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SIZE EFFECTS IN THE MEASUREMENT OF SOIL STRENGTH PARAMETERS

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INTRODUCTION. The main concern of the Land Locomotion Laboratory, ATAC, is the relationship of a vehicle to the terrain over which it travels. Once an insight into this relationship has been attained, the problems encountered from the initial design of a vehicle to its ultimate use in the field can be more easily grasped and rationally solved. The solution of engineering problems depends upon the selection of the relevant variables and a description of the functional relationship among these variables. The selection of the vehicle variables are within broad limits at the disposal of the designer, but for the terrain or soil, the selection of suitable variables becomes much more complicated. Soil is probably one of the most complex of all engineering materials [1]. Researchers in soil mechanics have added much to the knowledge of the mechanical and physical characteristics of soils, but as yet, no fully satisfactory general theory is available.

Land locomotion is an engineering application of soil mechanics to off-road vehicular operation. Its objective is to determine the relationships of vehicles, or more precisely, of the wheel and track, to the strength properties of the soil. In land locomotion research, one of the important questions is the nature of the pressure-sinkage relationship. The Bekker equation

$$(1) \quad p = \left(\frac{k_c}{b} + k_\phi \right) z^n$$

represents a family of curves with three unknown constants, k_c , k_ϕ , and n , and two variables, p the pressure under a loaded area, and z the sinkage* [2].

* b is a known constant, the plate width. p is per unit area (i.e., square inch).

A set of these constants which will approximately describe pressure-sinkage observations can be obtained [3]. To determine these constants, pressure-sinkage experiments were performed in the laboratory with footings of various sizes and shapes. The constants obtained were used to predict the sinkages for other loaded areas. It was found that the predictions were adequate for tracked vehicles, where the relative shapes of the loaded areas were similar. Rectangular test footings with a length/width ratio greater than 5 were used for determining the soil parameters used in the prediction equations. The basic equation includes only the width term b . For cohesionless soils, the ultimate bearing strength is dependent on the width only for long loaded areas [4]. To determine what the minimum length had to be before a footing was not considered long, laboratory tests were conducted with footings of varying L/b ratios. Acceptable results for the pressure-sinkage relationship were obtained when the L/b ratio was greater than 5. Consequently, all pressure-sinkage measurements were taken with footings of at least an L/b ratio of 5 or greater.

The ultimate equations in which these soil strength parameters are to be used apply to the general case of predicting vehicle performance for both tracked and wheeled vehicles. As noted above, the predictions of tracked vehicle sinkage and motion resistance have been generally satisfactory. For wheeled vehicles, however, improvements are needed. One of the differences to be noted between a tracked and wheeled vehicle is the shape of the loaded area. In most cases, tracked vehicles have a contact area of relatively long length as compared to width. Such a length-width ratio is not the usual situation for wheeled vehicles at moderate sinkages. Most tires will have a contact area where the L/b ratio is close to 1 or 2.* Therefore, it was thought that the shape of the loaded area when L/b was less than 5 might have effects on the pressure-sinkage relationship. A clearer understanding of the pressure-sinkage relationship in this region would provide us with an improved soil-vehicle model with broader and more useful applications. Consequently, a test program was undertaken by the Land Locomotion Laboratory to investigate further the pressure-sinkage relation.

*For example, the Army 6 x 6 5-ton truck normally carries an 11.00 x 20 tire. At one inch sinkage, $L/b = 1$ and at eight inch sinkage $L/b = 2.33$.

DESCRIPTION OF TEST PROGRAM. The test program to study the effect of plate size on the vertical soil strength or sinkage parameters was divided into two parts. The first part comprised a study of the reliability or repeatability of the test results for the equipment and mixing techniques that were to be used in the tests. The second part covered the measurement of the load-sinkage curves while using different sized plates.

In carrying out the experiments a laboratory model bevameter is used to drive a constant speed hydraulic piston arrangement which pushes a plate into the soil in a bin. The depth of sinkage and the force on a load cell are plotted electrically by an X-Y plotting device. The curve is traced on linear graph paper. From this graph values for p and z may be read off and plotted on log-log paper. The slope of the least square fitted line on the double-log plot gives an estimate of the parameter n . The constant term in such a fitted equation is the logarithm of

$$\frac{k_c}{b} + k_\phi$$

where b is the width of the plate used. Thus it is seen that k_c and k_ϕ cannot be estimated by any straight forward statistical technique. Use of two different b values, however, will permit setting up two simultaneous equations in k_c and k_ϕ .

