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ELECTROMAGNETIC BACKSCATTERING FROM A TRIANGULAR DIELECTRIC CYLINDER

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INTRODUCTION

Boundary integral equations (IEs) are an accurate and versatile tool in the electromagnetic scattering by smooth cylinder [1], especially when it is combined with analytical preconditioning [2]. An original approach to the E-wave scattering by PEC polygons was developed in [3] to study a triangular prism. Here the boundary of the scatterer was conformally mapped on a circular cylinder [4], and a log-singular IE was treated by a simple moment method. We combine this idea with the method of analytical regularization (MAR) and study the scattering by triangular dielectric cylinders. Applications of this analysis are expected in the design of microwave and optoelectronic prism sensors and couplers.

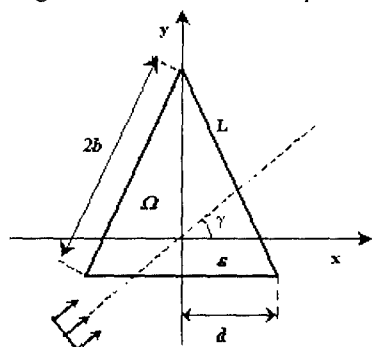


Fig. 1: Scattering geometry and notations

FORMULATION AND INTEGRAL EQUATIONS

The geometry of the problem is shown in Fig.1. A plane electromagnetic wave $H^i = e^{ik \cos \gamma x + i k \sin \gamma y}$ is incident on a uniform dielectric cylinder whose cross-section is an isosceles triangle of the base $2d$ and height h , having relative permittivity ϵ . The field functions have to satisfy the Helmholtz equation in free space and in dielectric, for $r \notin \Omega$ ($r \in \text{ext } \Omega$) and $r \in \Omega$, respectively. On the contour L the transmission conditions should be satisfied. The scattered field must also satisfy the Sommerfeld radiation condition at infinity and condition of local power finiteness.

We will seek the solution of the problem in the form of a pair of single-layer potentials with unknown densities $\phi \in C(\partial\Omega)$ and $\psi \in C(\partial\Omega)$. By using the conditions on L and the properties of the normal derivative of a single-layer potential when crossing the contour of integration [1], the following set of singular IEs is obtained:

$$\begin{cases} \int_L \phi(r') G_\epsilon(r, r') ds' - \int_L \psi(r') G(r, r') ds' = H^i(r) \\ \left[\frac{\phi(r)}{2\epsilon} + \frac{1}{\epsilon} \int_L \phi(r') \frac{\partial}{\partial n} G_\epsilon(r, r') ds' + \frac{\psi(r)}{2} - \int_L \psi(r') \frac{\partial}{\partial n} G(r, r') ds' \right] = \frac{\partial}{\partial n} H^i(r) \end{cases} \quad (2)$$

To facilitate building a fast algorithm, which is able to solve the scattering by cylinder with a contour arbitrarily close to triangle, we shall use the following continuous parametric approximation of curve L :

$$x(t) = \frac{d}{M} \left(\sin t + \sum_{k=1}^N a_k \sin(3k-1)t \right), \quad y(t) = \frac{b}{M} \left(-\cos t + \sum_{k=1}^N a_k \cos(3k-1)t \right), \quad (3)$$

$$a_k = \frac{\prod_{j=1}^k (5-3j)}{3^k (3k-1)k!} \quad (4)$$

One can obtain these formulas by conformal mapping of the region outside a regular polygon from Z -plane ($Z=x+iy$) to the (t,s) plane, where $0 \leq t \leq 2\pi$, and by using the transformation formula first derived in [5]: $\partial Z / \partial t = C(\cos(3t/2))^{2/3}$, where C is a complex coefficient depending on the size and orientation of the polygon. If $K \rightarrow \infty$ then the curve given by (3) tends to a triangle, however by truncating the series in (3) at the K -th terms a smoothed contour is obtained having the corner radius of curvature b inverse proportional to K . By changing the variables in (1) to t and t' , we arrive at IEs as follows (see also [2]):

$$\begin{cases} \int_0^{2\pi} p(t') K^\varepsilon(t, t') dt' - \int_0^{2\pi} q(t') K(t, t') dt' = f(t) \\ \frac{1}{\varepsilon} \frac{p(t)}{2} + \frac{q(t)}{2} + \int_0^{2\pi} p(t') M^\varepsilon(t, t') dt' - \int_0^{2\pi} q(t') M(t, t') dt' = \tilde{f}(t) \end{cases} \quad (5)$$

Here the following notations have been used:

$$K^\varepsilon(t, t') = \frac{i\varepsilon}{4} H_0^{(1)}(k\sqrt{\varepsilon} R(t, t')), \quad K(t, t') = \frac{i}{4} H_0^{(1)}(kR(t, t')), \quad (6)$$

$$M^\varepsilon(t, t') = \frac{ik}{4} \sqrt{\varepsilon} H_1^{(1)}(k\sqrt{\varepsilon} R(t, t')) \left(\frac{x(t) - x(t')}{R(t, t')} y'(t') - \frac{y(t) - y(t')}{R(t, t')} x'(t') \right), \quad (7)$$

$$M(t, t') = \frac{ik}{4} H_1^{(1)}(kR(t, t')) \left(\frac{x(t) - x(t')}{R(t, t')} y'(t') - \frac{y(t) - y(t')}{R(t, t')} x'(t') \right), \quad (8)$$

$R(t, t') = \sqrt{(x(t) - x(t'))^2 + (y(t) - y(t'))^2}$ is the distance between two points on L . Besides, in the right-hand parts we have:

$$f(t) = \exp(ik(x(t) \cos \gamma + y(t) \sin \gamma)) \quad (9)$$

$$\tilde{f}(t) = k(y'(t) \cos \gamma - x'(t) \sin \gamma) \exp(k(x(t) \cos \gamma + y(t) \sin \gamma)) \quad (10)$$

REGULARIZATION AND DISCRETIZATION

As a curve given by (3) is smooth, we can solve (5) by projecting them onto the set of global basis and testing functions $\{e^{im\theta}\}_{m=-\infty}^{+\infty}$. Here the key step is to split the IE kernels by adding and subtracting the terms corresponding to canonical scatterer, i.e., a circular cylinder [2]. Thanks to the fact that the singularities of actual kernels are the same as of canonical ones, and that angular exponents form a complete set of orthogonal eigenfunctions of the canonical operators, the resultant infinite-matrix equation is of the Fredholm second kind. It has a (2x2) block structure generated by (5) in obvious manner. Therefore this specialized Galerkin projection procedure can be regarded as an analytical preconditioner, which leads to an always-stable numerical solution whose accuracy is easily understood and controlled. Namely, if the intermediate computations have been done with superior accuracy, then the error in matrix inversion is controlled by the size N of each block, so that large enough N guarantees a uniform accuracy. An important point of the MAR-algorithm efficiency is fast computation of the matrix elements, which are the double Fourier transform coefficients of the twice-continuous functions formed by the differences of actual and canonical kernels of IEs. Here various versions of FFT and the like numerical algorithms can be successfully implemented. We shall demonstrate the features of the developed method by presenting the dependences of the matrix condition number and inversion error as a function of the matrix block size N for various triangle sizes, dielectric constants, peak curvatures of smoothing parameterization, and frequencies.

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