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TITLE: Resonant Spontaneous Bremsstrahlung of an Electron in the Field of the Nucleus and Two Light Waves

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TITLE: The Proceedings of the International Laser Physics Workshop [LPHYS'01] [10th] Held in Moscow, Russia on July 3-7, 2001 [Laser Physics. Volume 12, Number 2]

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Resonant Spontaneous Bremsstrahlung of an Electron in the Field of the Nucleus and Two Light Waves

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Abstract—The resonant spontaneous bremsstrahlung (when intermediate electron goes out on a mass surface) at scattering an electron on a nucleus in the field of two light waves propagating in one direction in interference region is theoretically considered. In the interference region the electron is correlated radiates and absorbs the same number of photons of the two waves. As a result of it, the electron interacts as though with one wave of the combination frequency $(\omega_1 \pm \omega_2)$. The differential cross section of the given process in requirements, when intensities of waves $\eta_{1,2} \ll 1$, is obtained. The resonance infinities were eliminated by Breit and Wigner procedure. It is shown that the resonant differential cross section of a spontaneous bremsstrahlung can on some orders of magnitude exceed the corresponding cross section without a laser field.

1. INTRODUCTION

Spontaneous bremsstrahlung by an electron scattered by a nucleus in the field of a plane electromagnetic wave has been investigated for a long time (see, e.g., [1–6]). Attention was focused primarily on the study of resonances associated with mass-shell effects in the Green's function of the intermediate electron in the plane-wave field. For the general relativistic case, the resonances were studied in [7] (see also the review in [8]).

Lately there has been an upsurge of interest in elementary quantum processes that occur in a superposition of several laser fields. Nonresonance spontaneous bremsstrahlung at scattering an electron on a nucleus in the field of two light waves is investigated recently [9, 10]. It was investigated in detail spontaneous bremsstrahlung in the noninterference and interference regions, and for the latter we predict the emergence of a spontaneous interference bremsstrahlung effect in two cases: for equal linear polarization's of the waves and elliptical polarization of the waves. In the latter case the effect amounts to strong correlation of the exit angles of the electron and spontaneous photon, to a dependence of the emission spectrum on the energy and polar incidence angle of the initial electron, and to stimulated, correlated emission and absorption (due to wave interference) of photons of both waves. We also show that the differential cross section of spontaneous bremsstrahlung in the interference region can be much greater than the corresponding cross section in any other geometry.

In the present paper, the resonant spontaneous bremsstrahlung (when intermediate electron goes out on a mass surface) at scattering an electron on a nucleus in the field of two light waves propagating in one direction is theoretically considered. It is shown that, in resonance conditions, the given process of the second order on a fine structure constant effectively breaks up to two processes of the first order: process of scattering of an electron on a nucleus in the field of two waves and process of spontaneous radiation by an electron of a quantum in the field of two waves. In the interference region the electron is correlated radiates and absorbs the same number of photons of the two waves. As a result of it, the electron interacts as though with one wave of the combination frequency $(\omega_1 \pm \omega_2)$. The differential cross section of the given process in requirements, when intensities of waves $\eta_{1,2} \ll 1$, is obtained. The resonance infinities were eliminated by Breit and Wigner procedure. It is shown that the resonant differential cross section of a spontaneous bremsstrahlung can on some orders of magnitude exceed the corresponding cross section without a laser field. Here, we use the system of units where $\hbar = c = 1$.

