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SOME ASPECTS OF THE SLOW WAVES IN THE CIRCULAR WAVEGUIDE WITH AZIMUTHALLY MAGNETIZED FERRITE ROD

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ABSTRACT

The real Kummer function, the ordinary and modified difference Bessel functions are applied to derive the characteristic equation of slow $\hat{TE}_{0\hat{n}}$ modes in the circular waveguide, containing azimuthally magnetized ferrite cylinder and dielectric toroid. A lemma on a remarkable property of the real zeros of the equation is formulated, based on its numerical solution. Essential features of the slow $\hat{TE}_{01}$ mode are found out, investigating its graphically shown phase curves.

INTRODUCTION

A few of the attributes of the slow waves in the azimuthally magnetized circular ferrite waveguides are known only [1]. The task for their propagation is rather interesting since such modes cannot be sustained, if the filling is isotropic. Very attractive in view of the variety of expected results are the stratified configurations, not treated until now, the simplest of which is examined here.

BOUNDARY-VALUE ANALYSIS

The knowledge of the roots of the equation:

$$\frac{1-\hat{\alpha}^2}{2}\frac{\Phi(\hat{a},\hat{c};\hat{\rho}\hat{x}_0)}{\Phi(\hat{a}-1,\hat{c}-2;\hat{\rho}\hat{x}_0)} = \begin{cases} -\dfrac{1}{\hat{v}_0}\dfrac{\partial}{\partial\hat{v}_0}\ln\left|\dfrac{\partial}{\partial\hat{y}_0}bsn_{\hat{m}}(\hat{y}_0-\hat{v}_0)\right|, & \hat{q}>0 \\ -\dfrac{1-\hat{\rho}^2}{2\hat{\rho}^2}, & \hat{q}\equiv\hat{p}\equiv 0 \\ \dfrac{1}{\hat{w}_0}\dfrac{\partial}{\partial\hat{w}_0}\ln\left|\dfrac{\partial}{\partial\hat{u}_0}bsh_{\hat{m}}(\hat{u}_0-\hat{w}_0)\right|, & \hat{p}=j\hat{q}>0, \end{cases} \tag{1}$$

is a necessary condition for the study of eigenvalue spectrum and phase characteristics of the circular waveguide (radius $\hat{r}_0$) with azimuthally magnetized ferrite rod (radius $\hat{r}_1$) and dielectric toroid under slow $\hat{TE}_{0\hat{n}}$ modes excitation, if the axial switching wire is dimensionless. (All quantities, related to waves mentioned are real and are marked by a hat "^" which may be omitted provided no ambiguity might arise.) The ferrite has a permeability tensor with off-diagonal element $\hat{\alpha}=\gamma\hat{M}_r/\omega$ (γ - gyromagnetic ratio,

$\hat{M}_r$ - remanent magnetization, ω - angular frequency of the wave). The relative permittivities of inner and outer media are ε_r and ε_d, resp. The left-hand side of eqn. (1) involves real Kummer functions [2]. The first and third forms of its right-hand one are written by ordinary and modified difference Bessel functions [3]. The symbols $bsn_{\hat{m}}(\hat{y}_0 - \hat{v}_0)$ and $bsh_{\hat{m}}(\hat{u}_0 - \hat{w}_0)$ stand for the $\hat{m}$th order ordinary and modified difference Bessel sine ($\hat{m}$ - an integer) [3]. It holds $\hat{a} = 1.5 + \hat{k}$, $\hat{c} = 3$, $\hat{k} = \hat{\alpha}\bar{\hat{\beta}}/(2\bar{\hat{\beta}}_2)$, $\bar{\hat{\beta}}_2 = \left[\bar{\hat{\beta}}^2 - (1-\hat{\alpha}^2)\right]^{1/2}$, $\hat{x}_0 = 2\bar{\hat{\beta}}_2\bar{\hat{r}}_0$, $\hat{\rho} = \bar{\hat{r}}_1/\bar{\hat{r}}_0$, $\hat{y}_0 = \hat{q}\hat{x}_0$, $\hat{v}_0 = \hat{\rho}\hat{y}_0$, $\hat{q} = 0.5\{(\varepsilon_d/\varepsilon_r)[1-(2\hat{k}/\hat{\alpha}^2)]/(1-\hat{\alpha}^2)+(2\hat{k}/\hat{\alpha}^2)\}^{1/2}$, $\hat{u}_0 = \hat{p}\hat{x}_0$, $\hat{w}_0 = \hat{\rho}\hat{u}_0$, $\hat{p} = j\hat{q}$, $\hat{m} = 0$. The phase constant, radial wavenumber, guide and rod radii are normalized as follows $\bar{\hat{\beta}} = \hat{\beta}/(\beta_0\sqrt{\varepsilon_r})$, $\bar{\hat{\beta}}_2 = \hat{\beta}_2/(\beta_0\sqrt{\varepsilon_r})$, $\bar{\hat{r}}_0 = \beta_0\hat{r}_0\sqrt{\varepsilon_r}$, $\bar{\hat{r}}_1 = \beta_0\hat{r}_1\sqrt{\varepsilon_r}$; $\beta_0 = \omega\sqrt{\varepsilon_0\mu_0}$. The condition $\hat{q}$ ($\hat{p}$) – real positive or $\hat{q} \equiv \hat{p} \equiv 0$ needs application of the first (third) or second form of eqn. (1). A numerical proof is given to the statement:

