

UNCLASSIFIED

Defense Technical Information Center
Compilation Part Notice

ADP013959

TITLE: Electromagnetic Waves Scattering on an Unclosed Cone with an Isotropic One Inside

DISTRIBUTION: Approved for public release, distribution unlimited

This paper is part of the following report:

TITLE: 2002 International Conference on Mathematical Methods in Electromagnetic Theory [MMET 02]. Volume 2

To order the complete compilation report, use: ADA413455

The component part is provided here to allow users access to individually authored sections of proceedings, annals, symposia, etc. However, the component should be considered within the context of the overall compilation report and not as a stand-alone technical report.

The following component part numbers comprise the compilation report:

ADP013889 thru ADP013989

UNCLASSIFIED

ELECTROMAGNETIC WAVES SCATTERING ON AN UNCLOSED CONE WITH AN ISOTROPIC ONE INSIDE

Vladimir A. Doroshenko, Elena K. Semenova

Kharkov National University of Radioelectronics

14, Lenin av., Kharkov, 61166, Ukraine

Email: h_semenova@yahoo.com

ABSTRACT

The problem of electromagnetic wave scattering by a perfectly conducting thin long bicone is considered. The bicone consists of a slotted cone and an isotropic one. The solution method is based on using the Kontorovich-Lebedev integral transforms and the semi-inversion method. Both analytical and numerical results are presented.

INTRODUCTION

Cones and bicones are omnidirectional and super-wide band in radiation pattern and matching. The structure under consideration is a model of a specific bicone reflector and a slotted cone antenna. The task of this work is to study effects of slots and isotropic cone on scattering characteristics.

FORMULATION AND SOLUTION METHOD

Let us consider the scattering of incident electromagnetic waves from a thin perfectly conducting long circular slotted bicone. The geometry of the bicone configuration is shown in Fig.1; (r, θ, φ) are spherical coordinates with the origin at the bicone tip. The bicone structure Σ consists of a semi-infinite cone with periodical longitudinal slots $\Sigma_2: \theta = \gamma_2$ and an isotropic one $\Sigma_1: \theta = \gamma_1$ ($\Sigma = \Sigma_1 \cup \Sigma_2$). The period $l = 2\pi/N$ and the slot width d are angular values. The source of an incident field is a magnetic radial dipole (the time dependence is assumed to be $\exp(i\omega t)$) that is located at the point $B_0(r_0, \theta_0, \varphi_0)$, $\gamma_2 < \theta_0$. The vectors $\vec{E}$ and $\vec{H}$ of total fields satisfy the system of Maxwell equations, the boundary condition on the bicone: $\vec{E}_{\tan}|_{\Sigma} = 0$, the condition of finite stored energy and the infinity condition. The conditions mentioned above guarantee the uniqueness of solution. In order to find it, it is convenient to use Debye potential ϑ , which satisfies the three-dimensional homogeneous Helmholtz equation outside the bicone and the source, the Neumann boundary condition on the bicone, the principle of ultimate absorption, and the edge condition in the proximity of boundary singularities. In accordance with the structure of the total fields $\vec{E} = \vec{E}^{(i)} + \vec{E}^{(s)}$, $\vec{H} = \vec{H}^{(i)} + \vec{H}^{(s)}$, we represent ϑ in the form $\vartheta = \vartheta^{(i)} + \vartheta^{(s)}$, where indices (i) and (s) correspond to dipole fields and fields scattered by the bicone respectively. We look for the solution in the form

$$\vartheta^{(s)}(r, \theta, \varphi) = -\frac{1}{2} \int_0^{+\infty} \tau \sinh \pi \tau e^{\pi \tau} \hat{\mathcal{G}}_{\tau}^{(s)}(\theta, \varphi) \frac{H_{\pi}^{(2)}(kr)}{\sqrt{r}} dr,$$

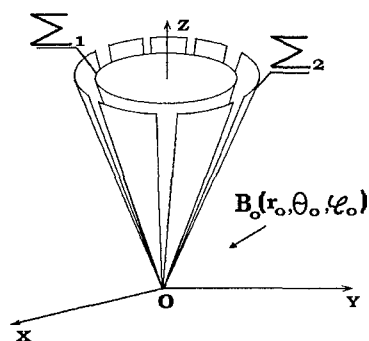


Fig.1 Bicone structure

$$\hat{\mathcal{G}}_{\tau}^{(s)} = - \sum_{m=-\infty}^{+\infty} a_{m\tau} \frac{d}{d\gamma_2} P_{-1/2+\tau}^m(\cos\gamma_2) P_{-1/2+\tau}^m(\cos\theta_0) V_{m\tau}(\theta, \varphi),$$