2. AMPLITUDE OF SPONTANEOUS BREMSSTRAHLUNG

We select the 4-vector potential of the external field in the form of the sum of two plane electromagnetic waves propagating parallel to the z axis:

$$A = A_1(\varphi_1) + A_2(\varphi_2), \tag{1}$$

where

$$A_j(\varphi_j) = \frac{F_j}{\omega_j}(e_{jx}\cos\varphi_j + \delta_j e_{jy}\sin\varphi_j). \tag{2}$$

Here is $\delta_1 = +1$, $\delta_2 = \pm 1$, $e_{jx} = (0, \mathbf{e}_{jx})$ and $e_{jy} = (0, \mathbf{e}_{jy})$ are the 4-vectors of wave polarization; F_j and ω_j are the

field strength and frequency of the first ($j = 1$) and second ($j = 2$) waves, and the argument φ_j has the form

$$\varphi_j = \omega_j(t - z), \quad j = 1, 2. \tag{3}$$

The amplitude of spontaneous bremsstrahlung by an electron scattered by a nucleus in the light wave (1) has the form (see Fig. 1)

$$S = \sum_{l_1 = -\infty}^{\infty} \sum_{l_2 = -\infty}^{\infty} S_{l_1 l_2}, \tag{4}$$

where the partial amplitude with emission ($l_1 > 0$, $l_2 > 0$) or absorption ($l_1 < 0$, $l_2 < 0$) $|l_1|$ of photons of the first wave and $|l_2|$ photons of the second is

$$S_{l_1 l_2} = -i \frac{8\pi^{5/2} Z e^3}{\sqrt{2\omega' \tilde{E}_i \tilde{E}_f}} \exp(i\phi_{if}) [\bar{u}_f H_{l_1 l_2} u_i] \frac{\delta(q_0)}{\mathbf{q}^2}, \tag{5}$$

$$H_{l_1 l_2}$$

$$= \sum_{s_1 = -\infty}^{\infty} \sum_{s_2 = -\infty}^{\infty} \left[M_{l_1-s_1, l_2-s_2}(\tilde{p}_f, \tilde{q}_i) \frac{\hat{\tilde{q}}_i + m_*}{\tilde{q}_i^2 - m_*^2} K_{s_1 s_2}(\tilde{q}_i, \tilde{p}_i) \right.$$

$$\left. + K_{s_1 s_2}(\tilde{p}_f, \tilde{q}_f) \frac{\hat{\tilde{q}}_f + m_*}{\tilde{q}_f^2 - m_*^2} M_{l_1-s_1, l_2-s_2}(\tilde{q}_f, \tilde{p}_i) \right]. \tag{6}$$

$$q = \tilde{p}_f - \tilde{p}_i + k' + l_1 k_1 + l_2 k_2, \tag{7}$$

$$\tilde{q}_i = \tilde{p}_i - k' - s_1 k_1 - s_2 k_2,$$
$$\tilde{q}_f = \tilde{p}_f + k' + s_1 k_1 + s_2 k_2. \tag{8}$$

In Eqs. (6)–(8), $k_1 = \omega_1 n = \omega_1(1, \mathbf{n})$ and $k_2 = \omega_2 n = \omega_2(1, \mathbf{n})$ are the 4-momenta of the photons of the first and second waves, $\tilde{p}_i$ and $\tilde{p}_f$ are the 4-quasimomenta of the electron before and after scattering, and m_* is the effective mass of the electron in the light wave (1):

$$\tilde{p}_{i,f} = p_{i,f} + (\eta_1^2 + \eta_2^2) \frac{m^2}{2(k_1 p_{i,f})} k_1,$$

$$m_* = m\sqrt{1 + \eta_1^2 + \eta_2^2}. \tag{9}$$

Here $p_{i,f} = (E_{i,f}, \mathbf{p}_{i,f})$ is the 4-momentum of the electron before and after scattering, and

$$\eta_{1,2} = \frac{eF_{1,2}}{m\omega_{1,2}} \tag{10}$$

is the classical Lorenz-invariant parameter characterizing the intensity of the first and second waves, respectively (see also [8, 9]).