Lemma 1: If $\xi_{\hat{k},\hat{n}}^{(\hat{c})}(\varepsilon_r, \varepsilon_d, \hat{\rho}, \hat{\alpha})$ be the $\hat{n}$th real positive root of characteristic equation in $\hat{x}_0$ provided $\hat{a}$, $\hat{c}$, $\hat{x}_0$ - real, $\hat{x}_0 > 0$, $\hat{c} = 3$, $\hat{k} < 0$ ($\hat{k} = \hat{a} - \hat{c}/2$), $\varepsilon_r > 0$, $\varepsilon_d > 0$, $0 < \hat{\rho} \le 1$, the infinite sequences of numbers $\left\{\xi_{\hat{k},\hat{n}}^{(\hat{c})}\right\}$ and $\left\{|\hat{k}|\xi_{\hat{k},\hat{n}}^{(\hat{c})}\right\}$ are convergent for $\hat{k} \to -\infty$ and their limits are 0 and $\hat{L}$, resp. where $\hat{L} = \hat{L}(\hat{c}, \varepsilon_r, \varepsilon_d, \hat{\rho}, \hat{\alpha}, \hat{n})$.

SOME PROPERTIES OF THE SLOW $\hat{\text{TE}}_{01}$ MODE

Using the roots $\xi_{\hat{k},1}^{(\hat{c})}$ of eqn. (1) and the relations between barred quantities, the structure's phase characteristics are drawn in Figs. 1, 2 for $\varepsilon_r = \varepsilon_d = 1$ and $\hat{\rho} = 0.9$. The analysis shows that propagation is possible for negative magnetization in two areas, subject to the terms: $\hat{\alpha}_{left}^{(1),(2)} < \hat{\alpha}^{(1),(2)} < \hat{\alpha}_{right}^{(1),(2)}$, $\bar{\hat{r}}_{0left}^{(1),(2)} < \bar{\hat{r}}_0^{(1),(2)} < \bar{\hat{r}}_{0right}^{(1),(2)}$, $\hat{k}_{left}^{(1),(2)} < \hat{k}^{(1),(2)} < \hat{k}_{right}^{(1),(2)}$, $\bar{\hat{\beta}}_{left}^{(1)} < \bar{\hat{\beta}}^{(1)} < \bar{\hat{\beta}}_{right}^{(1)}$, $\bar{\hat{\beta}}_{left}^{(2)} > \bar{\hat{\beta}}^{(2)} > \bar{\hat{\beta}}_{right}^{(2)}$ where $\hat{\alpha}_{left}^{(1)} = -1$, $\hat{\alpha}_{right}^{(1)} = 0$, $\hat{\alpha}_{left}^{(2)} = -\infty$, $\hat{\alpha}_{right}^{(2)} = -3$, $\bar{\hat{r}}_{0left}^{(1)} = \hat{L}^{(1)} / \left[|\hat{\alpha}^{(1)}|\left(1-[\hat{\alpha}^{(1)}]^2\right)^{1/2}\right]$, $\bar{\hat{r}}_{0left}^{(2)} = 0$, $\bar{\hat{r}}_{0right}^{(1),(2)} = +\infty$, $\hat{k}_{left}^{(1)} = -\infty$, $-1.5 < \hat{k}_{right}^{(1)} < -0.5$, $\hat{k}_{left}^{(2)} = \hat{\alpha}^{(2)}/2$, $\hat{k}_{right}^{(2)} = -1.5$, $\bar{\hat{\beta}}_{left}^{(1)} = \left(1-[\hat{\alpha}^{(1)}]^2\right)^{1/2}$, $\bar{\hat{\beta}}_{left}^{(2)} = +\infty$, $\bar{\hat{\beta}}_{right}^{(1),(2)} = \left\{\left(1-[\hat{\alpha}^{(1),(2)}]^2\right) / \left[1-\left(\hat{\alpha}^{(1),(2)}/\left(2\hat{k}_{right}^{(1),(2)}\right)\right)^2\right]\right\}^{1/2}$. The superscripts (1), (2) signify the zone to which given quantity is related; the subscripts "left", "right" – its limits. The symbols $\hat{TE}_{01}^{(1)}$ and $\hat{TE}_{01}^{(2)}$ are used in the first and second region, resp. In contrast to the ferrite case [1] the slowing down of waves grows, the transmission area of $\hat{TE}_{01}^{(1)}$ mode expands towards smaller $\bar{\hat{r}}_0^{(1)}$ (compare the enve-