PART I. We wished to determine the maximum number of penetrations that could be made with one preparation of the soil bin. For this purpose we set up a uniformity trial using only a 2" x 10" plate. Orientation and location of the penetrations was arranged as indicated in Figure 1. The three orientations shown were carried out in a randomized block design with six replicates.

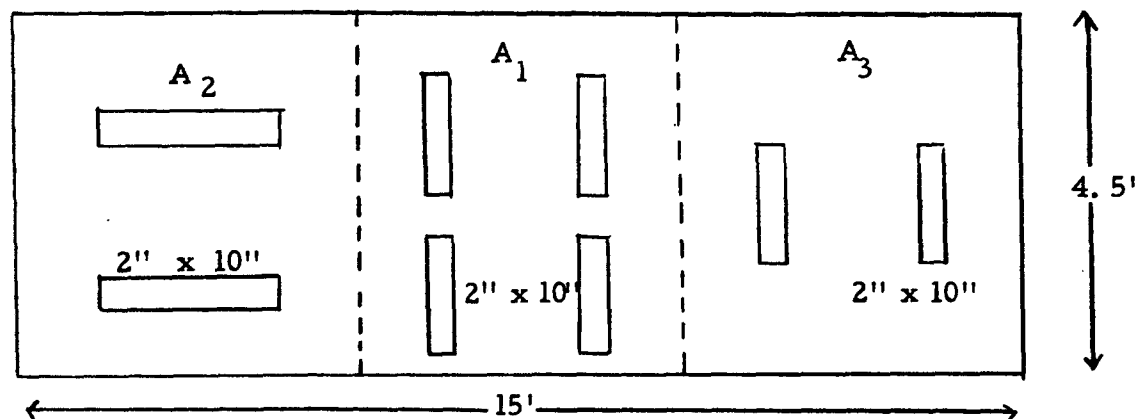


FIGURE 1: SAMPLE BLOCK LAYOUT FOR UNIFORMITY TRIALS

RESULTS FOR PART I: Mean pressures were computed for the orientations at 1", 2" and 3" depths of sinkage. Mean differences for the orientations at a given depth were found to be homogeneous (i. e., not significant). At the 2" depth of sinkage, the coefficient of variation was about 6 percent, quite a satisfactory value. Examination of the variability within the orientations showed that the variation within the arrangement A_1 was significantly greater than that for the other arrangements.

CONCLUSIONS FOR PART I: The experimental procedure would yield results of adequate reliability. The orientation A_1 appeared undesirable for taking pressure-sinkage measurements. Therefore, we decided to make the spacings between determinations as similar to the A_2 arrangement as practicable. Since a rectangular plate of size 3" x 10" was to be used in Part 2, it appeared necessary to "beef up the apparatus" to handle the greater pressures required to sink such a large plate.

PART 2. The second phase of the test program comprised measurement of the load-sinkage relation in dry sand with plates of varying sizes. Analysis of test data would provide estimates of the three parameters in the Bekker equation and their variations for the sizes and shapes of plates used. The results should show the dependence, if any, of the parameters on the size and shape of plate.

Many problems arose in the consideration of the second part of the test program. One point was that all plates should be tried in one mix of the bin. Thus, a complete block design would be preferred,

A second point is that the sinkage equation is a stress-strain relationship. It may also be described as a functional relation [5]. The problems of estimation which arise for the functional relation have been resolved by Dr. Joseph Berkson by introducing the "control variable" concept [6]. It appears that z may be taken as the control variable in our problem. This approach is contrary to the usual dependent-independent variable point of view, but Berkson has shown that the method is unbiased for estimating the functional relation if the errors in p are unbiased.

Thirdly, it was clear that a statistical analysis of the estimation procedure for the Bekker equation was needed. With transformation to logarithms of p and z it is assumed that the standard linear regression assumptions are valid in the transform [7]. Thus, estimation of the parameter n is quite straightforward. When n has been obtained, the procedure takes $z = 1$, hence, $\log z = 0$, and predicts a value of $\log p$, say P_o . Now, $\text{anti-log } P_o = p^* = k_\phi + k_c/b$. By taking two values of the plate width, b_1 and b_2 , and corresponding p_1^* and p_2^* values, the estimation equations for k_c and k_ϕ become

$$(2) \quad k_c = \frac{(p_1^* - p_2^*)b_1b_2}{b_2 - b_1}$$

and

$$(3) \quad k_\phi = \frac{b_2p_2^* - b_1p_1^*}{b_2 - b_1}$$

These results pointed out two things:

1. Widely spaced b values to permit use of large $b_2 - b_1$ would reduce the variance of the estimates of k_c and k_ϕ .
2. A formula for the variance of p^* is needed.