$$M_{r_1 r_2}(\tilde{p}_2, \tilde{p}_1) \sim I_{r_1 r_2}, \quad K_{s_1 s_2}(\tilde{p}_2, \tilde{p}_1) \sim I_{s_1 s_2}. \tag{11}$$
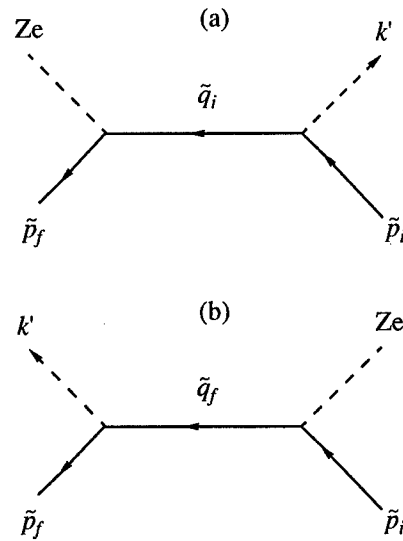

Fig. 1. Spontaneous bremsstrahlung by an electron scattered by a nucleus in the field of two light waves. The solid incoming and outgoing lines correspond to the wave functions of the initial and final electrons in the field of two waves (Volkov functions), the dashed lines correspond to the spontaneous photon k' and the "pseudophoton" of the nucleus, and the internal lines correspond to the Green's function of an electron in the field of two waves.

We note that the functions $I_{r_1 r_2}$ (11) can be expanded in Bessel functions J_r of integer order:

$$I_{r_1 r_2} \equiv I_{r_1 r_2}(\chi_1, \gamma_1; \chi_2, \gamma_2; \alpha_{12})$$

$$= \exp[-i(\chi_1 r_1 + \delta_2 \chi_2 r_2)]$$

$$\times \sum_{j = -\infty}^{\infty} \exp[i(\chi_1 - \delta_2 \chi_2)j] J_j(\alpha) J_{r_1-j}(\gamma_1) J_{r_2+\delta_2 j}(\gamma_2). \tag{12}$$

The arguments of the functions $I_{r_1 r_2}$ are

$$\gamma_j \equiv \gamma_j(\tilde{p}_2, \tilde{p}_1) = \eta_j m\sqrt{-g_j^2} = \eta_j m |\mathbf{g}_j''|,$$

$$g_j \equiv g_j(\tilde{p}_2, \tilde{p}_1) = \frac{\tilde{p}_2}{(k_j \tilde{p}_2)} - \frac{\tilde{p}_1}{(k_j \tilde{p}_1)}, \tag{13}$$

$$\alpha \equiv \alpha(\tilde{p}_2, \tilde{p}_1) = \eta_1 \eta_2 \frac{m^2}{\Omega} \left(\frac{1}{n\tilde{p}_1} - \frac{1}{n\tilde{p}_2} \right), \tag{14}$$

$$\Omega = \omega_1 - \delta_2 \omega_2, \quad \chi_j = \angle(\mathbf{e}_{jx}, \mathbf{g}_j'').$$

Note that the γ_j (13) are the well-known Bunkin–Fedorov multiphoton quantum parameters. The α (14) are quantum interference parameters, which determine the interference effects in the scattering of an electron by a nucleus and in the spontaneous emission of a photon by the electron in the field of two waves. Here we merely note that if $\alpha > 1$, correlated emission and absorption of photons of the two waves become impor-

tant. But if $\alpha \ll 1$, interference processes can be ignored $j = 0$ and the functions $I_{r_1 r_2}$ become the product of functions that determine the independent emission and absorption of photons of the first and second waves:

$$I_{r_1 r_2}(\chi_1, \gamma_1; \chi_2, \gamma_2; 0)$$
$$= \exp[-i(\chi_1 r_1 + \delta_2 \chi_2 r_2)] J_{r_1}(\gamma_1) J_{r_2}(\gamma_2). \quad (15)$$

3. AMPLITUDE OF SPONTANEOUS BREMSSTRAHLUNG IN THE INTERFERENCE REGION

In the interference region, the following relationships must hold in the general case of arbitrary wave intensities:

$$g_j^2(\tilde{p}_f, \tilde{q}_i) = g_j^2(\tilde{q}_i, \tilde{p}_i) = 0,$$
$$g_j^2(\tilde{p}_f, \tilde{q}_f) = g_j^2(\tilde{q}_f, \tilde{p}_i) = 0. \quad (16)$$