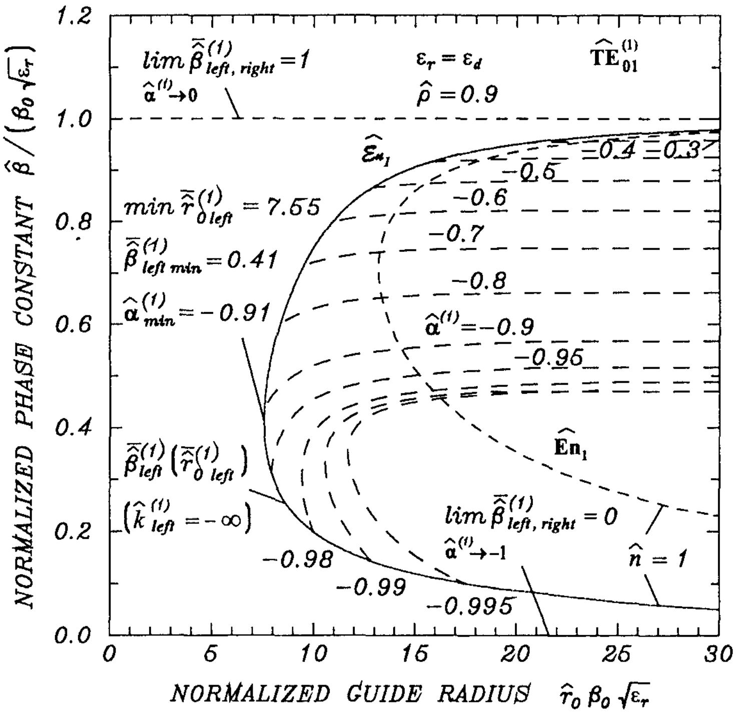

Fig.1. $\bar{\beta}^{(1)}\left(\bar{r}_0^{(1)}\right)$ *curves of* $\hat{TE}_{01}^{(1)}$ *mode for* $-1 < \hat{\alpha}^{(1)} < 0$.

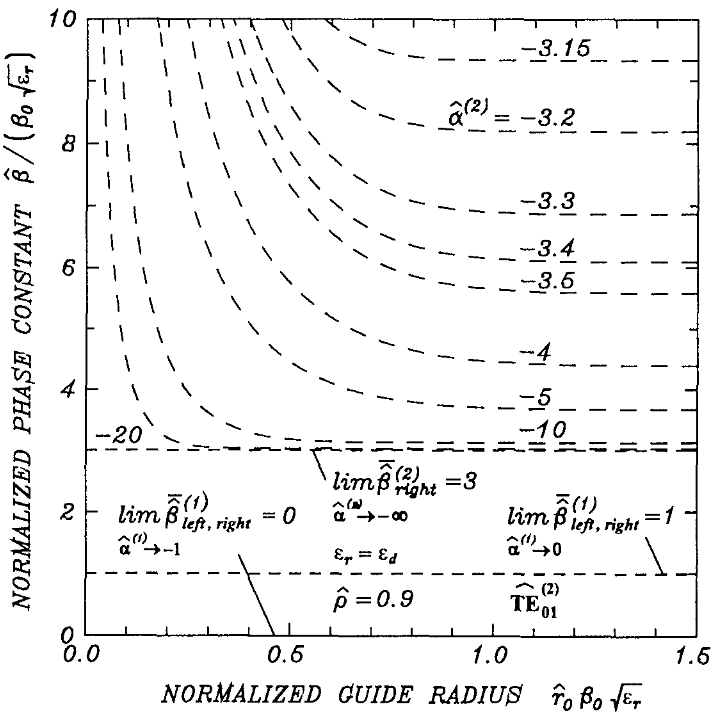

Fig.2. $\bar{\beta}^{(2)}\left(\bar{r}_0^{(2)}\right)$ *curves of* $\hat{TE}_{01}^{(2)}$ *mode for* $-\infty < \hat{\alpha}^{(2)} < -3$.

lopes, bounding it from the left $\hat{\mathcal{E}}_{n_1}$ and $\hat{E}n_1$, for the layered and ferrite geometry, resp.) and a domain of double-valuedness appears (cf. Fig. 1).

CONCLUSION

The propagation conditions are fixed and certain characteristics of the slow $\hat{TE}_{0\hat{n}}$ modes in the azimuthally magnetized ferrite-dielectric circular structure are explored. Inseparable part of the study is a lemma on its eigenvalue spectrum. The principles for solving a new class of boundary-value problems are outlined.

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