The variance formula for p^* presents some difficulty because p^* is a nonlinear function of P_o . We recall, however, that the choice of an appropriate experimental design will give us direct estimates of the experimental error for our estimated k_c and k_ϕ values so that we can bypass the variance formula problem.

DESIGN OF THE PART 2 EXPERIMENTS. From the statistical analysis it was clear that the largest possible difference in plate widths should be used to estimate the parameters k_c and k_ϕ . This consideration along with the desire for a complete block experiment already mentioned resulted in a revision of the choice of dimensions for the rectangular plates to be used in the experiment. The plate sizes actually selected are given in Table 1.

TABLE 1
LIST OF PLATE DIMENSIONS FOR PART 2 EXPERIMENTS

(in inches)

Rectangles:

1 x 4	2 x 4	3 x 4
1 x 6	2 x 6	3 x 6
1 x 8	2 x 8	3 x 8
1 x 10	2 x 10	3 x 10

Circles:

Diameters: 2 and 4.

The entire set of plates thus provided 14 treatments to be set up in a completely randomized block experiment where one block equals a mix of the soil bin. Each plate was to be used in a randomly selected plot of size about 2' x 2' to make a single measurement of the pressure-sinkage curve. It was further decided to complete six replicates which would yield 24 independent pairs of estimates of k_c and k_ϕ , but would, of course, yield 84 estimates of the parameter n , one estimate being obtained from each pressure-sinkage curve.

ANALYSIS OF RESULTS FOR PART 2. Analysis of the estimated values for n is straightforward and results are readily interpreted. For the k_c and k_ϕ values some difficulties arise. Plotting the means is a most useful device for aid in understanding the effects indicated by the analysis.

Figures 2 and 3 show the width and length effects on n without considering the interaction. In order to present the interaction effect, we show the usual type of plot for displaying an interaction. Parallel lines for the various lengths of plate would be indicative of no interaction.

Thus, the lack of parallelism exhibited in Figure 4 is indicative of the nature and source of the interaction. The major pattern, however, is still that shown in Figures 2 and 3. There appears to be a decrease in the n values with an increase in area of plate whether the area increase is due to change in length or change in width. The width effect is much smaller in going from 2" to 3", but the length effect is about the same at all widths.

In considering the analysis of the estimated k_c and k_ϕ values it will be useful to recall equations (2) and (3) which show how the estimates are obtained. There are some obvious points to be noted from these equations, but we shall defer them until later. Each experimental unit or plot in the soil bin yields a p versus z relation as drawn by the x, y plotter while a single plate is sunk into the soil. Then results from two different plate widths have to be combined to obtain a single estimate of k_c or k_ϕ . We may combine 1" and 2" widths or widths of 2" and 3" or 1" and 3". As shown above, the latter is the best choice, but we see that a single replicate or "set" will only give us four such independent estimates.

By keeping the width difference constant and varying the length we can pair within one set these four pairs of plates:

$$\begin{array}{llll} 1 \times 4 & \text{and} & 3 \times 4 & 1 \times 8 \text{ and } 3 \times 8 \\ 1 \times 6 & \text{and} & 3 \times 6 & 1 \times 10 \text{ and } 3 \times 10 \end{array}$$

to give us four pairs of estimates for k_c and k_ϕ . Utilizing the six "sets" gave us 24 values for analysis.* Estimates obtained from these pairings may be analyzed for the length effect alone. But we would also like to study the width of plate effect on these two parameters and look for interaction, if any, as we did for the parameter n .

First, we consider the estimates of k_c . The mean values obtained for the pairs just listed were:

*A set consists of six replicates carried out at the same depth. Eighteen replicates were actually completed at each of two depths. Aggregation over six replicates forms a set. Thus, there are a total of six sets.