Thus, in the interference region, the parameters γ_j vanish and the functions $I_{r_1 r_2}$ in (12), which determine the amplitude of spontaneous bremsstrahlung, become the Bessel functions J_r of integer order:

$$I_{r_1 r_2}(0, 0; 0, 0; \alpha) = J_r(\alpha), \quad r = \frac{1}{2}(r_1 - \delta_2 r_2). \quad (17)$$

$$\begin{cases} q \longrightarrow q_{(0)} = \tilde{p}_f - \tilde{p}_i + k' + lK \\ \tilde{q}_i \longrightarrow \tilde{q}_{i(0)} = \tilde{p}_i - k' - sK, \quad K = (\Omega, \mathbf{K}) = k_1 - \delta_2 k_2 \\ \tilde{q}_f \longrightarrow \tilde{q}_{f(0)} = \tilde{p}_f + k' + sK. \end{cases} \quad (18)$$

$$\begin{cases} q \longrightarrow q_{(1)} = \tilde{p}_f - \tilde{p}_i + k' + lK \\ \qquad -\frac{1}{2}(1 + \delta_2)k_1 - \frac{1}{2}(1 - \delta_2)k_2, \\ \tilde{q}_i \longrightarrow \tilde{q}_{i(1)} = \tilde{p}_i - k' - sK \\ \qquad +\frac{1}{2}(1 + \delta_2)k_1 + \frac{1}{2}(1 - \delta_2)k_2, \\ \tilde{q}_f \longrightarrow \tilde{q}_{f(1)} = \tilde{p}_f + k' + sK \\ \qquad -\frac{1}{2}(1 + \delta_2)k_1 - \frac{1}{2}(1 - \delta_2)k_2. \end{cases} \quad (19)$$

Equations (18) and (19) suggest that in the interference region the electron, during electron deceleration at the nucleus and spontaneous emission of a photon, emits (absorbs), in a correlated manner, equal numbers of photons of the first and second waves ($l_2 = -\delta_2 l_1$, $s_2 = -\delta_2 s_1$) or numbers that differ by 1 ($l_2 = -\delta_2 l_1 - 1$, $s_2 = -\delta_2 s_1 - 1$); processes in which the numbers of emitted

(absorbed) photons of both differ by more than one are suppressed.

The final formula for the amplitude of spontaneous bremsstrahlung by an electron scattered by a nucleus in the interference region is (see also [9])

$$S = \sum_{l=-\infty}^{\infty} [S_l^{(0)} + S_l^{(1)}], \quad (20)$$

where $S_l^{(0)}$ and $S_l^{(1)}$ are the partial amplitudes with correlated emission (absorption) of equal numbers of photons of both waves and of numbers of such photons that differ by 1.

4. RESONANT SPONTANEOUS BREMSSTRAHLUNG IN THE INTERFERENCE REGION

Let us consider resonant SB in conditions, when

$$\eta_1 \eta_2 \ll 1, \quad \eta_{1,2}^2 \ll 1. \quad (21)$$

We find that the kinematics of electron scattering and spontaneous-photon ejection for the amplitudes of a and b is the same. Here the scattering of the electron and the emission of the spontaneous photon occur in a single plane formed by the initial electron momentum and the direction of propagation of both waves. The corresponding azimuth angles are equal, while the polar angles are given by

$$a_f = a_i, \quad a_{i,f} = \frac{|\mathbf{p}_{i,f}|}{\kappa_{i,f}} \sin\theta_{i,f}, \quad (22)$$

for electron scattering, and

$$\cot\frac{\theta'}{2} = a_i, \quad (23)$$

for the exit angle of the spontaneous photon.