$$V_{m\tau} = \begin{cases} \sum_{n=-\infty}^{+\infty} [\beta_{mn} P_{-1/2+\tau}^{m+nN}(\cos\theta) + \xi_{mn} P_{-1/2+\tau}^{m+nN}(-\cos\theta)] e^{i(m+nN)\varphi}, & \gamma_1 < \theta < \gamma_2, \\ \sum_{n=-\infty}^{+\infty} \eta_{mn} P_{-1/2+\tau}^{m+nN}(-\cos\theta) e^{i(m+nN)\varphi}, & \gamma_2 < \theta < \pi; \end{cases}$$

where $H_{\tau}^{(2)}(kr)$ is the Hankel function, $P_{\tau}^m(\cos\theta)$ is the associated Legendre function, $a_{m\tau}$ are given; β_{mn} , ξ_{mn} , η_{mn} are unknown coefficients and expressed via y_n . The boundary condition imposed on the bicone and the field continuity condition on the slots yield the dual series equations those are reduced to the matrix equation of the Fredholm second kind [1] like $(I - A)Y = B$, $Y = \{y_n\}$. Coefficients y_n are independent of the wave-number k ; it is convenient for finding the field both near the vertex ($kr \ll 1$) and far from it ($kr \gg 1$). The solution of the matrix equation exists, is unique, and can be approximated by solving a truncated matrix equation. For a cone with narrow slots ($d_2/l \ll 1$) one can solve the equation by method of successive approximations.

ANALYTICAL RESULTS

For a cone with narrow slots we obtain the asymptotic expansion of potential $\mathcal{G}^{(s)}$ at a large distance from the slots in terms of parameter $(1-u) \ll 1$,

$u = \cos(\pi d_2/l)$ in the following form ($\varphi_0 = 0, \theta_0 = \pi, m = 0$)

$$\mathcal{G}^{(s)} = \int_0^{+\infty} b_{\tau} \frac{H_{\tau}^{(2)}(kr)}{\sqrt{r}} \frac{F_{\tau}}{F_{\tau} + \zeta} \frac{P_{-1/2+\tau}(-\cos\theta)}{\frac{d}{d\gamma_2} P_{-1/2+\tau}(-\cos\gamma_2)} d\tau - \zeta \int_0^{+\infty} b_{\tau} \frac{H_{\tau}^{(2)}(kr)}{\sqrt{r}} \frac{F_{\tau}}{F_{\tau} + \zeta} \times$$

$$\times \sum_{p \neq 0} \frac{1}{|p|} \varepsilon_p \frac{P_{-1/2+\tau}(-\cos\theta)}{\frac{d}{d\gamma_2} P_{-1/2+\tau}(-\cos\gamma_2)} d\tau + \zeta \int_0^{+\infty} b_{\tau} \frac{H_{\tau}^{(2)}(kr)}{\sqrt{r}} \frac{A_{\tau} F_{\tau}}{F_{\tau} + \zeta} \sum_{n \neq 0} \frac{P_{-1/2+\tau}^{nN}(-\cos\theta)}{\frac{d}{d\gamma_2} P_{-1/2+\tau}^{nN}(-\cos\gamma_2)} e^{inN\varphi} d\tau +$$

$$+ O((1-u)), \gamma_2 < \theta < \pi; \quad C_{\tau}^M = \frac{\frac{d}{d\gamma_1} P_{-1/2+\tau}^M(\cos\gamma_1) \frac{d}{d\gamma_2} P_{-1/2+\tau}^M(-\cos\gamma_2)}{\frac{d}{d\gamma_1} P_{-1/2+\tau}^M(-\cos\gamma_1) \frac{d}{d\gamma_2} P_{-1/2+\tau}^M(\cos\gamma_2)}$$

$$A_{\tau} = \frac{c h \pi \tau}{\pi \sin^2 \gamma_2 (\tau^2 + \frac{1}{4}) P_{-1/2+\tau}^{-1}(\cos\gamma_2) P_{-1/2+\tau}^{-1}(-\cos\gamma_2)} \frac{1}{1 - C_{\tau}^M|_{M=0}}, \quad F_{\tau} = \frac{1}{A_{\tau} - \frac{1}{N} \sum_{p \neq 0} \frac{1}{|p|} \varepsilon_p},$$

$$\zeta = -\frac{N}{\ln((1-u)/2)}, \quad \varepsilon_n = O(N^{-1}(n+v)^{-2}), \quad nN \gg 1, -1/2 \leq v < 1/2, \quad b_{\tau} \text{ is given.}$$

NUMERICAL RESULTS

We'll discuss the far field scattering characteristics of the bicone based on numerical examples of φ -plane scattering patterns. Let the source be at the z -axis ($\varphi_0 = 0, \theta_0 = \pi, m = 0$) and $N = 1$. Thus Fig.2 depicts the dependence of the far field scattering

on the angle γ_2 for an alone slotted cone (the isotropic cone is absent). The ray $\varphi = 0^\circ$ coincides with the slot axis. The slot lobe is symmetrical with respect to the slot axis; the main lobe of the slot radiation is centered in the direction $\varphi = 0^\circ$. The effects of the isotropic cone on the scattered far field are shown in the Fig.3 - Fig.5.