ESTIMATED n VALUES, BEKKER PRESSURE-SINKAGE EQUATION rectangular plate, dry sand

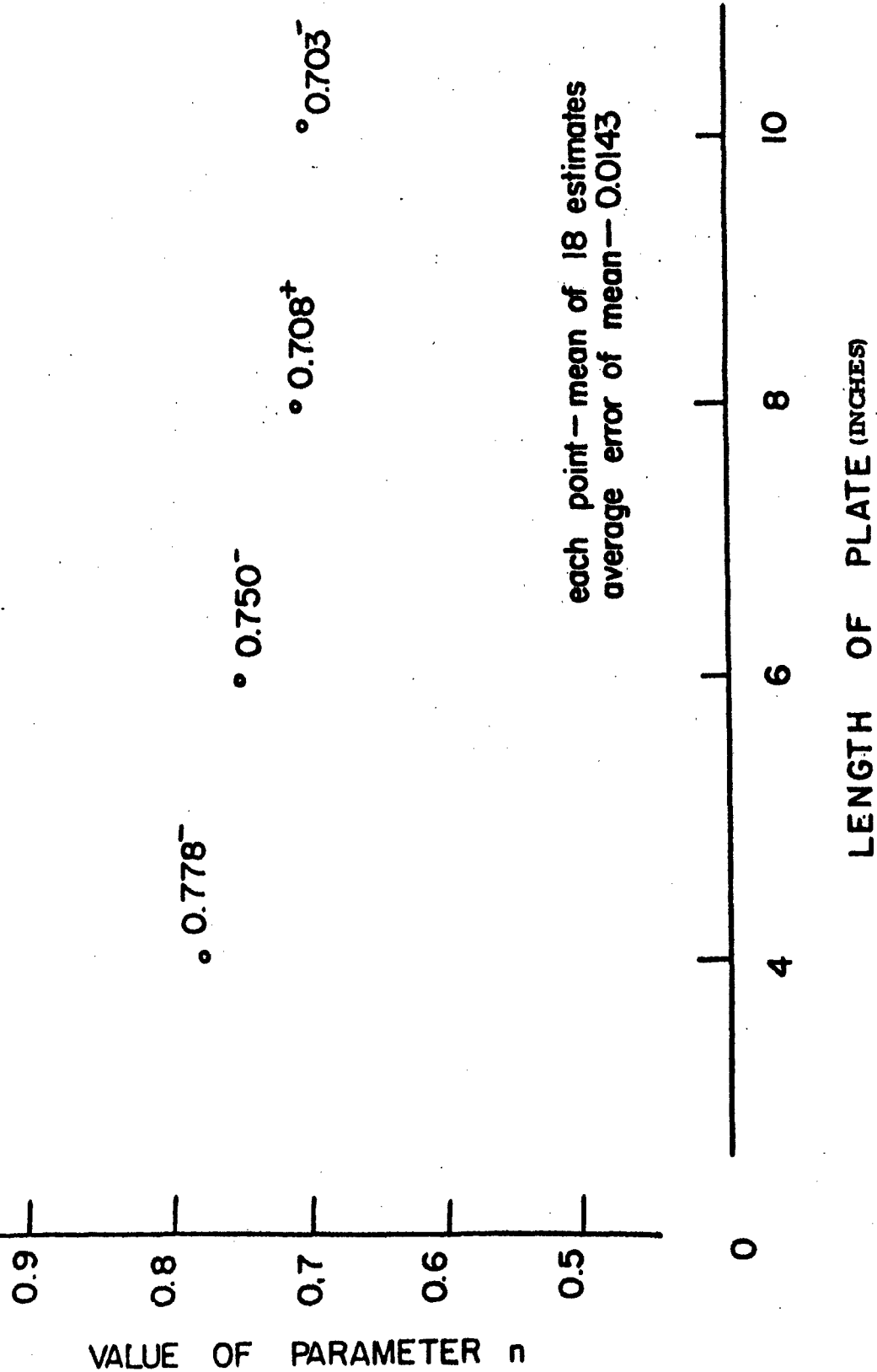


Figure 2

ESTIMATED n VALUES, BEKKER PRESSURE - SINKAGE EQUATION
rectangular plates, dry sand

