This paper is part of the following report:

TITLE: 2002 International Conference on Mathematical Methods in Electromagnetic Theory [MMET 02]. Volume 2

To order the complete compilation report, use: ADA413455

DISPERSIVE CHARACTERISTICS OF A WAVEGUIDE WITH PERIODIC IMPEDANCE OF WALLS

V. Bondarev[1], L. Logacheva[2]

[1]National Mining University
49026 Dnepropetrovsk, No.2 Oboyanskaya Str., apt.6, nbond@fregat.com
[2]Zaporozhye National Technical University

ABSTRACT

The present article is devoted to some problems of implementation of concept of a periodic impedance of walls in the process of studying wave propagation in waveguides. It enables to approach the calculation of different waveguide devices from the unified position. The implementation of such approach is shown on an example of absorbing filter of harmonics representing a sequently located round holes in a narrow or wide wall. It is shown that such tasks can be solved as Helmholtz equation with impedance boundary conditions for tangent components of electric and magnetic fields on the wall's surface.

SURFACE IMPEDANCE OF A NARROW WAVEGUIDE WALL WITH THE SYSTEM OF HOLES

Let's study a rectangular waveguide having a periodic series of round holes on a narrow wall along the line parallel to axis z .

Let's introduce a rectangular system of coordinates which beginning coincides with one of the tops of the waveguide and cylinder system of coordinates which beginning coincides with the center of the first hole circle. Then the chosen systems of coordinates can be connected in the following way:

$$z = l + \rho \cdot \cos\varphi, \quad y = d + \rho \cdot \sin\varphi. \tag{1}$$

where l –distance from waveguide aperture to the first hole centre, d – distance from the wide waveguide wall to the holes system centre.

From the task geometry it is clear that the surface impedance is equal to zero everywhere on the narrow wall except for the holes surfaces. Let's assume that on the holes the surface impedance coincides with the characteristic resistance of the secondary round waveguides connected to the holes.

Let's disintegrate the surface impedance in the plane YOZ into the double Fourier series on the orthogonal system of functions in the rectangular $(0 <= y <= b, 0 <= z <= h, h -$ distance between the holes centers) .

$$Z_S(y,z) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} V_{mn} \cdot (A_{mn}\cos\frac{m\delta}{b}y \cdot \cos\frac{n\delta}{h}z + B_{mn}\sin\frac{m\delta}{b}y \cdot \cos\frac{n\delta}{h}z +$$

$$+ C_{mn}\cos\frac{m\delta}{b}y \cdot \sin\frac{n\delta}{h}z + D_{mn}\sin\frac{m\delta}{b}y \cdot \sin\frac{n\delta}{h}z) \tag{2}$$

where A_{mn}, B_{mn}, C_{mn}, D_{mn} – disintegration coefficient of Fourier series. Value V_{mn} is equal to 1, ½, ¼, depending on the values of m and n.

Coefficients B_{mn}, D_{mn} are equal to zero and coefficients A_{mn}, C_{mn} are found from the following relations:

$$A_{mn} = \frac{16\pi Z_{\ni}}{bh} \cos \frac{m\pi}{b} d \cdot \cos \frac{n\pi}{h} l \cdot \sum_{q=0} S_q^{(m,n)},$$

$$m \neq n.$$

(3)

$$C_{mn} = \frac{16\pi Z_{\ni}}{bh} \cos \frac{m\pi}{b} d \cdot \sin \frac{n\pi}{h} l \cdot \sum_{q=0} S_q^{(m,n)},$$

$$m \neq n.$$

where

$$S_p^{(m,n)} = \frac{1}{2} \cdot \frac{\frac{n\pi}{h} R \cdot J_0(\frac{m\pi}{b} R) J_{-1}(\frac{n\pi}{h} R) - \frac{m\pi}{b} R \cdot J_{-1}(\frac{m\pi}{b} R) J_0(\frac{n\pi}{h} R)}{\left(\frac{m\pi}{b}\right)^2 - \left(\frac{n\pi}{h}\right)^2},$$

(4)

CALCULATION OF PROPAGATION CONSTANT OF A RECTANGULAR WAVEGUIDE WITH AN IMPEDANCE NARROW WALL

Let's study the magnetic waves propagation in a rectangular waveguide with a narrow impedance wall. As follows from the above said the impedance of a narrow wall is stipulated by the presence of round holes on it. A field in such a waveguide is determined through a longitudinal component Hz which satisfies the Helmholtz equation

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma_\perp^2 H_z = 0,$$

(5)

as well as boundary conditions on a narrow wall with $E_y = -Z_s(y,z)H_z$.
On other walls having ideal conductivity the component Ey = 0.
Due to the fact that the surface impedance is a periodic function from coordinate z the field components will be represented in the form of Floquet series

$$H_z = \sum_{n=0}^{\infty} \sum_{s=-\infty}^{\infty} A_n^{(s)} \cos\gamma_{xn}^{(s)} x \cdot \cos\frac{n\eth}{b} y \cdot e^{-j\beta_s z},$$

(6)

$$E_y = -j\omega\mu_a \sum_{n=0}^{\infty} \sum_{s=-\infty}^{\infty} \frac{\gamma_{xn}^{(s)}}{\gamma_{\perp n}^{(s)2}} A_n^{(s)} \sin\gamma_{xn}^{(s)} x \cdot \cos\frac{n\eth}{b} y \cdot e^{-j\beta_s z}.$$

where

$$\beta_{\rm s} = \beta + p\frac{2\pi}{h}, \ \gamma_{\rm xn}^{(s)\,2} = k^2 - \beta_s^2 - \left(\frac{n\pi}{b}\right)^2,$$

Using the boundary condition and expressions for the field components (6) as well as the orthogonality of the transverse eigenfunctions of the waveguide from (5) it is possible to receive the following system of homogeneous linear algebraic equations in regard to the field amplitudes

$$\sum_{k=0}^{\infty}\left\{\left[\left(Z_{po}^{+} + Z_{po}^{-}\right)U_k^{(s)} - (1+\delta_{nk})\delta_{nk}\cdot V_k^{(s)}\right]A_k^{(s)} + \right.$$
$$\left. +\frac{1}{2}\sum_{r=1}^{\infty}\left(Z_{pr}^{+}U_k^{(s-r)}A_k^{(s-r)} + Z_{pr}^{-}U_k^{(s+r)}A_k^{(s+r)}\right)\right\} = 0, \tag{8}$$

where

$$Z_{\rm pr}^{-} = A_{pr} - jC_{pr}, \ Z_{\rm pr}^{+} = A_{pr} + jC_{pr}.$$

$$U_k^{(s)} = \cos\gamma_{xk}^{(s)}a;$$

$$V_k^{(s)} = j\cdot\frac{\gamma_{xn}^{(s)}}{\gamma_{\perp k}^{(s)2}}\sin\gamma_{xk}^{(s)}a.$$

δnk – the Kronecker symbol.

Non-zero solution of the system (8) enables to receive the dispersive equation in a matrix form from which we determine the propagation constants in the structure under study. Properties of body and surface waves in an impedance waveguide are examined by numerical method in a wide range of parametres changes.