In (22) and (23) it is denoted:

$$\kappa_{i,f} = E_{i,f} - |\mathbf{p}_{i,f}| \cos\theta_{i,f},$$
$$\theta_{i,f} = \angle(\mathbf{n}, \mathbf{p}_{i,f}), \quad \theta' = \angle(\mathbf{n}, \mathbf{k}'). \quad (24)$$

Analyzing the kinematics conditions (22), we see that

$$\tan\frac{\theta_f}{2}$$
$$= \frac{1}{a_i(v_f^{-1} + 1)}\left[1 \pm \sqrt{\frac{(\omega'_{max} - \omega')(2E_i - \omega'_{max} - \omega')}{(E_i - \omega')^2 - m^2}}\right], \quad (25)$$

where

$$\omega'_{max} = E_i - m\sqrt{1 + a_i^2} \quad (26)$$

is the maximum frequency of the spontaneous photon. The solution (25) shows that the radiative spectrum in

the interference region is bounded from above by ω'_{max}, in contrast to the noninterference region, where the maximum frequency of the spontaneous photon is $(E_i - m)$. Note that the upper limit (124) of the frequency of the spontaneous photon depends heavily on the energy and incidence angles of the initial electron. If we consider it a function of the polar angle of the initial electron, it has minimum value 0 at the polar angle

$$\theta_{i*} = \arccos v_i \qquad (27)$$

(θ_{i*} is the critical angle, at which the electron ceases to emit photons), and a maximum value $(E_i - m)$ at $\theta_i = 0$, π. Hence, as $\theta_i \longrightarrow \theta_{i*}$ (from both the right and left), emission rapidly decreases $\omega'_{max} \longrightarrow 0$, and in a narrow range of angles near the critical angle $(\theta_i - \theta_{i*})^2 \ll 1$ it essentially vanishes ($\omega'_{max}/m \ll 1$). Thus, the electron emits photons if its polar incidence angle lies in the range

$$0 \leq \theta_i < \theta_{i*} \quad \text{and} \quad \theta_{i*} < \theta_i \leq \pi. \qquad (28)$$

Note that the critical angle θ_{i*} increases with decreasing initial electron velocity, taking values from $\theta_{i*} \approx \sqrt{2(1 - v_i)} \ll 1$ (for ultrarelativistic electron energies) to $\theta_{i*} \approx \pi/2$ (for nonrelativistic electron energies).

We now ascertain the angle at which the spontaneous photon emerges. From (23) we see that the exit angle of the spontaneous photon depends heavily on the energy and incidence angles of the initial electron. If we consider the exit angle of the spontaneous photon as a function of the polar angle of the initial electron, at the critical angle $\theta_i = \theta_{i*}$ it has a minimum value (at which there is no emission) equal to

$$\theta'_{min} = 2\arcsin(m/E_i) = 2\theta_{i*}. \qquad (29)$$

Therefore at the given energy initial electron and various incidence angles of the initial electron, the exit angle of the spontaneous photon is made in an interval

$$\theta'_{min} < \theta' \leq \pi. \qquad (30)$$

In conditions of a resonance, intermediate electron becomes real, i.e., accordingly for the diagram (a) and (b) (see Fig. 1) are satisfied:

$$\tilde{q}^2_{i(0)} = m^2_*, \quad \tilde{q}^2_{f(0)} = m^2_* . \qquad (31)$$

Let us consider a resonance for the diagram (a) (see Fig. 2). In this case we shall receive:

$$\tilde{p}_i + |s|K = \tilde{q}_{i(0)} + k',$$
$$q_{(0)} = \tilde{p}_f - \tilde{q}_{i(0)} + (l + |s|)K. \qquad (32)$$

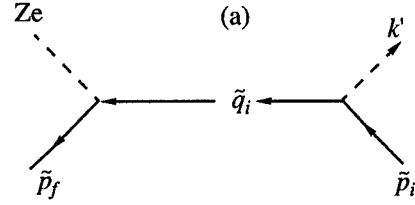From Eqs. (31), (32), (23) we shall receive frequency of a spontaneous photon and also exit angles of