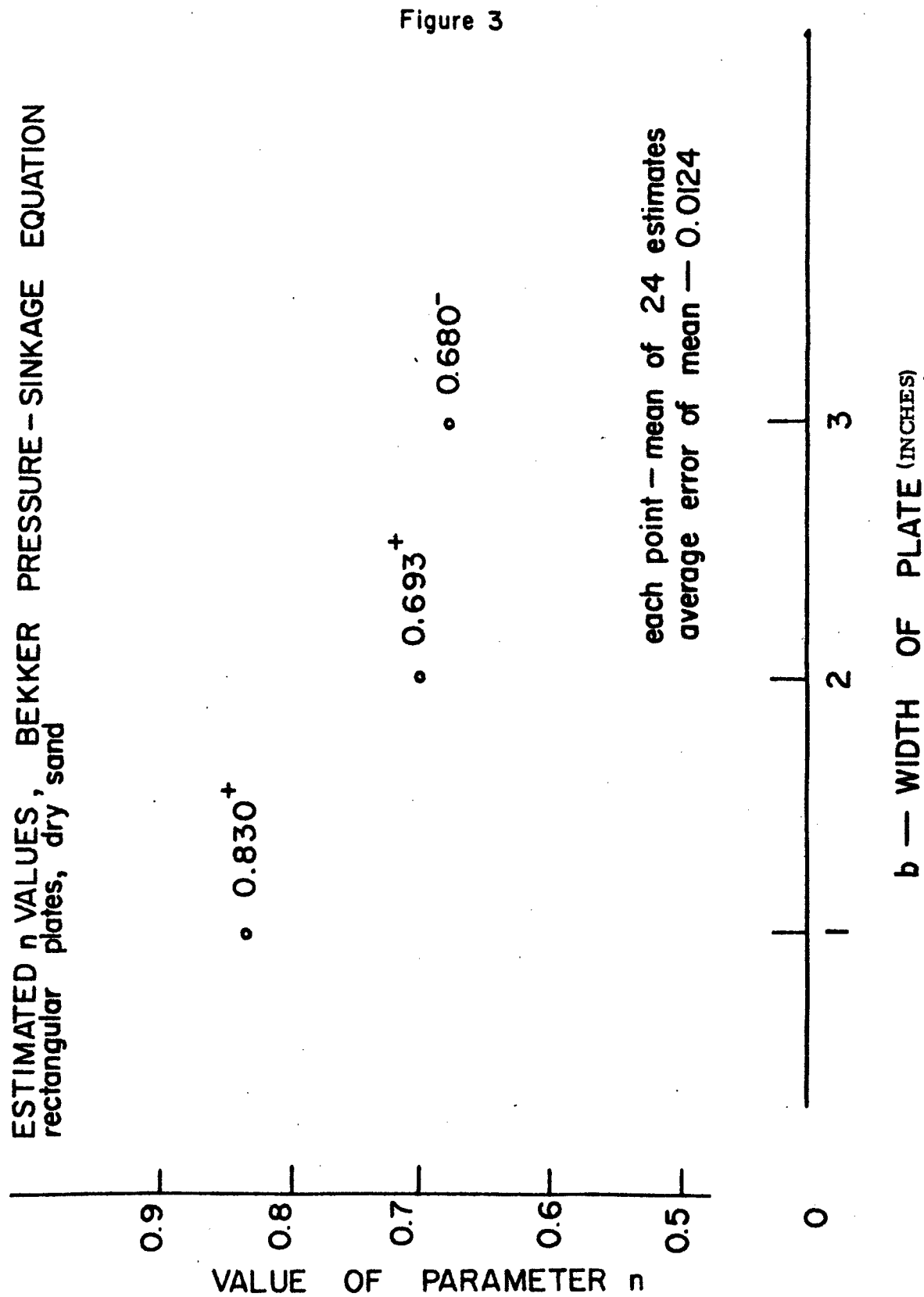
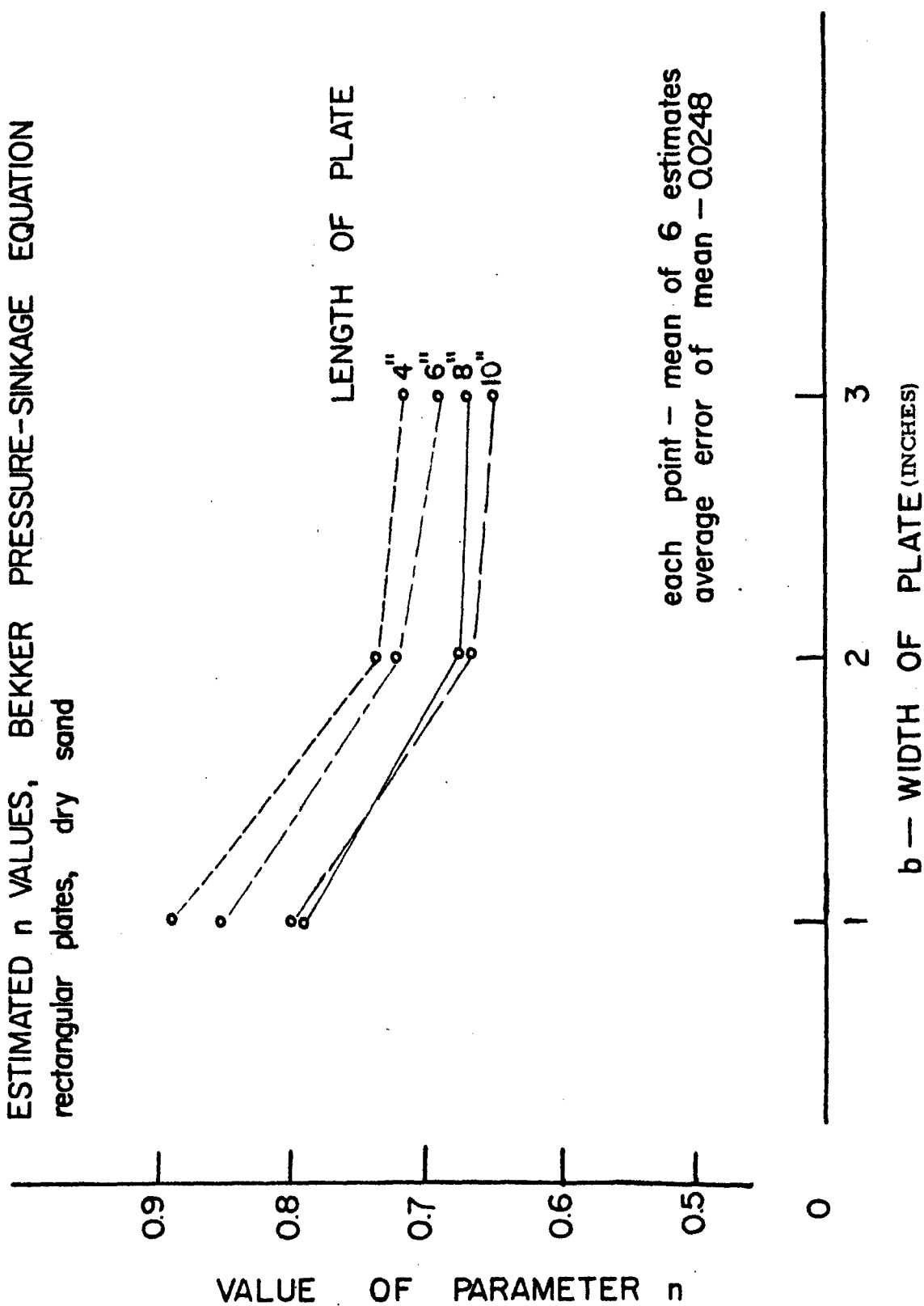


