, y)) = 1, \text{ in slot.} \quad (5)$$

In addition, functions $V_1(x, y)$ and $V_2(x, y)$ must fit the Meixner and radiation conditions on any compact set in the xOy plane. Here $\lambda = \frac{\varepsilon_1^2 - \varepsilon_2^2}{\varepsilon_1}$, $\tau = \frac{\varepsilon_2}{\varepsilon_1}$, and functions $V_1(x, y)$ and $V_2(x, y)$ are related to the scattered field component $H_z(x, y)$ as follows

$$H_z(x, y) = \begin{cases} V_1(x, y); & x > 0, \\ \left(\varepsilon_1^2 - \varepsilon_2^2\right) V_2(x, y); & x < 0. \end{cases}$$

Satisfying boundary conditions (3)-(5) gives the system of dual series equations

$$\begin{cases} \sum_{(n, n \neq 0)} \frac{1 + \tau_n}{1 + \tau_n + \lambda} |n| x_n e^{\frac{2\pi}{l} i n y} = \frac{\chi i}{1 + \sqrt{\lambda}} (x_0 + 2) + \sum_{(n, n \neq 0)} \frac{1 + \tau_n}{1 + \tau_n + \lambda} |n| x_n \delta_n e^{\frac{2\pi}{l} i n y}, & \text{on metal,} \\ \sum_{(n)} x_n e^{\frac{2\pi}{l} i n y} = 0, & \text{in slot,} \end{cases} \quad (6)$$

with $x_0 = b_0 \left(1 + \frac{1}{\sqrt{\lambda}}\right) - 2$, where $b_0 = \sqrt{\lambda} (1 - a_0)$, $\tau_n = \text{sign}(n)$.

For all $n \neq 0$, $x_n = \left(1 + \frac{\xi'_n + i\tau_n}{\lambda \xi_n}\right) b_n$, $\xi_n = \sqrt{\frac{\chi^2}{n^2} - 1}$, $\xi'_n = \sqrt{\lambda \frac{\chi^2}{n^2} - 1}$, $a_n = -\frac{\xi'_n + i\tau_n}{\lambda \xi_n} b_n$.

The values to find are amplitudes a_n and b_n of the diffraction spectra: a_0 and b_0 are, respectively, the reflection and transmission coefficients.

The authors' analytical regularization procedure suggested in [1] makes it possible to convert (6) into the infinite system of linear algebraic equations of the type

$$x_n = \sum_{m=-\infty}^{+\infty} B_{nm} x_m + w_n. \quad (7)$$

The matrix elements look like

$$B_{nm} = \begin{cases} \frac{\chi i (1 + \varepsilon_1 - \varepsilon_2)}{1 + \sqrt{\lambda}} A_{00}; & n = 0, m = 0, \\ (-1)^n A_{n0} \frac{\chi i (1 + \varepsilon_1 - \varepsilon_2)}{1 + \sqrt{\lambda}}; & n \neq 0, m = 0, \\ (-1)^m |m| \delta_m \Lambda_m A_{0m}; & n = 0, m \neq 0, \\ (-1)^{n+m} |m| \delta_m \Lambda_m A_{nm}; & n \neq 0, m \neq 0, \end{cases} \quad w_n = \begin{cases} \frac{2\chi i (1 + \varepsilon_1 - \varepsilon_2)}{1 + \sqrt{\lambda}} A_{00}; & n = 0 \\ \frac{2\chi i (1 + \varepsilon_1 - \varepsilon_2)}{1 + \sqrt{\lambda}} (-1)^n A_{n0}; & n \neq 0. \end{cases}$$

The A_{nm} expressions can be found in [1] and $\Lambda_n = \begin{cases} 1; & n > 0, \\ \frac{1 + \varepsilon_1 - \varepsilon_2}{1 + \varepsilon_1 + \varepsilon_2}; & n < 0. \end{cases}$ For large m ,

the smallness parameter $\delta_m \approx \left(\frac{\chi}{2m}\right)^2 \times \begin{cases} 1 + \varepsilon_1 - \varepsilon_2; & m > 0 \\ 1 + \varepsilon_1 + \varepsilon_2; & m < 0. \end{cases}$

So, it has been shown that matrix $\|B_{mn}\|_{m,n=-\infty}^{+\infty}$ generates the Hilbert-Schmidt operator in l_2 , and $w_n \in l_2$. Hence the solution of (7) can be obtained by truncation with any preassigned accuracy.