Fig. 2. Resonant spontaneous bremsstrahlung by an electron scattered by a nucleus in the field of two light waves.
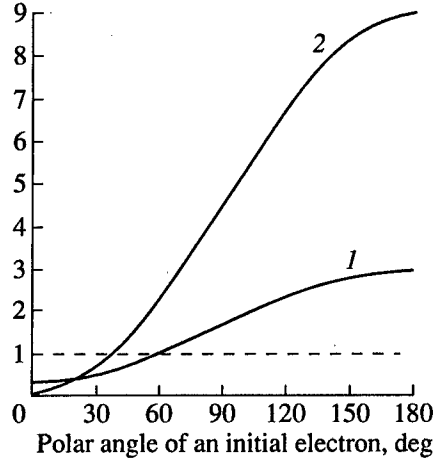

Fig. 3. Dependence of the frequency of a spontaneous photon on a polar incidence angle of the initial relativistic electron: curve 1 corresponds to the energy initial electron $E_i = 0.59$ MeV ($v_i = 0.5$); curve 2, $E_i = 0.85$ MeV ($v_i = 0.8$); crossing of the dashed line with curves 1 and 2 gives a critical angles near to which radiation is suppressed.

a final electron in conditions of a resonance:

$$\omega'_{res} = |s|(\omega_1 - \delta_2\omega_2)\left(\frac{2E_i\kappa_i}{m^2} - 1\right),$$
$$\tan\frac{\theta_f}{2} = \frac{(1 - v_i)}{(1 + v_i)}\cot\frac{\theta_i}{2} \qquad (33)$$

for the relativistic energy initial electron, and

$$\omega'_{res} = |s|(\omega_1 - \delta_2\omega_2)\left(\frac{2E_i}{m}\sin\frac{\theta_i}{2}\right)^2, \qquad (34)$$

$$\theta_f = \begin{cases} \theta_i - \dfrac{m^2}{2E_i^2}\cot(\theta_i/2)\cos^2(\theta_i/2) \approx \theta_i \\[4mm] \dfrac{m^2}{2E_i^2}\cot(\theta_i/2) \ll 1, \end{cases} \qquad (35)$$

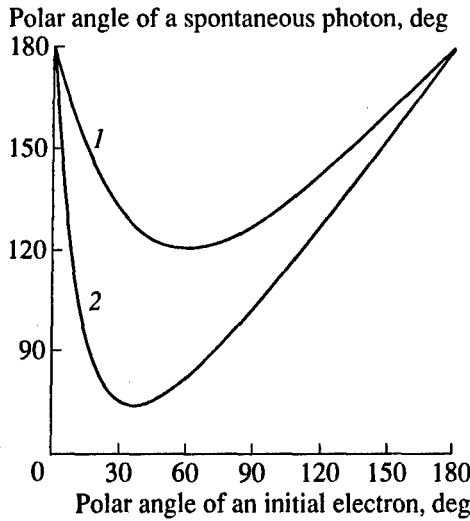$$\theta' = \theta_i + \frac{m^2}{2E_i^2}\cot(\theta_i/2)\cos\theta_i \approx \theta_i \qquad (36)$$

Polar angle of a spontaneous photon, deg

Fig. 4. Dependence of the exit angle of the spontaneous photon on a polar incidence angle of the initial relativistic electron: curve *1* corresponds to the energy initial electron E_i = 0.59 MeV (v_i = 0.5); curve 2, E_i = 0.85 MeV (v_i = 0.8).
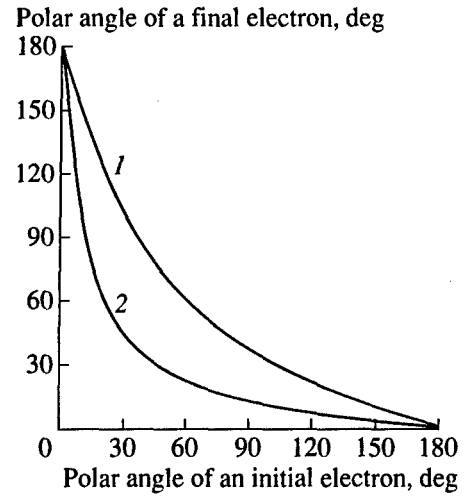
Polar angle of a final electron, deg