Figure 3

Figure 4

ESTIMATED n VALUES, BEKKER PRESSURE-SINKAGE EQUATION
rectangular plates, dry sand



Pairs:

1 x 4	1 x 6	1 x 8	1 x 10
and	and	and	and
3 x 4	3 x 6	3 x 8	3 x 10

Mean Value of k_c

-3.305	-4.030	-4.000	-4.166
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Relevant mean squares abstracted from the analysis of variance [8] for comparing these means are:

Source	d. f.	M. S.
Length of Plate	3	0.8977
Length of Plate by Depth	3	1.0526
Length of Plate by Sets within Depth	12	0.2262

Since the $P(F_{3,12} \geq 3.89) = 0.05$, we may conclude that there is a length effect on k_c and that the length effect is not uniform at the two depths studied (i. e., $F = 3.97 = 0.89770.2262$).

To extend our analyses other plate pairings were studied. Finally, it seemed in our judgment that the following pairs were most useful for securing information on the width effect plus a little more length information:

Pairs:

1 x 4	1 x 6	2 x 4	2 x 6
and	and	and	and
2 x 8	2 x 10	3 x 8	3 x 10

Mean Value of k_c

-2.463	-2.495	-5.169	-5.063
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Note that the pairs selected show a constant length difference of 4 inches. Furthermore, the length differences are balanced for comparing 1 and 2 inch widths with 2 and 3 inch widths for the study of the plate width effect in estimating k_c . It appears that the latter effect is large. Relevant mean squares from the analysis of variance are as follows:

Source	d. f.	M. S.
Width Pairs	1	41.717
Width Pairs by Depth	1	7.183

Length Differences within Width Pairs

Length in 1 and 2	1	0.003
Length in 2 and 3	1	0.034

Length Difference by Depth

in 1 and 2	1	0.006
in 2 and 3	1	5.914
Experimental Error	12	1.529

For comparing the width pairs we obtain an F ratio = $41.717/1.529 > 27$. We conclude that the parameter k_c depends on the choice of plate widths. In estimating k_c , pairing of 1 and 2 inch widths gives a larger value (algebraically) than pairing 2 and 3 inch widths. Looking at it another way the data show that changing the length/width ratio for the plates from

the range 4 to 6 to a range of 2 to 3 materially alters the k_c value obtained.