No wave transmission is found if $\chi = \sqrt{\chi_c^2 + \chi_p^2}$ and $\chi = \frac{1}{2}(\sqrt{\chi_c^2 + 4\chi_p^2} \pm \chi_c)$ because at these frequencies the plasma acts as a perfect reflector.

In the long-wave region, the reflection and transmission coefficients take the form

$$a_0 \approx \frac{\chi i A_{00}(1 + \varepsilon_1 - \varepsilon_2) + 1 - \sqrt{\lambda}}{\chi i A_{00}(1 + \varepsilon_1 - \varepsilon_2) - 1 - \sqrt{\lambda}}, \quad b_0 \approx \frac{2\sqrt{\lambda}}{1 + \sqrt{\lambda} - \chi i A_{00}(1 + \varepsilon_1 - \varepsilon_2)}. \quad (8)$$

$$\text{For } \frac{1 + \varepsilon_1 - \varepsilon_2}{1 + \varepsilon_1 + \varepsilon_2} > 0, \quad A_{00} = \frac{e^{-2\beta\theta}}{\pi} R_\sigma(\beta, \theta) \frac{2\theta\varepsilon_2}{1 + \varepsilon_1 - \varepsilon_2} - \frac{e^{-\pi\beta}}{2c\hbar(\pi\beta)} (e^{-2\beta\theta} R_\sigma(\beta, \theta) + e^{2\beta\theta} R_\sigma(-\beta, \theta)),$$

$$\text{where } \beta = \frac{1}{2\pi} \ln \frac{1 + \varepsilon_1 - \varepsilon_2}{1 + \varepsilon_1 + \varepsilon_2}, \quad \theta = \pi \left(1 - \frac{d}{l}\right); \quad R_\sigma(\beta, \theta) = \sum_{n=-\infty, n \neq 0}^{+\infty} \frac{(-1)^n}{n} P_{n-1}(-\beta, \theta) \quad \text{with}$$

$P_n(\beta, \theta)$ being the Pollachek polynomials. Notice that if χ_p and χ_c are both zero together, expressions (8) turn into the standard Lamb formulae for a grating at no plasma medium.

$$\text{For } \frac{1 + \varepsilon_1 - \varepsilon_2}{1 + \varepsilon_1 + \varepsilon_2} < 0,$$

$$A_{00} = \frac{e^{-\pi\tilde{\beta}}}{2sh(\pi\tilde{\beta})} (1 - e^{2\tilde{\beta}\theta}) - \frac{2\varepsilon_2}{\varepsilon_2 - \varepsilon_1 - 1} \frac{\theta}{\pi} + \frac{e^{\tilde{\beta}(\theta - \pi)\theta}}{\pi} \int_0^\pi \sin(\tilde{\beta} \ln \frac{\sin \frac{\theta + \varphi}{2}}{\sin \frac{\theta - \varphi}{2}}) d\varphi,$$

where $\tilde{\beta} = \frac{1}{2\pi} \ln \frac{\varepsilon_2 - \varepsilon_1 - 1}{\varepsilon_2 + \varepsilon_1 + 1}$. The integral in A_{00} presents no calculation problems since

it can be represented as a well convergent series expansion in the polynomials B_n given

$$\text{in [1]. For } \tilde{\beta} = 0, \quad A_{00} = -\frac{\theta}{\pi}.$$

REFERENCES

- [1] A.V. Brovenko, P.N. Melezhik, and A.Ye. Poyedinchuk, The regularization method to a class of dual series equations, *Ukrainian Math. Zh.*, 2001, v.53, no.10, pp.1320-1327 (in Ukrainian).