Fig. 5. Dependence of the exit angle of the final electron on a polar incidence angle of the initial relativistic electron: curve *1* corresponds to the energy initial electron E_i = 0.59 MeV (v_i = 0.5); curve 2, E_i = 0.85 MeV (v_i = 0.8).

for the ultrarelativistic energy initial electron.

Let us emphasize, that the ultrarelativistic energy initial electron are limited to size

$$\frac{E_i}{m} \le \frac{m}{4|s|\Omega \sin^2(\theta_i/2)} \sim \frac{m}{\Omega} \sim (10^5 - 10^6). \qquad (37)$$

In Fig. 3 we show the frequency dependence of a spontaneous photon on a polar incidence angle of the initial relativistic electron. In Fig. 4 we show the dependence of the exit angle of the spontaneous photon on a polar incidence angle of the initial relativistic electron. In Fig. 5 we show the dependence of the exit angle of the final electron on a polar incidence angle of the initial relativistic electron. Finally, in Fig. 6 we show the dependence of the frequency of a spontaneous photon on a polar incidence angle of the initial ultrarelativistic electron.

The resonance infinities were eliminated by Breit and Wigner procedure:

$$m \longrightarrow \mu_i = m - i\Gamma_i, \qquad (38)$$

where

$$\Gamma_i = \frac{q_{i(0)}^0}{2m} W_\gamma \sim \frac{e^2}{\hbar c}(\eta_1^2 + \eta_2^2)\Omega\frac{E_i}{m}. \qquad (39)$$

The resonant differential cross section of a spontaneous bremsstrahlung is given by

$$d\sigma_{res} \sim \frac{1}{\Gamma_i} dW^{(1)} d\sigma_s(q_{i(0)}), \qquad (40)$$

where $dW^{(1)}$ is the probability of spontaneous radiation by an electron of a quantum in the field of two waves

and $d\sigma_s$—the differential cross section of scattering of an electron on a nucleus in the field of two waves.

May be shown that the resonant differential cross section of a spontaneous bremsstrahlung can on some orders of magnitude exceed the corresponding cross section without a laser field ($d\sigma_*$),

$$R = \frac{d\sigma_{res}}{d\sigma_*} \sim \frac{\hbar c}{e^2}\left(\frac{E_i}{m}\right)^2 < \frac{\hbar c}{e^2}\left(\frac{m}{\Omega}\right)^2 < 10^7 - 10^8. \qquad (41)$$
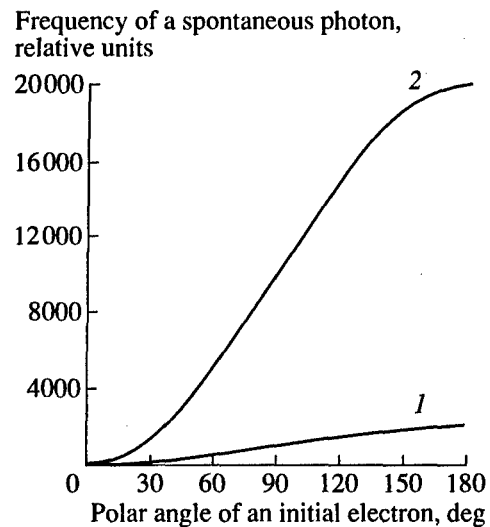
Frequency of a spontaneous photon, relative units

Fig. 6. Dependence of the frequency of a spontaneous photon on a polar incidence angle of the initial ultrarelativistic electron: curve *1* corresponds to the energy initial electron E_i = 11.43 MeV; curve 2, E_i = 36.12 MeV.

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