Similar analyses for the estimates of the parameter $k\phi$ were carried out as just described for k_c . We note that the $k\phi$ estimates were positive, mostly in the range +7 to +10. Our conclusions about the effect of plate width on the parameter $k\phi$ were the same as reached in regard to k_c , but no length effect was detected.

With these results available we may return to the further consideration of the method used to obtain the estimates. First, we observe that if there were no width of plate effect in the p versus z relation, then $k\phi$ would be equal to, say, p^* . In the notation used above, no width of plate effect would mean $P_{01} = P_{03}$, if one inch and three inch plate widths were used, and, hence, $p_1^* = p_3^* =$ a common value, p^* . Observed p_1^* and p_3^* values would differ, of course, due to experimental variation, but average values would be equal. Further, when $k\phi = p^*$, then the solution for k_c is zero. The parameter k_c was once regarded as a measure of cohesiveness of the soil and, hence, might be zero for sand [3]. The present experiments, in which mason's sand was used certainly do not agree with this point of view about the parameter k_c .

Recognizing then that the present model for the p versus z relation does admit a width of plate effect our analyses so far have indicated the magnitude of this effect. In addition, we have obtained some indication of a length effect on k_c although none appeared for $k\phi$.

Second, with respect to the method of obtaining the estimates of k_c and $k\phi$, we point out that the estimates obtained are correlated. This correlation cannot be avoided because the values are based on the solution of two simultaneous equations. In fact, the estimates also would have been correlated if we had been able to obtain them by a direct least squares procedure. In the least squares case, however, it is usually easy to write down the covariance of the estimates and, hence, to find the correlation if desired. For the two simultaneous equations in k_c and $k\phi$, matrix methods can be applied to find the covariance matrix for k_c and $k\phi$. The result obtained for the covariance of k_c and $k\phi$ is

$$(4) \quad \text{cov}(k_c, k_\phi) = (b_2^{-1} - b_1)^2 (b_1^2 b_2 V(p_1^*) + b_1 b_2^2 V(p_2^*))$$

where b_2 is the width of the larger plate and $V()$ denotes variance of the enclosed quantity.

It is interesting to examine the consequences for different choices of plate widths in estimating k_c and k_ϕ . As a first approximation it may be adequate to assume $V(p_1^*) \cong V(p_2^*)$. If this assumption is made, the following correlation values are obtained:

Plate widths paired: (widths in inches)		
1 and 3	1 and 2	2 and 3
Value of correlation of k_c and k_ϕ		
-0.8944	-0.9487	-0.9805

Observed values of the correlation between k_c and k_ϕ will, of course, differ from these theoretical values because of experimental variation and lack of equivalence of $V(p_1^*)$ and $V(p_2^*)$. It is clear, however, that the above results further support the use of maximum difference in plate widths for estimation of these parameters. Actual plots of the k_c and k_ϕ pairs support the high correlation of these estimates.

These considerations and the analyses for k_c and k_ϕ made it seem desirable to return to an analysis of the 84 p^* values which correspond to the 84 n values analyzed earlier. Such an analysis should give a clearer assessment of the length, width, and interaction effects than we have obtained from analysis of the k_c and k_ϕ values. Any effects found in this analysis must then be present in the k_c and k_ϕ values because of the method of derivation of these estimates from the p^* values.

The means for the p^* values are presented in Table 2. Table 3 following gives the analysis of variance for the p^* values. From the table of means it is seen that the width-of-plate effect is much larger than the length-of-plate effect. Both effects, however, may be judged significant from the analysis of variance results. It is interesting to note that the length by width interaction mean square is so small; no interaction is indicated. This result is in contrast to the analysis of the n values, Table 4, for which the interaction effect was judged significant.