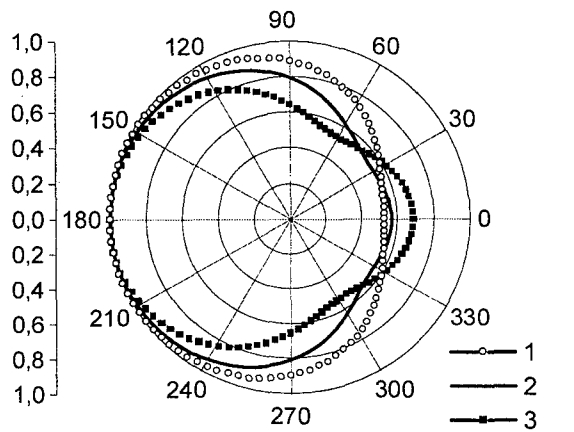


Fig.2 φ -plane scattering patterns for the alone slotted cone $\Sigma_2 (\gamma_1 = 0)$ $d_2 = 60^\circ$ for different values of γ_2 .
1. - $\gamma_2 = \pi/6$, 2. - $\gamma_2 = \pi/8$, 3. - $\gamma_2 = \pi/16$.

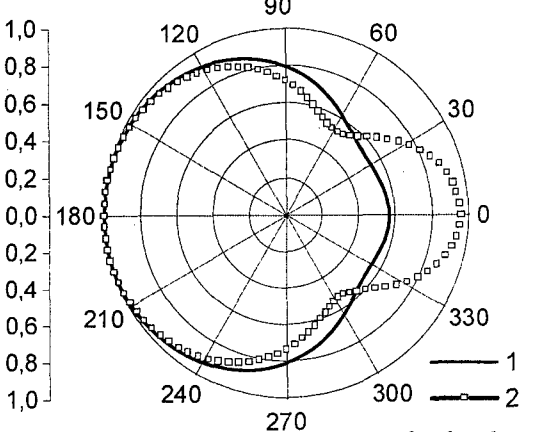


Fig.3 φ -plane scattering patterns for the alone cone (1) and the bicone (2) $\gamma_1 = \pi/16$, $\gamma_2 = \pi/8$, $d_2 = 60^\circ$.

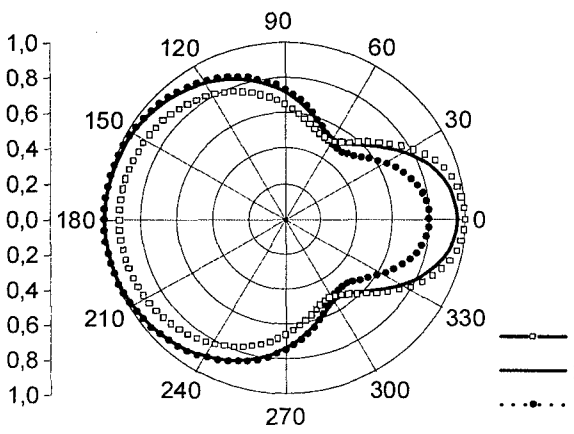


Fig.4 φ -plane scattering patterns. Scattered far field dependence on the angle γ_1 .
1. - $\gamma_1 = \pi/14$, 2. - $\gamma_1 = \pi/16$, 3. - $\gamma_1 = \pi/20$.

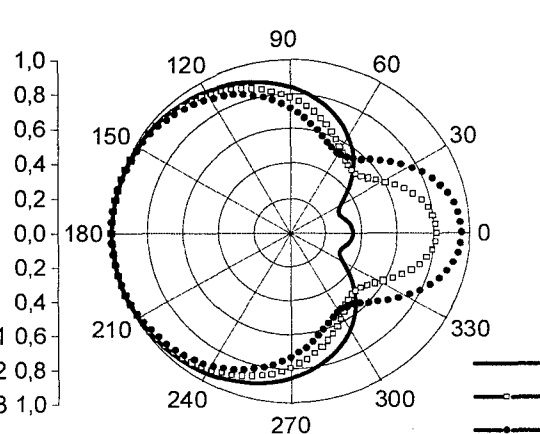


Fig.5 φ -plane scattering patterns. Scattered far field dependence on the slot width d_2 .
1. - $d_2 = 5^\circ$, 2. - $d_2 = 30^\circ$, 3. - $d_2 = 60^\circ$.

CONCLUSIONS

The problem of exciting the slotted cone with the isotropic one inside by the magnetic radial dipole has been considered. The analytical solution for narrow slots is analyzed. The scattered field structure contains the isotropic cone contribution and the narrow slots one. Scattered field patterns are given to investigate slots effects and the isotropic cone presence both.