CONCLUSIONS FOR PART 2. A large series of experiments have been conducted to study the pressure versus sinkage relation and to estimate the parameters in this relation. The experiments were carried out using a relatively homogeneous soil material, dry mason's sand. Bearing plates used comprised two circles of 2 and 4 inch diameters and 12 rectangular plates varying in size from 1 x 4 to 3 x 10 inches (refer Table 1 for list of plate sizes.) Thus, the length over width ratio of the plates ranged from a maximum of 10 to 1 down to 4 to 3, or from long narrow plates to almost square plates. This range of length over width ratios was selected to cover the range from tracked vehicles to wheeled vehicles with tires.

From the estimates of k_c , k_ϕ , and n plus the estimated pressures for one inch sinkage our analyses show that:

- (1) The parameter n varies with the width and length of plate. For one inch width of plate the n value was 0.83; at three inch width, the value was 0.68. Between 2" and 3" width there was little change. With length, n varied from 0.78 to 0.70 over lengths of 4" and 10". The decrease was nearly uniform over the 6" interval.
- (2) The estimated pressures for one inch of sinkage (the p^* values) show variation with both length and width of plate but no interaction of the factors is indicated. (Refer to Tables 2 and 3.)
- (3) The parameter k_c decreases algebraically with increase in plate length. The algebraic change, however, was much greater when estimates were compared from pairing of plates of 1" and 2" widths with estimates obtained by pairing 2" and 3" widths.
- (4) The parameter k_ϕ showed little or no response to length of plate but a large response to width of plate.

- (5) The estimates of k_c and k_ϕ are highly correlated. This correlation is negative so that when k_c increases in value k_ϕ decreases in value.

From these analyses it appears that the p versus z relation in the general form $p = (k_\phi + k_c/b)z^n$ is inadequate to predict the pressure-sinkage response. Although not pointed out previously in this paper, it should be mentioned that our analyses to date are based only on the experimental results for sinkage in the range 0.6" to 2". Perhaps it should be added that Dr. Bekker has not claimed that his equation would be adequate in the entire L/b region we have studied.

FUTURE WORK. While much has been learned, there is clearly need for the following:

- a. Similar laboratory experiments in other soil media.
- b. Experiments with greater depth differences in the soil bins to assess the depth effect, if any, on the parameters under homogeneous soil conditions.
- c. Revision of the model to take account of the dimensions of the bearing surface.
- d. Improvement of the model to cover a wider range in depth of sinkage, say, from at least 0.5 to 5.0 inches.

Table 2

TABLE OF MEANS FOR p^* VALUES
 (estimated pressure for one inch sinkage)
 summarized by plate dimensions (inches)

<u>SIZE OF PLATE</u>	<u>p^*</u>	<u>COMBINED MEANS</u>	<u>p^*</u>
Circles		all Circles	6.865
2" diameter	5.509		
4" "	8.222		
Rectangles			
1 x 4	5.188	Lengths (over all Widths)	
1 x 6	4.889	4	6.429
1 x 8	4.701	6	6.378
1 x 10	4.737	8	6.296
2 x 4	6.706	10	6.141
2 x 6	6.671		
2 x 8	6.420	Widths (over all Lengths)	
x 10	6.170	1	4.928
3 x 4	7.392	2	6.492
3 x 6	7.576	3	7.512
3 x 8	7.568		
3 x 10	7.515		

Table 3

ANALYSIS OF VARIANCE OF p^* VALUES
(estimated pressure for one inch sinkage)

<u>SOURCE OF VARIATION</u>	<u>DEGREES OF FREEDOM</u>	<u>MEAN SQUARE</u>
Plates	(13)	8.4927
Circles vs Rectangles	1	2.5279
Between Circles	1	22.0784
Among Rectangles	(11)	7.6181
Widths	2	40.9546
Lengths	3	0.4964
Length by Width	6	0.0668
Plates by Depth	(13)	0.2290
Experimental error	52	0.1793

Table 4

ANALYSIS OF VARIANCE OF n VALUES

(estimated from six replicates grouped together for fitting the pressure-sinkage equation in double-log form)

<u>SOURCE OF VARIATION</u>	<u>Degrees of FREEDOM</u>	<u>MEAN SQUARE</u>
Plates (13)		
Circles vs Rectangles	1	0.0112
Among Circles	1	0.1248
Among Rectangles (11)		
Widths	2	0.1113
Lengths	3	0.0228
Lengths by Widths	6	0.0193
Plates by Depth	13	0.0010
Experimental Error	52	0.0037

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